

Chromodynamics and the Lattice

The Standard Model (SM) summarises our current best understanding of particle physics. It is exceptionally accurate, but clearly incomplete.

- No energy frontier colliders likely the next decade(s)
- Indirect detection of new physics best option
- CKM matrix parametrises weak flavour change
- Strictly unitary in SM
- Non unitarity implies new physics
- Use precise results from LHCb, Belle II, BES III
- Future improvements from HL-LHC, FCC-ee
- Requires improved theoretical precision to determine

Quantum Chromodynamics (QCD) is the part of the SM that describes strong force interactions.

- Dominant interaction for hadrons
- Strongly coupled at low energies
- Renders perturbative methods non-viable
- Requires entirely different method

Lattice QCD is an ab initio nonperturbative method for calculating QCD results. It approximates spacetime as a finite discrete set of points. A typical calculation would involve the following steps:

- Generate gauge ensemble with size, dimension, fields
- Measure observable on each configuration
- Extract physical quantities via fitting
- Repeat for range of lattice spacings and volumes
- Extend to zero-spacing, infinite-volume, physical mass

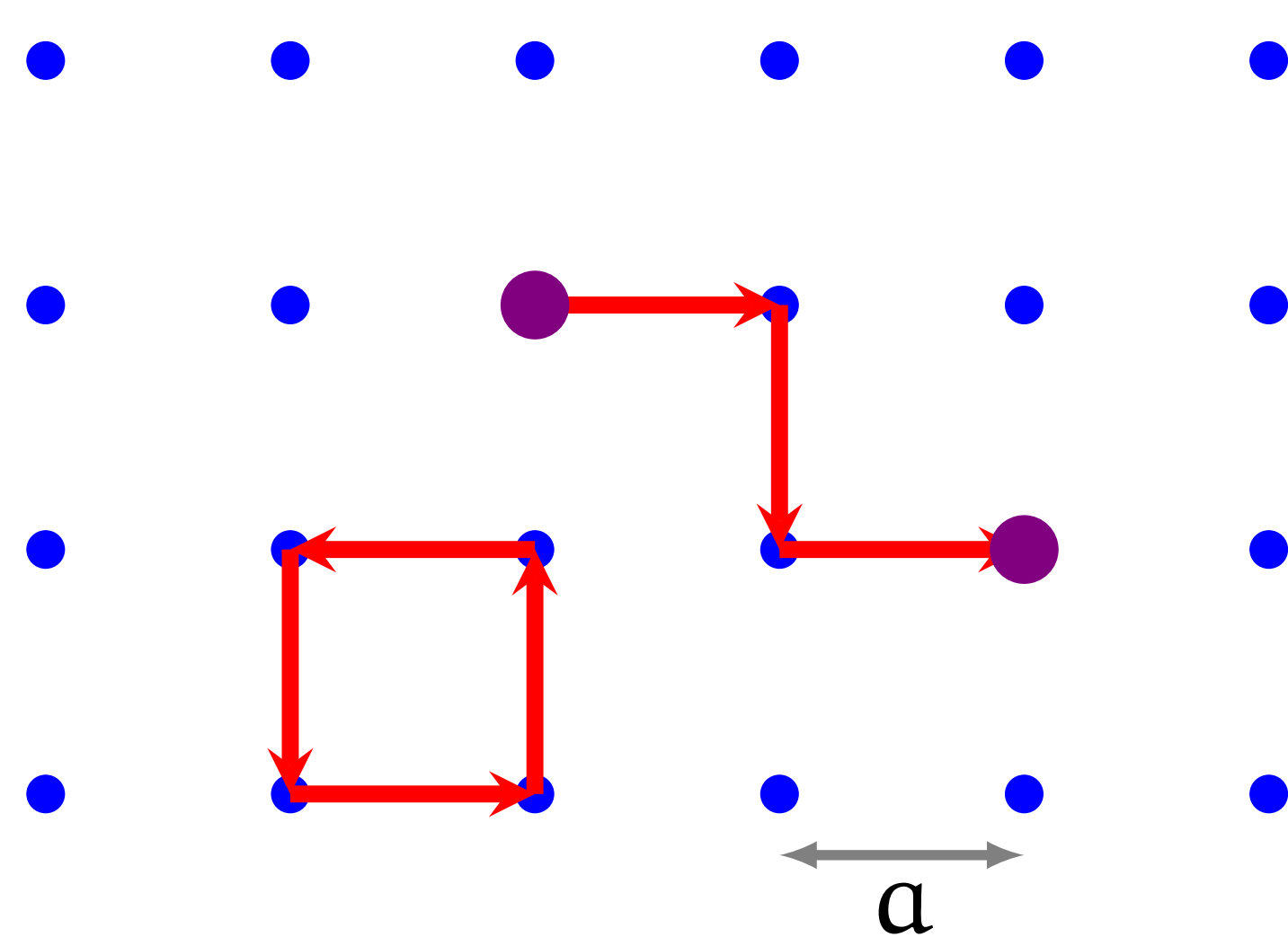


Figure 1: Two dimensional schema of a lattice. Quarks situated on lattice sites, gauge field on links.

Semileptonic Decays

We compute exclusive semileptonic decay rates for range of processes and will address major tension between different lattice results [1].

- Exclusive: specific particles in final state
- Flavour changing transitions probe CKM
- Parameterised by form factors depending on q^2

