Supercooling in Confining Theories

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Durham

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Based on work with Prateek Agrawal, Vazha Loladze, John March-Russell, and Mario Reig

arXiv: 2508.10091 [hep-ph]
arXiv: 2504.00199 [hep-ph] (JHEP 10 (2025) 066)

Flow of today's discussion

- Motivation
- Historical Overview
  - ⋆ Lattice
  - ⋆ Holography
- Computations in Softly-broken Supersymmetric Model
- Results
- Concluding Remarks

Motivation

1. Cosmological first order phase transitions proceed with bubble nucleation— expansion and collision and emit gravitational waves.

    i. The rate of bubble nucleation is

    S. Coleman, Phys. Rev. D 15 (1977) 2929
    C.G.Callan, S.Coleman, Phys.Rev.D 16 (1977) 1762, A.D.Linde, Nucl.Phys.B 216 (1983) 42

    $$\Gamma_{\text{nuc}} \sim e^{-S_b} \longrightarrow \text{Euclidean Bounce}$$

    ii. Gravitational Wave detection— LISA— near Electroweak scale

    ★ Cosmologically, the supercooling is related to duration of phase transition completion

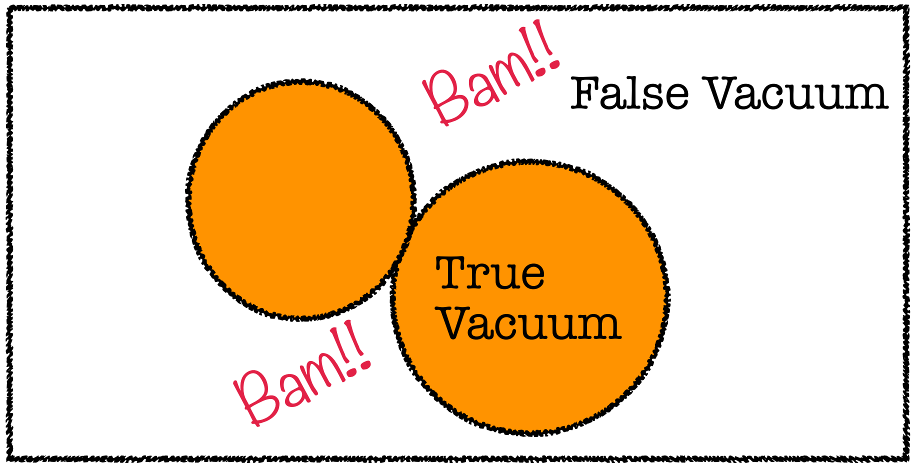
    ★ Large supercooling— stronger GWs

    ★ Chance to study transitions in various Beyond the Standard Model (BSM) physics

    C.Grojean, G. Servant, Phys.Rev. D 75 (2207) 043507
    LISA Cosmology Working Group, JCAP 10 (2024) 020; ...

2. Understand intrinsic aspects of thermal phase transitions in strongly-coupled theories— connection to lattice data (interpretation and predictions)

In this talk, we will focus on transition in $SU(N)$ gauge theory



$SU(N)$ on the Lattice

- Calculate thermodynamic quantities near

  $T = T_{\rm cr}$

  B. Lucini, M. Teper, and U. Wenger, JHEP 02 (2005) 033,

  A. Salami, T. Rindlisbacher, and K. Rummukainen, 2502.01396 [hep-lat]

Latent Heat: $Q_{\rm h} = \left(0.7731 - 0.959(58)/N^2\right)^4 N^2 T_{\rm cr}^4$,

Wall tension: $\sigma_{\rm BW} = \left(0.0189(11) - 0.190(19)/N^2\right) N^2 T_{\rm cr}^3$

- Thin wall bounce action:

Small number

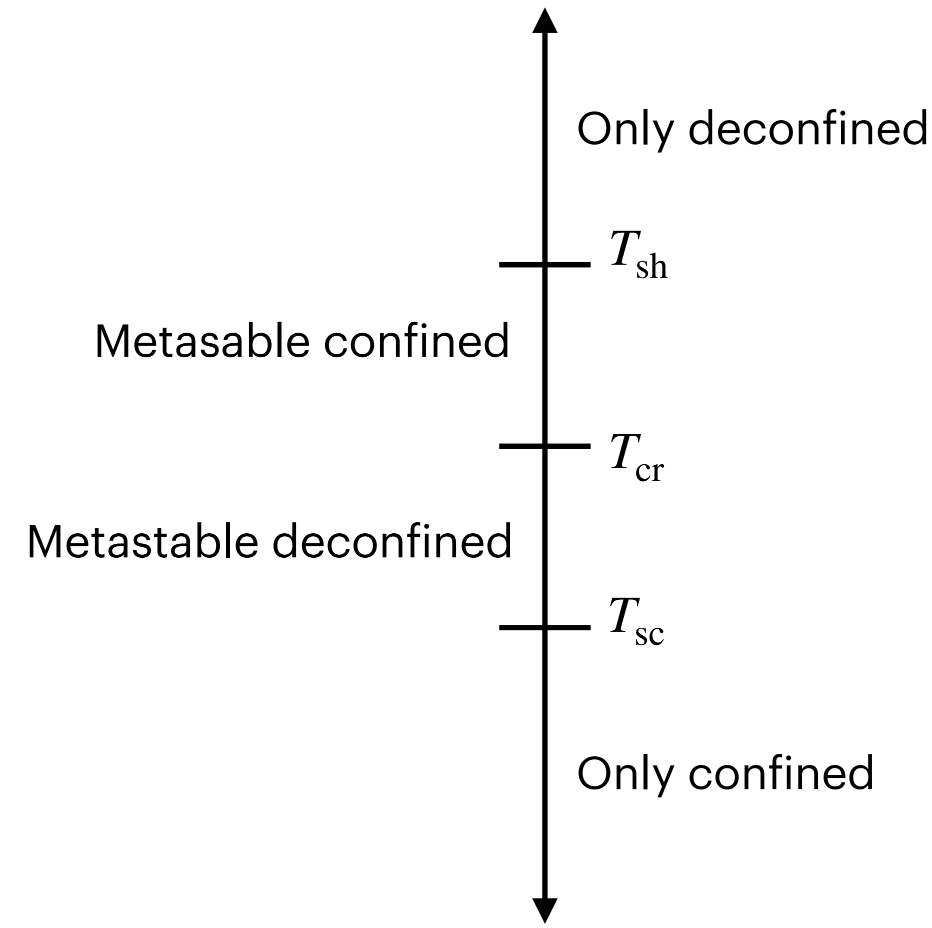
$$S_{\rm b} = \frac{16\pi}{3}\frac{\sigma_{\rm BW}^3}{Q_{\rm h}^2 T_{\rm cr}}\frac{1}{\epsilon^2} \approx 8.86\times 10^{-4}\frac{N^2}{\epsilon^2},$$

