A numerical analysis of axion-saxion quintessence

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YTF 2025
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CAMBRIDGE COMMONWEALTH EUROPEAN AND INTERNATIONAL TRUST
CONACYT

Collaborators

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  - Theoretical arguments
- **Review of quintessence**
  - Single field
  - Multi-field
- **Albrecht-Skordis potential**
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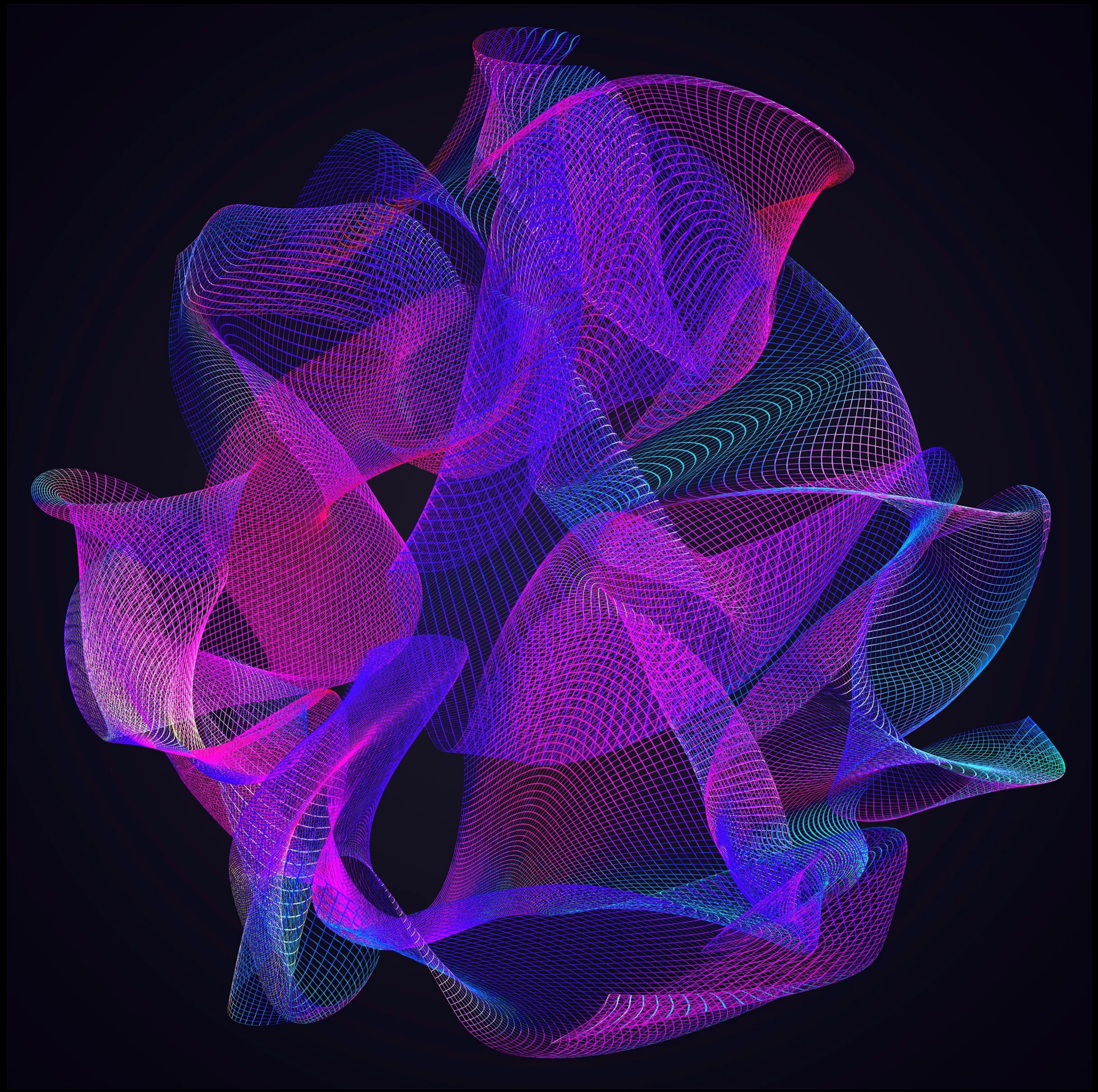

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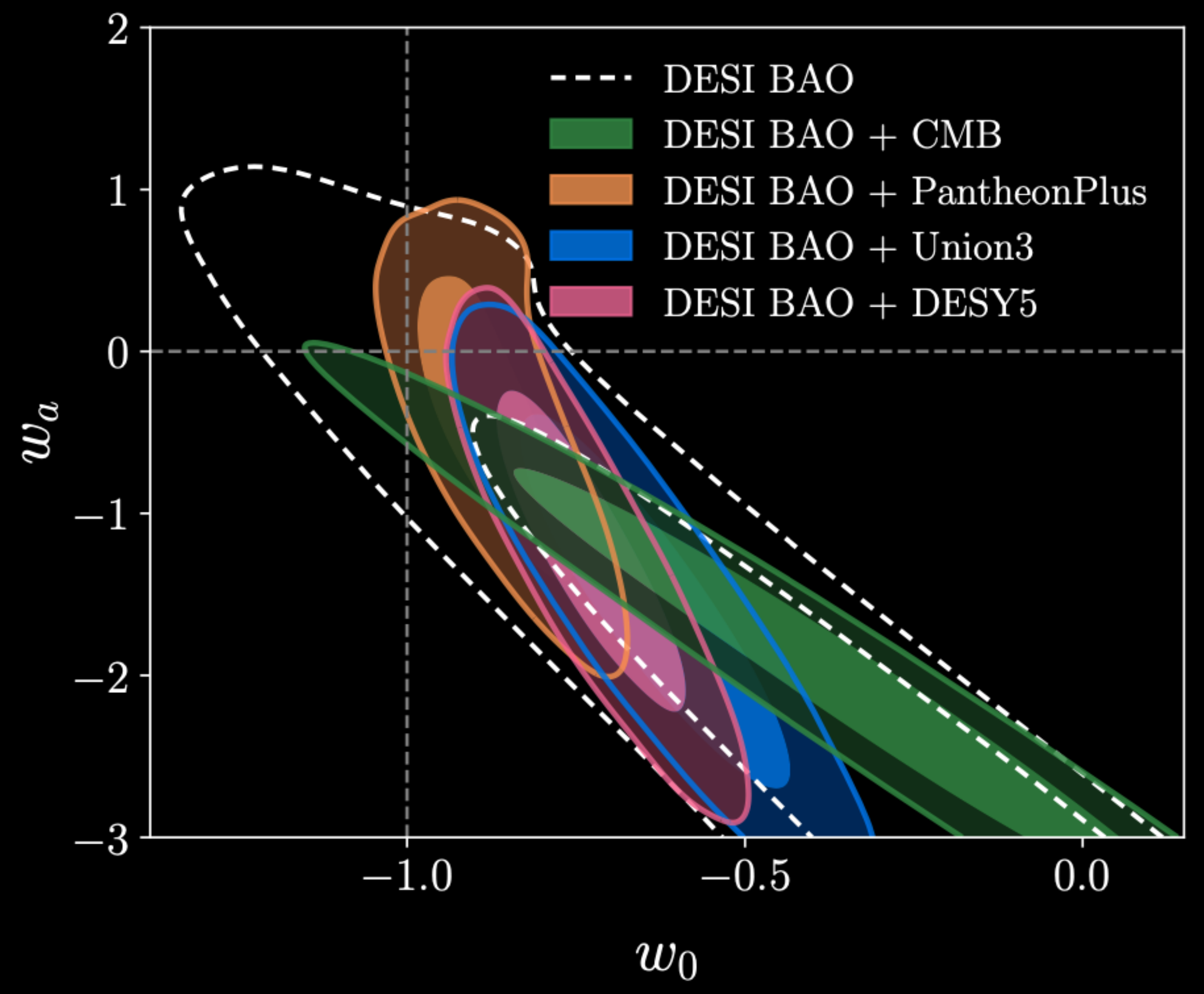

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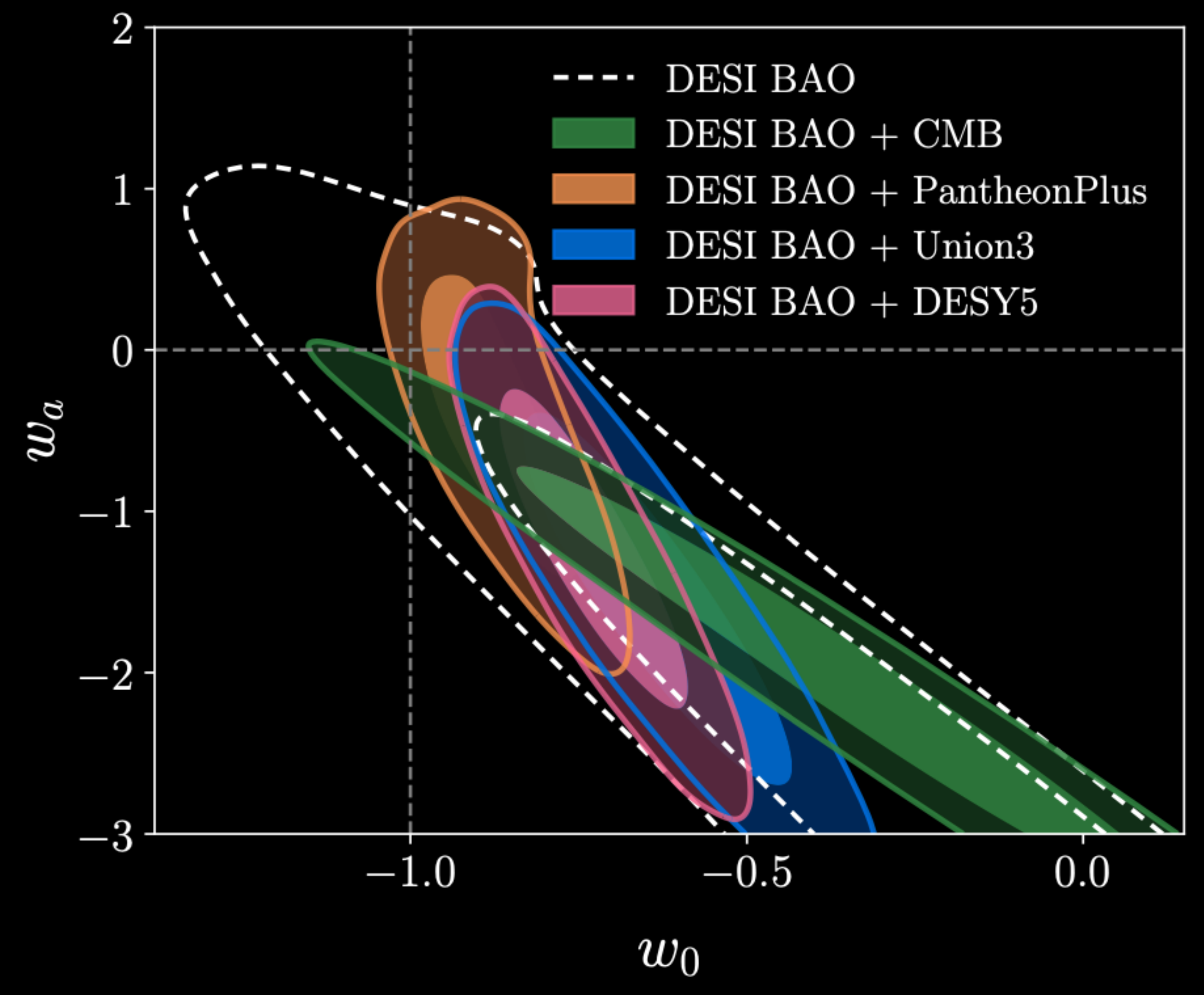

