



JHEP 05 (2025) 148 & arXiv:2511.20585

$$D \rightarrow P\eta'$$

**Prime candidates for analysing  
 $SU(3)_F$  in charm**

Ulrich Nierste, Stefan Schacht, Keri Vos

**Carolina Bolognani**

Nonleptonic Decays of Heavy Mesons  
March 24, 2026, Durham

# Overview

- Hadronic charm decays
- The  $SU(3)_F$  symmetry of QCD
- Treatment of  $\eta - \eta'$  mixing in  $SU(3)_F$  analyses of heavy flavour decays
- Branching ratio analysis of  $D \rightarrow P\eta'$  decays with linear  $SU(3)_F$ -breaking  
JHEP 05 (2025) 148
- CPV analysis of  $D \rightarrow P\eta'$  decays  
arXiv:2511.20585

CB, Ulrich Nierste, Stefan Schacht, Keri Vos

# Why hadronic charm decays?

- Access to up-type quark sector
- Cabibbo sub-matrix  $\Rightarrow$  small CP violation

$$V_{CKM} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix} + \mathcal{O}(\lambda^6)$$

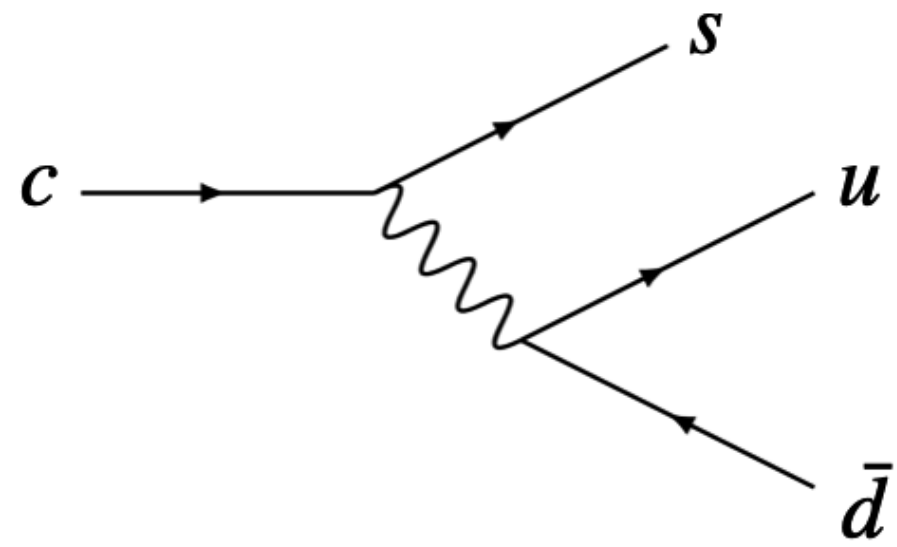
- Not so heavy quark mass  $\Rightarrow$  unreliable expansion parameter?

$$\frac{\Lambda_{QCD}}{m_c} \approx 0.3 \quad \frac{\Lambda_{QCD}}{m_b} \approx 0.1$$

$$\alpha_s(m_c) \approx 0.35 \quad \alpha_s(m_b) \approx 0.22$$

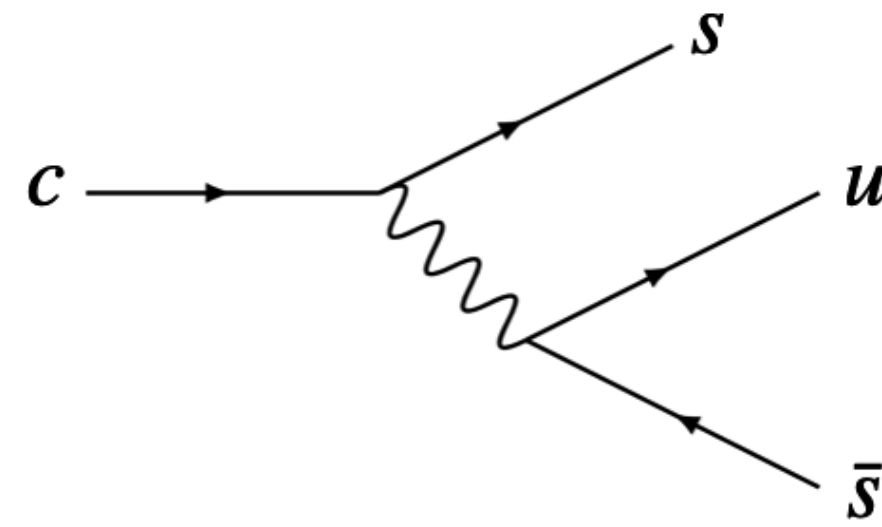
Without first principle calculations we can still use symmetries!

# What hadronic charm decays?



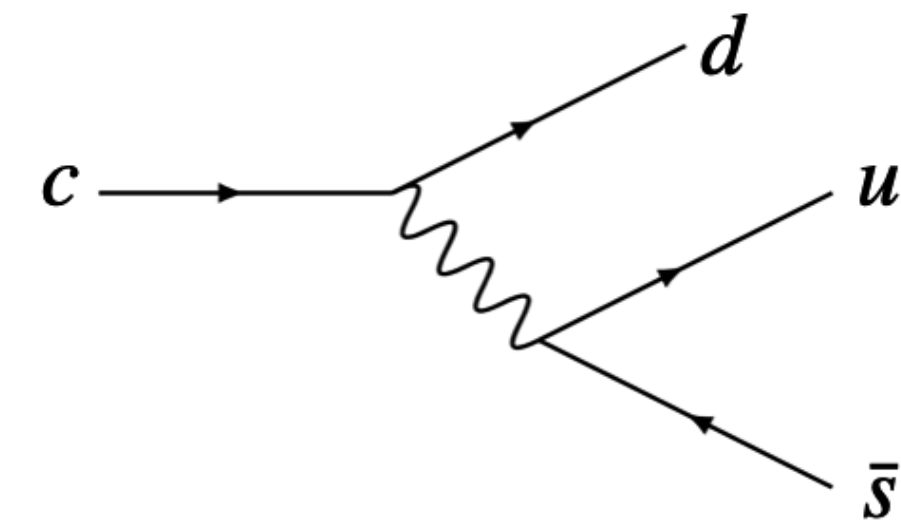
$$V_{cs}^* V_{ud} \sim \mathcal{O}(1)$$

Cabibbo-favoured



$$V_{cs}^* V_{us} \sim V_{cd}^* V_{ud} \sim \mathcal{O}(\lambda)$$

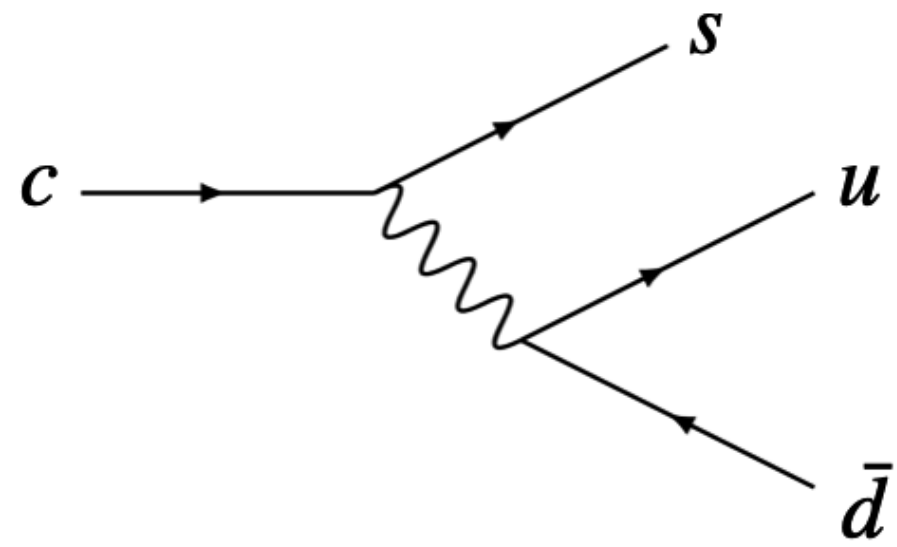
singly Cabibbo-suppressed



$$V_{cd}^* V_{us} \sim \mathcal{O}(\lambda^2)$$

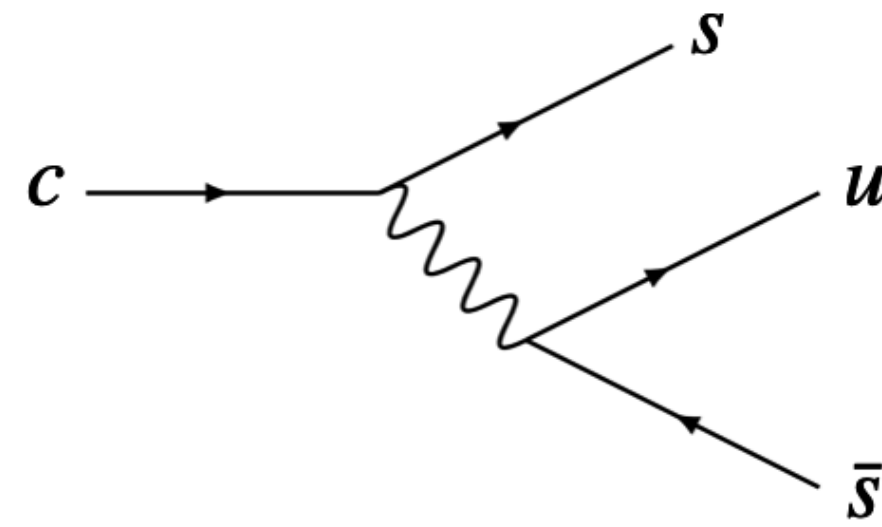
doubly-Cabibbo-suppressed

# What hadronic charm decays?



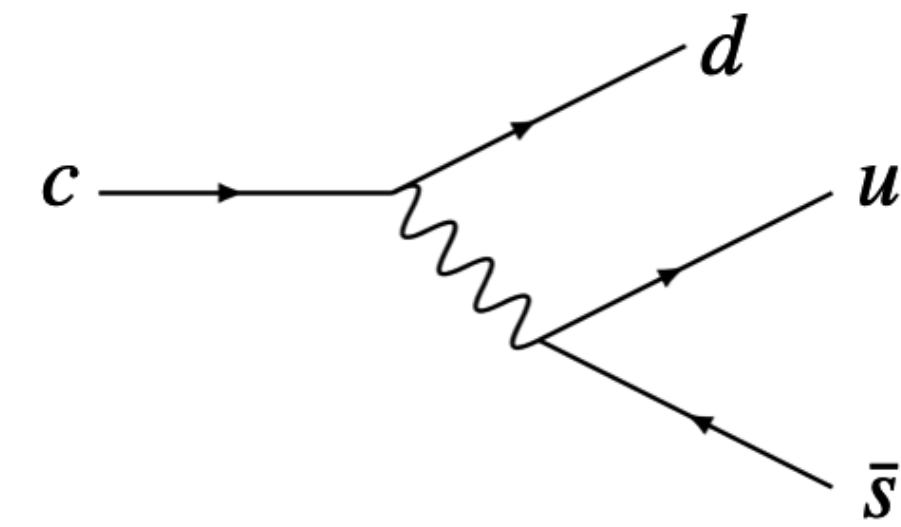
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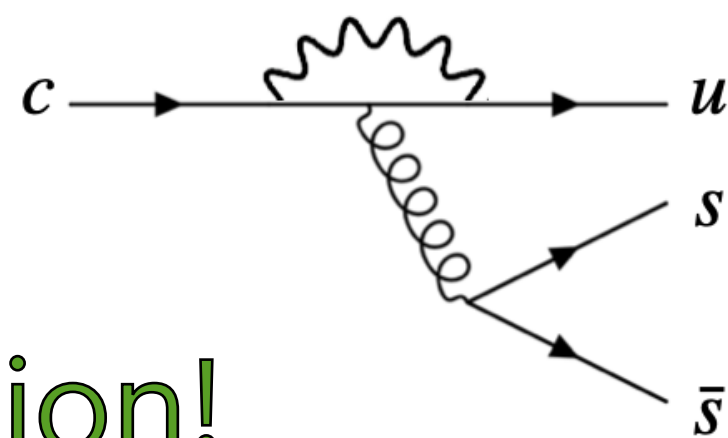
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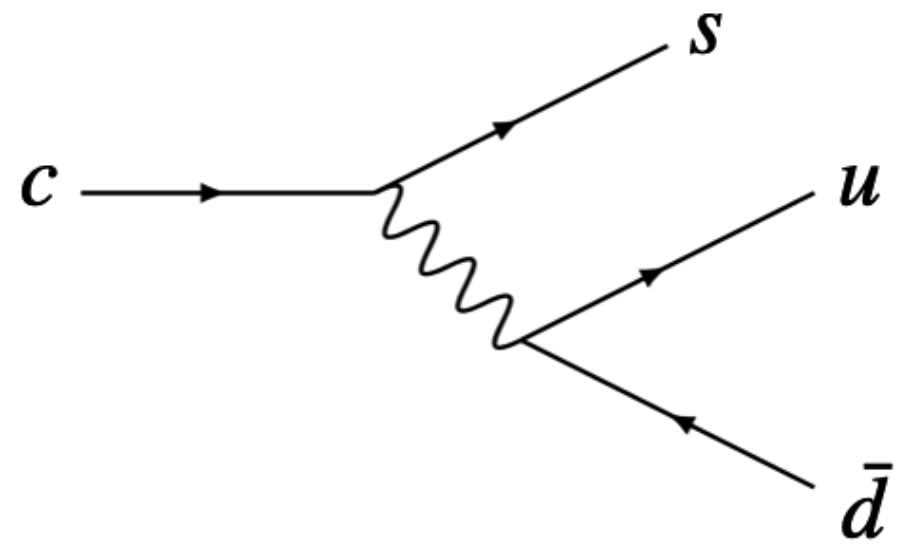
doubly-Cabibbo-suppressed

Additional penguin topologies due to  $q\bar{q}$  pair



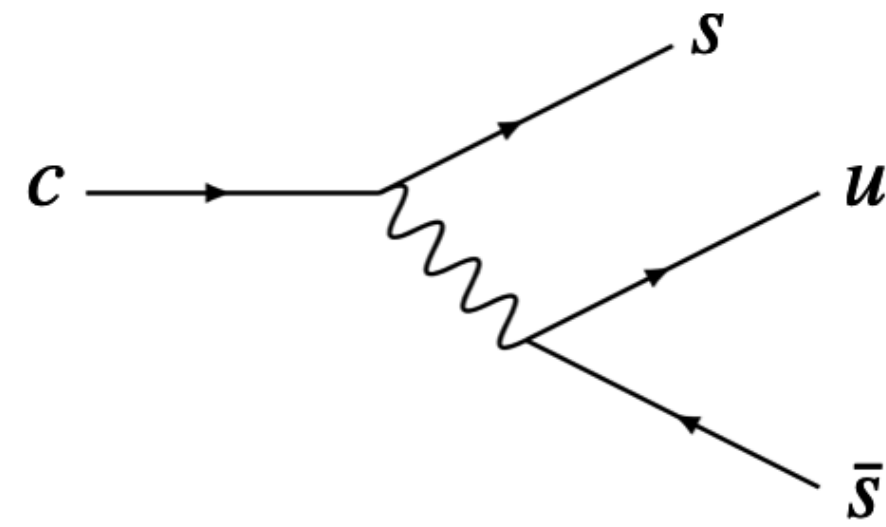
CP violation!

