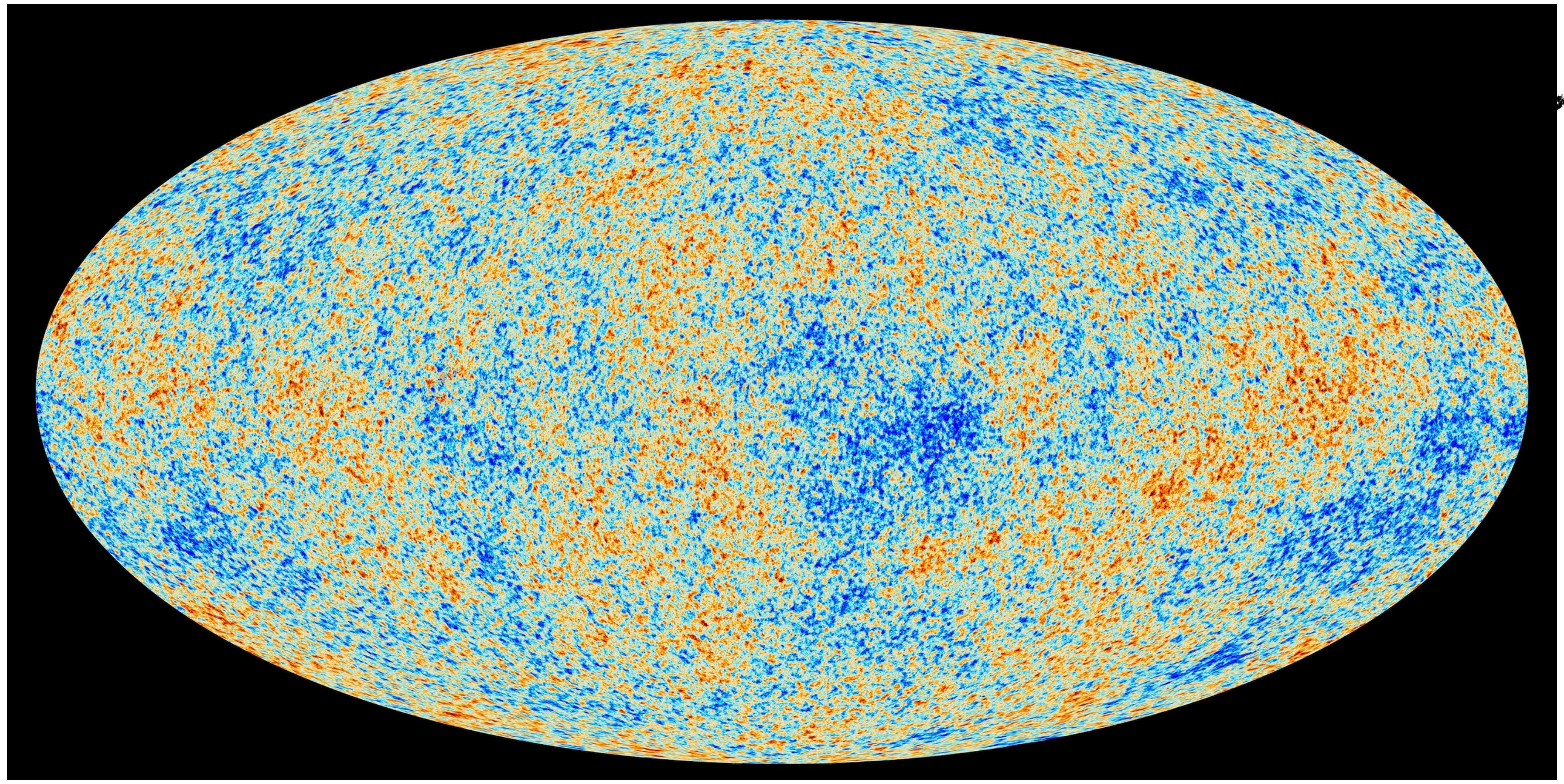
A Simple Model for Early Universe

Vatsalya Vaibhav
with Latham Boyle and Neil Turok,
Higgs Centre for Theoretical Physics

YTF 2025, Durham

The Cosmic Microwave Background



- Homogenous
- Isotropic
- Spatially flat

(almost!)

