EWP-Tree Relations and the $B \to \pi K$ Puzzle

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Talk based on the paper "Isospin-based EWP-tree Relations," 2510.13969, B. Bhattacharya, M. Bouchard, A. Jean, D.L. and I. Ray.

Indirect Searches for NP

Essentially all research in particle physics today involves looking for physics beyond the SM. How to search for NP? Two ways. (1) Direct searches. But LHC has found no new particles. (2) Indirect searches: look for (sets of) observables whose measurements disagree with the predictions of the SM. The subject of my talk today – in fact, the subject of this entire workshop – falls into this category.

Sounds simple, but is problematic. What is the "proof of NP detection?" Direct searches: 5σ. Indirect searches: ???

Sociology/PR: if find an indirect signal of NP, have to convince the physics community that this discrepancy should be taken seriously.

If indirect signal seen, $\exists$ question: can QCD effects explain the anomaly? (Almost impossible to calculate such effects with any certainty.)

Example: for 10+ years $\exists$ several observables in decays governed by the transition $b \to s\mu^+\mu^-$ whose measured values disagree with the SM (e.g., in the angular distribution of $B \to K^*\mu^+\mu^-$). $\exists$ some theory input ($B \to K^*$ form factors). Some say these theoretical uncertainties are "under control," others say that they are important.

Another observable (tests lepton flavour universality):

$$R_{K^{(*)}} \equiv \frac{\mathcal{B}(B \to K^{(*)}\mu^+\mu^-)}{\mathcal{B}(B \to K^{(*)}e^+e^-)} \ .$$

The SM predicts $R_{K^{(*)}} = 1$ with essentially no uncertainty. Pre-2022: measurements found $R_{K^{(*)}} \simeq 0.85$ with small error. This disagrees with the SM, and there was lots of theoretical activity trying to understand these results. 2022: LHCb remeasures $R_{K^{(*)}}$, finds it now agrees with the SM. Interest in the $b \to s\ell^+\ell^-$ anomalies drops considerably.

Point: can study nonleptonic decays of heavy mesons with the hope of finding an anomaly, i.e., a disagreement with the SM. However: must minimize theoretical input (e.g., saying a measurement disagrees with a QCDf calculation is unconvincing). QCD corrections must be dealt with as completely and rigorously as possible. Also, ideally: if an anomaly is found, come up with a NP explanation that can be tested via direct NP searches.

Methodology (see also talks by B. Bhattacharya and A. Jean tomorrow):

1. Assume a flavour symmetry (isospin, U spin, $SU(3)_F$).
2. Find all decays related by this symmetry, express their amplitudes in terms of RMEs using the Wigner-Eckart theorem.
3. Take all observables measured in these decays, perform a fit to see how well the SM explains the data, assuming no symmetry breaking. Note: apart from symmetry breaking, there are no other QCD corrections $\Longrightarrow$ this is rigorous, group-theoretically.
4. Include first-order symmetry breaking, redo the fit. At this stage, all leading-order QCD corrections are taken into account.
5. If there is still an anomaly, try to construct a NP explanation.