$$q^2 := (p_\mu^I - p_\mu^F)^2$$

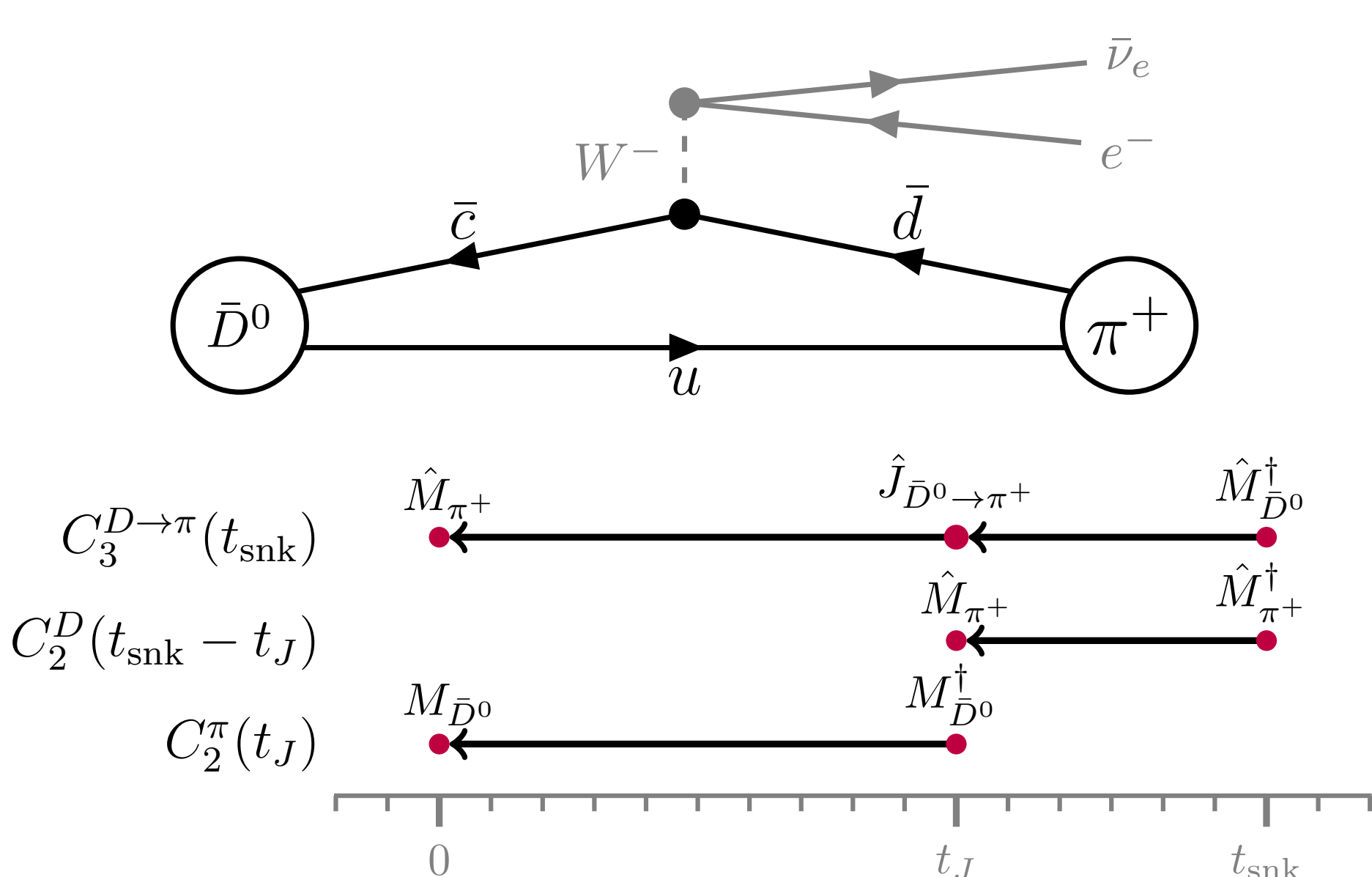