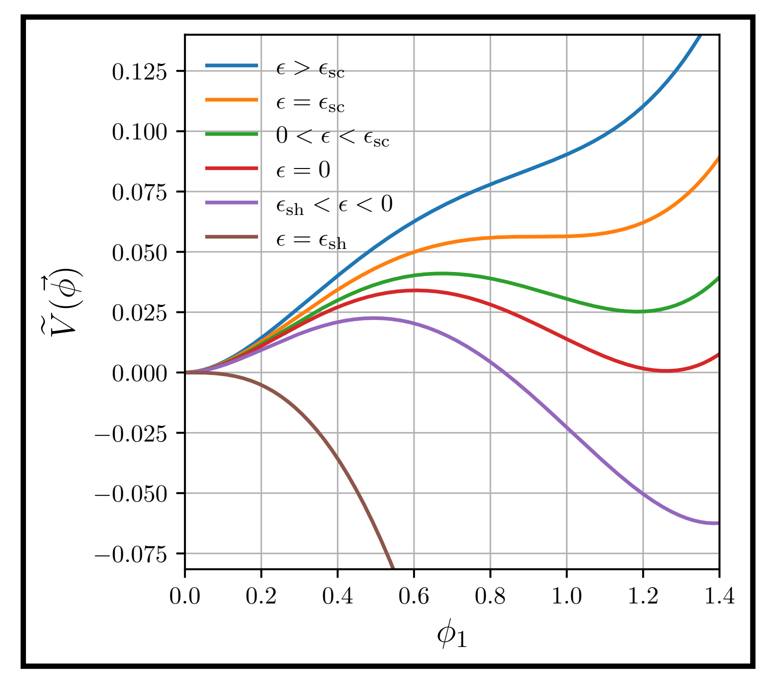
Supercooling

$$\epsilon = 1 - \frac{T}{T_{\rm cr}}$$

- For cosmological transitions near EW

  $\epsilon_{\rm n} \approx 2.5\times 10^{-3} N$





Example plot

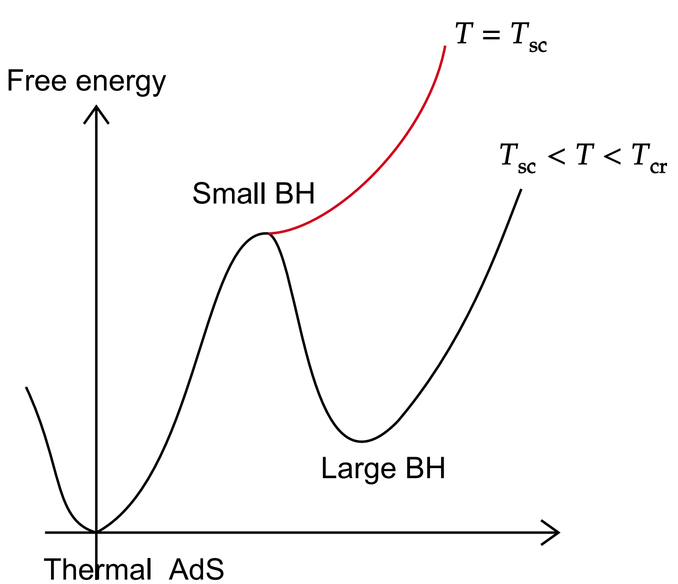
Hints from Holography

- Indirect way to learn about strongly-coupled gauge theories — $\mathrm{AdS}_{d+1} - \mathrm{QFT}_d$.

J.Maldacena Int.J.Theor.Phys. 38, 1999

- Below $T < T_{sc}$ no black hole solution exists— has to go to Thermal AdS_{d+1}

$$\epsilon_{sc} = 1 - \frac{\sqrt{d(d-2)}}{d-1} \approx \frac{1}{2d^2}$$



$$\begin{tikzcd}
\text{Deconfined} \arrow[r, leftrightarrow] \arrow[d] & \text{Black hole/ brane} \arrow[d, "\text{Hawking-Page Transition}"] \\
\text{Confined} \arrow[r, leftrightarrow] & \text{Thermal AdS}
\end{tikzcd}$$

$$\mathcal{Z} \approx \sum_{\mathrm{geo}} e^{-S_{\mathrm{geo}}}$$

S. W. Hawking and D. N. Page, Commun.Math.Phys. 87, 1983
E. Witten, Adv.Theor.Math.Phys. 2, 1998

- maximal supercooling of *few percent*
- Observed in other holographic setups

A.Buchel, Nucl.Phys.B 820, '09
Y.Bea et.al, arXiv: 2112.15478 [hep-th]
E. Morgate, N.Ramburg, P.Schwaller, Phys.Rev.D 107, '23

Note: Some setups do have large supercooling due to conformal behaviour (for example Randall-Sundrum models)

P.Creminelli, A.Nicolis, R.Rattazzi, JHEP 03 (2002) 051
P. Agrawal, GRK, V.Loladze, M.Reig JHEP 10 (2025) 066

$SU(N)$ Yang-Mills

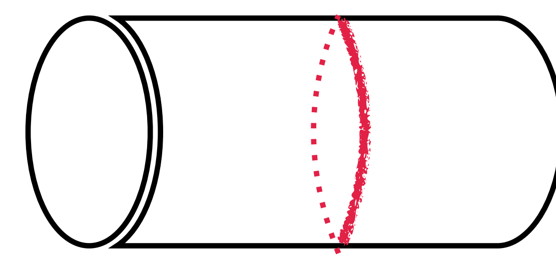
Formology

4d theory: $\mathbb{Z}_N^{(1)}$ one-form global symmetry

D. Gaiotto et.al. JHEP 05 (2017) 091

$\mathbb{R}^3 \times S^1$

Thermal Circle



$T^{-1} = L$

3d theory

$$\Omega(x) = \mathrm{Tr}\left(\mathscr{P}\exp\left(i\oint A_4(x, x_4)\ dx_4\right)\right) = \mathrm{Tr}\exp\left(iA_4(x)L\right)$$

$\mathbb{Z}_N^{(1)}$ one-form global symmetry

$\mathbb{Z}_N^{(0)}$ effective zero-form global symmetry (Polyakov Loops, Ω)

Order parameter

- Deconfined $\langle\Omega\rangle \neq 0$— Spontaneously broken
- Confined $\langle\Omega\rangle = 0$— Restored

