Supersymmetric AdS Solitons and their Marginal Deformations

Madison Hammond Swansea University

Based on work with Dimitrios Chatzis, Georgios Itsios, Carlos Nunez, Dimitrios Zoakos,

[2506.10062, 2511.18128], + To appear

Motivation

- Study strongly interacting QFTs with features resembling QCD using holography.
- D-branes used with features like confinement, chiral symmetry breaking and a mass gap are accompanied by a singularity in the gravity dual ⇒ supergravity breaks down.
- In some cases the singularities are resolved, but the UV theory may be not well-defined (Wrapped brane setups).

Gauge/gravity duality tells us that the gravity side will reach singularities in the IR,

We aim to resolve the issue of singularities in the IR whilst still maintaining the UV field theory, we do this by constructing **Smooth** asymptotically AdS deformed backgrounds dual to SCFT_4 in the UV, which flows to a gapped & confining SQFT_3 in the IR.
Implement a twisted compactification and use the soliton solution containing a metric, 2 scalar fields and 3 U(1) gauge fields

$$\mathrm{d}s_5^2 = \frac{r^2\lambda^2(r)}{L^2}\Big(- \mathrm{d}t^2 + \mathrm{d}z^2 + \mathrm{d}w^2 + L^2 F(r)\overbrace{\mathrm{d}\phi^2}^{S^1}\Big) + \frac{\mathrm{d}r^2}{r^2\lambda^4(r)F(r)},$$

$$\Phi_1 = \sqrt{\frac{2}{3}}\ln\lambda^{-6}(r), \quad \Phi_2 = 0, A^1 = A^2 = Q\left[\lambda^6(r) - \lambda^6(r_\star)\right] L\,\mathrm{d}\phi, \quad A^3 = Q\left[\frac{1}{\lambda^6(r)} - \frac{1}{\lambda^6(r_\star)}\right] L\,\mathrm{d}\phi,$$

$$F(r) = \frac{1}{L^2} - \frac{\varepsilon Q^2\ell^2 L^2}{r^4}\left[1 - \lambda^{-6}(r)\right], \quad \lambda^6(r) = \frac{r^2 + \varepsilon\ell^2}{r^2}, \quad \varepsilon = \pm 1.$$

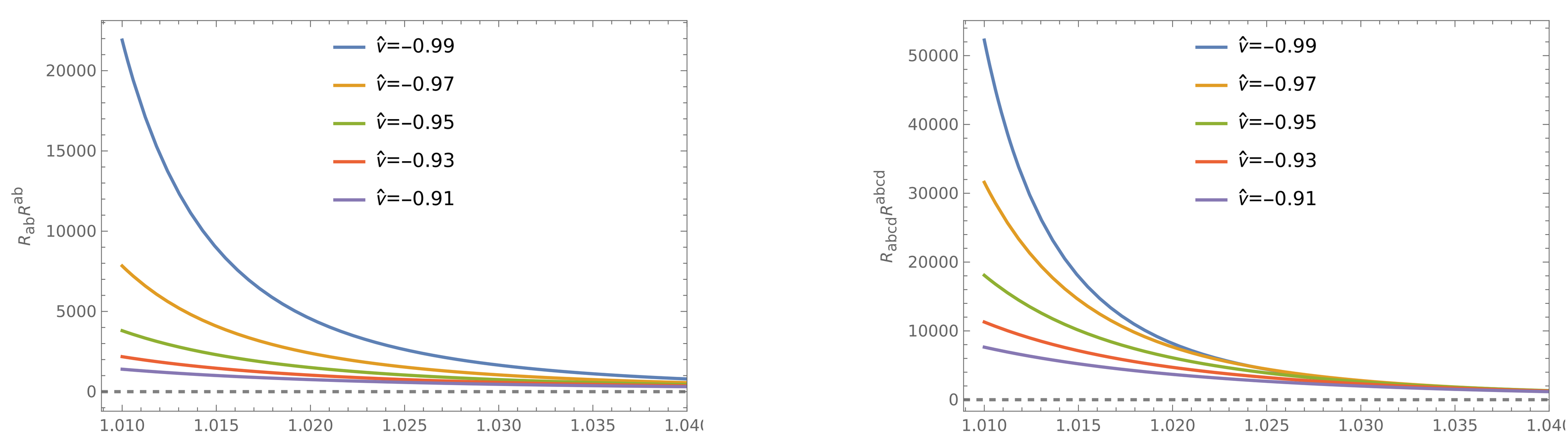
- Cigar like solution, $F(r_\star) = 0$
- As $r \to r_\star$ $S^1[\phi] \to 0$ and the space terminates smoothly.
- As $r \to \infty$ we recover AdS_5
- $\mathrm{U}(1)^3_{\mathcal{R}}$ made explicit with the isometries of $S^1[\phi_i]$.
- Twisted compactification: $\mathrm{D}\phi_i = \mathrm{d}\phi_i + L^{-1}A^i_\phi \mathrm{d}\phi$.
- Mixing of $\mathrm{U}(1)_{\mathcal{R}}$ with $\mathrm{SO}(2)_{\text{Poincaré}}$, results in partial SUSY preservation, preserves four supercharges