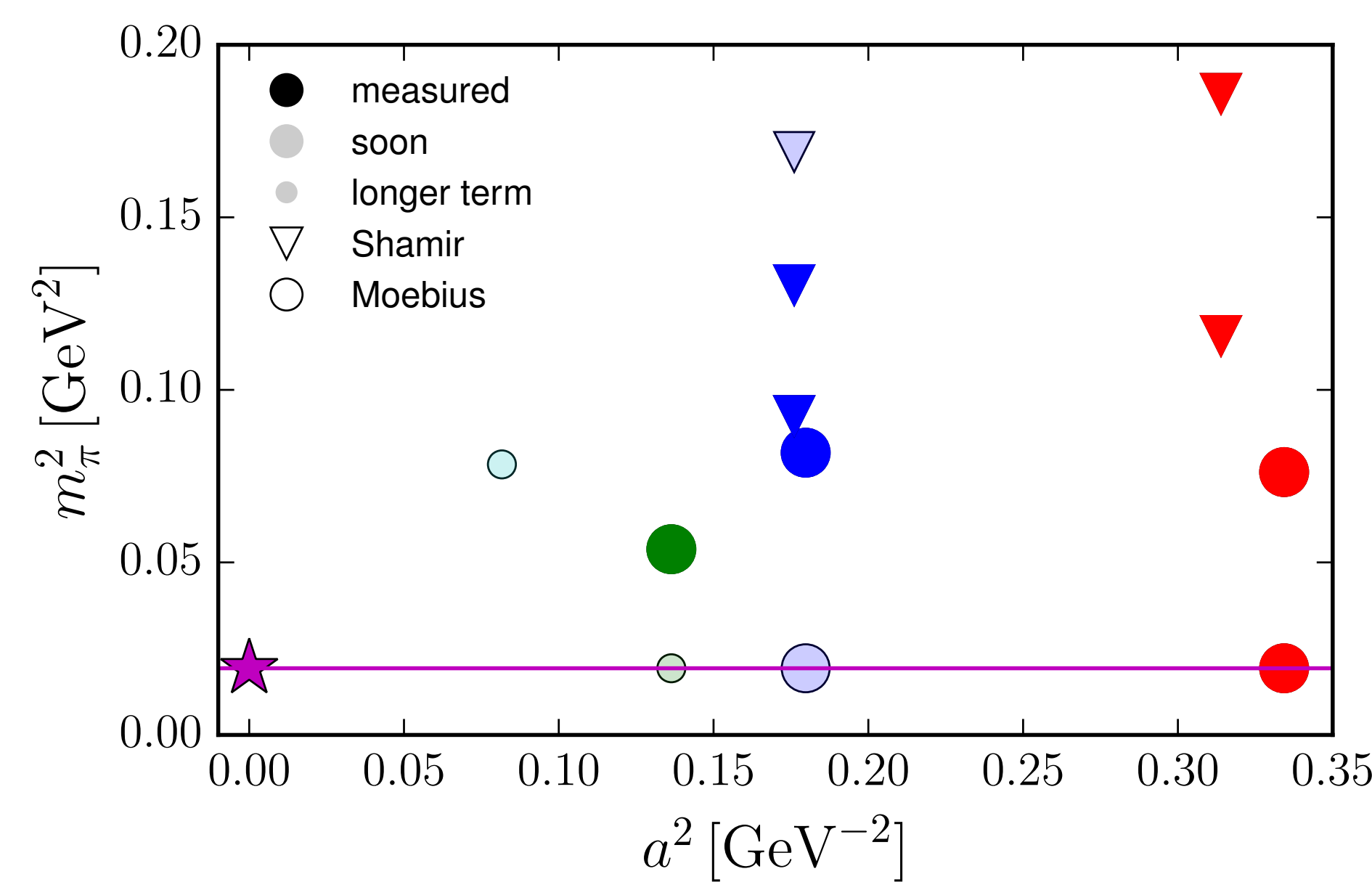


