Rapidity Dependent Beam Functions

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Aims of Calculations

- Produce an addition piece to the leading jet P_T beam functions to incorporate an additional pseudo-rapidity cut

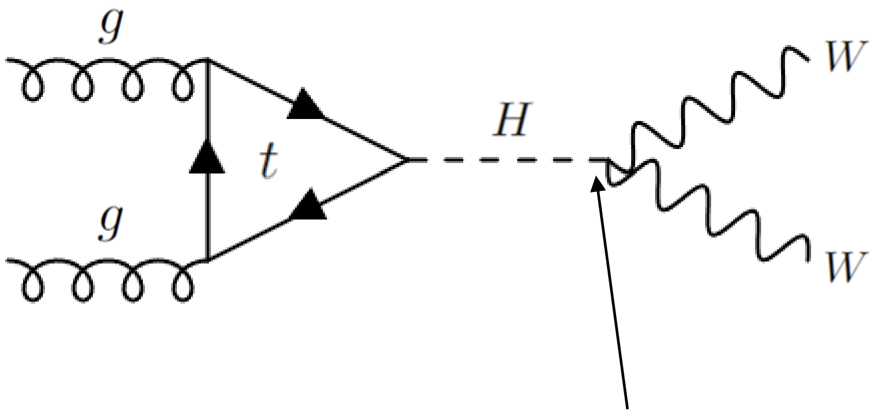
- Produce a correction to the resummed prediction for the 0-jet WZ production to alleviate tension between the theory and experimental result

Jet Vetoes

Class of observable used to separate final states by number of final state jets. A common jet veto is the leading jet transverse momentum P_{Tj}.

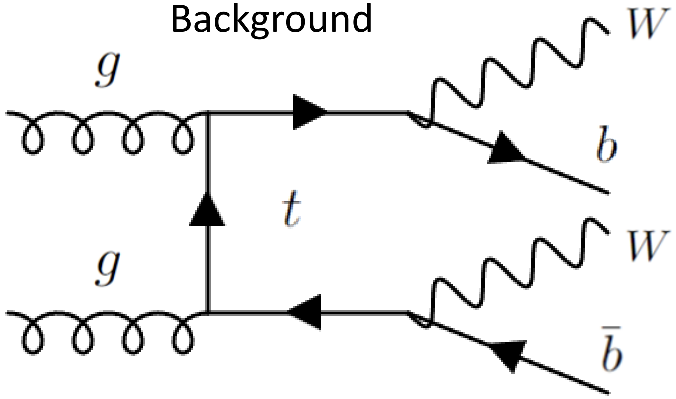
Can separate hard processes, cut out background events and study QCD radiation. For example:

Signal



Same initial states, final states differ by number of jets.

Background



Can be used to study Higgs coupling

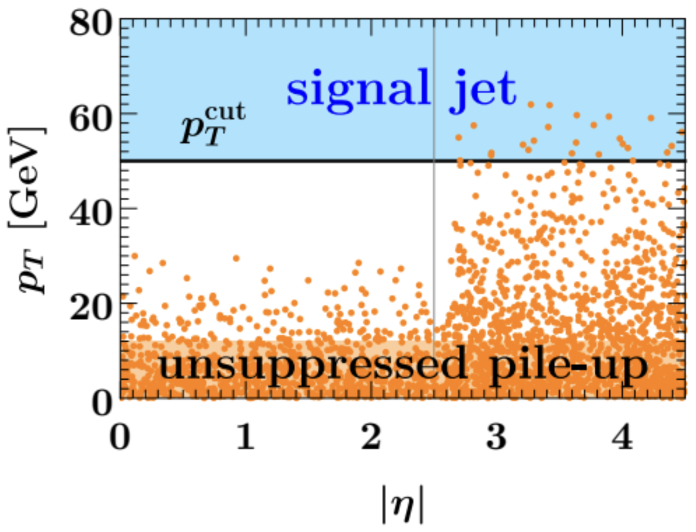
Can be removed with by imposing a veto on jets.

Rapidity Dependent Jet Vetoes

Useful to control the tightness of the P_T cut depending on kinematics of jet.

Due to a lack of tracking information, high rapidity, low P_T jets are hard to resolve experimentally.

A rapidity dependent jet veto allows tighter P_T cuts in central rapidites and looser P_T cuts at forward rapidities to remove sensitivity to these low P_T jets.

