Supersymmetric AdS Solitons, Coulomb Branch Flows, Twisted Compactifications and their Marginal Deformations

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December 2025

Based on work with Dimitrios Chatzis, Georgios Itsios, Carlos Nunez and Dimitrios Zoakos
arXiv: 2506.10062, 2511.18128 + to appear

Motivation

• Since gauge/ gravity duality, want to study strongly interacting QFTs using holography.

• Confinement, symmetry breaking and a mass gap are accompanied by a singularity in the gravity dual $\Rightarrow$ supergravity breaks down at the singularity.

• In some cases the singularities are resolved, but the UV may have issues $\Rightarrow$ we want to cure these problems

[Maldacena '97]

$\mathcal{N}=4$ SYM

$\mathcal{O}_{\Delta<4}$

RG flow

Confinement, etc.

$AdS_5 \times S^5$

F_5 ,

F_3, H_3, F_1, Φ

Singularities

- Generalise / extend idea of gauge/gravity duality via:
  - Wrapped branes on internal cycles
  - D-branes at tip of deformed conifolds.

In the IR these have no singularity, but are not well-defined in the UV.

To cure these we construct

Smooth, asymptotically AdS deformed backgrounds dual to SCFT_4 (UV) which flow to gapped & confining SQFT_3 (IR) via a **twisted compactification** using the soliton of Anabalon-Nastase-Oyarzo ’24.

Plan of the talk

- Construct smooth asymptotically AdS backgrounds dual to $4\mathrm{d}$ SCFT flowing to a gapped and confining $3\mathrm{d}$ QFT.

- Lots of observables can be calculated and for the different CFTs, some observables show a universal behaviour.

- TsT procedure used to construct marginal deformations.

Based on the works

- 2506.10062, 2511.18128 [Chatzis, Hammond, Itsios, Nunez, Zoakos]
- to appear, [Hammond + Itsios]

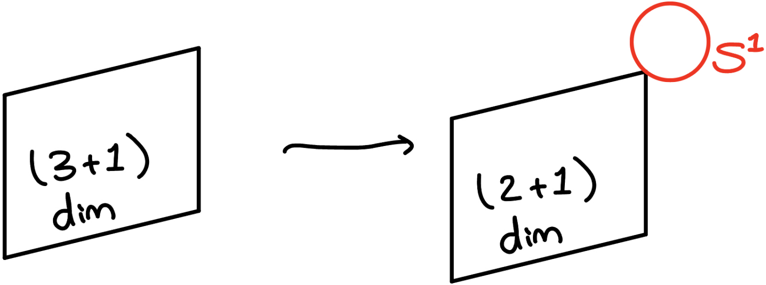
Anabalón-Ross 5d Soliton (2021)

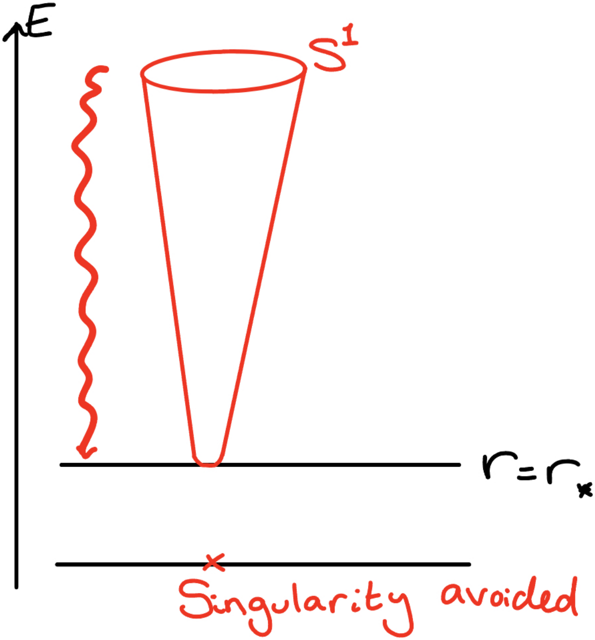
- Solution to $5d$ minimally gauged supergravity.

