Hadronic B decays: Symmetries and Anomalies

Talk based on 2311.18011(PRL), 2505.11492(PRD), 2510.13969(JHEP accepted)

Bhubanjyoti Bhattacharya
(bbhattach@ltu.edu)

Lawrence Technological University

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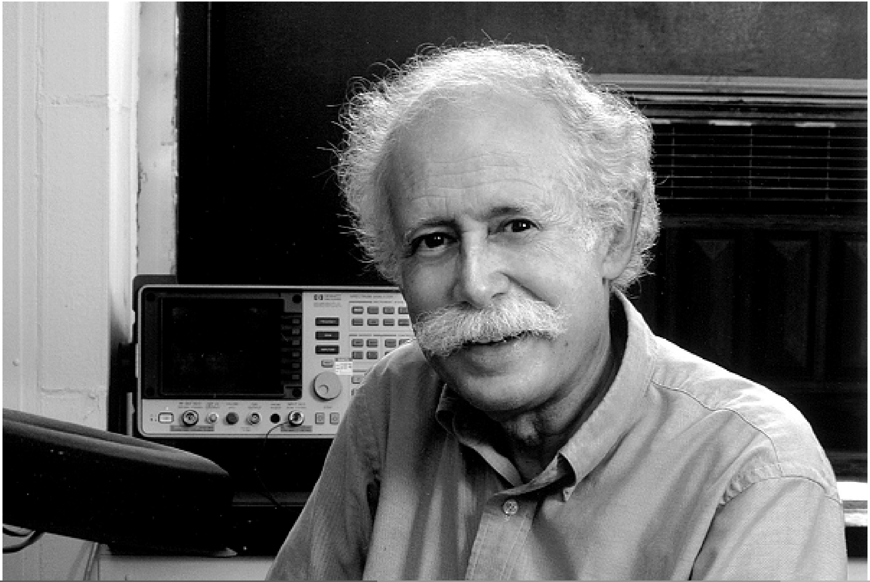
Nonleptonic decays of heavy mesons
IPPP Durham
March 24, 2026
bit.ly/i3pmar26

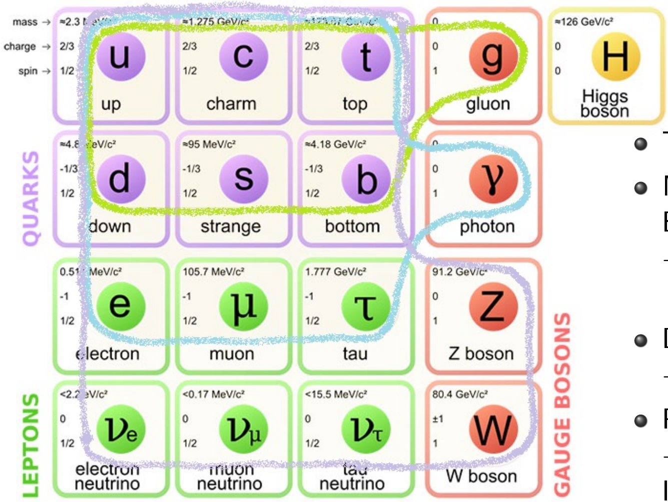
In memory of Jon Rosner



1941-2025

Photo: UChicago Physics

The Standard Model and beyond



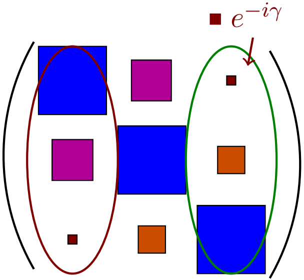
- The Standard Model is incomplete!
- May require new particles/symmetry
  Baryon-asymmetry problem
  → CP Violation + other criteria
  Sakharov, 1967
- Direct searches @ energy frontier
  → new particles/interactions
- Puzzles/Anomalies
  → SM prediction ≠ Expt.
  Intensity frontier ⇔ Energy frontier

Why hadronic B decays?

- Semileptonic decays of mesons $\Rightarrow$ measure magnitude of CKM elements
- To study CKM phases $\Rightarrow$ need interference of weak-decay amplitudes
- Example: γ in $B \to DK$ and $B \to \bar{D}K$ or α in $B \to \pi\pi$
- Challenges: non-perturbative effects in QCD, amplitudes not necessarily factorizable
- Goal: eliminate the necessity for theory input; identify inconsistencies

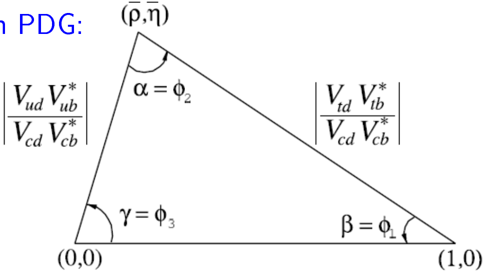
CKM Unitarity and the angle γ/ϕ_3

$$V_{\text{CKM}} = \begin{matrix} & d & s & b \\ u & V_{ud} & V_{us} & V_{ub} \\ c & V_{cd} & V_{cs} & V_{cb} \\ t & V_{td} & V_{ts} & V_{tb} \end{matrix}$$

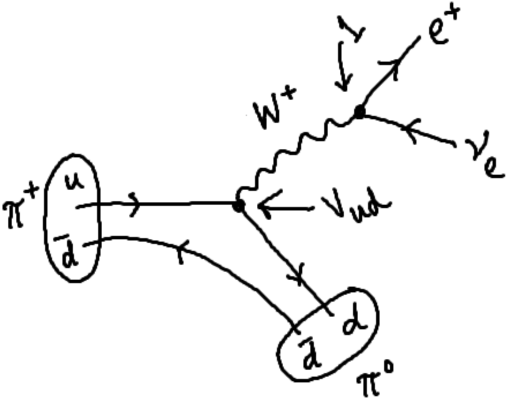


- Empirically close to diagonal

  $\rightarrow$ smaller elements farther from diagonal
- $V_{ij}V_{ik}^* = \delta_{jk} \qquad \sum_i |V_{ij}|^2 = 1$
- $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$
- Taken from PDG:

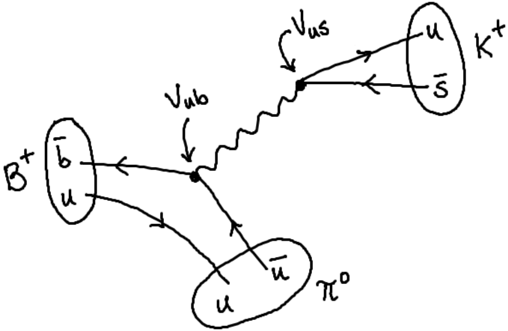


Measuring the CKM matrix



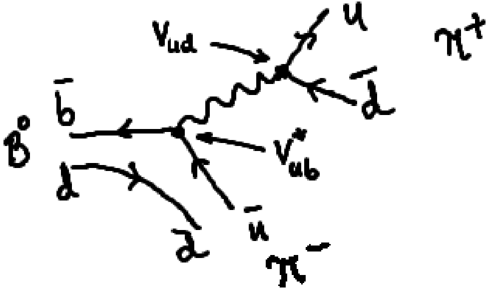
- Decay rate $\propto \quad |V_{ud}|^2$
- Factorizable current: hadron vs lepton
- QCD effects hidden in form factors

- Phase measurement tricky: need CPV
- CPV from amplitude interference
- Strong phases from quark rescattering
- Non-factorizable effects important



A typical hadronic decay amplitude

- Consider the decay $B_d^0 \to \pi^+\pi^-$: $\mathcal{A}(B_d^0 \to \pi^+\pi^-) = V_{ub}^* V_{ud} A_u + V_{tb}^* V_{td} A_t \approx -Te^{i\gamma} - P_{tc}$



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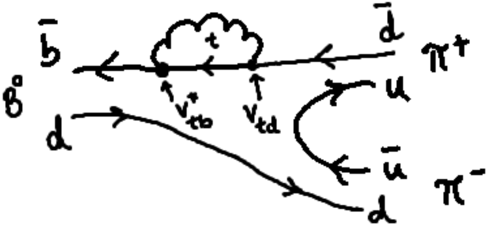

$$\propto V_{ub}^* V_{ud} = \lambda_u^{(d)} \leftarrow e^{i\gamma} \qquad\qquad \propto V_{tb}^* V_{td} = \lambda_t^{(d)}$$

How can we extract information? What information can we extract?

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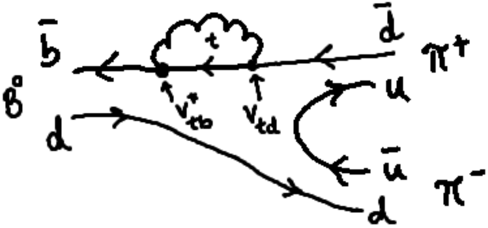

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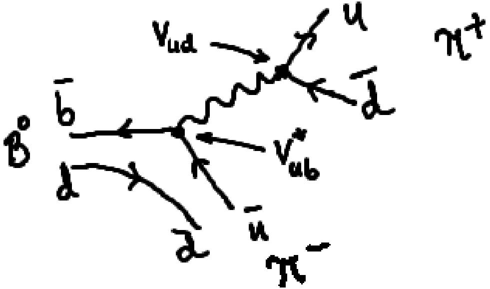

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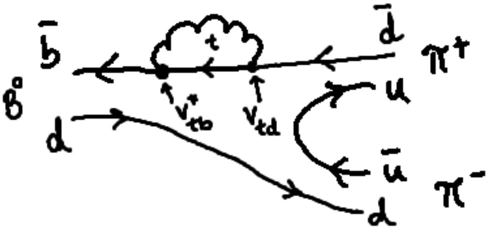

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Extracting information

- $\mathcal{A}(B \to f) = |a| + |b|\, e^{i\delta}\, e^{i\phi} \quad \to \quad \Gamma \propto |\mathcal{A}|^2$

  $\overline{\mathcal{A}}(\overline{B} \to \overline{f}) = |a| + |b|\, e^{i\delta}\, e^{-i\phi} \quad \to \quad \overline{\Gamma} \propto |\overline{\mathcal{A}}|^2$

  – 4 parameters: 2 magnitudes $(|a|, |b|)$, 1 strong phase (δ), 1 weak phase (ϕ)

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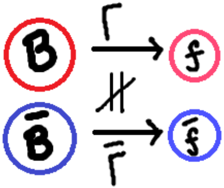