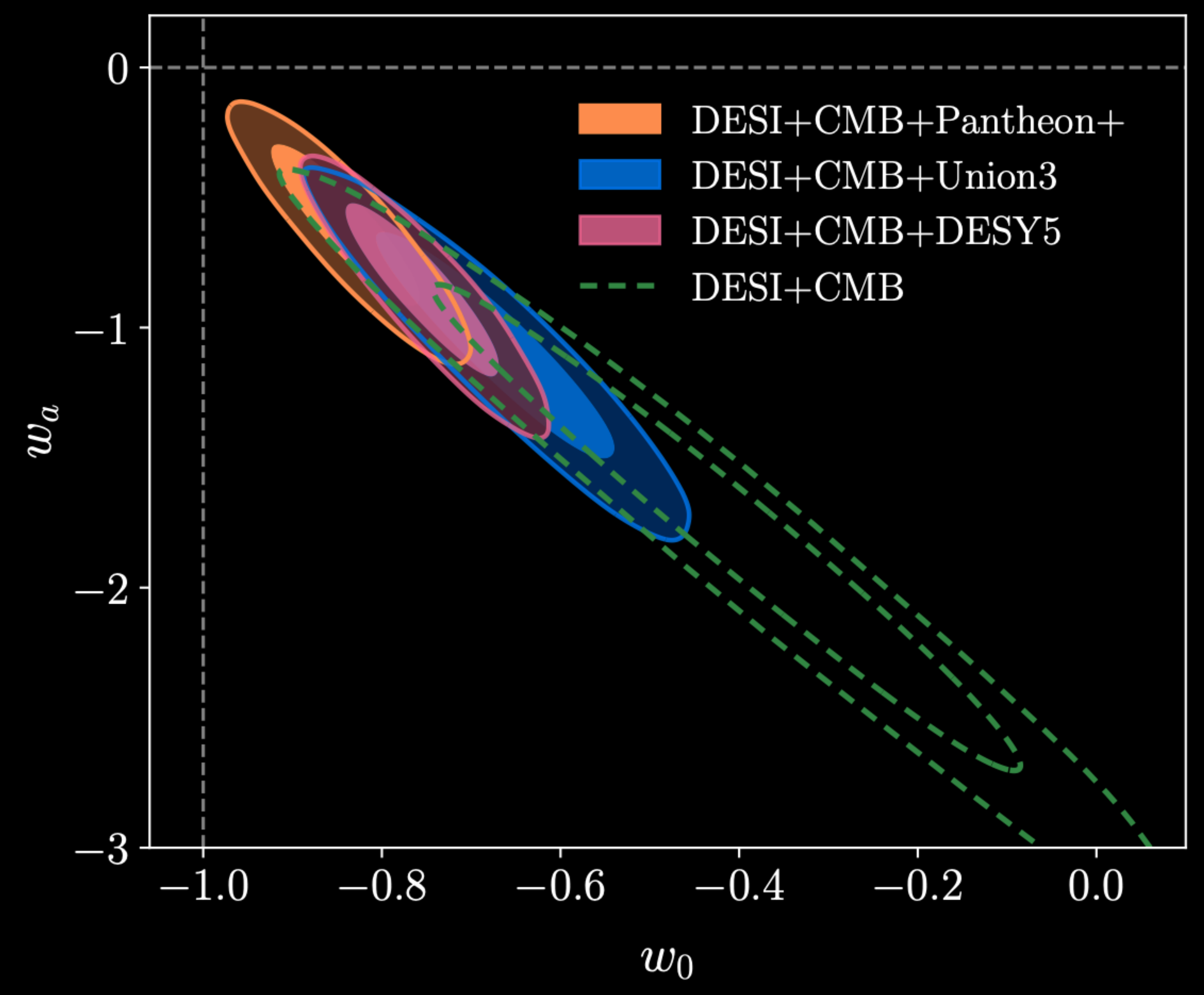

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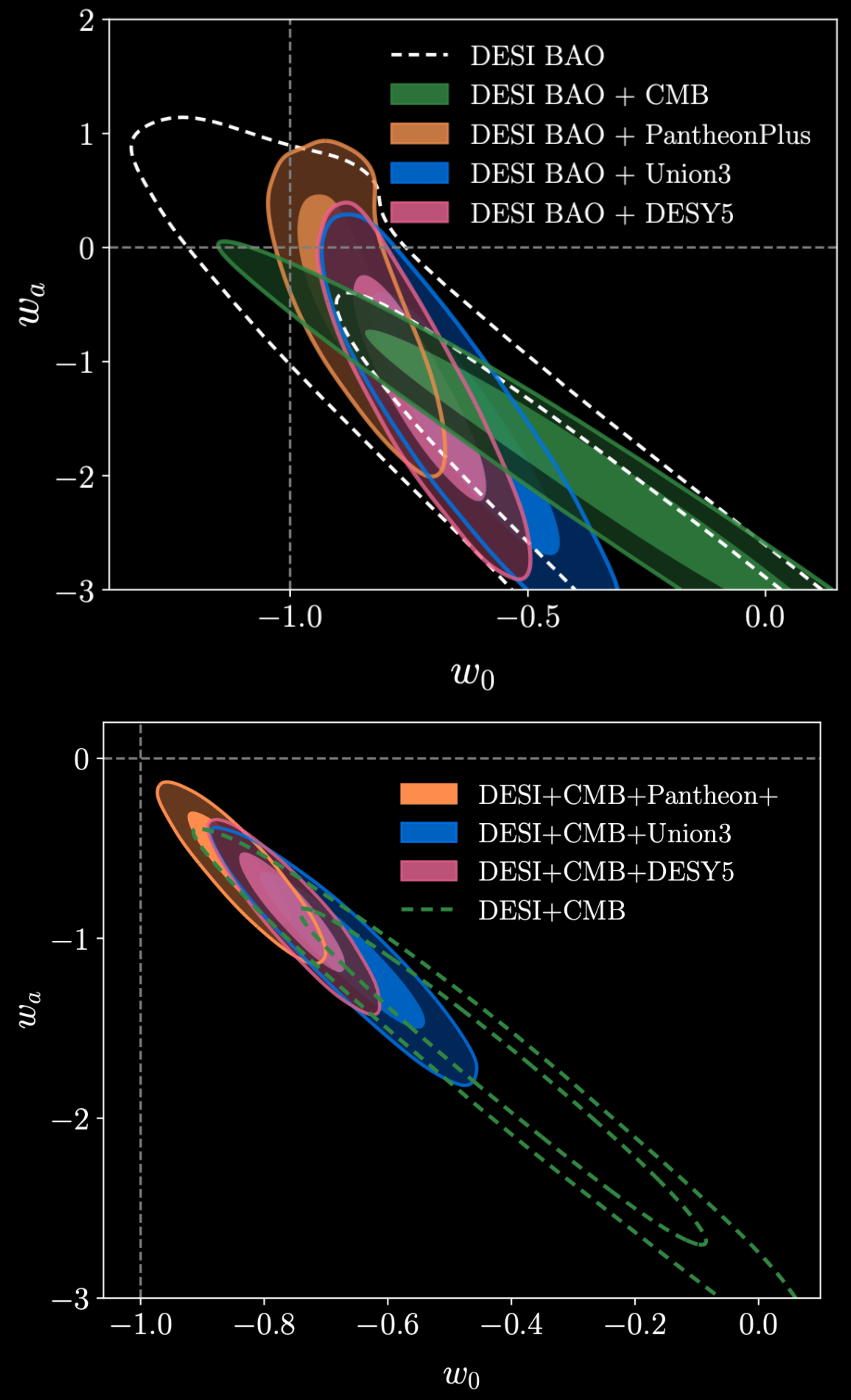

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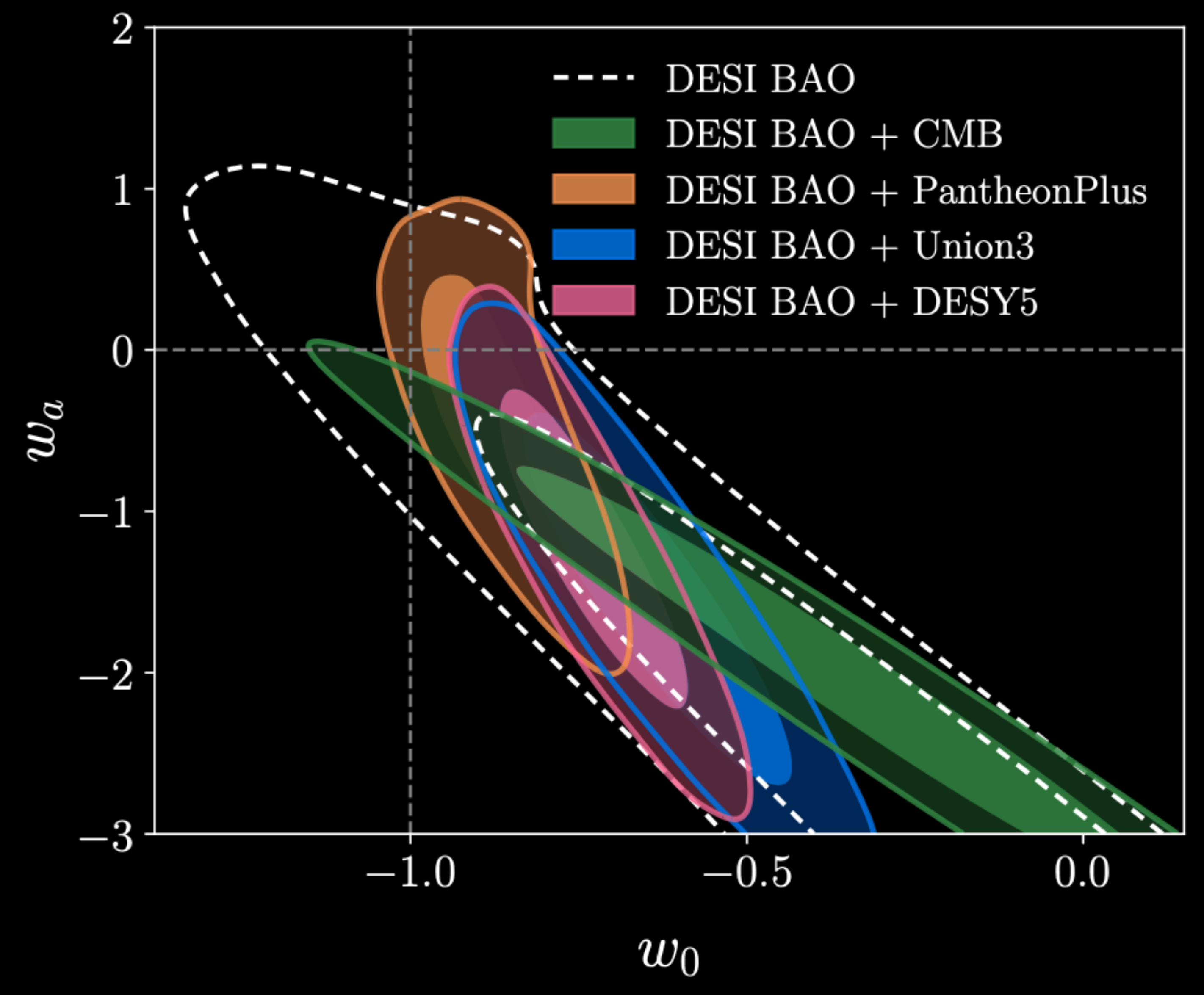
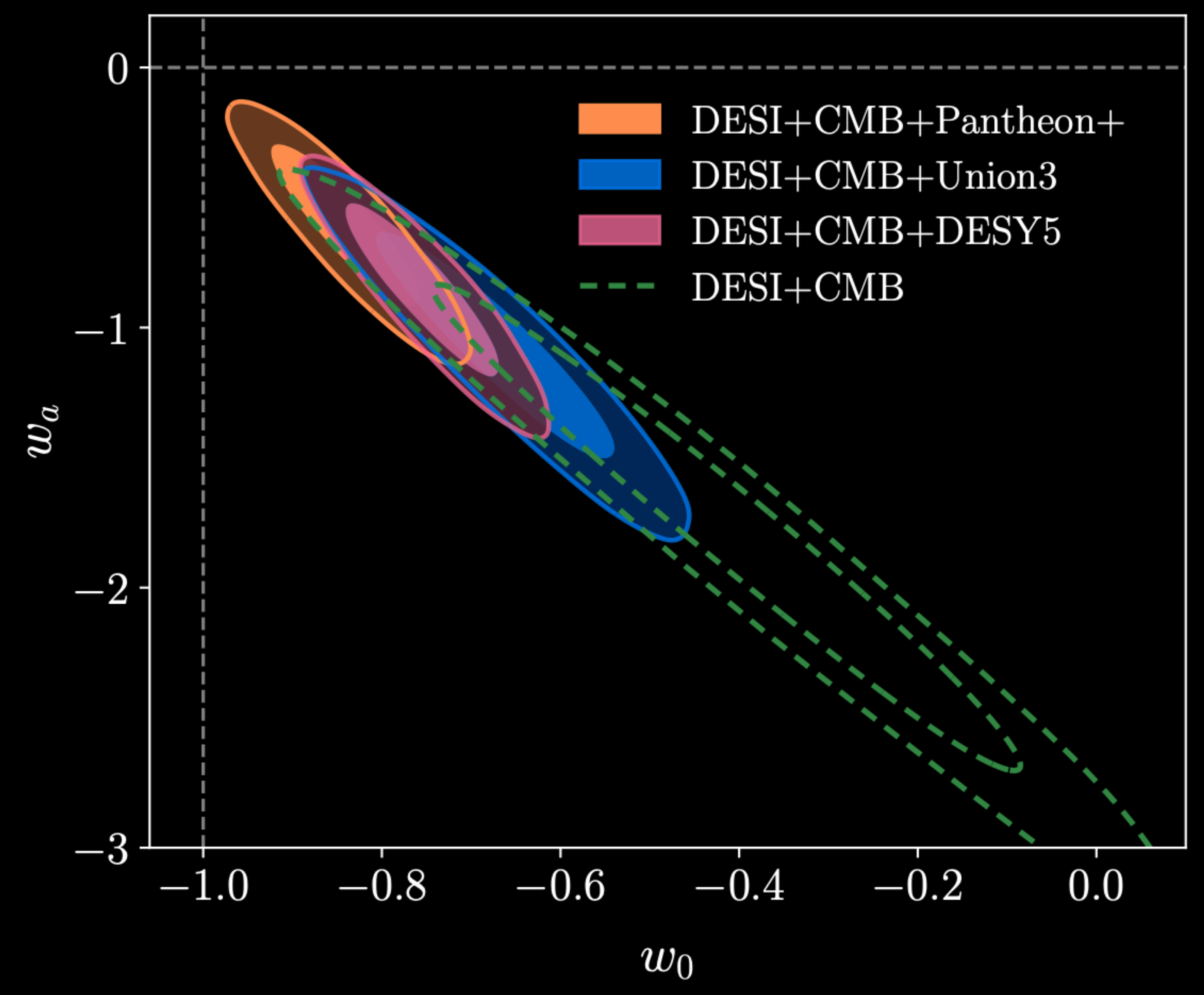
- DESI BAO data are consistent with ΛCDM on their own.
- Dynamical dark energy is preferred at
  - a) $2-3\sigma$ when two of them are combined
  - b) $3-4\sigma$ all of them combined

A model of dynamical dark energy is compulsory to explain data.