# What hadronic charm decays?



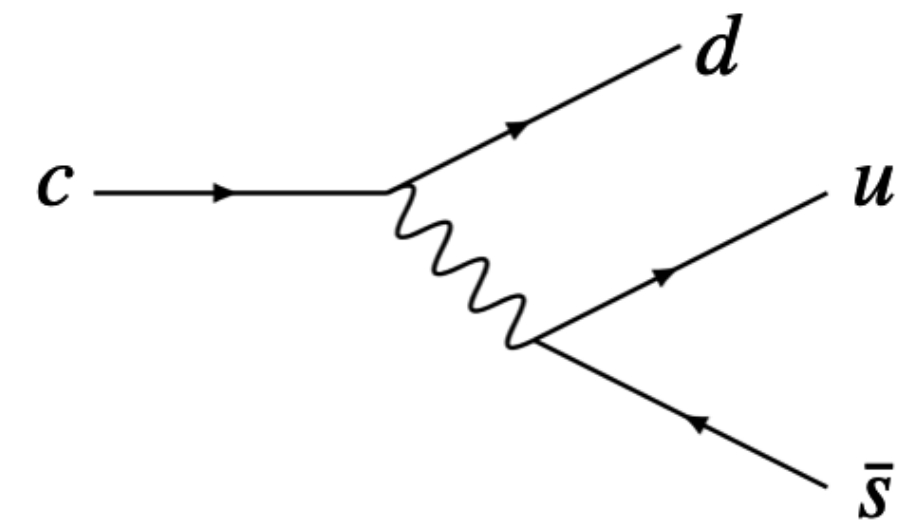
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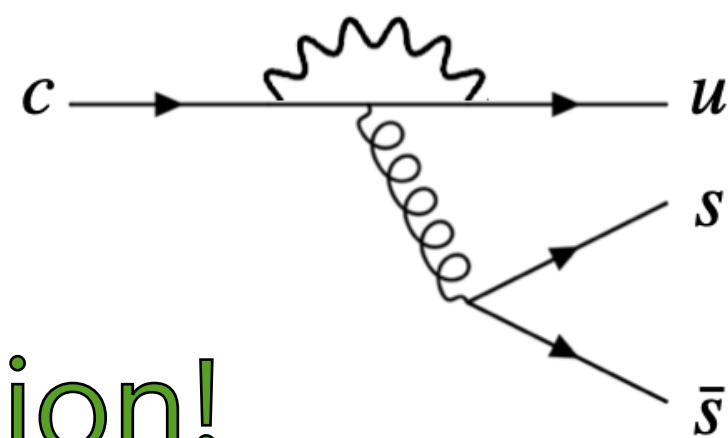
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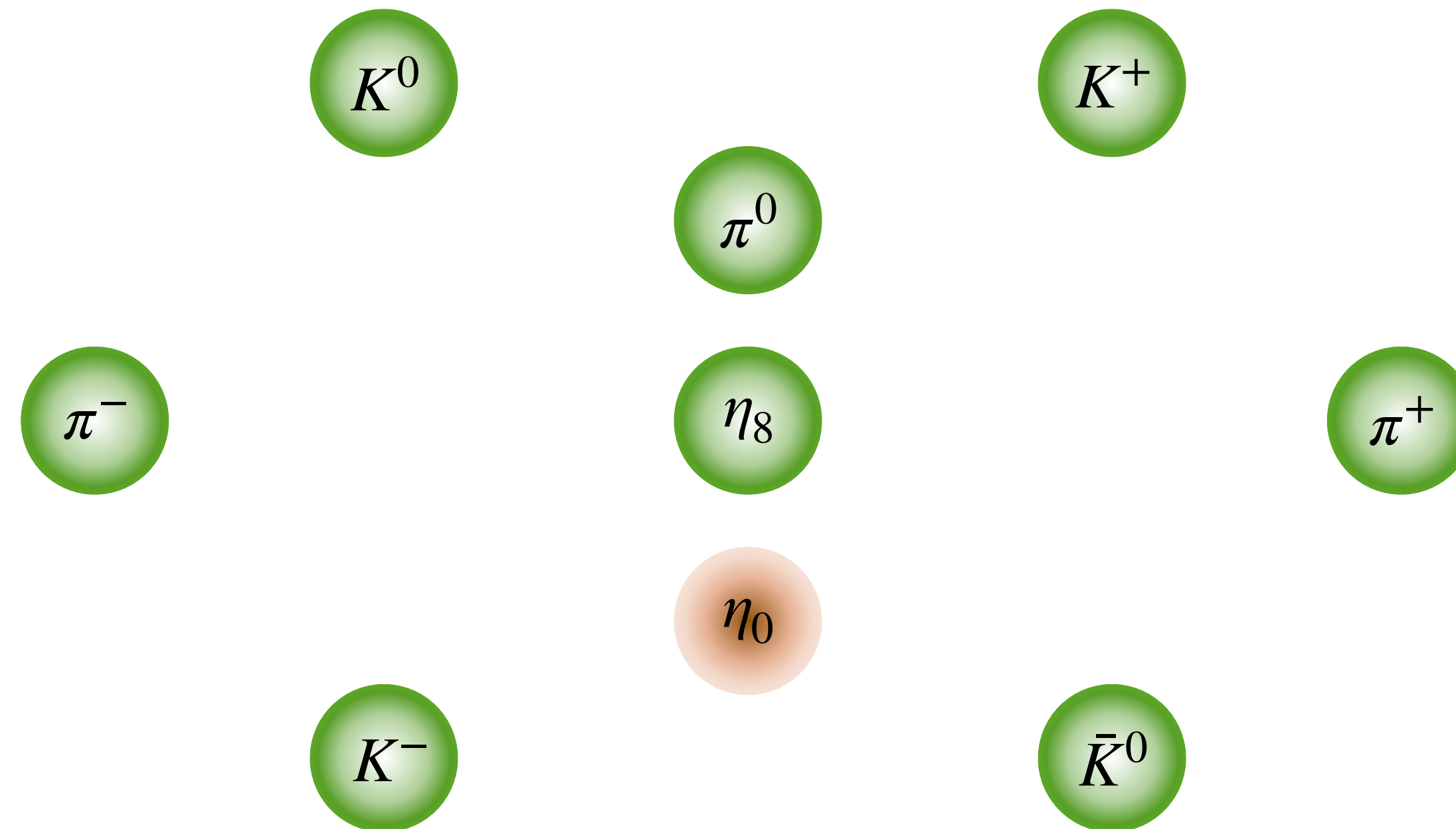
Suppressed complex phase in interference of

$$\lambda_q = V_{cq}^* V_{uq}$$

$$\text{Im} \frac{\lambda_d}{\lambda_s} < \text{Im} \frac{\lambda_b}{\lambda_s} \sim -6 \cdot 10^{-4}$$

# How? The $SU(3)_F$ symmetry of QCD!

- Approximately  $m_u = m_d = m_s$
- Pseudoscalars by combining three quarks as  $q\bar{q}$  pairs :  $3 \otimes \bar{3} = 8 \oplus 1$



- $SU(3)_F$  symmetry connects amplitudes involving different pseudoscalars
- Verify compatibility of  $D \rightarrow PP$  data with SM assumptions and predict more!

# What has been done

- Global  $SU(3)_F$  analyses of  $D \rightarrow PP$  branching fractions done for decades
  - ➔  $SU(3)_F$  breaking can be included!

M. Gronau, O. F. Hernandez, D. London, J. L. Rosner, Phys. Rev. D 52 (1995) 6356

Y. Grossman, D. J. Robinson, JHEP 04 (2013) 067

S. Müller, U. Nierste, S. Schacht, Phys. Rev. D 92 (2015) 014004

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- Unphysical states  $\eta_8$  and  $\eta_0 \Rightarrow \eta - \eta'$  mixing neglected or inconsistent!

$$|\eta_8\rangle = |\eta\rangle \cos\theta + |\eta'\rangle \sin\theta$$

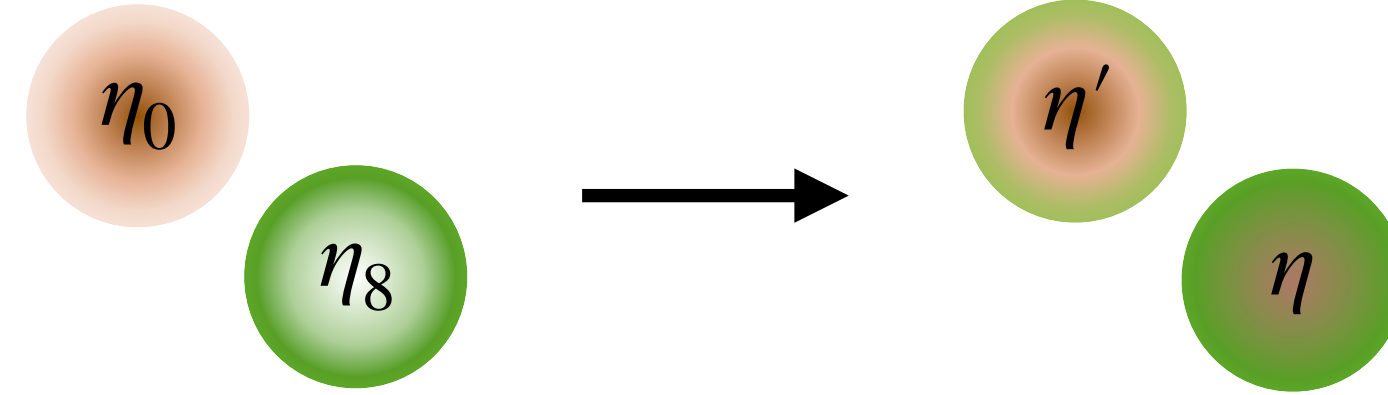
$$|\eta_0\rangle = -|\eta\rangle \sin\theta + |\eta'\rangle \cos\theta$$

Universal mixing angle treatment  
**inconsistent** and gives **poor**  
description of data !

H. Leutwyler, Nucl. Phys. B Proc. Suppl. 64 (1998) 223

T. Feldmann, P. Kroll, B. Stech, Phys. Rev. D 58 (1998) 114006

# $SU(3)_F$ and the unphysical states



- Mixing due to similar quantum numbers and the breaking of  $SU(3)_F$

$$|\eta_0\rangle = -|\eta\rangle \sin\theta + |\eta'\rangle \cos\theta$$

$\theta$  is an  $SU(3)_F$ -breaking parameter not related to any observable!  
Other effects at play...

- Mass differences not related to  $SU(3)_F$   
➔  $U(1)$  axial anomaly → contributions to singlet mass

E. Witten, Nucl. Phys. B149 (1979) 285  
G. Veneziano, Nucl. Phys. B 59 (1979) 213

- Appropriate description of  $\eta - \eta'$  mixing requires more parameters!

# $SU(3)_F$ decouples unphysical states

- Definition of states understood under  $SU(3)_F$

$$\theta = \mathcal{O}(\varepsilon) \Rightarrow \varepsilon \sim \frac{f_\pi}{f_K} - 1 \sim 20 - 30 \%$$

$$|\eta\rangle = |\eta_8\rangle + \mathcal{O}(\varepsilon)$$

$$|\eta'\rangle = |\eta_0\rangle + \mathcal{O}(\varepsilon)$$

- Not the matrix elements!

$$\begin{aligned} \langle P_\eta | H | D \rangle &= \cos \theta \langle P_{\eta_8} | H | D \rangle - \sin \theta \langle P_{\eta_0} | H | D \rangle \\ \langle P_{\eta'} | H | D \rangle &= \sin \theta \langle P_{\eta_8} | H | D \rangle' + \cos \theta \langle P_{\eta_0} | H | D \rangle' \end{aligned}$$

Different!

$$\langle P_{\eta_0} | H | D \rangle' \neq \langle P_{\eta_0} | H | D \rangle$$

Differences are  $\mathcal{O}(1)$  i.e. not  $SU(3)_F$ -breaking

$\Rightarrow SU(3)_F$  decouples  $D \rightarrow P_{\eta'}$  from octet only final states!  $\leftarrow$

# The amplitudes

- CKM-leading and subleading contributions to SCS decays

$$\mathcal{A}^{SCS} = \lambda_{sd} \mathcal{A}_{sd} - \frac{\lambda_b}{2} \mathcal{A}_b$$
$$\lambda_{sd} = \frac{\lambda_s - \lambda_d}{2} \simeq \lambda_s \simeq -\lambda_d \in \mathbb{R}$$
$$\text{Re} \frac{\lambda_b}{\lambda_{sd}} \sim 10^{-4}$$