- $f \neq \overline{f}$ Only 2 observables
- Measure Γ and $\overline{\Gamma}$; or

$$\mathcal{B}_{\rm CP} = \frac{\Gamma + \overline{\Gamma}}{2} \text{ and } A_{\rm CP} = \frac{\Gamma - \overline{\Gamma}}{\Gamma + \overline{\Gamma}}$$

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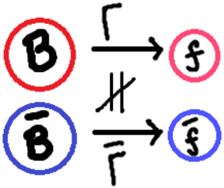

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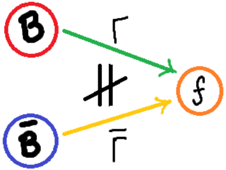

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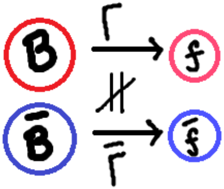

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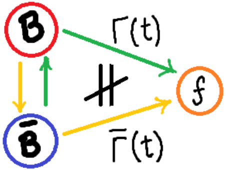
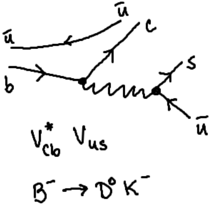
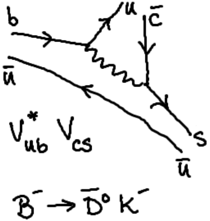


- $f = \overline{f}$: same 2 observables
- Additional observable(s) from B-$\overline{B}$ mixing
- $S_{\rm CP}$ from $\dfrac{\Gamma(t) - \overline{\Gamma}(t)}{\Gamma(t) + \overline{\Gamma}(t)}$

Measuring a CKM phase

- Consider $B^- \to D^0 K^-$ and $B^- \to \overline{D}^0 K^-$ with $D^0 \to f$
- Anti-decays are $B^+ \to \overline{D}^0 K^-$ and $B^+ \to D^0 K^+$;   No CP Violation in D decays
- Total number of observables in the B decays: $\Gamma, \overline{\Gamma}$ for each decay $\to$ 4





Only tree-level contributions in the SM

Highly-suppressed Loop (box diagrams)

Brod and Zupan (2013)

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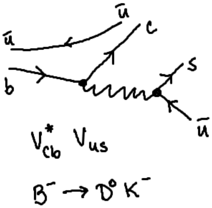

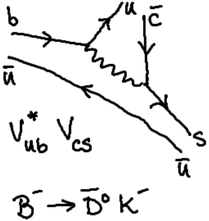

- GLW method (f_{CP} such as $\pi^+\pi^-$)
- 3 theory parameters: $|r_B|, \delta, \gamma$
- Extract γ from a fit
- ADS method (f such as $K^+\pi^-$)
- GGSZ method (f such as $K_S\pi\pi$)

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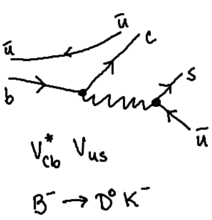

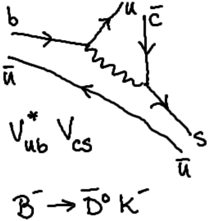

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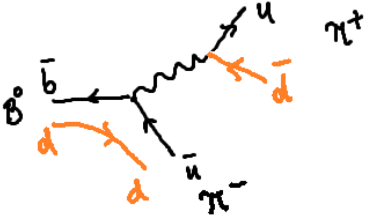
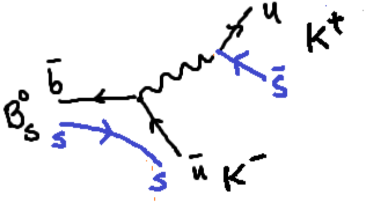
- Extract γ from a fit
- ADS method (f such as $K^+\pi^-$)
- GGSZ method (f such as $K_S\pi\pi$)
- Recent work in $B \to DK$ shows:
  - $\to$ large deviation from SU(3) hypothesis
  - $\to \sim 30\%$ ~~SU(3)~~ accounts for deviation

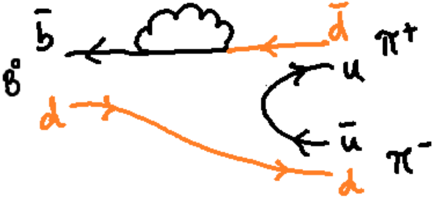
Schacht et al., 2403.04878

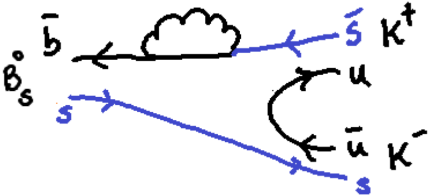
The utility of flavor symmetries

Same # of parameters, double the observables







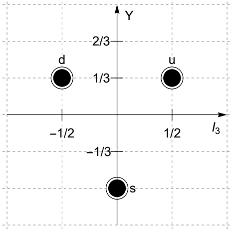


$B_d^0 \to \pi^+\pi^- \quad (\Delta S = 0)$

$B_s^0 \to K^+K^- \quad (\Delta S = 1)$

Flavor-SU(3) symmetry: SU(3)$_F$

- 3 light quarks, u, d, s, much lighter than b quark
- u, d, s = SU(3)$_F$ triplet; State $\rightarrow |\mathbf{irrep}, Y, I, I_3\rangle$
- $|u\rangle = \left|\mathbf{3}, \frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right\rangle$, $|d\rangle = \left|\mathbf{3}, \frac{1}{3}, \frac{1}{2}, -\frac{1}{2}\right\rangle$, $|s\rangle = \left|\mathbf{3}, -\frac{2}{3}, 0, 0\right\rangle$



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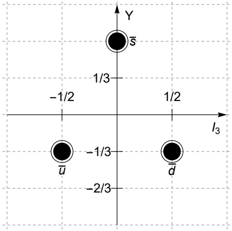

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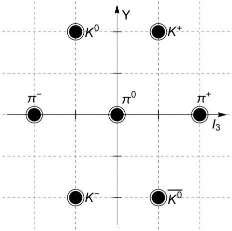

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- $\left|\bar{d}\right\rangle = \left|\mathbf{3}^*, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right\rangle$; Y = hypercharge, I = Isospin
- $\mathbf{3} \times \mathbf{3}^* \ = \ \mathbf{1} + \mathbf{8}$: *These are the 3 pions, 4 kaons,* η, η'
- $|\pi^+\rangle = \left|u\bar{d}\right\rangle = |\mathbf{8}, 0, 1, 1\rangle$; Similarly other pions and kaons are also octets
- Apply to two-body final states: *Identical bosons* $\to$ *symmetrize*

$$|PP\rangle_{\rm sym} = (\mathbf{8} \times \mathbf{8})_{\rm sym} = \mathbf{1} + \mathbf{8} + \mathbf{27} = 36$$

The Hamiltonian under QCD

- $H = \lambda_u \underbrace{(c_1\mathcal{O}_1 + c_2\mathcal{O}_2)}_{\text{tree,(V}-\text{A)}\times\text{(V}-\text{A)}} - \lambda_t \underbrace{\sum_i^{3-6} c_i\mathcal{O}_i}_{\text{penguins}}$

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- $\mathcal{O}_1 = (\bar{b}u)_{V-A}(\bar{u}q)_{V-A}, \quad \mathcal{O}_2 = (\bar{b}q)_{V-A}(\bar{u}u)_{V-A}$ $\qquad V \equiv \gamma^\mu, \qquad A \equiv \gamma^\mu\gamma^5$
- $\mathcal{O}_{3(5)} = (\bar{b}\underset{\underset{f_2}{\downarrow}}{q})_{V-A} \overbrace{\left\{(\bar{u}\underset{\underset{f_4}{\downarrow}}{u})_{V\overset{-}{(+)}A} + (\bar{d}\underset{\underset{f_4}{\downarrow}}{d})_{V\overset{-}{(+)}A} + (\bar{s}\underset{\underset{f_4}{\downarrow}}{s})_{V\overset{-}{(+)}A}\right\}}^{\mathbf{1}} \quad \underset{f_2 \leftrightarrow f_4}{\Rightarrow} \mathcal{O}_{4(6)}$

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- Sizes of WCs ($\mu = m_b$): $c_1 = 1.139$ $\quad c_2 = -0.307$ $\quad c_3 = 0.013$
  BBL, hep-ph/9512380 $\quad c_4 = -0.030$ $\quad c_5 = 0.009$ $\quad c_6 = -0.038$

The Hamiltonian under QCD and QED

- $H = \lambda_u \underbrace{(c_1\mathcal{O}_1 + c_2\mathcal{O}_2)}_{\text{tree,(V−A)×(V−A)}} - \lambda_t \left(\underbrace{\sum_i^{3-6} c_i\mathcal{O}_i}_{\text{penguins}} + \underbrace{c_7\mathcal{O}_7 + c_8\mathcal{O}_8}_{\text{EWP,(V−A)×(V+A)}} + \underbrace{c_9\mathcal{O}_9 + c_{10}\mathcal{O}_{10}}_{\text{EWP,(V−A)×(V−A)}} \right)$

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- $\mathcal{O}_{9(7)} = (\bar{b}\underset{\underset{f_2}{\downarrow}}{q})_{V-A}\left\{ e_u(\bar{u}\underset{\underset{f_4}{\downarrow}}{u})_{V\underset{(+)}{-}A} + e_d(\bar{d}\underset{\underset{f_4}{\downarrow}}{d})_{V\underset{(+)}{-}A} + e_s(\bar{s}\underset{\underset{f_4}{\downarrow}}{s})_{V\underset{(+)}{-}A} \right\} \underset{f_2 \leftrightarrow f_4}{\Rightarrow} \mathcal{O}_{8(10)}$

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- $\mathcal{O}_{9(7)} = (\bar{b}\underset{\substack{\downarrow \\ f_2}}{q})_{V-A}\left\{ e_u(\bar{u}\underset{\substack{\downarrow \\ f_4}}{u})_{V\underset{(+)}{-}A} + e_d(\bar{d}\underset{\substack{\downarrow \\ f_4}}{d})_{V\underset{(+)}{-}A} + e_s(\bar{s}\underset{\substack{\downarrow \\ f_4}}{s})_{V\underset{(+)}{-}A} \right\} \quad \underset{f_2 \leftrightarrow f_4}{\Rightarrow} \mathcal{O}_{8(10)}$

- Fortunately, $e_u = -2e_d = -2e_s$!