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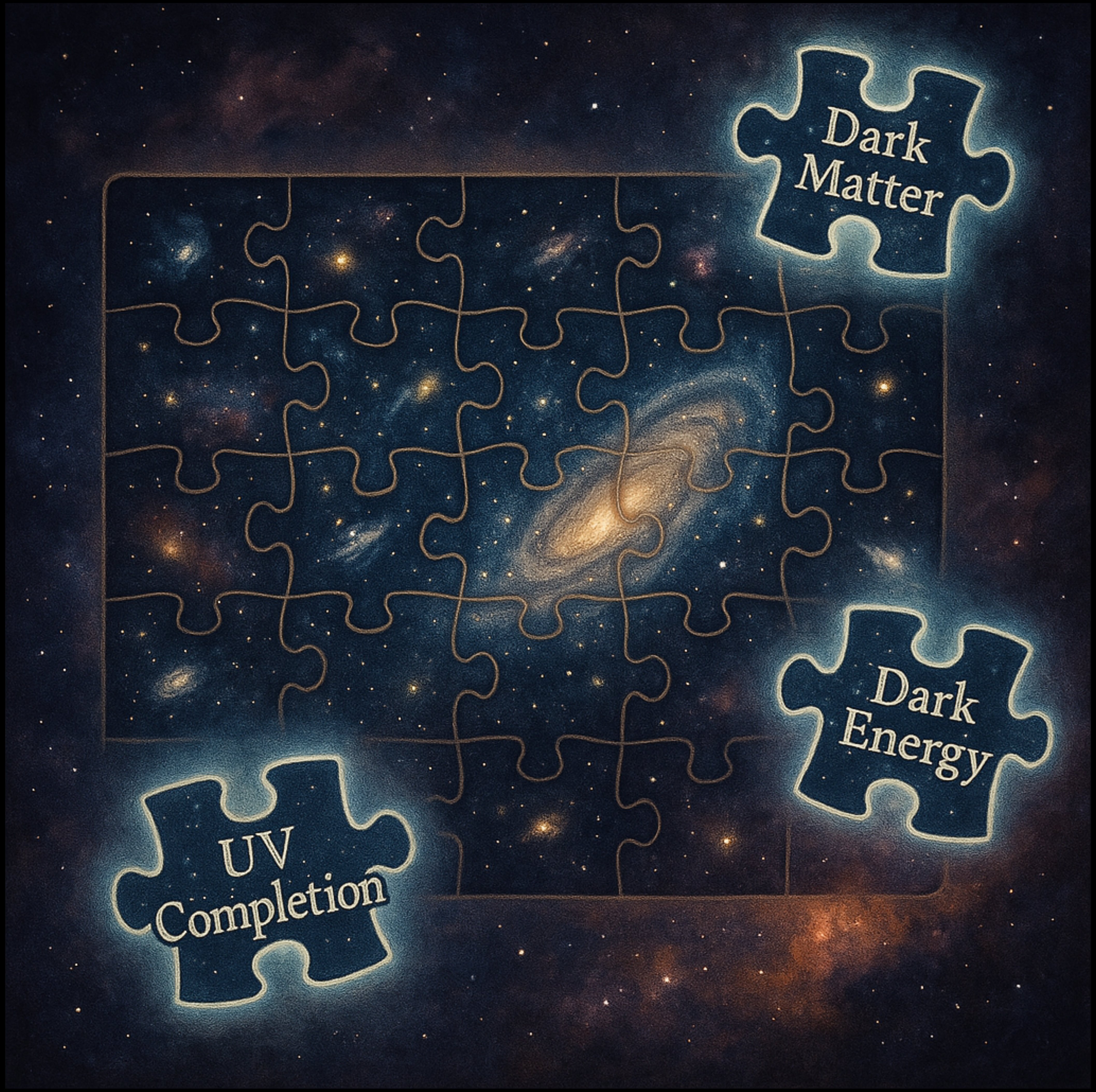

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In string compactifications, the **axio-dilaton** couple arises naturally with the target-space metric,

$$\mathscr{G}_{ab}(\phi)\partial_\mu\phi^a\partial^\mu\phi^b = \partial_\mu\varphi\partial^\mu\varphi + W^2(\varphi)\partial_\mu\mathfrak{a}\partial^\mu\mathfrak{a},$$

where φ and $\mathfrak{a}$ are pseudo-Goldstone bosons of and approximate spacetime scaling and internal shift symmetry, respectively. Moreover, the scaling condition entails $W(\varphi) = e^{-\xi\varphi}$.

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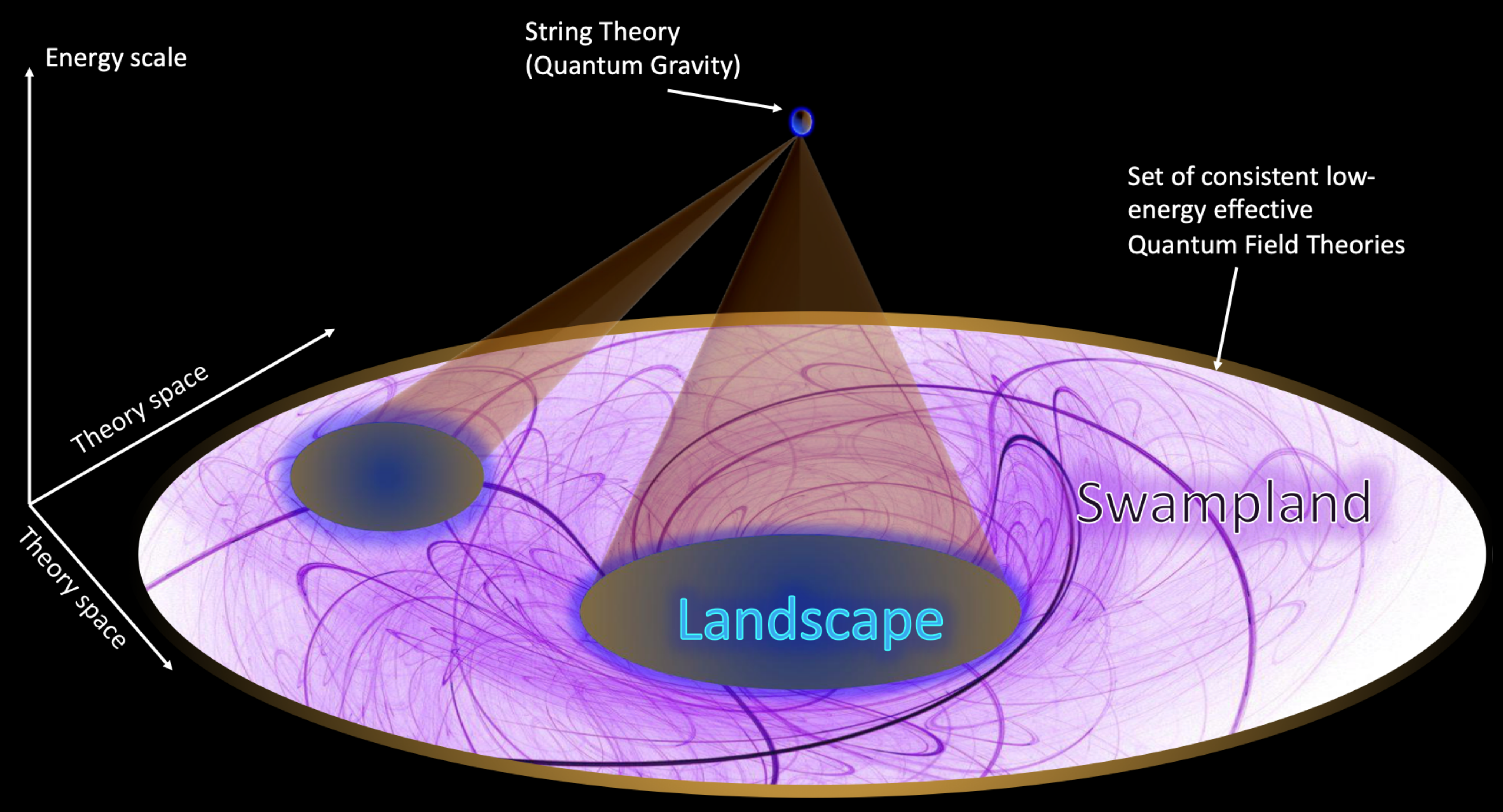

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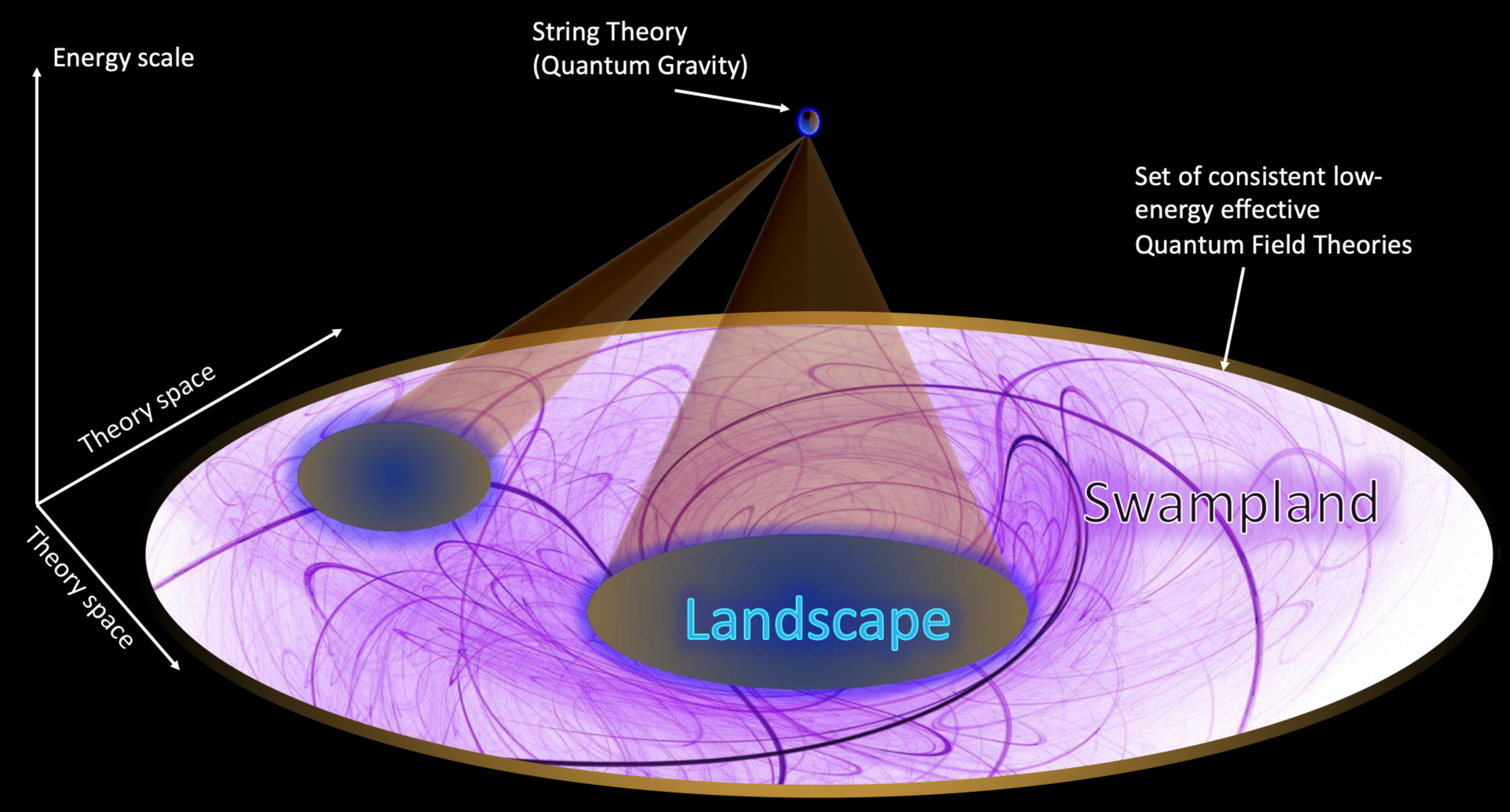