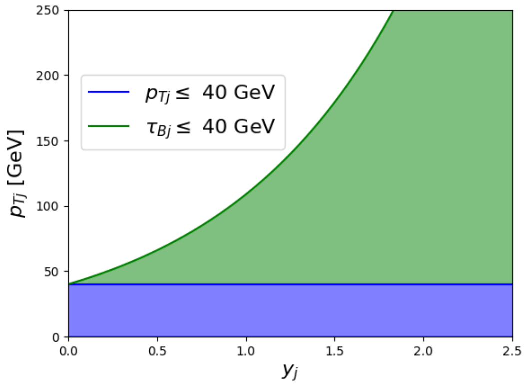


Michel, Pietrulewicz, Tackman, arXiv:1810.12911

Rapidity Dependent Jet Vetoes

The cut can be smoothy loosened via some continuous weighing function or via a discontinuous step at some given rapidity cut

$$\mathcal{T}_{Bj} = m_{Tj} e^{-|y_j - Y|}$$

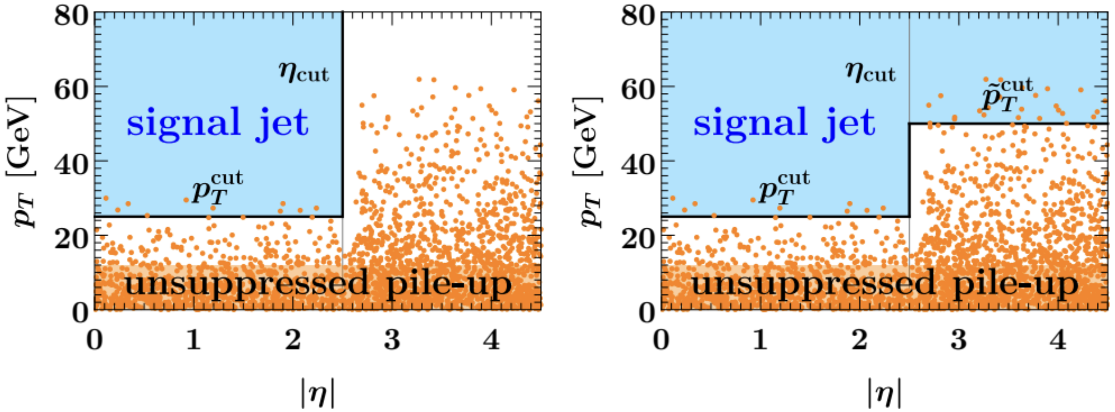


Tackmann, Walsh, Zuberi, arXiv:1206.4312

Gangal, Stahlhofen, Tackmann, arXiv:1412.4792

Gangal, Gaunt, Tackmann, Vryonidou, arXiv:2003.04323

Clark, Gangal, Gaunt, arXiv:2504.06353



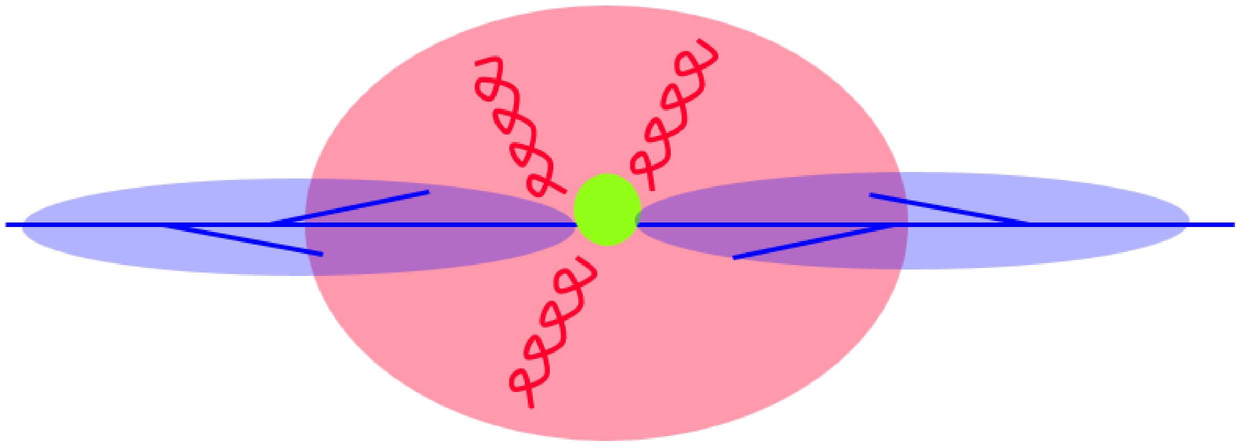
Michel, Pietrulewicz, Tackman, arXiv:1810.12911