$$\mathrm{d}s^2 = \frac{r^2}{l^2}\left(-\mathrm{d}t^2 + \mathrm{d}w^2 + \mathrm{d}z^2\right) + \frac{\mathrm{d}r^2}{f(r)} + \underbrace{f(r)\,\mathrm{d}\phi^2}_{S^1}$$

$$f(r) = \frac{r^2}{l^2} - \frac{\mu}{r^2} - \frac{q^2}{r^4}, \qquad A = q\left(\frac{1}{r^2} - \frac{1}{r_*^2}\right)\mathrm{d}\phi$$

$$f(r_\star) = 0 \Rightarrow \text{ space ends } \textbf{smoothly} \text{ at } r = r_\star$$




E
S^1
$r = r_*$
Singularity avoided

5d SUSY soliton: A solution in 5d gauged SUGRA

Solution contains: A metric, 2 scalar fields and 3 $\mathrm{U}(1)$ gauge fields, [ANO '24]

$$\mathrm{d}s_5^2 = \frac{r^2\lambda^2(r)}{L^2}\left(-\mathrm{d}t^2 + \mathrm{d}z^2 + \mathrm{d}w^2 + L^2 F(r)\overbrace{\mathrm{d}\phi^2}^{S^1}\right) + \frac{\mathrm{d}r^2}{r^2\lambda^4(r)F(r)},$$

$$\Phi_1 = \sqrt{\frac{2}{3}}\ln\lambda^{-6}(r), \quad \Phi_2 = 0, \qquad \zeta(r,\theta) = \sqrt{1 + \frac{\varepsilon\ell^2}{r^2}\cos^2\theta}$$

$$A^1 = A^2 = Q\left[\lambda^6(r) - \lambda^6(r_\star)\right] L\,\mathrm{d}\phi, \quad A^3 = Q\left[\frac{1}{\lambda^6(r)} - \frac{1}{\lambda^6(r_\star)}\right] L\,\mathrm{d}\phi,$$

$$F(r) = \frac{1}{L^2} - \frac{\varepsilon Q^2\ell^2 L^2}{r^4}\left[1 - \lambda^{-6}(r)\right], \quad \lambda^6(r) = \frac{r^2 + \varepsilon\ell^2}{r^2}, \quad \varepsilon = \pm 1.$$

- Cigar-like, when $F(r_\star) = 0$ space ends smoothly at $r = r_\star$, $S^1[\phi] \to 0$.
- $r \to \infty$, goes to AdS_5.
- Preserves four supercharges.

A quick look at the geometry

Define dimensionless quantities

$$\xi = \frac{r}{r_\star} \geq 1, \quad \hat{\nu} = \varepsilon \frac{\ell^2}{r_\star^2} \geq -1 \quad \Rightarrow F(\xi) = \frac{(\xi^2 - 1)\left(\xi^2 - \xi_+^2\right)\left(\xi^2 - \xi_-^2\right)}{L^2 \xi^4 (\hat{\nu} + \xi^2)}$$

with

$$\xi_\pm^2 = \frac{-(1+\hat{\nu}) \pm \sqrt{(\hat{\nu}+1)(\hat{\nu}-3)}}{2}$$

• The geometry invariants are **finite** but grow extremely large as $\hat{\nu} \to -1^+$.

• The supergravity needs higher curvature corrections in the action that are as important as R near $\hat{\nu} \approx -1^+$.

Singularity when $\hat{\nu} = -1$ at $r = r_\star = \ell$

Invariants of the geometry:

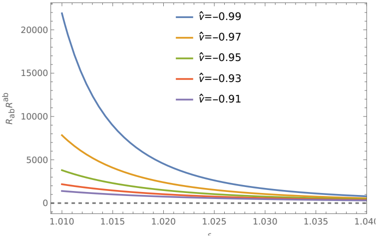


Figure: $R_{ab}R^{ab}$ vs ξ for $\hat{\nu} \approx -1$

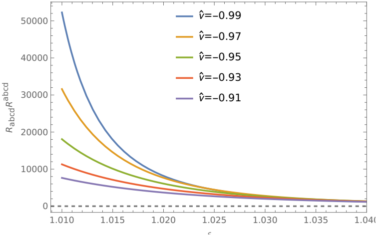


Figure: $R_{abcd}R^{abcd}$ vs ξ for $\hat{\nu} \approx -1$

[Hammond, Chatzis, Itsios, Nunez, Zoakos]