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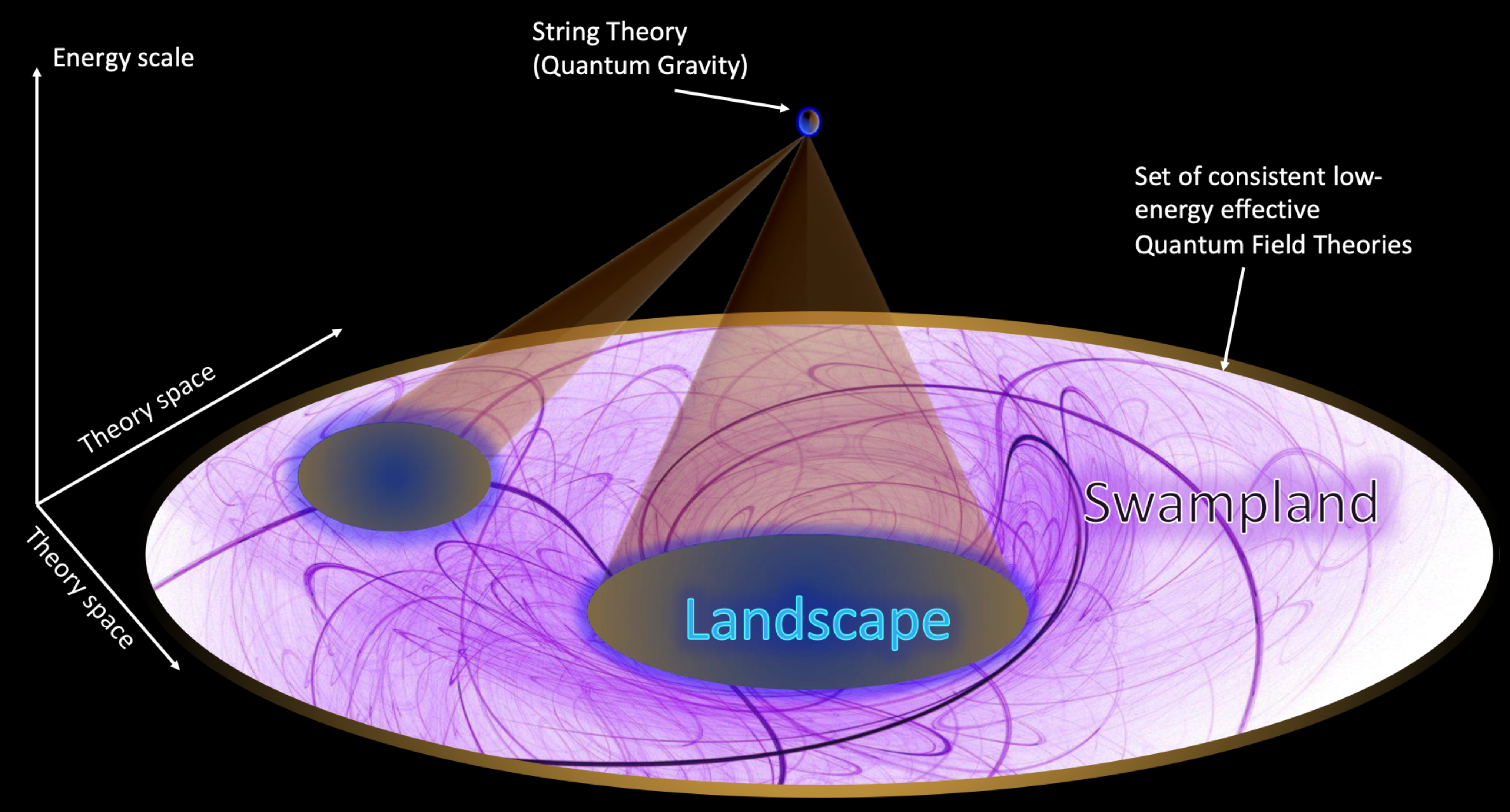

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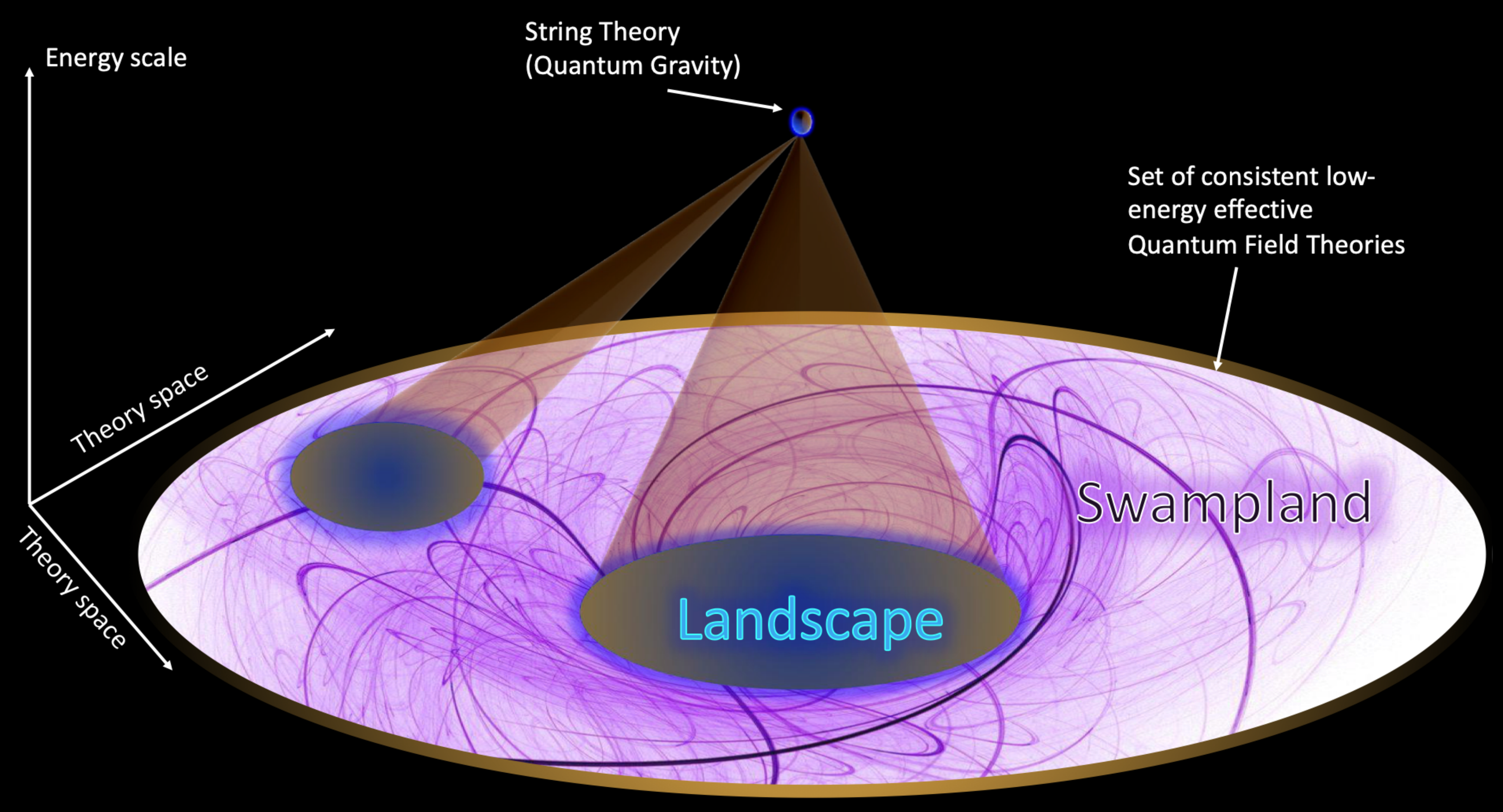

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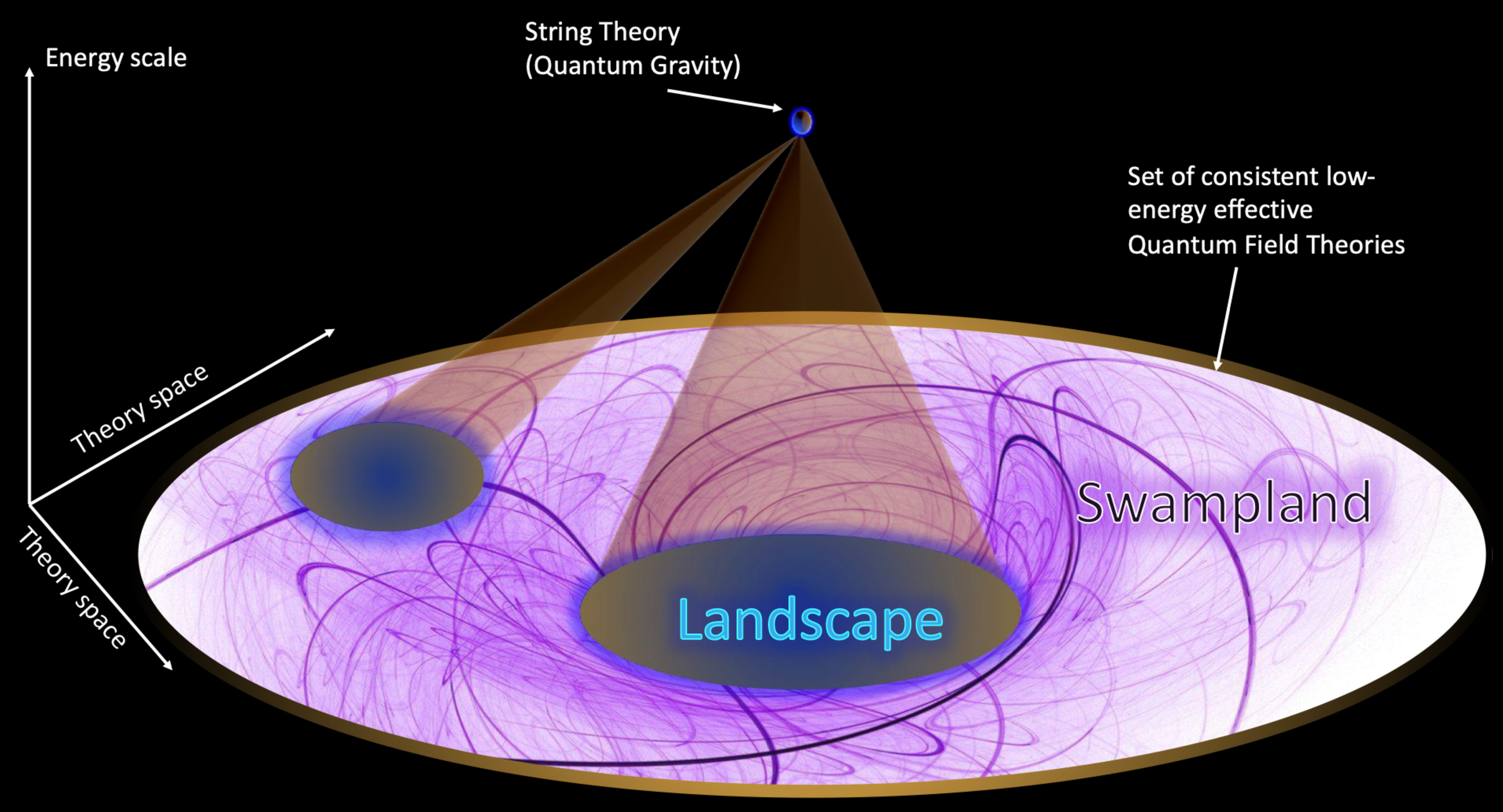

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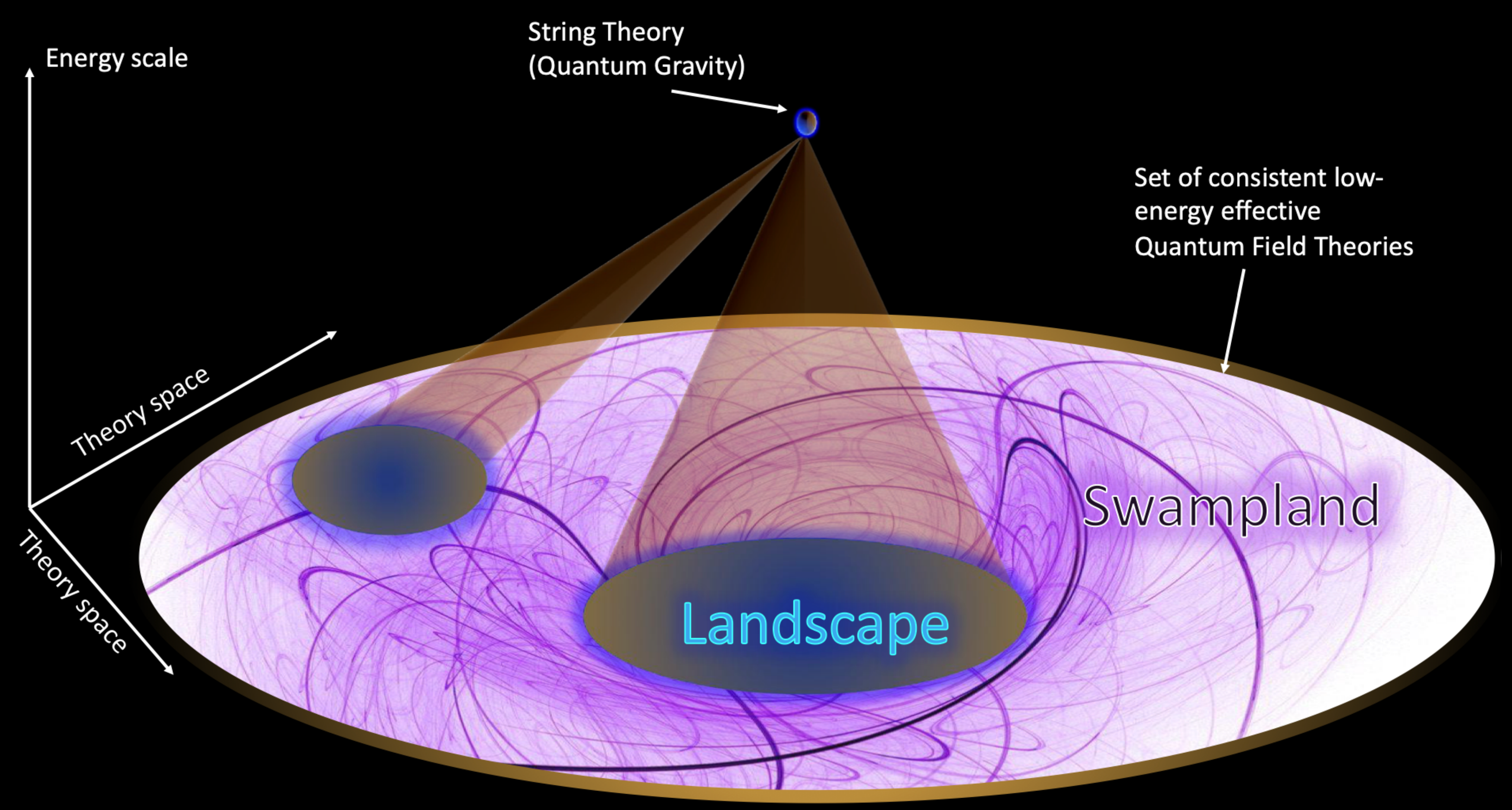