SCET Factorisation

The below cut cross section for quark-initiated processes can be factorised in SCET as follows:

$$\sigma\left(p_{Tj} < p_T^{\rm cut}\right) = H_{qq'}\left(Q,\mu\right) B_q\left(Q, p_T^{\rm cut}, x_q, \mu, \nu\right) B_{q'}\left(Q, p_T^{\rm cut}, x_{q'}, \mu, \nu\right) S\left(p_T^{\rm cut}, \mu, \nu\right) + \mathcal{O}\left(\frac{p_T^{\rm cut}}{Q}\right)$$

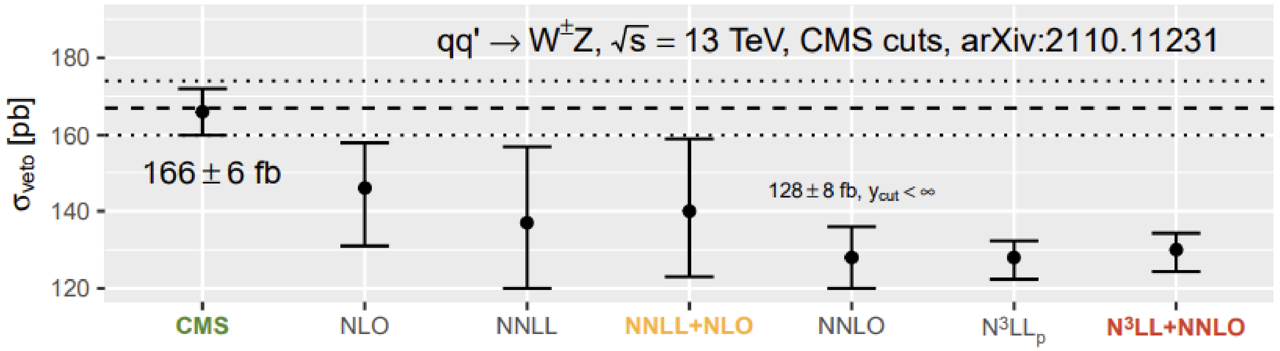
Tackmann, Walsh, Zuberi, arXiv:1206.4312



Motivation: WZ Production at the LHC

The factorisation formula does not include any rapidity cut in the jet definition

This can lead to mismatch between theory and experimental predictions where such a cut is often used



in this case, and that the use of y_{cut}-dependent beam functions is necessary to provide a reliable theoretical prediction. Overall, these results highlight the importance of using appropriate

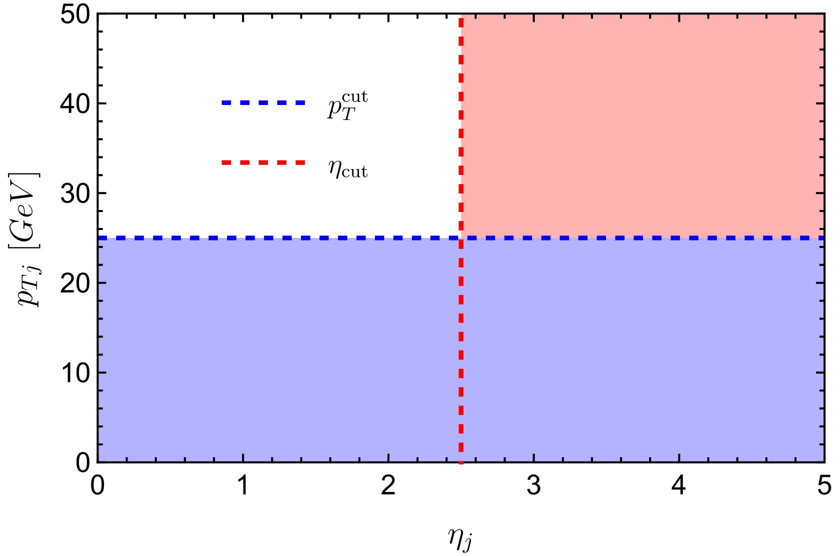
Campbell, Ellis, Neumann, Seth arXiv:2301.11768

