Recent Developments in the Dispersive Evaluation of Hadronic Vacuum Polarisation Contributions to Muon $g - 2$

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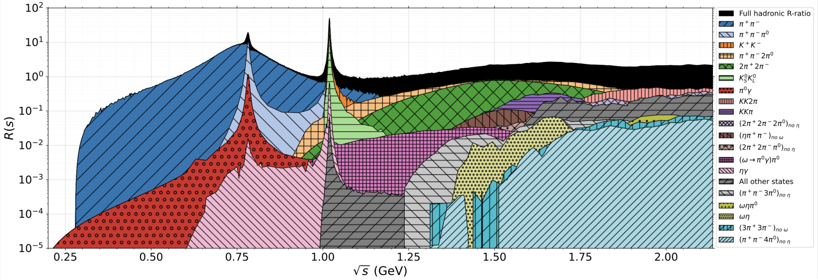
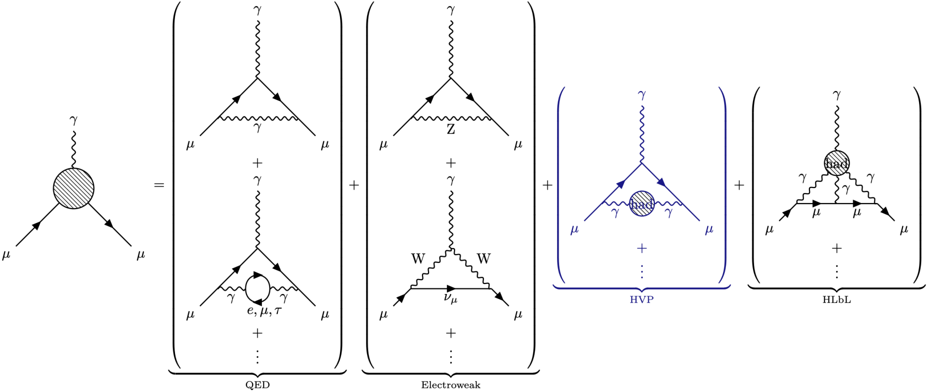


The Anomalous Magnetic Moment of the Muon

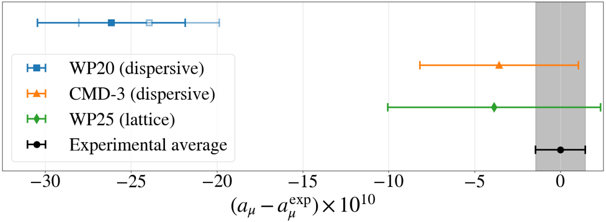


- Dirac equation $\implies$ muon gyromagnetic ratio is double the classical – i.e. $g = 2$.
- QFT $\implies$ this corresponds to the tree level diagram and higher order corrections exist.
- Experimental measurement of the muon anomaly at 124ppb!
- These can be perturbative (QED, EW) or **non-perturbative**...

Hadronic Vacuum Polarisation

Problem: QCD is non-perturbative at low $\sqrt{s}$.
Implication: HVP of photon cannot be calculated in loop integrals etc.

- Solution: discretise spacetime and perform lattice QCD calculation.
- Solution: Directly measure t-channel $e - \mu$ scattering.
  - → Upcoming MUonE experiment.
- Solution: dispersion integral over the $e^+e^- \to$ *hadrons* cross section.
  - → Dispersive/data-driven methods...
  - → The main focus of this presentation – including KNT**W** data compilation.



Dispersive Approach for a_μ^{HVP}

Problem: QCD is non-perturbative at low $\sqrt{s}$.
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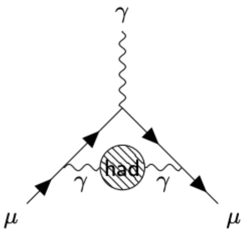
Solution: dispersion integral over the $e^+e^- \to$ *hadrons* cross section.

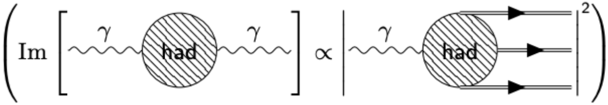


$$\propto \Pi^{had.}_{\mu\nu}(q^2)$$ *by definition*

$$\propto \Pi^{had.}(q^2)$$ *due to gauge invariance*

$$\propto \int \frac{\mathrm{Im}\left\{\Pi^{had.}(s)\right\}}{s\,(s - q^2 - i\varepsilon)} ds$$ *by analyticity and Cauchy's theorem*

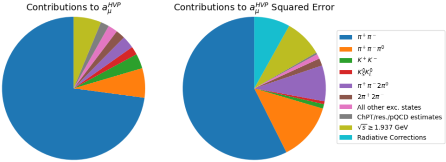
$$\propto \int \frac{\sigma^0_{had.}(s)}{s - q^2 - i\varepsilon} ds$$ *by unitarity $\Longrightarrow$ the Optical Theorem*