11d
M-theory
Lin-Lunin-Maldacena

11d
electrostatic

Type IIA
10d

10d
Type IIB
deformed
$AdS_5 \times S^5$

5d Soliton
[ANO]

[Hammond + Itsios to appear]

TsT giving new
IIB backgrounds

Deformed $\mathrm{AdS}_5 \times S^5$ in Type IIB

Dual to a deformation of $\mathcal{N} = 4$ SYM_4 flowing to $\mathcal{N} = 2$ QFT_3

$$\mathrm{d}s^2_{10} = \overbrace{\frac{\zeta(r,\theta)r^2}{L^2}\Big[-\mathrm{d}t^2 + \mathrm{d}w^2 + \mathrm{d}z^2 + L^2 F(r)\mathrm{d}\phi^2 + \frac{L^2\mathrm{d}r^2}{F(r)r^4\lambda^6(r)}}^{\text{deformed } \mathrm{AdS}_5 \ [t,w,z,\phi]} \underbrace{+L^4\mathrm{d}\theta^2\Big] + \frac{L^2}{\zeta(r,\theta)}\left[\cos^2\theta\mathrm{d}\psi^2 + \cos^2\theta\sin^2\psi\mathrm{D}\phi_1^2 + \cos^2\theta\cos^2\psi\mathrm{D}\phi_2^2 + \lambda^6(r)\sin^2\theta\mathrm{D}\phi_3^2\right]}_{\text{deformed } S^5 \ [\theta,\psi,\phi_1,\phi_2,\phi_3]},$$

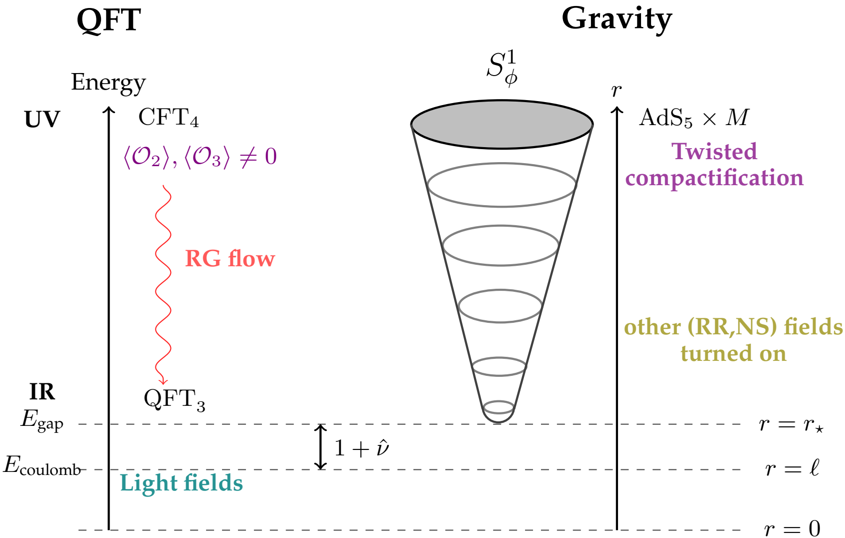
$$\& \quad F_5 = \star F_5$$

$\mathrm{U}(1)^3_{\mathcal{R}}$ made explicit with the isometries of $S^1[\phi_i]$.

"twisted compactification": $\qquad \mathrm{D}\phi_i = \mathrm{d}\phi_i + L^{-1}A^i_\phi \mathrm{d}\phi, \qquad i = 1, 2, 3$

Mixing of $\mathrm{U}(1)_{\mathcal{R}}$ with $\mathrm{SO}(2)_{\text{Poincaré}}$, results in partial SUSY preservation.

Interpretation in the dual QFT



TsT Procedure

Now I will show a TsT deformation of the type IIB background.
[Hammond + Itsios, to appear]

Where Θ_1, Θ_2 coordinates of the spacetime geometry,

Step 1: Perform a T-duality along Θ_1.

Step 2: Implement the coordinate shift $\Theta_2 \rightarrow \Theta_2 + \gamma\,\Theta_1$, where γ is a real deformation parameter.

Step 3: Perform a second T-duality along Θ_1.

