2504.05209

# Exploring Hadronic $B$ decays through $SU(3)_F$ symmetry and factorizable corrections

**Marta Burgos Marcos**

In collaboration with: **Keri Vos (Maastricht University - Nikhef)**

**Méril Reboud (CNRS, IJCLab, Orsay)**

Nonleptonic Decays of Heavy Mesons Workshop, March 2026

# Why looking at hadronic $B$ meson decays?

## Experimentally

- $B \to hh$ decays abundant at LHCb, Belle II, …
- **Plenty** of available **experimental data.** Especially CP asymmetries
- **Precision is improving significantly**: theoretical models can be contrasted

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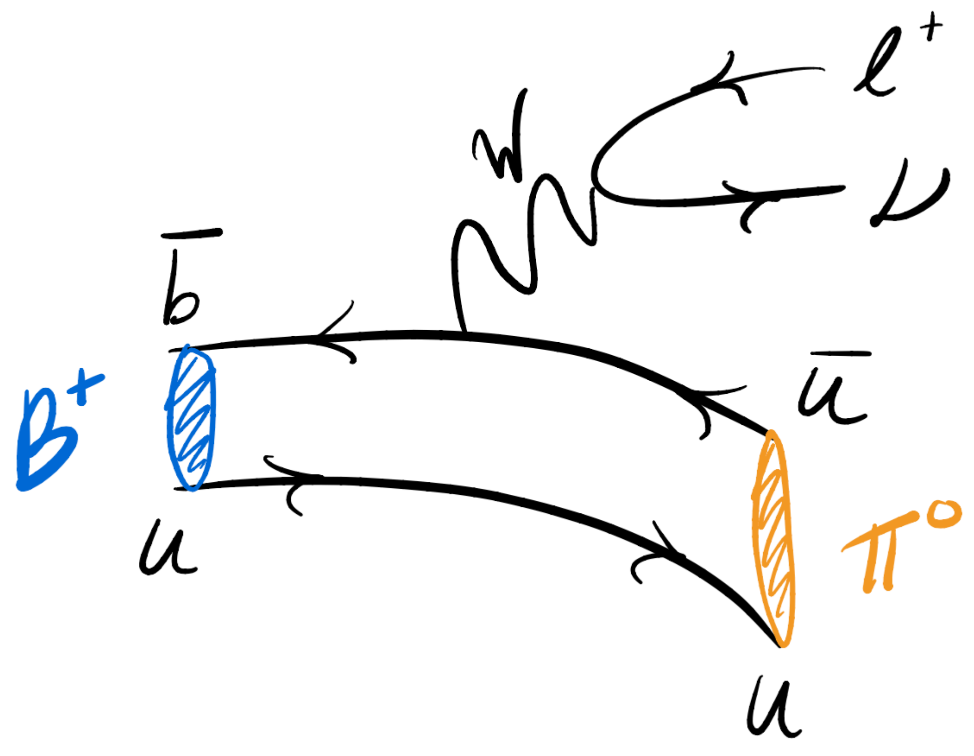
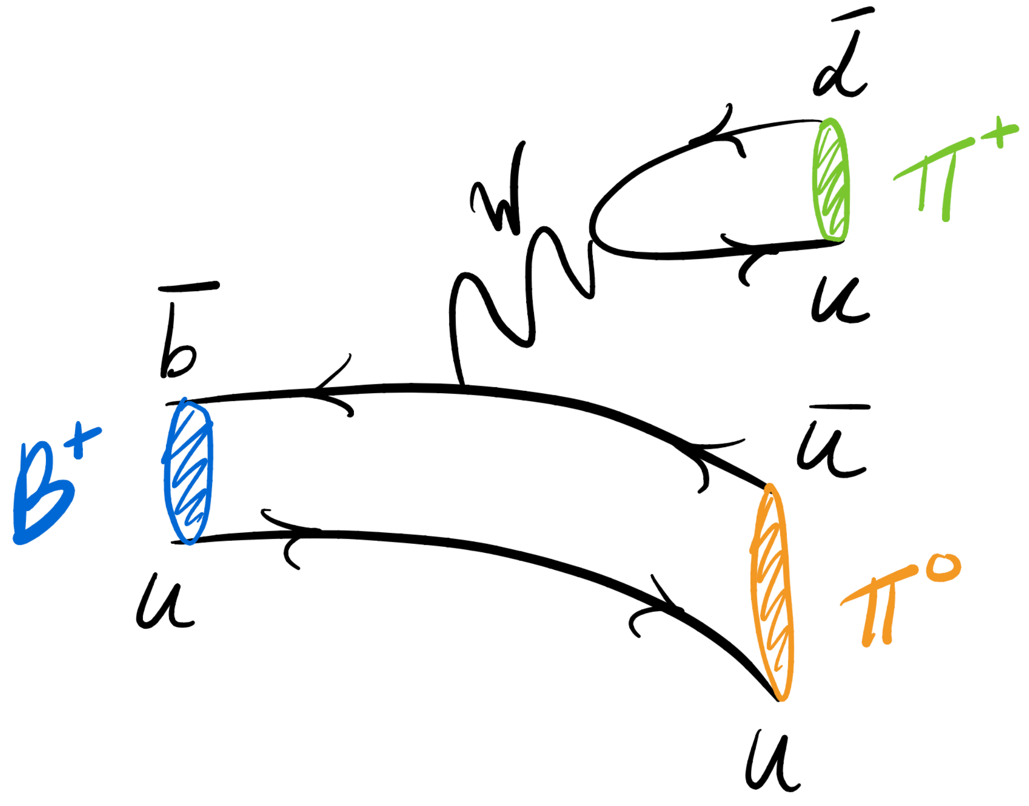
## Theoretically

- Theoretical predictions governed by **uncertainties:**

**Semileptonic** **vs.** **Hadronic**

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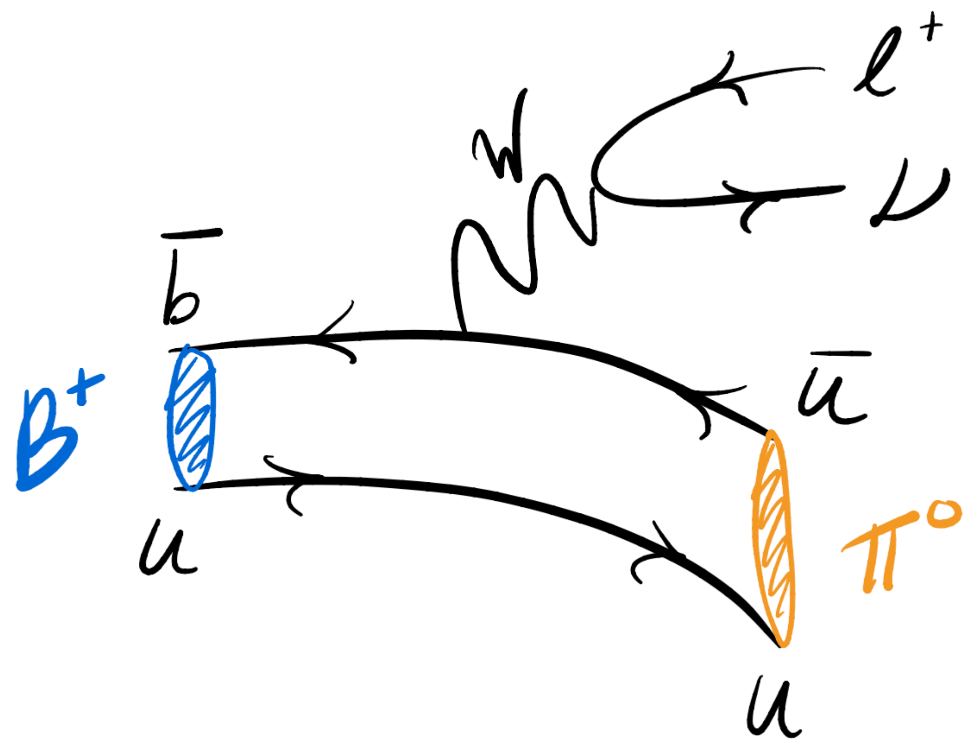


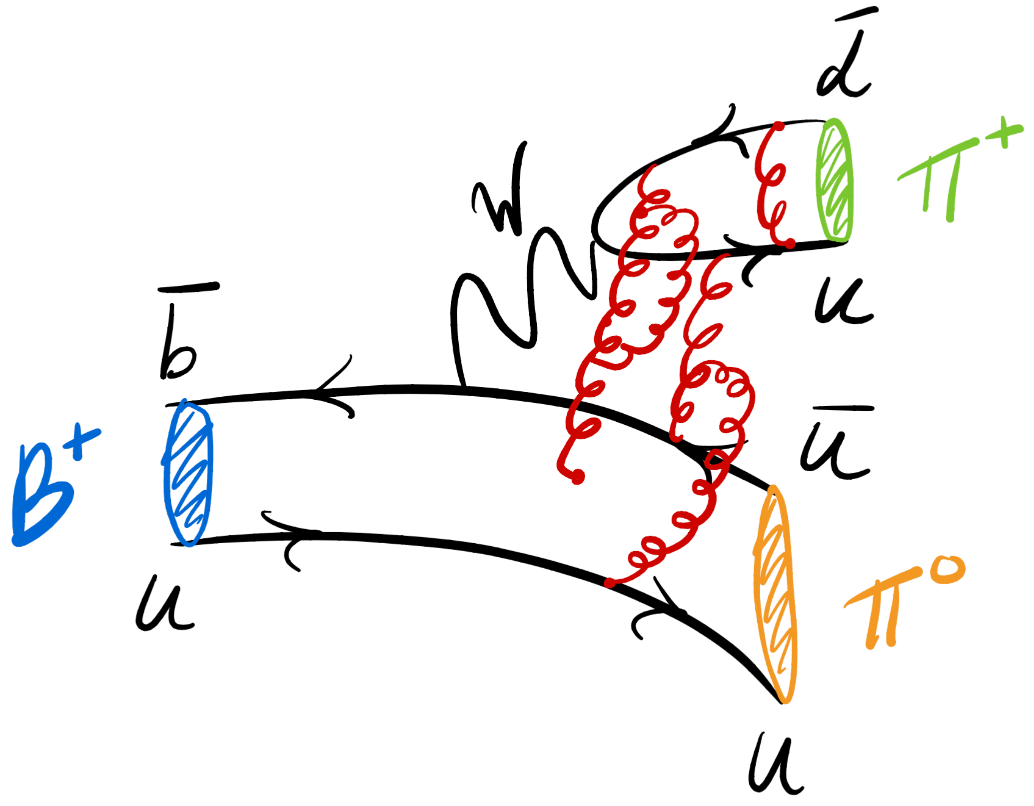


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# How can we describe $B \to PP$ decays?

# 1. Topological parametrization

Parametrize all $B \to PP$ decays in terms of topological coefficients

Used in e.g. Gronau, Hernández, London and Rosner 9504326
Huber and Tetlalmatzi-Xolocotzi 2111.06418
Berthiaume et al 2311.18011
Bhattacharya et al 2505.11492

→

Each coefficient represents a different topology

Equivalent to $SU(3)_F$ irreducible rep.:
He, Wang 1803.04227v1

Any two-body $B$ decay can be expressed as: $A(B \to PP) = \lambda_u^{(q)} A_u + \lambda_c^{(q)} A_c + \lambda_t^{(t)} A_t$

$$\lambda_i^{(q)} = V_{ib}^* V_{iq}$$

$$q = d, s$$

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**CKM unitarity!**

$$\lambda_u^{(q)} + \lambda_c^{(q)} + \lambda_t^{(q)} = 0$$

$$\lambda_i^{(q)} = V_{ib}^* V_{iq}$$

$$q = d, s$$

For every **tree** topology contributing to a decay we have its **penguin** counterpart:

$$A(B \to PP) = \lambda_u^{(q)} \mathcal{T} + \lambda_c^{(q)} \mathcal{P}$$

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**Tree amplitude**

$$\mathscr{T} = \mathbf{T}\ B_i (M)^i_j \bar{H}^{jl}_k (M)^k_l + \mathbf{C}\ B_i (M)^i_j \bar{H}^{lj}_k (M)^k_l + \mathbf{A}\ B_i \bar{H}^{il}_j (M)^j_k (M)^k_l$$
$$+ \mathbf{E}\ B_i \bar{H}^{li}_j (M)^j_k (M)^k_l + \mathbf{T_P} B_i (M)^i_j (M)^j_k \bar{H}^{lk}_l + \mathbf{T_{PA}} B_i \bar{H}^{li}_l (M)^j_k (M)^k_j$$

**Penguin amplitude**

$$\mathscr{P} = \mathbf{P_T}\ B_i (M)^i_j \tilde{H}^{jl}_k (M)^k_l + \mathbf{P_C}\ B_i (M)^i_j \tilde{H}^{lj}_k (M)^k_l + \mathbf{P_A}\ B_i \tilde{H}^{li}_l (M)^j_k (M)^k_j$$
$$+ \mathbf{P_{TE}}\ B_i \tilde{H}^{ji}_k (M)^k_l (M)^l_j + \mathbf{P}\ B_i (M)^i_j (M)^j_k \tilde{H}^k + \mathbf{P_{TA}}\ B_i \tilde{H}^{il}_j (M)^j_k (M)^k_l$$

**B-meson vector**

$$B_i = (B^+, B^0, B^0_s)$$

**Pseudo-scalar meson octet + singlet**

$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^- & K^- \\ \pi^+ & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \bar{K}^0 \\ K^+ & K^0 & -2\frac{\eta_8}{\sqrt{6}} \end{pmatrix} + \begin{pmatrix} \frac{\eta_0}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{\eta_0}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{\eta_0}{\sqrt{3}} \end{pmatrix}$$

**Flavor tensor**

$$H^{j,k}_i$$

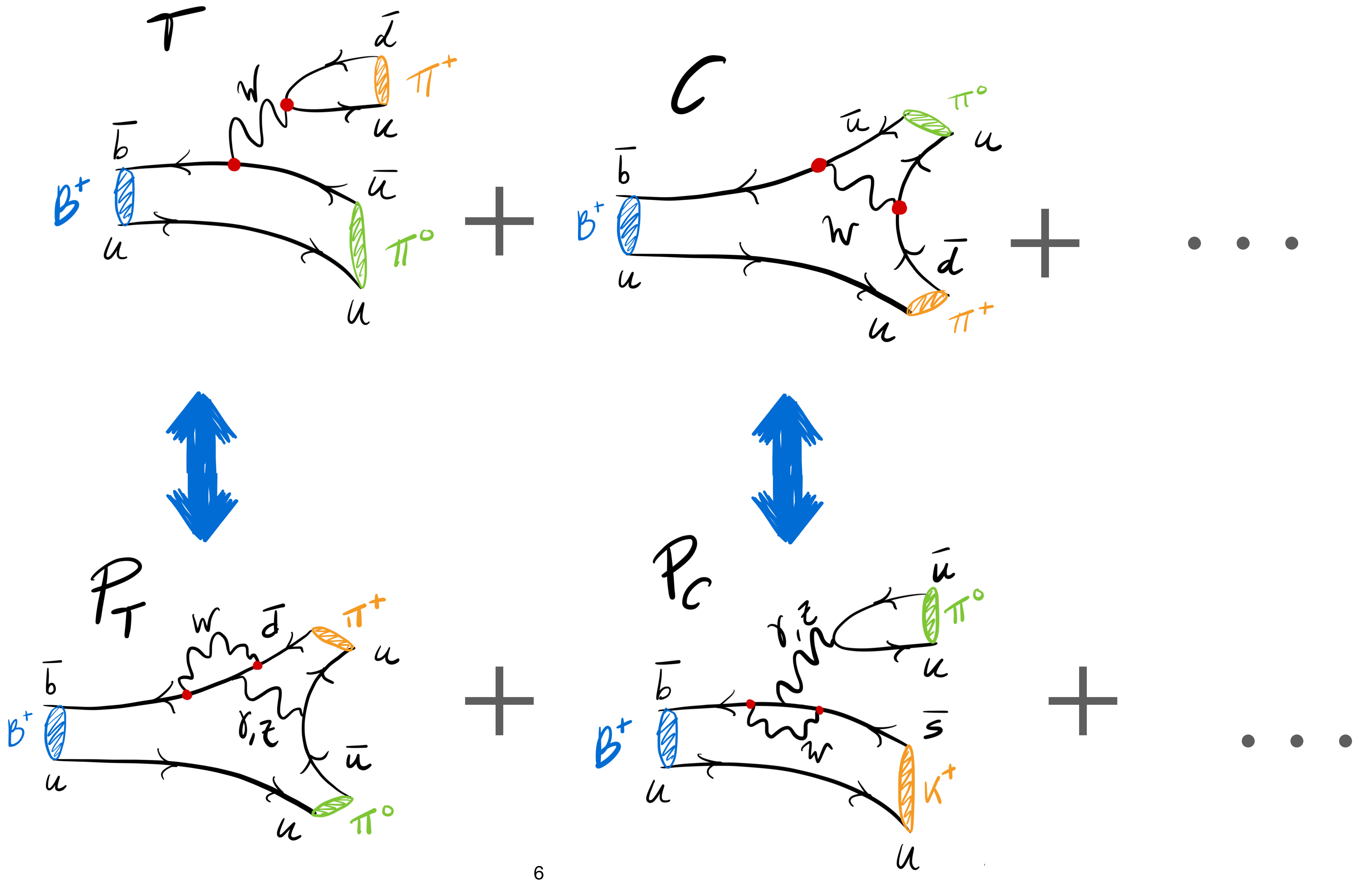
# 1. Topological parametrization

**Tree amplitude** :

$\mathcal{T} \sim V_{ub}^* V_{uq}$

**Penguin amplitude** :

$\mathcal{P} \sim V_{cb}^* V_{cq}$



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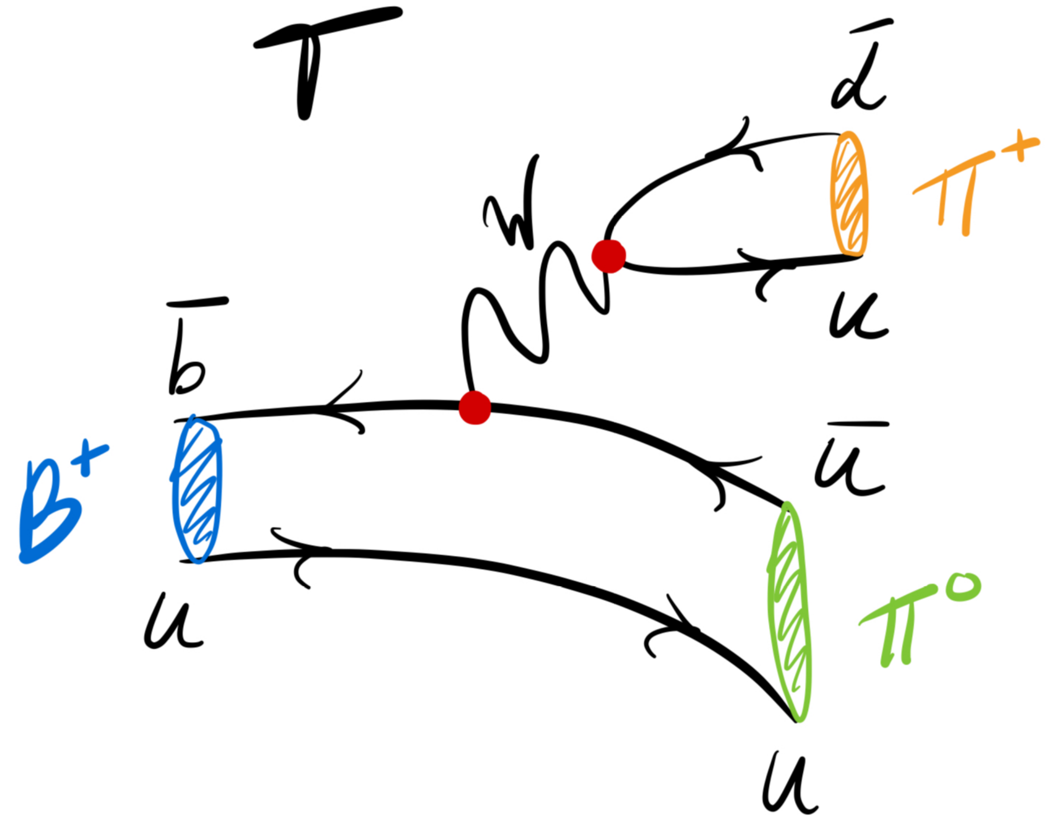
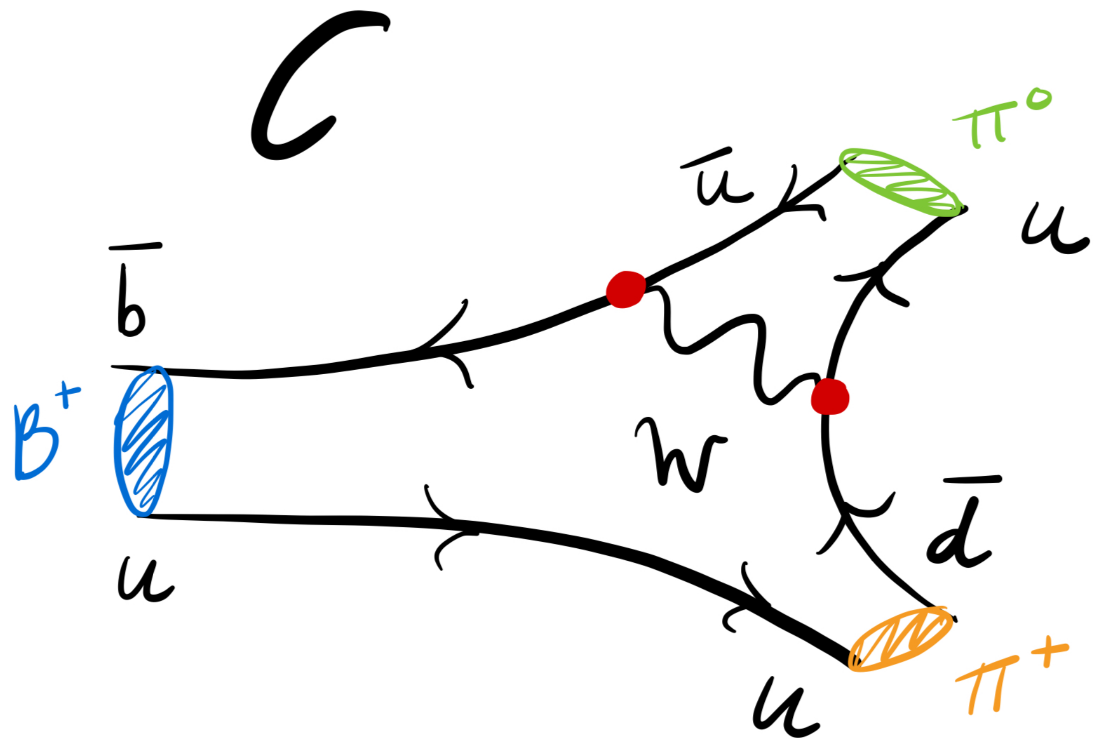
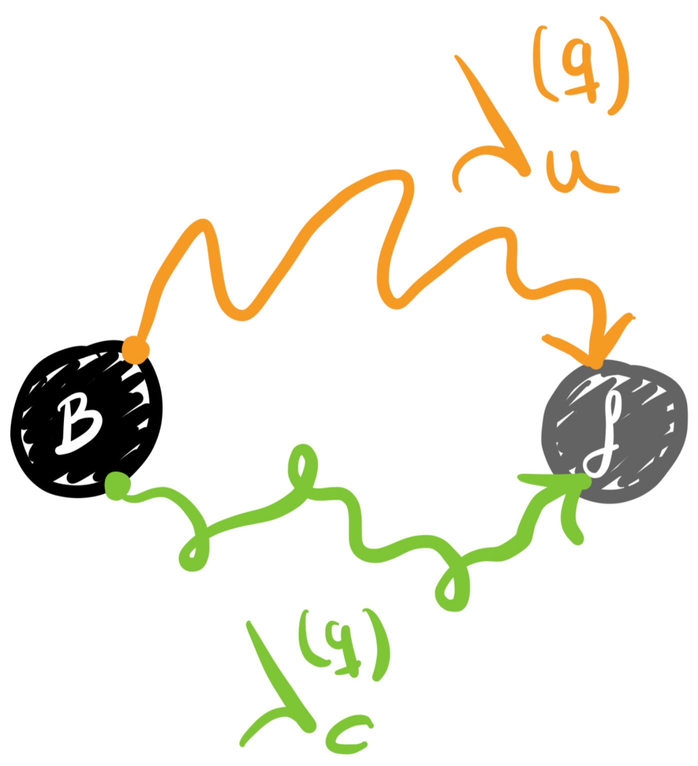
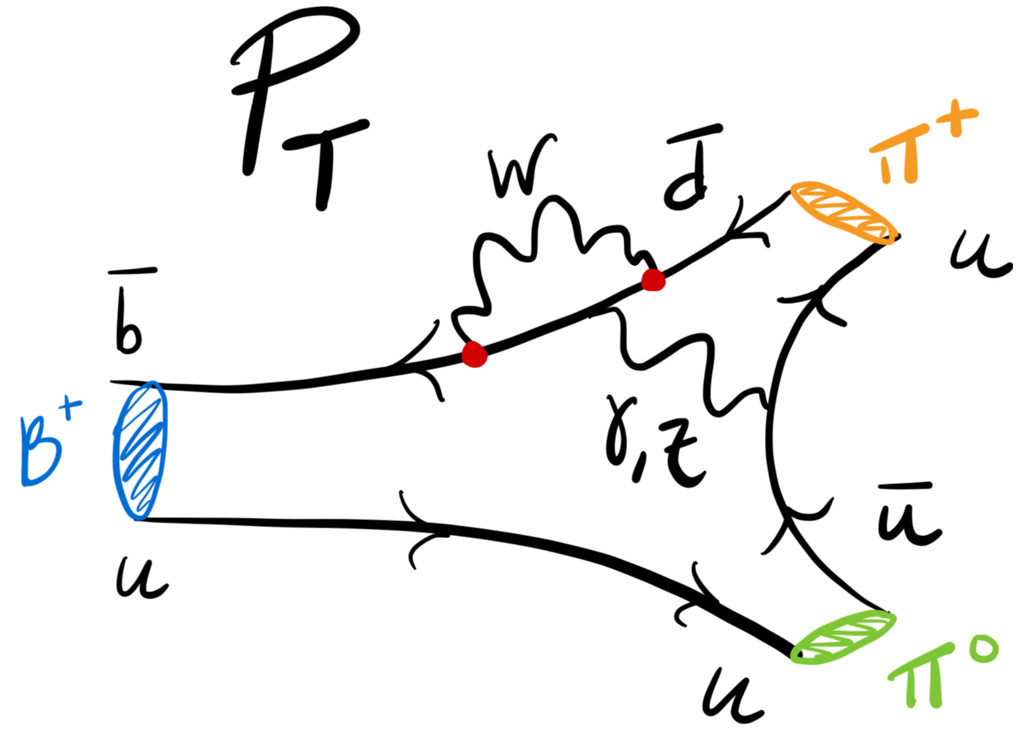