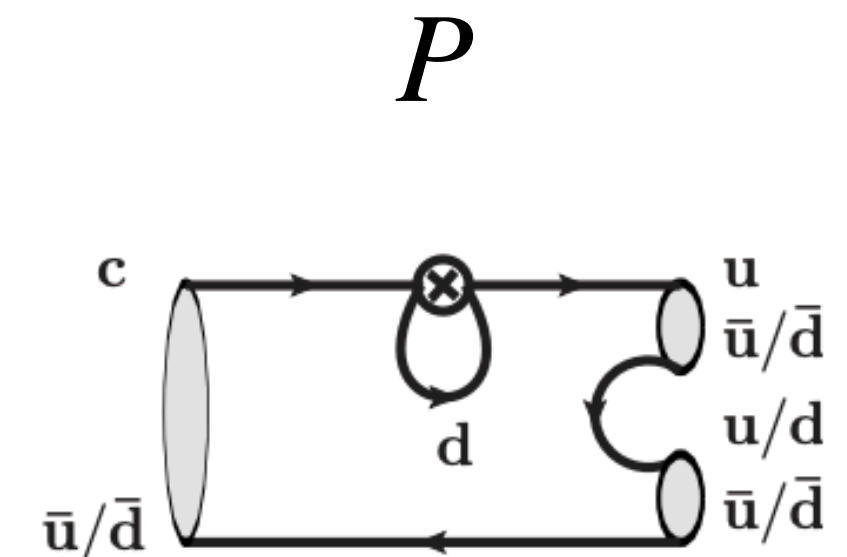
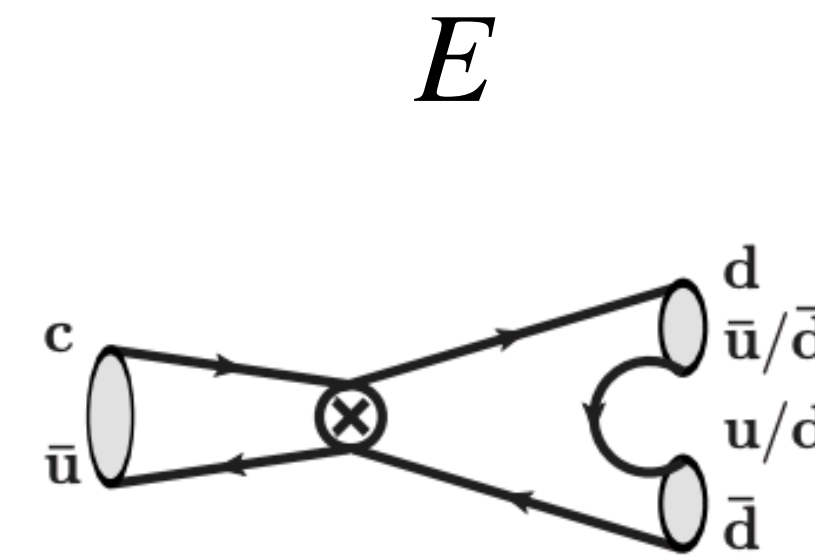
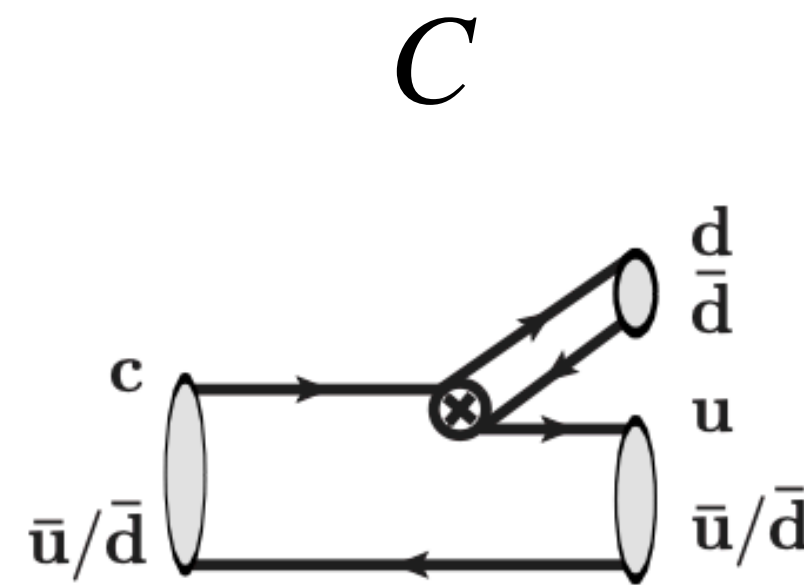
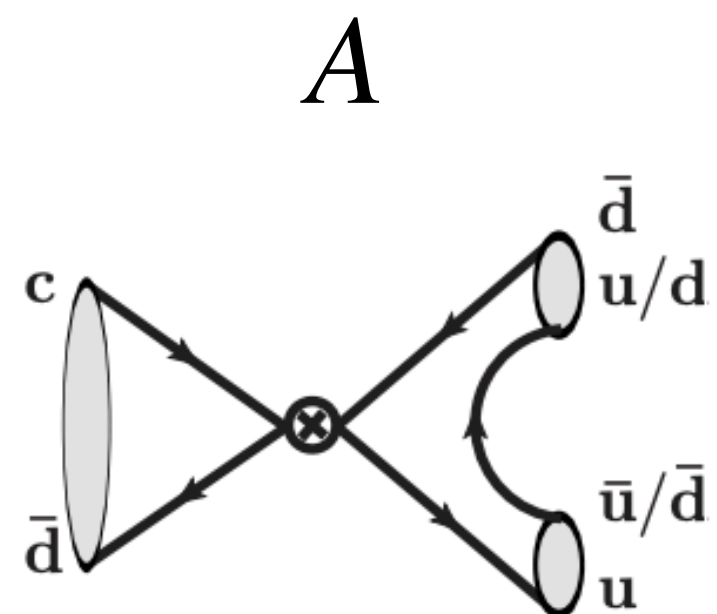
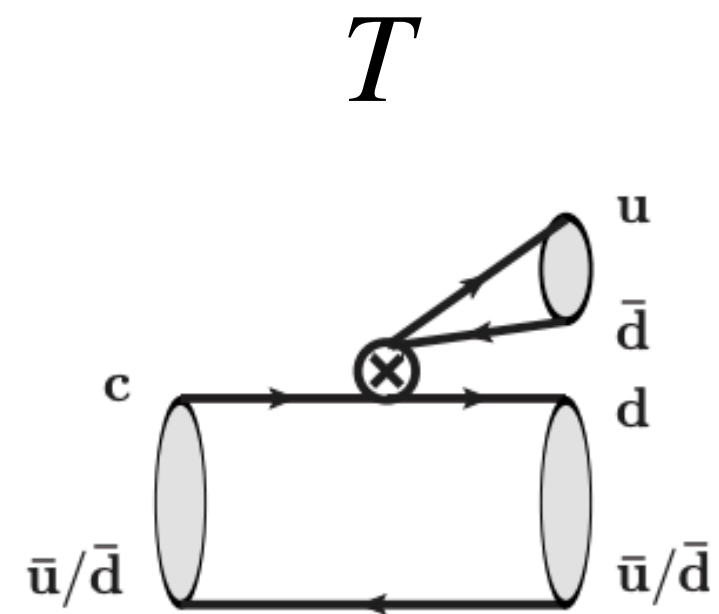
- **Branching ratios** sensitive only to CKM-leading in SCS  $\Rightarrow \mathcal{A}^{SCS} \simeq \lambda_{sd} \mathcal{A}_{sd}$
- Global analysis!

$$\mathcal{A}^{CF}(d) = V_{cs}^* V_{ud} \mathcal{A}(d)$$
$$\mathcal{A}^{SCS}(d) = \lambda_{sd} \mathcal{A}(d)$$
$$\mathcal{A}^{DCS}(d) = V_{cd}^* V_{us} \mathcal{A}(d)$$

Common parameters!

# Topological parametrisation

$$\mathcal{A}(d) = \sum_i c_i^d \mathcal{T}_i$$

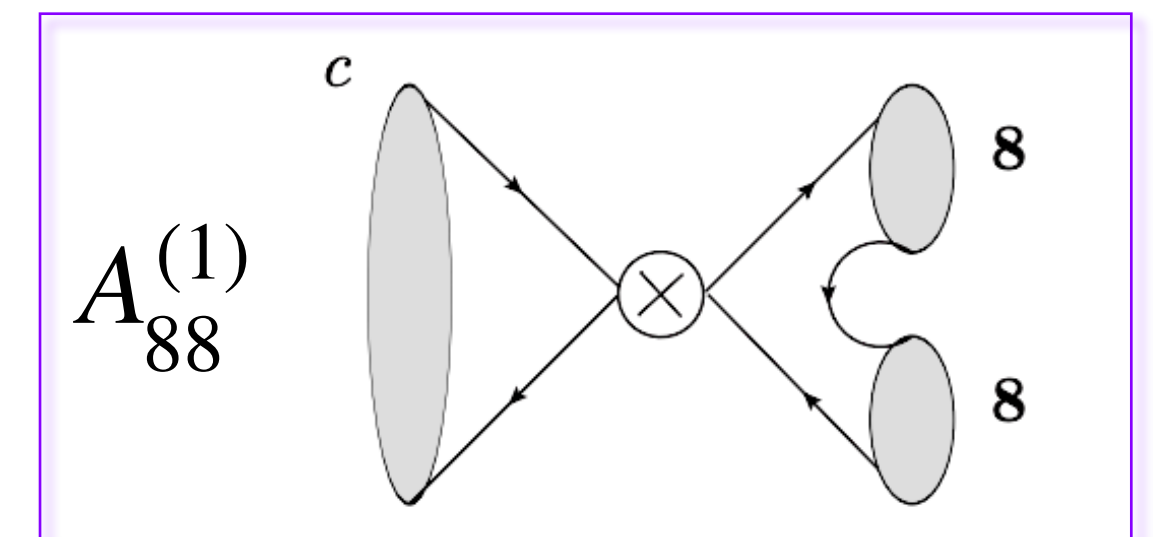
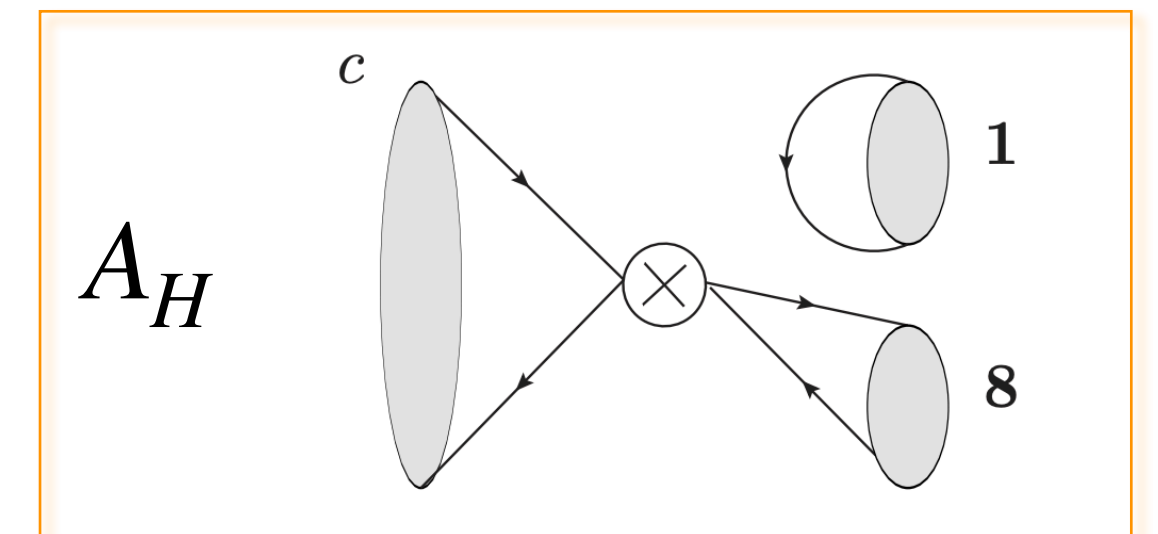
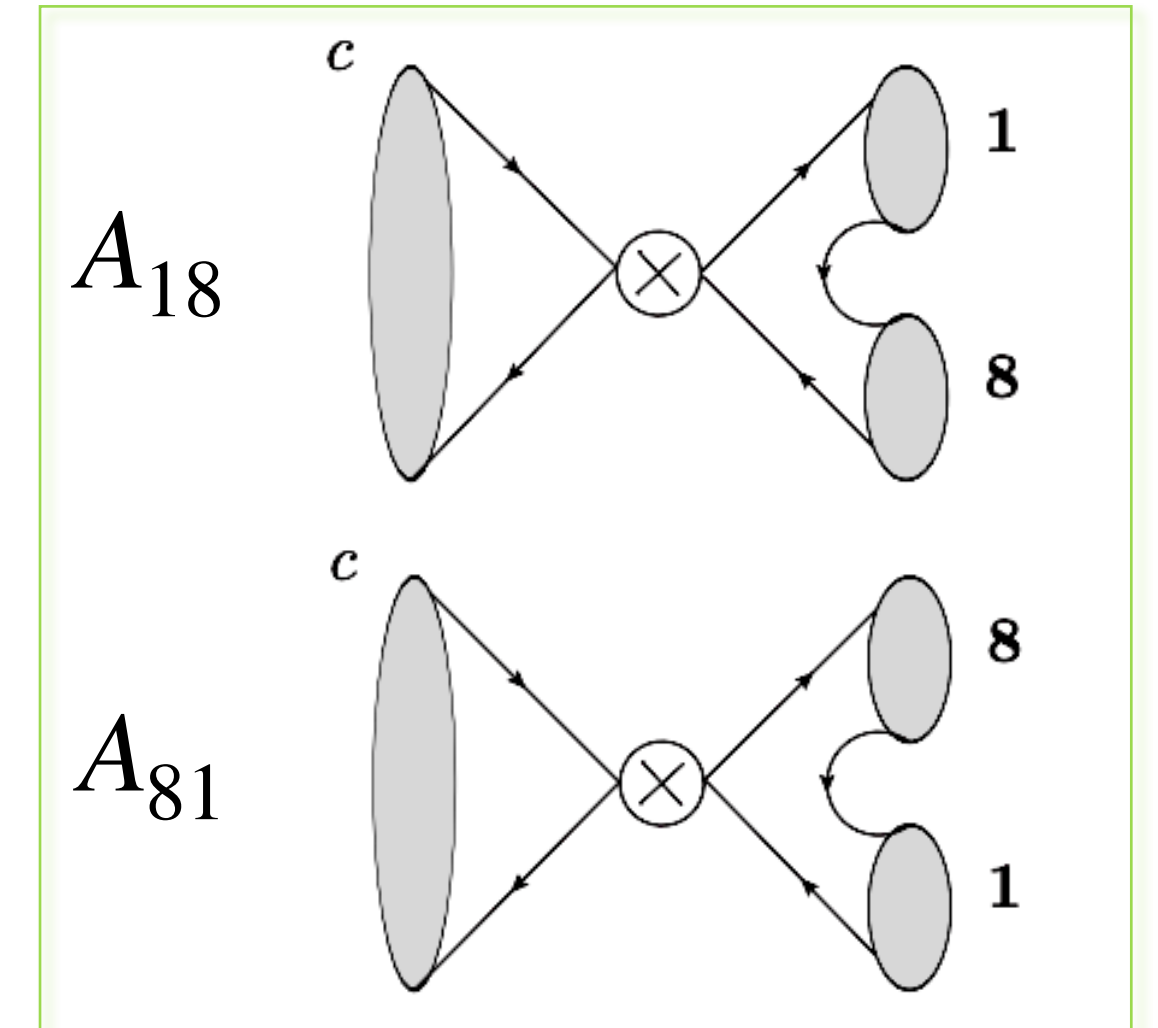


- Account for  $SU(3)_F$ -breaking corrections  $(m_s - m_d)s\bar{s}$  to the diagrams

$$T_1^{(1)} \quad c \text{---} \text{---} s \quad - \quad c \text{---} \text{---} d \quad = \quad c \text{---} \text{---} \text{---} \text{---}$$

# Remarks on topological parametrisation

- Differentiate which quark contributes to the  $\eta_0$  singlet
- Hairpin diagrams  $\rightarrow$  OZI suppression not imposed
  - $\rightarrow$  naturally get contributions only to singlets
  - $\rightarrow$  accommodates possible glueball components
- Octet component in  $\eta'$  and singlet component in  $\eta$  are  $SU(3)_F$ -breaking



# CKM-leading amplitudes in $SU(3)_F$ limit

Decay ampl. $\mathcal{A}(d)$	$T_{18}$	$A_{18}$	$A_{81}$	$A_H$	$C_{81}$	$E_{18}$	$E_{81}$	$E_H$
SCS								
$\mathcal{A}(D^0 \rightarrow \pi^0 \eta')$	0	0	0	0	$\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$	$\sqrt{\frac{3}{2}}$
$\mathcal{A}(D^0 \rightarrow \eta \eta')$	0	0	0	0	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$-\frac{3}{\sqrt{2}}$
$\mathcal{A}(D^+ \rightarrow \pi^+ \eta')$	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	0	0	0	0
$\mathcal{A}(D_s^+ \rightarrow K^+ \eta')$	$\frac{1}{\sqrt{3}}$	$\sqrt{\frac{1}{3}}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	0	0	0	0
CF								
$\mathcal{A}(D^0 \rightarrow \bar{K}^0 \eta')$	0	0	0	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$
$\mathcal{A}(D_s^+ \rightarrow \pi^+ \eta')$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	0	0	0	0
DCS								
$\mathcal{A}(D^0 \rightarrow K^0 \eta')$	0	0	0	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$
$\mathcal{A}(D^+ \rightarrow K^+ \eta')$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	0	0	0	0

charged
neutral

- Neutral and charged modes separated

- Redefinitions respect  $1/N_C$  counting

A. Buras, J. M. Gerard, R. Ruckl, Nucl. Phys. B 268 (1986) 16

$$\hat{T}_{18} = T_{18} + A_{18} + A_{81} + 3A_H$$

$$\hat{C}_{81} = C_{81} + E_{18} + E_{81} + 3E_H$$

# CKM-leading amplitudes sum rules

- Linear dependencies are absorbed into redefined parameters  
 → Keep power counting!