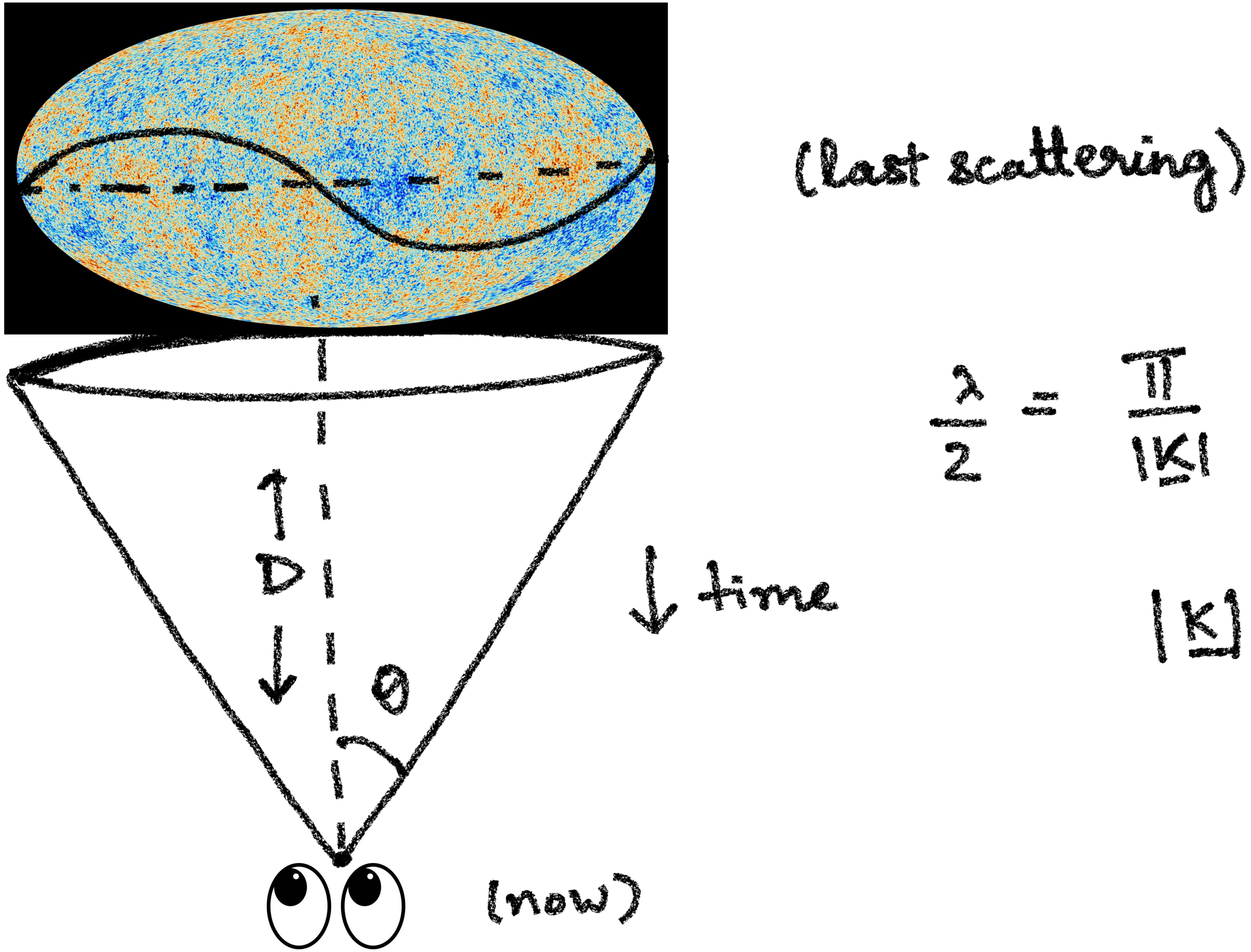
Temperature Fluctuations!