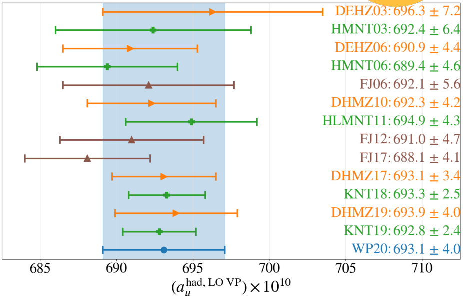


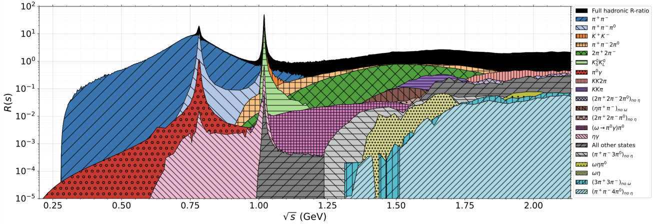
- For $>$ 50 years, low energy $e^+e^- \to$ *hadrons* data have been collected...

Hadronic Data and Dispersion Integral

- $\sim 11\,300$ data points from ~ 250 datasets in > 50 hadronic channels.
- Dominated ($\sim 73\%$) by $e^+e^- \to \pi^+\pi^-$.

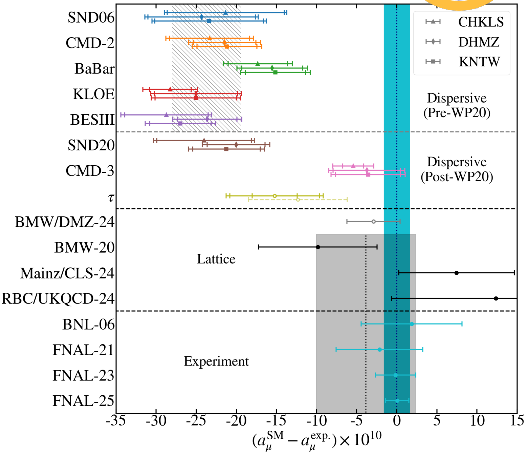


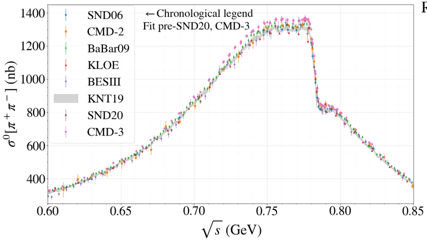




Present Status

- Severe tensions in $\pi^+\pi^-$ stymie dispersive a_μ^{HVP}.
- Historically handled $\sim 2.5\sigma$ tension between KLOE, BaBar[a].
- Recent CMD-3 $\pi^+\pi^-$ spectrum $\sim$ **5σ** higher than older spectra.
- No evidence of faulty data, new preliminaries corroborate tension.
- Cannot presently combine data...





- WP25 quotes only lattice – story **not** over!
- Handle tensions in new combination to fully understand $g-2$ experiment implications...

[a] See penultimate slide...

Blinding and the New KNTW Analysis

- KNTW philosophy is to be data-driven $\implies$ minimal modelling assumptions.
- Combination procedure:
  - All non-defective data used.
  - Radiative corrections from robust routines.
  - Clustering – dynamic data-driven combination.
  - Fitting – incorporate full correlation information whilst avoiding bias.
  - Integrate, e.g.

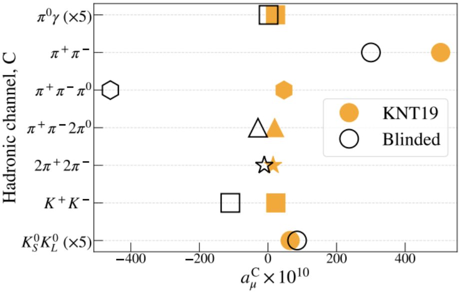
$$a_\mu^{\text{HVP,LO}} = \frac{1}{4\pi^3}\int_{s_{th}}^{\infty} ds \left\{ \sigma^0(s) K_\mu(s) \right\}$$

- Analysis choices:
  - Re-binning procedure;
  - Fitting procedure - correlations;
  - Use of additional constraints;
  - Error inflation;
  - Interpolation/integration...

$\implies$ changes in central value.