Supersymmetric Setup

$\mathcal{N} = 1$ super Yang-Mills on $\mathbb{R}^3 \times S^1$

E. Poppitz, T.Schafer, M.Unsal, JHEP 10 (2012) 115, JHEP 03 (2013) 087

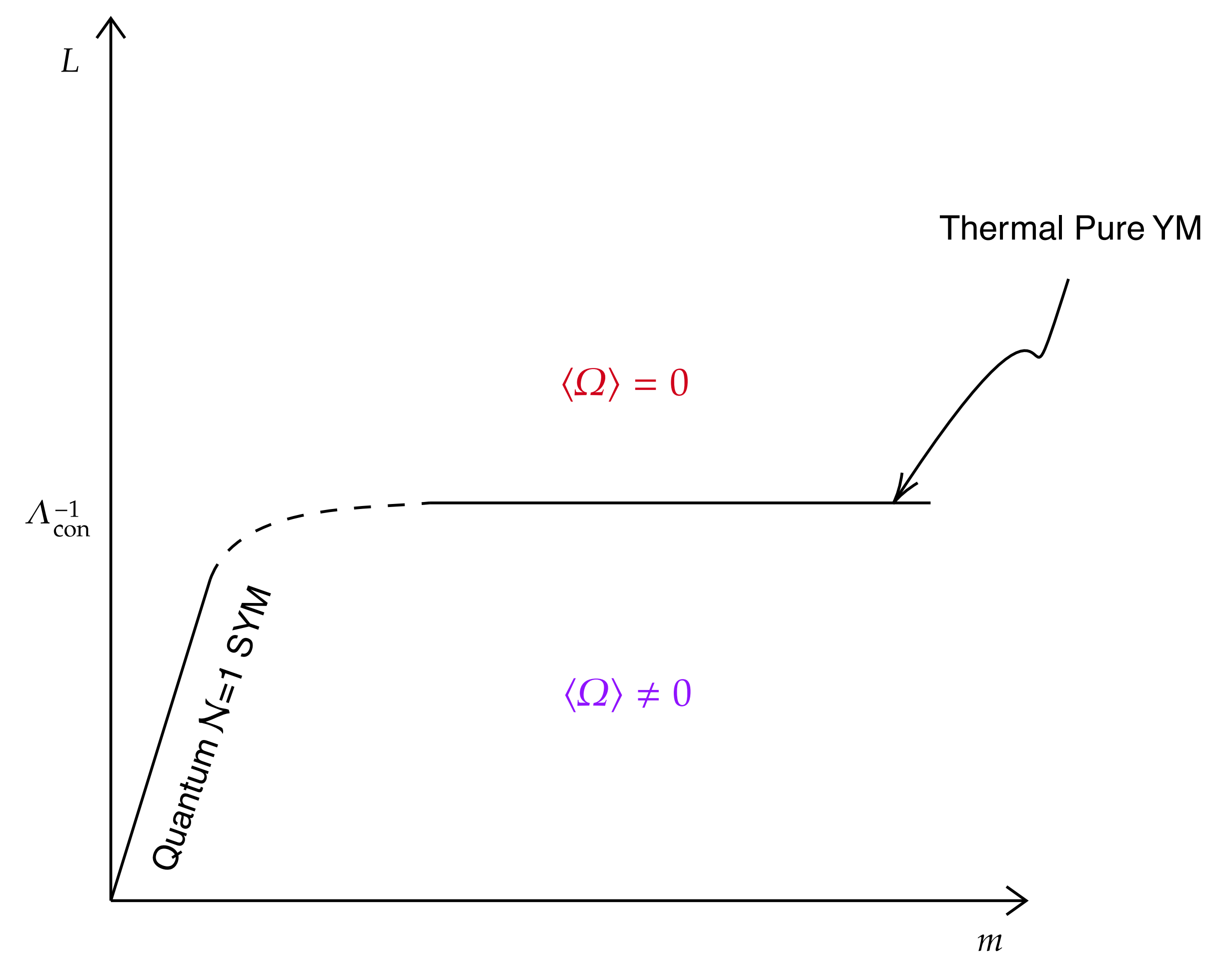
(Gauge Bosons + gauginos)

↓

Break softly using gaugino mass term "$m\lambda\lambda$".

↓

Considering theory with the circle size $L^{-1} \gg \Lambda_{\rm con}$.



In the $L^{-1} \gg \Lambda_{\rm con}$, we can study the low energy 3d theory in the weak coupling.

Setup continues

- If $A_4(x)$ gets a non-zero vacuum expectation value— This Higgses the theory from $SU(N) \to U(1)^{N-1}$

- Work in the gauge with fields lying in Cartan of $SU(N_c)$ i.e.
$\overrightarrow{A}_4 = A_4^a H^a$

- Parameterise the vacuum expectation value of A_4 as

Weyl Vector—

Centre Symmetric point

$$\overrightarrow{A}_4 = \frac{2\pi}{NL}\vec{\rho} + \frac{g_4^2}{4\pi NL}\overrightarrow{\phi}$$

$N-1$ fields

- The Polyakov loop is defined as

$$\Omega(x) = \mathrm{Tr}\left(\mathscr{P}\exp\left(i\oint A_4(x, x_4)\ dx_4\right)\right) = \mathrm{Tr}\exp\left(iA_4(x)L\right)$$

Setup still continues

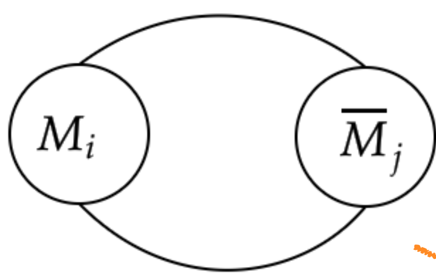
- If $\overrightarrow{A}_4$ is stabilised at the centre symmetric point when $\overrightarrow{\phi} = 0$

$$\langle \Omega \rangle = 0$$

- This shows that there can be a $\langle \Omega \rangle = 0 \leftrightarrow \langle \Omega \rangle \neq 0$ transition
- Need to study the potential for $\overrightarrow{\phi}$ — dynamical quantities parametrising the transition
- Different from studies related to Polyakov loop effective models.