Decay ampl. $\mathcal{A}(d)$	$\hat{T}_{18}$	$\hat{T}_{18,1}^{(1)}$	$\hat{T}_{18,2}^{(1)}$	$\hat{T}_{88}^{(1)}$	$\hat{C}_{18,1}^{(1)}$	$\hat{C}_{81}$	$\hat{C}_{81,1}^{(1)}$	$\hat{C}_{81,2}^{(1)}$	$\hat{C}_{88}^{(1)}$
SCS									
$\mathcal{A}(D^0 \rightarrow \pi^0 \eta')$	0	0	0	0	$\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$	0	0	$-\frac{1}{\sqrt{3}}$
$\mathcal{A}(D^0 \rightarrow \eta \eta')$	0	0	0	0	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{2}}{3}$	$-\frac{\sqrt{2}}{3}$	-1
$\mathcal{A}(D^+ \rightarrow \pi^+ \eta')$	$-\frac{1}{\sqrt{3}}$	0	0	$-\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{3}}$	0	0	0	$-\sqrt{\frac{3}{2}}$
$\mathcal{A}(D_s^+ \rightarrow K^+ \eta')$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$-\sqrt{\frac{2}{3}}$	$\frac{1}{\sqrt{3}}$	0	0	0	$-\sqrt{\frac{3}{2}}$
CF									
$\mathcal{A}(D^0 \rightarrow \bar{K}^0 \eta')$	0	0	0	0	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{6}}$
$\mathcal{A}(D_s^+ \rightarrow \pi^+ \eta')$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	0	$-\sqrt{\frac{2}{3}}$	0	0	0	0	0
DCS									
$\mathcal{A}(D^0 \rightarrow K^0 \eta')$	0	0	0	0	0	$\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{6}}$
$\mathcal{A}(D^+ \rightarrow K^+ \eta')$	$\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{6}}$	0	0	0	0	0

- Sum rules

$SU(3)_F$  limit

$$\mathcal{A}(D^+ \rightarrow K^+ \eta') = \mathcal{A}(D_s^+ \rightarrow \pi^+ \eta') = \mathcal{A}(D_s^+ \rightarrow K^+ \eta') = -\mathcal{A}(D^+ \rightarrow \pi^+ \eta')$$

$$\mathcal{A}(D^0 \rightarrow K^0 \eta') = \mathcal{A}(D^0 \rightarrow \bar{K}^0 \eta') = \sqrt{2} \mathcal{A}(D^0 \rightarrow \pi^0 \eta') = -\sqrt{\frac{2}{3}} \mathcal{A}(D^0 \rightarrow \eta \eta')$$

$SU(3)_F$ -breaking

$$\mathcal{A}(D^+ \rightarrow K^+ \eta') + \mathcal{A}(D_s^+ \rightarrow \pi^+ \eta') - \mathcal{A}(D_s^+ \rightarrow K^+ \eta') + \mathcal{A}(D^+ \rightarrow \pi^+ \eta') = 0$$

$$\mathcal{A}(D^0 \rightarrow K^0 \eta') + \mathcal{A}(D^0 \rightarrow \bar{K}^0 \eta') - \frac{\mathcal{A}(D^0 \rightarrow \pi^0 \eta')}{\sqrt{2}} + \sqrt{\frac{3}{2}} \mathcal{A}(D^0 \rightarrow \eta \eta') = 0$$

# Fit to Branching Ratio data

- Redefinitions  $\rightarrow$  cannot constrain individual  $SU(3)_F$ -breaking topologies
- Constrain total  $SU(3)_F$ -breaking effect at the amplitude level

$$\left| \frac{\mathcal{A}_{SU(3)}(D \rightarrow P\eta')}{\mathcal{A}_{SU(3)\text{-lim}}(D \rightarrow P\eta')} \right| \leq \varepsilon$$

- Complex topologies  $\rightarrow$  17 fit parameters , 8 measurements, 8 constraints
- $\chi^2$  minimisation with excess of parameters?

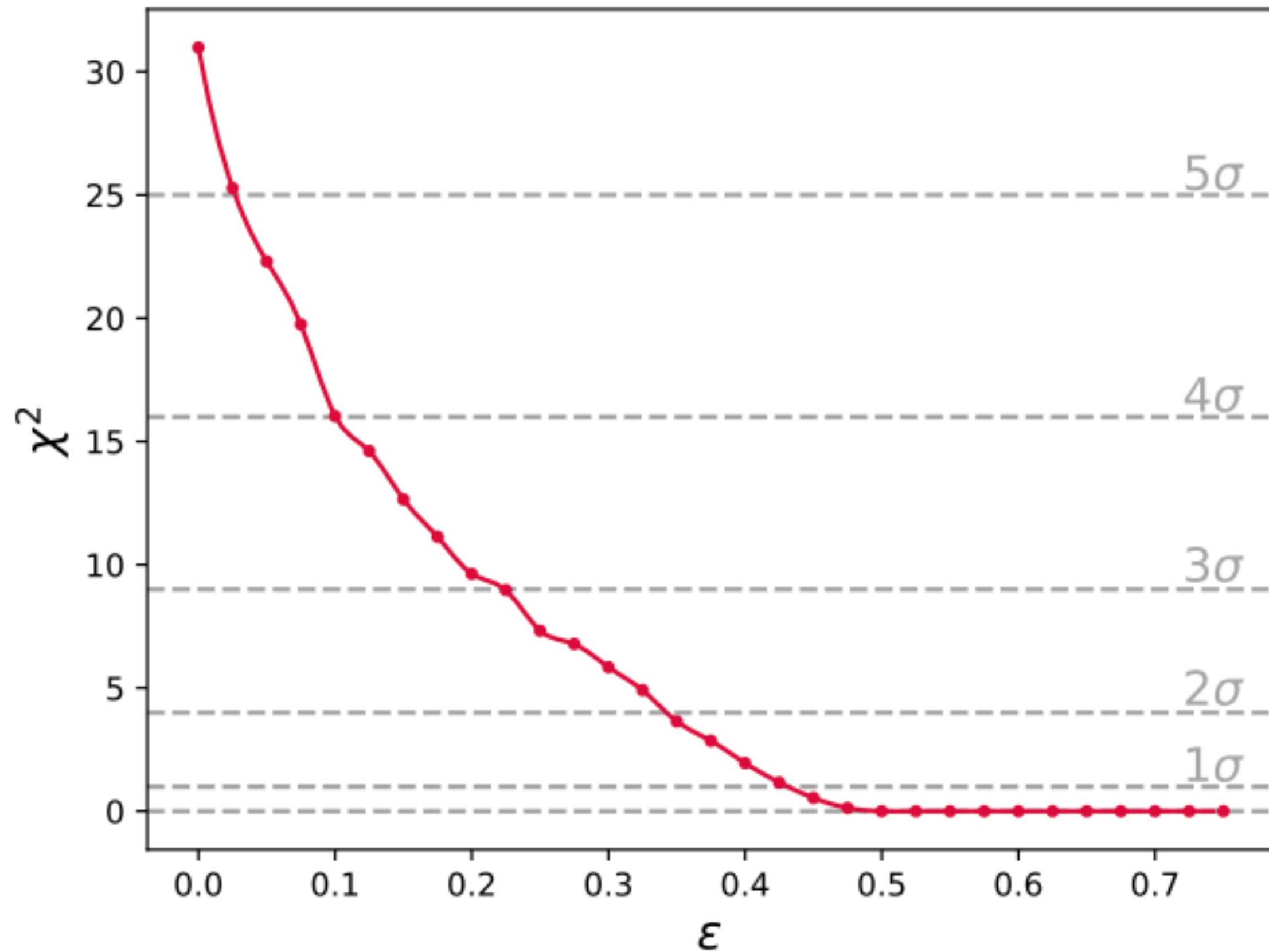
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- Complex topologies  $\rightarrow$  17 fit parameters , 8 measurements, 8 constraints
- $\chi^2$  minimisation with excess of parameters!
  - $\rightarrow$  Interpretation with respect to perfect reproduction of data  $\chi^2 = 0$

# Test of $SU(3)_F$ from Branching Ratio data



➔  $SU(3)_F$  limit ruled out  $> 5\sigma$

➔ Perfect fit for  $\epsilon \geq 50\%$  : violates assumption of linear  $SU(3)_F$ -breaking

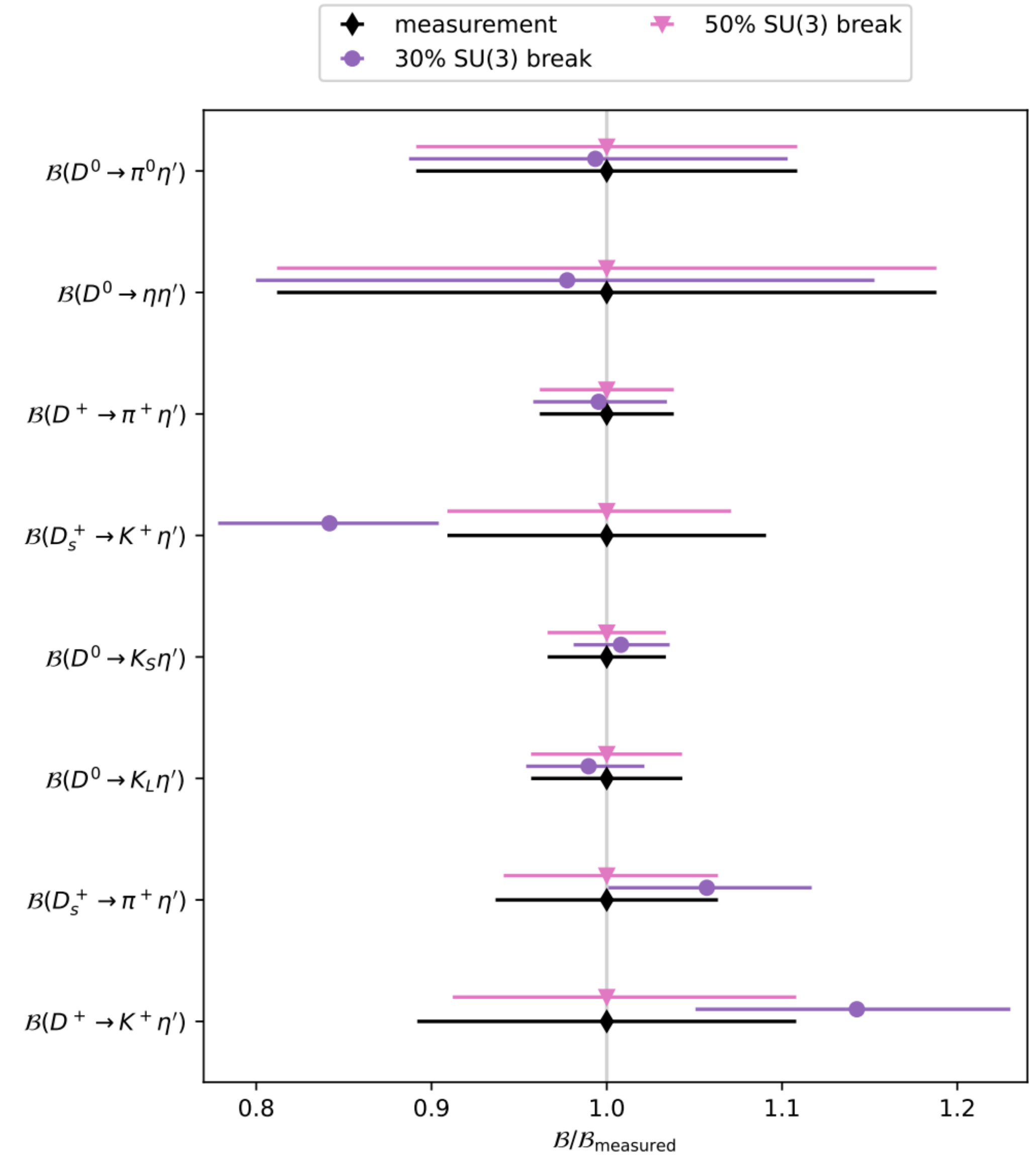
➔ Nominal result at  $\epsilon = 30\%$

▶ Slight tension of  $2.5\sigma$

# Branching Ratio predictions from fit

Redefinitions limit interpretation of underlying physical parameters

- Uncertainties  $> 10\%$  for  $D^0 \rightarrow \pi^0 \eta'$  and  $D^0 \rightarrow \eta \eta'$ ...
- Predictions differ by  $\sim 1\sigma$  for  $D_{(s)}^+ \rightarrow K^+ \eta'$



