Exploiting Perpendicular Momentum Distributions in Semileptonic Decays

Using a $\bar{B}_s^0 \to D_s^+ \mu^- \bar{\nu}$ case study

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Motivation

Why study semileptonic decays?

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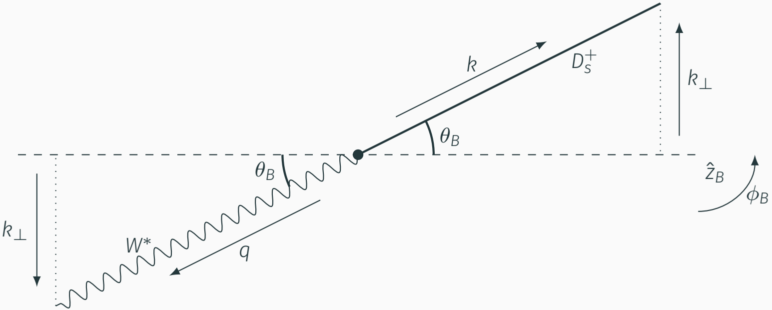
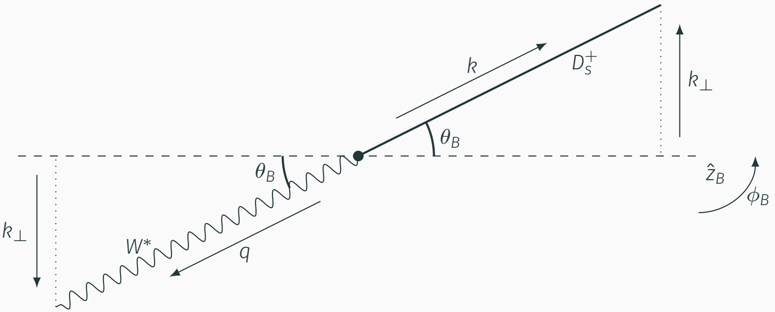
- The presence of final state neutrinos make q^2 reconstruction difficult at LHCb.
- The traditional method involves solving a quadratic equation, introducing a twofold ambiguity.
- This causes efficiency losses and potential mismodelling issues.

A different observable: $k_\perp$



- $k_\perp\big|_B$ - transverse projection of D_s^+ momentum onto the $\bar{B}_s^0$ flight direction, $\hat{z}_B$, defined in the $\bar{B}_s^0$ rest frame.
- Invariant under $\hat{z}_B$-boosts - $k_\perp\big|_B = k_\perp\big|_{\mathrm{lab}}$.
- Determined uniquely from visible quantities.

Aims of this work



- Derive $d\Gamma/dk_\perp$ from existing theory defined in q^2.

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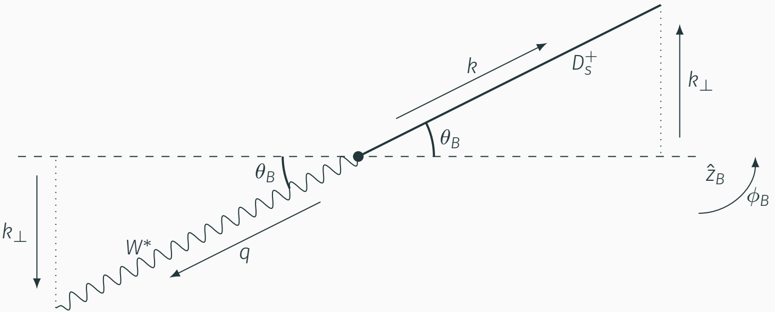