Wilson Loops and the Geometry

The background is smooth however near the singularity, the geometry invariants become finite but very large and the Wilson loop detects these effects. To see this, define dimensionless quantities:

$$\xi = \frac{r}{r_\star} \geq 1, \quad \hat{\nu} = \varepsilon\frac{\ell^2}{r_\star^2} \geq -1, \quad F(\xi) = \frac{(\xi^2 - 1)\left(\xi^2 - \xi_+^2\right)\left(\xi^2 - \xi_-^2\right)}{L^2\xi^4(\hat{\nu} + \xi^2)} \Rightarrow \quad \xi_\pm^2 = \frac{-(1+\hat{\nu}) \pm \sqrt{(\hat{\nu}+1)(\hat{\nu}-3)}}{2}$$

For $\hat{\nu} \in (-1, \infty)$ no real roots $\xi_\pm$. Singularity when $\hat{\nu} = -1$ at $r = r_\star = \ell$



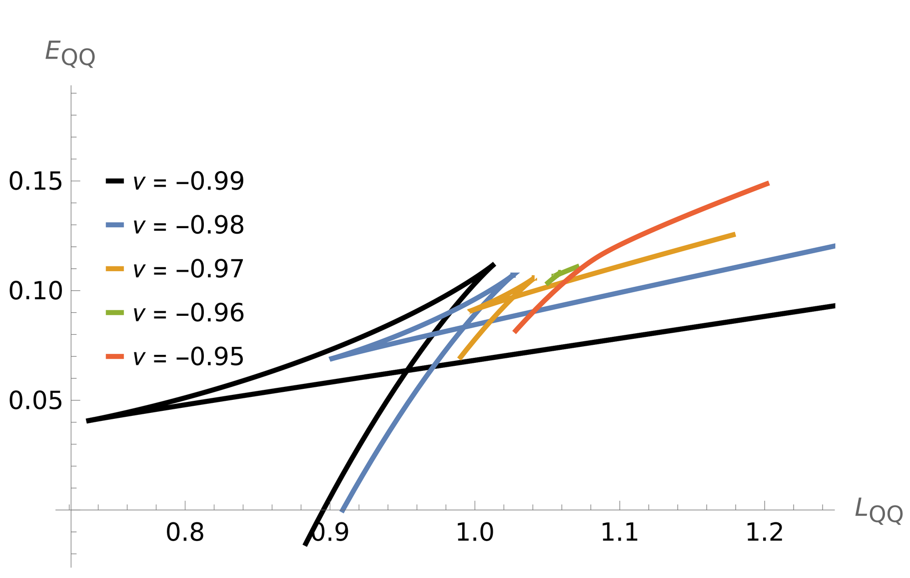
Plots of $R_{ab}R^{ab}$ vs ξ for $\hat{\nu} \approx -1$ and $R_{abcd}R^{abcd}$ vs ξ for $\hat{\nu} \approx -1$.