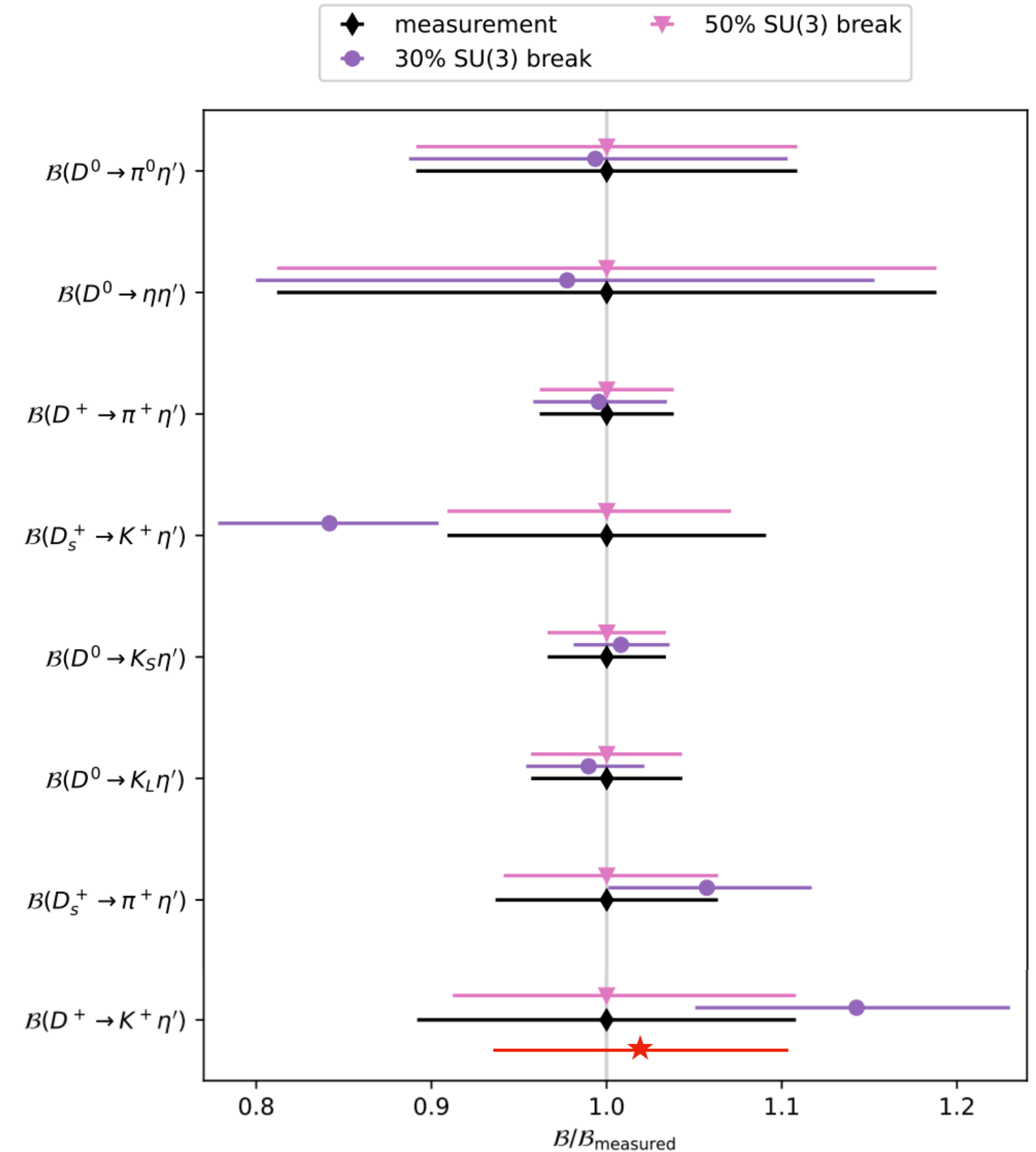
CB, UN, SS, KV, JHEP 05 (2025) 148

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- Predictions differ by  $\sim 1\sigma$  for  $D_{(s)}^+ \rightarrow K^+ \eta'$
- **New** result in-line with our predictions

BESIII, JHEP 10 (2025) 178



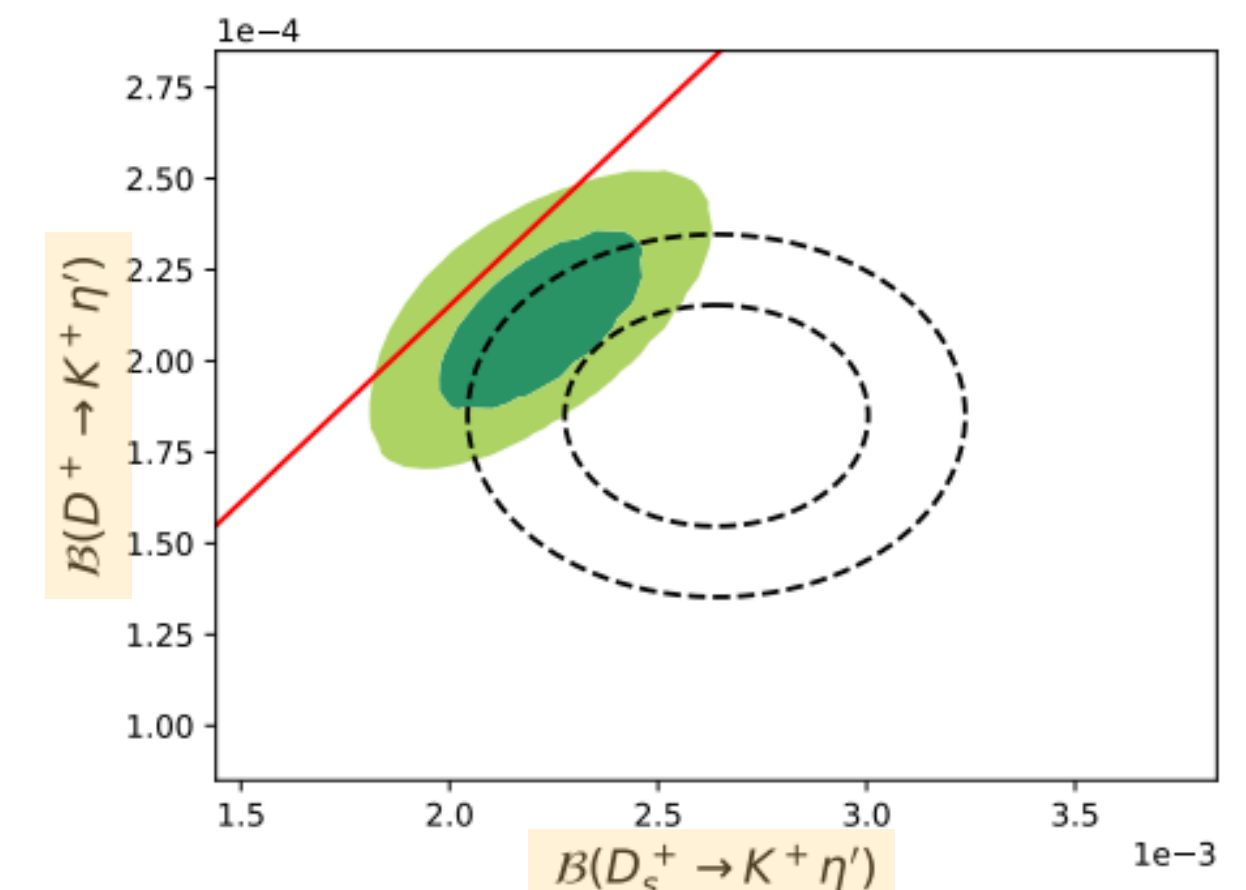
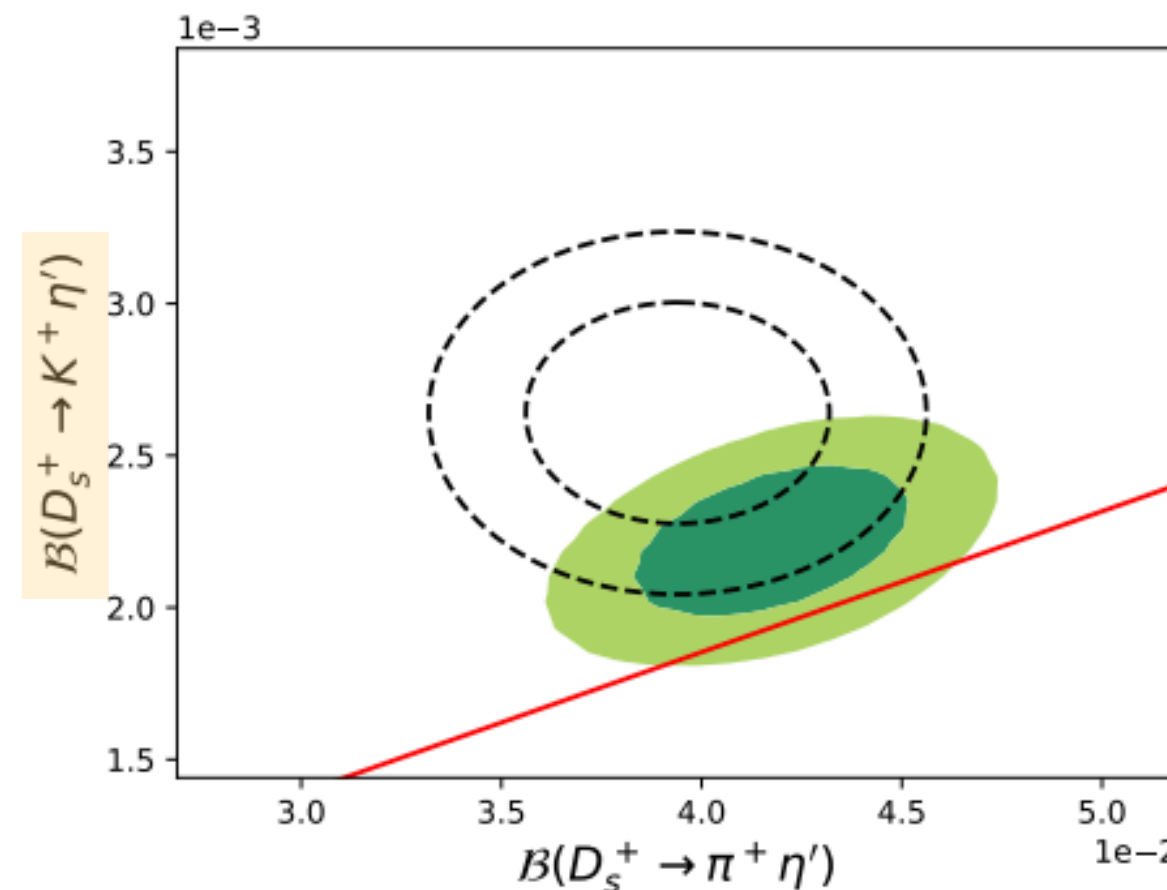
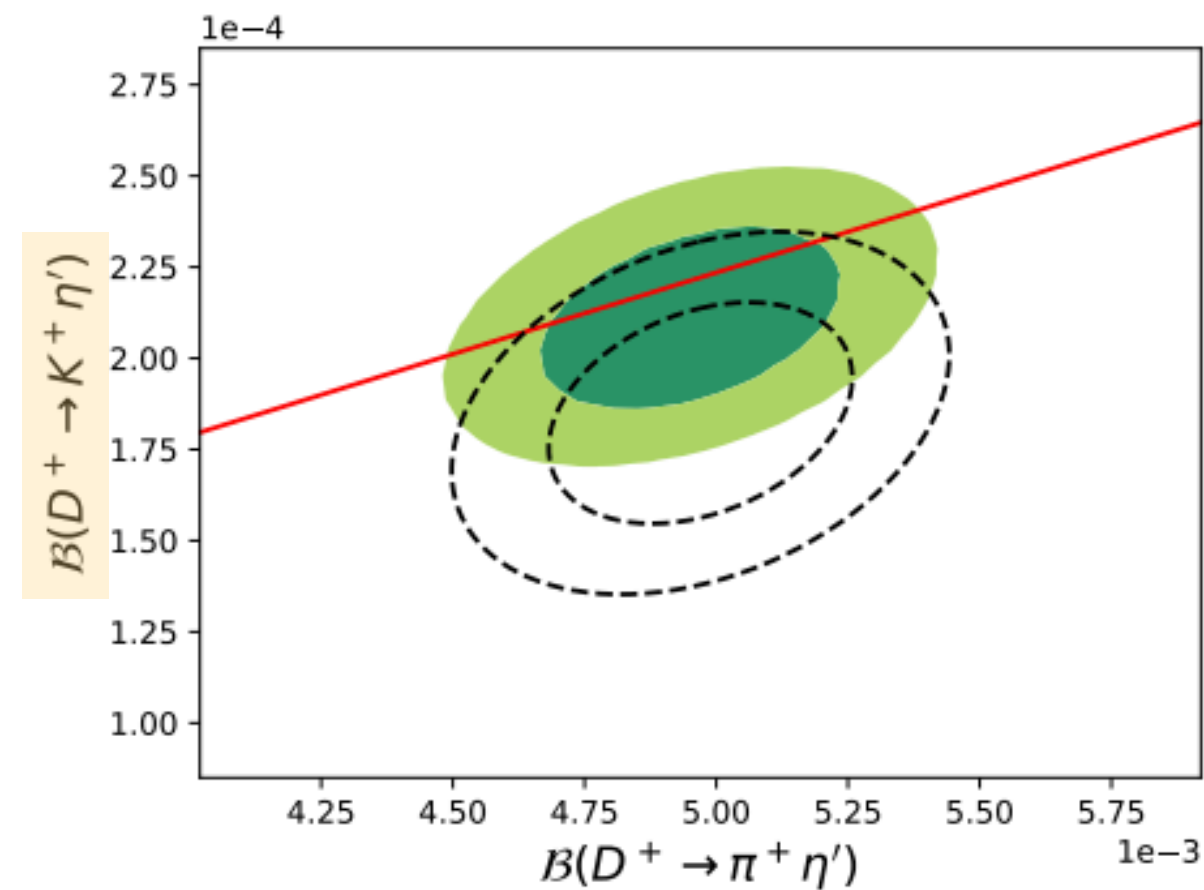
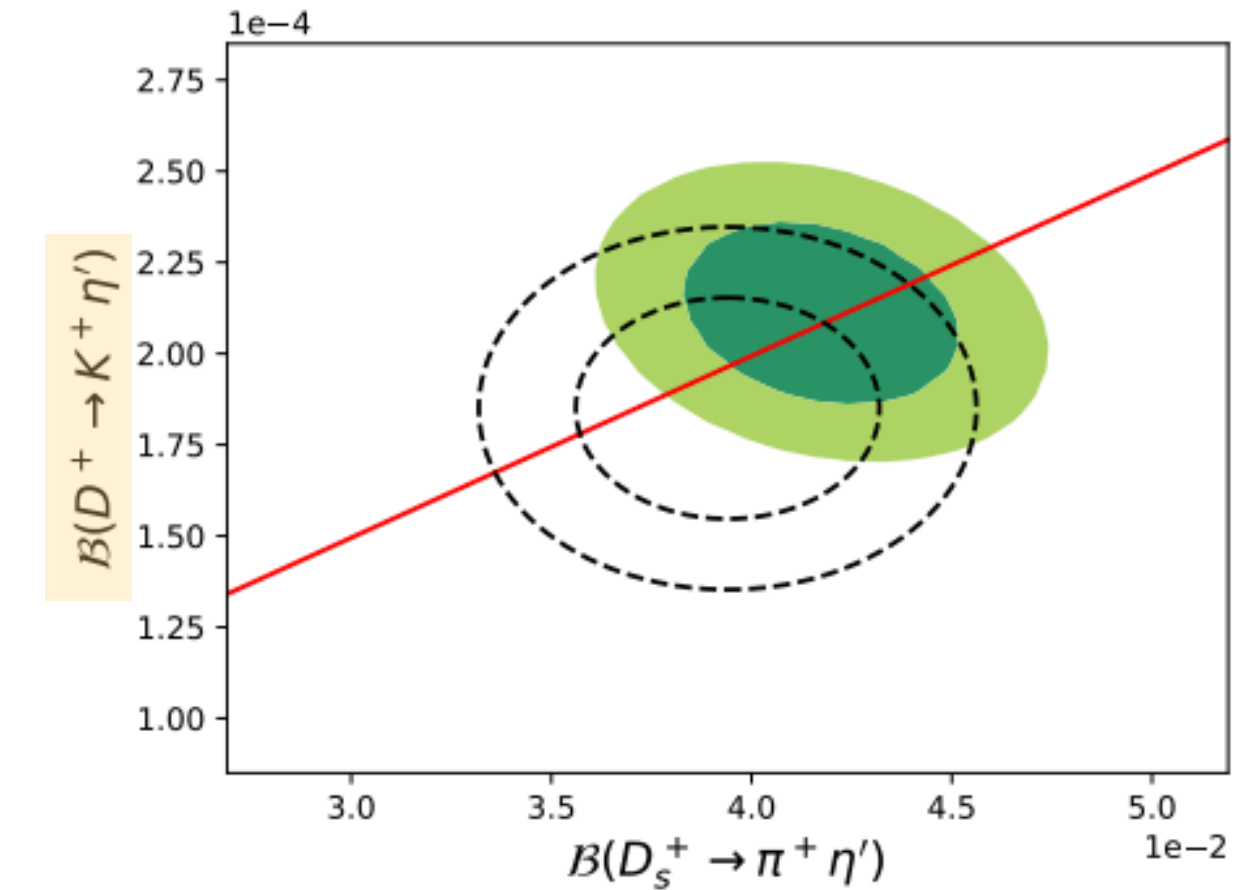
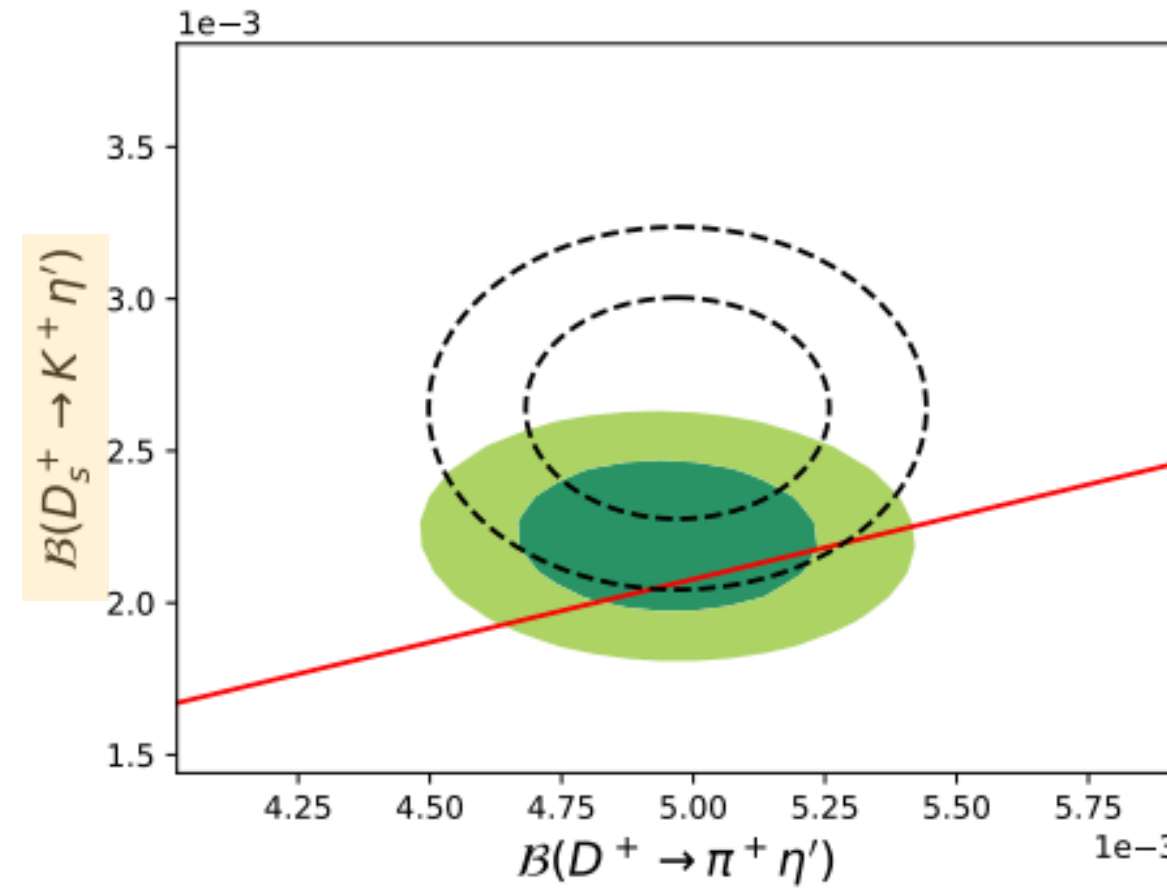
CB, UN, SS, KV, JHEP 05 (2025) 148