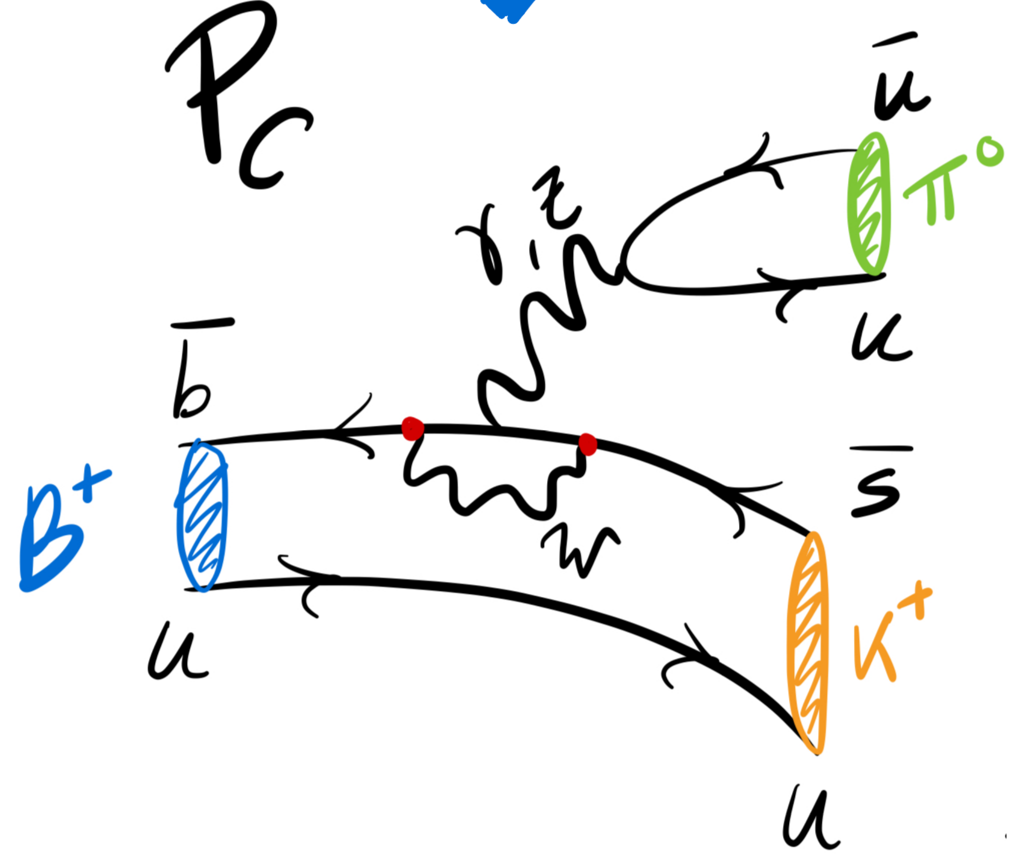



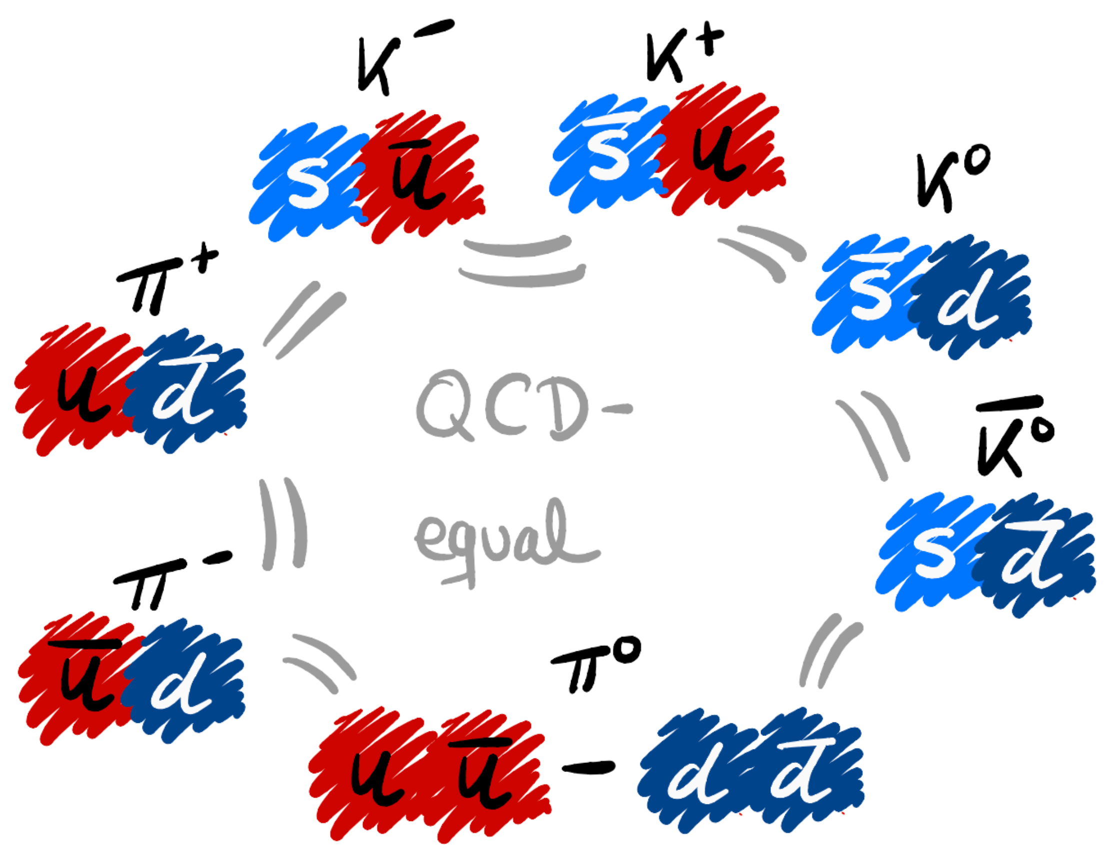

Amplitude interference
**necessary for CP Violation**

**Penguin amplitude**
$\mathcal{P} \sim V^*_{cb} V_{cq}$



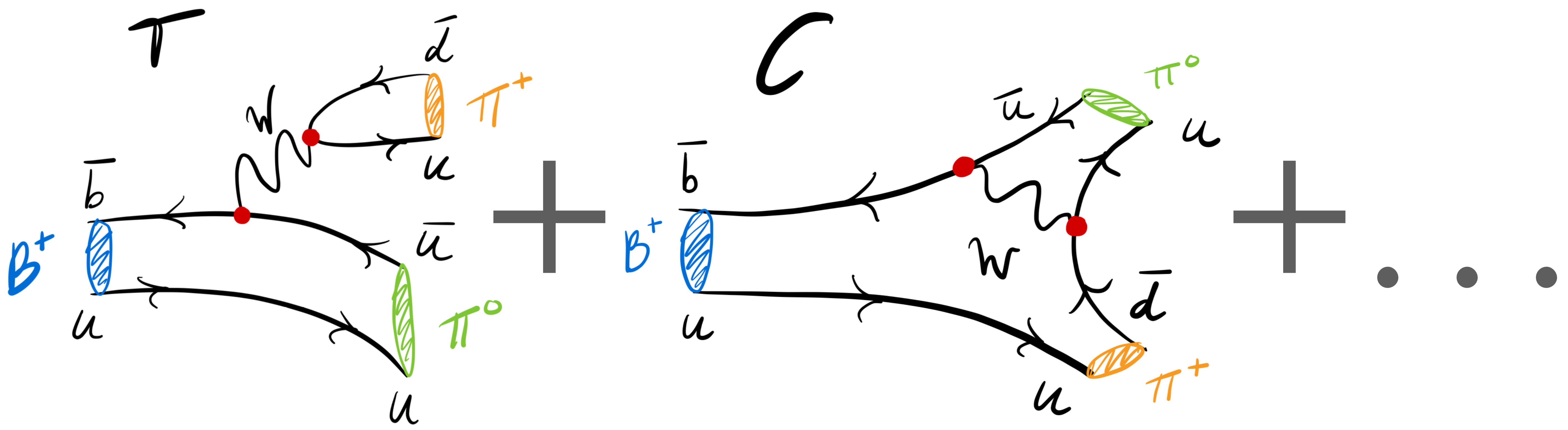



# 2. $SU(3)_F$ Symmetry




We can relate same coefficients in different decays:

$$B^+ \to \pi^0 \pi^+$$




$$A(B^+ \to \pi^0 \pi^+) = V_{ub}^* V_{ud} (T + C + \dots)$$

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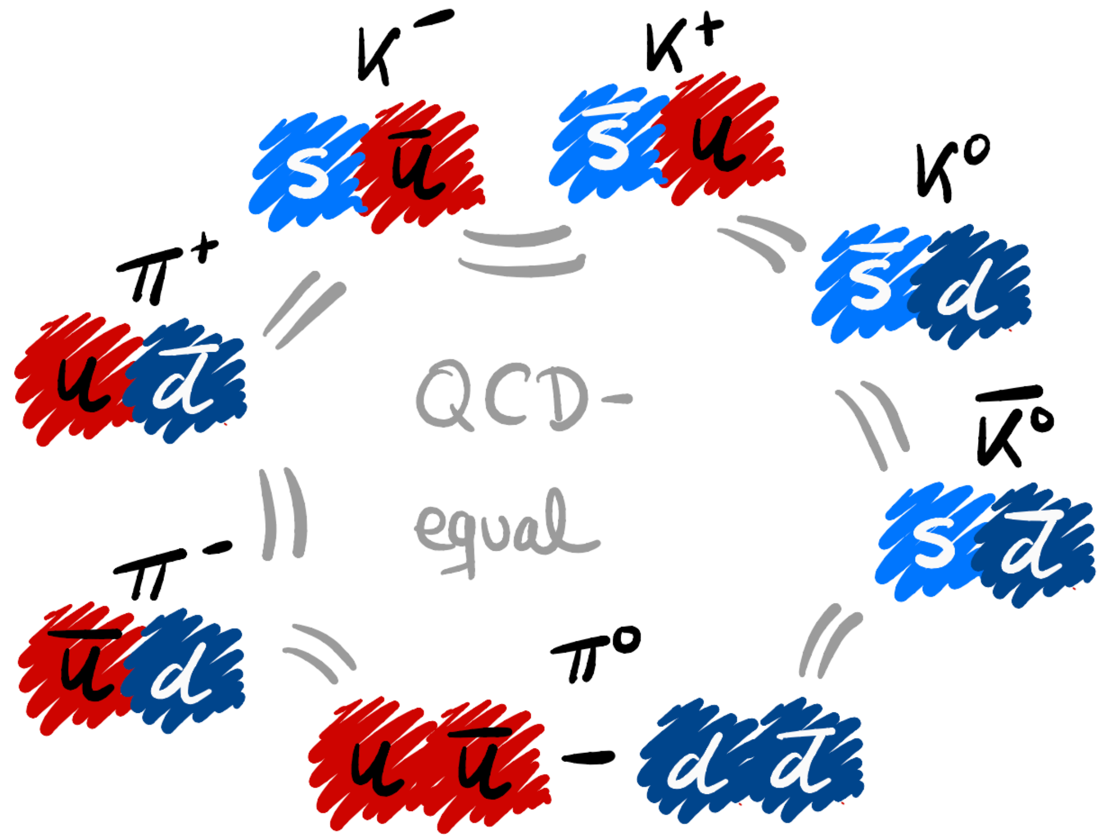


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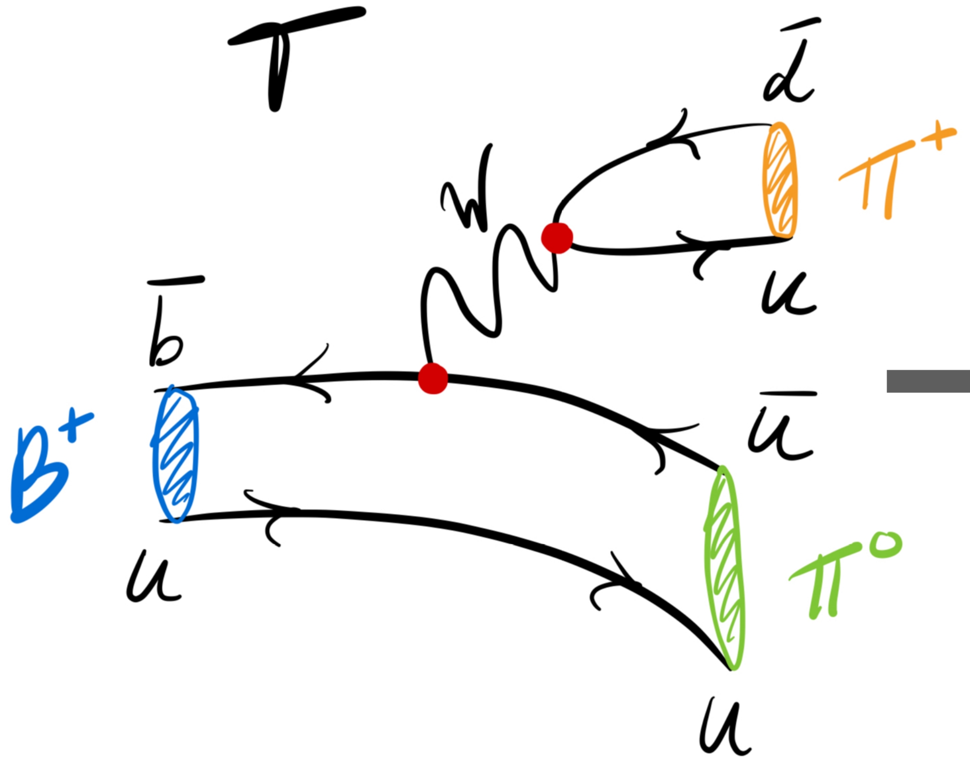
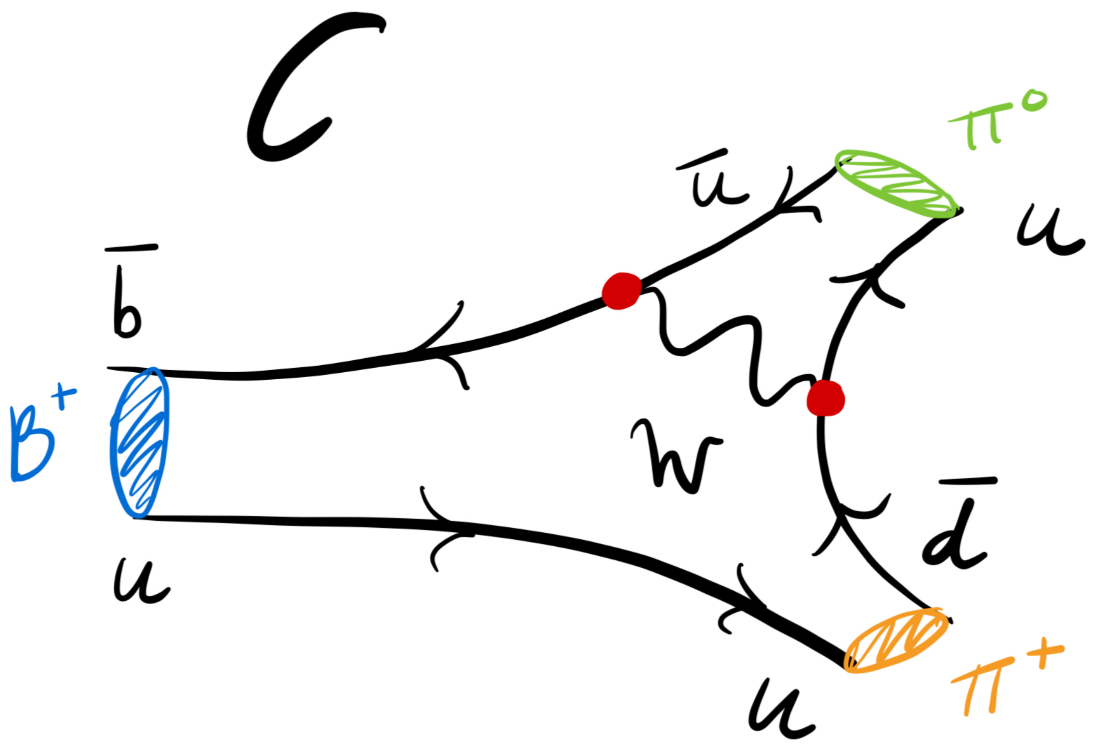
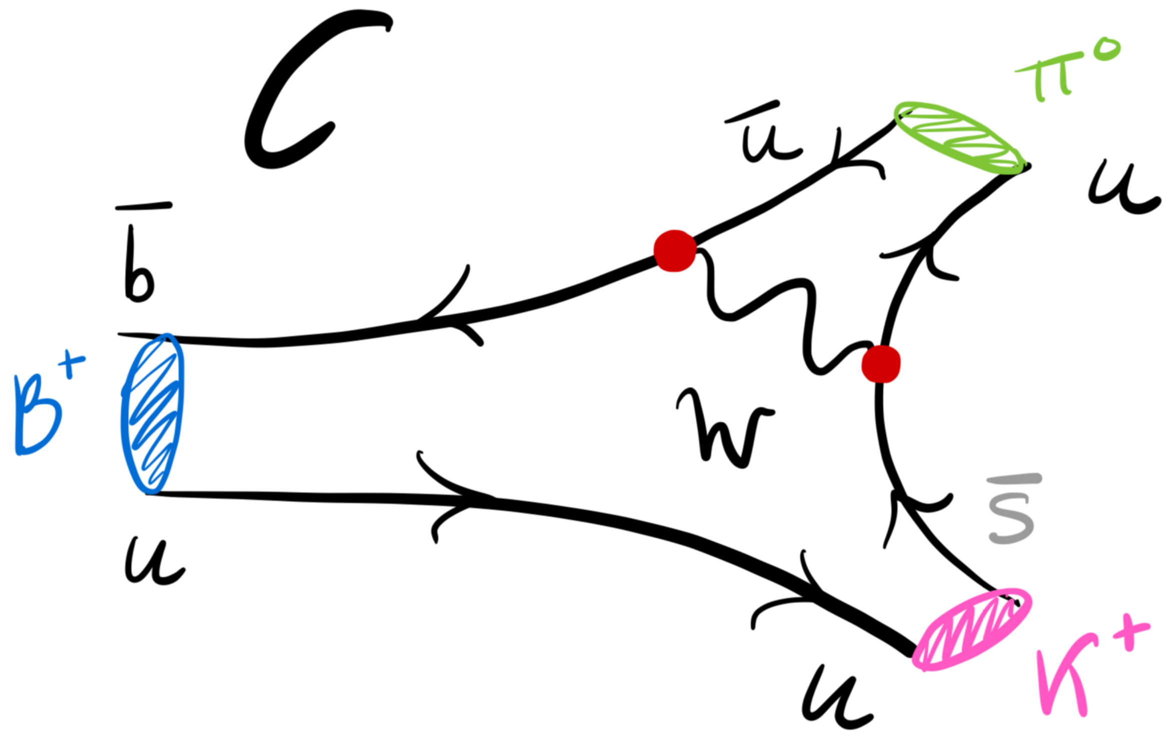



$+$




$+ \dots$

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$$A(B^+ \to \pi^0 \pi^+) = V^*_{ub} V_{ud} (T + C + \dots)$$