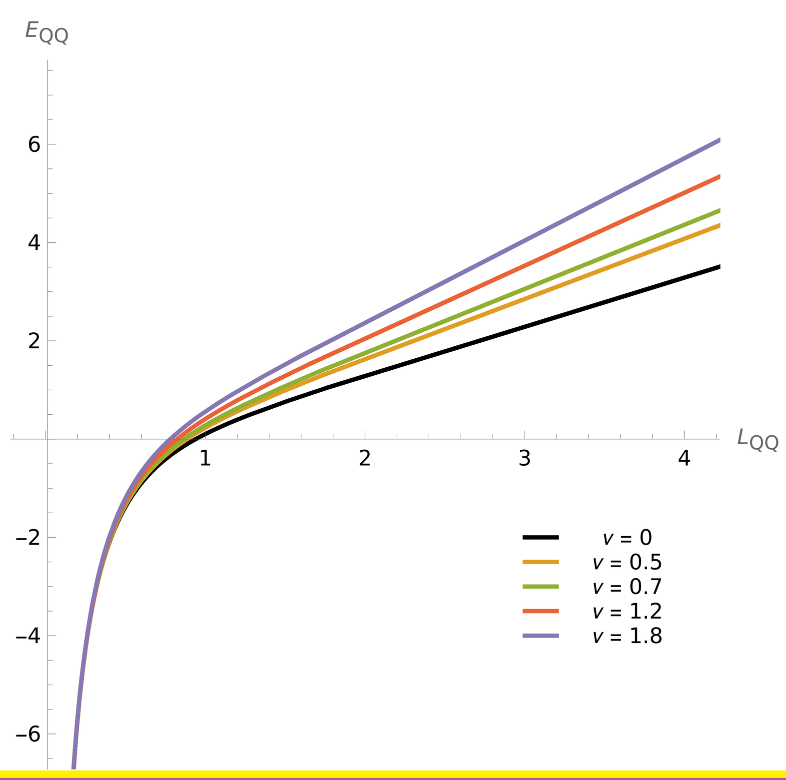
The curvature invariants are **finite** but grow extremely large as $\hat{\nu} \to -1^+$, so supergravity needs higher curvature corrections in the action in this region at order of R.

$$r = r(w): \quad \mathrm{S}_{\mathrm{NG}} = \frac{1}{2\pi}\int \mathrm{d}t\mathrm{d}w\sqrt{-\mathrm{det}g_{\mathrm{ind}}} = \frac{\mathcal{T}}{2\pi}\int_{-L_{QQ}/2}^{L_{QQ}/2}\mathrm{d}w\sqrt{\mathcal{F}^2 + \mathcal{G}^2 r'^2} \qquad \mathcal{F} = \frac{r^2\lambda^3(r)}{L^2}, \quad \mathcal{G} = \frac{1}{L\sqrt{F(r)}}$$

The Wilson loops have **universal behaviour** $\mathrm{S}_{\mathrm{IIB,IIA,M}} = \hat{\mathcal{N}}_{\mathrm{IIB,IIA,M}} \times \left(\int_{-L_{QQ}/2}^{L_{QQ}/2}\mathrm{d}w\sqrt{\mathcal{F}^2 + \mathcal{G}^2\mathbf{r}'^2}\right)$

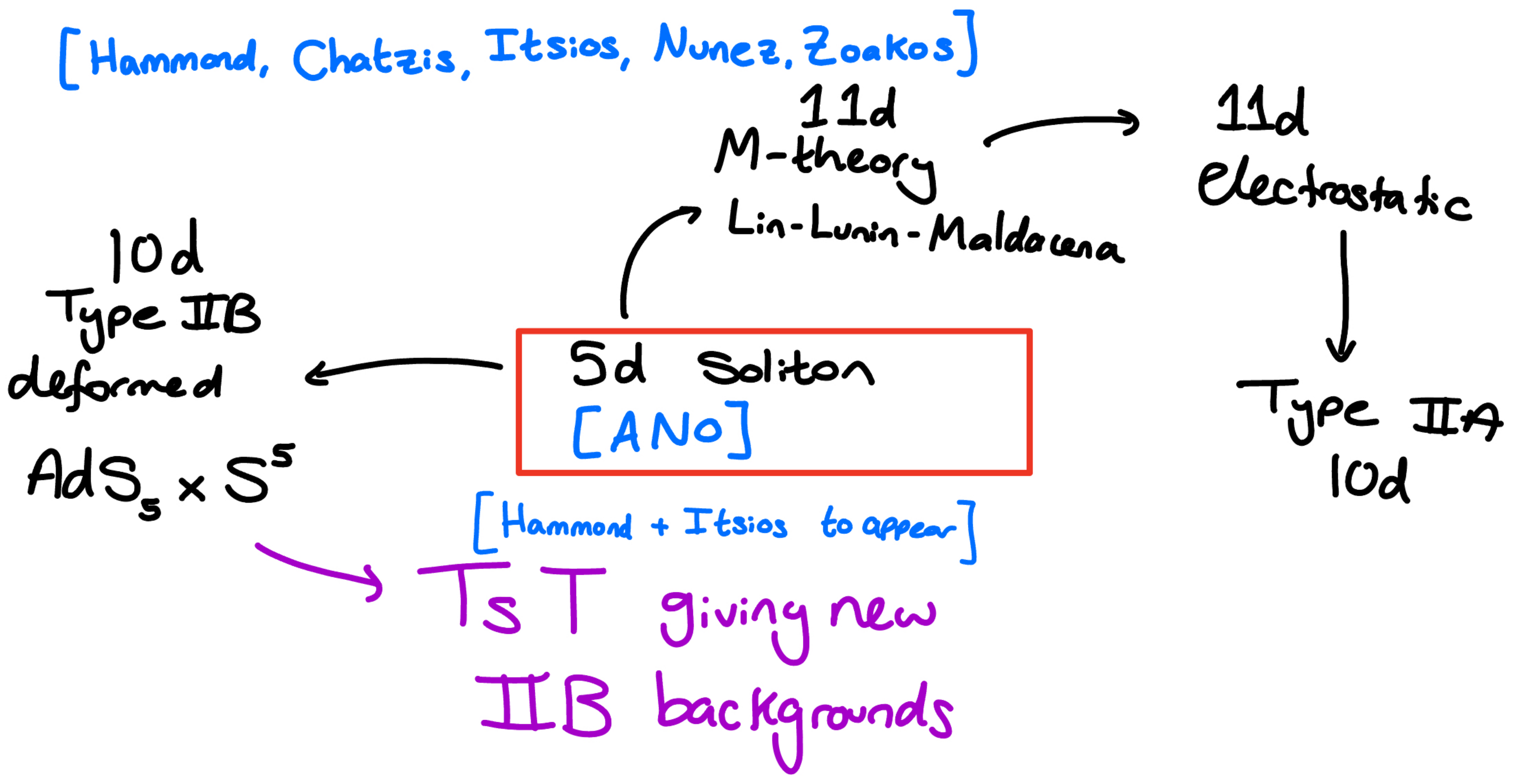
Background is **smooth** for $\hat{\nu} > -1$, the Wilson loop detects higher curvature corrections needed for $\hat{\nu} \approx -1$.