Figure 2: top: Quark flow diagram for $D \rightarrow \pi$. bottom: Relevant correlation functions on the lattice. Quark component structurally identical for other semileptonic decays.

Our Data



We have a significant amount of data over a range of ensembles and decay processes.

- $N_f = 2 + 1$ sea domain wall fermions (DWF)
- 3 lattice spacings
- $c \rightarrow l, s$ via sequential solves
- l, s and c -quarks use DWF
- Induce definite momenta via Fourier transform
- Include initial state at rest and with momentum
- Allows better q^2 coverage, more currents with signal
- $\mathcal{O}(5)$ source-sink separations in fw and bw direction
- $\mathcal{O}(10)$ final state momenta
- Multiple source positions and all mode averaging

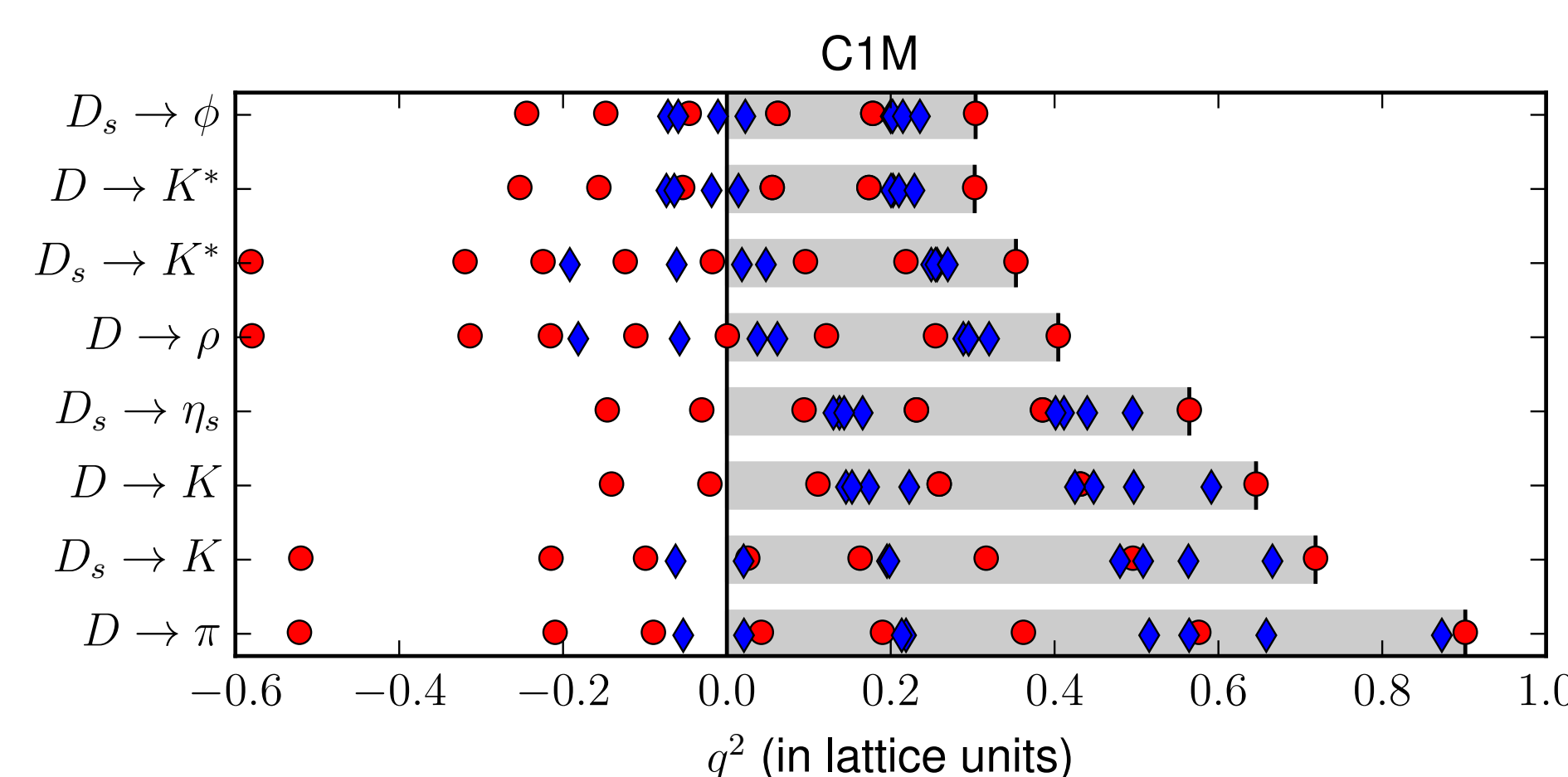


Figure 3: Range of q^2 values covered by dataset. Red circles (blue diamonds) initial state at rest (with momentum).

Blinding

To reduce bias 3pt data was blinded by applying

$$C_{3,\text{blind}}^{I \rightarrow F} = (1 + \alpha_{I \rightarrow F} + \beta_{I \rightarrow F}(\vec{p}_i^2 + \vec{p}_f^2)) C_3^{I \rightarrow F},$$