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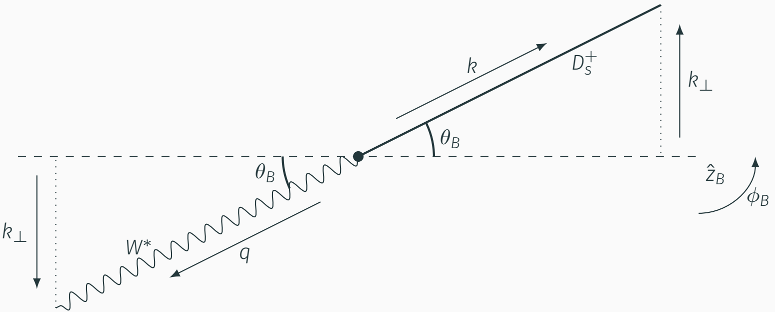
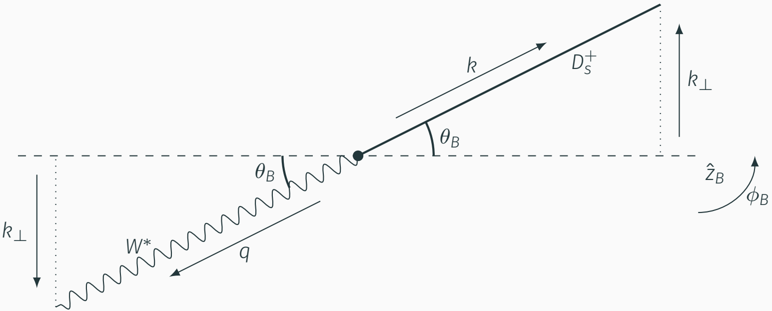


- Derive $d\Gamma/dk_\perp$ from existing theory defined in q^2.
- Build an approximate response matrix for LHCb's detector effects.
- Use it to extract $|V_{cb}|$ and form factor information.

Theory

Kinematic geometry



$$k_\perp^2 = \frac{\lambda(M_{B_s}^2\,,\, M_{D_s}^2\,,\, q^2)}{4M_{B_s}^2}\,\sin^2\theta_B$$

Källén function: $\lambda(a\,,\,b\,,\,c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ac$

Theory input

In the SM and for massless leptons, the differential branching ratio with q^2 reads:

$$\frac{d\mathcal{B}}{dq^2} = \frac{G_F^2 M_{D_s}^2 \lambda(M_{B_s}^2\,,\, M_{D_s}^2\,,\, q^2)}{48\pi^3 M_{B_s}} |V_{cb}|^2 |f_+(q^2)|^2$$

Where the full integrated branching ratio is defined as:

$$\mathcal{B} = \iint dq^2 d\cos\theta_B \frac{d^2\mathcal{B}}{dq^2\, d\cos\theta_B} = \iint dk_\perp^2\, d\cos\theta_B \frac{d^2\mathcal{B}}{dk_\perp^2\, d\cos\theta_B}$$

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We are examining the (pseudo)scalar initial state $\bar{B}_s^0$ meson so:

$$\frac{d^2\mathcal{B}}{dq^2\, d\cos\theta_B} = \frac{1}{2}\frac{d\mathcal{B}}{dq^2}$$

There is only one physical branch for q^2 in terms of $k_\perp^2$ and $\cos\theta_B$

$$q^2(k_\perp^2, \cos\theta_B) = M_{B_s}^2 + M_{D_s}^2 - 2M_{B_s}\sqrt{\frac{k_\perp^2}{1-\cos\theta_B} + M_{D_s}^2}$$

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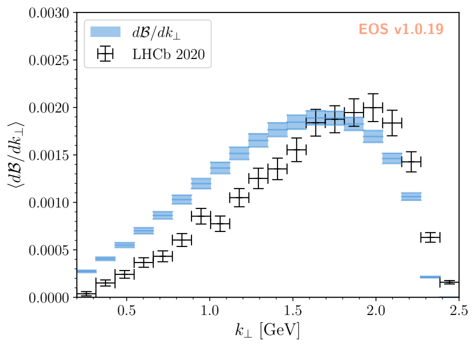
Allowing us to perform the necessary change of variables

$$\frac{d\mathcal{B}}{dk_\perp^2} = \frac{1}{2}\int_{-1}^{+1} d\cos\theta_B \left|\frac{\partial(q^2, \cos\theta_B)}{\partial(k_\perp^2, \cos\theta_B)}\right| \frac{d\mathcal{B}}{dq^2}\, \Theta(q^2 - m_\ell^2)\Big|_{q^2=q^2(k_\perp^2, \cos\theta_B)}$$

How does this compare to measurement?