- Need to make sure we make optimal choices $\implies$ exhaustive reanalysis.
- Avoid bias $\implies$ blinded analysis.
- Channel dependent blinding kernel B_i:

$$a_\mu^{\text{blind}}[i] = \frac{1}{4\pi^3}\int_{s_{th}}^{\infty} ds \left\{ \sigma_i^0(s) K_\mu(s) B_i(s) \right\}$$

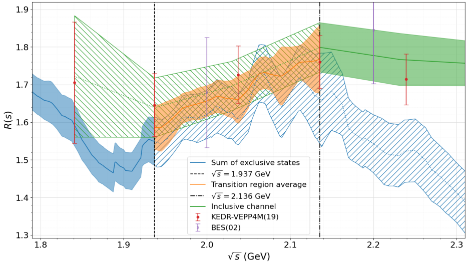


KNTW- Phys. Rev. D 111 L011901 (2025)

"Re-Baselining"

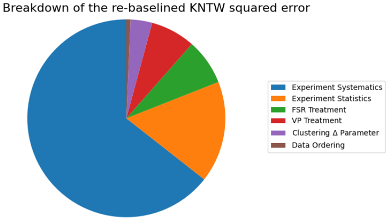
Correction/Completion

- Full code rewrite FORTRAN→Python.
- (*Minor*) Corrections of KNT19 analysis:
  - Checks of database against literature.
  - More detailed systematic covariance matrix construction.
- Completions of KNT19 analysis features:
  - Lagrange polynomial interpolation of all resonances.
  - Exclusive/inclusive transition region.



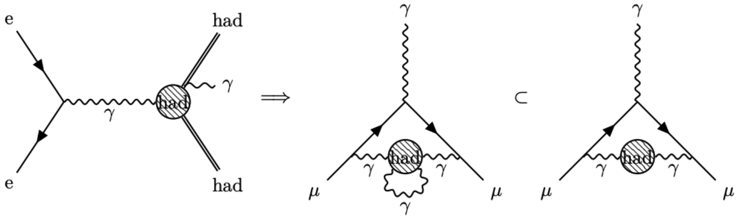
Method Systematics

- Quantify quality of current method.
- KNTW clustering procedure (simplified)
  - Optimised cluster with Δ for each channel.
  - Work through ordered points – may join clusters if with Δ, else form new.
  - Continues until fully rebinned.
- Two 'unfixed' aspects: Δ and ordering.
- Straightforward to estimate systematics.
- $<5\%$ of KNT19 squared error.



FSR Corrections

Must work with "**bare**" cross sections.
Requires **final state radiation** corrections.



- Emission of photons by final-state hadrons corresponds to photons in hadronic insertion.
- Technically one order in α higher but mathematically indistinguishable.
- No procedure to handle separately $\Longrightarrow$ use 'FSR-inclusive' cross sections.

- Experimentally measured cross sections are not always FSR inclusive.
- Can 'correct' cross sections to be FSR inclusive – use scalar QED (sQED) for dominant pions.
- Technically require finite 'hard' corrections for photons with $E_\gamma \gtrsim \Lambda$.

FSR Studies – Inclusive Channel

- Inclusive channel can be ∼described by pQCD (away from c, b resonances).
- Correction for sum of $q\bar{q}$ states:

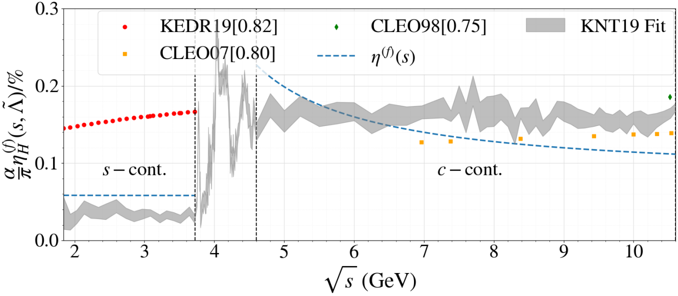
$$\frac{R^{(\gamma)}_{q\bar{q}}(s)}{R_{q\bar{q}}(s)} = 1 + \frac{\alpha}{\pi} Q_q^2 \eta^{(f)}(s, m_q^2)$$

- Sufficient (w. conservative uncertainty).
- Alone yields significant uncertainty reduction.
- **However** 16/19 datasets are already expected to be FSR inclusive.
- Remainder require *hard* FSR correction.

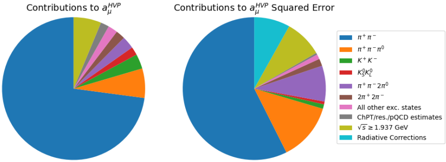
$$\eta_H^{(f)}(s, m_q^2, \Lambda) \sim \int_{4m_q^2}^{s(1-\frac{2\Lambda}{\sqrt{s}})} \rho(s, s')ds'$$

for known Λs, integrand function.

- $\sim 33\%$ drop in $\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$ uncertainty!