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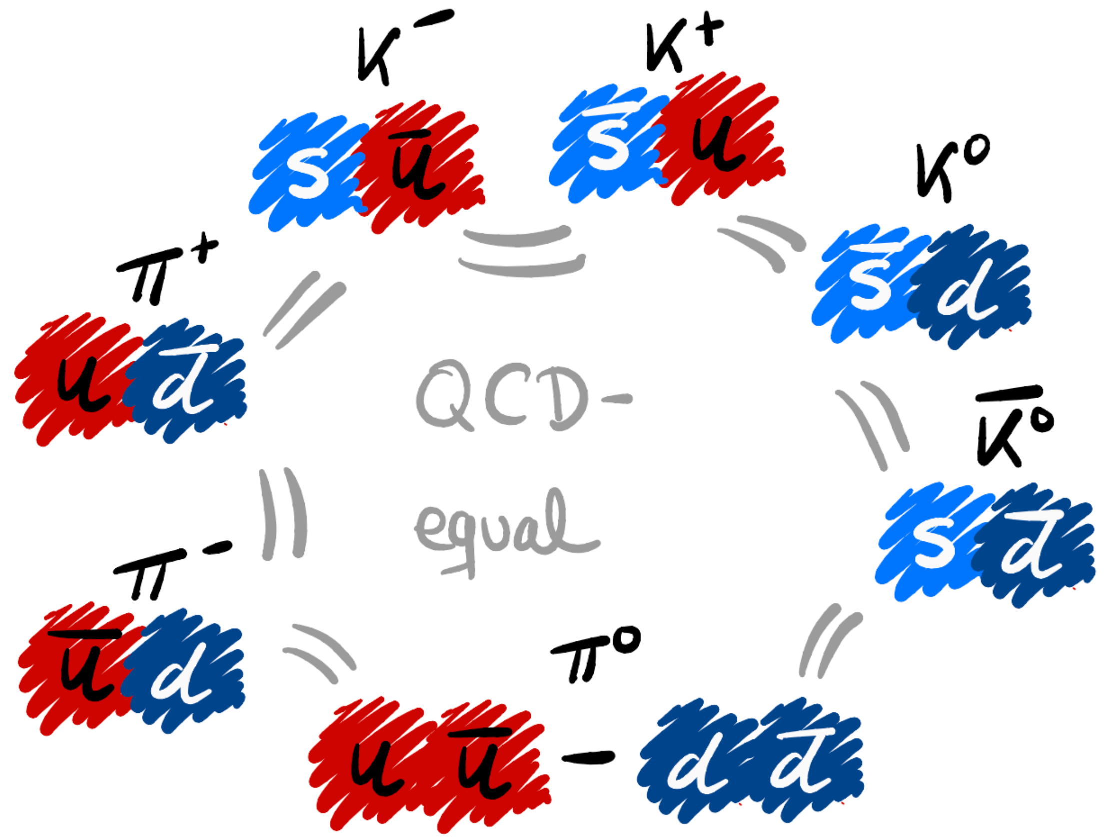


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$$\boxed{\textbf{Same } C \text{ Under } SU(3)_F}$$

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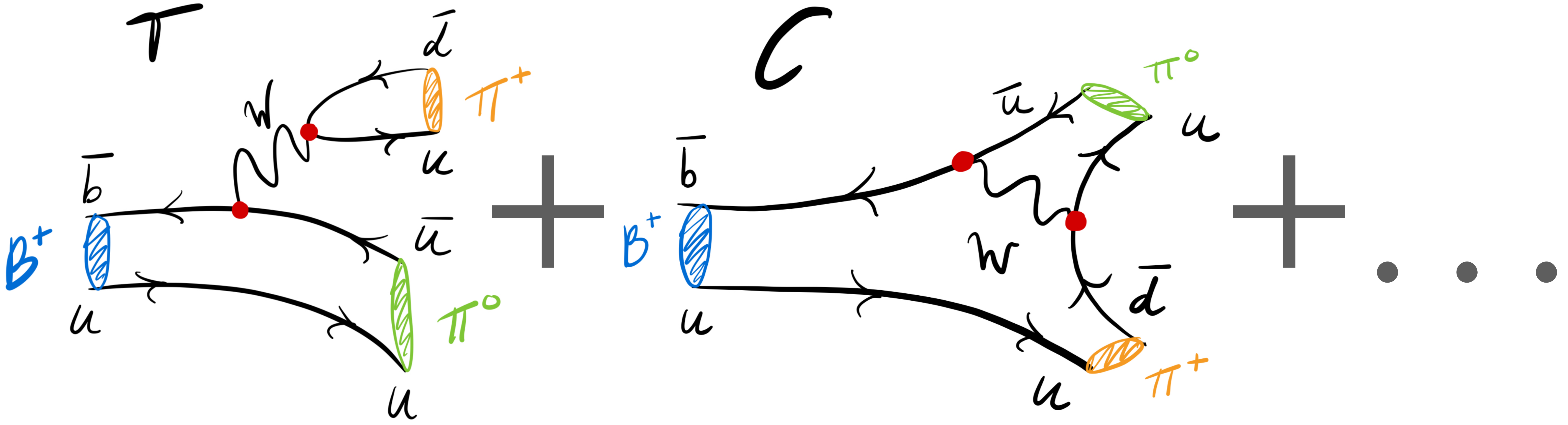


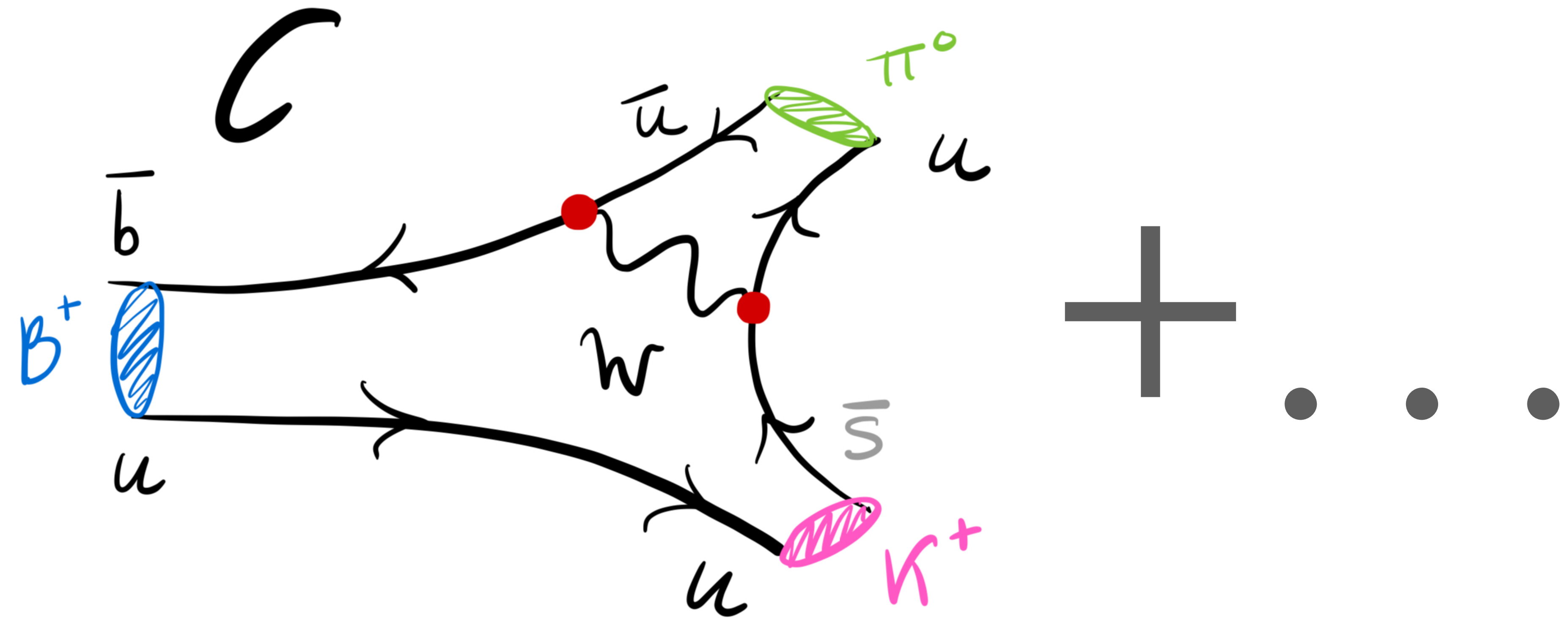


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# 3. Extract the Coefficients from Experimental Data

Express **observables** in terms of the amplitudes under **topological parameterization**

+

**Experimental results** from 16 decay modes

=

**Fit** the values for the **topological coefficients** in $SU(3)_F$ symmetry

Recent SU(3) analysis: Huber and Tetlalmatzi-Xolocotzi 2111.06418, Berthiaume et al 2311.18011, Bhattacharya et al 2505.11492

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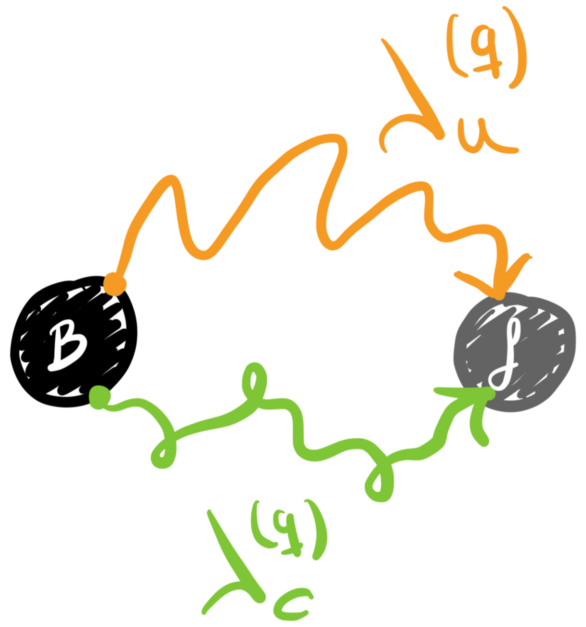
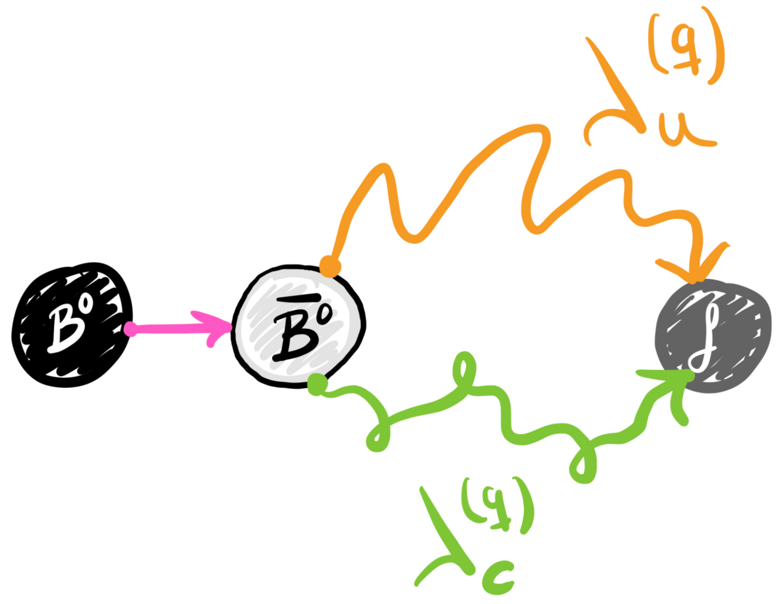
Branching ratios

Direct CP asymmetry: $\mathscr{A}_{CP}^{dir}$



Mixing-induced CP asymmetry: $\mathscr{A}_{CP}^{mix}$



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No distinction between $\Delta S = 0$ and $\Delta S = 1$ amplitudes

No constraint on the coefficients' magnitudes

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# 4. Fit predictions

The fit gives a $\chi^2 \simeq 32.3$ for 15 degrees of freedom, and a **p-value of 0.58%**. The fit is not satisfactory and we conclude that $SU(3)_F$ **cannot describe the experimental data**

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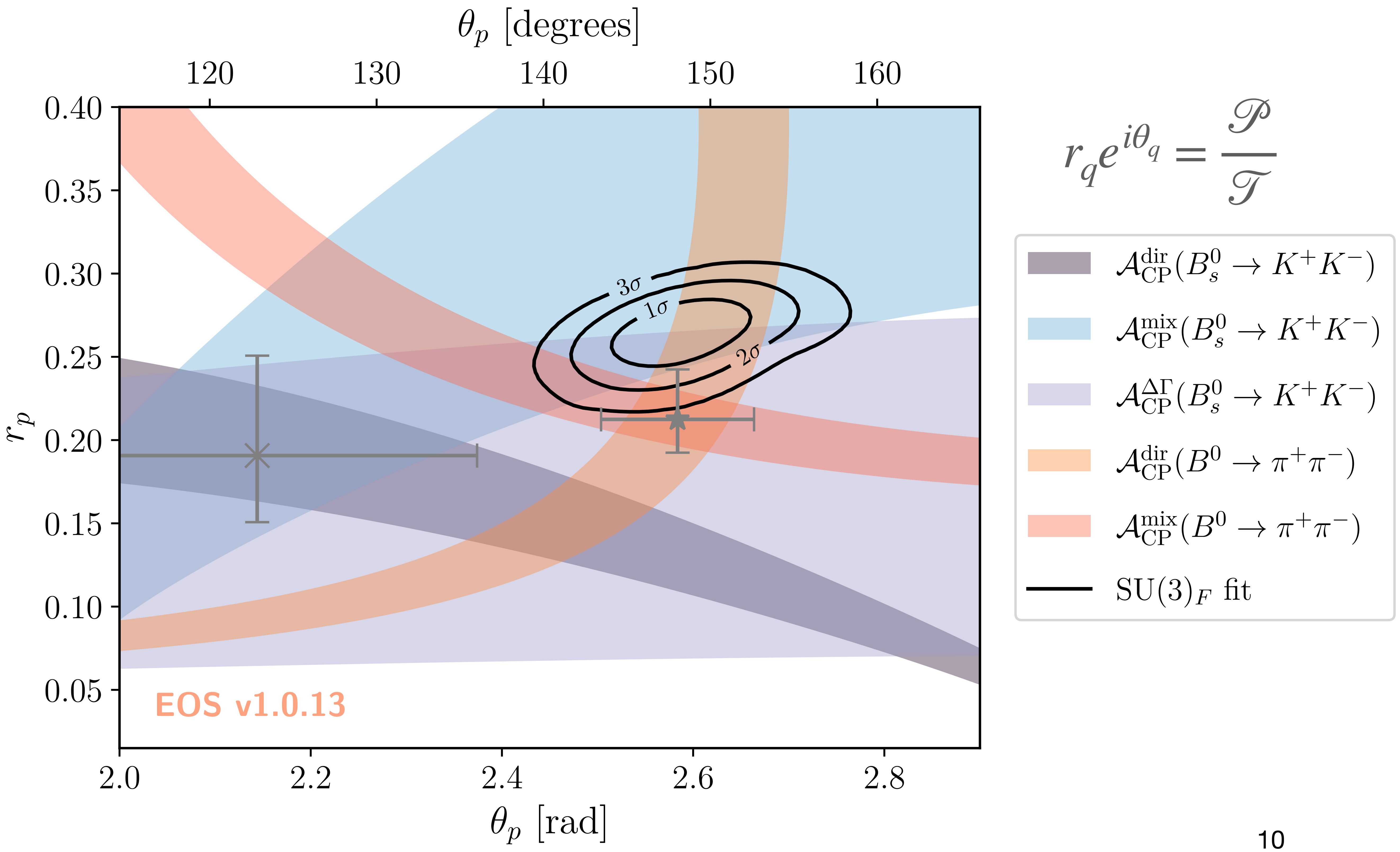
Certain modes highlight the limitations of exact $SU(3)_F$ symmetry in global analysis




$B_s^0 \to K^+K^-$ and $B^0 \to \pi^+\pi^-$ are U-spin partners (same topologies, different CKM elements).

Experimental data show $SU(3)_F$ breaking

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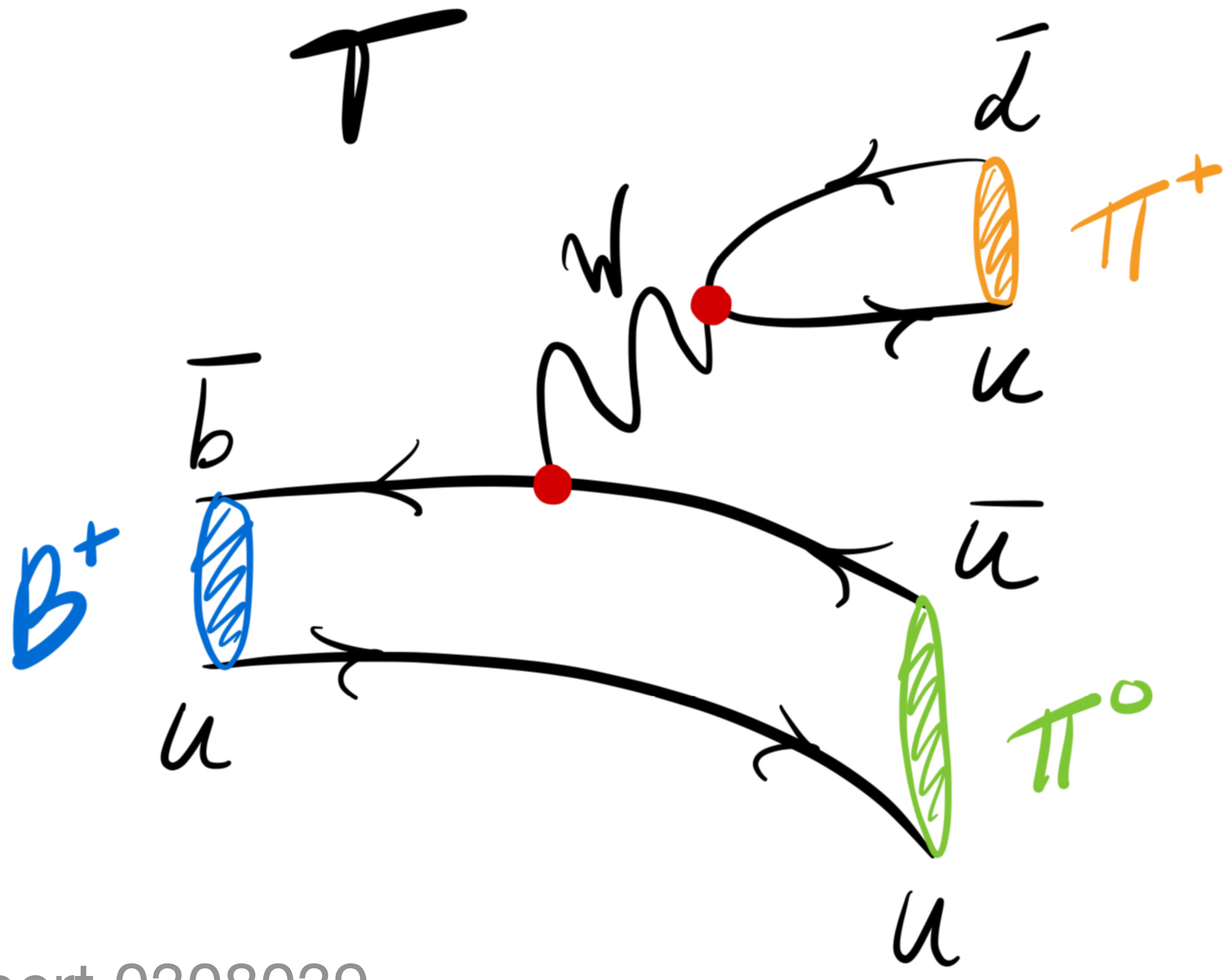
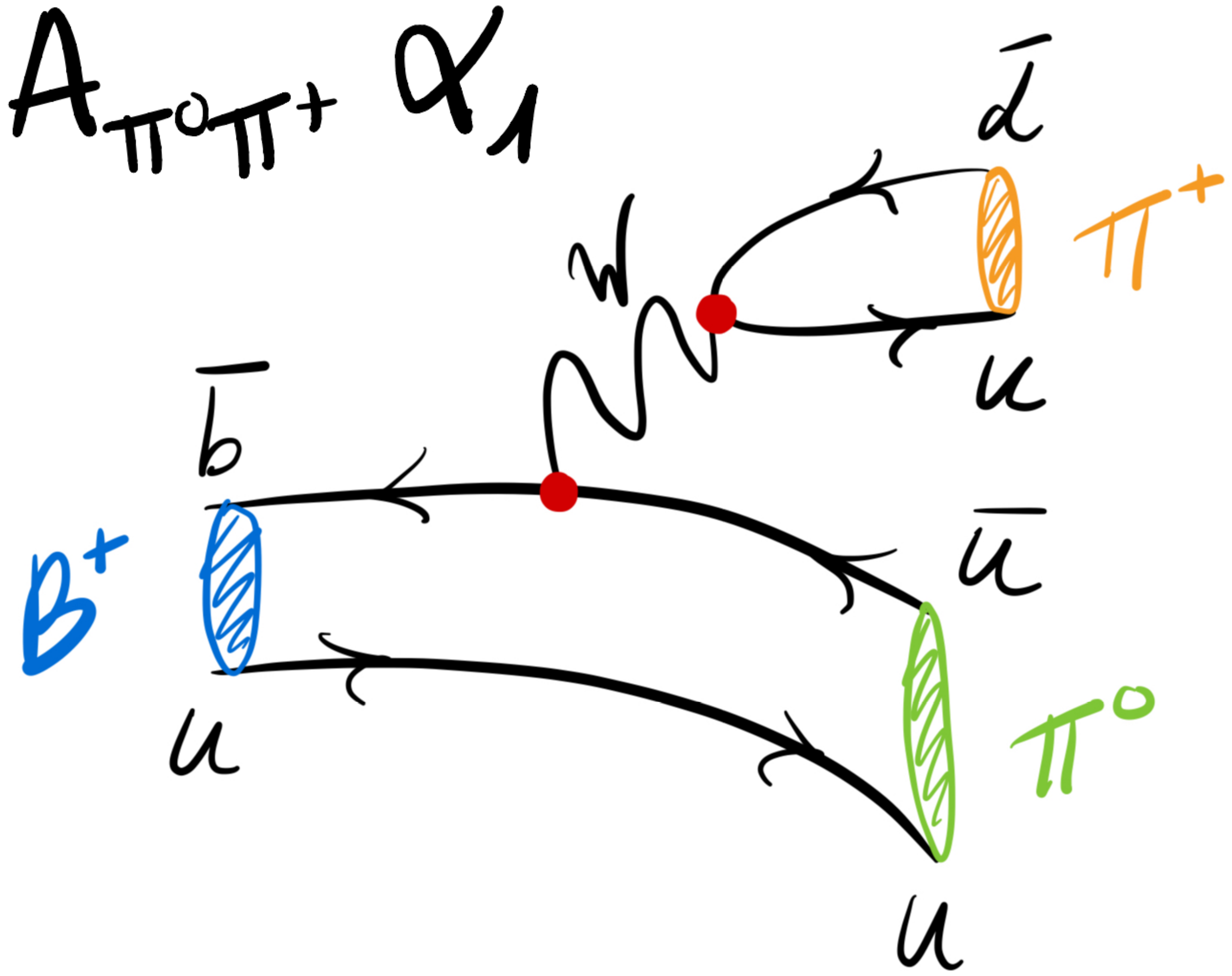
Factorizable $SU(3)_F$ breaking allows us to account for the different masses of the mesons without adding (almost) any new coefficients