The rate of $\bar{B}_s^0 \to D_s^+ \mu^- \bar{\nu}$ as measured by the LHCb experiment [arxiv:2001.03225], overlaid with the theoretical prediction

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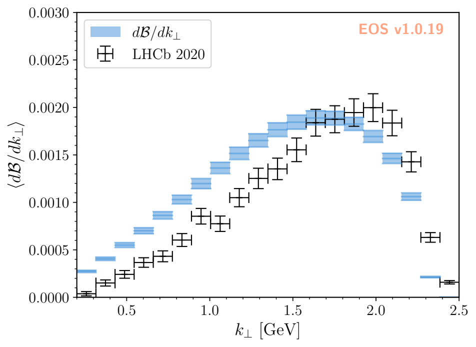

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Very poor visual agreement!

We must account for substantial detector effects

Detector Modelling

Forward Modelling

Forward modelling is expressed as the convolution

$$P^{\mathrm{det}}(k_\perp^{\mathrm{det}}) = \iint dk_\perp^{\mathrm{det}}\, dk_\perp^{\mathrm{th}}\, A(k_\perp^{\mathrm{th}}, k_\perp^{\mathrm{th}} - k_\perp^{\mathrm{det}})\, P^{\mathrm{th}}(k_\perp^{\mathrm{th}})$$

$$A(k_\perp^{\mathbf{th}}, k_\perp^{\mathbf{th}} - k_\perp^{\mathbf{det}}) = \varepsilon(k_\perp^{\mathbf{th}}) r(k_\perp^{\mathbf{th}} - k_\perp^{\mathbf{det}})$$

P^{th}: The theoretical model we have

P^{det}: The detector level model we want

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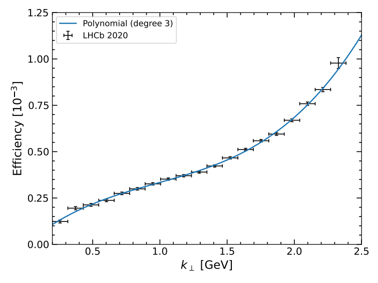
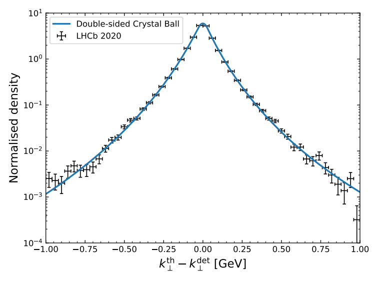
$\varepsilon(k_\perp^{\mathrm{th}})$: Detector efficiency

$r(k_\perp^{\mathrm{th}} - k_\perp^{\mathrm{det}})$: Detector resolution

Determined by LHCb with Monte Carlo simulations

Efficiency & Resolution

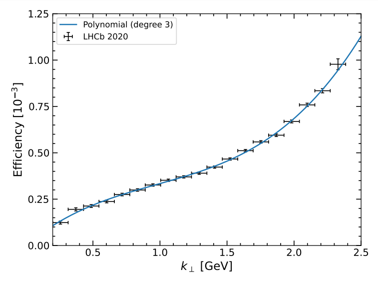


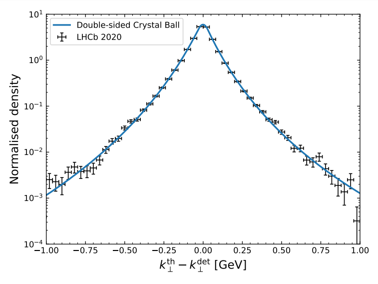


Efficiency fit with Legendre polynomials:

$$\hat{\varepsilon}(k_\perp) = \frac{1}{2}\left[1 + \sum_{n=1}^{n=3} \frac{\varepsilon_n}{\varepsilon_0} P_n(\zeta)\right]$$

Efficiency & Resolution





Resolution fit with Double Sided Crystal Ball distribution:

$$r(k_\perp^{\mathrm{th}} - k_\perp^{\mathrm{det}}) = \begin{cases} a_L\left(b_L - \frac{\Delta_{k_\perp} - \mu}{\sigma}\right)^{-n_L} & \text{for } \frac{\Delta_{k_\perp} - \mu}{\sigma} \leq -\alpha_L, \\ \exp\left\{-\frac{\left(\Delta_{k_\perp} - \mu\right)^2}{2\sigma^2}\right\} & \text{for } -\alpha_L < \frac{\Delta_{k_\perp} - \mu}{\sigma} \leq \alpha_R, \\ a_R\left(b_R + \frac{\Delta_{k_\perp} - \mu}{\sigma}\right)^{-n_R} & \text{for } \alpha_R < \frac{\Delta_{k_\perp} - \mu}{\sigma}, \end{cases}$$