- Sizes of WCs ($\mu = m_b$): $c_1 = 1.144$ $c_2 = -0.308$ $c_3 = 0.014$
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- Independent RMEs: $V_{ub}^* V_{us} \quad \to \quad 5, \quad V_{tb}^* V_{ts} \quad \to \quad 2$
- Each RME is a complex number: 7 independent RMEs = 13 real parameters

EWP-Tree relations of GPY *(see D. London's talk)*

- $$H = \lambda_u \underbrace{(c_1\mathcal{O}_1 + c_2\mathcal{O}_2)}_{\text{tree},(\text{V}-\text{A})\times(\text{V}-\text{A})} - \lambda_t \left(\underbrace{\sum_i^{3-6} c_i\mathcal{O}_i}_{\text{penguins}} + \underbrace{c_7\mathcal{O}_7 + c_8\mathcal{O}_8}_{\text{EWP},(\text{V}-\text{A})\times(\text{V}+\text{A})} + \underbrace{c_9\mathcal{O}_9 + c_{10}\mathcal{O}_{10}}_{\text{EWP},(\text{V}-\text{A})\times(\text{V}-\text{A})} \right)$$

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- $\mathcal{O}_{10} = \frac{3}{2}\left[\frac{2}{3}(\bar{b}u)_{V-A}(\bar{u}q)_{V-A} - \frac{1}{3}(\bar{b}d)_{V-A}(\bar{d}q)_{V-A} - \frac{1}{3}(\bar{b}s)_{V-A}(\bar{s}q)_{V-A}\right]$

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- Similarly, one finds: $2\mathcal{O}_9 = 3\mathcal{O}_2 - \mathcal{O}_3$ If $\langle\mathcal{O}_3\rangle = 0$, then $2\langle\mathcal{O}_9\rangle = 3\langle\mathcal{O}_2\rangle$
- EWP RMEs are not independent of Tree RMEs! *G. Paz, hep-ph/0206312*

Implementing EWP-Tree relations

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- Identical RMEs from $\mathbf{6}$ and $\mathbf{15}^*$ in $\mathcal{O}_{1,2}$ and $\mathcal{O}_{9,10}$
- RMEs: $\mathbf{3}^* \to A, \mathbf{6} \to R, \mathbf{15}^* \to P$: $A_1^q, A_8^q, R_8^q, P_8^q, P_{27}^q$ $(q = d, s)$ *(Zeppenfeld, 1981)*
- EWP-tree relations: $R_8^t \propto R_8^u$, $P_8^t \propto P_8^u$, $P_{27}^t \propto P_{27}^u$ $\leftarrow$ RME relations

  Diagram relations $\to$ $P_{EW} \approx \kappa T$, $P_{EW}^C \approx \kappa C$, with $\kappa \sim 1.4\%$

$B \to PP$ decay amplitudes in SU(3)

Decays		$\lambda_u^{(q)}$					$\lambda_t^{(q)}$				
		c_1, c_2					$c_3,\ c_4,\ c_5,\ c_6,\ c_9,\ c_{10}$		$c_9,\ c_{10}$		
		A_1	A_8	R_8	P_8	P_{27}	B_1	B_8	R_8	P_8	P_{27}
$\Delta S = 0$	$B^+ \to \bar{K}^0 K^+$	0	$-\frac{\sqrt{3}}{\sqrt{5}}$	$-\frac{1}{\sqrt{5}}$	$\frac{3\sqrt{3}}{5}$	$\frac{2\sqrt{3}}{5}$	0	$-\frac{\sqrt{3}}{\sqrt{5}}$	$\frac{3}{\sqrt{5}}$	$\frac{9\sqrt{3}}{5}$	$\frac{6\sqrt{3}}{5}$
	$B^+ \to \pi^0 \pi^+$	0	0	0	0	$\sqrt{6}$	0	0	0	0	$3\sqrt{6}$
	$B^+ \to \eta_8 \pi^+$	0	$\frac{\sqrt{2}}{\sqrt{5}}$	$\frac{\sqrt{2}}{\sqrt{15}}$	$-\frac{3\sqrt{2}}{5}$	$\frac{3\sqrt{2}}{5}$	0	$\frac{\sqrt{2}}{\sqrt{5}}$	$-\frac{\sqrt{6}}{\sqrt{5}}$	$-\frac{9\sqrt{2}}{5}$	$\frac{9\sqrt{2}}{5}$
	$B^0 \to K^0 \bar{K}^0$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{\sqrt{15}}$	$-\frac{1}{\sqrt{5}}$	$-\frac{3\sqrt{3}}{5}$	$\frac{\sqrt{3}}{10}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{\sqrt{15}}$	$\frac{3}{\sqrt{5}}$	$-\frac{9\sqrt{3}}{5}$	$\frac{3\sqrt{3}}{10}$
	$B^0 \to \pi^+ \pi^-$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{\sqrt{15}}$	$-\frac{1}{\sqrt{5}}$	$-\frac{\sqrt{3}}{5}$	$\frac{7\sqrt{3}}{10}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{\sqrt{15}}$	$\frac{3}{\sqrt{5}}$	$-\frac{3\sqrt{3}}{5}$	$\frac{21\sqrt{3}}{10}$
	$B^0 \to K^- K^+$	$\frac{1}{2\sqrt{3}}$	$-\frac{2}{\sqrt{15}}$	0	$-\frac{2\sqrt{3}}{5}$	$-\frac{\sqrt{3}}{10}$	$\frac{1}{2\sqrt{3}}$	$-\frac{2}{\sqrt{15}}$	0	$-\frac{6\sqrt{3}}{5}$	$-\frac{3\sqrt{3}}{10}$
	$B^0 \to \pi^0 \pi^0$	$-\frac{1}{2\sqrt{6}}$	$-\frac{1}{\sqrt{30}}$	$\frac{1}{\sqrt{10}}$	$\frac{\sqrt{3}}{5\sqrt{2}}$	$\frac{13\sqrt{3}}{10\sqrt{2}}$	$-\frac{1}{2\sqrt{6}}$	$-\frac{1}{\sqrt{30}}$	$-\frac{3}{\sqrt{10}}$	$\frac{3\sqrt{3}}{5\sqrt{2}}$	$\frac{39\sqrt{3}}{10\sqrt{2}}$
	$B^0 \to \pi^0 \eta_8$	0	$\frac{1}{\sqrt{5}}$	$\frac{1}{\sqrt{15}}$	1	0	0	$\frac{1}{\sqrt{5}}$	$-\frac{\sqrt{3}}{\sqrt{5}}$	3	0
	$B^0 \to \eta_8 \eta_8$	$-\frac{1}{2\sqrt{6}}$	$\frac{1}{\sqrt{30}}$	$-\frac{1}{\sqrt{10}}$	$-\frac{\sqrt{3}}{5\sqrt{2}}$	$-\frac{3\sqrt{3}}{10\sqrt{2}}$	$-\frac{1}{2\sqrt{6}}$	$\frac{1}{\sqrt{30}}$	$\frac{3}{\sqrt{10}}$	$-\frac{3\sqrt{3}}{5\sqrt{2}}$	$-\frac{9\sqrt{3}}{10\sqrt{2}}$
	$B_s^0 \to \pi^+ K^-$	0	$\frac{\sqrt{3}}{\sqrt{5}}$	$-\frac{1}{\sqrt{5}}$	$\frac{\sqrt{3}}{5}$	$\frac{4\sqrt{3}}{5}$	0	$\frac{\sqrt{3}}{\sqrt{5}}$	$\frac{3}{\sqrt{5}}$	$\frac{3\sqrt{3}}{5}$	$\frac{12\sqrt{3}}{5}$
	$B_s^0 \to \pi^0 \bar{K}^0$	0	$-\frac{\sqrt{3}}{\sqrt{10}}$	$\frac{1}{\sqrt{10}}$	$-\frac{\sqrt{3}}{5\sqrt{2}}$	$\frac{3\sqrt{6}}{5}$	0	$-\frac{\sqrt{3}}{\sqrt{10}}$	$-\frac{3}{\sqrt{10}}$	$-\frac{3\sqrt{3}}{5\sqrt{2}}$	$\frac{9\sqrt{6}}{5}$
	$B_s^0 \to \eta_8 \bar{K}^0$	0	$-\frac{1}{\sqrt{10}}$	$\frac{1}{\sqrt{30}}$	$-\frac{1}{5\sqrt{2}}$	$\frac{3\sqrt{2}}{5}$	0	$-\frac{1}{\sqrt{10}}$	$-\frac{\sqrt{3}}{\sqrt{10}}$	$-\frac{3}{5\sqrt{2}}$	$\frac{9\sqrt{2}}{5}$

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Decays		$\lambda_u^{(q)}$						$\lambda_t^{(q)}$							
		T	C	P_{uc}	A	PA_{uc}	E	P_{tc}	PA_{tc}	P_{EW}^{T}	P_{EW}^{C}	P_{EW}^{A}	P_{EW}^{E}	$P_{EW}^{P_u}$	$P_{EW}^{PA_u}$
$\Delta S = 0$	$B^+ \to \bar{K}^0 K^+$	0	0	1	1	0	0	1	0	0	$-\frac{1}{3}$	0	$\frac{2}{3}$	$-\frac{1}{3}$	0
	$B^+ \to \pi^0 \pi^+$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0	0	0	0	0	0	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0	0	0	0
	$B^+ \to \eta_8 \pi^+$	$-\frac{1}{\sqrt{6}}$	$-\frac{1}{\sqrt{6}}$	$-\frac{2}{\sqrt{6}}$	$-\frac{2}{\sqrt{6}}$	0	0	$-\frac{2}{\sqrt{6}}$	0	$-\frac{1}{\sqrt{6}}$	$-\frac{1}{3\sqrt{6}}$	0	$-\frac{4}{3\sqrt{6}}$	$\frac{2}{3\sqrt{6}}$	0
	$B^0 \to K^0 \bar{K}^0$	0	0	1	0	1	0	1	1	0	$-\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
	$B^0 \to \pi^+ \pi^-$	-1	0	-1	0	-1	-1	-1	-1	0	$-\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
	$B^0 \to K^+ K^-$	0	0	0	0	-1	-1	0	-1	0	0	$-\frac{1}{3}$	0	0	$\frac{1}{3}$
	$B^0 \to \pi^0 \pi^0$	0	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$
	$B^0 \to \pi^0 \eta_8$	0	0	$-\frac{1}{\sqrt{3}}$	0	0	$\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	0	0	$\frac{1}{3\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{3\sqrt{3}}$	$\frac{1}{3\sqrt{3}}$	0
	$B^0 \to \eta_8 \eta_8$	0	$\frac{1}{3\sqrt{2}}$	$\frac{1}{3\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$	$\frac{1}{3\sqrt{2}}$	$\frac{1}{3\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{9\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	$-\frac{1}{9\sqrt{2}}$	$-\frac{1}{9\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$
	$B_s^0 \to \pi^+ K^-$	-1	0	-1	0	0	0	-1	0	0	$-\frac{2}{3}$	0	$\frac{1}{3}$	$\frac{1}{3}$	0
	$B_s^0 \to \pi^0 \bar{K}^0$	0	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0	0	0	$\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	0	$-\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	0
	$B_s^0 \to \eta_8 \bar{K}^0$	0	$-\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$	0	0	0	$\frac{1}{\sqrt{6}}$	0	$-\frac{1}{\sqrt{6}}$	$-\frac{1}{3\sqrt{6}}$	0	$-\frac{1}{3\sqrt{6}}$	$-\frac{1}{3\sqrt{6}}$	0