**$SU(3)_F$ symmetry: $T$** → **Factorizable $SU(3)_F$ breaking: $A_{M_1M_2}\ \alpha_1$**




$$A_{\pi^0\pi^+} = M_{B^+}^2\, F_0^{B^+\to\pi^0}(m_{\pi^+}^2)\, f_{\pi^+}$$

Beneke, Neubert 0308039,
Beneke, Buchalla, Neubert, Sachrajda 0006124

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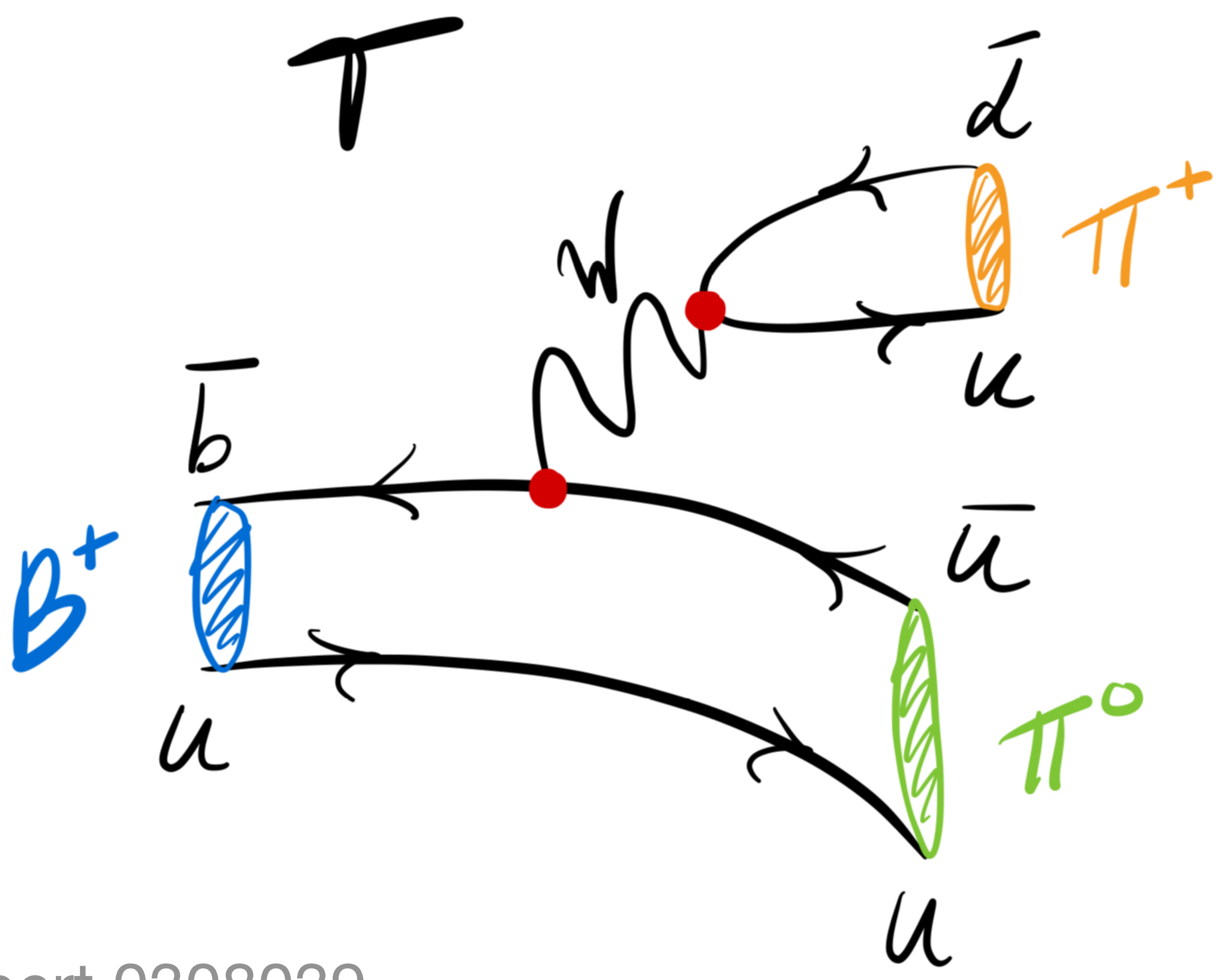


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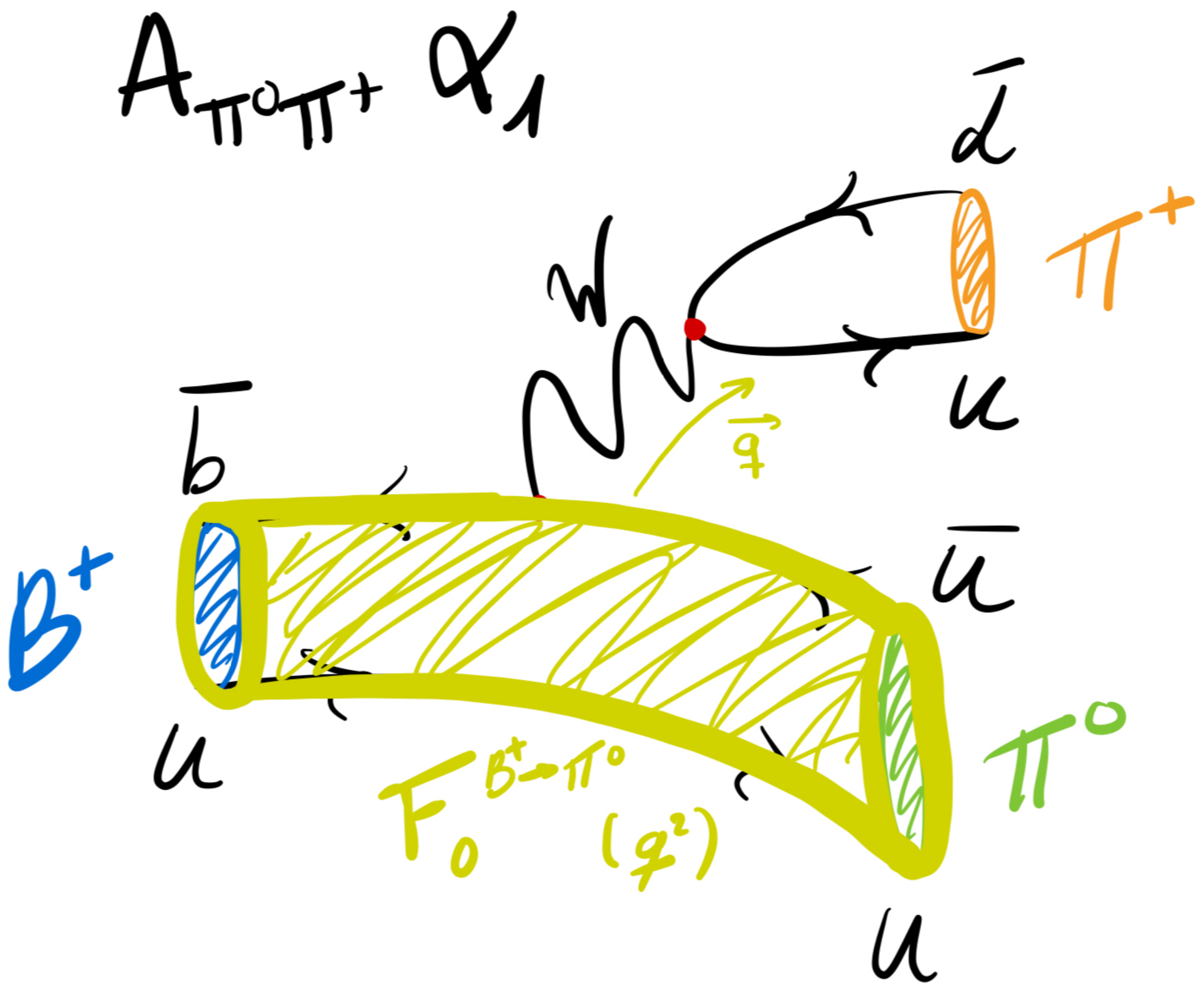


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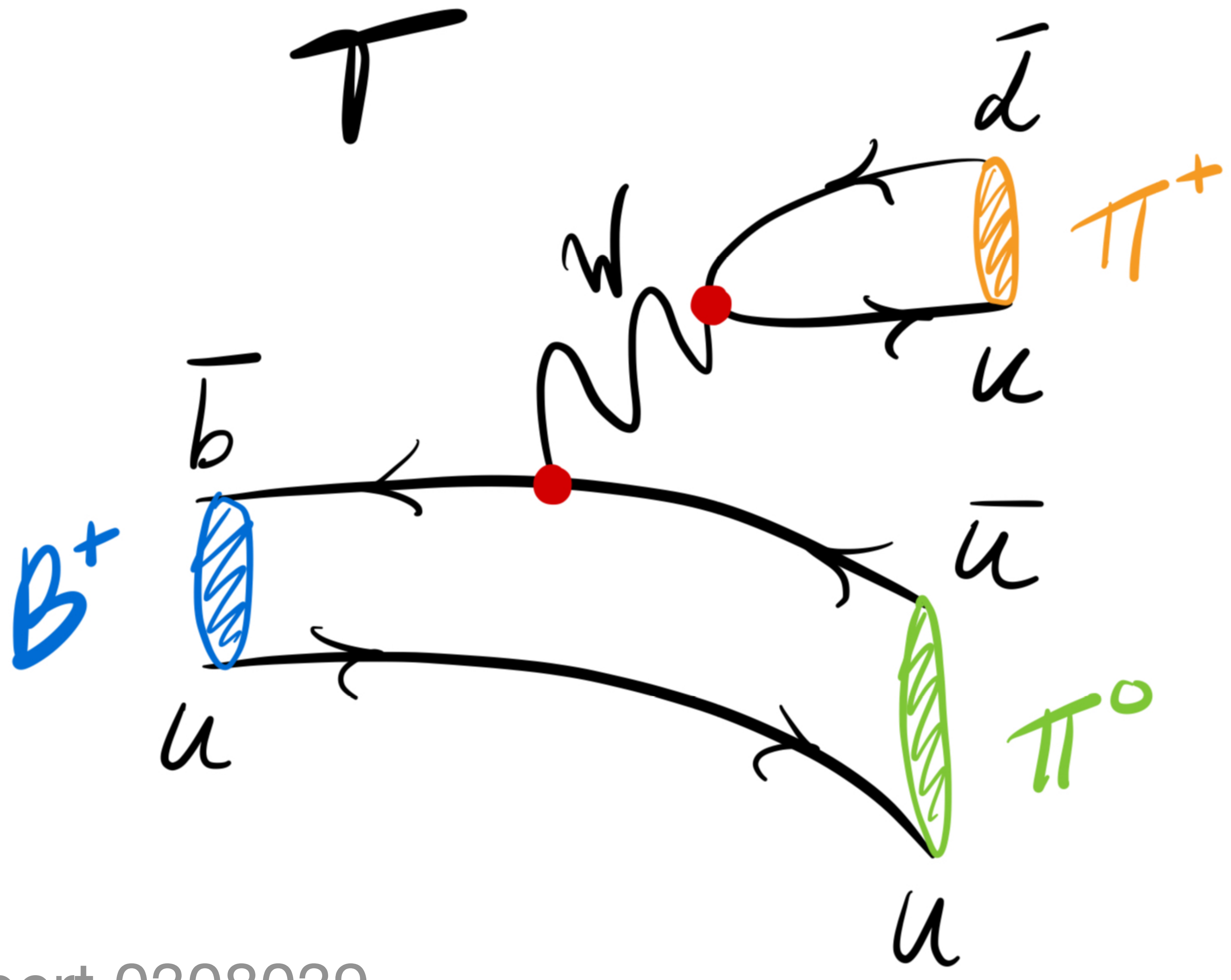


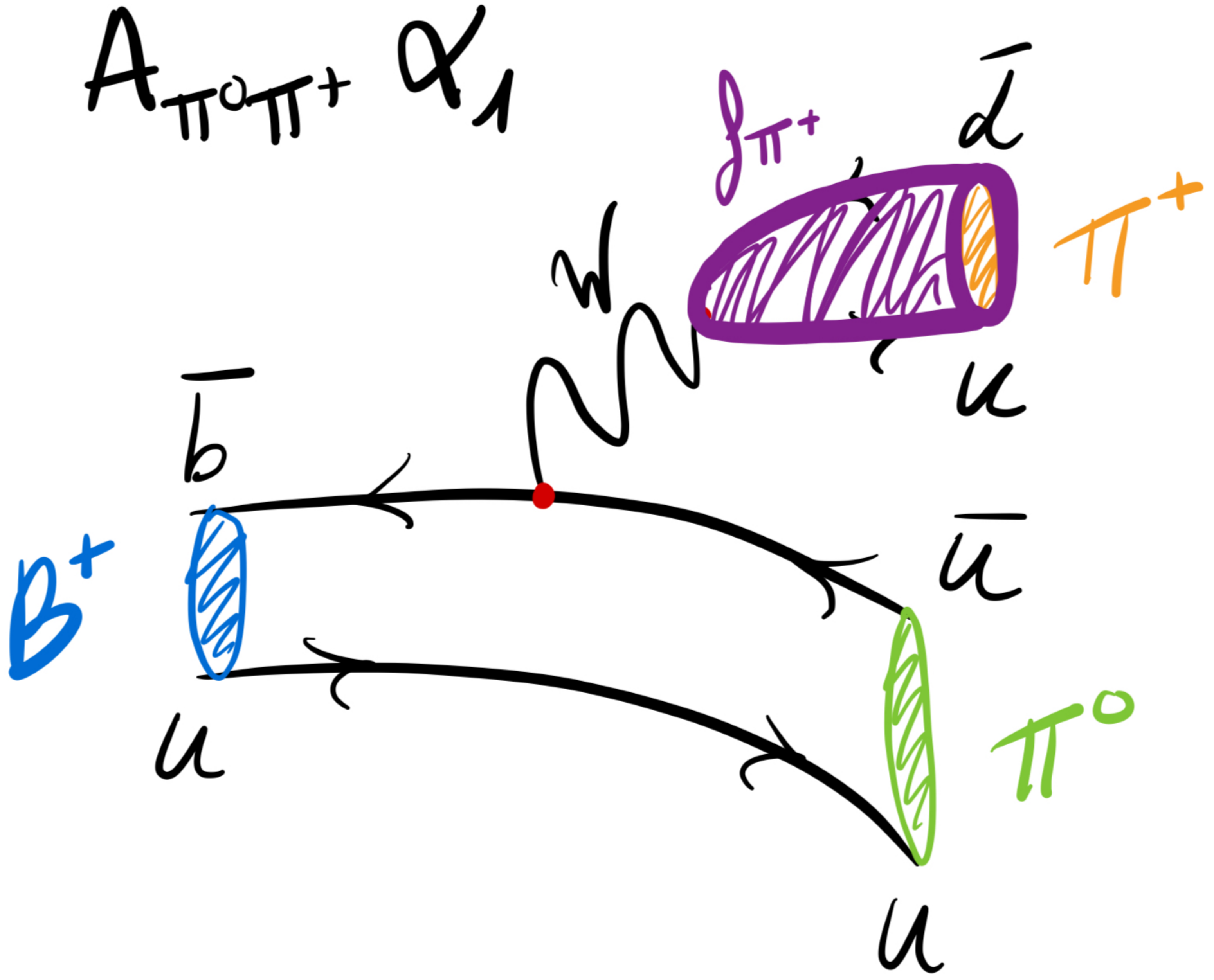


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$A_{M_1M_2}$ : $SU(3)$ **breaking**, but **known**! (Fixed, no new coefficients)

$\alpha_1$ : $SU(3)$ **symmetric**, fitted from experimental data

Beneke, Neubert 0308039,
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$$T \Rightarrow A_{M_1M_2}\tilde{\alpha}_1$$

$$P_T \Rightarrow A_{M_1M_2}\tilde{\alpha}^c_{4,EW}$$

$$C \Rightarrow A_{M_1M_2}\tilde{\alpha}_2$$

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$$+\ldots \qquad +\ldots$$

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**Inspired by QCD Factorization, more on that later!**

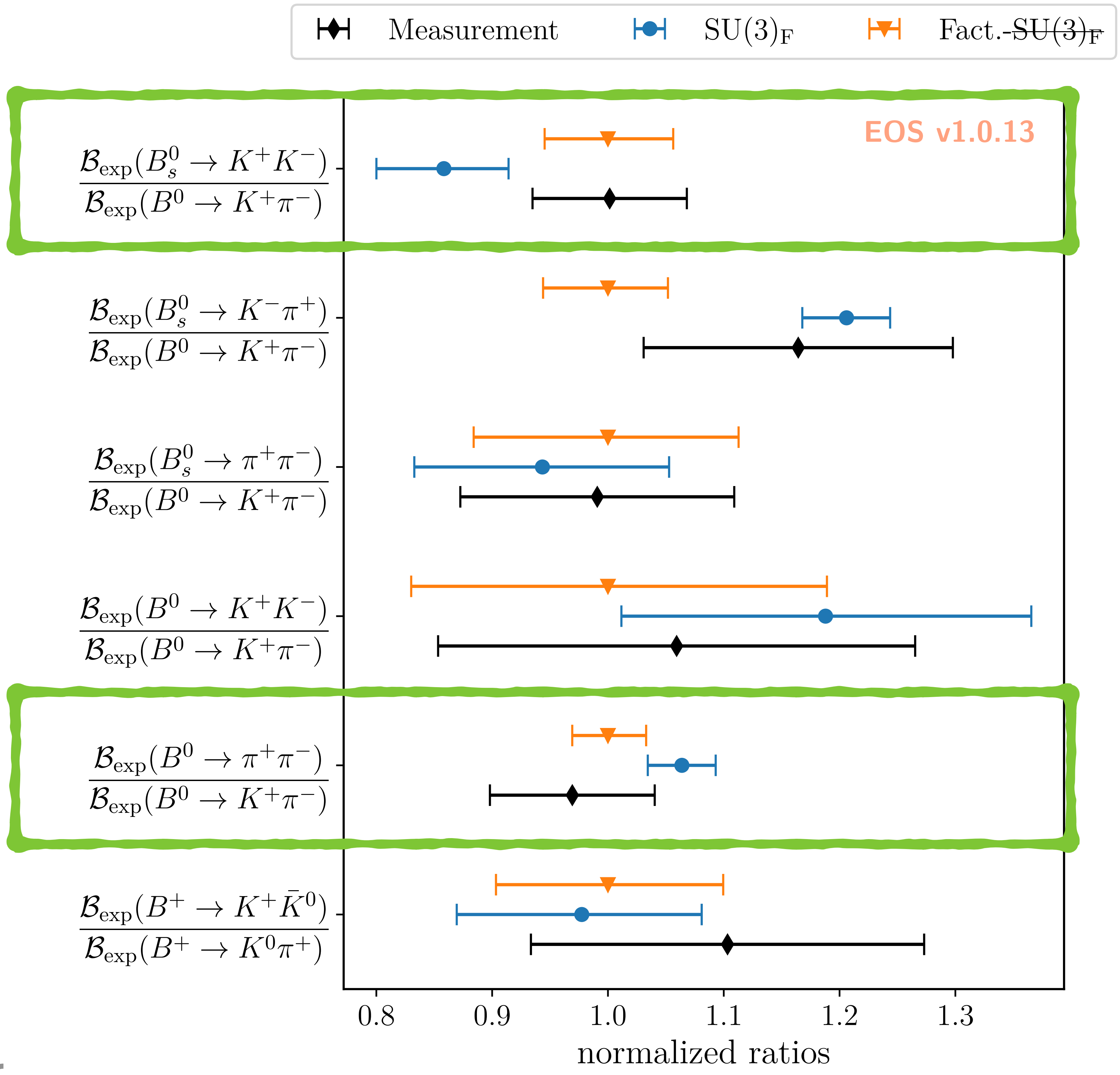
2111.15428

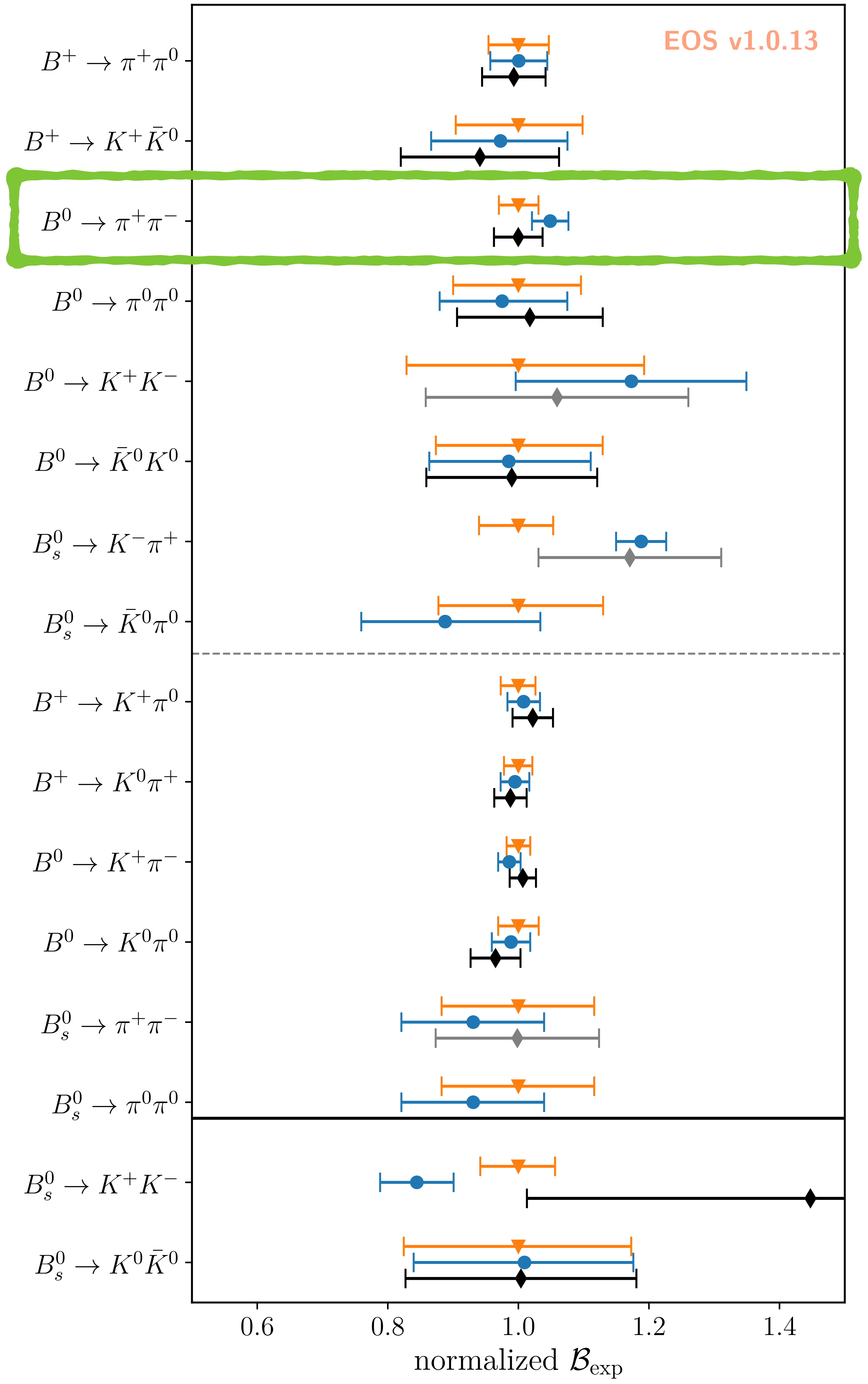
# Results: Factorizable $SU(3)_F$ breaking

**MBM, Reboud, Vos 2504.05209**

Factorizable $SU(3)_F$ breaking describes data almost perfectly, with a **p-value of 31%**