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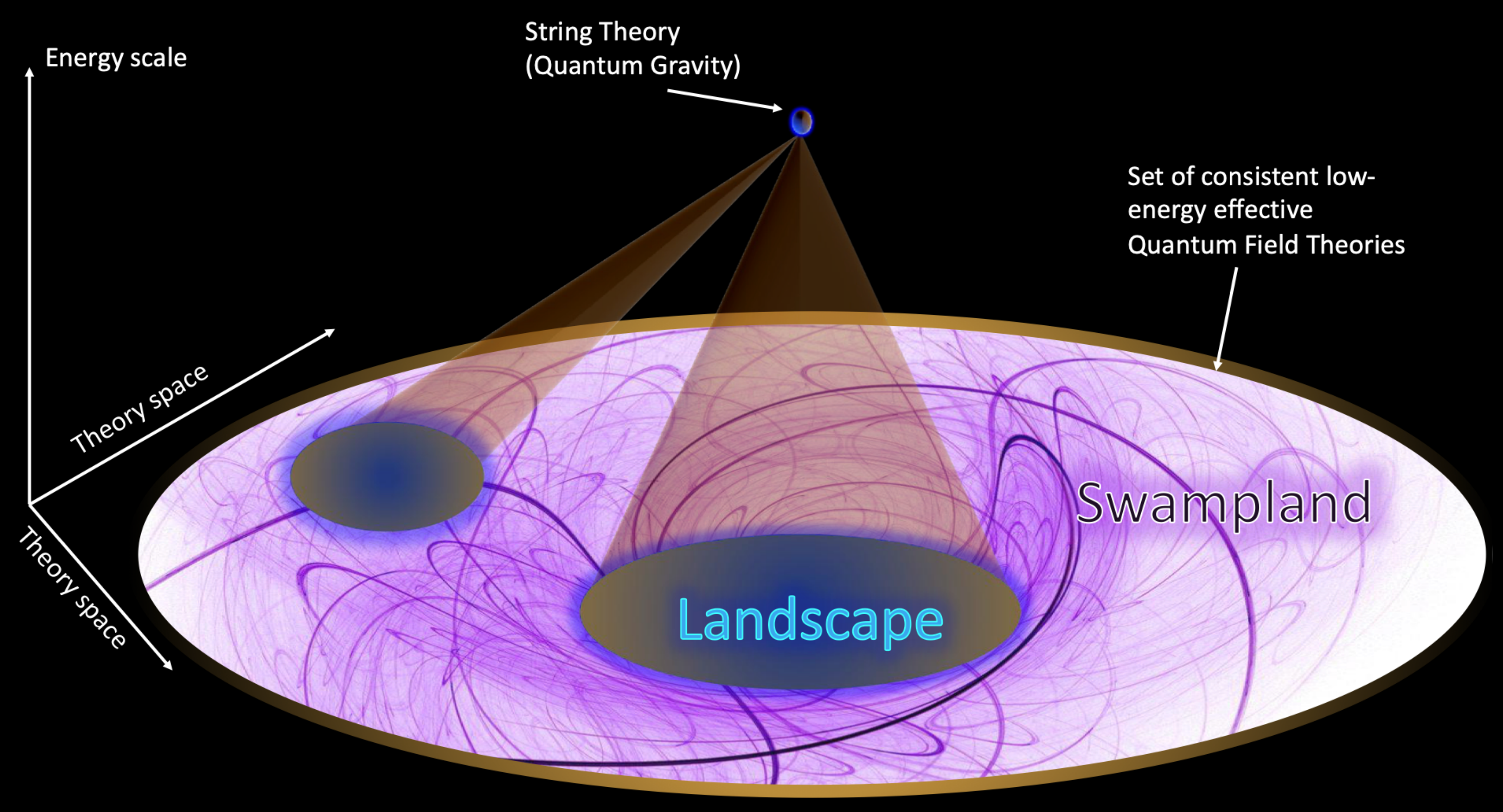

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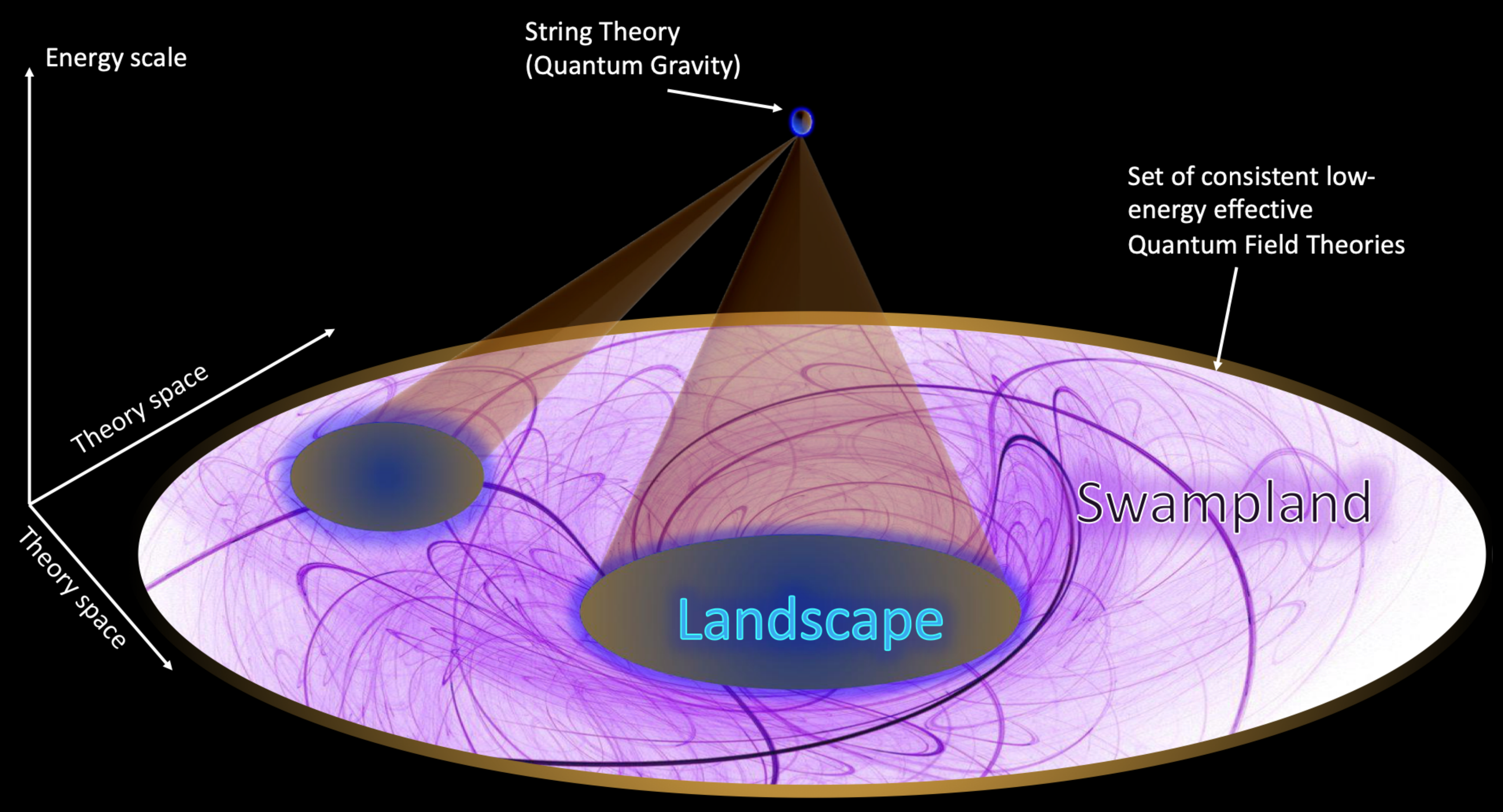

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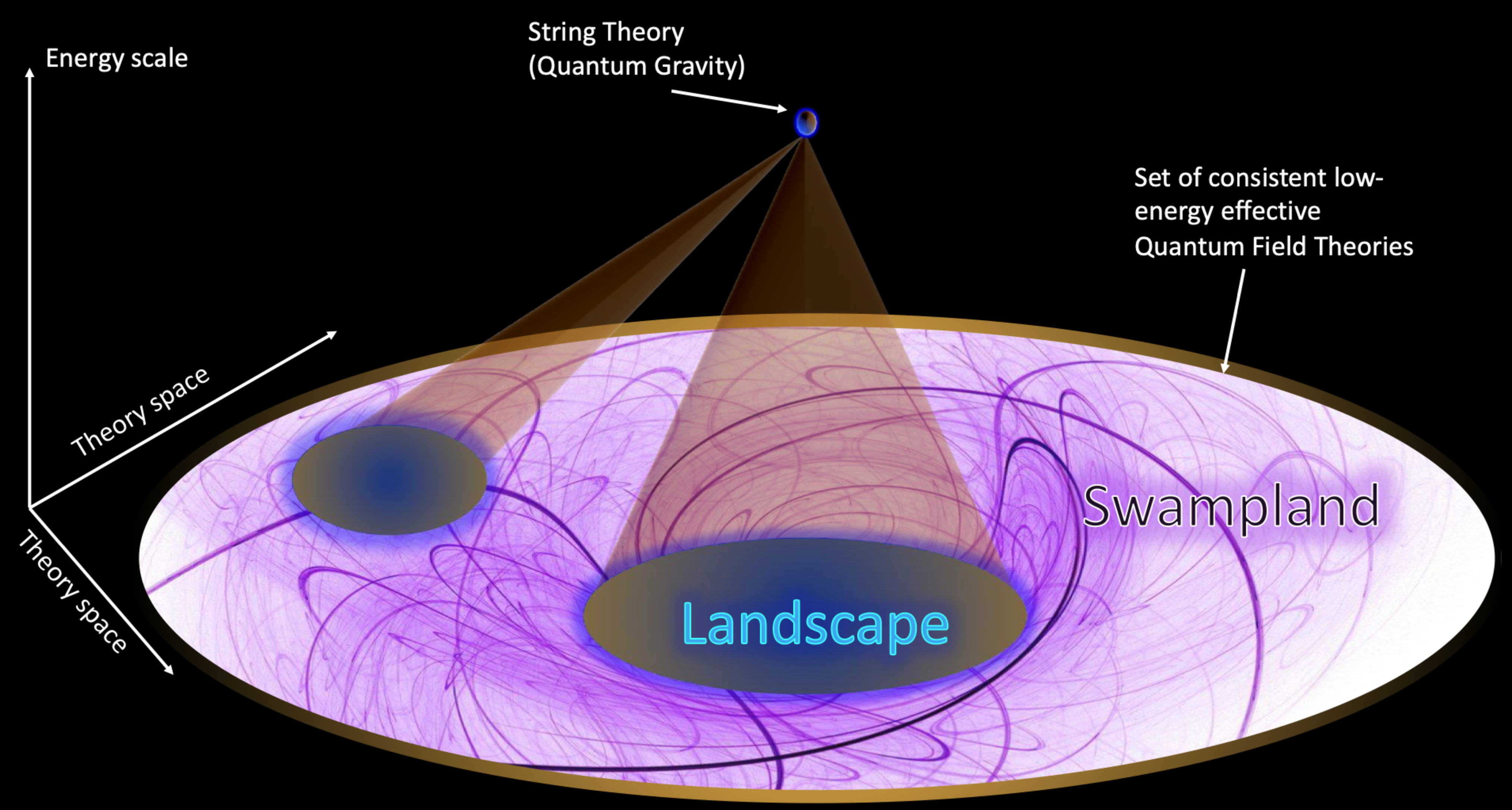