Flavour $SU(3)$

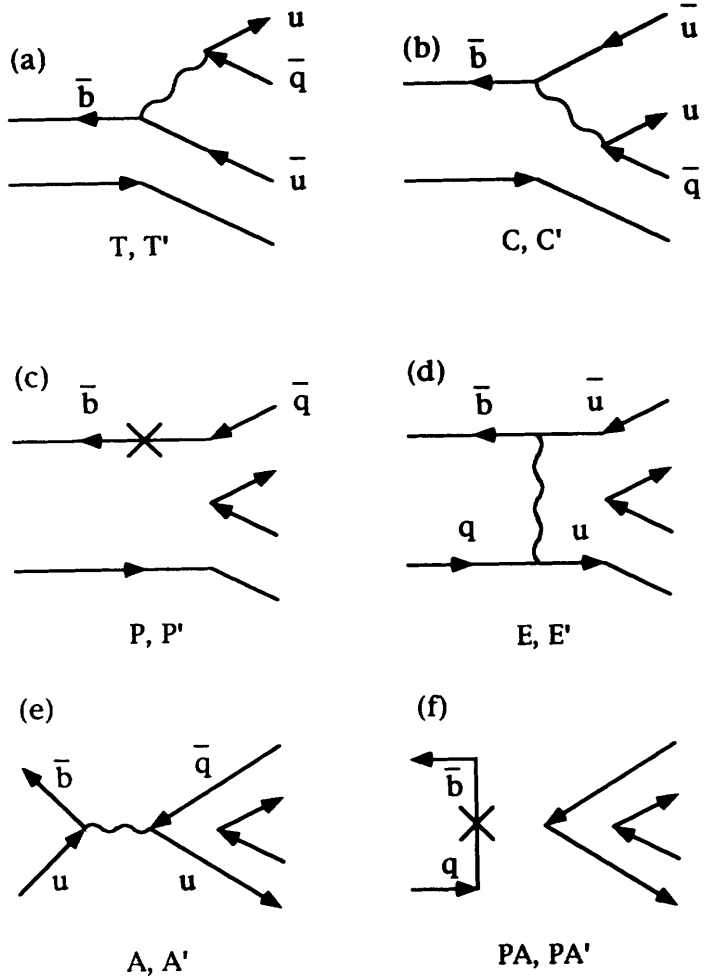
Consider $B \to PP$ decays, $P \in \{\pi, K\}$. $\exists$ 16 decays, 8 $\Delta S = 0$ and 8 $\Delta S = 1$ The decays are related to one another by $SU(3)_F$.

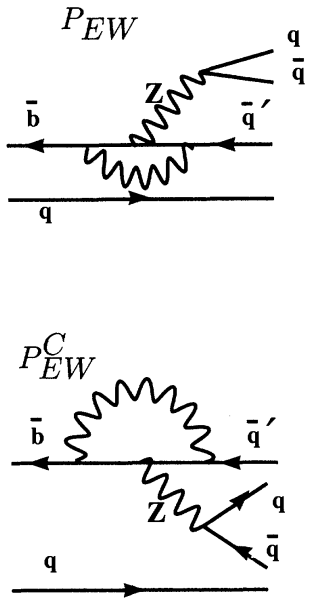
Identify RMEs:
Under $SU(3)_F$, (u, d, s) is a $\mathbf{3} \Longrightarrow$ Initial state $B = (B^+, B^0, B_s^0)$ also a $\mathbf{3}$.
Decays involve $\bar{b} \to \bar{q}u\bar{u}$ or $\bar{b} \to \bar{q}(d\bar{d} + s\bar{s})$ $(q = d, s)$, so the weak Hamiltonian transforms as a $\mathbf{3}^*$, $\mathbf{6}$ or $\mathbf{15}^*$. Final-state Ps must be symmetrized (all members of the $\mathbf{8}$ are considered to be identical particles), with $(\mathbf{8} \otimes \mathbf{8})_{\rm sym} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27}$.

This leads to five RMEs:

$$S_1 = \langle \mathbf{1}||H_W(\mathbf{3}^*)||\mathbf{3}\rangle \ , \ S_8 = \langle \mathbf{8}||H_W(\mathbf{3}^*)||\mathbf{3}\rangle \ ,$$
$$R_8 = \langle \mathbf{8}||H_W(\mathbf{6})||\mathbf{3}\rangle \ , \ P_8 = \langle \mathbf{8}||H_W(\mathbf{15}^*)||\mathbf{3}\rangle \ , \ P_{27} = \langle \mathbf{27}||H_W(\mathbf{15}^*)||\mathbf{3}\rangle \ .$$

Diagrams vs. RMEs: can parametrize amplitudes using topological diagrams (T, C, P, etc.). Can show that diagrams are equivalent to RMEs. Caveat: there are more diagrams than RMEs $\Longrightarrow$ have to carefully use appropriate combinations of diagrams when performing fits.





Advantages of diagrams: can estimate relative sizes of diagrams. If we keep all diagrams, the analysis is equivalent to using RMEs, and is rigorous, group-theoretically; if we make assumptions about the relative sizes of diagrams, this is theoretical input.

It is often useful to use both parametrizations, first to get a rigorous result for the fit (RMEs), and then to compare it with the result when theoretical input is added (diagrams).

Reminder: charged-current interactions follow the unitary CKM matrix, V_{CKM}. T, C, E, A diagrams $\propto V_{ub}^* V_{uq}$. However, the P diagram:

$$\begin{aligned} P &= P_u V_{ub}^* V_{uq} + P_c V_{cb}^* V_{cq} + P_t V_{tb}^* V_{tq} \\ &= P_{uc} V_{ub}^* V_{uq} + P_{tc} V_{tb}^* V_{tq} \ , \end{aligned}$$

where $P_{uc} \equiv P_u - P_c$, $P_{tc} \equiv P_t - P_c$.