Marginal Deformation [Hammond + Itsios, to appear]

Two T-dualities along ϕ_1 and a shift along ϕ_2.
The frame components e^7 and e^8 are mapped

$$(e^7, e^8) \mapsto \frac{1}{W}\,(e^7, e^8)\,, \qquad W(r,\theta,\psi) = \sqrt{1 + \gamma^2 L^4 \frac{\cos^4\theta\,\cos^2\psi\,\sin^2\psi}{\zeta^2}}\,.$$

$$\begin{aligned} ds^2 = {} & \frac{\zeta}{L^2}\Big(r^2\big(-dt^2 + dw^2 + dz^2 + L^2 F\,d\phi^2\big) + L^2\frac{dr^2}{r^2 F \lambda^6} + L^4 d\theta^2\Big) \\ & + \frac{L^2}{\zeta}\Bigg(\cos^2\theta\,d\psi^2 + \frac{\cos^2\theta\,\sin^2\psi}{W^2}\Big(d\phi_1 + \frac{A_1}{L}\Big)^2 + \frac{\cos^2\theta\,\cos^2\psi}{W^2}\Big(d\phi_2 + \frac{A_2}{L}\Big)^2 \\ & + \lambda^6\sin^2\theta\Big(d\phi_3 + \frac{A_3}{L}\Big)^2\Bigg)\,. \end{aligned}$$

NS sector of the deformed solution contains a non-trivial dilaton and two-form

$$\Phi = -\ln W\,, \qquad B_2 = -\sqrt{W^2 - 1}\,e^7 \wedge e^8\,.$$

Page Charges

- Quantised, conserved charges $\Rightarrow$ **integer** number of branes.
- Associated to the presence of Dp branes and they are defined on a $(8-p)$-dimensional cycle Σ_{8-p} transverse to the brane,

$$Q_{Dp} = \frac{1}{(2\pi)^{7-p}} \int_{\Sigma_{8-p}} \widehat{F}_{8-p} \,.$$

$$\hat{F} := F \wedge e^{-B_2}$$

Page Charges: Marginal Deformation

D3 Charge

- $D3$ Page charge is not affected by the TsT transformation. The RR sector of the undeformed solution contains only the self-dual five-form.
- 5d cycle transverse to the $D3$ branes spanned by $\Sigma_5 = (\theta, \psi, \phi_1, \phi_2, \phi_3)$.
- Asymptotic behaviour at large r, where the geometry approaches $AdS_5 \times S^5$:

$$\widehat{F}_5\Big|_{\Sigma_5} = -2L^4 \cos^3\theta \sin\theta \sin(2\psi) d\theta \wedge d\psi \wedge d\phi_1 \wedge d\phi_2 \wedge d\phi_3 \,.$$

$$Q_{D3} = \frac{1}{(2\pi)^4} \int_{\Sigma_5} \widehat{F}_5 = \frac{L^4}{4\pi} = N \in \mathbb{N} \,.$$

Page Charge for D5 branes

- TsT generates an F_3, so we have $D5$ and $D3$ branes.

D5 Charge

- $D5$ branes are transverse to the three-cycle spanned by $\Sigma_3 = (\theta, \psi, \phi_3)$.
- Asymptotic behaviour of $\widehat{F}_3$ at large r:

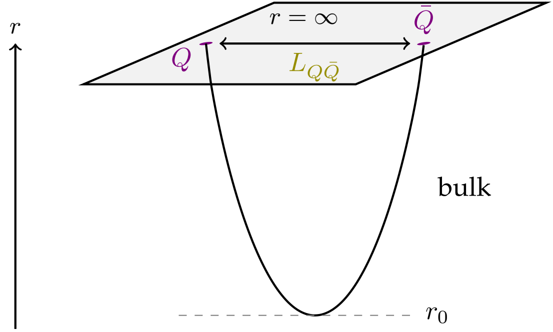
$$\widehat{F}_3\Big|_{\Sigma_3} = -2\gamma L^4 \cos^3\theta \sin\theta \sin(2\psi) d\theta \wedge d\psi \wedge d\phi_3 \,.$$

$$Q_{D5} = \frac{1}{(2\pi)^2}\int_{\Sigma_3} \widehat{F}_3 = \gamma \frac{L^4}{4\pi} = \gamma N = M \in \mathbb{N} \qquad \gamma \in \mathbb{Q}.$$

Note the condition enforced on $\gamma \in \mathbb{Q}$ the deformation parameter.