Fact. $SU(3)_F$ improves predictions compared to full $SU(3)_F$




Experimental data from LHCb, Belle II and BaBar

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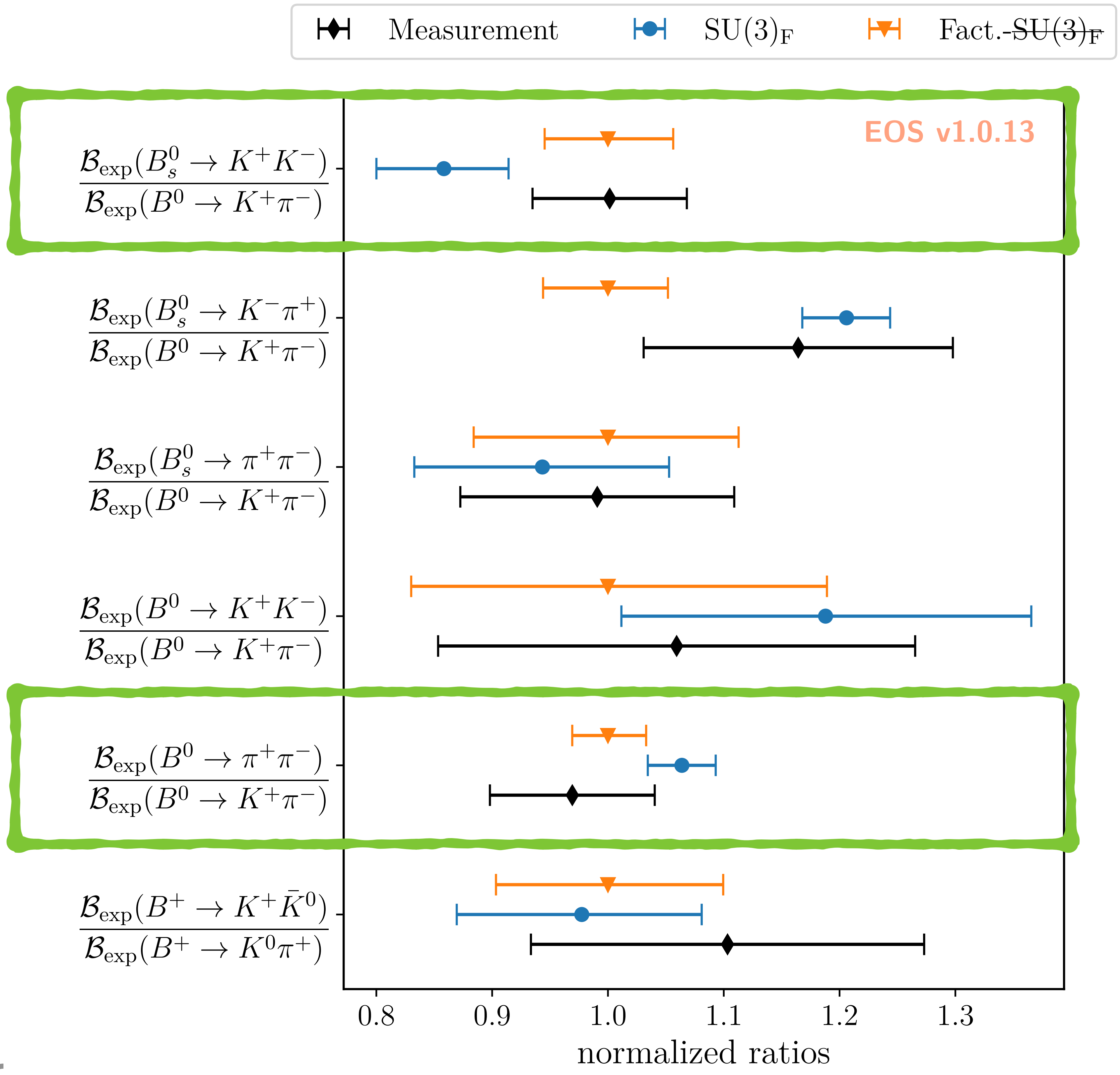


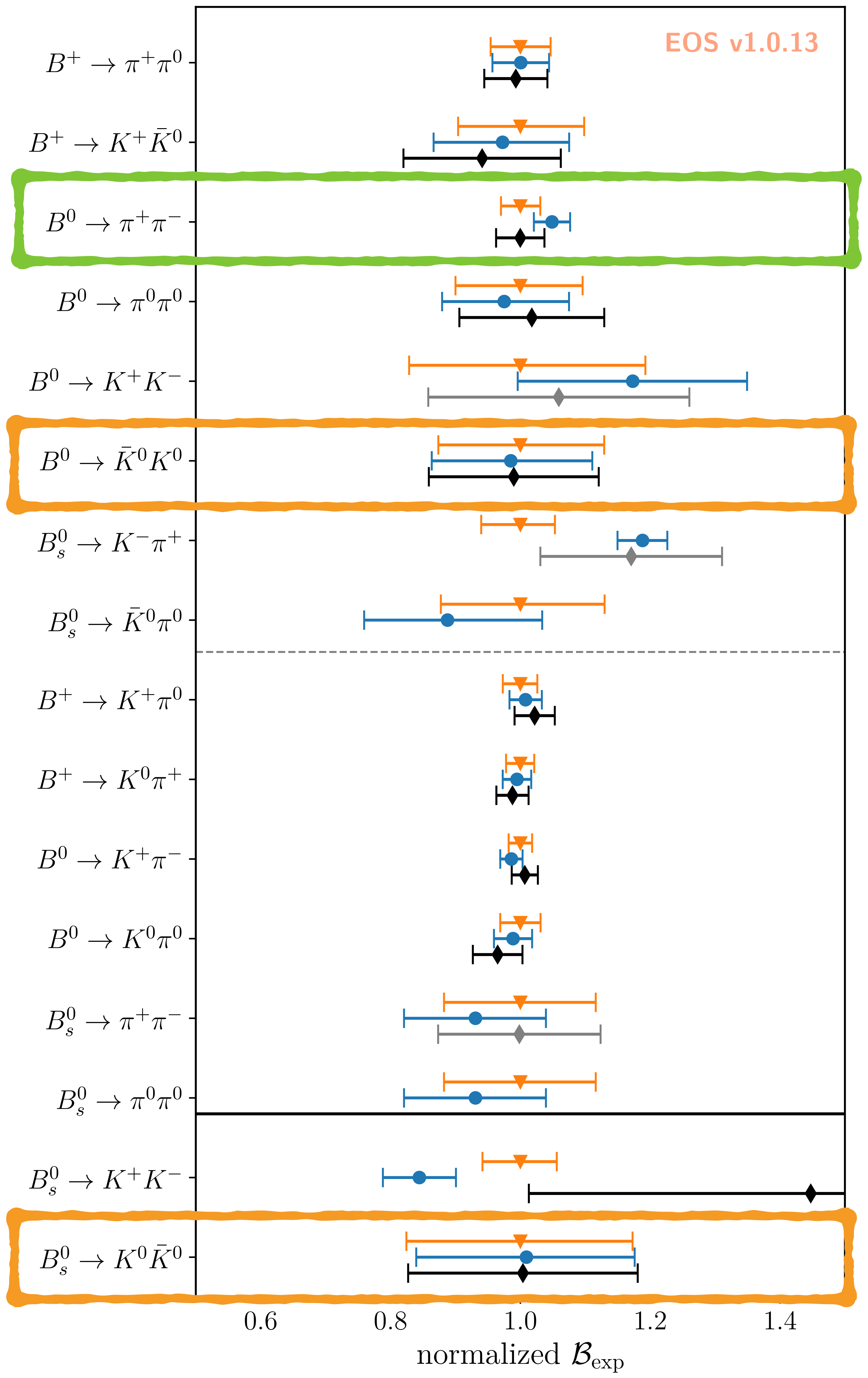


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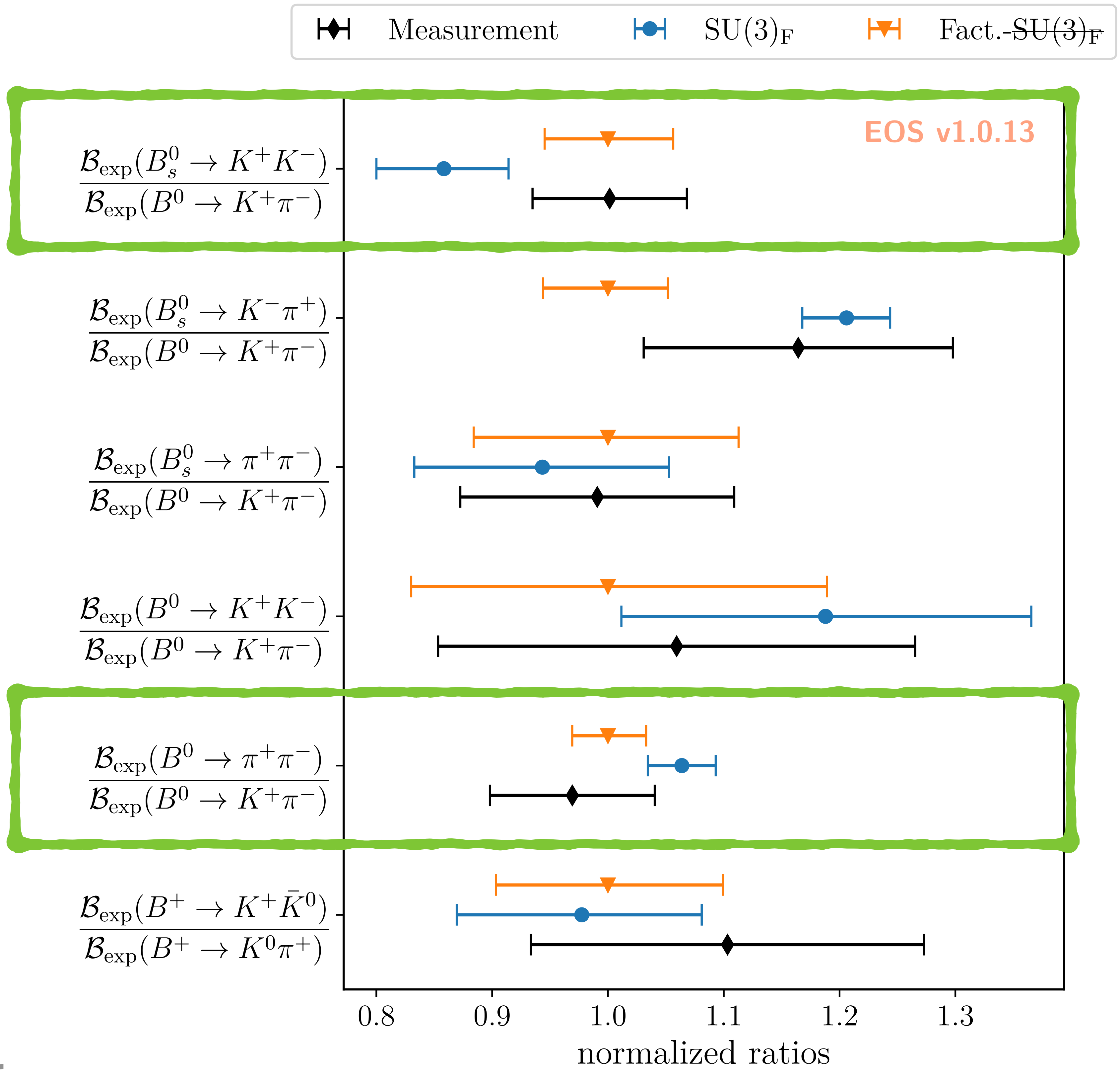
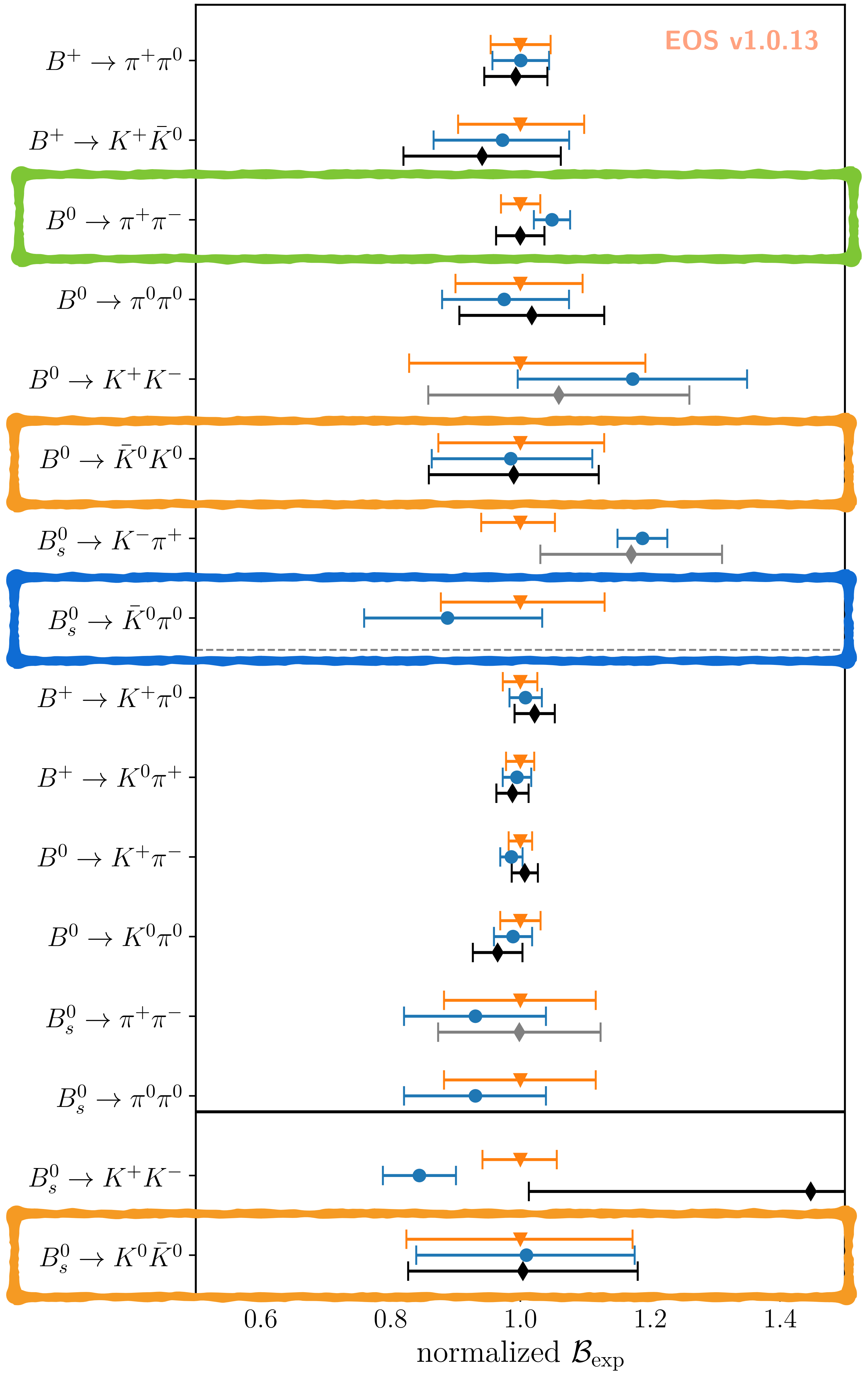
Obtain predictions for decays that have not been measured yet




Experimental data from LHCb, Belle II and BaBar

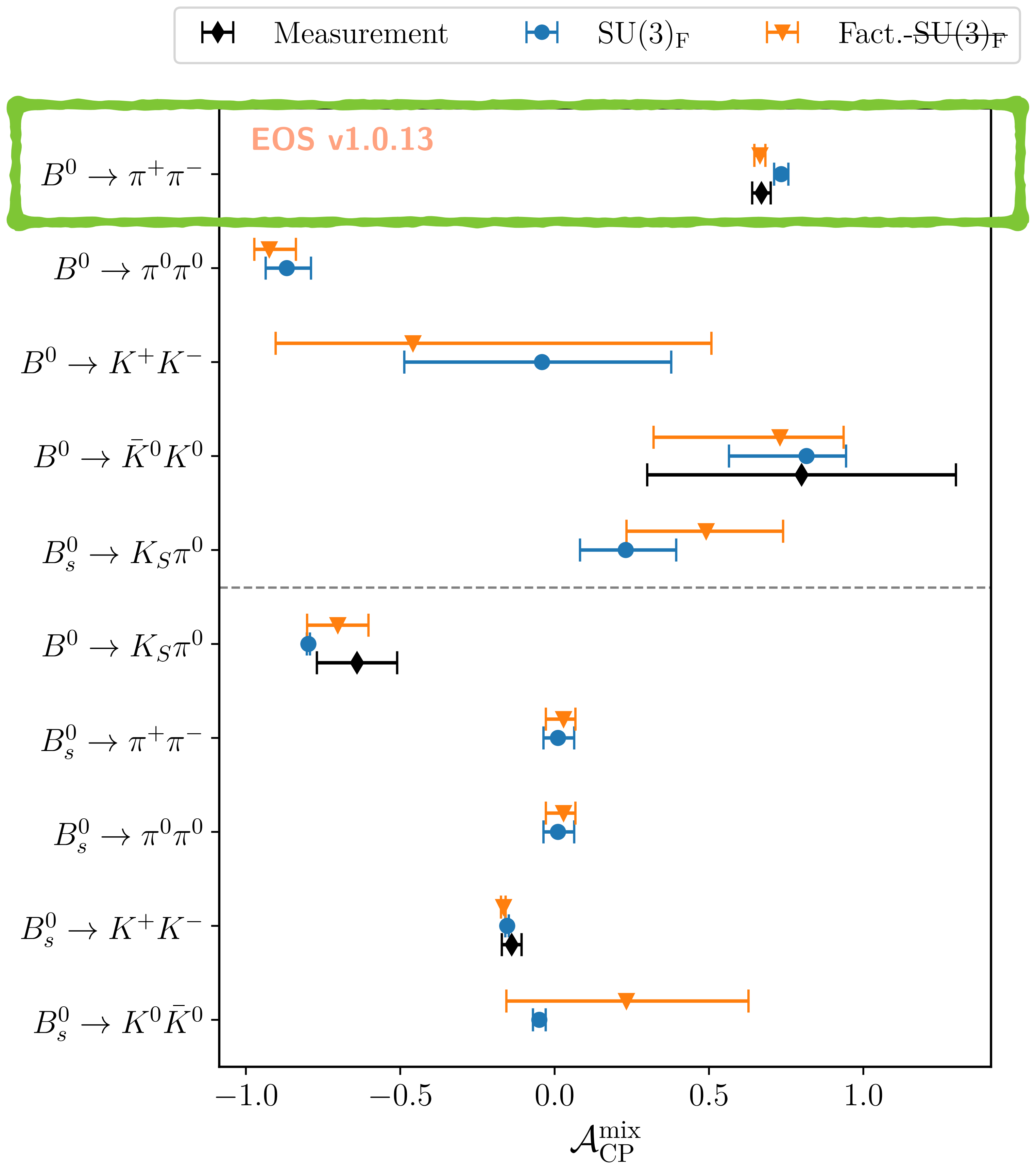
2111.15428

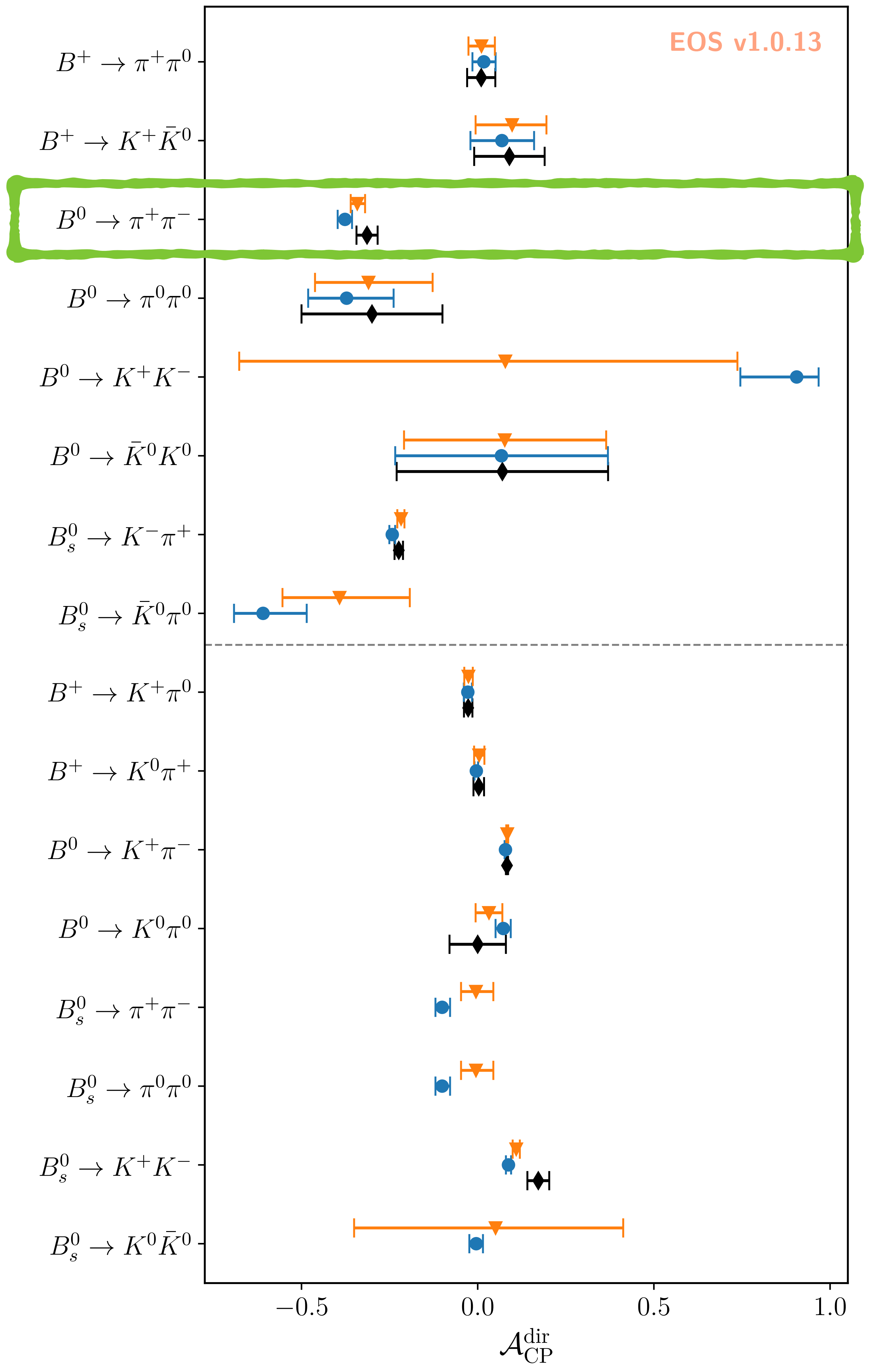
# Results: Factorizable $SU(3)_F$ breaking

**MBM, Reboud, Vos 2504.05209**

Factorizable $SU(3)_F$ breaking describes data almost perfectly, with a **p-value of 31%**

Fact. $SU(3)_F$ improves predictions compared to full $SU(3)_F$

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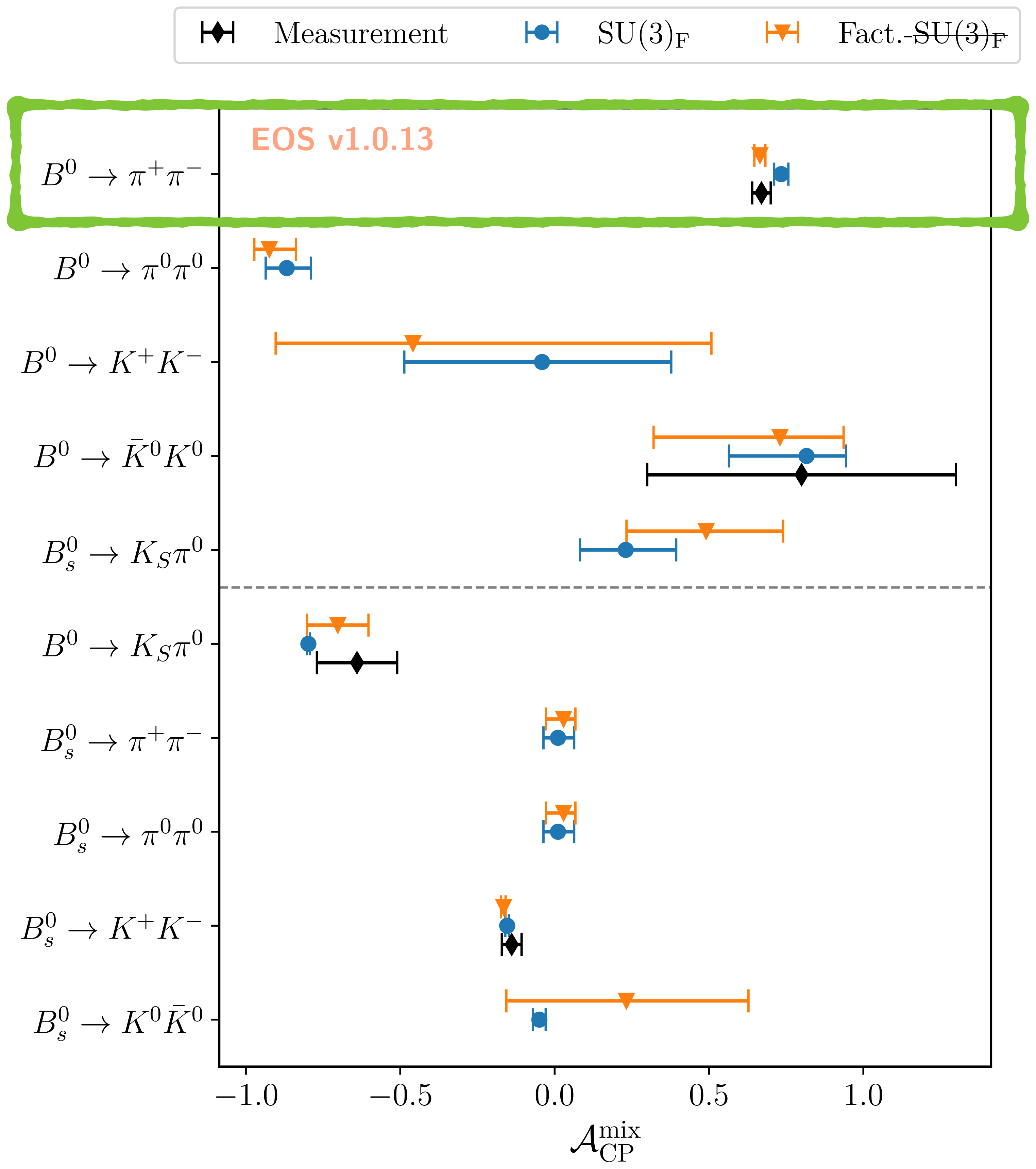