We define $\lambda_u^{(q)} \equiv V_{ub}^* V_{uq}$ and $\lambda_t^{(q)} \equiv V_{tb}^* V_{tq}$. All diagrams are proportional to $\lambda_u^{(q)}$ or $\lambda_t^{(q)}$.

Similarly, all RMEs can be split into pieces proportional to $\lambda_u^{(q)}$ and $\lambda_t^{(q)}$:

$$S_1 = \lambda_u^{(q)} A_1 - \lambda_t^{(q)} B_1 \ , \quad S_8 = \lambda_u^{(q)} A_8 - \lambda_t^{(q)} B_8 \ ,$$
$$R_8 = \lambda_u^{(q)} R_8^u - \lambda_t^{(q)} R_8^t \ ,$$
$$P_8 = \lambda_u^{(q)} P_8^u - \lambda_t^{(q)} P_8^t \ , \quad P_{27} = \lambda_u^{(q)} P_{27}^u - \lambda_t^{(q)} P_{27}^t \ .$$

So there are really 10 $SU(3)_F$ RMEs in $B \to PP$ decays.

The Weak Hamiltonian

The weak Hamiltonian that governs charmless $B \to PP$ decays is

$$H_W = \frac{G_F}{\sqrt{2}} \left(\lambda_u^{(q)} \sum_{i=1}^{2} c_i Q_i^{(q)} - \lambda_t^{(q)} \sum_{i=3}^{10} c_i Q_i^{(q)} \right) \quad , \qquad q = d, s \ .$$

$Q_{1,2}^{(q)}$ are tree operators, $\propto \lambda_u^{(q)}$. $Q_{3\text{-}6}^{(q)}$ are gluonic penguin operators and $Q_{7\text{-}10}^{(q)}$ are EWP operators, all $\propto \lambda_t^{(q)}$.

Now, $Q_{7,8}^{(q)}$ are $(V-A) \times (V+A)$ EWP operators,
$Q_{9,10}^{(q)}$ are $(V-A) \times (V-A)$ EWP operators.

Observation: $c_{7,8}$ much smaller than $c_{9,10} \Longrightarrow$ neglect $Q_{7,8}$.
$\Longrightarrow$ EWPs are purely $(V-A) \times (V-A)$.

Look more closely:

$$Q_2 = (\bar{b}q)(\bar{u}u) \ ,$$
$$Q_3 = (\bar{b}q)[(\bar{u}u) + (\bar{d}d) + (\bar{s}s)] \ ,$$
$$Q_9 = \tfrac{3}{2}\left[(\bar{b}q)\left[\tfrac{2}{3}(\bar{u}u) - \tfrac{1}{3}(\bar{d}d) - \tfrac{1}{3}(\bar{s}s)\right]\right] \ .$$

All are $(V-A)\times(V-A)$.

This leads to

$$Q_9 = \frac{3}{2}Q_2 - \frac{1}{2}Q_3 \ . \ \text{ Similarly, } Q_{10} = \frac{3}{2}Q_1 - \frac{1}{2}Q_4 \ .$$

$\Longrightarrow \exists$ relations between the tree, gluonic penguin and EWP operators.

But note: gluonic penguin operators ($Q_{3,4}$) involve only $\bar{b}\to\bar{q}$, so they transform purely as $\mathbf{3}^*$. On the other hand, tree and EWP operators ($Q_{1,2}$ and $Q_{9,10}$) transform as $\mathbf{3}^*$, $\mathbf{6}$ or $\mathbf{15}^*$.

$\Longrightarrow$ tree and EWP operators contribute to the RMEs $\langle \mathbf{8}||H_W(\mathbf{6})||\mathbf{3}\rangle$, $\langle \mathbf{8}||H_W(\mathbf{15}^*)||\mathbf{3}\rangle$, $\langle \mathbf{27}||H_W(\mathbf{15}^*)||\mathbf{3}\rangle$, but gluonic penguin operators do not.