The Cosmic Microwave Background

Gravity

Thermo

The Cosmic Microwave Background

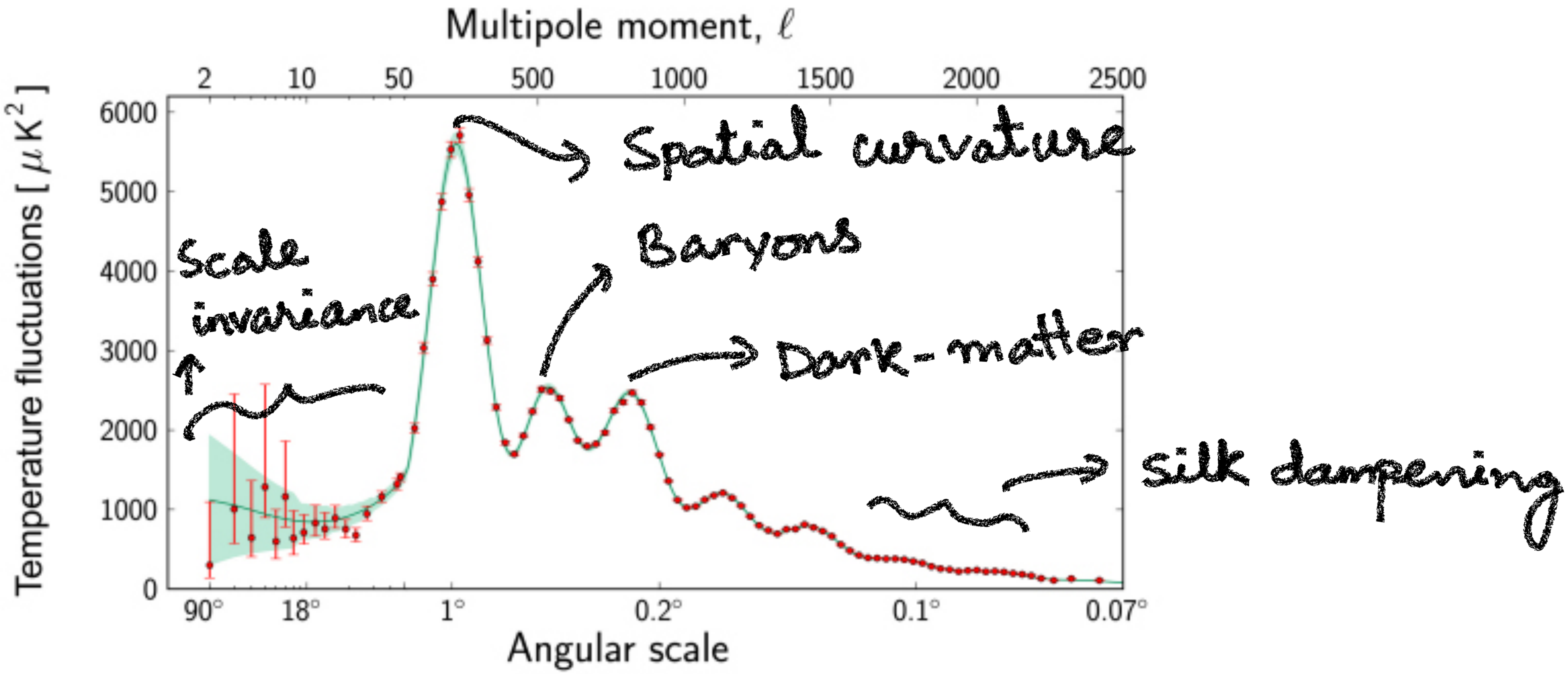


$$\frac{\lambda}{2} = \frac{\pi}{|\underline{k}|} \simeq D\theta = \frac{D\pi}{\ell}$$