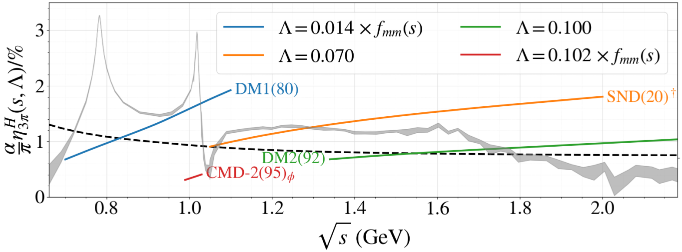


FSR Studies – Three Pion Channel



- Adding FSR in $\pi^+\pi^-\pi^0$ & inclusive $\implies$ FSR in $>$ 90% of contributions.

- Martin Hoferichter et al: derived FSR correction for 3π.
- Can similarly derive hard correction.
- c.f. Inclusive channel - most datasets already FSR inclusive.
- FSR excluding datasets are largely irrelevant to combination.
- Implication: radiative uncertainties reduced $\times$**3**.



Impact of Correlations – Introduction

- Historic difference between KNTW/DHMZ $a_\mu[\pi^+\pi^-]$ at $\sim 1\sigma$.
- Understanding: KNTW full systematic correlations vs DHMZ local correlations only.
- Test this assumption and generate an uncertainty with **decorrelation procedures**:

Global

$$\tilde{C}_{ij} = \alpha C_{ij} + (1-\alpha)\mathrm{diag}\left[C_{ij}\right]$$

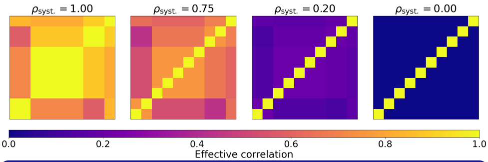
$$0 \leq \alpha \leq 1$$

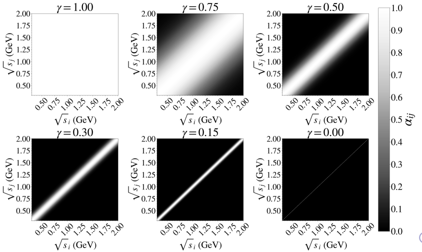
(Motivated by systematics as normalisations.)

Local

$$\tilde{C}_{ij} = \alpha_{ij}(\gamma) C_{ij}$$

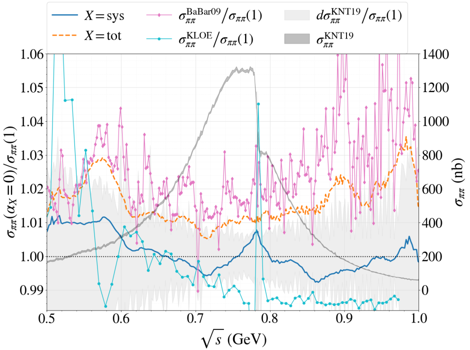
$$\alpha_{ij} = \exp\left[-\left(\frac{1-\gamma}{\gamma} \times \frac{\sqrt{s_i}-\sqrt{s_j}}{250\ \mathrm{MeV}}\right)^2\right]$$





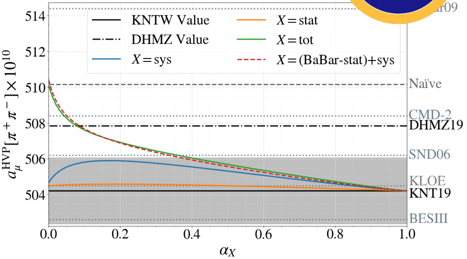
Study conclusion: global decorrelation produces larger spectral deviations!!

Impact of Correlations – Results ($a_\mu[\pi^+\pi^-]$)

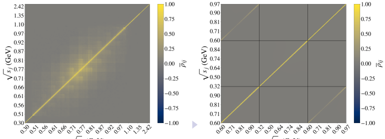


- Effect of syst. corr.s is **small!!**
- Replace old proxy 'KLOE-BaBar' systematic with 5× smaller uncertainty.
- All channels: $\pm 1.08\times10^{-10}$: increases error by 16.6% (vs KB 57.2%).

More details: see KNTW paper
arXiv2601.XXXXX



- Stays 'KLOE' dominated. Syst. variation does not replicate DHMZ or naïve value.
- Replicated varying *statistical* corr.s.
- Explanation: BaBar stat. corr.s are considerable and (accurately) including them allows a lower lying high quality fit.



Conclusions

- The muon anomaly provides a means to test the SM at high precision.
- Dispersive methods (using experimental data) provide non-perturbative terms.
- The present SM status of a_μ is unclear, as is the new physics case.
- Thorough analyses are ongoing to understand the dispersive tensions.

The muon anomaly puzzle is not solved yet!