FLAVOUR HIERARCHIES VIA NON-MINIMAL IRREPS

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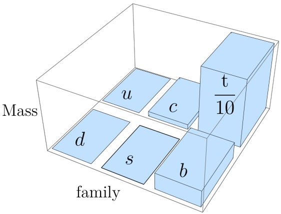
Only the Yukawa couplings break the SM flavor symmetries

$$\mathrm{U}(3)_Q \times \mathrm{U}(3)_u \times \mathrm{U}(3)_d \times \mathrm{U}(3)_L \times \mathrm{U}(3)_e$$

Why is this breaking hierarchical?



Isidori, Kaon 2025

Analyse via a **spurion** – a dynamical field which is the only parameter to explicitly break global symmetries.

Framework	Spurion	Flavor Puzzle?
Minimal Flavour Violation (MFV)	Yukawa is $3_Q \times \overline{3}_{u(d)}$ spurion	No NP FCNCs, but only accommodates hierarchy

BSM APPROACHES TO THE FLAVOUR PUZZLE

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Froggatt–Nielsen (FN)	Additional $U(1)_{FN}$ Spurion (flavon) gets a vev Fermion charges generation dependent	Generates hierarchy, but charges under $U(1)$ are ad-hoc

$$\mathcal{L} \supset y_{ij}\overline{Q}_i H d_j \left(\frac{\theta}{\Lambda}\right)^{x_{ij}}, \text{ for flavon } \theta \text{ and charge } x_{ij}$$

NEW: FLAVOR, ACCIDENTALLY

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Accidental Hierarchy (AH)	Spurion in non-minimal irrep of $SU(3)$ flavour symmetries e.g. $\mathbf{6}_Q \times \overline{\mathbf{6}}_{u(d)}$	Can a random $\mathcal{O}(1)$ spurion accidentally lead to flavour hierarchies?

Essentially, how do we make the Yukawa ($\mathrm{y} \sim \mathbf{3}_{\mathrm{Q}} \times \overline{\mathbf{3}}_{\mathrm{u(d)}}$) from copies of our spurion ($\mathcal{Y} \sim \mathbf{6}_{\mathrm{Q}} \times \overline{\mathbf{6}}_{\mathrm{u(d)}}$) ?

$$\mathrm{y} \sim \mathcal{Y}^2\overline{\mathcal{Y}} + \epsilon\left(\mathcal{Y}^4 + \mathcal{Y}\overline{\mathcal{Y}}^3\right) + \epsilon^2\left(\overline{\mathcal{Y}}^5 + \mathcal{Y}^3\overline{\mathcal{Y}}^2\right) + \ldots .$$

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$$\begin{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} \mathcal{O}(1) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \mathcal{O}(1) \\ 0 & \mathcal{O}(1) & 0 \end{pmatrix} \\ \begin{pmatrix} \mathcal{O}(1) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & \mathcal{O}(1) \\ 0 & 0 & \mathcal{O}(1) \\ \mathcal{O}(1) & \mathcal{O}(1) & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \mathcal{O}(1) \\ 0 & \mathcal{O}(1) & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{pmatrix}$$

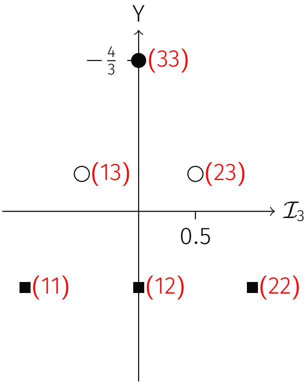
$$\mathcal{Y}$$

$$\Rightarrow \mathrm{y} \approx \begin{pmatrix} \mathcal{O}(\epsilon) & \mathcal{O}(\epsilon) & 0 \\ 0 & 0 & \mathcal{O}(\epsilon) \\ 0 & \mathcal{O}(\epsilon) & \mathcal{O}(1) \end{pmatrix} + \mathcal{O}(\epsilon^2)$$

$$\boxed{\text{Eigenvalues: } \left(\mathcal{O}(1), \mathcal{O}(\epsilon), \mathcal{O}(\epsilon^2)\right)}$$

BACKUP: CONSTRUCTING THE SPURION

The spurion $\mathcal{Y}^{\alpha\beta}_{\mathrm{ab}}$ transforms under indices $(\alpha\,\beta)$ of $\mathrm{SU(3)_Q}$ and (a b) of $\mathrm{SU(3)_{u(d)}}$



$$\begin{pmatrix} (11) & (12) & (13) \\ (21) & (22) & (23) \\ (31) & (32) & (33) \end{pmatrix}$$

Indices of the **6** in terms of flavour charges and how they form the matrices that build up the spuiron $\mathcal{Y}$. (12) = (21) etc. from symmetry.

BACKUP: UNIQUE SPURION MEANS UNIQUE PHENO

Consider the pattern of flavour violation in kaon mixing for dim-6 SMEFT operators.

	MFV	AH	FN
	sd		
c_{LL}	$y_c^4 s_{12}^2$	ϵ^2	$\dot{\epsilon}^2$
c_{LR}	$y_c^4 s_{12}^2 y_d y_s$	ϵ^1	1
c_{RR}	$y_c^4 s_{12}^2 y_d^2 y_s^2$	1	$\dot{\epsilon}^2$

Upshot? Frameworks generating same SM structure can have very different NP structure.