where $\alpha_{I \rightarrow F}$ and $\beta_{I \rightarrow F}$ are blinding factors, for the process I decaying to F .

- α between -0.5 and 0.5
- β between -1.0 and 1.0
- Different for each decay channel
- Momentum factor blinds q^2 dependence of form factor
- Allows partial unblinding for q_{max}^2 benchmark result

Excited States and Laplace Filtering

On the lattice we produce two and three-point functions.

$$C_2(t) = \sum_n A_n^2 e^{-E_n t}$$

$$C_3^{I \rightarrow F}(t_J, t_{\text{snk}}) = \sum_{n,m} A_n^F \langle F_n | J | I_m \rangle A_m^I e^{-E_n^F t_J - E_m^I (t_{\text{snk}} - t_J)}$$

- Operators induce 'all' excitations, get tower of states
- Analysis often limited by excited state contributions
- Laplace filtering mitigates this [2]:
 - Preserves exponential modes
 - Can tune to remove/suppress low excited states
 - Linear transform of mode amplitudes

$$D_\lambda C(t) = -C(t-1) + (2 + \lambda^2)C(t) - C(t+1)$$

$$D_\lambda C_2(t) = \sum_n (\lambda^2 - \tilde{E}_n^2) A_n^2 e^{-E_n t}$$

Analysis

Effective mass flatness is a proxy for ground state dominance. Laplace filter improves the onset to the plateau.