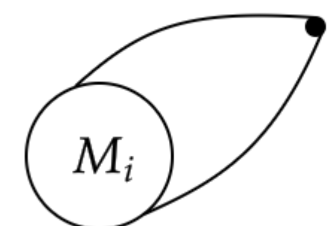
The Potential

In the soft breaking— leading contribution to potential comes from non-perturbative physics— Monopole instantons and Bions (Monopole instanton— Anti-monopole instanton bound state)



Magnetic bions $M_i\overline{M}_j$

Neutral bions $M_i\overline{M}_i$



$$V\left(\vec{\phi}\right) = V_0 \sum_{i=1}^{N_c} e^{-\vec{\alpha}_i\cdot\vec{\phi}}\left(\overbrace{e^{-\vec{\alpha}_i\cdot\vec{\phi}} - e^{-\vec{\alpha}_{i+1}\cdot\vec{\phi}}}^{\text{Centre-Stabilizing}} - \kappa\frac{4\pi^2}{N^2}\frac{L_{\mathrm{cr}}^2}{L^2}\right)$$

Roots of $SU(N)$

$SU(2)$ Example

- Although $SU(2)$ has second order phase transition, it is useful to understand the structure.
- If $A_4^3(x)$ gets a vev then $SU(2) \to U(1)$

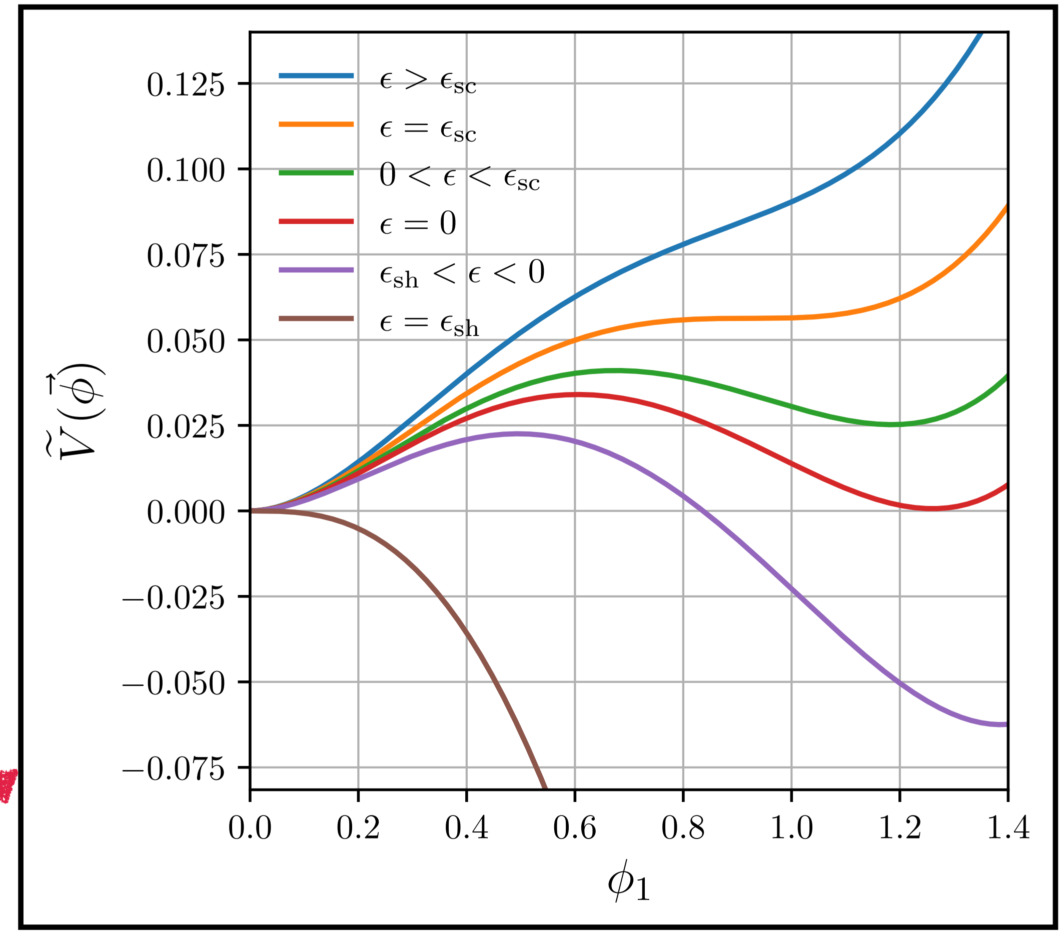
$$\Omega = \text{Tr}\begin{pmatrix} e^{-iA_4^3 L/2} & 0 \\ 0 & e^{iA_4^3 L/2} \end{pmatrix} = 2\cos\left(\frac{A_4 L}{2}\right)$$

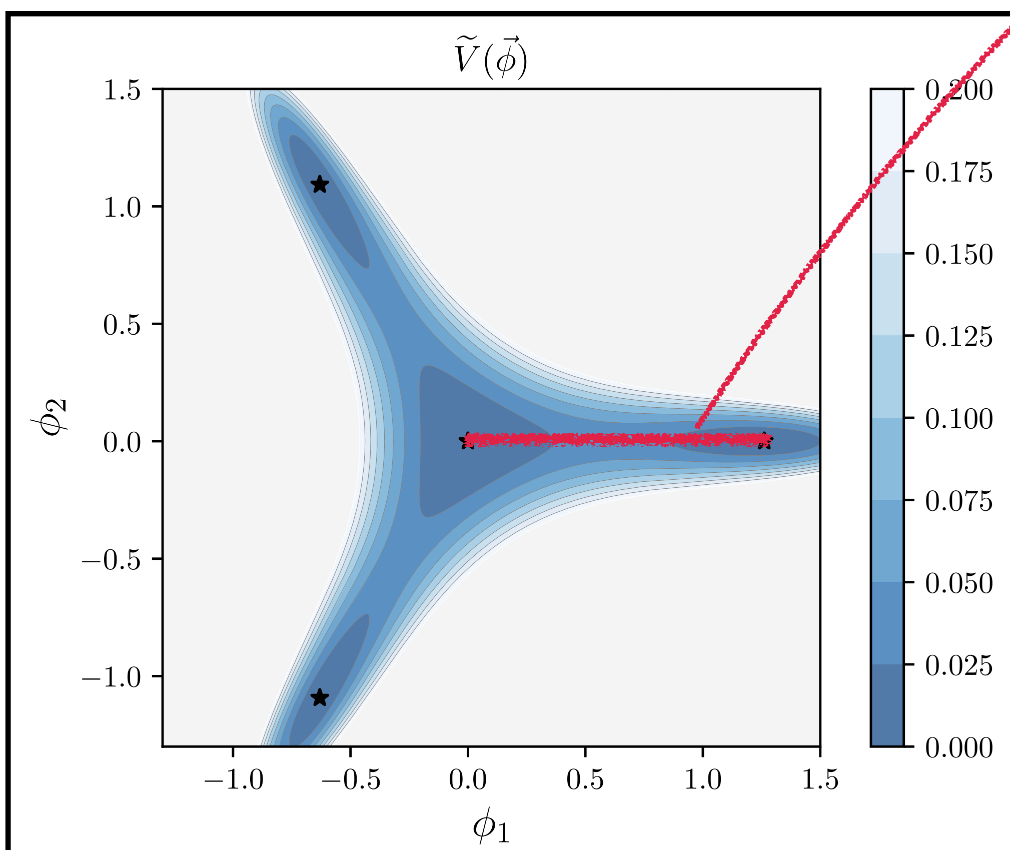
- Centre-symmetric point: $A_4^3 = \pi/L$

$$V(\phi) = 2V_0\left(\cosh(2\phi) - \kappa\pi^2\frac{L_{\rm cr}^2}{L^2}\sinh(\phi)\right)$$