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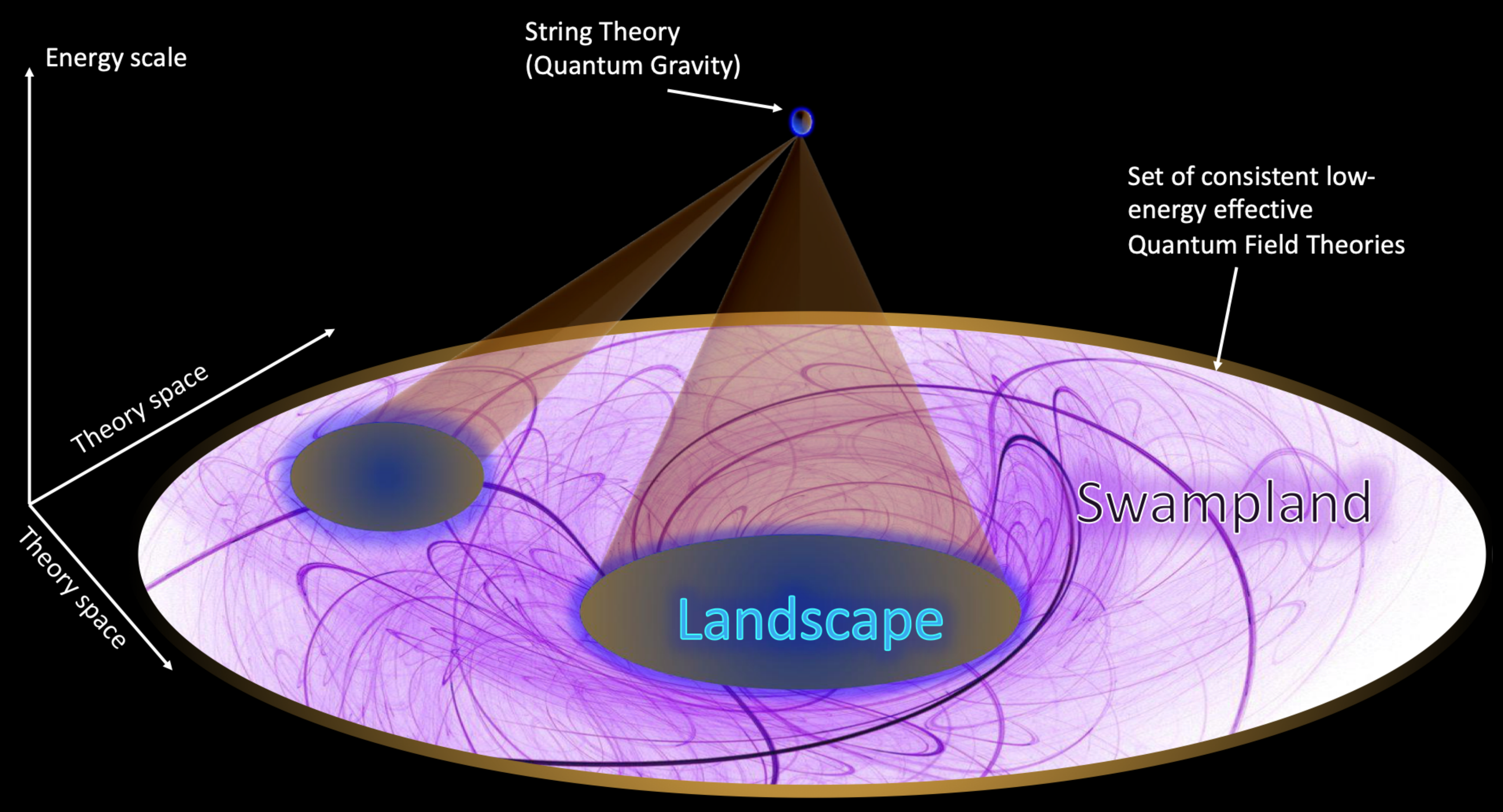

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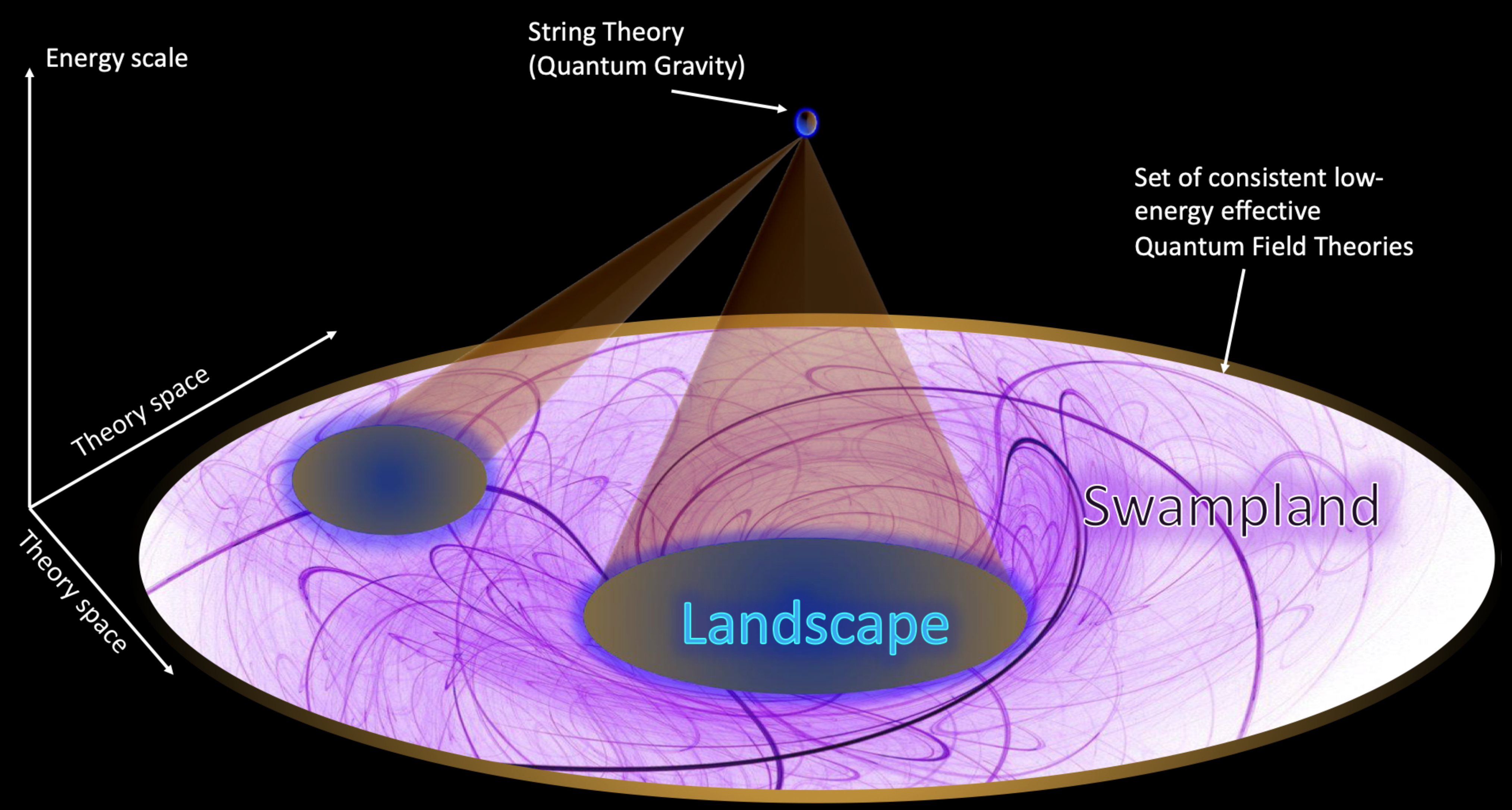

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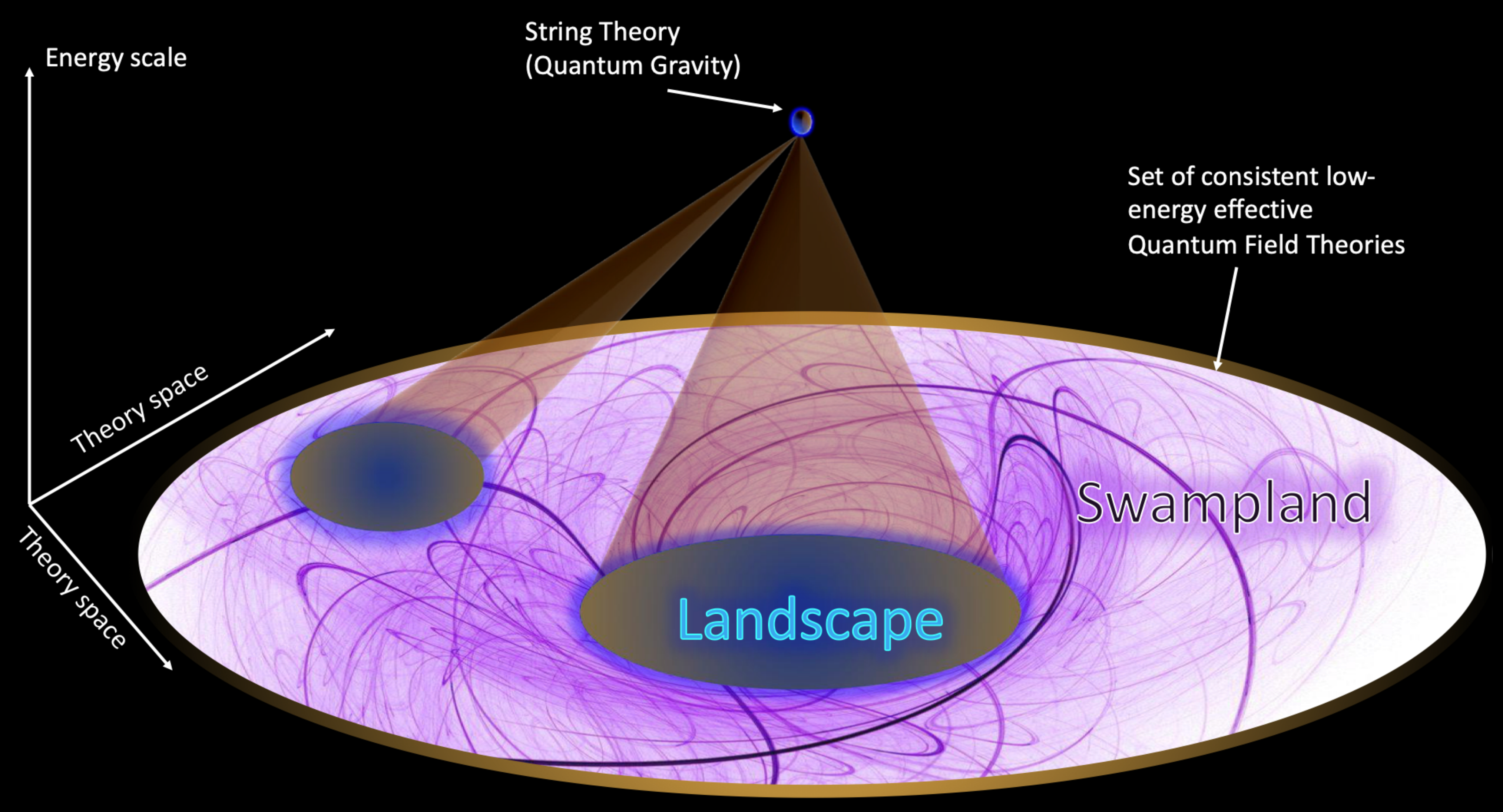

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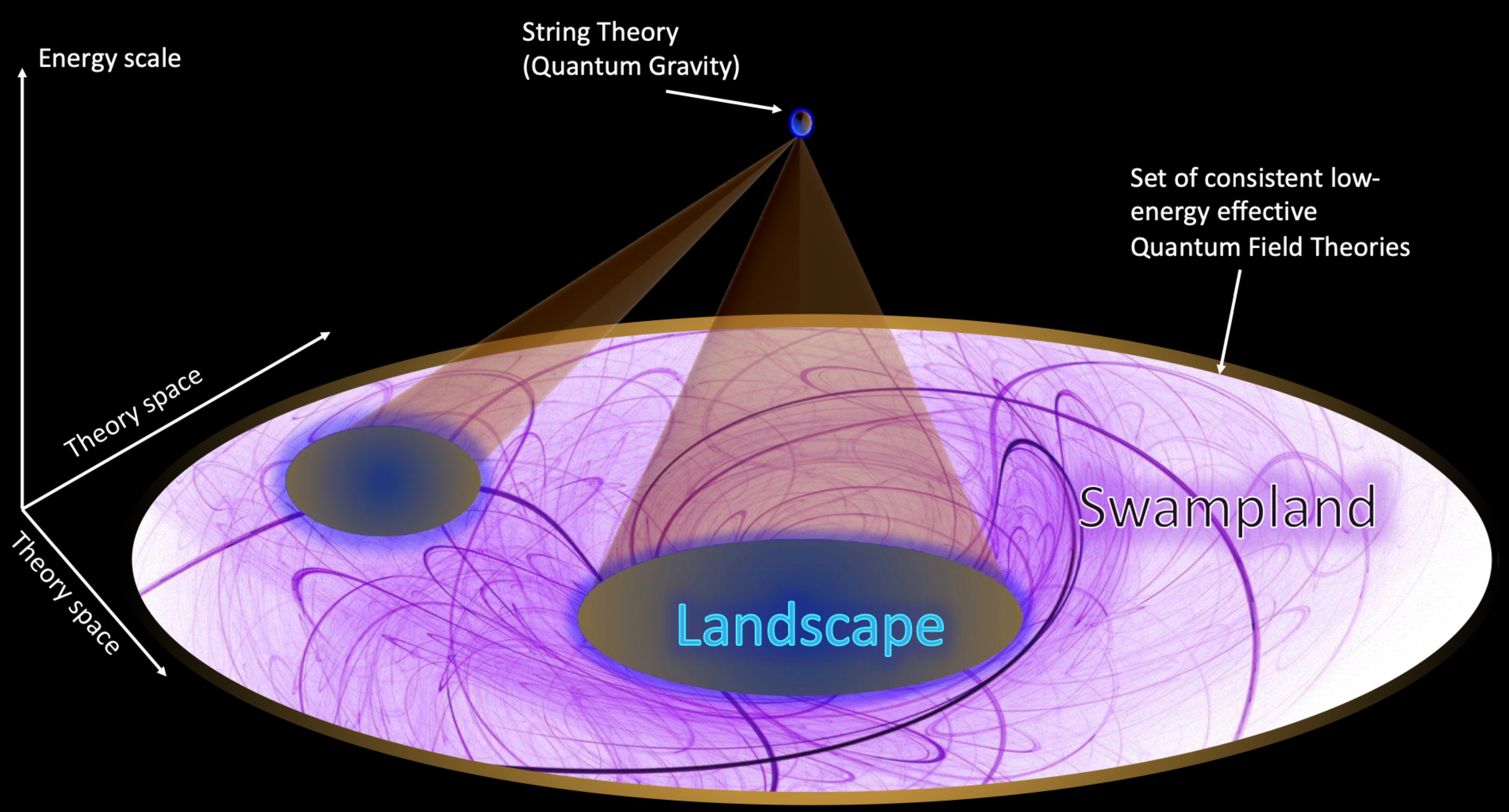

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If the conjectures were correct, not only would dS vacua be absent in our theory of quantum gravity, but also effectively single-field slow-roll quintessence models.