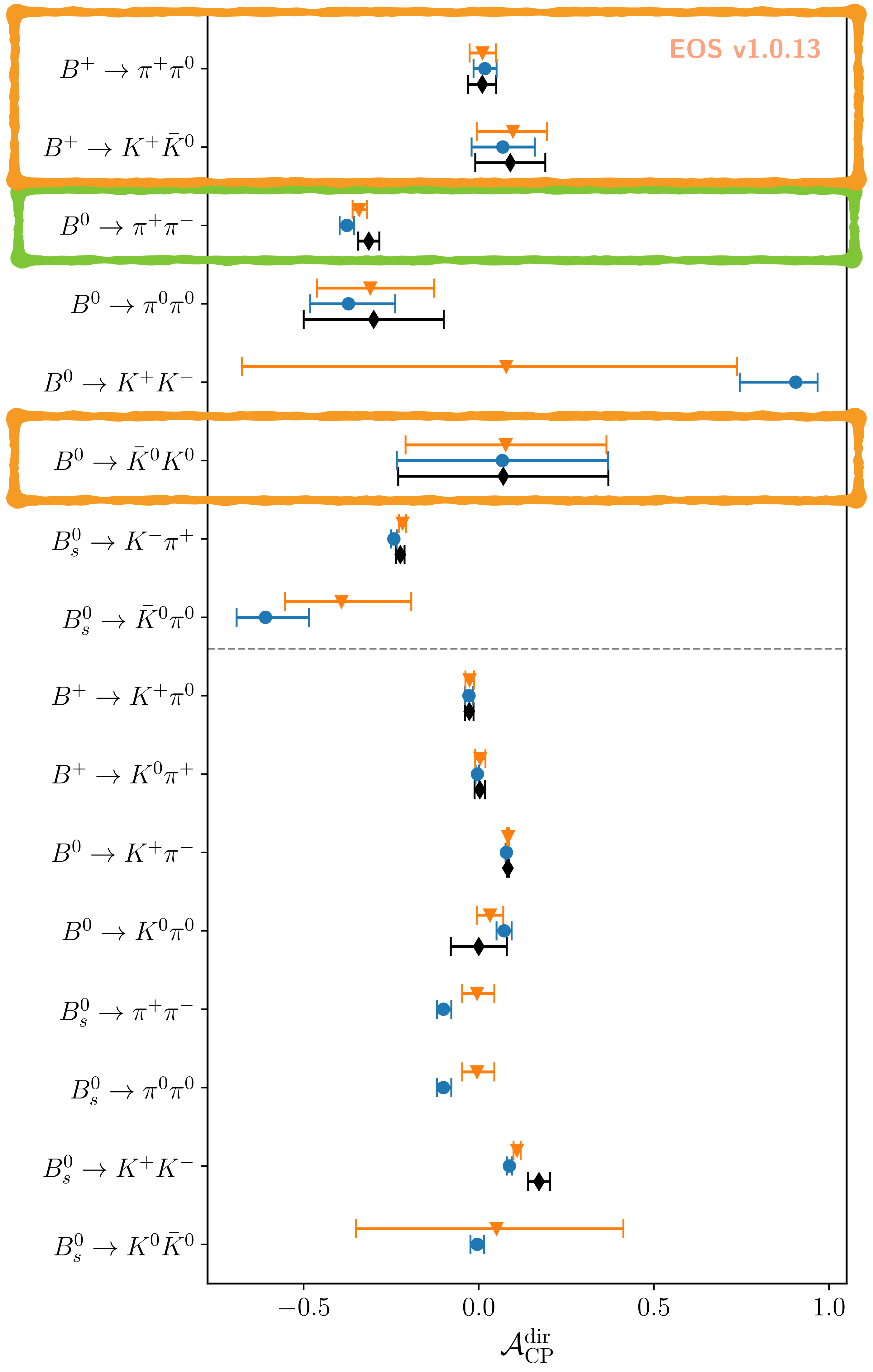


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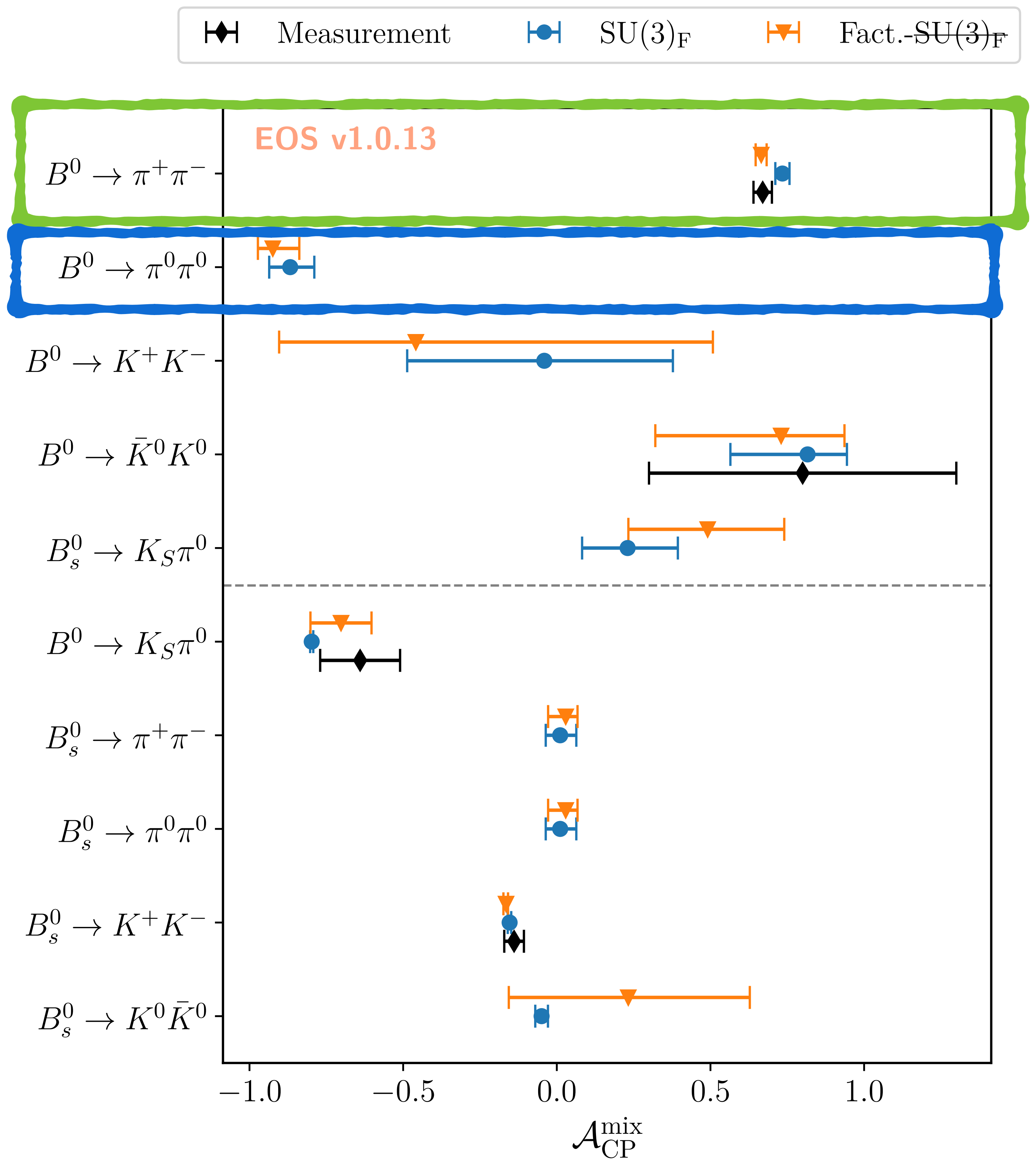


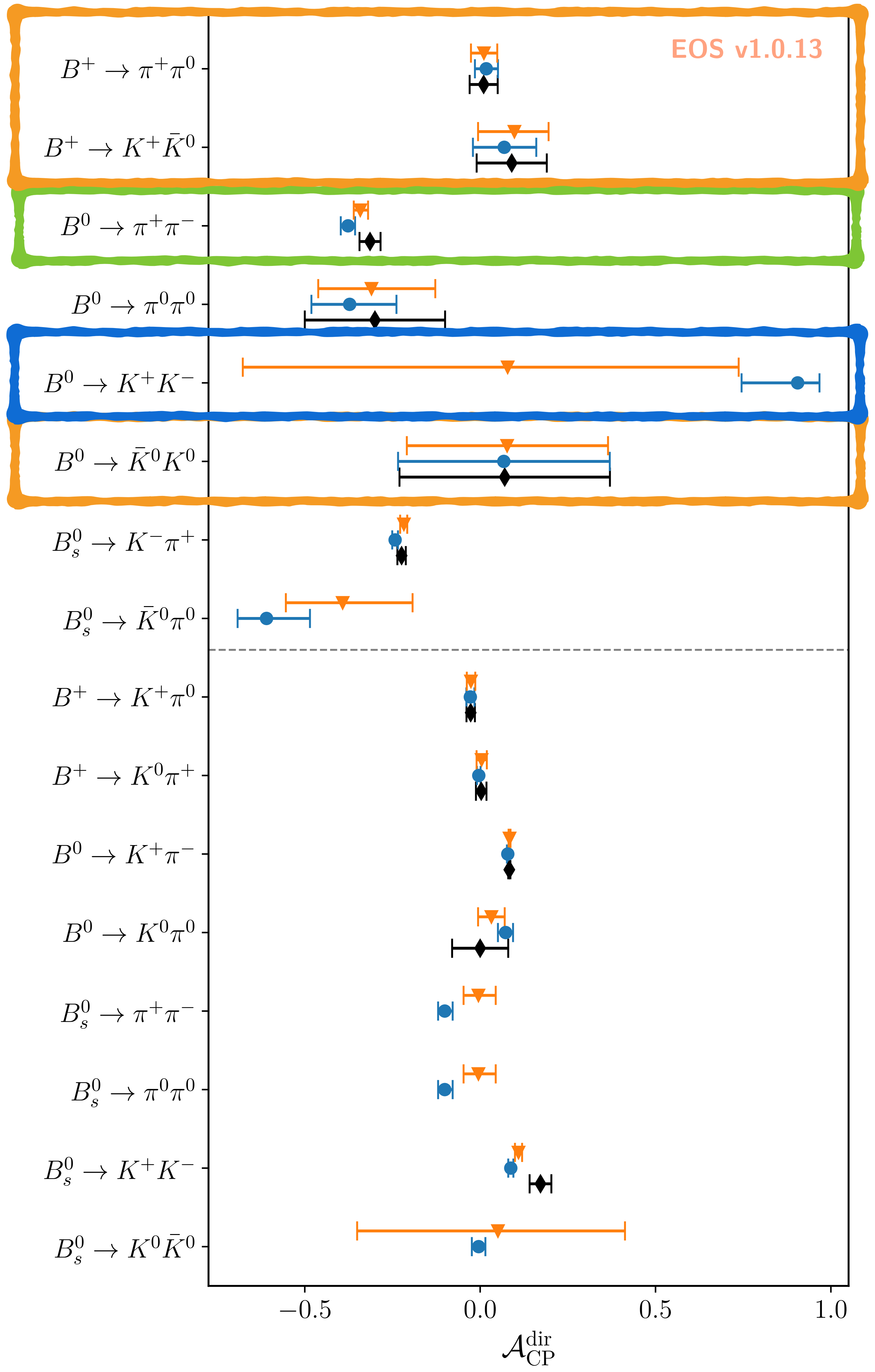


Experimental data from LHCb, Belle II and BaBar

2111.15428

# Results: Factorizable $SU(3)_F$ breaking

MBM, Reboud, Vos 2504.05209

Factorizable $SU(3)_F$ breaking provides an excellent description of the data. Small tensions in $\mathscr{A}_{\mathrm{CP}}(B_s^0 \to K^+K^-)$, $\mathscr{A}_{\mathrm{CP}}^{\mathrm{mix}}(K_S^0\pi^0)$ and $\mathscr{A}_{\mathrm{CP}}^{\mathrm{dir}}(B^0 \to \pi^+\pi^-)$ that do not exceed $1.5\sigma$

Updates / new measurements would further test the validity of this approach: $B_{(s)}^0 \to K^0\bar{K}^0, B^0 \to K^+K^-, \ldots$

What about the coefficients? Can we improve our understanding of QCD?

# What do we learn about QCD?

# Factorizable $SU(3)_F$ breaking vs QCDF

Beneke, Neubert 0308039,
Beneke, Buchalla, Neubert, Sachrajda 0006124

Fact. $SU(3)_F$ breaking parametrisation is inspired by QCD Factorization:

| QCDF | Fact. $\cancel{SU(3)}_F$ |
|---|---|
| Method to calculate theoretically amplitudes for hadronic processes at higher orders | Purely data-driven |

- Expand in orders of $\alpha_s$ and $\Lambda_{QCD}/m_b$
- Use the large mass of the $b$-quark to factorize the final state. Then correct for gluon interactions
- End-point divergences arise in hard-spectator corrections and annihilation topologies: require model-dependent treatment

# Factorizable $SU(3)_F$ breaking vs QCDF

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Fact. $SU(3)_F$ breaking parametrisation is inspired by QCD Factorization:

| QCDF | Fact. $\cancel{SU(3)}_F$ |
|---|---|
| Theoretical determination | Purely data-driven |
| Coefficients depend on the initial and final states mesons: $\alpha_i(B_q, M_1 M_2)$ | $SU(3)_F$ symmetry in coefficients $\alpha_i$ |

$$\frac{|\alpha_1(B^0, \pi\pi)|}{|\alpha_1(B_s^0, KK)|} \simeq 1 \qquad \frac{|\alpha_2(B^0, \pi\pi)|}{|\alpha_2(B_s^0, KK)|} \simeq 0.78$$

# Factorizable $SU(3)_F$ breaking vs QCDF

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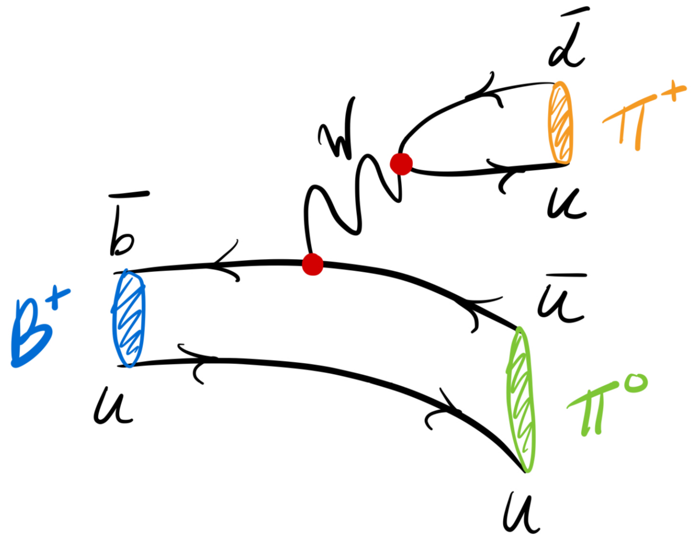
| QCDF | Fact. $\cancel{SU(3)_F}$ |
|---|---|
| Theoretical determination | Purely data-driven |
| Coefficients depend on the initial and final states mesons | $SU(3)_F$ symmetry in coefficients |

In both cases we use the same expression to construct the amplitudes

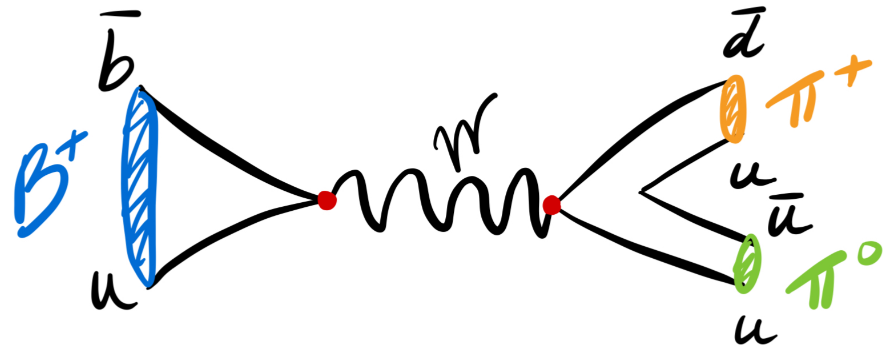
$$\mathscr{A}(\bar{B} \to M_1 M_2) = A^{B_q}_{M_1 M_2} \sum_{i=1}^{3} \left\{ \lambda_u^{(p)} \mathscr{T}^i + \lambda_c^{(p)} \mathscr{P}^i \right\}$$

Trees and penguins: $\alpha$'s




$$+ B^{B_q}_{M_1 M_2} \sum_{i=4}^{6} \left\{ \lambda_u^{(p)} \mathscr{T}^i + \lambda_c^{(p)} \mathscr{P}^i \right\},$$

Annihilation: $b$'s



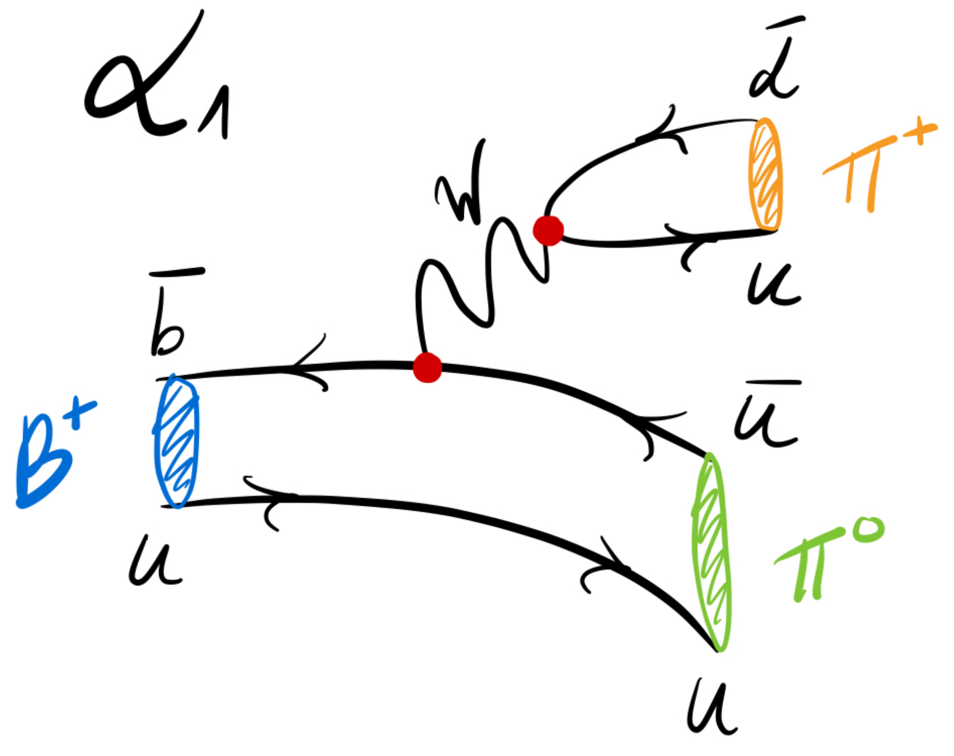
# Coefficient results: Fact. $SU(3)_F$ breaking

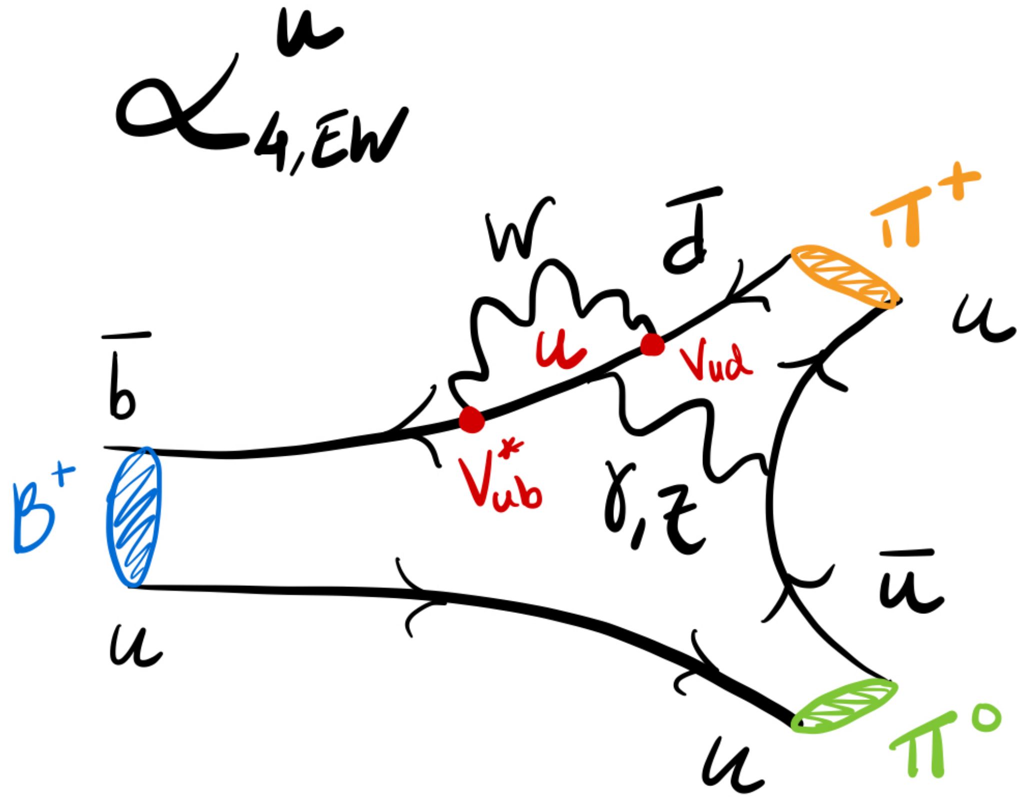
MBM, Reboud, Vos 2504.05209

- However, **individual QCDF coefficients** are mostly **unconstrained** in the fit

  → Intrinsic blind directions: certain coefficient combinations always enter together at amplitude level, making them **individually unobservable**

$$\tilde{\alpha}_1 = \alpha_1 + \frac{3}{2}\alpha^u_{4,\mathrm{EW}}$$



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2111.15428

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$$\tilde{\alpha}_1 \equiv \alpha_1 + \frac{3}{2}\alpha^u_{4,EW}, \quad \tilde{\alpha}_2 \equiv \alpha_2 + \frac{3}{2}\alpha^u_{3,EW}$$

$$\tilde{\beta}_1 \equiv \beta_1 + \frac{3}{2}\beta^u_{4,EW}, \quad \tilde{\beta}_2 \equiv \beta_2 + \frac{3}{2}\beta^u_{3,EW},$$

$$\tilde{\alpha}^r_4 \equiv \alpha^r_4 + \beta^r_3 - \frac{1}{2}\left(\alpha^r_{4,EW} + \beta^r_{3,EW}\right), \quad \tilde{\beta}^r_4 \equiv \beta^r_4 - \frac{1}{2}\beta^r_{4,EW},$$

Redundancy in the parameterisation, no assumptions

# Coefficient results: Fact. $SU(3)_F$ breaking

**MBM, Reboud, Vos 2504.05209**

However, **individual QCDF coefficients** are mostly **unconstrained** in the fit

- Intrinsic blind directions: certain coefficient combinations always enter together at amplitude level, making them **individually unobservable**

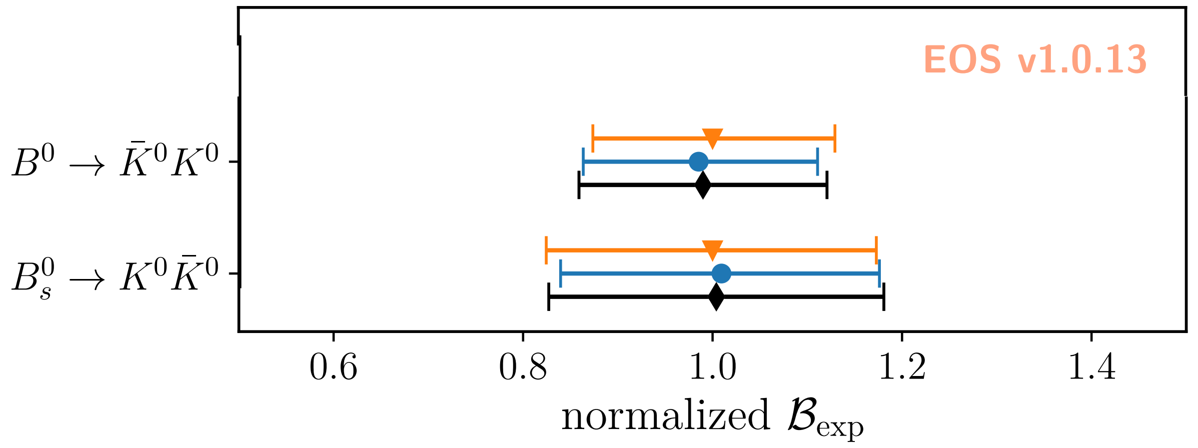
- Approximate blind directions: **data limitations** lead to poor individual constraints

$$\left.\begin{array}{l} B^0 \to \pi^+\pi^- \\ B_s^0 \to K^-\pi^+ \\ B^0 \to \pi^-K^+ \\ B_s^0 \to K^+K^- \end{array}\right\} \sim \tilde{\alpha}_1 + \tilde{\alpha}_4^u$$

- $\tilde{\alpha}_1$**:** never alone

- $\tilde{\alpha}_4^u$**:** could be extracted from $B_{(s)}^0 \to K^0\bar{K}^0$, but…

**We are saturating exp. uncertainty!**



2111.15428

# Coefficient results: Fact. $SU(3)_F$ breaking

**MBM, Reboud, Vos 2504.05209**

More results in backup!