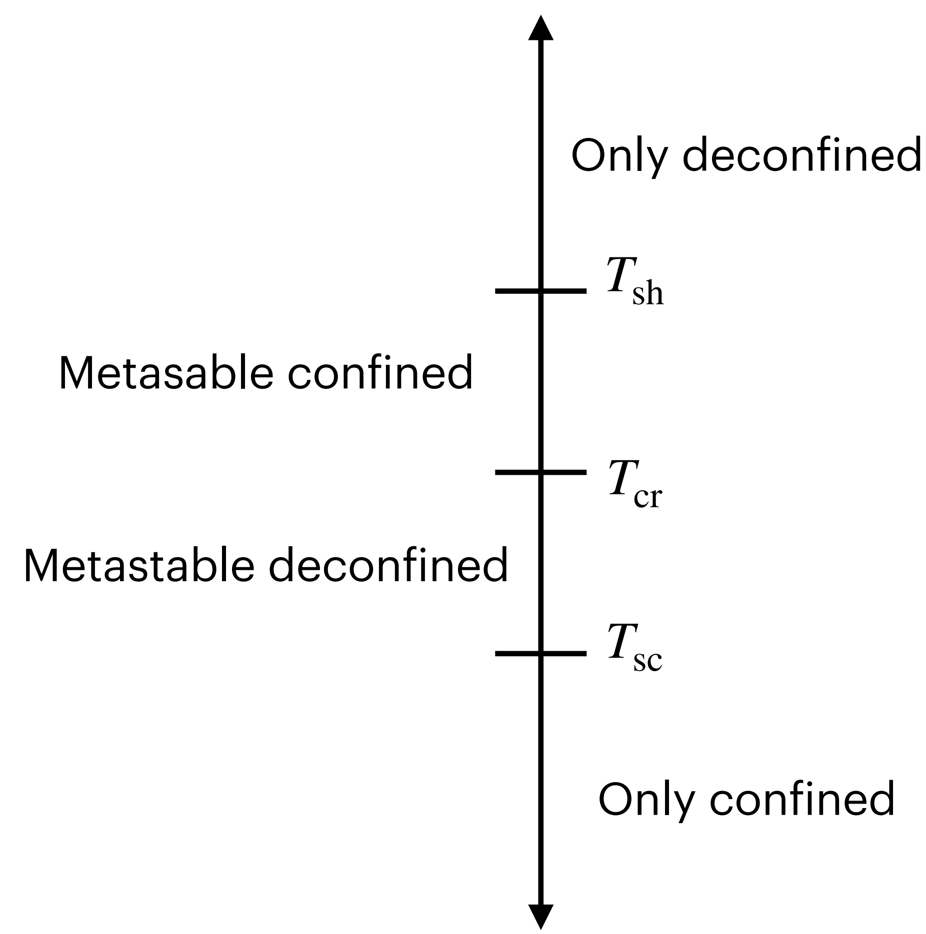
The Potential

- $\epsilon = 1 - \dfrac{T}{T_{\rm cr}} = 1 - \dfrac{L_{\rm cr}}{L}$
- $\epsilon = 0$ at critical value
- Below $T_{\rm sc}$ only confined minimum
- Above $T_{\rm sh}$ only deconfined minima