Calculation: Overview

Most of the phase space is shared between the standard leading jet transverse momentum beam functions and the rapidity dependent beam functions

The calculation has been done at NLO (see arXiv:1810.12911), we aim to complete the NNLO calculation

$$B_q\left(p_T^{\rm cut}, \eta_{\rm cut}\right) = B_q\left(p_T^{\rm cut}\right) + \Delta B_q\left(p_T^{\rm cut}, \eta_{\rm cut}\right)$$



$$\begin{aligned}\mathcal{M}_{\Delta B}^{RR} =&\theta(\Delta R < R)\theta(\eta_t > \eta_{\rm cut})\theta(k_{t\perp} > p_T^{\rm cut}) \\ &+\theta(\Delta R > R)\left[\theta(k_{2T} < p_T^{\rm cut})\theta_{k_1} + \theta(k_{1T} < p_T^{\rm cut})\theta_{k_2} + \theta_{k_1}\theta_{k_2}\right]\end{aligned}$$

$$\theta_{k_i} = \theta(y_i > \eta_{\rm cut})\theta(k_{iT} > p_T^{\rm cut})$$

Calculation: Simple Addition

We further split the calculation by introducing a simplified measurement where all emissions are clustered regardless of the separation:

$$\mathcal{M}^{\text{Simple}} = \theta\left(\eta_t > \eta_{\text{cut}}\right)\theta\left(k_{t\perp} > p_T^{\text{cut}}\right) \qquad \mathcal{M}^{\mathcal{A}} = \mathcal{M}^{\text{Total}} - \mathcal{M}^{\text{Simple}}$$

This simple measurement contribution will contain all the real-virtual and virtual-virtual contributions and act as a ‘counter term’ for divergences from collinear splitting’s

Calculation: Simple Addition

Previously existing fully differential beam functions in the virtuality and transverse momentum of the incoming quark can be used to calculate the simple addition contribution:

$$B_i(t, x, \vec{k}_\perp^{\,2}, \mu) = \sum_j \int_x^1 \frac{\mathrm{d}z}{z} \mathcal{I}_{ij}\Big(t, \frac{x}{z}, \vec{k}_\perp^{\,2}, \mu\Big) f_j(z, \mu) \Big[1 + \mathcal{O}\Big(\frac{\Lambda_{\mathrm{QCD}}^2}{t}, \frac{\Lambda_{\mathrm{QCD}}^2}{\vec{k}_\perp^{\,2}}\Big)\Big]$$

Gaunt, Stahlhofen, arXiv:1409.8281

As the total transverse momentum and rapidity of all emissions is related to the virtuality and transverse momentum of the incoming quark, we can directly integrate over the phase space to find the simple addition:

$$I_G[f(t, \vec{k}_\perp^2)] = \theta\left(\frac{\zeta_{\mathrm{cut}}}{1+\zeta_{\mathrm{cut}}} > x\right) \int_{\frac{x}{1-x}(p_T^{\mathrm{cut}})^2}^{\frac{1-x}{x}(\zeta_{\mathrm{cut}} p_T^{\mathrm{cut}})^2} \mathrm{d}t \int_{(p_T^{\mathrm{cut}})^2}^{\frac{1-x}{x}t} \mathrm{d}(\vec{k}_\perp^2) f(t, \vec{k}_\perp^2)$$

$$\Delta B_q^{\mathrm{Simple}}\left(p_T^{\mathrm{cut}}, \zeta_{\mathrm{cut}}, x_q, \mu\right) = I_G\left[B_q\left(t, \vec{k}_\perp^{\,2}, x_q, \mu\right)\right] \qquad \zeta_{\mathrm{cut}} = \frac{x_q E_{\mathrm{CM}}}{p_T^{\mathrm{cut}}} e^{-\eta_{\mathrm{cut}}}$$

