Data driven bounds on QCDF amplitudes,

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with T. Huber

forthcoming publication

with T. Huber, X.-Q. Li, W.-S. Fang and E. Malami

CPPS, Theoretische Physik 1,
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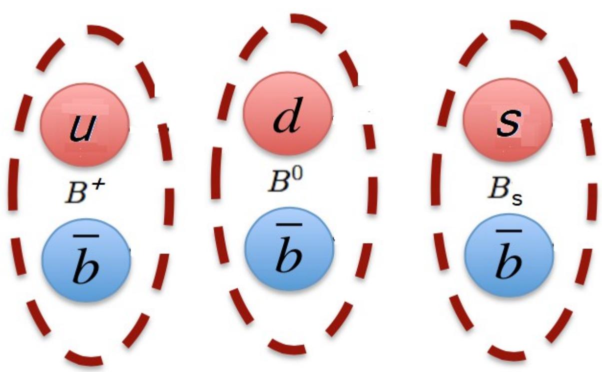
Nonleptonic Decays of Heavy Mesons

Durham, 25th March 2026

Non-leptonic B meson decays

In this talk, we will focus on the description of *B* mesons decays into pairs of light pseudoscalar mesons

$$B \to \pi\pi, \qquad B \to K\pi, \qquad B \to KK, \qquad B \to \eta\eta$$



$$q_i \otimes \overline{q}_j \rightarrow 3 \otimes \overline{3} = 8 \oplus 1$$

$$i, j \in [u, d, s]$$

$$B = (B^+, B_d^0, B_s^0)$$

$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_q}{\sqrt{2}} + \frac{\eta'_q}{\sqrt{2}} & \pi^- & K^- \\ \pi^+ & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_q}{\sqrt{2}} + \frac{\eta'_q}{\sqrt{2}} & \bar{K}^0 \\ K^+ & K^0 & \eta_s + \eta'_s \end{pmatrix},$$

Non-leptonic B meson decays

The decay amplitudes can be decomposed in *two sectors depending on the CKM structure*

$$\mathcal{A}^{\mathrm{TDA}} = i\,\frac{G_F}{\sqrt{2}}\left[\mathcal{T}^{\mathrm{TDA}} + \mathcal{P}^{\mathrm{TDA}}\right],$$

$$\lambda_u^{(D)} \qquad \lambda_t^{(D)}$$

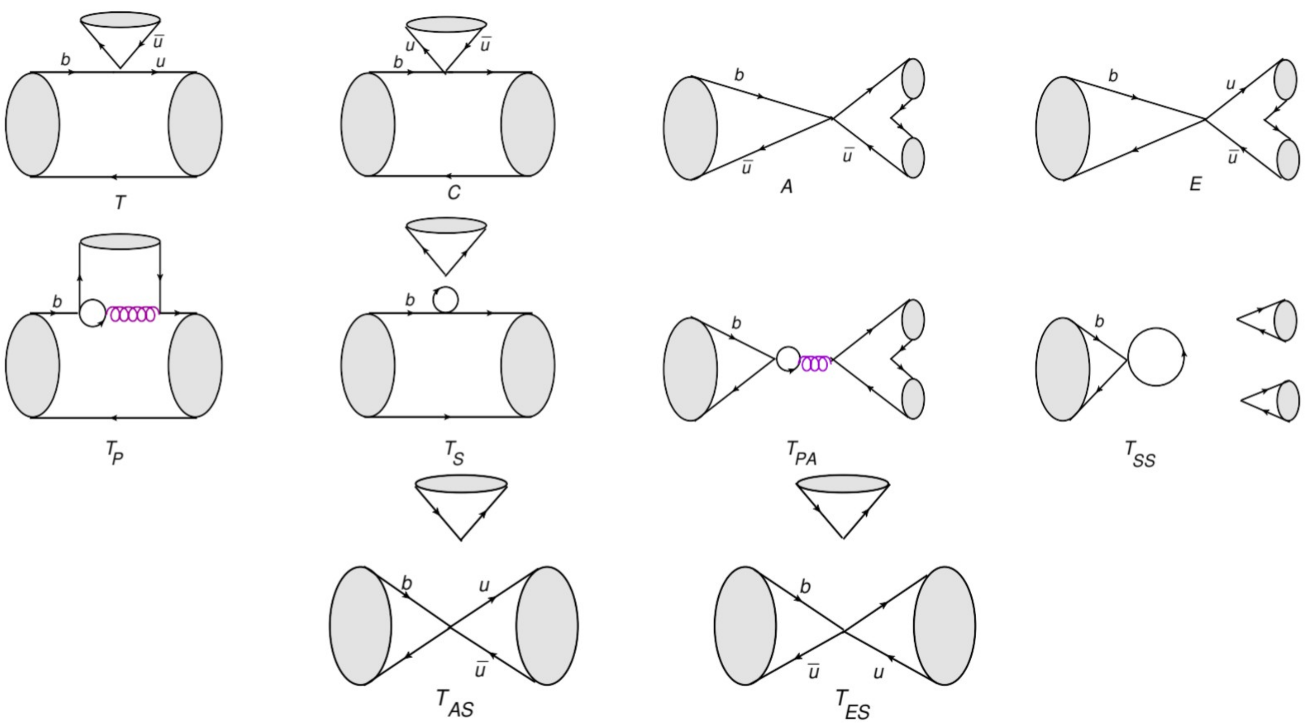
$$\lambda_u^{(D)} = V_{ub}\,V_{uD}^*, \qquad \lambda_t^{(D)} = V_{tb}\,V_{tD}^* \qquad D = d, s$$