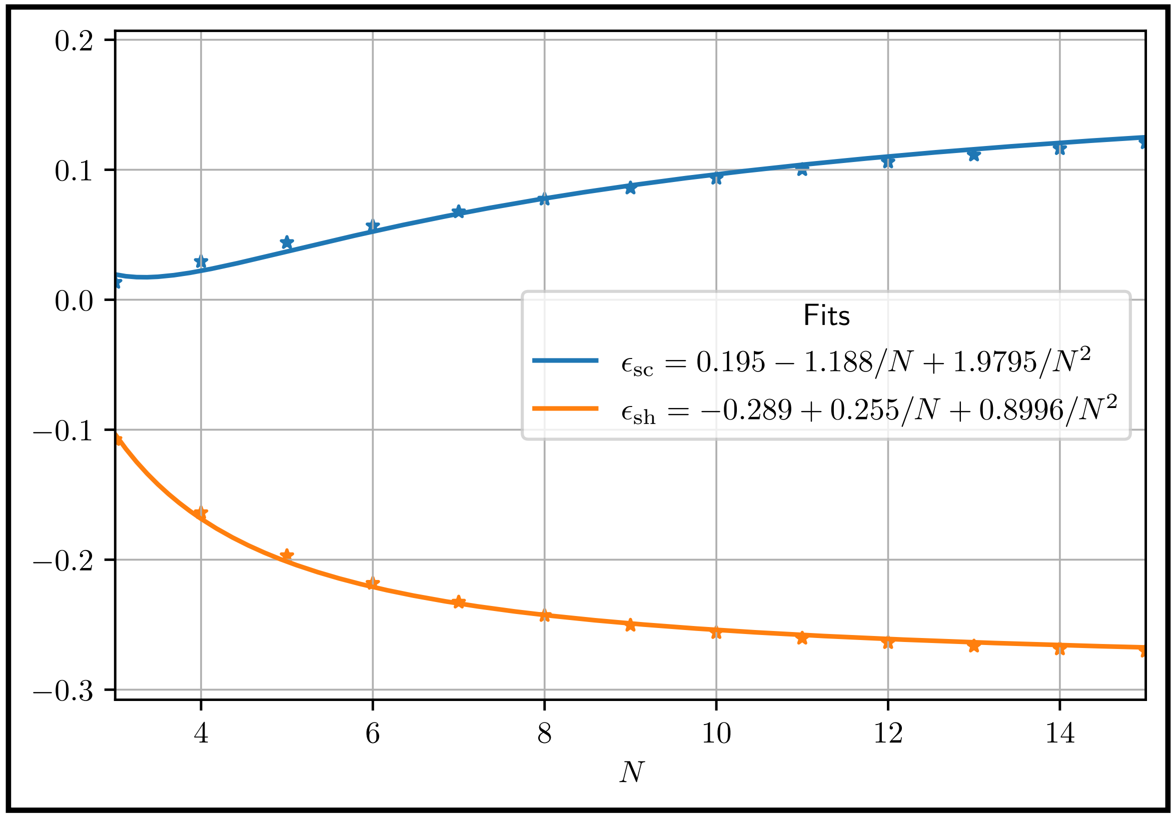


Sample plot for $SU(3)$ at $\epsilon = 0$



Maximal supercooling and superheating

One can vary the radius to see the behaviour of the minima



Numerically evaluating

$\epsilon_{\rm sc}$ and $\epsilon_{\rm sh}$ using the potential

for Super-Yang Mills

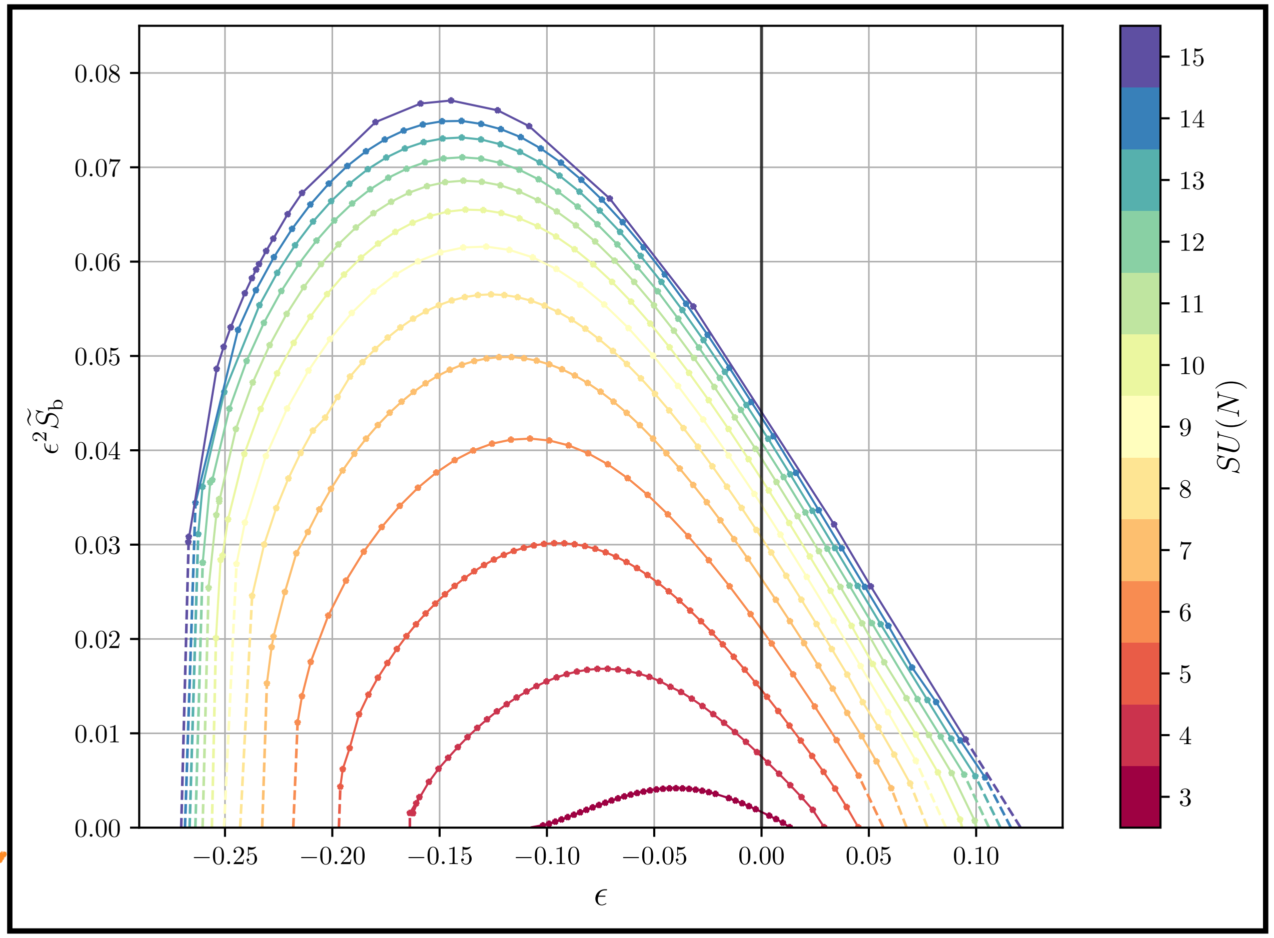
The Bounce

- Multi-field space trajectory
- Calculate bounce using `FindBounce`

V.Guada, M.Nemevsek, M.Pintar, Comput.Phys.Commun 256 (2020) 107480

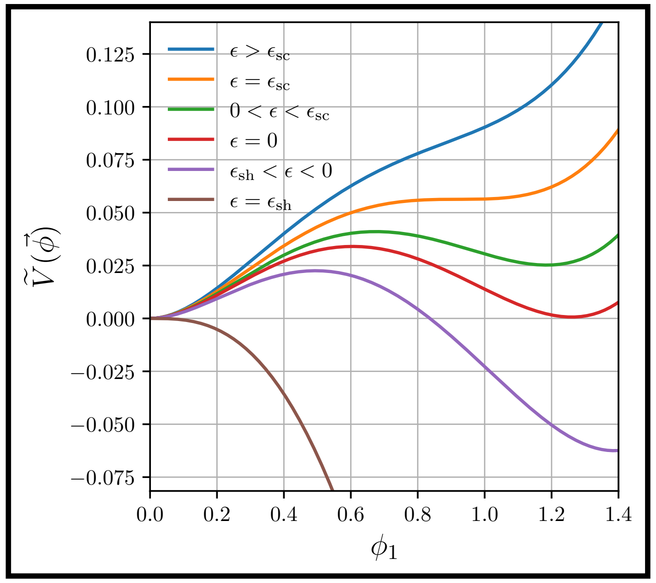
- Parametrize the action as

$$S_{\rm b} = -\frac{\Lambda^3 N^2}{\sigma_{\rm str}^2 L_{\rm cr}} \underbrace{h(\epsilon, N)\frac{(\epsilon - \epsilon_{\rm sc})(\epsilon - \epsilon_{\rm sh})}{\epsilon^2}}_{\widetilde{S}_{\rm b}}$$