Calculation: Renormalization

By considering the renormalization of all beam functions involved, a total expression for the addition can be found:

$$\begin{aligned}\Delta\mathcal{I}_{ij}^{(2)}(\eta_{\mathrm{cut}}, p_T^{\mathrm{cut}}, z) &= I_G\left[\mathcal{I}_{ij}^{(2)}(\vec{k}_\perp^2, t, z)\right] + I_{\mathcal{A}}\left[\mathcal{M}^{\mathcal{A}}\mathcal{A}_{ij}^{(2)}\right] + I_{\mathcal{A}}\left[\mathcal{M}^{y\to\eta}\mathcal{A}_{ij}^{(2)}\right]\\ &+2\sum_k I_G\left[\mathcal{I}_{ik}^{(1)}(\vec{k}_\perp^2, t, z)\otimes_z f_{k/j}^{(1)}(z)\right] + I_G\left[Z_i^{B(1)}(t,z)\otimes_t \mathcal{I}_{ij}^{(1)}(\vec{k}_\perp^2, t, z)\right]\\ &-2\sum_k \Delta\mathcal{I}_{ik}^{(1)}(\eta_{\mathrm{cut}}, p_T^{\mathrm{cut}}, z)\otimes_z f_{k/j}^{(1)}(z) - Z_i^{B(1)}(p_T^{\mathrm{cut}}, z)\times\Delta\mathcal{I}_{ij}^{(1)}(\eta_{\mathrm{cut}}, p_T^{\mathrm{cut}}, z)\end{aligned}$$

Most terms can be calculated in a similar manner to the simple addition; the amplitude terms are the most interesting

Calculation: Amplitude Terms

The remaining terms can be calculating using known double-real emission amplitudes (see arXiv:1401.5478, arXiv:1405.1044)

These terms require multiple subtraction term to remove divergences

The goal it to calculate these terms semi-numerically

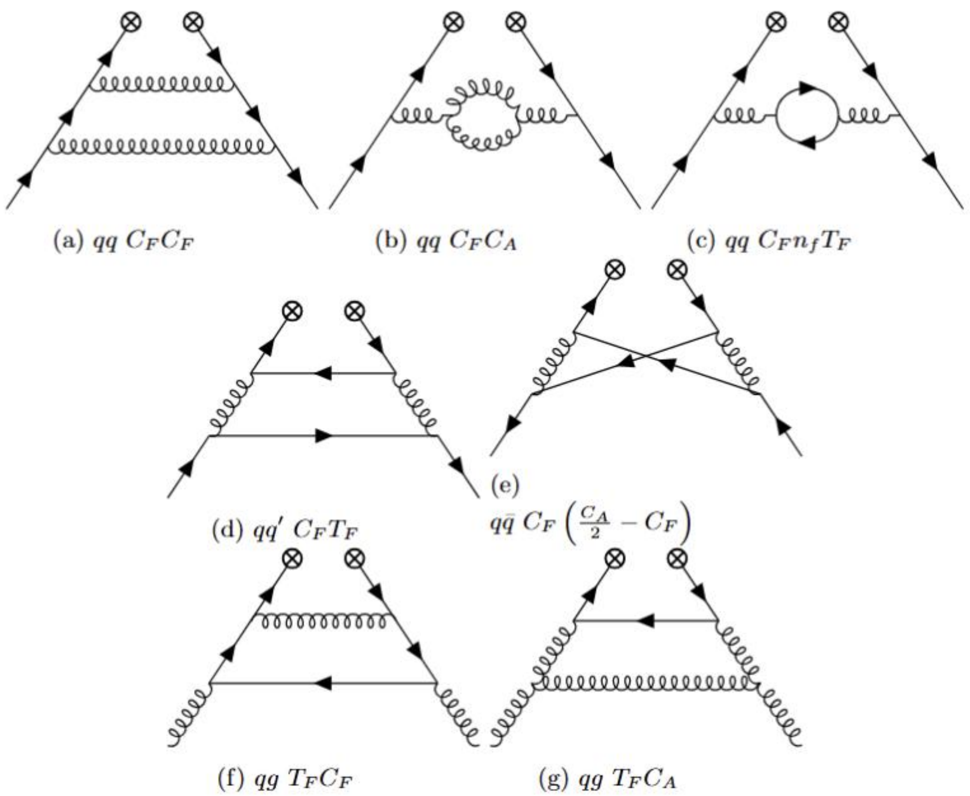


Figure made by Emmet Byrne

Calculation: Amplitude Terms

Subtraction terms are calculated by analytically integrating the measurement and amplitude simplified in various divergent limits

$$I_{\mathcal{A}}[\mathcal{M}^{\mathcal{A}}\mathcal{A}_{ij}^{(2)} - \sum_{\hat{\mathrm{L}}} \hat{L}[\mathcal{M}^{\mathcal{A}}\mathcal{A}_{ij}^{(2)}]] + I_{\mathcal{A}}[\sum_{\hat{\mathrm{L}}} \hat{L}[\mathcal{M}^{\mathcal{A}}\mathcal{A}_{ij}^{(2)}]]$$