Wilson loops

We embed a probe string with its endpoints representing a $Q\bar{Q}$ pair at $r = \infty$.



$$r = r(w): \quad \mathrm{S}_{\mathrm{NG}} = \frac{1}{2\pi}\int \mathrm{d}t\mathrm{d}w\sqrt{-\mathrm{det}g_{\mathrm{ind}}} = \frac{\mathcal{T}}{2\pi}\int_{-L_{\mathrm{QQ}}/2}^{L_{\mathrm{QQ}}/2} \mathrm{d}w\sqrt{\mathcal{F}^2 + \mathcal{G}^2 r'^2}$$

$$\mathcal{F} = \frac{r^2\lambda^3(r)}{L^2}, \quad \mathcal{G} = \frac{1}{L\sqrt{F(r)}}$$

Wilson loops

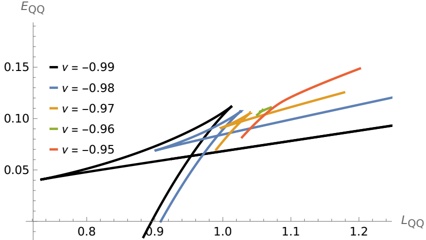
[Chatzis, Hammond, Itsios, Nunez, Zoakos '25]

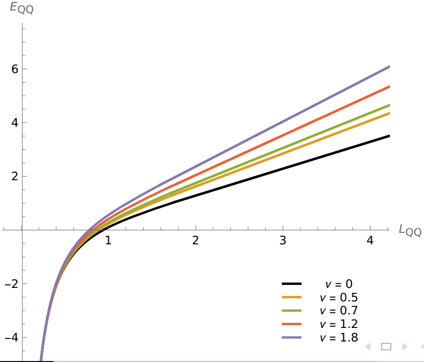
The Wilson loops and some other observables have **universal behaviour**.

$$\mathrm{S}_{\mathrm{IIB,IIA,M}} = \hat{\mathcal{N}}_{\mathrm{IIB,IIA,M}} \times \left(\int_{-L_{QQ}/2}^{L_{QQ}/2} \mathrm{d}w \sqrt{\mathcal{F}^2 + \mathcal{G}^2 \mathbf{r}'^2} \right)$$

Even though the background is **smooth** ($\hat{\nu} > -1$), the Wilson loop detects higher curvature corrections needed for $\hat{\nu} \approx -1$ by the appearance of a (non-physical) phase transition.

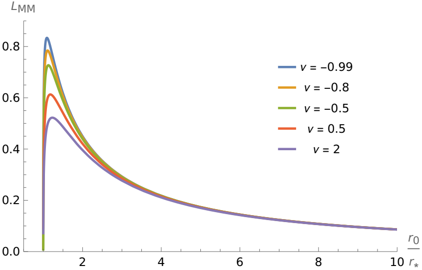
Wilson loops

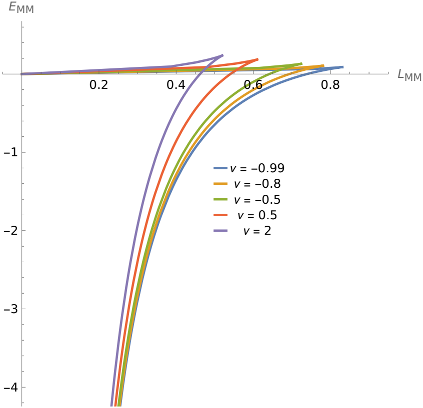




’t Hooft loops in original background

We use Dp ($p = 3, 5, 7$) probes extended in $[t, w, \phi, \dots]$ with $r = r(w)$.