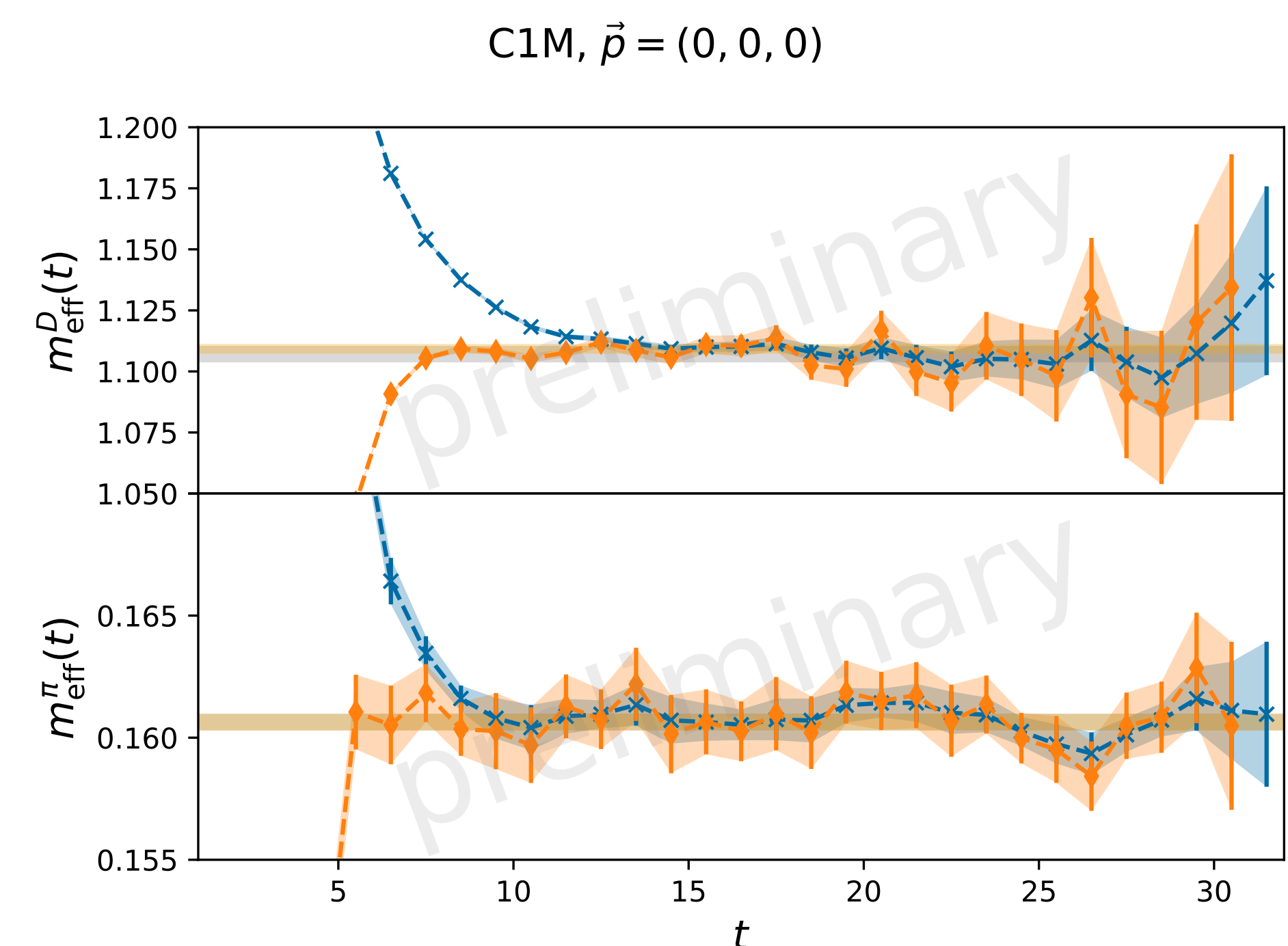


Figure 4: Effective masses for two-point functions. Original: blue crosses, Laplace filtered: orange diamonds.

Construct a suitable ratio which asymptotes to ground state matrix element.

$$R^{I \rightarrow F}(t_J, t_{\text{snk}}) := \frac{C_3^{I \rightarrow F}(t_J, t_{\text{snk}})}{C_2^F(t_J) C_2^I(t_{\text{snk}} - t_J)}$$

Filtering the two-point functions which enter the ratio improves approach to the ground state.