$$\lambda_u^{(D)} + \lambda_c^{(D)} + \lambda_t^{(D)} = 0$$

Topological description

$$
\begin{aligned}
\mathcal{T}^{\text{TDA}} = {} & T\, B_i (M_1)^i_j\, \bar{H}^j_k (M_2)^k_l + C\, B_i (M_1)^i_j\, \bar{H}^l_k (M_2)^k_l + A\, B_i\, \bar{H}^l_j (M_1)^j_k (M_2)^k_i \\
& + E\, B_i\, \bar{H}^{li}_j (M_1)^j_k (M_2)^k_l + T_{ES}\, B_i\, \bar{H}^{ij}_l (M_1)^l_j (M_2)^k_k + T_{AS}\, B_i\, \bar{H}^{ji}_l (M_1)^l_j (M_2)^k_k \\
& + T_S\, B_i (M_1)^i_j\, \bar{H}^{lj}_l (M_2)^k_k + T_{PA}\, B_i\, \bar{H}^{li}_l (M_1)^j_k (M_2)^k_j + T_P\, B_i (M_1)^i_j (M_2)^j_k\, \bar{H}^{lk}_l \\
& + T_{SS}\, B_i\, \bar{H}^{li}_l (M_1)^j_j (M_2)^k_k + (M_1 \leftrightarrow M_2)\,,
\end{aligned}
$$

There is a diagrammatic correspondence for each amplitude



Topological description

$$\mathcal{P}^{\rm TDA} = P\, B_i (M_1)^i_j (M_2)^j_k\, \tilde{H}^k + P_T\, B_i (M_1)^i_j\, \tilde{H}^{jl}_k (M_2)^k_l + S\, B_i (M_1)^i_j\, \tilde{H}^j (M_2)^k_k$$
$$+ P_C\, B_i (M_1)^i_j\, \tilde{H}^{lj}_k (M_2)^k_l + P_{TA}\, B_i\, \tilde{H}^{il}_j (M_1)^j_k (M_2)^k_l + P_A\, B_i\, \tilde{H}^i (M_1)^j_k (M_2)^k_j$$
$$+ P_{TE}\, B_i\, \tilde{H}^{ji}_k (M_1)^k_l (M_2)^l_j + P_{AS}\, B_i\, \tilde{H}^{ji}_l (M_1)^l_j (M_2)^k_k + P_{SS}\, B_i\, \tilde{H}^i (M_1)^j_j (M_2)^k_k$$
$$+ P_{ES}\, B_i\, \tilde{H}^{ij}_l (M_1)^l_j (M_2)^k_k + (M_1 \leftrightarrow M_2)$$

$$\bar{H}^{12}_1 = \lambda_u^{(d)}, \quad \bar{H}^{13}_1 = \lambda_u^{(s)}, \qquad \bar{H}^2 = \lambda_u^{(d)}, \quad \bar{H}^3 = \lambda_u^{(s)},$$
$$\tilde{H}^{12}_1 = -\lambda_t^{(d)}, \quad \tilde{H}^{13}_1 = -\lambda_t^{(s)}, \qquad \tilde{H}^2 = -\lambda_t^{(d)}, \quad \tilde{H}^3 = -\lambda_t^{(s)},$$

The flavour structure allows a systematic decomposition of the physical decay amplitudes.