- Action going to zero as

$$\epsilon \to \{\epsilon_{\rm sc}, \epsilon_{\rm sh}\}$$



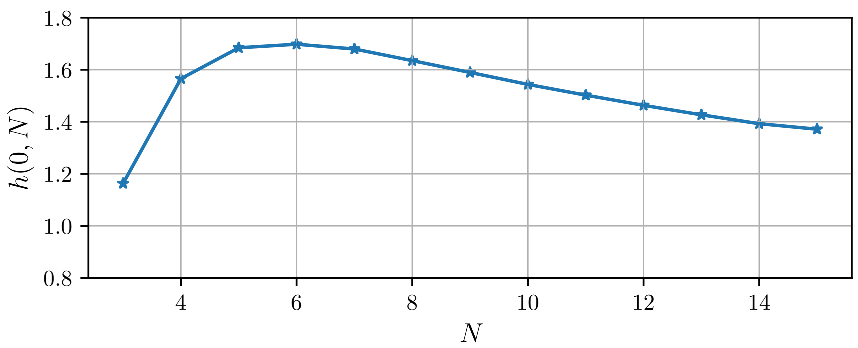
- This is because the barrier is vanishing at these points— the phase ceases to be metastable

Going to thermal $SU(N)$ Yang-Mills

- Lattice gives us data near the critical temperature— this corresponds to $\epsilon \to 0$ limit.

- In this limit, the bounce action in Super Yang-Mills case is

$$\lim_{\epsilon \to 0} \epsilon^2 S_\mathrm{b} = -\frac{\Lambda^3 N^2}{\sigma_\mathrm{str}^2 L_\mathrm{cr}} h(0,N) \epsilon_\mathrm{sc} \epsilon_\mathrm{sh}$$



- Continuity conjecture— this transition is continuously connected to Yang-Mills thermal phase transition.

E. Poppitz, T.Schafer, M.Unsal, JHEP 10 (2012) 115, JHEP 03 (2013) 087

- This motivates us to map $(T_\mathrm{cr} \sim \Lambda)$ C.Allton, M.Teper, A.Trivini JHEP 07 (2008) 021

$$\lim_{\epsilon \to 0} \epsilon^2 S_\mathrm{b}^\mathrm{YM} \sim -\frac{T_\mathrm{cr}^4 N^2}{\sigma_\mathrm{str}^2} \epsilon_\mathrm{sc}^\mathrm{YM} \epsilon_\mathrm{sh}^\mathrm{YM}$$

Going to thermal $SU(N)$ Yang-Mills

$$\lim_{\epsilon\to 0}\epsilon^2 S_{\rm b}^{\rm YM} \sim -\frac{T_{\rm cr}^4 N^2}{\sigma_{\rm str}^2}\epsilon_{\rm sc}^{\rm YM}\epsilon_{\rm sh}^{\rm YM} \sim 8.86\times 10^{-4}N^2 \quad \text{(From thin-wall action)}$$

Using the Lattice data: $T_{\rm cr}\approx 0.36\sigma_{\rm str}$ B.Lucini, M.Teper, U.Wenger, JHEP 02 (2005) 033, B. Lucini, A.Rago, E.Rinaldi, Phys. Lett. B 712 (2012)

$$-\epsilon_{\rm sc}^{\rm YM}\epsilon_{\rm sh}^{\rm YM} \sim 6.8\times 10^{-3} \Rightarrow \boxed{\epsilon_{\rm sc}^{\rm YM}\lesssim 0.08}$$

Small *maximal* supercooling! The deconfined phase becomes unstable quickly below the critical temperature!

Concluding Remarks

- What we did:
  - ★ Connect a small number observed in Lattice simulations to an intrinsic property of gauge theory using analytical approach
  - ★ Predict *maximal* supercooling in thermal deconfinement- confinement transition in $SU(N_c)$ gauge theory— testable on lattice R. Seppa, K.Rummukainen, D.J.Weir [arXiv:2501.17593]
  - ★ Seems to be a generic in confining theories (where we don't have conformality— need more investigation towards this)
- Implication to Gravitational Waves:
  - ★ Small supercooling— weak gravitational waves
  - ★ Exact value will depend on exact simulations
  - ★ Friction, *bubble stalling* etc— challenges in strongly coupled theories
- Ongoing follow-up: (with Vazha Loladze, Prateek Agrawal, John March-Russell, Arthur Platschorre)
  - ★ Dependence on θ-parameter on transitions (gives rich metastable vacuum structure)
  - ★ Using similar approaches to study other properties of gauge theories (using SQCD)
  - ★ Understand implications in the cases where we don't have centre symmetries