New Backgrounds

We uplift this $5d$ supersymmetric soliton and create marginal deformations of it via the TsT procedure



IIB Background: Dual to a deformation of $\mathcal{N} = 4$ SYM_4 flowing to $\mathcal{N} = 2$ QFT_3

$$\mathrm{d}s_{10}^2 = \overbrace{\frac{\zeta(r,\theta)}{L^2}\left[r^2(-\mathrm{d}t^2 + \mathrm{d}w^2 + \mathrm{d}z^2 + L^2F(r)\mathrm{d}\phi^2) + \frac{L^2\mathrm{d}r^2}{F(r)r^2\lambda^3(r)}\right]}^{\text{deformed AdS}_5\ [t,w,z,\phi]}$$
$$\underbrace{+\frac{L^2}{\zeta(r,\theta)}\left[\zeta(r,\theta)^2\mathrm{d}\theta^2 + \cos^2\theta\mathrm{d}\psi^2 + \cos^2\theta\sin^2\psi\mathrm{D}\phi_1^2 + \cos^2\theta\cos^2\psi\mathrm{D}\phi_2^2 + \lambda^6(r)\sin^2\theta\mathrm{D}\phi_3^2\right]}_{\text{deformed } S^5\ [\theta,\psi,\phi_1,\phi_2,\phi_3]},$$

$$F_5 = \star F_5$$

Uplift to 11d: $\mathrm{AdS}_5 \times \mathcal{M}_6$ solutions in M-theory with $\mathrm{SU}(2)_{\mathcal{R}} \times \mathrm{U}(1)_{\mathcal{R}}$ dual to non-Lagrangian $\mathcal{N} = 2$ CFT_4:

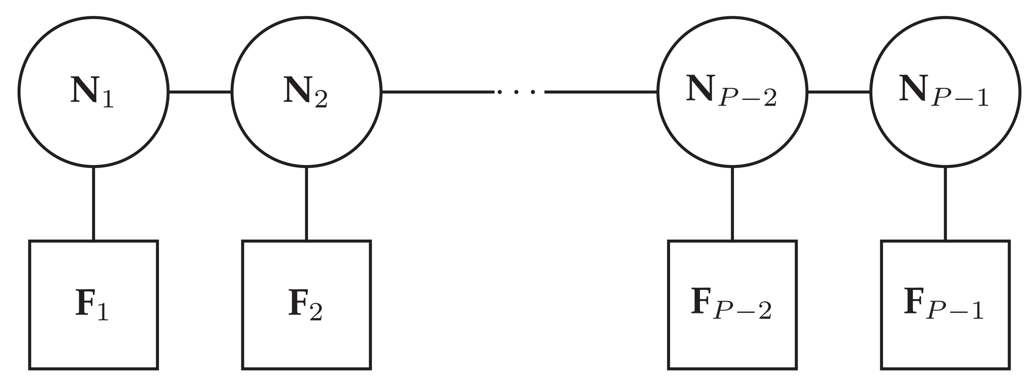
$$\frac{\mathrm{d}s_{11}^2}{\kappa^{2/3}} = e^{2\hat{\lambda}}\mathcal{Z}\Big\{\frac{4}{X}\mathrm{d}s_5^2 + \frac{y^2e^{-6\hat{\lambda}}}{\mathcal{Z}^3}\overbrace{\mathrm{D}\mu^i\mathrm{D}\mu^i}^{\mathbb{S}^2[\theta,\varphi]} + \frac{4X^3}{\mathcal{Z}^3}\frac{\overbrace{\mathrm{D}\chi^2}^{\mathbb{S}^1}}{1 - y\partial_y D_0} - \frac{\partial_y D_0}{y}\Big[\underbrace{\mathrm{d}y^2}_{\text{compact}} + e^{D_0}\underbrace{(\mathrm{d}v_1^2 + \mathrm{d}v_2^2)}_{\text{Riemann surface}}\Big]\Big\},$$

$$G_4, \qquad \mathcal{Z} = \mathcal{Z}(X, \hat{\lambda}, r), \quad X = \lambda^2(r), \quad \hat{\lambda} = \hat{\lambda}(\partial_y D_0), \qquad \nabla^2_{(v_1,v_2)}D_0 + \partial_y^2 e^{D_0} = 0$$

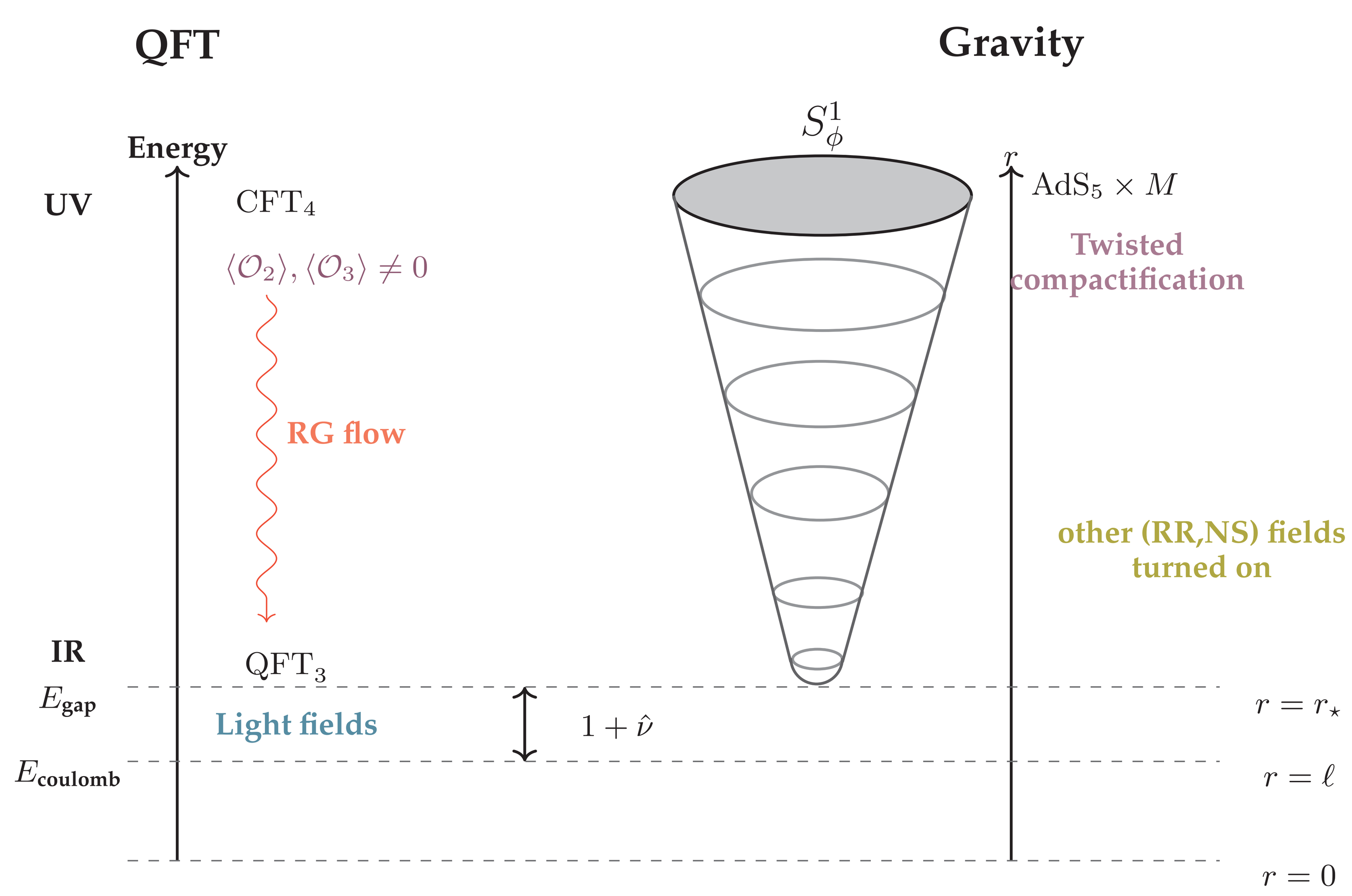
- Solution fully determined by $D_0(v_1, v_2, y)$.