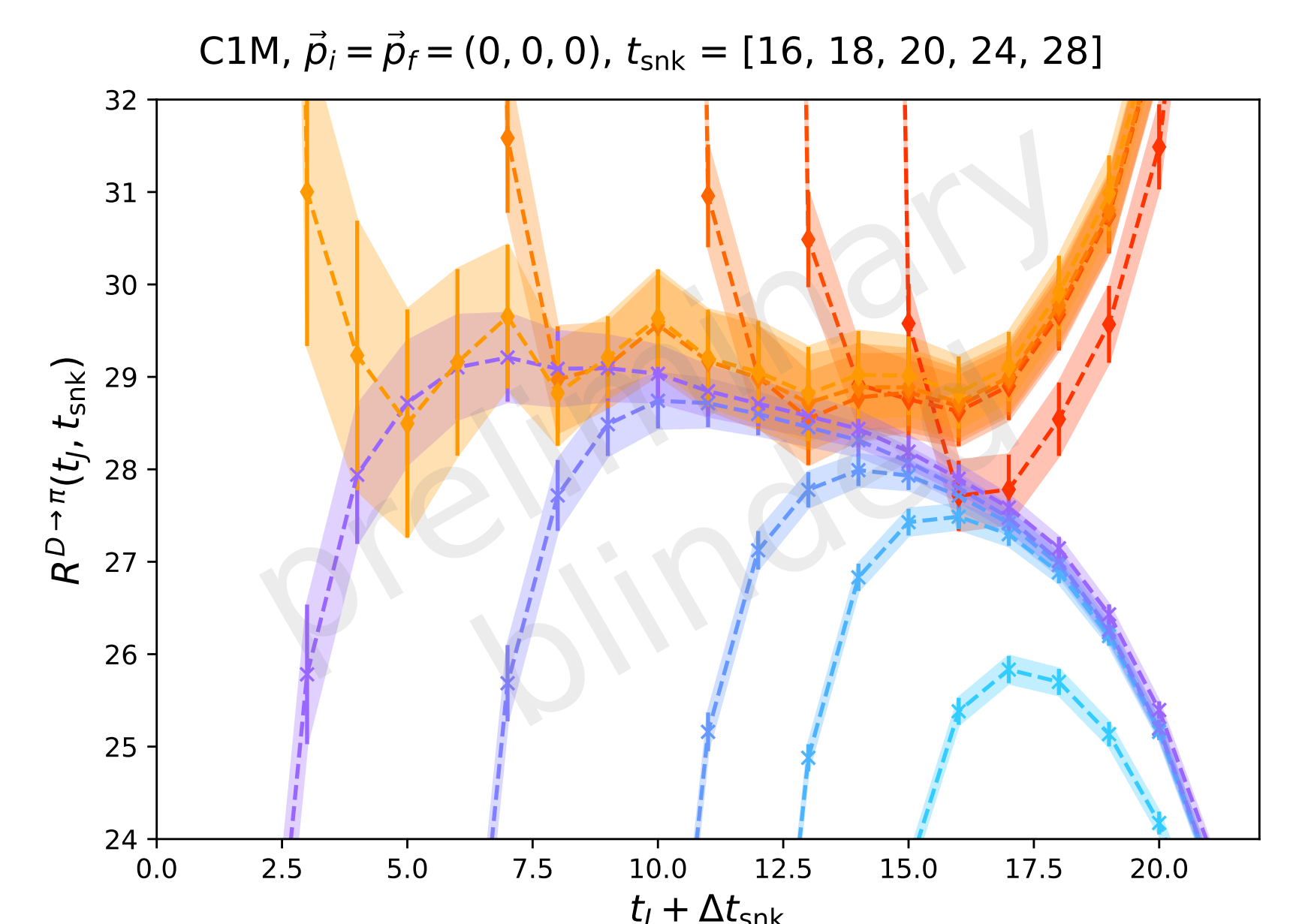


Figure 5: Ratios of three-point and two-point functions, corresponds to matrix element. Original data purple-blue crosses, Laplace filtered yellow-orange diamond.

Future Work

Many threads and people working in parallel, on multiple aspects and targets.

- Advance/systematise fitting via unified analysis code
- perform $a \rightarrow 0, L \rightarrow \infty, m_q \rightarrow m_q^{\text{phys}}$
- Calculation of q_{max}^2 as benchmark quantity [3].
- Full q^2 -dependence and determination of CKM.
- Extend analysis to vector final states
- $b \rightarrow l, s, c$ using RHQ b -quarks (data exists)

Acknowledgments

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This project employs the libraries Grid (github.com/paboyle/grid) and Hadrons (github.com/aportelli/hadrons).

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References

- [1] Y. Aoki et al. *Phys. Rev. D*. arXiv: 2411.04268 [hep-lat].
- [2] A. Portelli et al. *Phys. Rev. D*. arXiv: 2508.11541 [hep-lat].
- [3] J. T. Tsang et al. *Eur. Phys. J. ST*. arXiv: 2310.02705 [hep-lat].