# Charged modes BR correlations from fit

Experimental result -----

$SU(3)_F$  symmetry limit ———

- Deviations in  $D_{(s)}^+ \rightarrow K^+ \eta'$  from experimental results show pull towards  $SU(3)_F$  limit

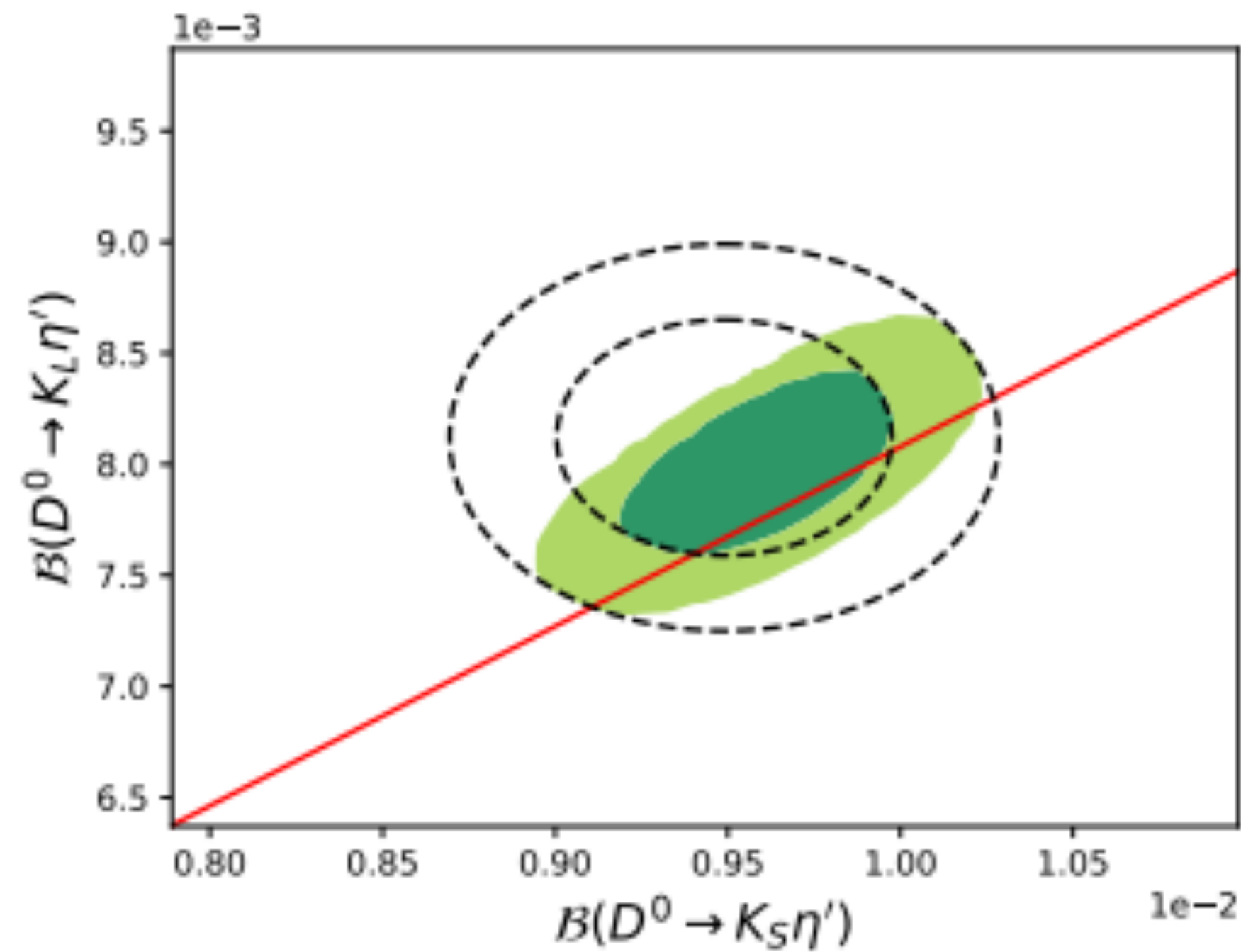


# Neutral modes BR correlations from fit

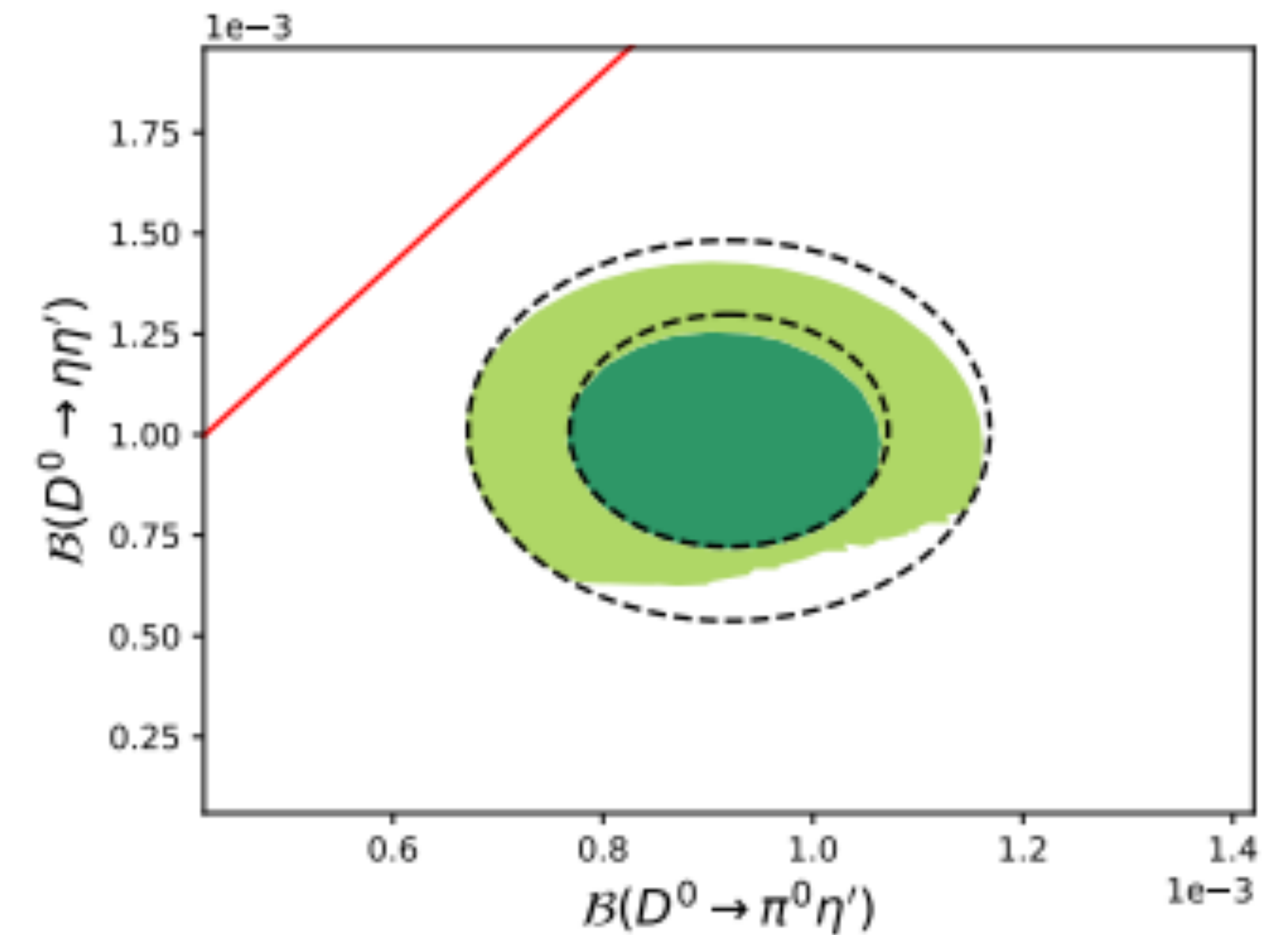
Experimental result -----

$SU(3)_F$  symmetry limit ———

- Strong theory correlation for physical  $K_{S,L}$  modes



- SCS neutral modes responsible for deviation from  $SU(3)_F$  limit



# What about CPV?

- CKM-leading and subleading interference  $\Rightarrow$  CPV in SCS decays!

$$\mathcal{A}^{SCS} = \lambda_{sd} \mathcal{A}_{sd} - \frac{\lambda_b}{2} \mathcal{A}_b$$

$$a_{CP}^{\text{dir}} = \text{Im} \frac{\lambda_b}{\lambda_{sd}} \text{Im} \frac{\mathcal{A}_b}{\mathcal{A}_{sd}}$$

$$\text{Im} \frac{\lambda_b}{\lambda_{sd}} \sim -6 \cdot 10^{-4}$$

- **A charming puzzle!**

$$a_{CP}^{\text{dir}}(D^0 \rightarrow K^- K^+) = (7.7 \pm 5.7) \cdot 10^{-4}$$

$$a_{CP}^{\text{dir}}(D^0 \rightarrow \pi^- \pi^+) = (23.2 \pm 6.1) \cdot 10^{-4}$$

LHCb Collaboration,  
Phys. Rev. Lett. 131 (2023) 9, 091802

➔ Perturbative QCD estimates  $|\mathcal{A}_b/\mathcal{A}_{sd}| \sim 0.1$

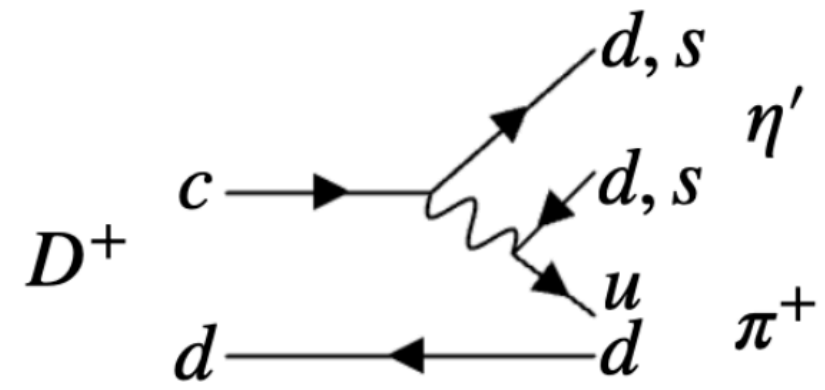
A. Khodjamirian, A. Petrov,  
Phys. Lett. B 774 (2017) 235-242