We can still extract plenty of information from coefficient relations

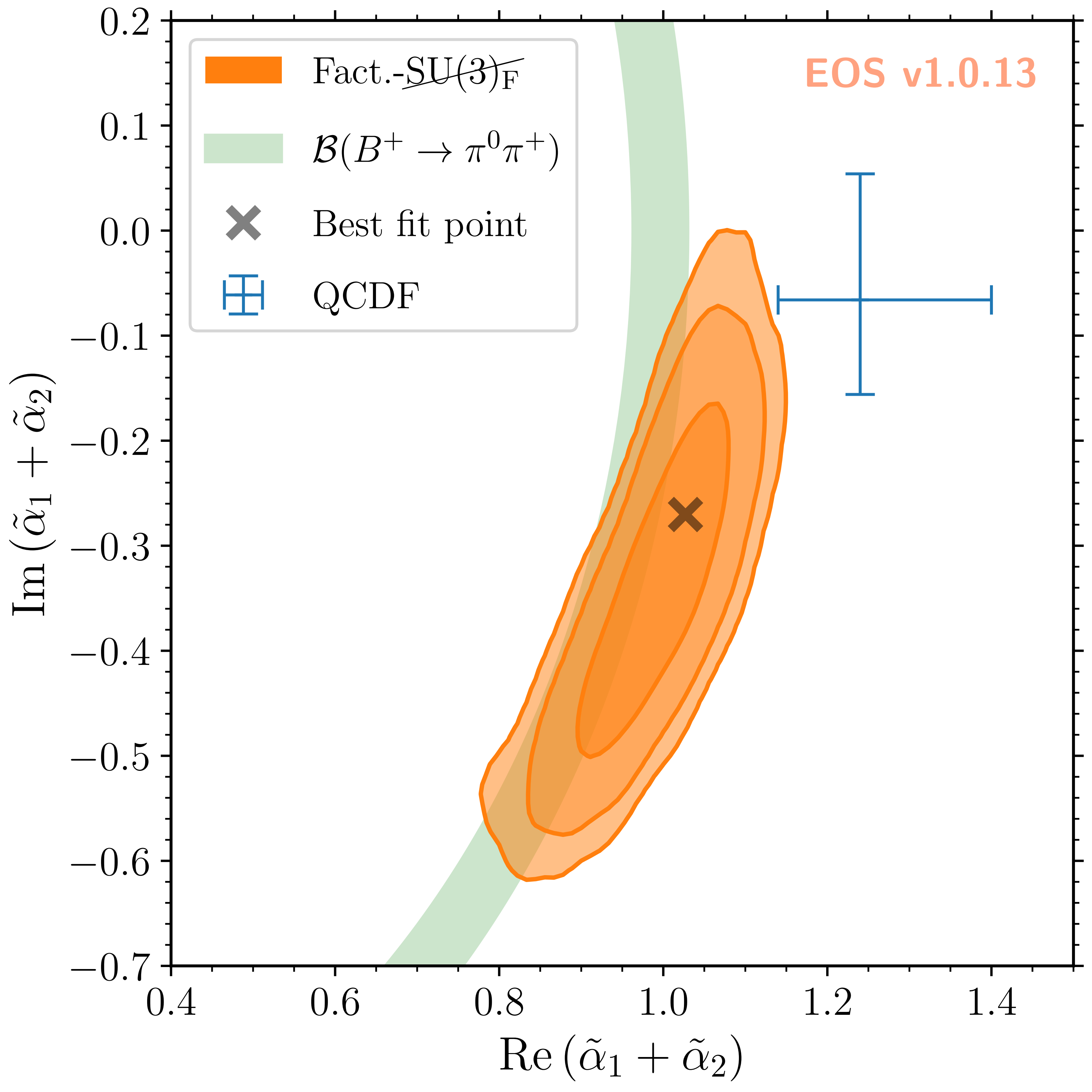
Only certain coefficients have been calculated up to NNLO in QCDF

$$\alpha_1(\pi\pi) = 1.000^{+0.029}_{-0.069} + 0.011^{+0.023}_{-0.050} i$$

$$\alpha_2(\pi\pi) = 0.240^{+0.217}_{-0.125} - 0.077^{+0.115}_{-0.078} i$$

$\rightarrow$ $\tilde{\alpha}_1 + \tilde{\alpha}_2$

Beneke, Huber, Xin-Qiang Li 0911.3655




Calculations of $a_4^u$ and $a_4^c$ by Bell, Beneke, Huber, Xin-Qiang Li 2002.03262

2111.15428

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  - Use well constrained relations and QCDF assumptions to extract individual coeffs.

Well constrained

**Preliminary**

$$\underbrace{\tilde{\alpha}_1 + \tilde{\alpha}_4^u}_{} - \underbrace{\frac{3}{2}\tilde{\alpha}_{4,\mathrm{EW}}^u - \tilde{\alpha}_4^c}_{} \overset{\mathrm{QCDF}}{=} \alpha_1 + \cancel{\delta_P}^{\mathcal{O}(10^{-2})} \Rightarrow \alpha_1 = (0.72 \pm 0.02) + (-0.041 \pm 0.002)i$$

$$\delta_P = \Delta P_4 + \Delta P_{10} + r_\chi\left(\Delta P_6 + \Delta P_8\right) \quad \text{with} \quad \Delta P_i = P_i^u - P_i^c$$

QCDF: $\alpha_1(\pi\pi) = 1.000^{+0.029}_{-0.069} + 0.011^{+0.023}_{-0.050}i$

# Coefficient results: Fact. $SU(3)_F$ breaking

2111.15428

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  - Hierarchies between coefficients: $\left|\frac{\alpha^c_{4,EW}}{\tilde{\alpha}_1}\right| \sim \left|\frac{\alpha^c_{3,EW}}{\tilde{\alpha}_2}\right| \sim \left|\frac{\tilde{\alpha}^c_4}{\tilde{\alpha}^u_4}\right| \sim [10^{-4}, 10^{-2}]$

Data is consistent with small EW corrections!

Surprisingly $\tilde{\alpha}^c_4 \ll \tilde{\alpha}^u_4$

$$\left|\frac{\tilde{\alpha}^c_4}{\tilde{\alpha}^c_4}\right| \sim 1 \pm \mathcal{O}(5\%)_{\text{QCDF,NLO}}$$

2111.15428

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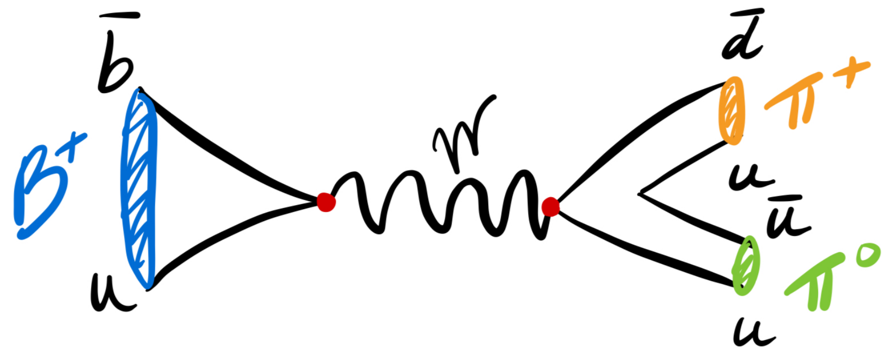
  - In progress: QCDF predictions for **annihilation** modes ($b$ coefficients) are **model dependent:** Can we improve the modelling?



# Conclusions

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- $\boldsymbol{SU(3)}$ **flavor symmetry** is a useful approximation that allows us to make predictions on **hadronic decays**, for which we do not have a strict theoretical framework

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- Factorizable $SU(3)_F$ breaking can be included with (almost) **no extra coefficients**

- In this approach, data can be explained **almost perfectly**, with a p-value of 31%.

- Certain relations ($\alpha_1$, $\left|\dfrac{\tilde{\alpha}_4^c}{\tilde{\alpha}_4^u}\right|$) in tension with QCDF predictions

# Conclusions

- $\boldsymbol{SU(3)}$ **flavor symmetry** is a useful approximation that allows us to make predictions on **hadronic decays**, for which we do not have a strict theoretical framework
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- In this approach, data can be explained **almost perfectly**, with a p-value of 31%.
- Certain relations ($\alpha_1$, $\left|\dfrac{\tilde{\alpha}_4^c}{\tilde{\alpha}_4^u}\right|$) in tension with QCDF predictions
- Experimental updates in modes like $B^0 \to K^0\bar{K}^0$ or $B^0 \to \pi^0\pi^0$ could improve significantly our predictions and **solve** the **strong correlations** between coefficients
- Detailed comparison with QCDF and inclusion of $\eta, \eta'$ mesons in progress

# Backup

# $\eta - \eta'$: Flavor vs. Mass basis

In addition to pions and kaons, the pseudo scalar meson spectrum also includes $\eta_8$ and $\eta_1$

$$\eta_8 = \frac{uu + dd - 2 \cdot ss}{\sqrt{6}} \qquad \eta_1 = \frac{uu + dd + ss}{\sqrt{3}}$$

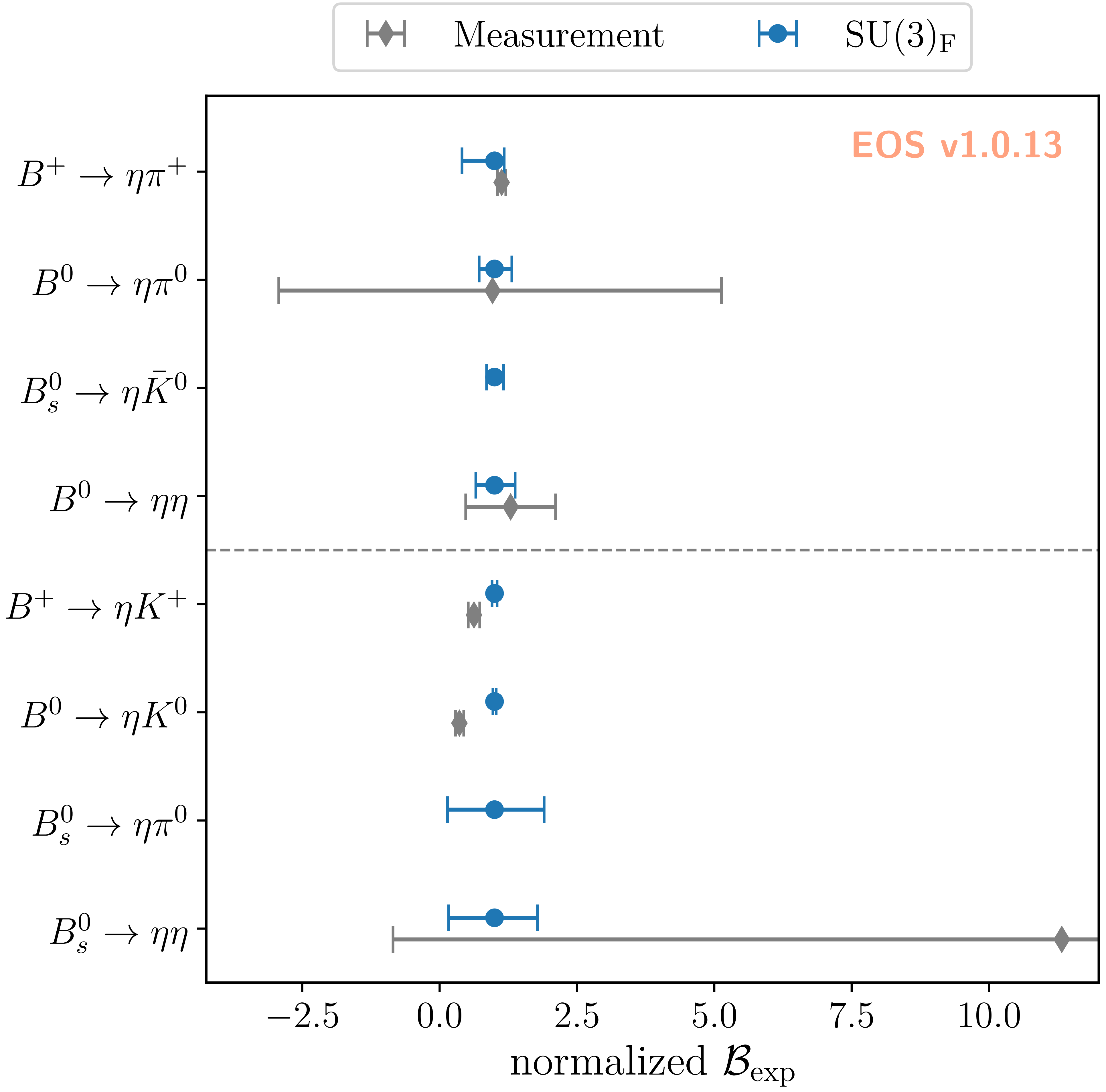
However, the mesons observed in experiments, $\eta$ and $\eta'$, are a mixture between these two

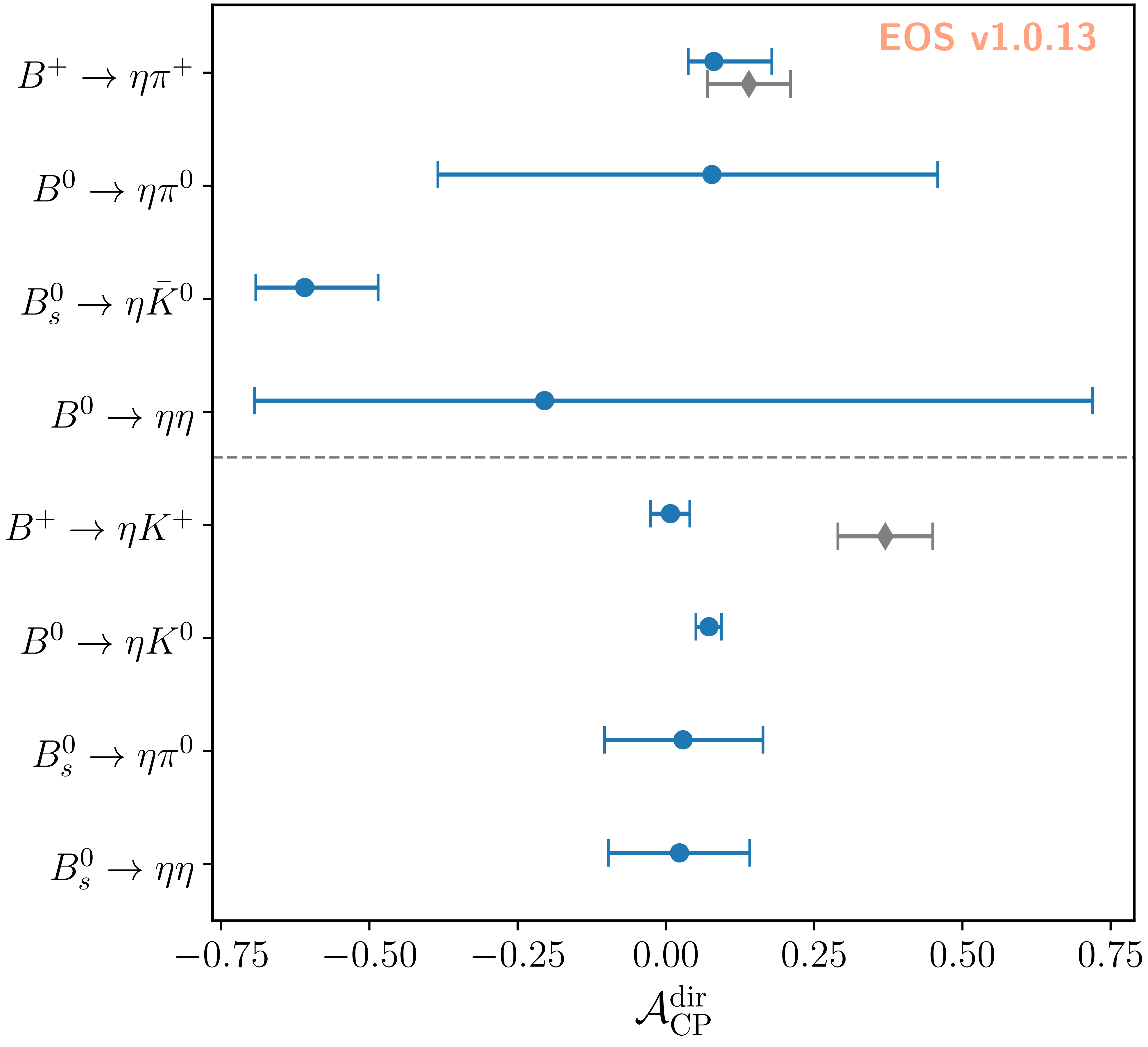
$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\eta_8\rangle \\ |\eta_1\rangle \end{pmatrix}$$

# $SU(3)$ limit in the $\eta - \eta'$ system

- In full $SU(3)$ flavor symmetry, $\theta = 0$ (therefore $\eta = \eta_8$ and $\eta' = \eta_1$ ) and $\eta_8$ meson is massless (like the pions and kaons.
- This is **not** the case for $\eta_1$, which remains massive even if the masses of the $u, d$ and $s$ are zero.
- In experiments, it is observed non-negligible mixing, $\theta \simeq -19°$: $\eta$ receives contributions form $\eta_1$
- $SU(3)$ breaking is needed to describe the $\eta - \eta'$ system with a certain level of accuracy
- Fact. $SU(3)_F$ breaking in $B \to \eta h$ would require inclusion of singlet coefficients and decoupling of form factors and decay constant into singlet and octet contributions

# $SU(3)_F$ postdictions for the $\eta$ system



See 2311.18011

# $SU(3)$ Flavor Symmetry

$q = u, d, s$

$D = d, s$

## 1. Fundamental representation of $SU(3)_F$, $q$:

$$\mathbf{3} = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

## 2. Transformation of each element in $\langle PP | \mathcal{O}_i | B \rangle$

- Light meson $P = q\bar{q}$ : $\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$
- $B$ meson $\bar{b}q$ : $\mathbf{3}$
- Hamiltonian operators $\mathcal{O}_i$ : $\bar{b} \to \bar{D}u\bar{u}$ and $\bar{b} \to \bar{D}$ : $\bar{\mathbf{3}}, \mathbf{6}$ and $\overline{\mathbf{15}}$

$\boxed{\text{How do we construct a matrix element } \langle PP | \mathcal{O}_i | B \rangle \text{ using } SU(3)_F\text{?}}$

## 3. Representation of final state:

$$\langle PP |$$

$$(\mathbf{8} \otimes \mathbf{8})_s = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27}$$

$$\mathcal{O}_i | B \rangle$$

- $\bar{\mathbf{3}} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8}_1$
- $\mathbf{6} \otimes \mathbf{3} = \mathbf{8}_2 \oplus \cancel{\mathbf{10}}$
- $\overline{\mathbf{15}} \otimes \mathbf{3} = \mathbf{8}_3 \oplus \cancel{\overline{\mathbf{10}}} \oplus \mathbf{27}$

## 4. Independent parameters

5 components for "tree"-like, 5 for "penguin" like:

$\boxed{\textbf{10 independent coefficients in any parameterisation}}$

# $SU(3)_F$ vs. Topological Parameterization

$$A_3^T = -\frac{A}{8} + \frac{3E}{8} + T_{PA},$$

$$C_3^T = \frac{1}{8}(3A - C - E + 3T) + T_P,$$

$$A_6^T = \frac{1}{4}(A - E),$$

$$C_6^T = \frac{1}{4}(-C + T),$$

$$B_{15}^T = \frac{T_{ES} + T_{AS}}{8},$$

$$B_3^T = T_{SS} + \frac{3T_{AS} - T_{ES}}{8},$$

$$D_3^T = T_S + \frac{1}{8}(3C - T_{AS} + 3T_{ES} - T),$$

$$B_6^T = \frac{1}{4}(T_{ES} - T_{AS}),$$

$$A_{15}^T = \frac{A + E}{8},$$

$$C_{15}^T = \frac{C + T}{8},$$

# Comparison with Berthiaume et al. 2311.18011

Seven Reduced Matrix Elements (RMEs)

$$
\begin{aligned}
\lambda_u^{(q)} &: \; A_1 = \langle \mathbf{1} || \mathbf{3}^*_{\mathbf{1}} || \mathbf{3} \rangle \;,\; A_8 = \langle \mathbf{8} || \mathbf{3}^*_{\mathbf{1}} || \mathbf{3} \rangle \;, \\
\lambda_t^{(q)} &: \; B_1 = \langle \mathbf{1} || \mathbf{3}^*_{\mathbf{2}} || \mathbf{3} \rangle \;,\; B_8 = \langle \mathbf{8} || \mathbf{3}^*_{\mathbf{2}} || \mathbf{3} \rangle \;, \\
\lambda_u^{(q)} \,\&\, \lambda_t^{(q)} &: \; R_8 = \langle \mathbf{8} || \mathbf{6} || \mathbf{3} \rangle \;,\; P_8 = \langle \mathbf{8} || \mathbf{15}^* || \mathbf{3} \rangle \;, \\
& \qquad P_{27} = \langle \mathbf{27} || \mathbf{15}^* || \mathbf{3} \rangle \;.
\end{aligned}
$$