Approximate Response Matrix

Since the measurement data provided in [arxiv:2001.03225] is provided in bins of $k_\perp^{\mathrm{det}}$, we bin the detector and theory level quantities as;

$$P_m^{\mathrm{det}} = \int_{\mathrm{bin}\, m} dk_\perp^{\mathrm{det}} P^{\mathrm{det}}(k_\perp^{\mathrm{det}}) \qquad P_n^{\mathrm{th}} = \int_{\mathrm{bin}\, n} dk_\perp^{\mathrm{th}} P^{\mathrm{th}}(k_\perp^{\mathrm{th}})$$

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Calculate the approximate relationship via

$$R_{mn} = \frac{P_m^{\rm det}\big|_n}{P_n^{\rm th}}, \quad P_m^{\rm det}\big|_n = \int_{{\rm bin}\, m} dk_\perp^{\rm det} \int_{{\rm bin}\, n} dk_\perp^{\rm th} A(k_\perp^{\rm th}, k_\perp^{\rm th} - k_\perp^{\rm det}) P^{\rm th}(k_\perp^{\rm th})$$

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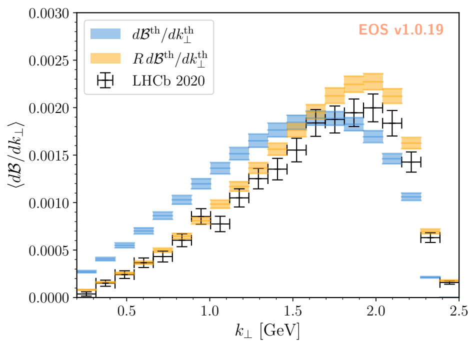
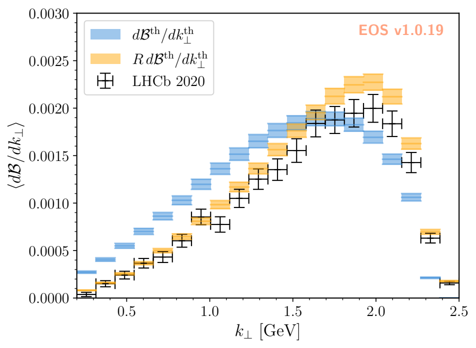
Gives our theory to detector-level measurements bin-by-bin

$$P_m^{\rm det} = \sum_n R_{mn} P_n^{\rm th}$$

Comparing theory and measurement



Much better visual agreement!



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How dependent on the underlying signal shape is this response matrix?

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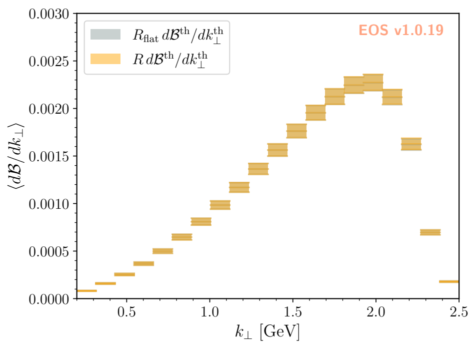
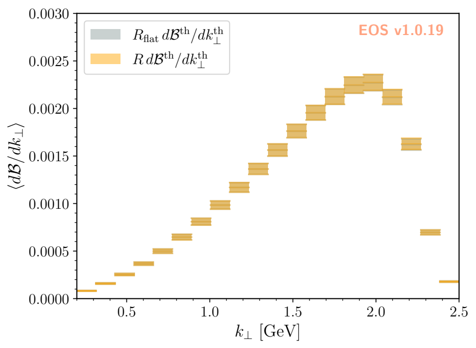


$$P_{\text{th}}^{\text{flat}}(q^2) = \frac{1}{q^2_{\text{max}} - q^2_{\text{min}}}$$

Almost no visual difference - eigenvalues agree within 0.4%



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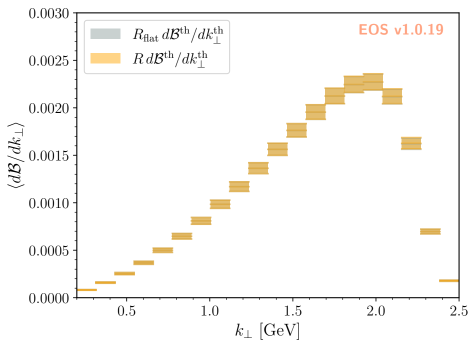

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Measurements of distributions in $k_\perp$ serve as useful cross-checks for discrepancies found between LQCD and experimental measurements of hadronic form factors.