$SU(3)_F$ EWP-Tree Relations

Recall:

$$R_8 = \lambda_u^{(q)} R_8^u - \lambda_t^{(q)} R_8^t \ ,$$
$$P_8 = \lambda_u^{(q)} P_8^u - \lambda_t^{(q)} P_8^t \ , \quad P_{27} = \lambda_u^{(q)} P_{27}^u - \lambda_t^{(q)} P_{27}^t \ .$$

The pieces $\propto \lambda_u^{(q)}$ come from trees, the pieces $\propto \lambda_t^{(q)}$ come from EWPs.

But for these RMEs, tree and EWP operators are proportional to one another, leading to:

$$R_8^t = \frac{3}{2} \frac{(c_9 - c_{10})}{(c_1 - c_2)} R_8^u \ , \quad P_8^t = \frac{3}{2} \frac{(c_9 + c_{10})}{(c_1 + c_2)} P_8^u \ , \quad P_{27}^t = \frac{3}{2} \frac{(c_9 + c_{10})}{(c_1 + c_2)} P_{27}^u \ .$$

These are the $SU(3)_F$ EWP-tree relations; they reduces the number of independent $SU(3)_F$ RMEs in $B \to PP$ decays from 10 to 7.

Can also write EWP-tree relations in terms of diagrams. Taking $(c_9 - c_{10})/(c_1 - c_2) = (c_9 + c_{10})/(c_1 + c_2)$ (better than 3%), they are

$$\begin{aligned} P_{EW} + P_{EW}^E &= -\frac{3}{2}\frac{(c_9 + c_{10})}{(c_1 + c_2)}(T + E) , \\ P_{EW}^C - P_{EW}^E &= -\frac{3}{2}\frac{(c_9 + c_{10})}{(c_1 + c_2)}(C - E) , \\ P_{EW}^A + P_{EW}^E &= -\frac{3}{2}\frac{(c_9 + c_{10})}{(c_1 + c_2)}(A + E) . \end{aligned}$$

The first two EWP-tree relations above can be added together to produce a "combined" EWP-tree relation:

$$P_{EW} + P_{EW}^C = -\frac{3}{2}\frac{(c_9 + c_{10})}{(c_1 + c_2)}(T + C) .$$

These results are not new. The combined EWP-tree relation was first derived in 1998 by Neubert and Rosner ([hep-ph/9808493], [hep-ph/9809311]). And the first two $SU(3)_F$ EWP-tree relations were derived in a paper by Gronau, Pirjol and Yan, later in 1998 ([hep-ph/9810482]) .

With these EWP-tree relations, we can quantify the effect of neglecting the $Q_{7,8}$ operators. These operators contribute to the same RMEs as $Q_{9,10}$. However, $Q_{7,8}$ are $(V-A)\times(V+A)$ operators, while $Q_{9,10}$ are $(V-A)\times(V-A)$. Including the contributions of $Q_{7,8}$, the above combined EWP-tree relation is modified to

$$P_{EW}+P_{EW}^{C}=-\frac{3}{2}\frac{(c_9+c_{10})}{(c_1+c_2)}\left[1+\frac{(c_7X_7+c_8X_8)}{(c_9+c_{10})}\right](T+C)\ ,$$

where $X_{7,8}$ are the ratios of $(V-A)\times(V+A)$ and $(V-A)\times(V-A)$ matrix elements.

X_7 and X_8 have both been calculated within QCDf for $B\to\pi K$ decays (M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, [hep-ph/0104110]), and $X_{7,8}=O(1)$ is found. Now, $(c_7+c_8)/(c_9+c_{10})\simeq 10\% \Longrightarrow$ the above correction is $\simeq 10\%$.

Point: assuming this holds for all $B\to PP$ decays, had $Q_{7,8}$ not been neglected, the EWP-tree relations would change by at most $\simeq 10\%$.

The $B \to \pi K$ Puzzle

The symmetry is isospin. $\exists$ 4 $B \to \pi K$ decays: $B^+ \to \pi^+ K^0$ $(+0)$, $B^+ \to \pi^0 K^+$ $(0+)$, $B^0 \to \pi^- K^+$ $(-+)$ and $B^0 \to \pi^0 K^0$ (00).