All available $B \to PP$ data by transition

17 $b \to d$ ($\Delta S = 0$) decays

Decay	$\mathcal{B}_{\rm CP}$	$C_{\rm CP}$	$S_{\rm CP}$
$B^+ \to K^+\overline{K}^0$	✓	✓	
$B^+ \to \pi^+\pi^0$	✓	✓	
$B^0 \to \pi^+\pi^-$	✓	✓	✓
$B^0 \to \pi^0\pi^0$	✓	✓	
$B^0 \to K^0\overline{K}^0$	✓	✓	✓
$B^0 \to K^+K^-$	✓		
$B_s^0 \to \pi^+K^-$	✓	✓	
$B_s^0 \to \pi^0\overline{K}^0$			
$B^+ \to \pi^+\eta^{(\prime)}$	✓	✓	
$B^0 \to \pi^0\eta^{(\prime)}$	✓		
$B^0 \to \eta^{(\prime)}\eta^{(\prime)}$	✓$^{()}$ ✓$''$		
$B_s^0 \to \overline{K}^0\eta^{(\prime)}$			

7 decays involve singlets

17 $b \to s$ ($\Delta S = 1$) decays

Decay	$\mathcal{B}_{\rm CP}$	$C_{\rm CP}$	$S_{\rm CP}$
$B^+ \to \pi^+K^0$	✓	✓	
$B^+ \to \pi^0K^+$	✓	✓	
$B^0 \to \pi^-K^+$	✓	✓	
$B^0 \to \pi^0K^0$	✓	✓	✓
$B_s^0 \to K^+K^-$	✓	✓	✓
$B_s^0 \to K^0\overline{K}^0$	✓		
$B_s^0 \to \pi^+\pi^-$	✓		
$B_s^0 \to \pi^0\pi^0$	✓		
$B^+ \to K^+\eta^{(\prime)}$	✓	✓	
$B^0 \to K^0\eta^{(\prime)}$	✓	✓$'$	✓$'$
$B_s^0 \to \pi^0\eta^{(\prime)}$			
$B_s^0 \to \eta^{(\prime)}\eta^{(\prime)}$	✓		

7 decays involve singlets

$B \to PP$ data excluding η, η'

- $\Delta S = 0$: $\bar{b} \to \bar{d}$ transitions
- 15 measurements available
- 7 RMEs $\to$ 13 hadronic parameters
- $\chi^2_{\rm min}/{\rm dof} = 1.1/2$; $p \sim 0.6$ good fit

Decay	$\mathcal{B}_{\rm CP}$	$C_{\rm CP}$	$S_{\rm CP}$
$B^+ \to K^+ \overline{K}^0$	✓	✓	
$B^+ \to \pi^+ \pi^0$	✓	✓	
$B^0 \to K^0 \overline{K}^0$	✓	✓	✓
$B^0 \to \pi^+ \pi^-$	✓	✓	✓
$B^0 \to \pi^0 \pi^0$	✓	✓	?
$B^0 \to K^+ K^-$	✓	?	?
$B_s^0 \to \pi^+ K^-$	✓	✓	
$B_s^0 \to \pi^0 \overline{K}^0$	?	?	?

- $\Delta S = 1$: $\bar{b} \to \bar{s}$ transitions
- 15 measurements available
- 7 RMEs $\to$ 13 hadronic parameters
- $\chi^2_{\rm min}/{\rm dof} = 1.6/2$; $p \sim 0.4$ good fit

Decay	$\mathcal{B}_{\rm CP}$	$C_{\rm CP}$	$S_{\rm CP}$
$B^+ \to \pi^+ K^0$	✓	✓	
$B^+ \to \pi^0 K^+$	✓	✓	
$B^0 \to \pi^- K^+$	✓	✓	
$B^0 \to \pi^0 K^0$	✓	✓	✓
$B_s^0 \to K^+ K^-$	✓	✓	✓
$B_s^0 \to K^0 \overline{K}^0$	✓	?	?
$B_s^0 \to \pi^+ \pi^-$	✓	?	?
$B_s^0 \to \pi^0 \pi^0$	✓	?	?

dof = #(observables) − #(real parameters)

Combined $B \to PP$ data: the anomaly *(see Alex Jean's talk)*

- In 2311.18011 (PRL): fit the entire set of $B \to PP$ data
- 30 observables, 26 parameters: fit gives $|T_s/T_d| = 12 \pm 4$
- SU(3) hypothesis: 30 observables, 13 parameters: fit gives $\chi^2_{\rm min}/{\rm dof} \sim 43/17$ (3.5σ)
- Fit with QCDf-inspired constraint $|C/T| = 0.2$
  - $\to$ $\Delta S = 1$ fit: $\chi^2_{\rm min}/{\rm dof} \sim 7/3$, $p \sim 0.07$
  - $\to$ $\Delta S = 0$ fit: $\chi^2_{\rm min}/{\rm dof} \sim 19/3$, $p \sim 3 \times 10^{-4}$ or 3.6σ away from SM SU(3)$_F$
  - $\to$ Combined fit: $\chi^2_{\rm min}/{\rm dof} \sim 56/18$, $p \sim 9 \times 10^{-6}$ or 4.4σ away from SM SU(3)$_F$
- Both fits find deviations in $B^0_s \to K^+K^-$ observables
- Deviations also in $B^+ \to \pi^0 K^+, B^0 \to \pi^- K^+, \pi^0 K^0, K^0\overline{K}^0$

The anomaly using RMEs

RMEs	$\chi^2/\text{d.o.f.} = 1.1/2$	$\chi^2/\text{d.o.f.} = 1.6/2$	$\chi^2/\text{d.o.f.} = 43/17$
	$\Delta S = 0$	$\Delta S = 1$	Full fit
$\lvert A_1 \rvert$	6.3 ± 0.7	72 ± 9	5.1 ± 0.6
$\lvert A_8 \rvert$	4.65 ± 0.25	44 ± 4	2.71 ± 0.20
$\lvert R_8 \rvert$	2.63 ± 0.25	2.50 ± 0.24	2.83 ± 0.14
$\lvert P_8 \rvert$	1.00 ± 0.23	1.06 ± 0.07	1.528 ± 0.010
$\lvert P_{27} \rvert$	3.3 ± 0.5	64 ± 4	3.89 ± 0.34
$\lvert B_1 \rvert$	0.65 ± 0.26	9.8 ± 3.3	0.60 ± 0.23
$\lvert B_8 \rvert$	2.62 ± 0.06	1.2 ± 0.4	2.61 ± 0.06

A factor of 10 breaking in parameters: $|B_1|, |A_1|, |A_8|, |P_{27}|$

(Table courtesy: Marianne Bouchard (UdeM))

The anomaly including η, η'

- In 2505.11492 (PRD): also include η, η' final states
- Final state: $|P_8\rangle$ contains (3 πs, 4 Ks, and η_8) + $|P_1\rangle$ (singlet part of ηs)
- More measurables but also additional RMEs: $\langle B| H |P_8 P_1\rangle$ (4 new), $\langle B| H |P_1 P_1\rangle$ (2 new)
- Independent RMEs: $V_{ub}^* V_{us} \quad \to \quad 5+4=9, \quad V_{tb}^* V_{ts} \quad \to \quad 2+2=4$
- Each RME is a complex number: 13 independent RMEs = 25 real parameters

Include η, η' in the final state

- $\Delta S = 0$: $\bar{b} \to \bar{d}$ transitions
- 15 + 6 measurements available
- 11 RMEs $\to$ 21 hadronic parameters
- $\chi^2_{\rm min} = 1.1$

Decay	$\mathcal{B}_{\rm CP}$	$C_{\rm CP}$	$S_{\rm CP}$
$B^+ \to \pi^+\eta$	✓	✓	
$B^+ \to \pi^+\eta'$	✓	✓	
$B^0 \to \pi^0\eta$	✓		
$B^0 \to \pi^0\eta'$	✓		
$B_s^0 \to \overline{K}^0\eta$			
$B_s^0 \to \overline{K}^0\eta'$			

- $\Delta S = 1$: $\bar{b} \to \bar{s}$ transitions
- 15 + 8 measurements available
- 11 RMEs $\to$ 21 hadronic parameters
- $\chi^2_{\rm min}/{\rm dof} = 1.5/2$; $p \sim 0.5$ good fit

Decay	$\mathcal{B}_{\rm CP}$	$C_{\rm CP}$	$S_{\rm CP}$
$B^+ \to K^+\eta$	✓	✓	
$B^+ \to K^+\eta'$	✓	✓	
$B^0 \to K^0\eta$	✓		
$B^0 \to K^0\eta'$	✓	✓	✓
$B_s^0 \to \pi^0\eta$			
$B_s^0 \to \pi^0\eta'$			

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$B^0 \to \pi^0\eta$	✓		
$B^0 \to \pi^0\eta'$	✓		
$B^0_s \to \overline{K}^0\eta$			
$B^0_s \to \overline{K}^0\eta'$			

- Other data: $\mathcal{B}_{\rm CP}$ in $B^0 \to \eta\eta, \eta'\eta'$
- + 2 data; + 2 RMEs; $\chi^2_{\rm min} = 1.1$

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$B^+ \to K^+\eta'$	✓	✓	
$B^0 \to K^0\eta$	✓		
$B^0 \to K^0\eta'$	✓	✓	✓
$B^0_s \to \pi^0\eta$			
$B^0_s \to \pi^0\eta'$			

- Other data: $\mathcal{B}_{\rm CP}$ in $B^0_s \to \eta\eta, \eta\eta', \eta'\eta'$
- + 3 data; + 2 RMEs; $\chi^2_{\rm min}/{\rm dof} = 2.4/1$