➔ Experimental results imply  $|\mathcal{A}_b/\mathcal{A}_{sd}| \sim \mathcal{O}(1)$

➔ Experimental results imply  $2.7\sigma$  deviation from  $SU(3)_F$

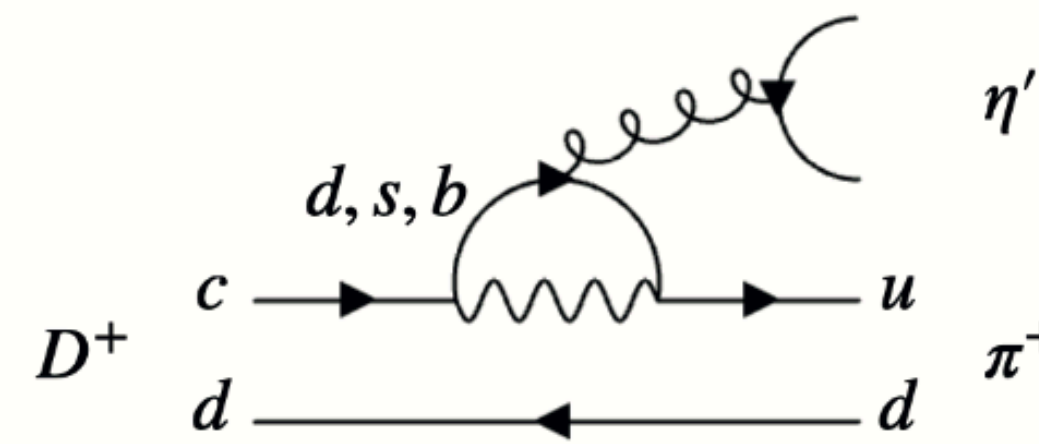
# CKM-subleading amplitudes

- Topologies not included in CKM-leading amplitudes



tree-level

$$\begin{aligned}
 &= \frac{1}{\sqrt{3}} (\lambda_d + \lambda_s) C_{18} \\
 &= \frac{1}{\sqrt{3}} \left( -\lambda_{sd} - \frac{\lambda_b}{2} + \lambda_{sd} - \frac{\lambda_b}{2} \right) C_{18} \\
 &= \frac{2}{\sqrt{3}} \left( -\frac{\lambda_b}{2} \right) C_{18}.
 \end{aligned}$$



penguins

$$\begin{aligned}
 &= 3 \frac{1}{\sqrt{3}} (\lambda_d P_{H,d} + \lambda_s P_{H,s} + \lambda_b P_{H,b}) \\
 &= \frac{3}{\sqrt{3}} \left[ \lambda_{sd} (P_{H,s} - P_{H,d}) \right. \\
 &\quad \left. - \frac{\lambda_b}{2} (P_{H,d} + P_{H,s} - 2P_{H,b}) \right] \\
 &= \frac{3}{\sqrt{3}} \left( \lambda_{sd} P_{H,\text{break}}^{(1)} - \frac{\lambda_b}{2} P_H \right).
 \end{aligned}$$

- Redefinitions respect power counting and  $1/N_c$

$$\bar{C}_{18} = C_{18} + \frac{P_{18} + P_{81} + 3P_H}{2}$$

# CPV sum rules

- Subleading amp. written in terms of leading amp. parameters and **new**

$$\begin{aligned}\mathcal{A}_b(d) &= c_{sd}^d \mathcal{A}_{sd}(d) + \Delta \mathcal{A}_b(d) \\ &= c_{sd}^d \mathcal{A}_{sd}(d) + \sum_i c_i^d \mathcal{T}_i,\end{aligned}$$

Decay ampl. $\mathcal{A}_b(d)$	$\mathcal{A}_{sd}$	$\bar{C}_{18}$	$\hat{C}_{81}$	$\hat{C}_{18,1}^{(1)}$	$\bar{C}_{18,3}^{(1)}$	$\hat{C}_{88}^{(1)}$	$\bar{C}_{11}^{(1)}$	$\bar{E}_{88}^{(1)}$
$D^0 \rightarrow \pi^0 \eta'$	1	$\sqrt{\frac{2}{3}}$	$-\sqrt{\frac{2}{3}}$	0	0	0	0	$-\frac{2}{\sqrt{3}}$
$D^0 \rightarrow \eta \eta'$	1	$\frac{\sqrt{2}}{3}$	$\frac{\sqrt{2}}{3}$	0	0	$\frac{2}{3}$	$\frac{4}{3}$	$\frac{4}{3}$
$D^+ \rightarrow \pi^+ \eta'$	-1	$\frac{2}{\sqrt{3}}$	0	$\frac{2}{\sqrt{3}}$	0	$-2\sqrt{\frac{2}{3}}$	0	$-2\sqrt{\frac{2}{3}}$
$D_s^+ \rightarrow K^+ \eta'$	1	$\frac{2}{\sqrt{3}}$	0	0	$\frac{2}{\sqrt{3}}$	$\sqrt{\frac{2}{3}}$	0	$\sqrt{\frac{2}{3}}$

- Sum rules only in  $SU(3)_F$  symmetry limit

**neutrals**  $a_{CP}^{\text{dir}}(D^0 \rightarrow \pi^0 \eta')|_{SU(3)_F\text{-lim.}} = -3 a_{CP}^{\text{dir}}(D^0 \rightarrow \eta \eta')|_{SU(3)_F\text{-lim.}}$  ,

**charged**  $a_{CP}^{\text{dir}}(D^+ \rightarrow \pi^+ \eta')|_{SU(3)_F\text{-lim.}} = - a_{CP}^{\text{dir}}(D_s^+ \rightarrow K^+ \eta')|_{SU(3)_F\text{-lim.}}$  .

- Modified: factor out  $SU(3)_F$ -breaking in CKM-leading part

$$\frac{a_{CP}^{\text{dir}}(D^0 \rightarrow \pi^0 \eta')}{a_{CP}^{\text{dir}}(D^0 \rightarrow \eta \eta')} = -3 \sqrt{\frac{\mathcal{P}(D^0 \rightarrow \pi^0 \eta')}{\mathcal{P}(D^0 \rightarrow \eta \eta')}} \sqrt{\frac{\mathcal{B}(D^0 \rightarrow \eta \eta')}{\mathcal{B}(D^0 \rightarrow \pi^0 \eta')}} ,$$

$$\frac{a_{CP}^{\text{dir}}(D^+ \rightarrow \pi^+ \eta')}{a_{CP}^{\text{dir}}(D_s^+ \rightarrow K^+ \eta')} = - \sqrt{\frac{\mathcal{P}(D^+ \rightarrow \pi^+ \eta')}{\mathcal{P}(D_s^+ \rightarrow K^+ \eta')}} \sqrt{\frac{\mathcal{B}(D_s^+ \rightarrow K^+ \eta')}{\mathcal{B}(D^+ \rightarrow \pi^+ \eta')}} ,$$

# Limitations from CPV measurements

- Measurements available only for charged modes

$$a_{CP}^{\text{dir}}(D^+ \rightarrow \pi^+ \eta') \Big|_{\text{HFLAV}} = (0.4 \pm 0.2)\%$$

$$a_{CP}^{\text{dir}}(D_s^+ \rightarrow K^+ \eta') \Big|_{\text{HFLAV}} = (6.0 \pm 18.9)\%$$

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- ➔ Single CLEO 2010 measurement based on ~50 events
- ➔ No constraining power
- ➔ Large central value incompatible with  $D^+ \rightarrow \pi^+ \eta'$  measurement
  - $D_s^+ \rightarrow K^+ \eta'$  shares same topologies + 1  $SU(3)_F$ -breaking
- ➔ Would artificially influence global minimum

**Not included in the analysis!**

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- ➔ Dominated by LHCb 2023 measurement
- ➔ Single data point: cannot constrain all parameters!
  - In  $SU(3)_F$  limit:

Ratio subleading to leading amplitudes

$$\left| \frac{\bar{C}_{18}}{\hat{T}_{18}} \right| \sin(\arg \bar{C}_{18} - \delta(D^+ \rightarrow \pi^+ \eta')) = -0.78 \pm 0.39$$

Unknown phase

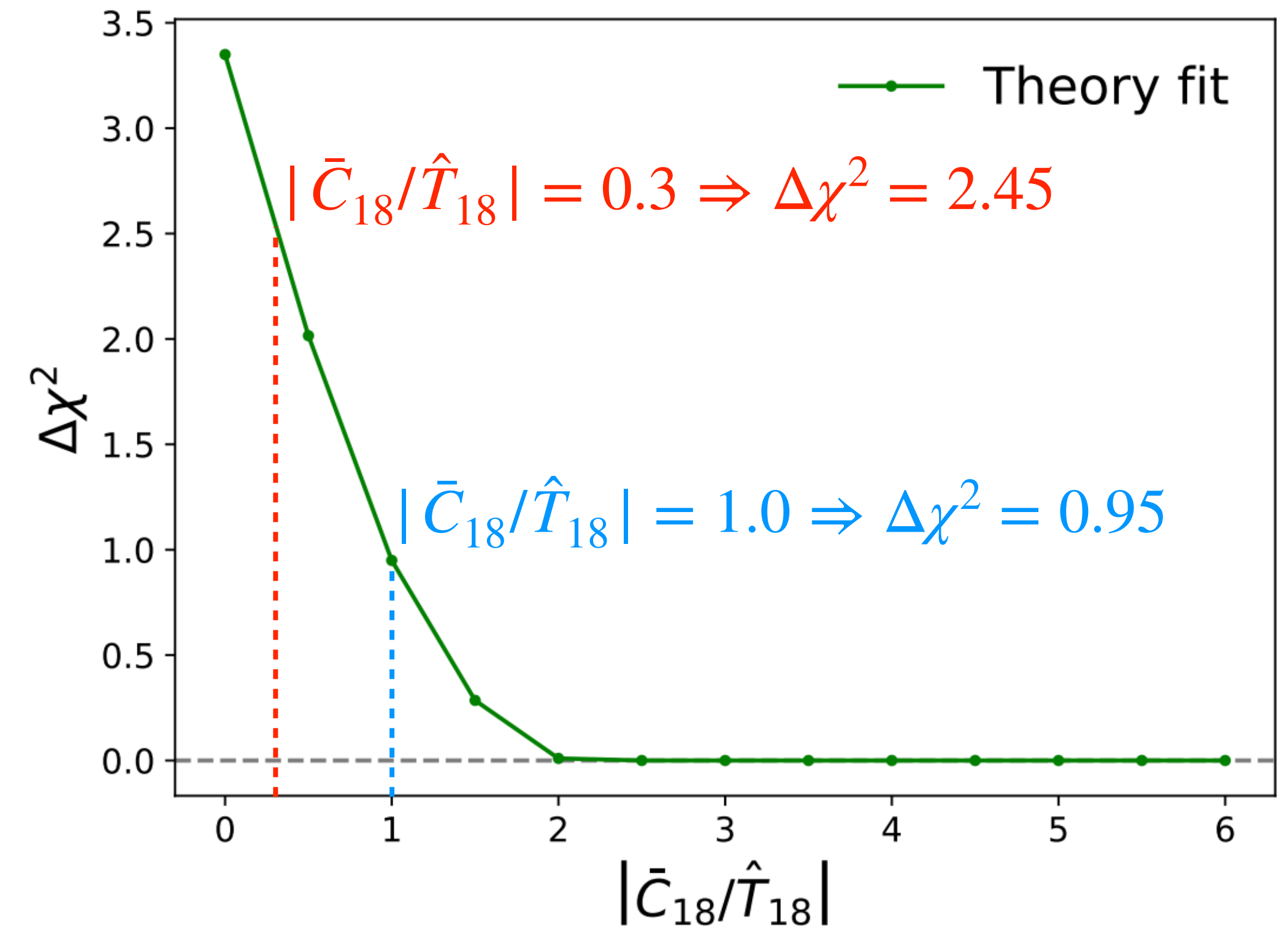
Leading amplitude phase  
Unconstrained by BR analysis

# Ratio of subleading to leading amplitudes

- Full analysis compared to BR only with

$$\varepsilon = 30\% \quad \rightarrow \quad \left| \frac{\mathcal{A}_{b,\text{SU}(3)}(D \rightarrow P\eta')}{\mathcal{A}_{b,\text{SU}(3)\text{-lim}}(D \rightarrow P\eta')} \right| \leq \varepsilon$$

- Scan shows increased tension for small values!
- No tension for values  $\geq 2$