Identify RMEs:
Initial state $B = (B^+, B^0)$ has isospin $\frac{1}{2}$. Decays involve $\bar{b} \to \bar{s}u\bar{u}$ or $\bar{b} \to \bar{s}d\bar{d}$, so the Hamiltonian has two pieces, H_W^0 and H_W^1. Final πK states have isospin $\frac{1}{2}$ or $\frac{3}{2}$

$\Longrightarrow$ there are three RMEs: $\langle \frac{1}{2} || H_W^0 || \frac{1}{2} \rangle$, $\langle \frac{1}{2} || H_W^1 || \frac{1}{2} \rangle$ and $\langle \frac{3}{2} || H_W^1 || \frac{1}{2} \rangle$.

With four decay amplitudes, $\exists$ a quadrilateral relation:

$$A^{+0} + \sqrt{2} A^{0+} = A^{-+} + \sqrt{2} A^{00} \ .$$

All RMEs can be split into pieces proportional to $\lambda_u^{(s)}$ and $\lambda_t^{(s)}$
$\Longrightarrow$ $\exists$ a total of 6 RMEs, 3 proportional to $\lambda_u^{(s)}$, 3 proportional to $\lambda_t^{(s)}$.

Can write $B \to \pi K$ amplitudes in terms of diagrams:

$$
\begin{aligned}
A^{+0} &= \lambda_u^{(s)} \widetilde{P_{uc}} + \lambda_t^{(s)} \left[\widetilde{P_{tc}} - \frac{1}{3} \widetilde{P_{EW}^C} \right] , \\
\sqrt{2} A^{0+} &= \lambda_u^{(s)} \left[-\widetilde{T} - \widetilde{C} - \widetilde{P_{uc}} \right] + \lambda_t^{(s)} \left[-\widetilde{P_{tc}} - \widetilde{P_{EW}} - \frac{2}{3} \widetilde{P_{EW}^C} \right] , \\
A^{-+} &= \lambda_u^{(s)} \left[-\widetilde{T} - \widetilde{P_{uc}} \right] + \lambda_t^{(s)} \left[-\widetilde{P_{tc}} - \frac{2}{3} \widetilde{P_{EW}^C} \right] , \\
\sqrt{2} A^{00} &= \lambda_u^{(s)} \left[-\widetilde{C} + \widetilde{P_{uc}} \right] + \lambda_t^{(s)} \left[\widetilde{P_{tc}} - \widetilde{P_{EW}} - \frac{1}{3} \widetilde{P_{EW}^C} \right] ,
\end{aligned}
$$

where the effective diagrams have "absorbed" A, P_{EW}^E and $P_{EW}^{P_u}$. The diagram E does not appear in $B \to \pi K$ decays.

As with RMEs, there are 3 diagrams proportional to $\lambda_u^{(s)}$ $(\widetilde{T}, \widetilde{C}, \widetilde{P_{uc}})$, 3 proportional to $\lambda_t^{(s)}$ $(\widetilde{P_{tc}}, \widetilde{P_{EW}}, \widetilde{P_{EW}^C})$. Each $\lambda_u^{(s)}$ $(\lambda_t^{(s)})$ RME can be written as a linear combination of the 3 $\lambda_u^{(s)}$ $(\lambda_t^{(s)})$ diagrams.

In the past, analyses of $B \to \pi K$ decays imposed the $SU(3)_F$ EWP-tree relations. Problem: it is necessary to neglect E and $A \Longrightarrow$ analyses are not rigorous, group theoretically.

Better: use isospin symmetry to derive EWP-tree relations, applicable to $B \to \pi K$. Point: gluonic penguin involves $\bar{b} \to \bar{s}(u\bar{u} + d\bar{d}) \Longrightarrow$ pure H_W^0 $\Longrightarrow$ does not contribute to $\langle \frac{1}{2} || H_W^1 || \frac{1}{2} \rangle$ and $\langle \frac{3}{2} || H_W^1 || \frac{1}{2} \rangle$.

Writing these RMEs as sums of two pieces,

$$\begin{aligned} \langle \frac{1}{2} || H_W^1 || \frac{1}{2} \rangle &= \lambda_u^{(s)} A_{1,1/2}^u - \lambda_t^{(s)} A_{1,1/2}^t \ , \\ \langle \frac{3}{2} || H_W^1 || \frac{1}{2} \rangle &= \lambda_u^{(s)} A_{1,3/2}^u - \lambda_t^{(s)} A_{1,3/2}^t \ , \end{aligned}$$

the pieces $\propto \lambda_u^{(s)}$ come from trees ($\widetilde{T}$, $\widetilde{C}$), the pieces $\propto \lambda_t^{(s)}$ come from EWPs ($\widetilde{P_{EW}}$, $\widetilde{P_{EW}^C}$).