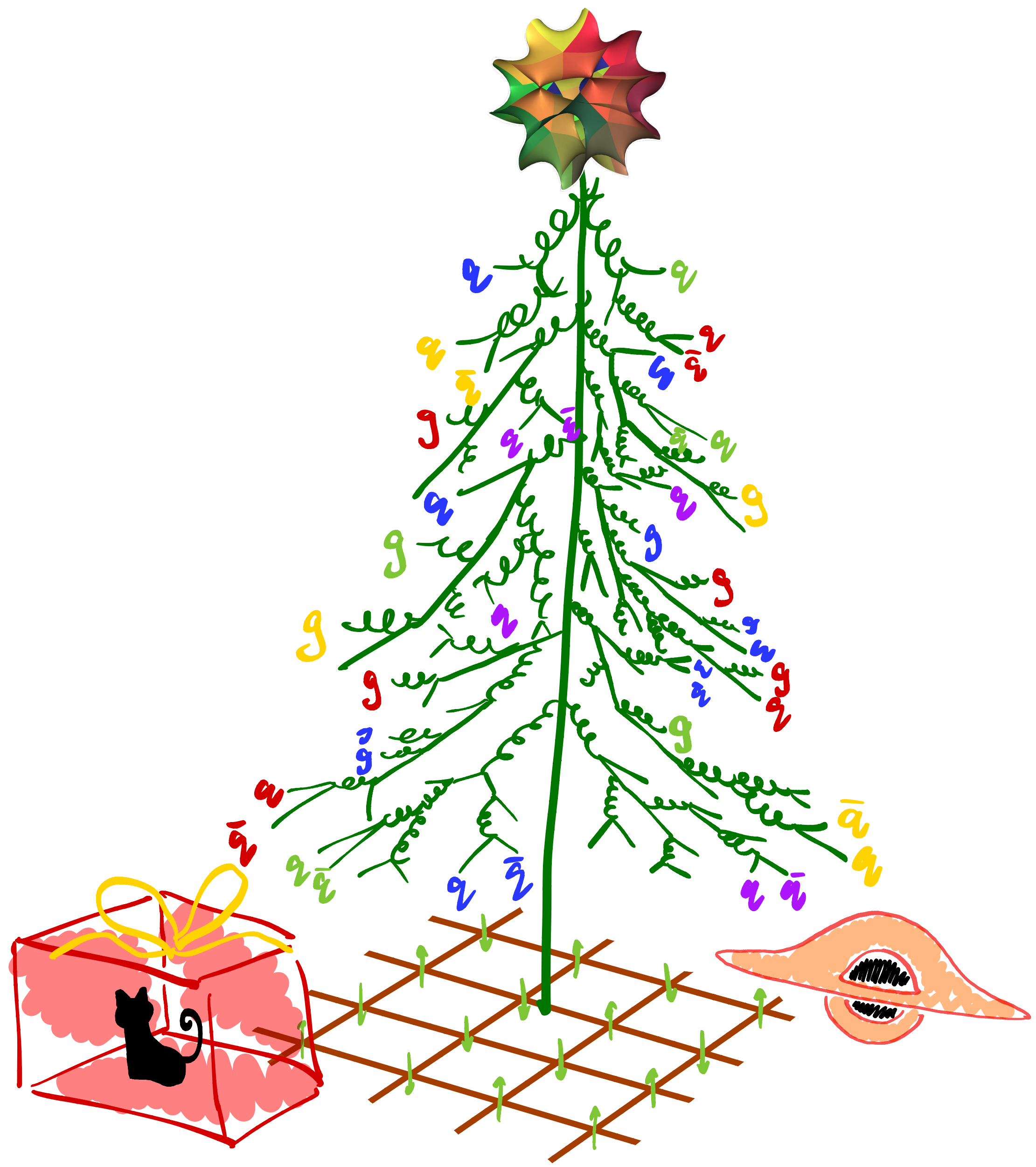


Image Credit- First year Theoretical Physics DPhil students @ Oxford

That’s all Folks

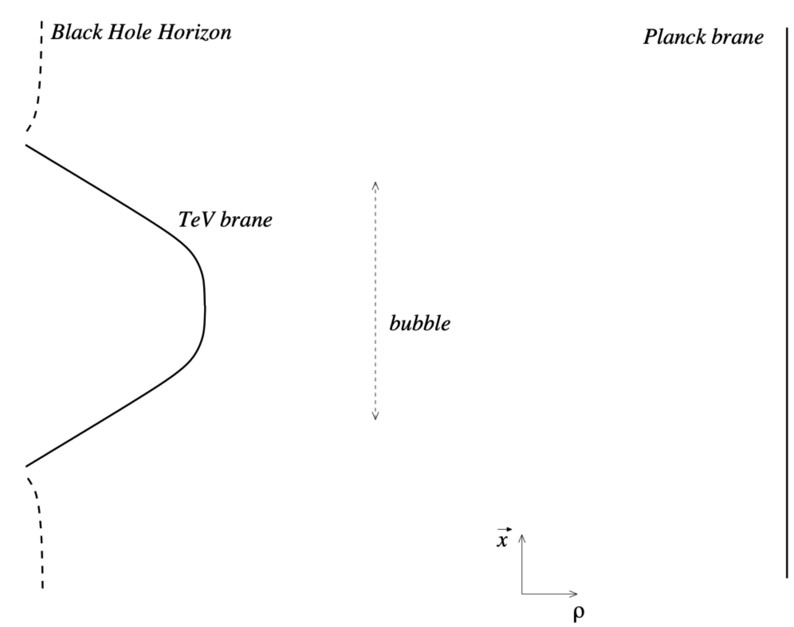
Backup slides

Holography but large supercooling

- Unlike the models mentioned earlier, in some holographic setup, there can be large supercooling, example— Randall-Sundrum Models
- Large supercooling due *almost* conformality— without stabilization, the theory is conformal. The Goldberger-Wise (GW) stabilisation breaks it very weakly. Hence the deformation becomes relevant in deep IR which can be much below the critical scale
- For GW stabilization, the bounce S_b is minimum of $O(4)$ and $O(3)$ actions

$$S_\mathrm{b} \sim \min\left\{\frac{N^2}{16\pi^2\lambda_f}, \frac{N^2}{8}\left(\frac{1}{\lambda_f}\right)^{3/4}\frac{1-\epsilon}{(1-(1-\epsilon)^4)^2}\right\}$$

P.Creminelli, A.Nicolis, R.Rattazzi, JHEP 03 (2002) 051
K.Agashe et.al JHEP 05 (2020) 086, JHEP 02 (2021) 051
R.K.Mishra, L.Randall, JHEP 06 (2024) 099
A.Azatov, M.Vanvlasselaer JHEP 09 (2020) 085
K Fujikura et.al JHEP 07 (2025) 215
P. Agrawal, GRK, V.Loladze, M.Reig JHEP 10 (2025) 066



Details of the action

- The potential is

$$V\left(\overrightarrow{\phi}\right) = \underbrace{V_0 \sum_{i=1}^{N_c} e^{-\vec{\alpha}_i \cdot \overrightarrow{\phi}} \left(\overbrace{e^{-\vec{\alpha}_i \cdot \overrightarrow{\phi}} - e^{-\vec{\alpha}_{i+1} \cdot \overrightarrow{\phi}}} - \kappa \frac{4\pi^2}{N^2} \frac{L_{\mathrm{cr}}^2}{L^2} \right)}_{\widetilde{V}}$$

$$V_0 = \frac{27}{8\pi} \frac{\Lambda^6}{v^3}, \quad \kappa = \frac{4m}{3\Lambda^3 L_{\mathrm{cr}}}, \quad v = \frac{2\pi}{N_{\mathrm{c}} L}$$

- With this, the action looks like

$$S_{\mathrm{b}} = \frac{(v/\Lambda)^3}{27\pi \ln^2(v/\Lambda)} \int d^3y \, \left(\frac{1}{2} (\partial_y \overrightarrow{\phi})^2 + \widetilde{V}(\overrightarrow{\phi}) \right)$$

- The string tension comes from calculating $\langle \Omega(0)\Omega(\vec{x})\rangle$ correlation that should scale as

$$\langle \Omega(0)\Omega(\vec{x})\rangle \sim e^{-\sigma_{\text{str}} r L} \text{ (in the confined phase)}$$

- In the weak coupling picture as $\Omega \sim \text{Tr}(\exp(i\overrightarrow{\phi}))$, this simplifies to calculating $\langle \phi_i \phi_i \rangle$ type correlation function which is proportional to mass of the scalar field. In the deep confined phase this goes as

M.M.Anber, V.Pellizzani, Phys. Rev. D 96 11 (2017) 114015

$$\sigma_{\text{str}} = \frac{9}{2} L \Lambda^3 \ln\left(\frac{v}{\Lambda}\right)$$

- Substituting this gives the form of action shown earlier.