Dimensional regularization and a rapidity regulator are used to extract the poles

$$\begin{aligned} C_2^- :& \quad k_{2T} \sim \lambda^1, \quad k_2^- \sim \lambda^0\,, \\ C_2^+ :& \quad k_{2T} \sim \lambda^1, \quad k_2^- \sim \lambda^2\,, \\ S_2 :& \quad k_{2T} \sim \lambda^1, \quad k_2^- \sim \lambda^1\,. \end{aligned}$$

The integration of the total minus the subtracted terms will now be finite and easily done numerically

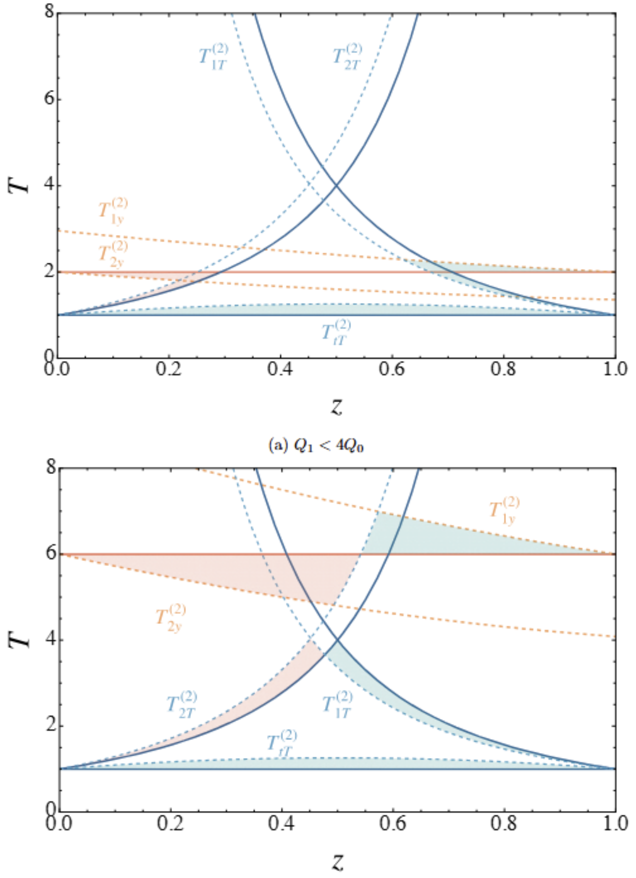
$$d = 4 - 2\epsilon, \left(\frac{\nu}{k_2^-}\right)^{\eta} \to \frac{1}{\epsilon^n}, \frac{1}{\eta^m}, \frac{1}{\epsilon^n \eta^m}$$

Calculation: Jet Radius Dependence

Calculating the full jet radius dependence analytically is beyond the scope of this calculation

By taking small R limits of the measurement and amplitudes, the log and leading powers of R can be extracted

The arrangements of different contributing regions in this calculation get very complex