Phenomenological Analysis

Statistical Model and Priors

Parametrise the hadronic matrix elements for $\bar{B}_s^0 \to D_s^+$ transitions using [arxiv:1503.05534]:

$$f_+(q^2) = \frac{1}{1 - q^2/M_{B_s^*}^2} \sum_{k=0}^{K} \alpha_k^{(+)} \left[z(q^2) - z(0)\right]^k$$

$$f_0(q^2) = \frac{1}{1 - q^2/M_{B_{s,0}}^2} \sum_{k=0}^{K} \alpha_k^{(0)} \left[z(q^2) - z(0)\right]^k$$

Where we are using the conformal map from q^2 plane to unit disk in z

$$z(q^2) \equiv \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}} \quad \text{where} \quad t_\pm = (M_{B_s} \pm M_{D_s})^2, \quad t_0 \equiv t_+ \left(q - \sqrt{1 - t_-/t_+}\right)$$

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- Use $f_+(0) = f_0(0)$ to replace $\alpha_0^{(0)}$ with a linear combination of the remaining expansion coefficients, and truncate the series to $K = 2$, giving **five** hadronic parameters
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- One free parameter - $|V_{cb}|$
- Use uniform priors, chosen wide enough to not cut off any peaks from the likelihood

Likelihood

LQCD results for f_+ and f_0 implemented as a five-dimensional multivariate Gaussian likelihood

- f_+ in three q^2 points,
- f_0 in two q^2 points

Use the 20 bins of $k_\perp$ from LHCb 2020

- Average efficiency $\langle \varepsilon \rangle$ already accounted for
- No published correlation information across the bins
- Supplement with a 20-dimensional diagonal covariance matrix

Results

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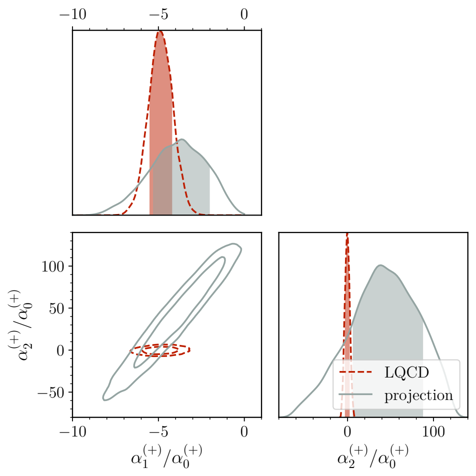
- Precision is limited by the normalisation to the absolute branching fraction of $\bar{B} \to D\,\mu^- \bar{\nu}$
- The posterior distribution for $\alpha_1^{(+)}/\alpha_0^{(+)}$ is narrower than its prior

  Sensitivity to $\alpha_2^{(+)}/\alpha_0^{(+)}$ is poor
- Using this parameter allows for independent analysis without relying on internal LHCb knowledge
- This type of measurement can now be included in global analyses

Projected Results



- After the completion of LHCb Run 3 data-taking, it is expected that there will be ten times as many samples and the covariance matrix is rescaled by a factor of 1/10
- Only at this points begins to become competitive with LQCD results
- 2D marginal posterior: LQCD and projected dataset provide complementary constraints
- Sensitivity to $\alpha_2^{(+)}/\alpha_0^{(+)}$ remains poor

Summary

- Derived $d\Gamma/dq^2$ to $d\Gamma/dk_\perp$ using the scalar nature of the $\bar{B}_s^0$ meson
- Constructed an approximate detector response matrix
- Found that the detector response matrix is largely independent of the underlying theory signal model
- Showed that $k_\perp$ distributions provide sensitivity to form-factor shape parameters, complementary to lattice QCD

Questions?