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$$\frac{\mathscr{L}}{\sqrt{-g}} = \frac{1}{2}\mathscr{G}_{ij}\partial\mu\phi^i\partial^\mu\phi^j - V(\phi) \quad \text{with} \quad d\mathfrak{s}^2 = d\phi_1^2 + W^2(\phi_1)d\phi_2^2 \quad \text{and} \quad W(\phi_1) = e^{-\xi\phi_1/M_p}.$$

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with $' \equiv d/d\ln a$ is the derivative with respect to the number of e-folds. Also, the scalar sector energy density is

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Dynamical system [S. Bahamonde et al, 2018]

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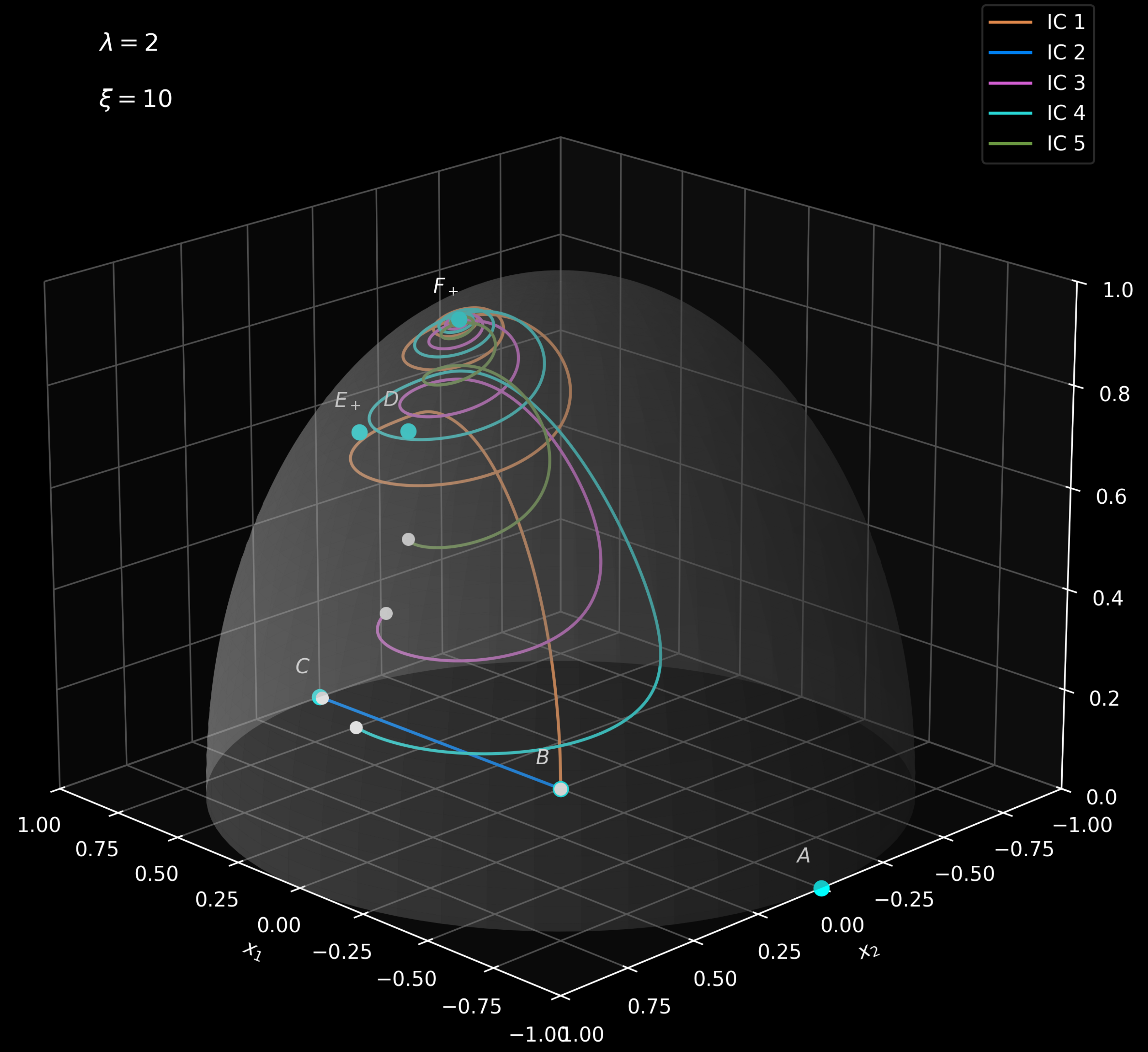
$$\Omega_\phi = x_1^2 + x_2^2 + y^2 \leq 1.$$

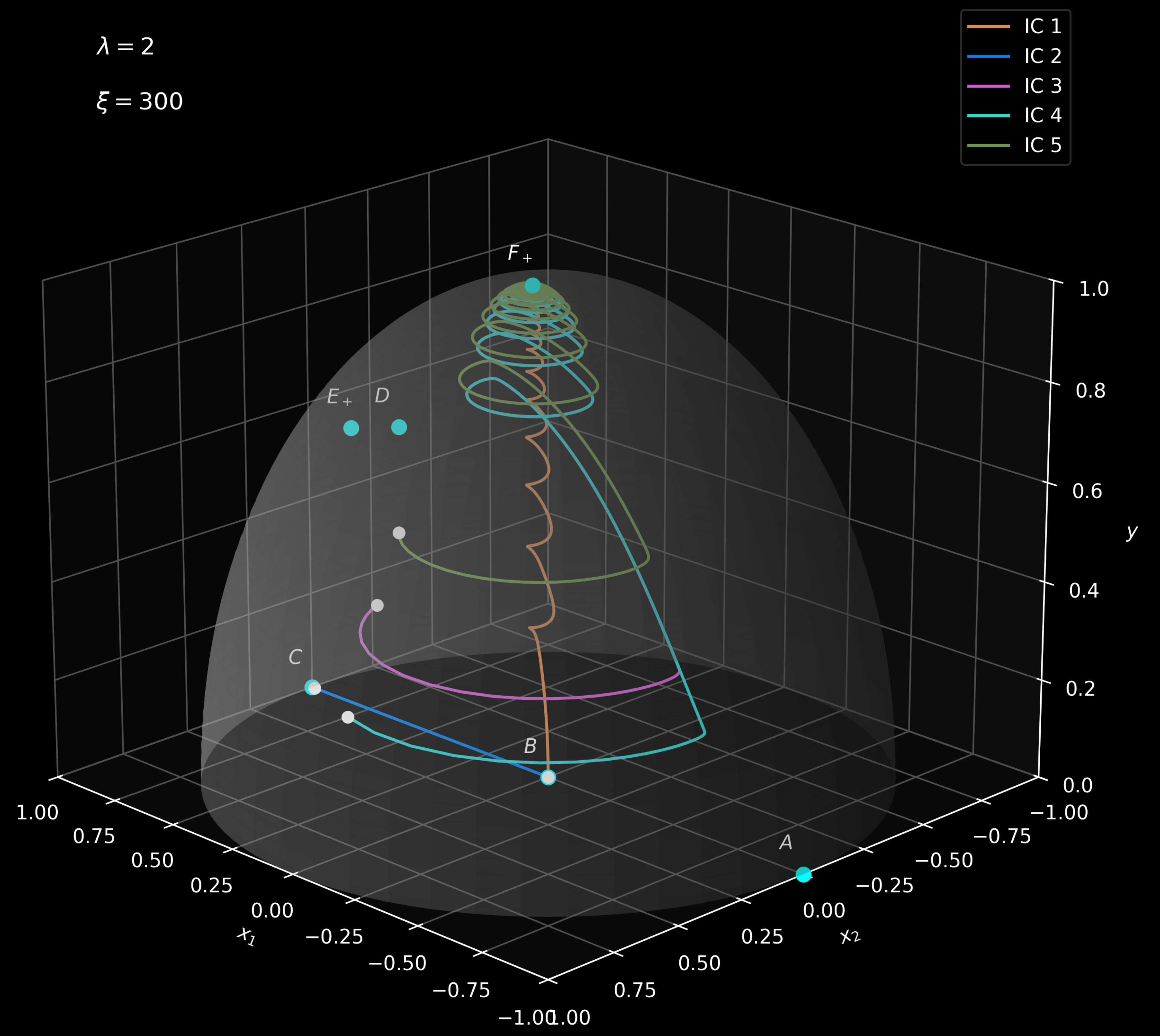
Dynamical system

Point	x_1	x_2	y	Ω_ϕ	ω_ϕ	Additional conditions
A	-1	0	0	1	1	None
B	0	0	0	0	indeterminate	None
C	1	0	0	1	1	None
D	$\frac{\sqrt{\frac{3}{2}\gamma}}{\lambda}$	0	$\frac{\sqrt{\frac{3}{2}\gamma(2-\gamma)}}{\lambda}$	$\frac{3\gamma}{\lambda^2}$	$-1+\gamma$	$(0<\lambda<\sqrt{6}\wedge 0<\gamma\leq\frac{\lambda^2}{3})\vee(\lambda\geq\sqrt{6})$
E_-	$\frac{\lambda}{\sqrt{6}}$	0	$-\sqrt{1-\frac{\lambda^2}{6}}$	1	$-1+\frac{\lambda^2}{3}$	$\lambda=\sqrt{6}$
E_+	$\frac{\lambda}{\sqrt{6}}$	0	$\sqrt{1-\frac{\lambda^2}{6}}$	1	$-1+\frac{\lambda^2}{3}$	$0<\lambda\leq\sqrt{6}$
F_-	$\frac{\sqrt{6}}{\lambda+2\xi}$	$-\frac{\sqrt{-6+\lambda^2+2\lambda\xi}}{\lambda+2\xi}$	$\frac{\sqrt{2\xi}}{\sqrt{\lambda+2\xi}}$	1	$\frac{\lambda-2\xi}{\lambda+2\xi}$	$(0<\lambda<\sqrt{6}\wedge\xi\geq\frac{6-\lambda^2}{2\lambda})\vee(\lambda\geq\sqrt{6})$
F_+	$\frac{\sqrt{6}}{\lambda+2\xi}$	$\frac{\sqrt{-6+\lambda^2+2\lambda\xi}}{\lambda+2\xi}$	$\frac{\sqrt{2\xi}}{\sqrt{\lambda+2\xi}}$	1	$\frac{\lambda-2\xi}{\lambda+2\xi}$	$(0<\lambda<\sqrt{6}\wedge\xi\geq\frac{6-\lambda^2}{2\lambda})\vee(\lambda\geq\sqrt{6})$

Physical fixed points of the two-field dynamical system with a barotropic fluid.