Joint $\Delta S = 0$ and $\Delta S = 1$ fit results

- $\chi^2_{\rm min}$ values from fits

Type	$\chi^2_{\rm min}/{\rm dof}$ (p-value)		
	$\Delta S = 0$	$\Delta S = 1$	Combined
Without η, η'	1.1/2 (0.6)	1.5/2 (0.5)	43/17 (5×10^{-4})
One η or η'	1.1	1.5/2 (0.5)	56/23 (1×10^{-4})
Including η and η'	1.1	2.4/1 (0.1)	61/24 (5×10^{-5})

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- Ratios of $\Delta S = 1$ and $\Delta S = 0$ diagrams $|D'/D|$

Type	$\|T'/T\|$	$\|C'/C\|$	full fit $\|C/T\|$
Without η, η'	13$\pm$2	9$\pm$2	0.9$\pm$0.2
One η or η'	13$\pm$2	9$\pm$2	0.9$\pm$0.2
Including η and η'	13$\pm$2	9$\pm$2	0.8$\pm$0.2

$|C/T| \sim 0.2$ expected
$|D'/D| = 1$ in SU(3)$_F$
$|D'/D| = 2 \Rightarrow 100\%$
SU(3) breaking

Joint $\Delta S = 0$ and $\Delta S = 1$ fit results

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$|C/T| \sim 0.2$ expected
$|D'/D| = 1$ in SU(3)$_F$
$|D'/D| = 2 \Rightarrow 100\%$
SU(3) breaking

- $\sim 1000\%$ SU(3) breaking observed

Effect of the EWP-tree relations

- SU(3) is broken, but not a 1000% breaking (Burgos Marcos et al., 2025)
- EWP-tree relations reduce the number of parameters
- 7 complex parameters with 3 EWP-tree relations
- Without EWP-tree relations there are 3 additional complex parameters
- Fits including additional parameters: $\chi^2_{\rm min} \sim 32/15(p = 6 \times 10^{-3})$

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- Fits including additional parameters: $\chi^2_{\rm min} \sim 32/15 (p = 6 \times 10^{-3})$
- Compare with fits including additional EWPs:

	$\lvert P_{EW}^{T}\rvert_{\rm fit}$	$\lvert P_{EW}^{C}\rvert_{\rm fit}$	$\lvert P_{EW}^{A}\rvert_{\rm fit}$
fit values	2.33$\pm$0.02	2.37$\pm$0.02	2.20$\pm$0.12
expected values	0.067$\pm$0.004	0.059$\pm$0.005	0.000$\pm$0.001

- EWPs are too large when floated

Origin of the EWP-tree relations

- Tree operators and EWP operators are related: $\mathcal{O}_{1,2}$(tree),$\mathcal{O}_{3-6}$(penguin),$\mathcal{O}_{9-10}$(EWP)

  $\mathcal{O}_9 = (3/2)\mathcal{O}_2 - (1/2)\mathcal{O}_3 \qquad \mathcal{O}_{10} = (3/2)\mathcal{O}_1 - (1/2)\mathcal{O}_4$

- Tree and EWP RMEs are related if penguins don't contribute to an RME
- SU(3) $\rightarrow$ Tree and EWP have $\mathbf{3}^*, \mathbf{6}, \mathbf{15}^*$, Penguins have $\mathbf{3}^*$
- So Tree and EWP RMEs for $\mathbf{6}$ and $\mathbf{15}^*$ are related

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- So Tree and EWP RMEs for $\mathbf{6}$ and $\mathbf{15}^*$ are related

- Apply this to isospin: 2510.13969

- $\bar{b} \to \underbrace{\bar{u}u\bar{d}}_{\mathbf{2}\times\mathbf{2}\times\mathbf{2}}$ and $\bar{b} \to \underbrace{\bar{u}u\bar{s}}_{\mathbf{2}\times\mathbf{2}\times\mathbf{1}}$ are different under isospin

  So expect different isospin EWP-tree relations!

Application to the $B \to \pi\pi$ isospin triangle & the CKM angle α

- $\bar{b} \to \bar{u}u\bar{d}$ transforms as $\mathbf{2} \times \mathbf{2} \times \mathbf{2} = \mathbf{2} + \mathbf{2} + \mathbf{4}$ ($\mathbf{2} \to I = 1/2$ and $\mathbf{4} \to I = 3/2$)
- 3 decays: $\sqrt{2}A(B^+ \to \pi^+\pi^0) = A(B^0 \to \pi^+\pi^-) + \sqrt{2}A(B^0 \to \pi^0\pi^0)$
- 2 RMEs: $A_{3/2} = \langle 2| \mathcal{O}_{3/2} |1/2\rangle$ and $A_{1/2} = \langle 0| \mathcal{O}_{1/2} |1/2\rangle$
- 6 observables depend on 3 diagrams (5 parameters) + α (Gronau & London, 1990)

  EWPs are suppressed, so no direct CP Asymmetry in $B^+ \to \pi^+\pi^0$

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  EWPs are suppressed, so no direct CP Asymmetry in $B^+ \to \pi^+\pi^0$
- No contributions to $A_{3/2}$ from penguins $\Rightarrow A^t_{3/2} \propto A^u_{3/2}$

  $\to$ EWP-tree relation (Neubert & Rosner, 1998)
- $A(B^+ \to \pi^+\pi^0) \propto V^*_{ub}V_{ud}A^u_{3/2} + V^*_{tb}V_{td}A^t_{3/2} \quad \to \delta$(strong phase) = 0
- Even in the presence of EWPs, under isospin, no direct CP-asymmetry in $B^+ \to \pi^+\pi^0$
- $\pi\pi$ isospin analysis works even when EWPs are included due to ***isospin*** EWP-tree relations

The $B \to K\pi$ system

- $\bar{b} \to \bar{u}u\bar{s}$ transforms as $\mathbf{2} \times \mathbf{2} = \mathbf{1} + \mathbf{3}$ ($\mathbf{1} \to I = 0$ and $\mathbf{3} \to I = 1$)
- 4 decays: $B^+ \to K^0\pi^+$, $B^+ \to K^+\pi^0$, $B^0 \to K^+\pi^-$, $B^0 \to K^0\pi^0$; 1 isospin quadrangle
- 3 RMEs: $A_{0,1/2} = \langle 1/2| \mathcal{O}_0 |1/2\rangle$, $A_{1,1/2} = \langle 1/2| \mathcal{O}_1 |1/2\rangle$, $A_{1,3/2} = \langle 3/2| \mathcal{O}_1 |1/2\rangle$
- 9 observables (4 $\times\mathcal{B}$, 4 $\times\mathcal{A}_{\rm CP}$, $S_{\rm CP}$ in $\pi^0 K^0$); 6 complex = 11 real parameters

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- 2 EWP-tree relations from SU(3): $P_{EW} \propto T, P_{EW}^C \propto C$ $\Rightarrow$ 7 independent parameters
- Many fits in the literature: $|C/T|$ large, new physics? $B \to K\pi$ puzzle

The $B \to K\pi$ system

- $\bar{b} \to \bar{u}u\bar{s}$ transforms as $\mathbf{2} \times \mathbf{2} = \mathbf{1} + \mathbf{3}$ ($\mathbf{1} \to I = 0$ and $\mathbf{3} \to I = 1$)
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- Many fits in the literature: $|C/T|$ large, new physics? $B \to K\pi$ puzzle
- Relationship connecting observables: Gronau, 2005

$$\delta_{\pi K} = \frac{\mathcal{B}^{+-}\mathcal{A}_{\rm CP}^{+-}}{\Pi^{+-}} + \frac{\mathcal{B}^{0+}\mathcal{A}_{\rm CP}^{0+}}{\Pi^{0+}} - \frac{2\,\mathcal{B}^{+0}\mathcal{A}_{\rm CP}^{+0}}{\Pi^{+0}} - \frac{2\,\mathcal{B}^{00}\mathcal{A}_{\rm CP}^{00}}{\Pi^{00}} \qquad \Pi = \text{phase space}$$

- $\delta_{\pi K} \propto {\rm Im}\left[(P_{EW} + P_{EW}^C)^*(T + C) + (P_{EW}^* C - P_{EW}^{C*} T)\right] \approx 0$

Applying isospin EWP-tree relations to $B \to K\pi$

- Diagrams in SU(3) contain information about all $B \to PP$ decays
- SU(3) EWP-tree relations are derived using information beyond $B \to \pi\pi, \pi K$
- SU(3) gives: $P_{EW} + P_{EW}^C \propto T + C$ (Neubert & Rosner, 1998)
- Isospin symmetry: penguins contribute only to $\mathcal{O}_{I=0}$

  $\Rightarrow$ 2 EWP-tree relations: $\underbrace{A^t_{1,3/2} \propto A^u_{1,3/2}}_{\text{Neubert \& Rosner}}$ and $\underbrace{A^t_{1,1/2} \propto A^u_{1/2,1}}_{\text{new}}$

- Combine the EWP-tree relations: $P_{EW} \propto C$ and $P_{EW}^C \propto T$
- Consequence: $\delta_{\pi K} = 0$ is exact in ***Isospin***

$$\delta_{\pi K} \propto \mathrm{Im}\left[(P_{EW} + P_{EW}^C)^*(T + C) + (P_{EW}^* C - P_{EW}^{C*} T)\right]$$

The effect of isospin EWP-tree relations on $B \to K\pi$ fits

	$SU(3)_F$ EWP-tree relations $\chi^2_{\rm min}/\text{d.o.f.} = 0.98/2$ p-value $= 0.61$	$B \to \pi K$ $SU(2)_I$ EWP-tree relations $\chi^2_{\rm min}/\text{d.o.f.} = 2.9/2$ p-value $= 0.24$
Parameter	Best-fit value	Best-fit value
$\lvert T\rvert$	59 ± 5	33 ± 13
$\lvert C\rvert$	62 ± 5	40 ± 10
$\lvert P_{uc}\rvert$	65.1 ± 1.0	38 ± 5
$\lvert P_{tc}\rvert$	0.66 ± 0.08	0.80 ± 0.04
δ_C	$(182.7 \pm 1.5)^\circ$	$(180.5 \pm 1.2)^\circ$
$\delta_{P_{uc}}$	$(182.43 \pm 0.19)^\circ$	$(175.6 \pm 1.6)^\circ$
$\delta_{P_{tc}}$	$(182.4 \pm 0.6)^\circ$	$(356.3 \pm 2.0)^\circ$