As before, for these RMEs, trees and EWPs are proportional to one another $\Longrightarrow \lambda_t^{(s)}$ RMEs $\propto \lambda_u^{(s)}$ RMEs:

$$A^t_{1,1/2} = \frac{3}{2}\,\frac{c_9 + c_{10}}{c_1 + c_2}\,A^u_{1,1/2} \quad , \qquad A^t_{1,3/2} = \frac{3}{2}\,\frac{c_9 + c_{10}}{c_1 + c_2}\,A^u_{1,3/2} \ .$$

Using the relations between RMEs and diagrams, these become

$$\widetilde{P_{EW}} = -\frac{3}{2}\frac{c_9 + c_{10}}{c_1 + c_2}\,\widetilde{C} \quad , \qquad \widetilde{P^C_{EW}} = -\frac{3}{2}\frac{c_9 + c_{10}}{c_1 + c_2}\,\widetilde{T} \ .$$

These are the isospin EWP-tree relations. They are applicable to $B \to \pi K$ decays, and reduce the number of independent RMEs/diagrams from 6 to 4. This corresponds to 7 unknown parameters (4 magnitudes, 3 relative strong phases).

9 observables have been measured in $B \to \pi K$ decays: 4 BRs, 4 direct CP asymmetries, and the (mixing-induced) indirect CP asymmetry in $B^0 \to \pi^0 K^0$. With 9 observables and 7 parameters, can perform fit to the data. Find $\chi^2_{\rm min}/{\rm d.o.f.} = 2.9/2$ (p-value $= 0.24$). This is a perfectly acceptable fit. So where's the "$B \to \pi K$ puzzle?"

Look at the best-fit values of the diagrams:

$$|\widetilde{T}| = 33 \pm 13 \ , \quad |\widetilde{C}| = 40 \pm 10 \ .$$

These effective diagrams are defined as $\widetilde{T} \equiv T - A$, $\widetilde{C} \equiv C + A$. But $|A|$ is expected to be $\ll |T|, |C|$, in which case $|\widetilde{C}/\widetilde{T}| \simeq |C/T|$. But the above values are problematic. Naively, we expect $|C/T| = 1/3$. In QCDf, $|C/T| = 0.2$ at NLO. But here, $|\widetilde{C}/\widetilde{T}| \simeq |C/T| = 1.2 \pm 0.6$.

With this in mind, redo the fit, fixing $|\widetilde{C}/\widetilde{T}| = 0.2$. Result: find $\chi^2_{\rm min}/{\rm d.o.f.} = 29.2/3$ (p-value $= 2.0 \times 10^{-6}$). This corresponds to a 4.8σ discrepancy with the SM.

This is a very poor fit. Although isospin-breaking corrections are tiny, theoretical input has been used $\Longrightarrow$ results may not be convincing to everyone. Still, this shows that, if one uses the isospin-based EWP-tree relations applicable to $B \to \pi K$ decays, there may be some tension with the SM in the $B \to \pi K$ system.

Isospin-based EWP-Tree Relations

Other applications: (1) for each $B \to \pi^i K^j$ mode, construct

$$\Delta^{ij} \equiv \frac{\mathcal{B}^{ij} A_{CP}^{ij}}{\Pi^{ij}} = |\overline{A}^{ij}|^2 - |A^{ij}|^2 \ .$$

$\mathcal{B}^{ij}$, A_{CP}^{ij} and Π^{ij} are respectively the CP-averaged branching ratio, the direct CP asymmetry, and the two-body phase-space factor.

Can combine the Δ^{ij} for the four decays and compute

$$\delta_{\pi K} = \Delta^{-+} + \Delta^{+0} - 2\,\Delta^{0+} - 2\,\Delta^{00} \ .$$

Can show that (i) leading-order terms in $\delta_{\pi K}$ cancel (isospin), and (ii) subleading terms also cancel if one uses the $SU(3)_F$ EWP-tree relations and one adds some theory input. M. Gronau, hep-ph/0508047 [hep-ph]

However, if one instead uses the isospin-based EWP-tree relations for $B \to \pi K$ decays, can show that $\delta_{\pi K} = 0$ is exact, i.e., additional theory input is unnecessary. Belle II: measurement of $\delta_{\pi K}$ is consistent with zero.