Dynamical system





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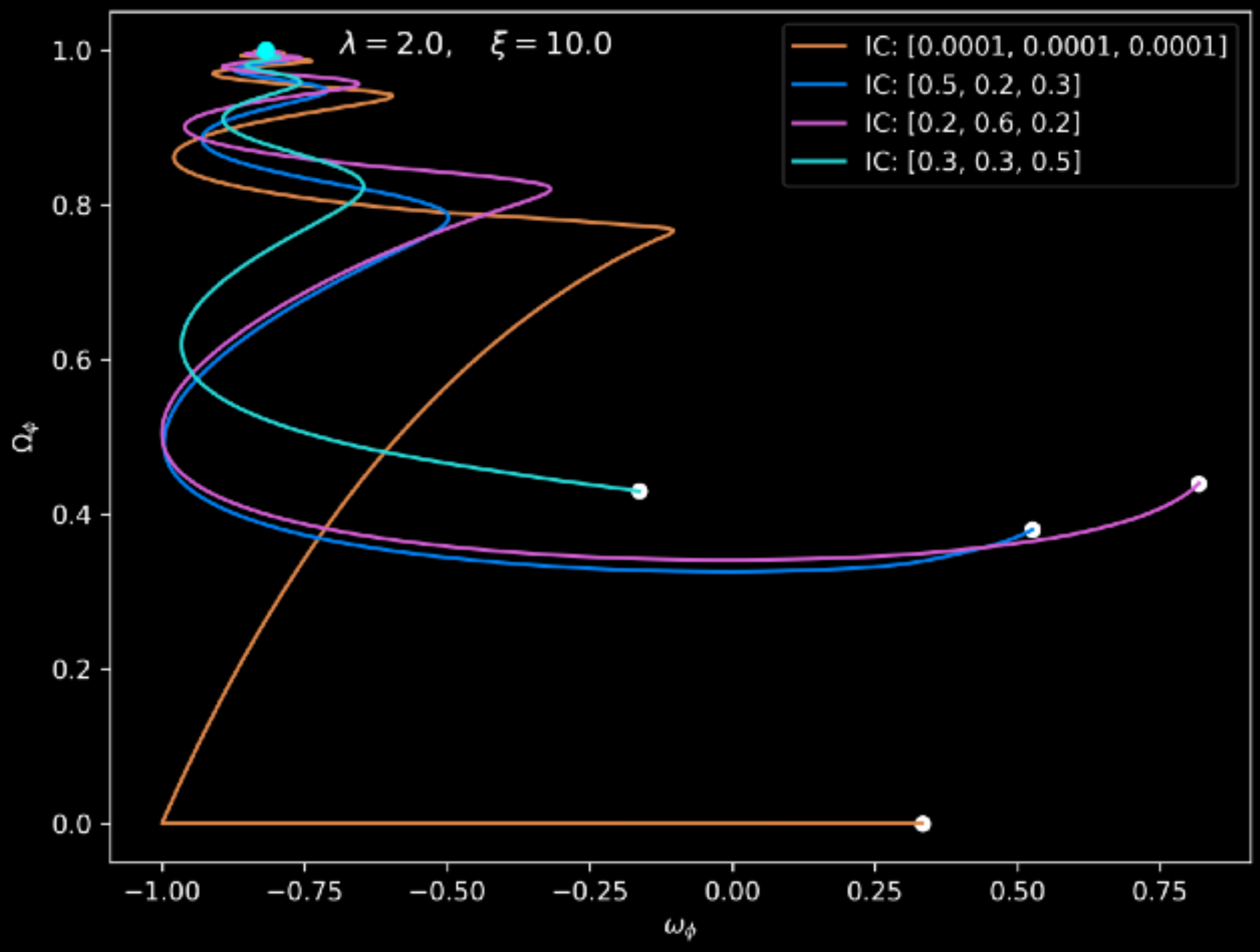
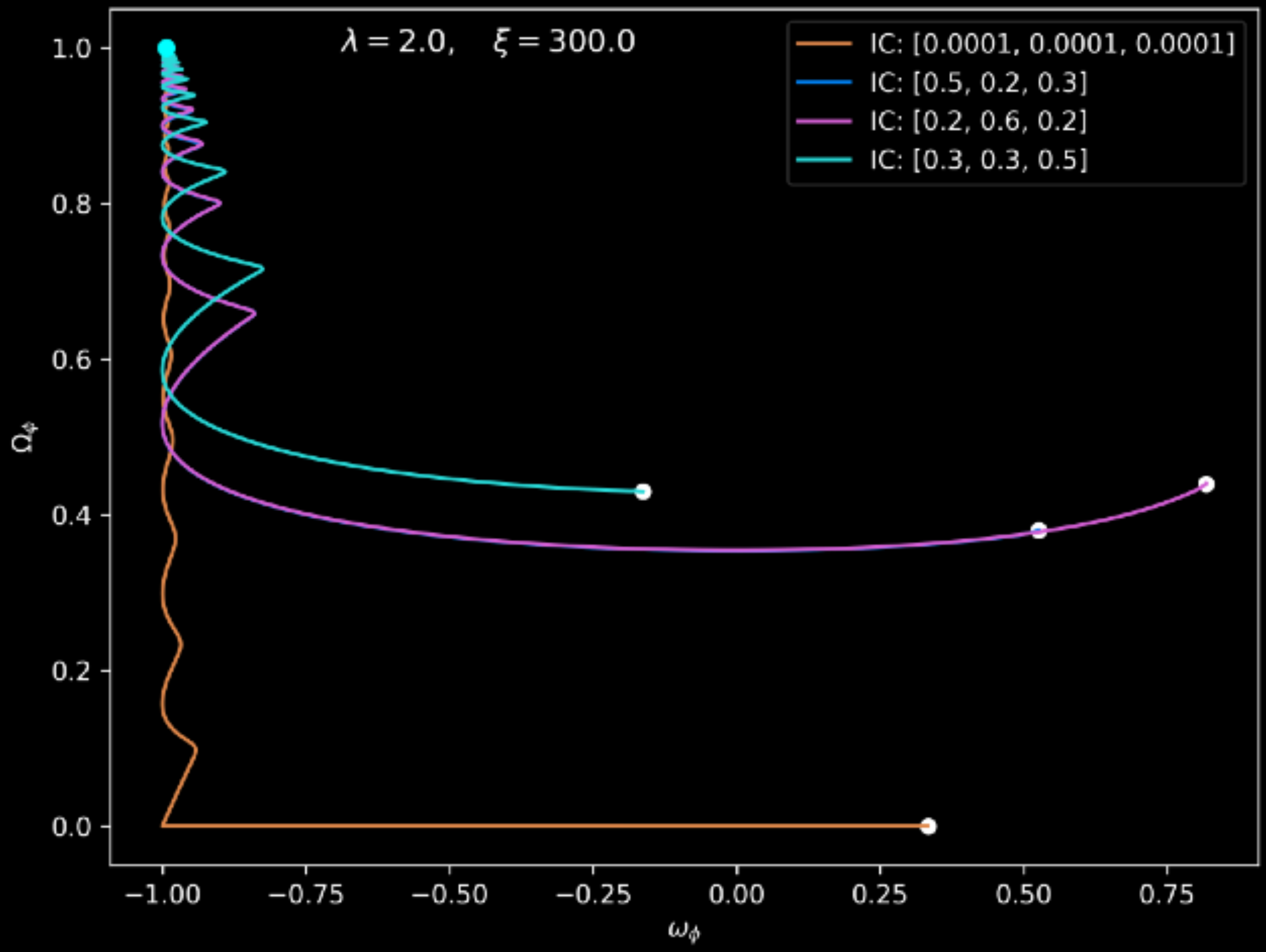
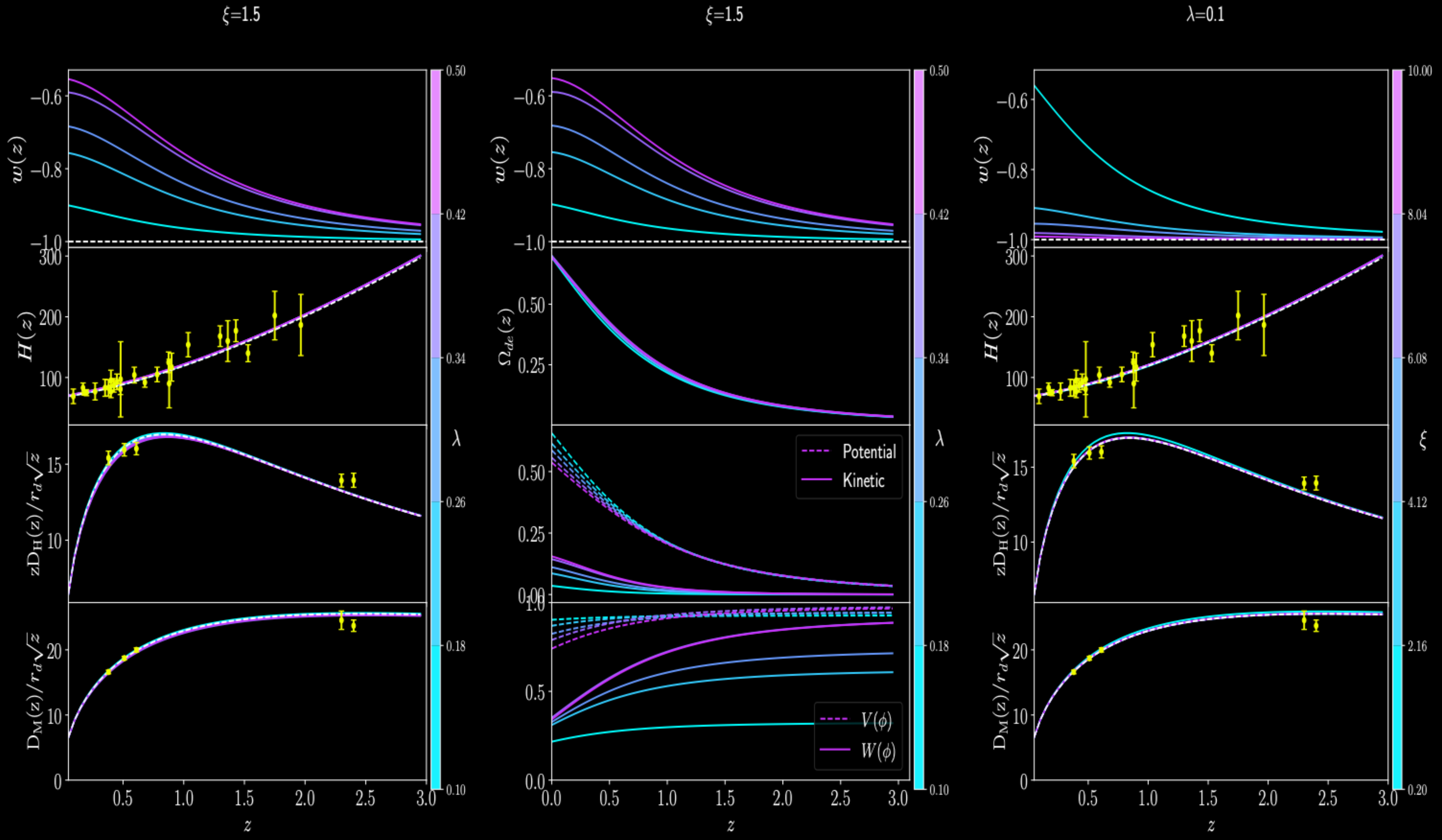




Numerical analysis



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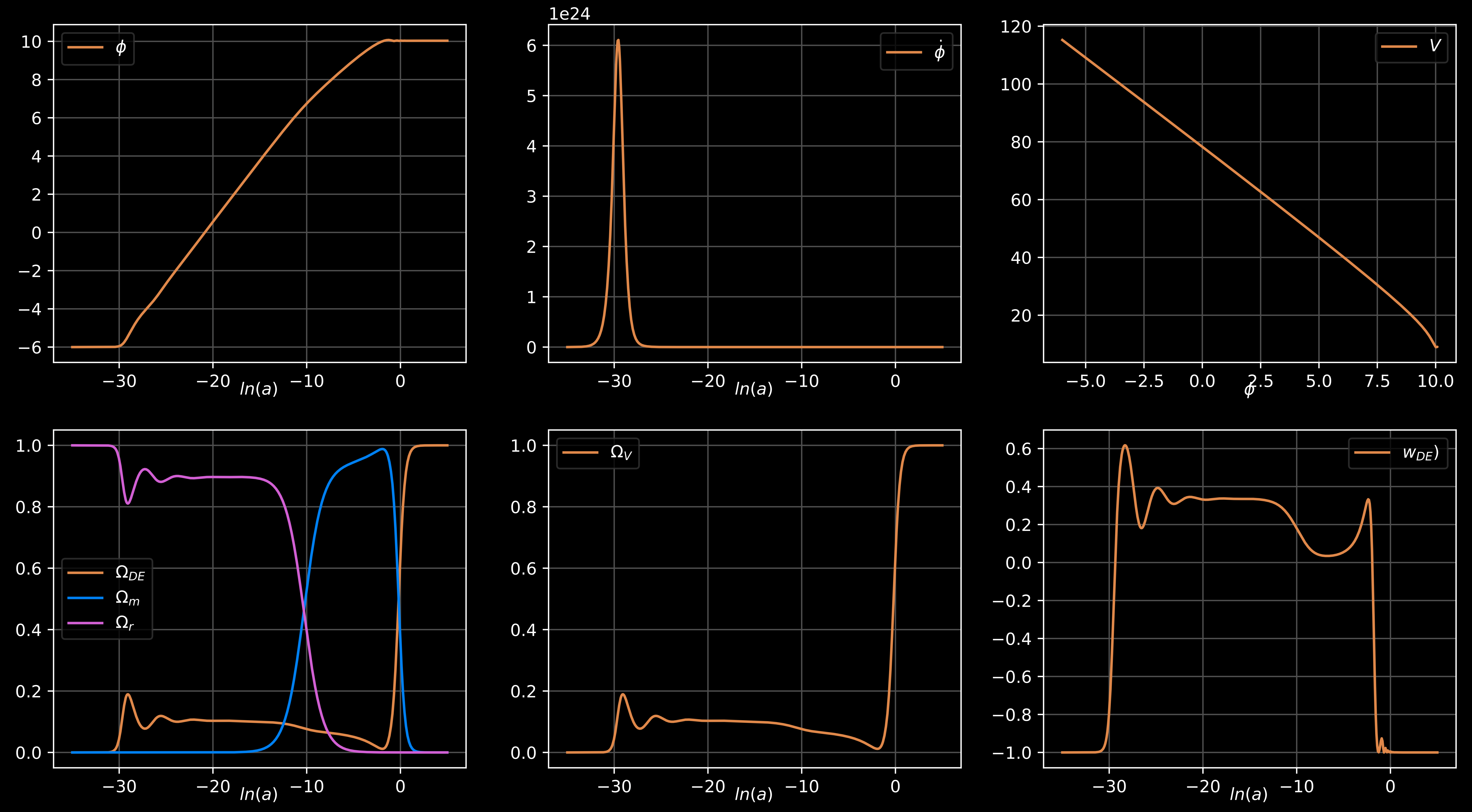
Numerical results



Conclusions

- **Multifield quintessence with exponential potential $V = e^{-\lambda\varphi_1}$ and KC $W = e^{\xi\phi_1}$**
  - Dynamical system analysis
  - Numerical analysis
  - In agreement with observations and swampland constraints.

- **Multifield quintessence with AS potential $V \sim \left[A + (\phi_1/M_p - B)^2\right] e^{-\lambda\phi_1/M_p}$, and KC $W = e^{\xi\phi_1}$**
  - Numerical analysis
  - In agreement with observations and swampland constraints.

Work in progress

- **Dynamical system analysis of the system with AS potential.**

- **Bayesian analysis**
  - Comparison of the exponential and AS potential to observations
  - Estimate what model is statistically preferred by experimental data.

- **Explanation of the crossing of the $w < -1$ line.**

Thank you

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DAMTP | University of Cambridge
(Applying for postdocs)