The effect of isospin EWP-tree relations on $B \to K\pi$ fits

	$SU(3)_F$ EWP-tree relations $\chi^2_{\rm min}/{\rm d.o.f.} = 15.6/3$ p-value $= 1.3 \times 10^{-3}$	$B \to \pi K$ $SU(2)_I$ EWP-tree relations $\chi^2_{\rm min}/{\rm d.o.f.} = 29.2/3$ p-value $= 2.0 \times 10^{-6}$
Parameter	Best-fit value	Best-fit value
$\|T\|$	10.0 ± 1.1	34 ± 5
$\|P_{uc}\|$	2.2 ± 2.9	2.1 ± 2.5
$\|P_{tc}\|$	1.232 ± 0.027	1.037 ± 0.023
δ_C	$(256.1 \pm 17)^\circ$	$(36 \pm 6)^\circ$
$\delta_{P_{uc}}$	$(395 \pm 18)^\circ$	$(183 \pm 12)^\circ$
$\delta_{P_{tc}}$	$(346.1 \pm 3.4)^\circ$	$(185.2 \pm 0.9)^\circ$

We have assumed: $|C/T| = 0.2$; ***Isospin*** fit is much worse

Application to $B \to VV$

- Time dependent decay amplitudes for $B \to VV$ can be written in terms of $K_n(t)$

$$K_n(t) = \frac{1}{2} e^{-\Gamma t} \left[a_n \cosh{(\Delta\Gamma/2)t} + b_n \sinh{(\Delta\Gamma/2)t} + c_n \cos{\Delta m t} + d_n \sin{\Delta m t} \right]$$

- Each $K_n(t)$ multiplies a specific angular function (see 1306.1911)
- For certain angular modes one finds:

n	$K_n(t)$	a_n	c_n
1	$\lvert A_0(t)\rvert^2$	$\lvert A_0\rvert^2 + \lvert\bar{A}_0\rvert^2$	$\lvert A_0\rvert^2 - \lvert\bar{A}_0\rvert^2$
2	$\lvert A_\parallel(t)\rvert^2$	$\lvert A_\parallel\rvert^2 + \lvert\bar{A}_\parallel\rvert^2$	$\lvert A_\parallel\rvert^2 - \lvert\bar{A}_\parallel\rvert^2$
3	$\lvert A_\perp(t)\rvert^2$	$\lvert A_\perp\rvert^2 + \lvert\bar{A}_\perp\rvert^2$	$\lvert A_\perp\rvert^2 - \lvert\bar{A}_\perp\rvert^2$
4	$\mathrm{Re}[A_\parallel(t)A_0^*(t)]$	$\mathrm{Re}[A_\parallel A_0^* + \bar{A}_\parallel \bar{A}_0^*]$	$\mathrm{Re}[A_\parallel A_0^* - \bar{A}_\parallel \bar{A}_0^*]$
5	$\mathrm{Im}[A_\perp(t)A_0^*(t)]$	$\mathrm{Im}[A_\perp A_0^* - \bar{A}_\perp \bar{A}_0^*]$	$\mathrm{Im}[A_\perp A_0^* + \bar{A}_\perp \bar{A}_0^*]$
6	$\mathrm{Im}[A_\perp(t)A_\parallel^*(t)]$	$\mathrm{Im}[A_\perp A_\parallel^* - \bar{A}_\perp \bar{A}_\parallel^*]$	$\mathrm{Im}[A_\perp A_\parallel^* + \bar{A}_\perp \bar{A}_\parallel^*]$

- $\mathrm{Im}[A_\perp A_\parallel^* - \bar{A}_\perp \bar{A}_\parallel^*]$ is a CP-odd observable
- Compare across all SU(3)-related $B \to VV$ decays $\to$ work in progress

Summary and Outlook

- Hadronic $B \to PP$ decays appear to significantly deviate from SU(3) symmetry
- $\Delta S = 0$ and $Delta S = 1$ fit parameters largely disagree
- EWP-tree relations in SU(3) play a key role
- Isospin must be applied to subsets of decays related by isospin
- New isospin EWP-tree relation found in $B \to K\pi$
- Observable relationship in $B \to K\pi$: exact under isospin
- Isospin fits demonstrate increased tension
- Next steps: SU(3) breaking – *Work in progress*
- Challenges with application of Wigner-Eckart and large number of RMEs
- Future data on additional modes will pave the way

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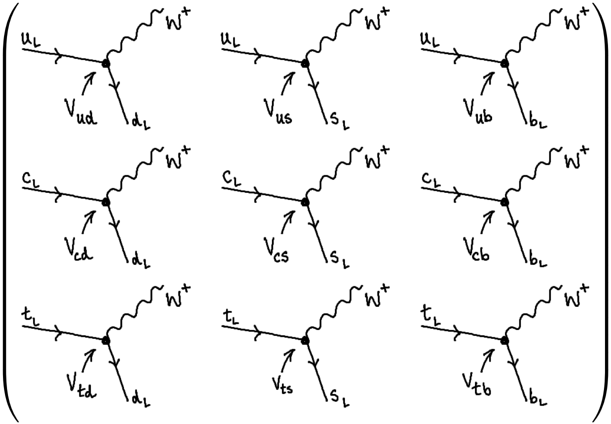
- Postdocs: S. Kumbhakar, I. Ray
- Faculty: D. London (UdeM)
- Support: US National Science Foundation (PHY-2310627)

Back-up Slides

CP Violation in the SM: The CKM paradigm

The Cabibbo-Kobayashi-Maskawa matrix
(1/2 Physics Nobel 2008)



CP Violation in the SM: The CKM paradigm

The Cabibbo-Kobayashi-Maskawa matrix
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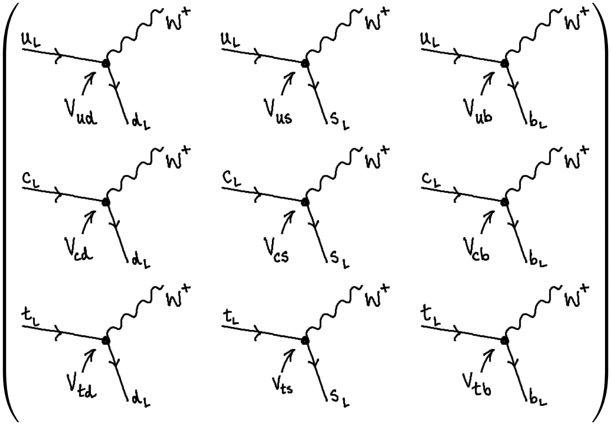

- SM quarks: $Q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}, \; d_R^i, \; u_R^i$
  $i = 1, 2, 3$ represent 3 generations
- $W_\mu^+ \; \left(\overline{u}_L^i \; \gamma^\mu \; V_{ij} \; d_L^j\right)$

CP Violation in the SM: The CKM paradigm

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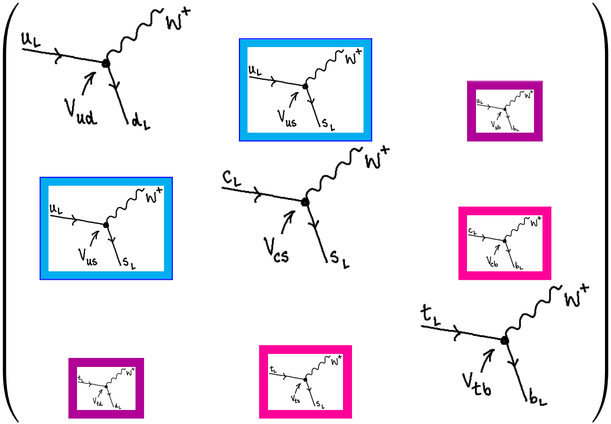

- SM quarks: $Q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}$, d_R^i , u_R^i
  $i = 1, 2, 3$ represent 3 generations
- $W_\mu^+ \; \left(\overline{u}_L^i \; \gamma^\mu \; V_{ij} \; d_L^j\right)$
- $\begin{pmatrix} u_L & c_L & t_L \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$

CP Violation in the SM: The CKM paradigm

The Cabibbo-Kobayashi-Maskawa matrix
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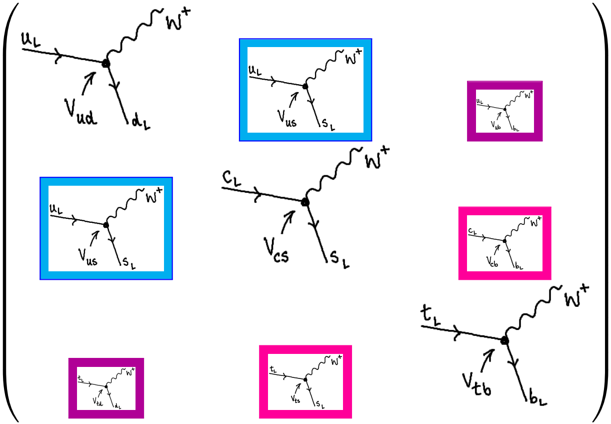

- SM quarks: $Q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}, \; d_R^i, \; u_R^i$
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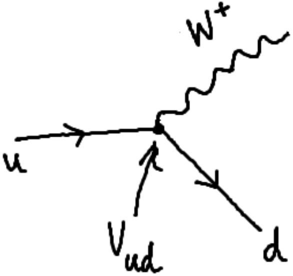
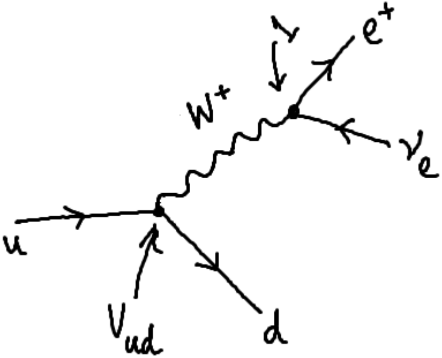
- $W_\mu^+ \; \left(\overline{u}_L^i \; \gamma^\mu \; V_{ij} \; d_L^j\right)$
- $\begin{pmatrix} u_L & c_L & t_L \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$
- Unitary matrix: $V^\dagger V = 1$
- 3×3 unitary matrix: 4 parameters
  3 real angles
  1 complex phase (CPV)