Electrostatic Notation: IIA Background When there is an extra SO(2) isometry we exchange
$(v_1, v_2) \leftrightarrow (r, \beta) \qquad D_0(r,\beta,y) \mapsto V(\sigma,\eta), \quad \sigma\partial_\sigma(\sigma\partial_\sigma V) + \sigma^2\partial_\eta^2 V = 0$ & b.c.
Laplace-like equation, easier to handle, reduce in ∂_β to give the type IIA solution:

$$\mathrm{d}s_{10}^2 = \tilde{f}_1^{\frac{3}{2}}\tilde{f}_5^{\frac{1}{2}}\left[4\tilde{f}\mathrm{d}s_5^2 + \tilde{f}_2\mathrm{D}\mu^i\mathrm{D}\mu^i + \tilde{f}_3\mathrm{D}\chi^2 + \tilde{f}_4(\mathrm{d}\sigma^2 + \mathrm{d}\eta^2)\right],$$
$$\& \ e^{\frac{4}{3}\Phi} = \tilde{f}_1\tilde{f}_5, \ldots, \qquad \tilde{f}_i = \tilde{f}_i(r, V, \textbf{derivatives})$$



Deformed versions of Gaiotto-Maldacena backgrounds dual to $\mathcal{N} = 2$ linear quivers in $4d$ flowing to $\mathcal{N} = 1$ QFT_3 in the IR.



TsT procedure giving Marginal Deformation of the IIB Background:

- **Step 1: Θ_1, Θ_2 coordinates of the spacetime geometry. Perform a T-duality along Θ_1.**
- **Step 2: Implement the coordinate shift $\Theta_2 \to \Theta_2 + \gamma\,\Theta_1$, where γ is a real deformation parameter.**
- **Step 3: Perform a second T-duality along Θ_1.**
- **T-dualities along ϕ_1, shift along ϕ_2, T-duality along ϕ_1,** $W(r,\theta,\psi) = \sqrt{1 + \gamma^2 L^4\frac{\cos^4\theta\,\cos^2\psi\,\sin^2\psi}{\zeta^2}}$.

$$ds^2 = \frac{\zeta}{L^2}\left(r^2\left(-dt^2 + dw^2 + dz^2 + L^2F\,d\phi^2\right) + L^2\frac{dr^2}{r^2F\lambda^6} + L^4 d\theta^2\right) + \frac{L^2}{\zeta}\Big(\cos^2\theta\,d\psi^2$$
$$+\frac{\cos^2\theta\,\sin^2\psi}{W^2}\left(d\phi_1 + \frac{A_1}{L}\right)^2 + \frac{\cos^2\theta\,\cos^2\psi}{W^2}\left(d\phi_2 + \frac{A_2}{L}\right)^2 + \lambda^6\sin^2\theta\left(d\phi_3 + \frac{A_3}{L}\right)^2\Big).$$

NS sector of the deformed solution contains a non-trivial dilaton and two-form

$$\Phi = -\ln W, \qquad B_2 = -\sqrt{W^2 - 1}\,\frac{L^2\cos^2\theta\sin\psi\cos\psi}{W^2\zeta}\left(d\phi_1 + \frac{A_1}{L}\right) \wedge \left(d\phi_2 + \frac{A_2}{L}\right).$$