′t Hooft Loops for the Marginal Deformations

The ′t Hooft loop calculated in the deformed backgrounds are expected to be different, as the probe brane extends in the directions of the internal space.

$$S_{D3} = T_{D3} \int \mathrm{d}^4\sigma \; e^{-\Phi} \sqrt{-\det g_{ind}}.$$

The ′t Hooft loop of a D3 probe embedded in $\Sigma_4 = [t, w, \phi, \phi_3]$,

Deformed $\mathrm{AdS}_5 \times S^5$:

$$S_{\mathrm{D}p} = \hat{\mathcal{N}}_{Dp} \int_{-L_{\mathrm{MM}}/2}^{L_{\mathrm{MM}}/2} \mathrm{d}w \; \sqrt{r^6 \lambda^6 F + L^2 r^2 r'^{\,2}}.$$

Marginal Deformation: **Not everything is universal**

$$S_{D3} = T_{D3} \; L \; \sin\theta \int \mathrm{d}^4\sigma \; W(r, \theta, \psi) \sqrt{r^6 \lambda^6 F + L^2 r^2 r'^{\,2}}.$$

Flow Central Charge

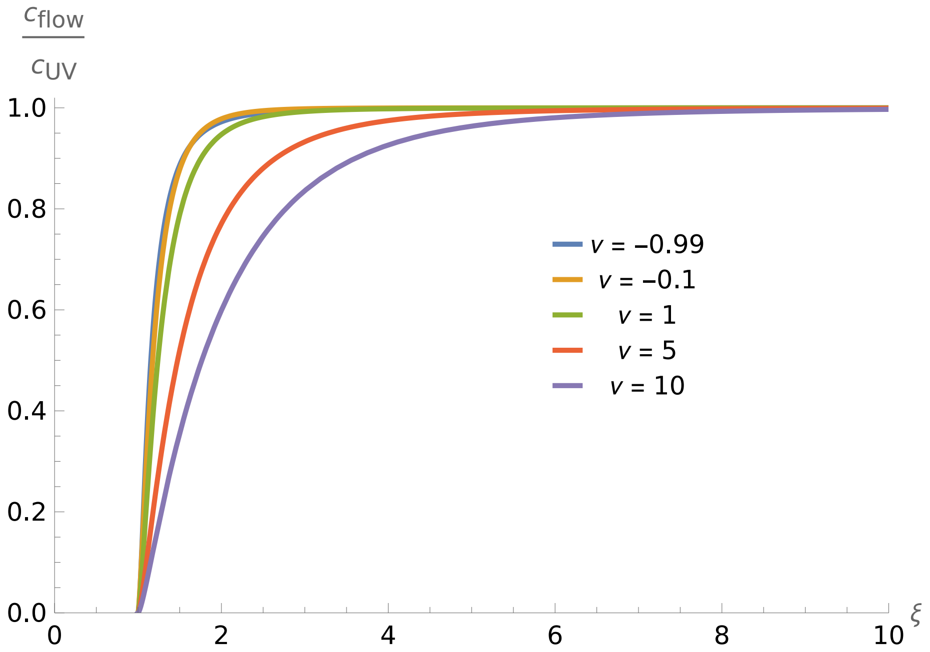
2506.10062, 2511.18128 [Chatzis, Hammond, Itsios, Nunez, Zoakos]

- Theory transitions from $4d \to 3d$, which breaks Lorentz invariance $\Rightarrow$ monotonicity theorems for a,c-functions require Lorentz invariance.

- This monotonic quantity measures the number of degrees of freedom as the system flows from $\mathrm{CFT}_4 \to \mathrm{QFT}_3$ despite breaking Lorentz invariance.

- Monotonic function becoming constant at the CFT fixed points.

$$c_{\text{flow},(2+1)} = \left(\frac{\sqrt{f(r)}}{1 + \frac{r f'(r)}{6}} \right)^3 \times c_{\text{4d SCFT}}. \tag{1}$$



$\frac{c_{\text{flow}}}{c_{\text{UV}}}$
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
ξ
$v = -0.99$
$v = -0.1$
$v = 1$
$v = 5$
$v = 10$

Conclusions

• We construct **smooth** asymptotically AdS backgrounds dual to 4d SCFT flowing to a gapped and confining 3d QFT.

• We calculate a range of observables and find that for the different CFTs, they show a universal behaviour.

• We further deform the type IIB background giving some marginal deformations and a solution dual to a non-commutative gauge theory.

arxiv

• 2506.10062, 2511.18128 [Chatzis, Hammond, Itsios, Nunez and Zoakos]

• to appear [Hammond + Itsios]