$$|\underline{k}| = \frac{\ell}{D}$$

The Cosmic Microwave Background

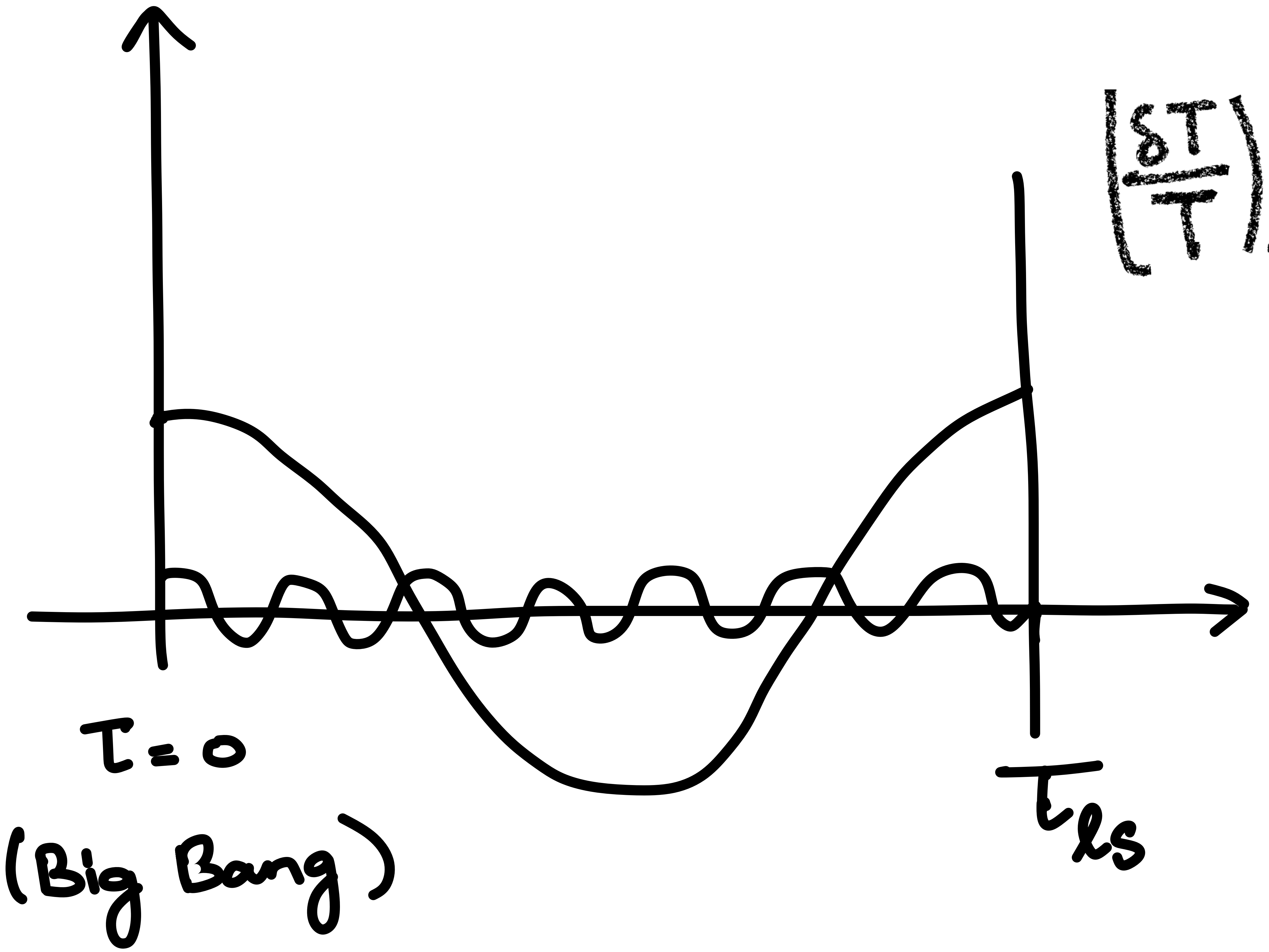
- Adiabatic
- Gaussian (mostly)
- Almost no tensor/vector perturbation



CMB Power spectrum: $P_\ell = \langle \Theta_\ell , \Theta_\ell \rangle$; $\left(\Theta_\ell = \frac{\delta T}{T} \right)$