In topological parameterisation: 9 topologies + 2 EW Penguin - Tree relations

$$
P_{EW}^{(C)} = -\frac{3}{4}\left[\frac{\Sigma_9}{\Sigma_1}(\widetilde{T} + \widetilde{C} \overset{+}{\scriptstyle (-)} \widetilde{A}) \overset{+}{\scriptstyle (-)} \frac{\Delta_9}{\Delta_1}(\widetilde{T} - \widetilde{C} - \widetilde{A})\right]
$$

where the 9 topologies come from the redefinition of coefficients

$$
\begin{aligned}
\widetilde{T} \equiv T + E \;,\quad \widetilde{C} \equiv C - E \;,\quad \widetilde{A} \equiv A + E \;, \\
\widetilde{P}_{uc} \equiv P_{uc} - E \;,\quad \widetilde{PA}_{uc} \equiv PA_{uc} + E \;.
\end{aligned}
$$

| Decay | $\lambda_u^{(d)}$ | | | | | $\lambda_t^{(d)}$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| Mode | $\widetilde{T}$ | $\widetilde{C}$ | $\widetilde{P}_{uc}$ | $\widetilde{A}$ | $\widetilde{PA}_{uc}$ | $P_{tc}$ | $PA_{tc}$ | $P_{EW}$ | $P_{EW}^C$ |
| $B^+ \to \overline{K}^0 K^+$ | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | $-\frac{1}{3}$ |
| $B^+ \to \pi^0 \pi^+$ | $-\frac{1}{\sqrt{2}}$ | $-\frac{1}{\sqrt{2}}$ | 0 | 0 | 0 | 0 | 0 | $-\frac{1}{\sqrt{2}}$ | $-\frac{1}{\sqrt{2}}$ |
| $B^0 \to K^0 \overline{K}^0$ | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | $-\frac{1}{3}$ |
| $B^0 \to \pi^+ \pi^-$ | $-1$ | 0 | $-1$ | 0 | $-1$ | $-1$ | $-1$ | 0 | $-\frac{2}{3}$ |
| $B^0 \to \pi^0 \pi^0$ | 0 | $-\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | 0 | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | $-\frac{1}{\sqrt{2}}$ | $-\frac{1}{3\sqrt{2}}$ |
| $B^0 \to K^+ K^-$ | 0 | 0 | 0 | 0 | $-1$ | 0 | $-1$ | 0 | 0 |
| $B_s^0 \to \pi^+ K^-$ | $-1$ | 0 | $-1$ | 0 | 0 | $-1$ | 0 | 0 | $-\frac{2}{3}$ |
| $B_s^0 \to \pi^0 \overline{K}^0$ | 0 | $-\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | 0 | 0 | $\frac{1}{\sqrt{2}}$ | 0 | $-\frac{1}{\sqrt{2}}$ | $-\frac{1}{3\sqrt{2}}$ |

# Fact. $SU(3)_F$ Coefficients

Beneke, Neubert 0308039,

$$\sum_{p=u,c} A_{M_1 M_2} \Big\{ \boldsymbol{B M_1} \left( \alpha_1 \boldsymbol{U}_p + \alpha_4^p + \alpha_{4,\mathrm{EW}}^p \,\hat{\boldsymbol{Q}} \right) \boldsymbol{M_2}\, \boldsymbol{\Lambda}_p$$

$$+ \boldsymbol{B M_1 \Lambda}_p \cdot \mathrm{Tr} \left[ \left( \alpha_2 \boldsymbol{U}_p + \alpha_3^p + \alpha_{3,\mathrm{EW}}^p \,\hat{\boldsymbol{Q}} \right) \boldsymbol{M_2} \right]$$

$$+ \boldsymbol{B} \left( \beta_2 \boldsymbol{U}_p + \beta_3^p + \beta_{3,\mathrm{EW}}^p \,\hat{\boldsymbol{Q}} \right) \boldsymbol{M_1 M_2 \Lambda}_p$$

$$+ \boldsymbol{B \Lambda}_p \cdot \mathrm{Tr} \left[ \left( \beta_1 \boldsymbol{U}_p + \beta_4^p + b_{4,\mathrm{EW}}^p \,\hat{\boldsymbol{Q}} \right) \boldsymbol{M_1 M_2} \right]$$

$$+ \boldsymbol{B} \left( \beta_{S2} \boldsymbol{U}_p + \beta_{S3}^p + \beta_{S3,\mathrm{EW}}^p \,\hat{\boldsymbol{Q}} \right) \boldsymbol{M_1 \Lambda}_p \cdot \mathrm{Tr}\boldsymbol{M_2}$$

$$+ \boldsymbol{B \Lambda}_p \cdot \mathrm{Tr} \left[ \left( \beta_{S1} \boldsymbol{U}_p + \beta_{S4}^p + b_{S4,\mathrm{EW}}^p \,\hat{\boldsymbol{Q}} \right) \boldsymbol{M_1} \right] \cdot \mathrm{Tr}\boldsymbol{M_2} \Big\}$$

# End-point divergences in QCDF

Beneke, Neubert 0308039,

End-point divergencies show up in hard spectator corrections:

$$H_i(M_1M_2) = \frac{B_{M_1M_2}}{A_{M_1M_2}} \frac{m_B}{\lambda_B} \int_0^1 dx \int_0^1 dy \left[ \frac{\Phi_{M_2}(x)\Phi_{M_1}(y)}{\bar{x}\bar{y}} + r_\chi^{M_1} \frac{\Phi_{M_2}(x)}{x} \frac{\Phi_{m_1}(y)}{\bar{y}} \right]$$

$$X_H^M = \int_0^1 \frac{dy}{\bar{y}} \Phi_p^M$$

And annihilation topologies:

$$\begin{aligned}
A_1^i &= \pi\alpha_s \int_0^1 dxdy \left\{ \Phi_{M_2}(x)\,\Phi_{M_1}(y) \left[ \frac{1}{y(1-x\bar{y})} + \frac{1}{\bar{x}^2 y} \right] + r_\chi^{M_1} r_\chi^{M_2}\, \Phi_{m_2}(x)\,\Phi_{m_1}(y)\, \frac{2}{\bar{x}y} \right\}, \\
A_1^f &= 0\,, \\
A_2^i &= \pi\alpha_s \int_0^1 dxdy \left\{ \Phi_{M_2}(x)\,\Phi_{M_1}(y) \left[ \frac{1}{\bar{x}(1-x\bar{y})} + \frac{1}{\bar{x}y^2} \right] + r_\chi^{M_1} r_\chi^{M_2}\, \Phi_{m_2}(x)\,\Phi_{m_1}(y)\, \frac{2}{\bar{x}y} \right\}, \\
A_2^f &= 0\,, \qquad (5 \\
A_3^i &= \pi\alpha_s \int_0^1 dxdy \left\{ r_\chi^{M_1}\, \Phi_{M_2}(x)\,\Phi_{m_1}(y)\, \frac{2\bar{y}}{\bar{x}y(1-x\bar{y})} - r_\chi^{M_2}\, \Phi_{M_1}(y)\,\Phi_{m_2}(x)\, \frac{2x}{\bar{x}y(1-x\bar{y})} \right\}, \\
A_3^f &= \pi\alpha_s \int_0^1 dxdy \left\{ r_\chi^{M_1}\, \Phi_{M_2}(x)\,\Phi_{m_1}(y)\, \frac{2(1+\bar{x})}{\bar{x}^2 y} + r_\chi^{M_2}\, \Phi_{M_1}(y)\,\Phi_{m_2}(x)\, \frac{2(1+y)}{\bar{x}y^2} \right\}.
\end{aligned}$$

# End-point divergences in QCDF

Beneke, Neubert 0308039,

End-point divergences are defined as: $X_H^M = \int_0^1 \frac{dy}{\bar{y}} \Phi_p^M$ with $\bar{y}$ the momentum of the antiquark

Perturbative computations of these interactions are expected to hold up to $\bar{y} \sim \Lambda_{QCD}/m_b$, and therefore:

$$X_H^M \sim \ln(m_b/\Lambda_{QCD})$$

Current modelling of these divergences assigns 100% uncertainty + complex phase to this estimation

$$X_H \approx (1 + \rho_H e^{i\phi_H})\ln(m_b/\Lambda_{QCD}) \qquad \text{with } \rho_H \leq 1$$

# Redefinitions in Fact. $SU(3)_F$

The parametrisation formula show redundancy in the number of coefficients:

$$\tilde{\alpha}_1 \equiv \alpha_1 + \frac{3}{2}\alpha^u_{4,EW}, \tilde{\alpha}_2 \equiv \alpha_2 + \frac{3}{2}\alpha^u_{3,EW}$$

$$\tilde{\beta}_1 \equiv \beta_1 + \frac{3}{2}\beta^u_{4,EW}, \tilde{\beta}_2 \equiv \beta_2 + \frac{3}{2}\beta^u_{3,EW},$$

$$\tilde{\alpha}^r_4 \equiv \alpha^r_4 + \beta^r_3 - \frac{1}{2}\left(\alpha^r_{4,EW} + \beta^r_{3,EW}\right), \tilde{\beta}^r_4 \equiv \beta^r_4 - \frac{1}{2}\beta^r_{4,EW},$$

Certain coefficients have been calculated up to NNLO in QCDF (See 0911.3655, 1507.03700)

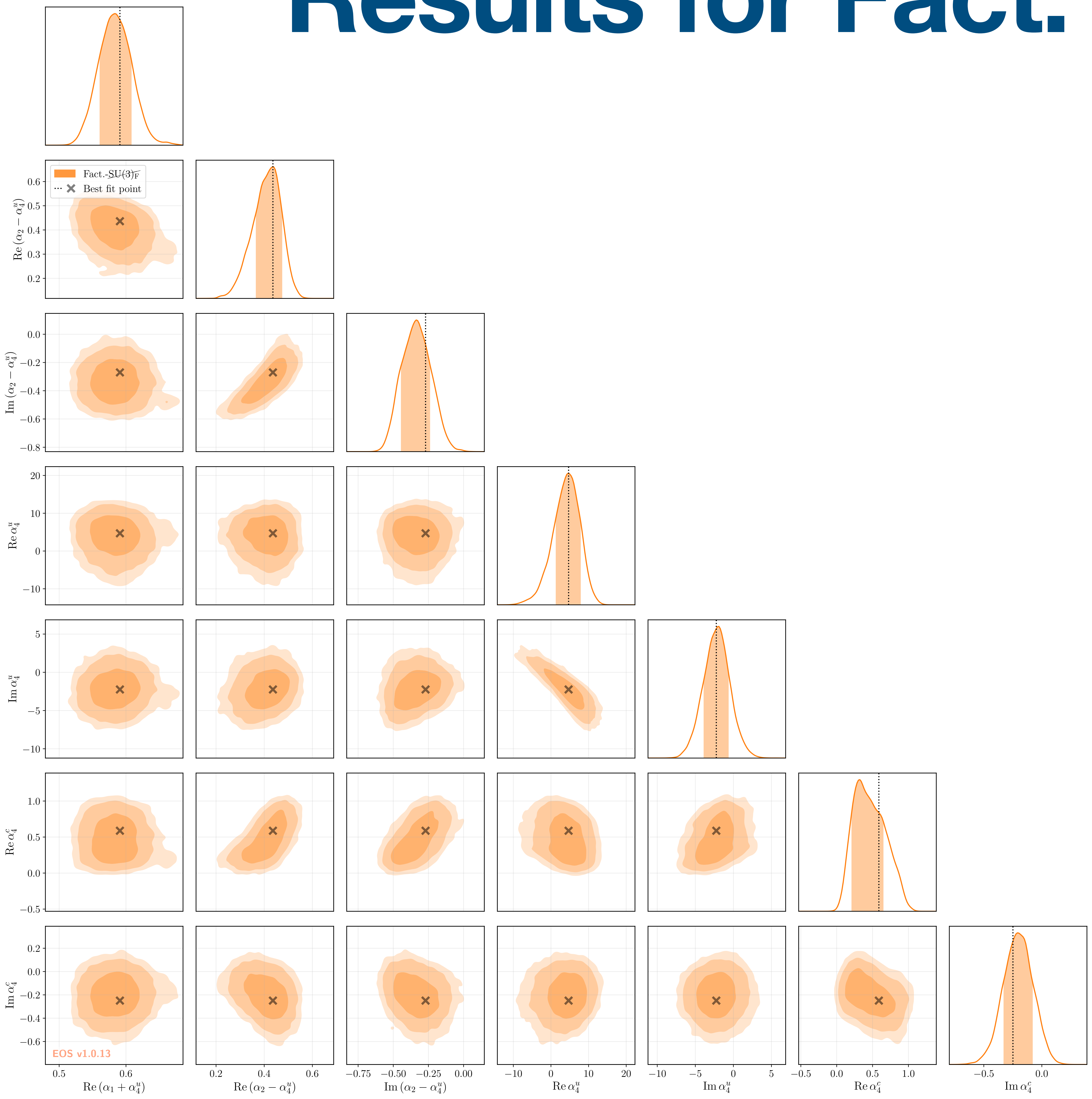
$$\alpha_1 = 1.000^{+0.029}_{-0.069} + 0.011^{+0.023}_{-0.050}i \qquad a^u_4 = -\left(2.46^{+0.49}_{-0.24} + 1.94^{+0.32}_{-0.20}i\right) \times 10^{-2}$$

$$\alpha_2 = 0.240^{+0.217}_{-0.125} - 0.077^{+0.115}_{-0.078}i \qquad a^c_4 = -\left(3.34^{+0.43}_{-0.27} + 1.05^{+0.45}_{-0.36}i\right) \times 10^{-2}$$

# Fact. $SU(3)_F$ Coefficients

| $b \to d$ **decay** | $\tilde{\alpha}_1$ | $\tilde{\alpha}_2$ | $\tilde{\alpha}_4^u$ | $\tilde{b}_2$ | $\tilde{b}_1$ | $\tilde{b}_4^u$ | $b \to s$ **decay** | $\tilde{\alpha}_1$ | $\tilde{\alpha}_2$ | $\tilde{\alpha}_4^u$ | $\tilde{b}_2$ | $\tilde{b}_1$ | $\tilde{b}_4^u$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $B^+ \to \pi^0\pi^+$ | $\frac{1}{\sqrt{2}}$ | 0 | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | 0 | 0 | $B^+ \to \pi^0 K^+$ | $\frac{1}{\sqrt{2}}$ | 0 | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | 0 | 0 |
| $B^+ \to \pi^+\pi^0$ | 0 | $\frac{1}{\sqrt{2}}$ | $-\frac{1}{\sqrt{2}}$ | $-\frac{1}{\sqrt{2}}$ | 0 | 0 | $B^+ \to K^+\pi^0$ | 0 | $\frac{1}{\sqrt{2}}$ | 0 | 0 | 0 | 0 |
| $B^+ \to \bar{K}^0 K^+$ | 0 | 0 | 0 | 0 | 0 | 0 | $B^+ \to K^0\pi^+$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $B^+ \to K^+\bar{K}^0$ | 0 | 0 | 1 | 1 | 0 | 0 | $B^+ \to \pi^+ K^0$ | 0 | 0 | 1 | 1 | 0 | 0 |
| $B^0 \to \pi^+\pi^-$ | 0 | 0 | 0 | 0 | 1 | 1 | $B_s \to K^+K^-$ | 0 | 0 | 0 | 0 | 1 | 1 |
| $B^0 \to \pi^-\pi^+$ | 1 | 0 | 1 | 0 | 0 | 1 | $B_s \to K^-K^+$ | 1 | 0 | 1 | 0 | 0 | 1 |
| $B^0 \to \pi^0\pi^0$ | 0 | $-1$ | 1 | 0 | 1 | 2 | $B_s \to \pi^0\pi^0$ | 0 | 0 | 0 | 0 | 1 | 2 |
| $B^0 \to K^+K^-$ | 0 | 0 | 0 | 0 | 1 | 1 | $B_s \to \pi^+\pi^-$ | 0 | 0 | 0 | 0 | 1 | 1 |
| $B^0 \to K^-K^+$ | 0 | 0 | 0 | 0 | 0 | 1 | $B_s \to \pi^-\pi^+$ | 0 | 0 | 0 | 0 | 0 | 1 |
| $B^0 \to K^0\bar{K}^0$ | 0 | 0 | 1 | 0 | 0 | 1 | $B_s \to \bar{K}^0 K^0$ | 0 | 0 | 1 | 0 | 0 | 1 |
| $B^0 \to \bar{K}^0 K^0$ | 0 | 0 | 0 | 0 | 0 | 1 | $B_s \to K^0\bar{K}^0$ | 0 | 0 | 0 | 0 | 0 | 1 |
| $B_s \to \pi^+K^-$ | 0 | 0 | 0 | 0 | 0 | 0 | $B^0 \to K^+\pi^-$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $B_s \to K^-\pi^+$ | 1 | 0 | 1 | 0 | 0 | 0 | $B^0 \to \pi^-K^+$ | 1 | 0 | 1 | 0 | 0 | 0 |
| $B_s \to \pi^0\bar{K}^0$ | 0 | 0 | 0 | 0 | 0 | 0 | $B^0 \to \pi^0 K^0$ | 0 | 0 | $-\frac{1}{\sqrt{2}}$ | 0 | 0 | 0 |
| $B_s \to \bar{K}^0\pi^0$ | 0 | $\frac{1}{\sqrt{2}}$ | $-\frac{1}{\sqrt{2}}$ | 0 | 0 | 0 | $B^0 \to K^0\pi^0$ | 0 | $\frac{1}{\sqrt{2}}$ | 0 | 0 | 0 | 0 |

# Results for Fact. $SU(3)_F$ coefficients




$$\begin{cases} \tilde{\alpha}_1 + \tilde{\alpha}_4^u = 0.584 \pm 0.24 \\ \dfrac{3}{2}\alpha^c_{4,\mathrm{EW}} + \tilde{\alpha}_4^c = -\left(0.102 \pm 0.001\right) + \left(0.044 \pm 0.002\right) i \end{cases}$$

$$\begin{cases} \tilde{\alpha}_2 - \tilde{\alpha}_4^u = \left(0.414^{+0.053}_{-0.065}\right) - \left(0.34 \pm 0.11\right) i \\ \dfrac{3}{2}\alpha^c_{3,EW} - \tilde{\alpha}_4^c = \left(0.141^{+0.031}_{-0.024}\right) - \left(0.059 \pm 0.010\right) i \end{cases}$$

$$\begin{cases} \tilde{b}_1 + 2\tilde{b}_4^u = -\left(3.7^{+9.6}_{-8.2}\right) + \left(13.7^{+6.9}_{-11.9}\right) i \\ \dfrac{3}{2}b^c_{4,EW} + 2\tilde{b}_4^c = -\left(1.5^{+1.4}_{-1.5}\right) + \left(11.0^{+4.6}_{-5.2}\right) i \end{cases}$$

# Theoretical vs. Experimental Branching Ratio

Experimental Branching ratio is given by:

$$\langle \Gamma(B_s(t) \to f) \rangle \equiv \Gamma(B_s^0(t) \to f) + \Gamma(\bar{B}_s^0(t) \to f) = R_{\mathrm{H}}^f e^{-\Gamma_{\mathrm{H}}^{(s)} t} + R_{\mathrm{L}}^f e^{-\Gamma_{\mathrm{L}}^{(s)} t},$$

Theoretical Branching ratio is given by:

$$\langle \Gamma(B_s(t) \to f) \rangle \big|_{t=0} \equiv \Gamma(B_s^0 \to f) + \Gamma(\bar{B}_s^0 \to f)$$

Relation between Theoretical and Experimental BR (effects up to $\mathcal{O}(7\%)$):

$$\mathrm{BR}\left(B_s \to f\right)_{\mathrm{theo}} = \left[\frac{1 - y_s^2}{1 + \mathcal{A}_{\Delta\Gamma}^f\, y_s}\right] \mathrm{BR}\left(B_s \to f\right)_{\mathrm{exp}}$$

# Role of interference in CP violation

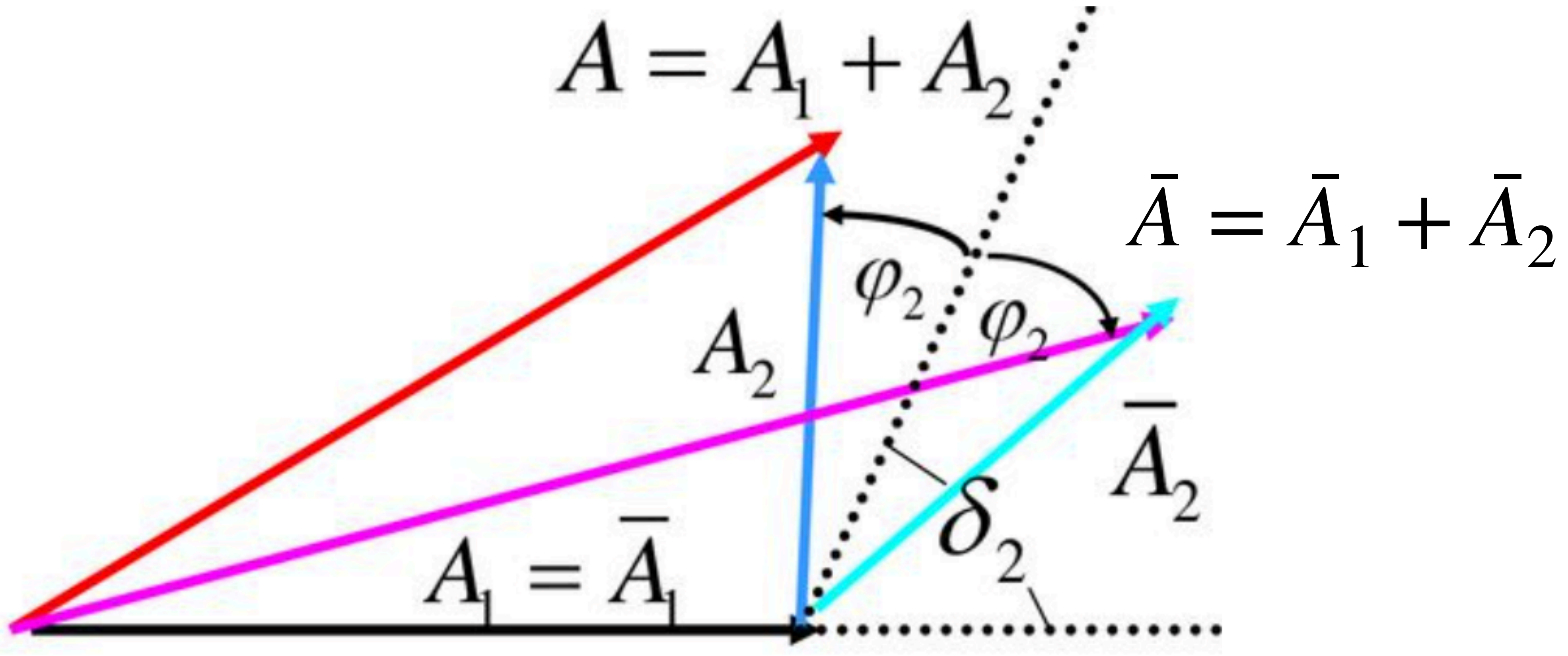
Consider the amplitude of a certain process $B \to f$. CP violation arises if for a certain observable $\mathcal{O}$ we have $\mathcal{O}(B \to f) \neq \mathcal{O}(\bar{B} \to \bar{f})$

Observables depend only on the **modulus** of the amplitude

Only weak phases are shifted when they are CP conjugated

We can define de amplitude of the process as $A = A_1 + A_2$

CP violation only appears if $A_1$ and $A_2$ have a relative weak phase, $\phi_2$, and strong phase, $\delta_2$




$A_1 = |A_1|$

$A_2 = e^{i\delta_2} e^{i\phi} |A_2|$