(2) $\exists$ 3 $B \to \pi\pi$ decays: $B^+ \to \pi^+\pi^0$, $B^0 \to \pi^+\pi^-$ and $B^0 \to \pi^0\pi^0$; amplitudes related by isospin. Initial state $B = (B^+, B^0)$ has isospin $\frac{1}{2}$. Decays involve $\bar{b} \to \bar{d}u\bar{u}$ or $\bar{b} \to \bar{d}d\bar{d} \Longrightarrow$ Hamiltonian has two pieces, $H_W^{1/2}$ and $H_W^{3/2}$. Final $\pi\pi$ states (symmetrized) have isospin 0 or 2. $\Longrightarrow \exists$ two RMEs, $\langle 0||H_W^{1/2}||\frac{1}{2}\rangle$ and $\langle 2||H_W^{3/2}||\frac{1}{2}\rangle$.

With three decay amplitudes, $\exists$ a triangle relation for $B \to \pi^i\pi^j$:

$$\sqrt{2}A^{+0} = A^{+-} + \sqrt{2}A^{00} \ .$$

Gluonic penguin operator is $\bar{b} \to \bar{d}(u\bar{u} + d\bar{d})$, purely $H_W^{1/2} \Longrightarrow$ does not contribute to $\langle 2||H_W^{3/2}||\frac{1}{2}\rangle$. Write

$$\langle 2||H_W^{3/2}||\frac{1}{2}\rangle = \lambda_u^{(d)} A_{3/2,2}^u - \lambda_t^{(d)} A_{3/2,2}^t \ .$$

Get EWP-tree relation:

$$A_{3/2,2}^t = \frac{3}{2}\frac{(c_9 + c_{10})}{(c_1 + c_2)} A_{3/2,2}^u \ .$$

In terms of $B \to \pi\pi$ effective diagrams, the EWP-tree relation is

$$(\widetilde{P_{EW}} + \widetilde{P_{EW}^C}) = -\frac{3}{2}\frac{c_9 + c_{10}}{c_1 + c_2}(\widetilde{T} + \widetilde{C}) \ .$$

$B \to \pi\pi$ decays are used to extract the CP phase α. The decay $B^+ \to \pi^+\pi^0$ is pure $\langle 2||H_W^{3/2}||\frac{1}{2}\rangle$, and the standard method for getting α assumes this RME is purely $\lambda_u^{(d)}$. However, this is not the case:

$$\langle 2||H_W^{3/2}||\frac{1}{2}\rangle = \lambda_u^{(d)}(\widetilde{T} + \widetilde{C}) - \lambda_t^{(d)}(\widetilde{P_{EW}} + \widetilde{P_{EW}^C}) \ .$$

This problem is usually dealt with by simply neglecting the EWPs, arguing that they are small (e.g., see 1705.02981 by the CKMfitter group). But this isn't necessary – because of the above isospin EWP-tree relation, the $\lambda_t^{(d)}$ contribution $(\widetilde{P_{EW}} + \widetilde{P_{EW}^C})$ is related to the $\lambda_u^{(d)}$ contribution $(\widetilde{T} + \widetilde{C})$ $\Longrightarrow$ α can be extracted cleanly from $B \to \pi\pi$ decays even including EWPs.

Summary

$B \to PP$ decays assuming $SU(3)_F$ symmetry: if neglect contributions of $Q_{7,8}$ (holds to $\simeq 10\%$), $\exists$ EWP-tree relations at the level of RMEs and diagrams. This result is not new – it was first shown in 1998.

$\exists$ four $B \to \pi K$ decays, amplitudes related by isospin. For many years, analyses of the $B \to \pi K$ puzzle imposed $SU(3)_F$ EWP-tree relations. This requires neglecting certain diagrams, so not rigorous, group-theoretically.

Better: use isospin-based EWP-tree relations, applicable directly to $B \to \pi K$ decays. When one does a fit to the data, one finds a perfectly acceptable fit. But if one adds some QCDf-motivated theoretical input, one finds an almost 5σ discrepancy with the SM.

Can also apply isospin-based EWP-tree relations to $B \to \pi\pi$ decays. Allows the extraction of the CP phase α without making any assumptions about the size of EWPs in such decays.