Seeding the amplitude for power spectrum

Case1: Big Bang is a mirror (standing waves) Boyle & Turok 1803.08928

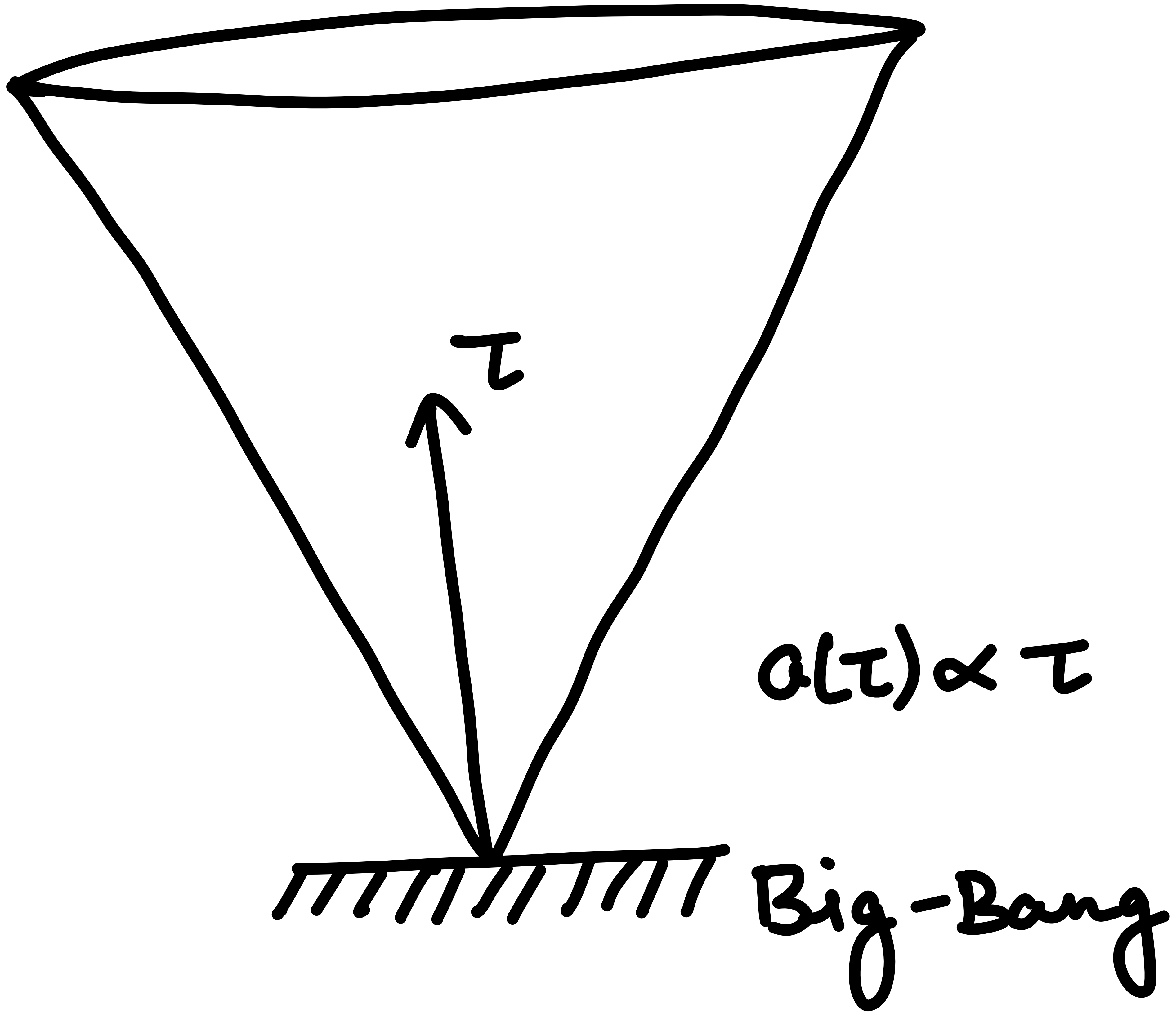


$$\left(\frac{\delta T}{T}\right)_k + \phi_k \approx A_k \cos\left(k \int_0^{\tau_{ls}} c_s' \, d\tau'\right) - \frac{3}{4}\frac{\rho_B}{\rho_\gamma}\phi_k$$

Case2: Inflation (no waves)

Taking Radiation Domination seriously

$$ds^2 = a^2(\tau)\Big[-d\tau^2(1+\phi) + 2\,\xi_i\, d\tau\, dx^i + dx^i dx^j\big(\delta_{ij}(1-\Phi) + h_{ij}\big)\Big]$$



- $a(-\tau) = -a(\tau)$
- $\Phi(\tau, \underline{x}) = -\Phi(-\tau, \underline{x})$

  $\xi_i = 0$

  $h_{ij}(\tau, \underline{x}) = -h_{ij}(-\tau, \underline{x})$

A Few Puzzles

1) Why CMB power spectrum is scale-invariant at large wavelengths?