(a) $Q_1 < 4Q_0$

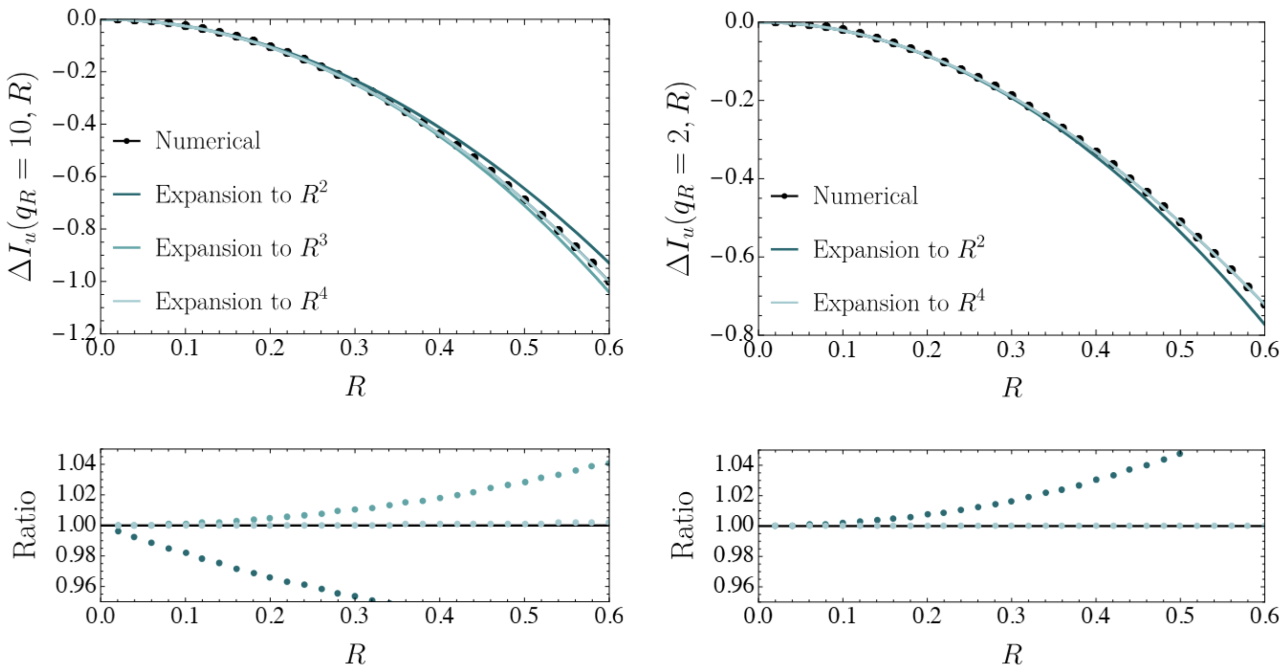
(b) $Q_1 > 4Q_0$

Figures made by Emmet Byrne

Calculation: Jet Radius Dependence

The log R term along with all terms up to R^2 have been calculated analytically (and up to R^4 in some channels)

By generating results at some small R to minimise unknown sub-leading terms, these leading analytic terms can be used to extrapolate to different R values



Figures made by Emmet Byrne

Calculation: Numerical Fitting

The remaining numerical parts of the calculation depend on 2 variables:

$$\{z, \zeta_{\text{cut}}\}$$

Numerical grids have been generated in these variables, which can be directly used with the analytic terms to produce predictions for the total addition to the beam function

There is ongoing work to produce an accurate 2-D fit over these variables to allow easier use of the beam functions

Global Slicing

The cancelation of all poles in the calculation is a strong check all terms have been included in the calculation

This however does not check the value of the final finite terms

To check these terms, we perform a global slicing check using a similar SCET factorisation formula for the rapidity dependent beam functions:

$$\sigma\left(p_{Tj} < p_T^{\mathrm{cut}}, \eta_{\mathrm{cut}}\right) = H_{q\bar{q}}(Q,\mu) B_q\left(Q, p_T^{\mathrm{cut}}, x_q, R, \mu, \nu\right) B_{\bar{q}}\left(Q, p_T^{\mathrm{cut}}, \eta_{\mathrm{cut}}, x_{\bar{q}}, R, \mu, \nu\right) S\left(p_T^{\mathrm{cut}}, R, \mu, \nu\right) + \mathcal{O}\left(\frac{p_T^{\mathrm{cut}}}{Q}, e^{-\eta_{\mathrm{cut}}}\right)$$

Michel, Pietrulewicz, Tackman, arXiv:1810.12911

Global Slicing

This below cut cross section can be combined with an above cut to obtain a prediction for the inclusive cross section for a test process (e.g. on-shell Z production)

$$\sigma_{\mathrm{FO}} = \sigma_{\mathrm{Below}}^{\mathrm{SCET}}\left(p_T^{\mathrm{cut}}\right) + \sigma_{\mathrm{Above}}\left(p_T^{\mathrm{cut}}\right) + \mathcal{O}\left(\frac{p_T^{\mathrm{cut}}}{Q}\right)$$

This above cut term can be easily calculated using programs such as MadGraph as it only contains NLO type divergences

$$\sigma_{\mathrm{FO}} = \sigma_{\mathrm{Below}}^{\mathrm{SCET}}\left(p_T^{\mathrm{cut}}, \eta_{\mathrm{cut}}\right) + \sigma_{\mathrm{Above}}\left(p_T^{\mathrm{cut}}, \eta_{\mathrm{cut}}\right) + \mathcal{O}\left(\frac{p_T^{\mathrm{cut}}}{Q}, e^{-\eta_{\mathrm{cut}}}\right)$$

$$\Delta\sigma_{\mathrm{Below}}^{\mathrm{SCET}}\left(p_T^{\mathrm{cut}}, \eta_{\mathrm{cut}}\right) = -\Delta\sigma_{\mathrm{Above}}\left(p_T^{\mathrm{cut}}, \eta_{\mathrm{cut}}\right) + \mathcal{O}\left(\frac{p_T^{\mathrm{cut}}}{Q}, e^{-\eta_{\mathrm{cut}}}\right)$$