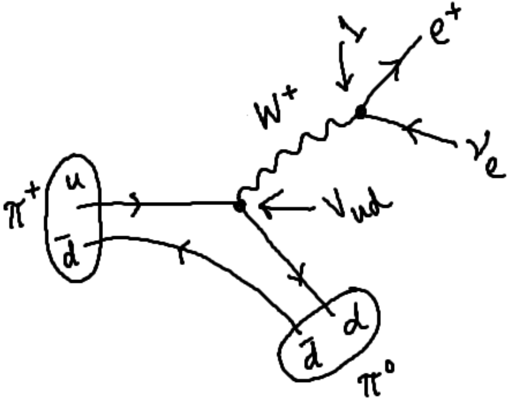
Measuring the CKM matrix



Measuring the CKM matrix

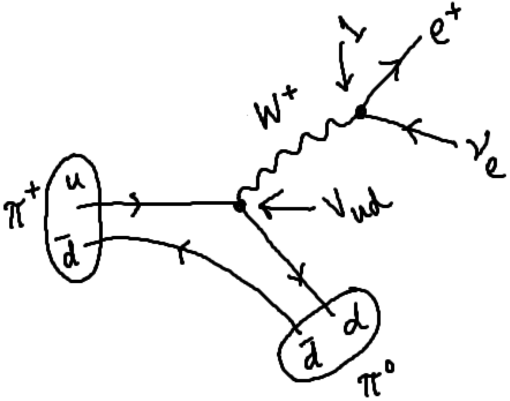


Measuring the CKM matrix

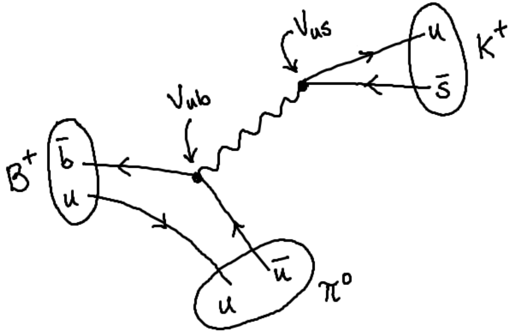


- Decay rate $\propto \quad |V_{ud}|^2$
- Factorizable current: hadron vs lepton
- QCD effects hidden in form factors

Measuring the CKM matrix

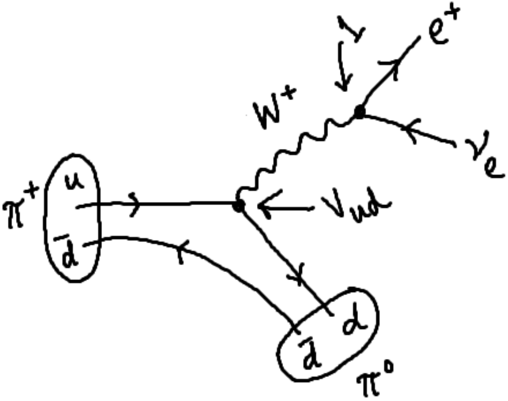


- Phase measurement tricky: need CPV



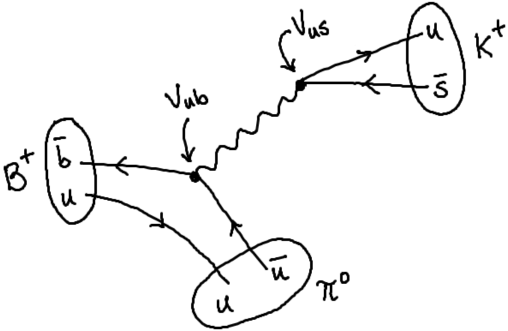
- Decay rate ∝ $|V_{ud}|^2$
- Factorizable current: hadron vs lepton
- QCD effects hidden in form factors

Measuring the CKM matrix



- Decay rate $\propto \quad |V_{ud}|^2$
- Factorizable current: hadron vs lepton
- QCD effects hidden in form factors

- Phase measurement tricky: need CPV
- CPV from amplitude interference
- Strong phases from quark rescattering
- Non-factorizable effects important



$B \to K\pi$: Isospin vs SU(3)

- 4 decays: $B^+ \to K^0\pi^+$, $B^+ \to K^+\pi^0$, $B^0 \to K^+\pi^-$, $B^0 \to K^0\pi^0$;
- Isospin quadrangle relationship: $A^{0+} + \sqrt{2}A^{+0} = A^{-+} + \sqrt{2}A^{00}$
- Isospin relation holds either way (isospin or SU(3)) $\Rightarrow$ 3 independent RMEs
- Isospin Hamiltonian: $\bar{b} \to \bar{u}u\bar{s} \quad \to \quad I = 0$ and $I = 1/2$
- $K\pi$ final state: $1/2 \times 1 = 1/2 + 3/2$
- 3 $(\times 2)$ isospin RMEs: $A^{u,t}_{0,1/2}, \; A^{u,t}_{1,1/2}, \; A^{u,t}_{1,3/2}$
- Penguin operators are iso-singlets $\Rightarrow I = 1$ RMEs get EWP-Tree relations:

  $A^t_{1,1/2} = A^u_{1,1/2}, \quad A^t_{1,3/2} = A^u_{1,3/2}$: 2 relations
- 4 remaining RMEs $\Rightarrow$ 7 real parameters

$B \to K\pi$: Isospin vs SU(3)

- 4 decays: $B^+ \to K^0\pi^+$, $B^+ \to K^+\pi^0$, $B^0 \to K^+\pi^-$, $B^0 \to K^0\pi^0$;
- Isospin quadrangle relationship: $A^{0+} + \sqrt{2}A^{+0} = A^{-+} + \sqrt{2}A^{00}$
- Isospin relation holds either way (isospin or SU(3)) $\Rightarrow$ 3 independent RMEs
- SU(3) Hamiltonian: $\bar{b} \to \bar{u}u\bar{s} \quad \to \quad \mathbf{3}^*_{\mathbf{1}}(A),\ \mathbf{3}^*_{\mathbf{2}}(A),\ \mathbf{6}(R),\ \mathbf{15}^*(P)$
- $K\pi$ final state: $(\mathbf{8}\times\mathbf{8})_{\rm sym} =$ ~~**1**~~ + **8** + **27**
- 4 $(\times 2)$ SU(3) RMEs: $A_8^{u,t},\ R_8^{u,t},\ P_8^{u,t},\ P_{27}^{u,t}$
- Penguin operators are SU(3) triplets $\Rightarrow R,\ P$ RMEs get EWP-Tree relations:

  $R_8^t = R_8^u, \quad P_8^t = P_8^u, \quad P_{27}^t = P_{27}^u$: 3 relations
- 5 remaining RMEs $\Rightarrow$ 9 real parameters $\Rightarrow$ make approximations to go down to 7
- Note: $\mathbf{15}^*(P)$ has both $I = 0$ and $I = 1$ components!
- Note: EWP-tree relations in SU(3) include effects of additional $B \to PP$ decays

Data on $\Delta S = 0$ decays

Decay	$\mathcal{B}_{CP}$ $(\times 10^{-6})$	$C_{\rm CP}$	$S_{\rm CP}$
$B^+ \to K^+ \overline{K}^0$	1.31±0.14	0.04±0.14†	
$B^+ \to \pi^+\pi^0$	5.59±0.31	0.008±0.035	
$B^0 \to K^0\overline{K}^0$	1.21±0.16†	0.06±0.26	-1.08±0.49
$B^0 \to \pi^+\pi^-$	5.15±0.19	0.311± 0.030	-0.666± 0.029
$B^0 \to \pi^0\pi^0$	1.55± 0.16	0.30±0.20	
$B^0 \to K^+K^-$	0.080±0.015		
$B_s^0 \to \pi^+K^-$	$5.90^{+0.87}_{-0.76}$	0.225±0.012	
$B_s^0 \to \pi^0\overline{K}^0$			

† data from the PDG

other data from HFLAV

Data on $\Delta S = 1$ decays

Decay	$\mathcal{B}_{CP}$ $(\times 10^{-6})$	$C_{\rm CP}$	$S_{\rm CP}$
$B^+ \to \pi^+ K^0$	23.52±0.72	-0.016 ± 0.015	
$B^+ \to \pi^0 K^+$	13.20±0.46	0.029±0.012	
$B^0 \to \pi^- K^+$	19.46±0.46	-0.0836 ± 0.0032	
$B^0 \to \pi^0 K^0$	10.06±0.43	-0.01 ± 0.10	0.57±0.17
$B_s^0 \to K^+ K^-$	$26.6^{+3.2}_{-2.7}$	-0.17 ± 0.03	0.14±0.03
$B_s^0 \to K^0 \overline{K}^0$	17.4±3.1		
$B_s^0 \to \pi^+ \pi^-$	$0.72^{+0.11}_{-0.10}$		
$B_s^0 \to \pi^0 \pi^0$	2.8±2.8*		

* data from Belle

other data from HFLAV

Fit results

Fit $\Delta S = 0$	$\lvert\widetilde{T}\rvert$	$\lvert\widetilde{C}\rvert$	$\lvert\widetilde{P}_{uc}\rvert$	$\lvert\widetilde{A}\rvert$
	4.0 ± 0.5	6.6 ± 0.7	3 ± 4	6 ± 5
	$\lvert\widetilde{PA}_{uc}\rvert$	$\lvert P_{tc}\rvert$	$\lvert PA_{tc}\rvert$	
	0.7 ± 0.8	0.8 ± 0.4	0.2 ± 0.4	

Fit $\Delta S = 1$	$\lvert\widetilde{T}'\rvert$	$\lvert\widetilde{C}'\rvert$	$\lvert\widetilde{P}'_{uc}\rvert$	$\lvert\widetilde{A}'\rvert$
	48 ± 14	41 ± 14	48 ± 15	81 ± 28
	$\lvert\widetilde{PA}'_{uc}\rvert$	$\lvert P'_{tc}\rvert$	$\lvert PA'_{tc}\rvert$	
	7 ± 4	0.78 ± 0.16	0.24 ± 0.04	

Fit $SU(3)_F$	$\lvert\widetilde{T}\rvert$	$\lvert\widetilde{C}\rvert$	$\lvert\widetilde{P}_{uc}\rvert$	$\lvert\widetilde{A}\rvert$
	4.7 ± 0.5	5.8 ± 0.6	2.1 ± 0.5	4.2 ± 0.7
	$\lvert\widetilde{PA}_{uc}\rvert$	$\lvert P_{tc}\rvert$	$\lvert PA_{tc}\rvert$	
	0.70 ± 0.09	1.15 ± 0.04	0.214 ± 0.018	

Electro-weak penguin operators

- EWP operators $\propto V_{tb}^* V_{ts}$
- Operators of the type $\mathbf{3}^* \times \mathbf{8} = \mathbf{3}^* + \mathbf{6} + \mathbf{15}^*$
- Reminder: tree operators also have $\mathbf{3}^* + \mathbf{6} + \mathbf{15}^*$
- Resulting RMEs in $\langle B | \, H \, | PP \rangle$ of the type:

  $\langle \mathbf{3}^* | \, \mathbf{6} \, | \mathbf{8} \rangle$, $\langle \mathbf{3}^* | \, \mathbf{15}^* \, | \mathbf{8} \rangle$, and $\langle \mathbf{3}^* | \, \mathbf{15}^* \, | \mathbf{27} \rangle$

  Identical in trees and EWPs
- This is the source of EWP-Tree relations
- Breaking EWP-Tree relations is effectively SU(3) breaking

Weak-phase information from B decays with tree + loop

- $\mathcal{A}(B \to f) = |a| + |b| e^{i\phi} e^{i\delta} \quad \to \quad \Gamma \propto |\mathcal{A}|^2$

  $\overline{\mathcal{A}}(\overline{B} \to \bar{f}) = |a| + |b| e^{-i\phi} e^{i\delta} \quad \to \quad \overline{\Gamma} \propto |\overline{\mathcal{A}}|^2$

  – 4 parameters: 2 magnitudes ($|a|, |b|$), 1 rel. strong phase (δ), 1 rel. weak phase (ϕ)
- 2 Observables: $\mathcal{B}_{\rm CP} = \dfrac{\Gamma + \overline{\Gamma}}{2\Gamma_B}$, $\quad C_{\rm CP} = \dfrac{\Gamma - \overline{\Gamma}}{\Gamma + \overline{\Gamma}}$ (direct CP asymmetry)
- For $B^0 \to f$ with $f = \bar{f}$ additional observable $S_{\rm CP}$ (indirect CP asymmetry)

  B-mixing: $|B\rangle_{\rm mass} = p\,|B\rangle + q\,|\bar{B}\rangle$ with $\lambda = \dfrac{q}{p}\dfrac{\bar{\mathcal{A}}}{\mathcal{A}} \Rightarrow S_f = \dfrac{2{\rm Im}[\lambda]}{1 + |\lambda|^2}$
- Information about q/p comes from $B - \bar{B}$ mixing (independent source)
- For B_s, additional observable $A^{\Delta\Gamma} = \dfrac{-2{\rm Re}[\lambda]}{1 + |\lambda|^2}$ (since $\Delta\Gamma_s$ is sizable)
- $C_{\rm CP} = \dfrac{1 - |\lambda|^2}{1 + |\lambda|^2} \quad \Rightarrow$ Identity: $(C_{\rm CP})^2 + (S_{\rm CP})^2 + (A^{\Delta\Gamma})^2 = 1$ (LHCb)

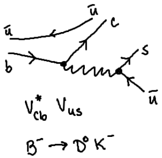
Motivation: Study Weak Interactions

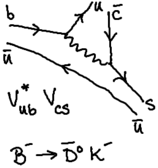
- Direct measurement of γ
  – GLW, ADS, GGSZ methods

- Theoretically clean, but statistics limited

- Theoretically clean $\Rightarrow$
  - more observables than parameters
  - obtain γ from fits
  - ***no theory input for hadronic parameters***

- Limited experimental precision
  - Current $\Delta\gamma \sim 7^\circ$ (LHCb-CONF-2022-003)
  - Long term LHCb target $\Delta\gamma \sim 1 - 2^\circ$





Only tree diagrams interfere in the SM

Highly-suppressed loop (box diagrams)

Brod and Zupan (2013)

- Unitarity triangle: $\gamma = \pi - \alpha - \beta$
- Additional methods? Include loops
- Crosschecks of the CKM paradigm

Weak-phase info using U-spin

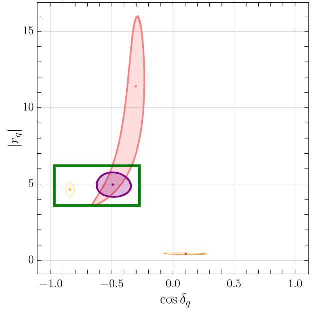
- R. Fleischer, hep-ph/9903456 (Phys. Lett. B 459 (1999) 306)
- 4 observables in $B_s \to K^+K^-$ and $B_d \to \pi^+\pi^-$: $C_{KK}, S_{KK}, C_{\pi\pi}, S_{\pi\pi}$
- $|q/p| \approx 1$ for $B^0_{d,s}$ (can check from semileptonic B decays);
  $\arg(q_s/p_s) \approx 2\beta_s$ $\to$ from $B_s \to J/\Psi\phi$
- Hadronic parameters same for both decays: $(|b/a|, \delta) \leftarrow$ 2 parameters
- Weak decay parameters: $\gamma, \beta_d \leftarrow$ Up to 2 parameters
- $C_{\pi\pi}, C_{KK}, S_{KK}$ sufficient to determine γ + 2 hadronic parameters
- Use $S_{\pi\pi}$ to also get β_d
- Data unavailable at the time

> The strategies proposed in this paper are very interesting for "second-generation" B-physics experiments performed at hadron machines, for example LHCb, where the very

Recent LHCb measurement and theory progress

- LHCb measurement of CP Asymmetries in $B_{s(d)} \to K^+K^-(\pi^+\pi^-)$: 1805.06759, 2012.05319
- Theory investigation of U-spin: Nir, Savoray, and Viernik, 2201.03573
- $C_{KK} = 0.172 \pm 0.031,\ \ S_{KK} = 0.139 \pm 0.032,\ \ C_{\pi\pi} = -0.32 \pm 0.04,\ \ S_{\pi\pi} = -0.64 \pm 0.04$
- Use β_d $(B_d \to J/\Psi K_s)$, β_s $(B_s \to J/\Psi\phi)$, γ $(B \to DK)$
- Find hadronic parameters for both decays $\Rightarrow$ test U-spin
- $\dfrac{|b_s/a_s|}{|b_d/a_d|} = 1.07,\ |a_s/a_d| = 1.26$ $\sim$ 30% U-spin breaking

  $\mathcal{O}(m_s/\Lambda_{\rm QCD}) \sim 30\%,\ f_K/f_\pi - 1 \sim 20\%$
- Result: NP + different orders of breaking at play



Is that it for U-Spin?

- What about other U-spin related decays? BB with others, 2211.06994
- Consider U-spin SU(2) subgroup of flavor SU(3)

  $\rightarrow$ quark doublet: (d, s); $\rightarrow$ antiquark doublet: $(\bar{s}, -\bar{d})$;

  $\rightarrow$ meson doublets: (π^-, K^-), (K^+, π^+), (B_d^0, B_s^0)
- Initial state: B doublet; Final state: Doublet×Doublet = Singlet(0) + Triplet(1)
- 6 decays possible: 3 decays each $\Delta S = 0(b \rightarrow d), 1(b \rightarrow s)$; 4 U-spin RMEs

Decay	Representation	$\mathcal{B}_{\rm CP}$	$C_{\rm CP}$	$S_{\rm CP}$
$B_d^0 \rightarrow \pi^+\pi^-$	$M_{1d}^{1/2} + M_{0d}^{1/2}$	$\sim 10^{-6}$	✓	✓
$B_d^0 \rightarrow K^+K^-$	$M_{1d}^{1/2} - M_{0d}^{1/2}$	$\sim 10^{-8}$	?	?
$B_s^0 \rightarrow \pi^+K^-$	$2\ M_{1d}^{1/2}$	$\sim 10^{-6}$	✓	
$B_s^0 \rightarrow K^+K^-$	$M_{1s}^{1/2} + M_{0s}^{1/2}$	$\sim 10^{-5}$	✓	✓
$B_s^0 \rightarrow \pi^+\pi^-$	$M_{1s}^{1/2} - M_{0s}^{1/2}$	$\sim 10^{-7}$	?	?
$B_d^0 \rightarrow K^+\pi^-$	$2\ M_{1s}^{1/2}$	$\sim 10^{-5}$	✓	

- Each $M_{xq}^{1/2}$ has two parts
- $M_{xq}^{1/2} = V_{ub}^* V_{uq} T_q^x + V_{cb}^* V_{cq} P_q^x$
- 12 measurements
- 4 yet to be measured
- 2 amplitude triangles: $\pi^+\pi^- + K^+K^- = \pi K$

Hints of U-spin breaking

- $\Delta S = 0 \Rightarrow q = d$, $\Delta S = 1 \Rightarrow q = s$

  $\rightarrow$ 7 hadronic parameters $\leftarrow T_q^x, P_q^x$ with $x = 0, 1$

  $\rightarrow$ 6 measurements available ✗

  $\rightarrow$ 2 future measurements $\Rightarrow \gamma$ can be extracted with β_q from independent source
- Apply U-spin! $\Rightarrow$ 8 parameters (γ + 7 hadronic for both $\Delta S = 0, 1$); 12 measurements ✓

Hints of U-spin breaking

- $\Delta S = 0 \Rightarrow q = d$, $\Delta S = 1 \Rightarrow q = s$

  $\rightarrow$ 7 hadronic parameters $\leftarrow T_q^x, P_q^x$ with $x = 0, 1$

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  $\rightarrow$ 2 future measurements $\Rightarrow \gamma$ can be extracted with β_q from independent source

- Apply U-spin! $\Rightarrow$ 8 parameters (γ + 7 hadronic for both $\Delta S = 0, 1$); 12 measurements ✓
- Bad Fit! $\chi^2_{\rm min} = 17.8$ for 4 dof. $\gamma = (67.6 \pm 3.4)^\circ$ close to $\gamma_{\rm direct}$
- U-spin relation(s): $-\dfrac{C^s_{\rm CP}\mathcal{B}^s_{\rm CP}F^d_{\rm PS}}{C^d_{\rm CP}\mathcal{B}^d_{\rm CP}F^s_{\rm PS}} = 1$
- $\mathcal{A}(B_d^0 \to \pi^+\pi^-) \approx \mathcal{A}(B_s^0 \to \pi^+K^-)$
- $\mathcal{A}(B_s^0 \to K^+K^-) \approx \mathcal{A}(B_d^0 \to \pi^-K^+)$

$\Delta S = 0$	$\Delta S = 1$	Relation	
$B_d^0 \to \pi^+\pi^-$	$B_s^0 \to K^+K^-$	2.90±0.69	✓
$B_s^0 \to \pi^+K^-$	$B_d^0 \to \pi^-K^+$	1.21±0.25	✓
$B_s^0 \to \pi^+K^-$	$B_s^0 \to K^+K^-$	3.43±0.91	✗
$B_d^0 \to \pi^+\pi^-$	$B_d^0 \to \pi^-K^+$	1.06±0.42	✗