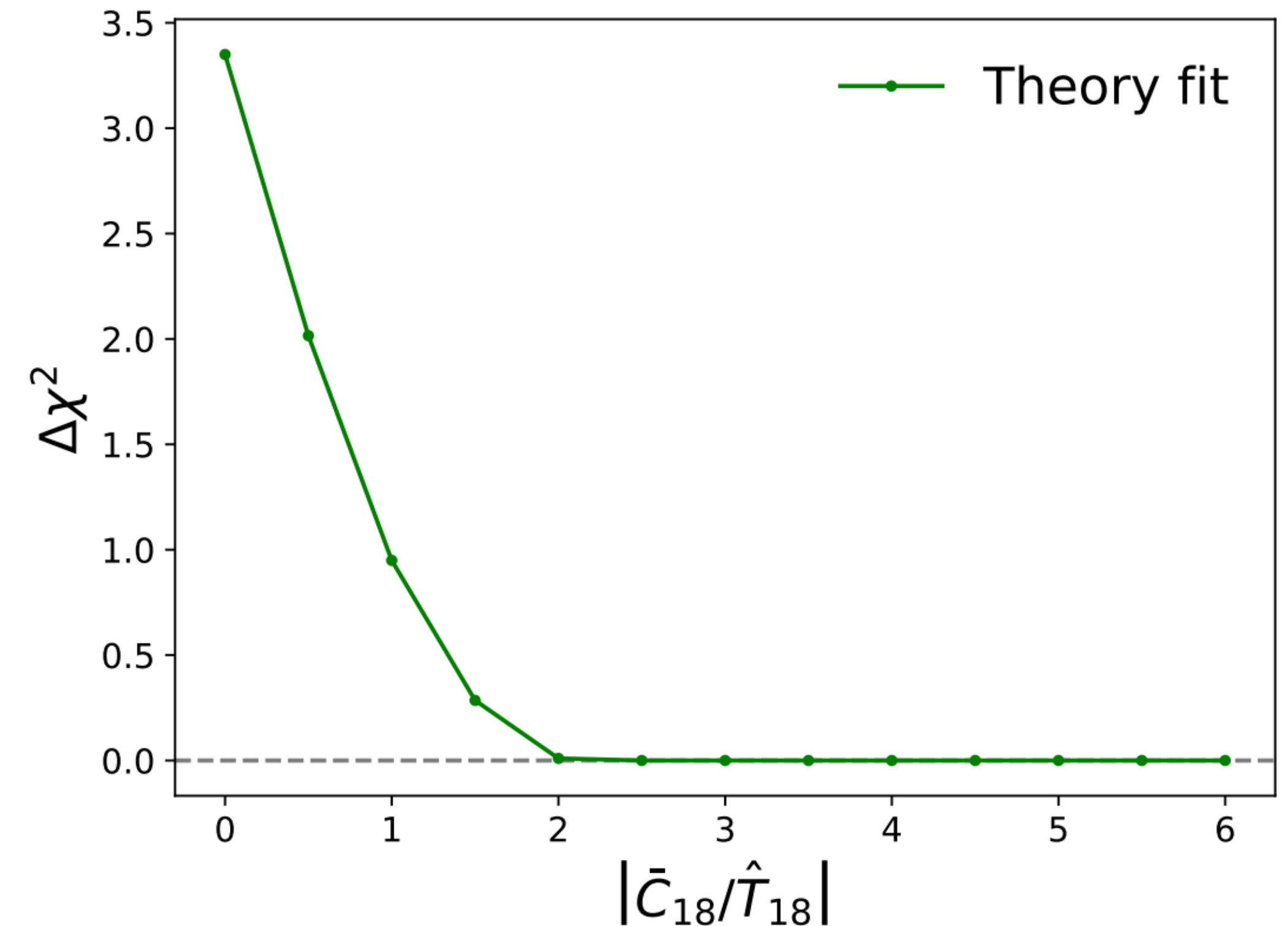


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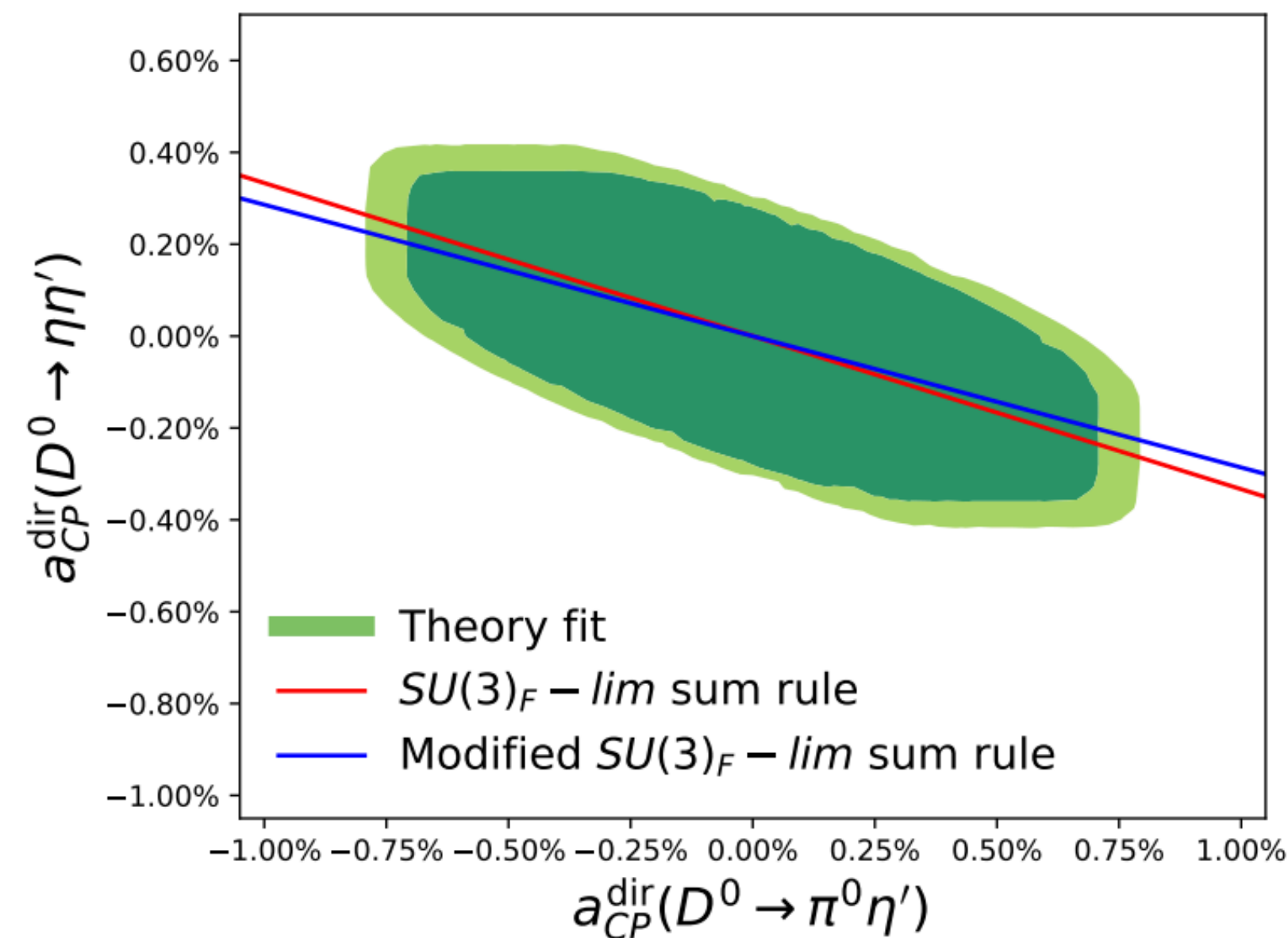
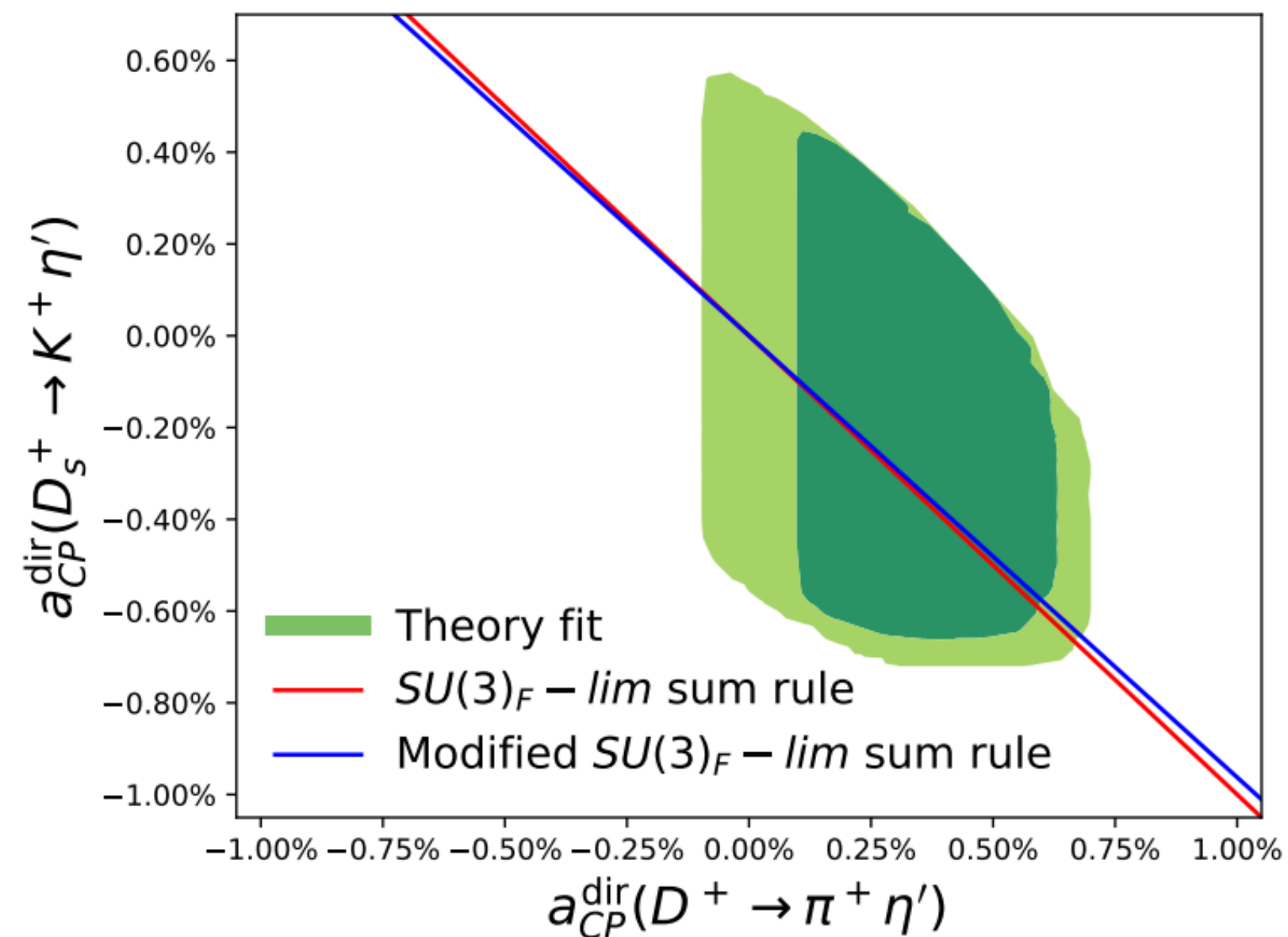


- Plateau due to single data point limitation
- Predictions will be dependent on allowed range for  $|\bar{C}_{18}/\hat{T}_{18}|$

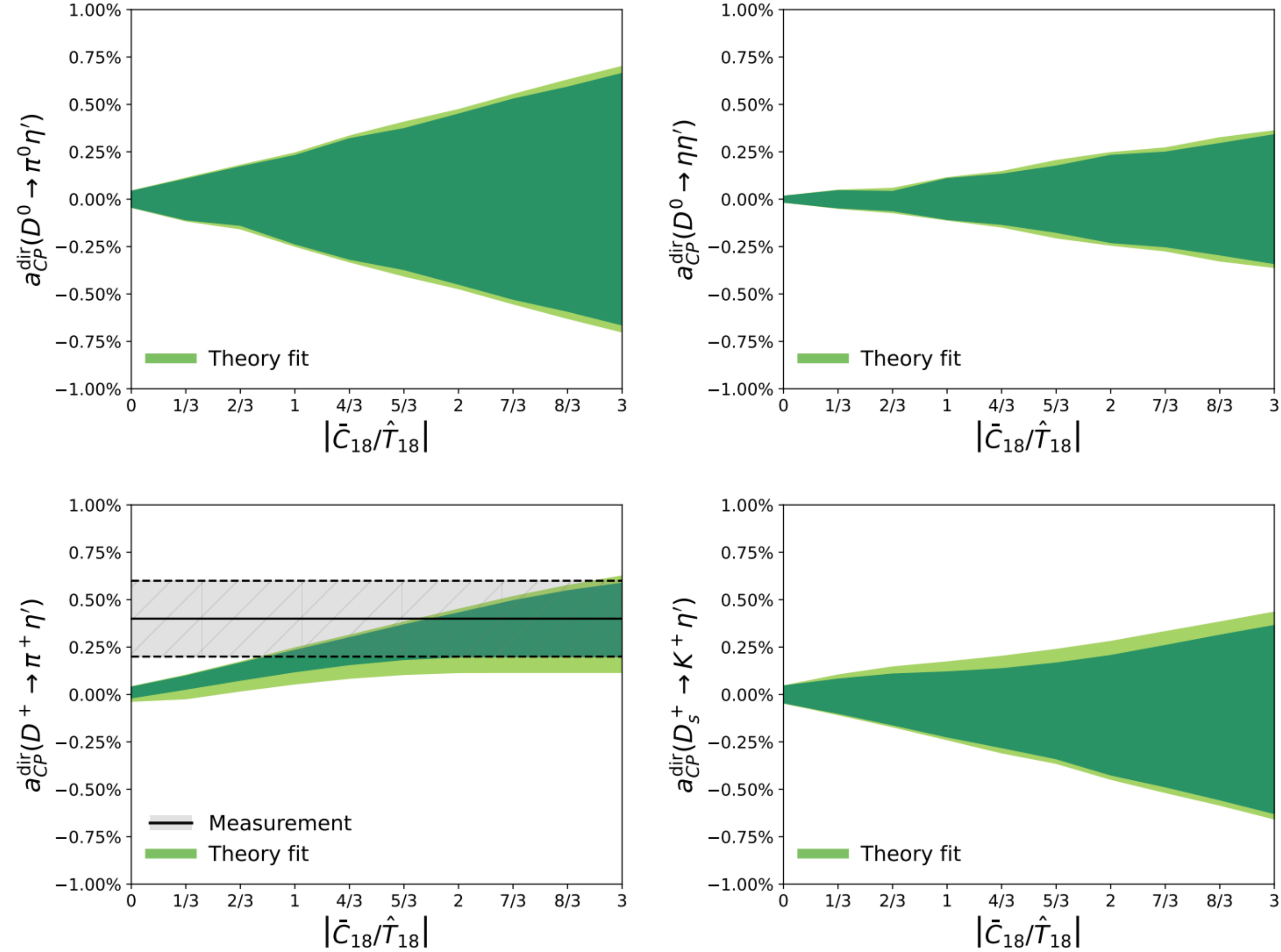
# Predictions for upper-bound on ratio

- Conservative assumption on technical bound  $|\bar{C}_{18}/\hat{T}_{18}| \leq 3$

Observable	Theory fit	Experiment
$a_{CP}^{\text{dir}}(D^0 \rightarrow \pi^0 \eta')$	$[-0.66, 0.66]\%$	—
$a_{CP}^{\text{dir}}(D^0 \rightarrow \eta \eta')$	$[-0.29, 0.29]\%$	—
$a_{CP}^{\text{dir}}(D^+ \rightarrow \pi^+ \eta')$	$(0.40^{+0.19}_{-0.20})\%$	$(0.40 \pm 0.20)\%$
$a_{CP}^{\text{dir}}(D_s^+ \rightarrow K^+ \eta')$	$[-0.62, 0.36]\%$	—



# Dependence of predictions on the ratio



- New measurements will allow for extraction of  $|\bar{C}_{18}/\hat{T}_{18}|$

# Conclusions

- $SU(3)_F$  limit does not accommodate these decays  $\Rightarrow$  due to  $D^0 \rightarrow \pi^0 \eta'$  and  $D^0 \rightarrow \eta \eta'$
- 30%  $SU(3)_F$ -breaking can describe data  $\Rightarrow$  small tension of  $2.5\sigma$
- Important to get updated BR measurements on  $D^0 \rightarrow \pi^0 \eta'$ ,  $D^0 \rightarrow \eta \eta'$ ,  $D_s^+ \rightarrow K^+ \eta'$  to confront  $SU(3)_F$  symmetry
- Lack of CPV measurements in these channels
  - ➔ Charged  $D_s^+ \rightarrow K^+ \eta'$  would constrain fit!
- We can constrain  $|\bar{C}_{18}/\hat{T}_{18}|$  with new measurements and probe  $1/N_c$  expectation

Obrigada!