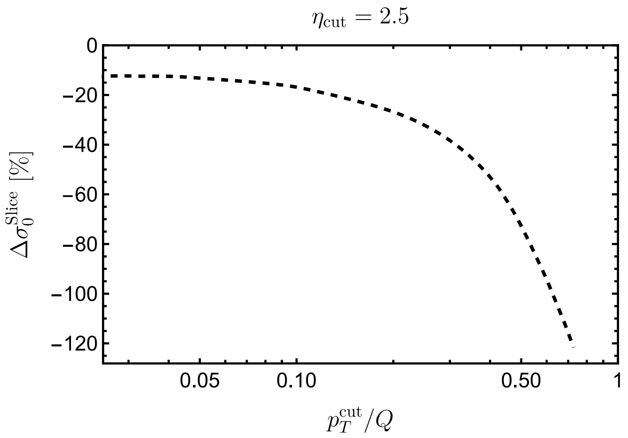
Performing this calculation with the standard and rapidity dependent beam functions allows an expected value for the addition term to be found

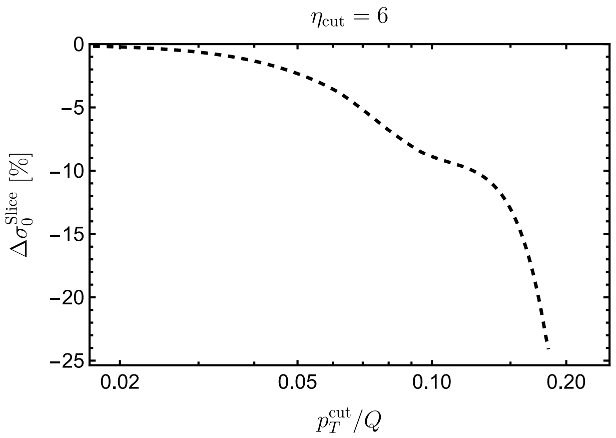
Global Slicing

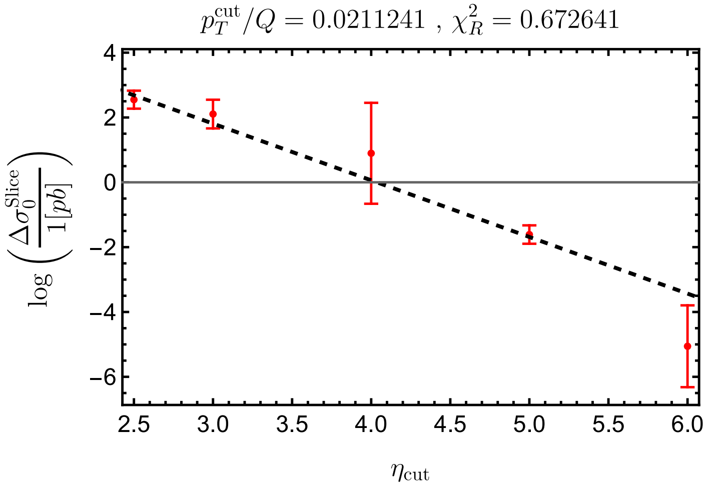
By choosing particular cut values, the power corrections can be reduced to check the below cut prediction and therefore the calculation of the beam functions

The size of deviation can also be compared to the expected size of the power corrections

The behaviour expected from the power corrections is seen validating the beam function calculation!







What’s Next?

Calculation of gluon rapidity dependent beam functions

Calculation of finite step rapidity dependent beam function

Calculation of addition to the WZ cross section to incorporate the rapidity cut into the jet definition

Summary

Successfully calculate the quark rapidity dependent beam function at NNLO

Working semi-analytic + numerical implementation with a final 2 variable fit in progress to simplify use

Predictions in WZ, gluon rapidity dependent beam functions and finite step veto all to come in the future

Any Questions?