2) Why are tensor modes suppressed to scalar modes?

3) What if the dark matter thermalized?

Quantum Corrections from Gravity

$$S_{grav} = \int d^4x \sqrt{-g} \left[\frac{m^2 R}{2} - m^2 \Lambda + \frac{1}{2f_0^2} R^2 - \frac{1}{2f_2^2} C^2 + \dots \right]$$

$$m^2 = \frac{1}{8\pi G}$$

Quantum Corrections from Gravity

$$S_{Grav} = \int d^4x \sqrt{-g} \left[\frac{m^2 R}{2} - m^2 \Lambda + \frac{1}{2 f_0^2} R^2 - \frac{1}{2 f_2^2} C^2 + \dots \right]$$

$$D_{\mu\nu\alpha\beta} = \frac{-2 i f_2^2}{q^2 (q^2 - m^2 f_2^2)} P^{(2)}_{\mu\nu\alpha\beta} + \frac{2 i}{m^2 (q^2 - m^2 f_0^2)} P^{(0)}_{\mu\nu\alpha\beta}$$

$$D^{IR}_{\mu\nu\alpha\beta} = \frac{2i}{m^2 q^2} P^{(2)}_{\mu\nu\alpha\beta} \quad \text{(Einstein gravity)}$$

Avoiding Dark-matter Overproduction

$$D^{(UV)}_{\mu\nu\alpha\beta} \longrightarrow \frac{-2i f_2^2 \, \rho^{(2)}}{q^4}$$

$$\Gamma = \begin{cases} \dfrac{T^5}{M_{pl}^4} & T \ll f_2 M_{pl} \\ T & T \gg f_2 M_{pl} \end{cases} \quad ; \quad H^2 = \frac{\pi^2}{90 g_*} \frac{T^4}{M_{pl}^2}$$

$\Gamma < H$ (Freeze out)

Constraint: $f_2 < 10^{-6}$

Scalar Power Spectrum and Tilt

- The comoving curvature perturbation $\mathcal{R}^{(3)}$ is given by the scalar mode of metric at large wavelength
- The amplitude is therefore defined by

$$\langle \phi(t,\underline{x}) \cdot \phi(t,\underline{y}) \rangle = \int \frac{d^3 q}{q^3} \, \Delta_s \, e^{-i \underline{q} \cdot (\underline{x} - \underline{y})}$$

Scalar Power Spectrum and Tilt

- The power spectrum is scale invariant at large wavelengths but not exactly

- spectral tilt measures the deviation from scale invariance

$$\frac{d\ln\Delta_s}{d\ln q} = n_s - 1 \Rightarrow \Delta_s = A\left(\frac{k}{k_0}\right)^{n_s-1} \; ; \; n_s \simeq 1$$

Scalar Power Spectrum and Tilt

- Remember that

$$\langle \phi(t,\underline{x}), \phi(t,\underline{y}) \rangle = \int \frac{d^3q}{m^2} \frac{2i}{(q^2+m^2f_0^2)^{1/2}}$$

- m^2 runs!! $m^2 \rightarrow \frac{M_{Pl}^2}{8\pi} + \left(\xi - \frac{1}{6}\right) n_s R^2$

- $\Delta_s = \frac{1}{(8\xi)(1+8\xi f_0^2)^{1/2}}$; $n_s - 1 = \frac{d \ln \Delta_s}{d \ln q}$

Scalar to tensor ratio

- $\Delta_s \approx \dfrac{1}{\delta \xi} \doteq 10^{-10}$

- $\dfrac{d \ln \Delta_s}{d \ln k} = n_s - 1 \simeq 0.97$ if $f_0 \sim 10^{-10}$

- $\Delta_t \propto f_2^2 = 10^{-12}$ (within experimental bounds)

$\Rightarrow \Delta_t / \Delta_s = 0.01$

Thanks!