$$\{T, C, A, E, T_{ES}, T_{AS}, T_S, T_{PA}, T_P, T_{SS}\}$$

The subamplitudes

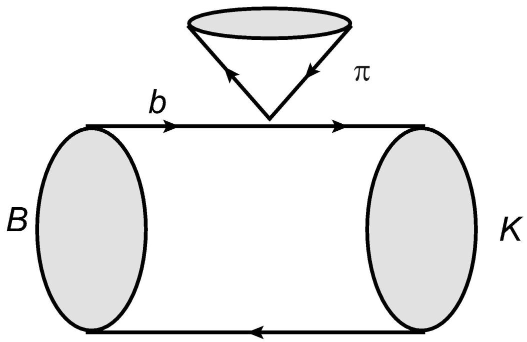
$$\{P,\ P_T,\ S,\ P_C,\ P_{TA},\ P_A,\ P_{TE},\ P_{AS},\ P_{SS},\ P_{ES}\}$$

can then be obtained from fits to data

He and Wang 1803.04227

Dynamical approach

Consider $B \to K\pi$



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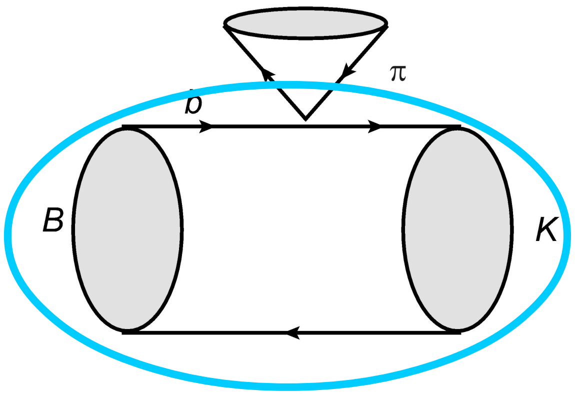

Naive factorization

$$\langle K\pi | Q | B \rangle \sim F_{B\to K} f_{\pi}$$

Dynamical approach

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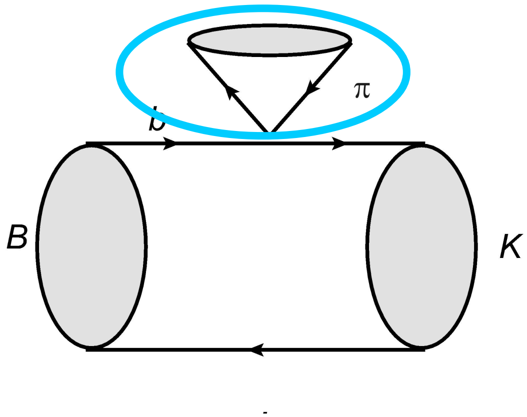

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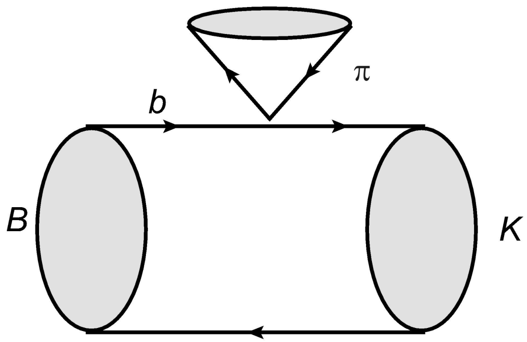

$$\langle K\pi | Q | B \rangle \sim F_{B\to K} f_{\pi}$$

A well accepted formalism to describe these processes theoretically is

QCD-factorization:

Beneke, Buchalla, Neubert, Sachrajda, 9905312

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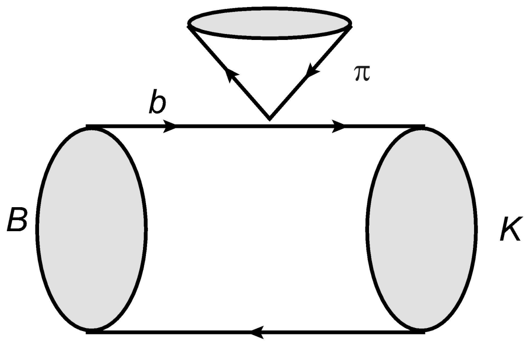

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$$\langle M_1 M_2|\hat{Q}_i|B\rangle = \sum_j F_j^{B\to M_1}(0)\int_0^1 du T_{ij}^I(u)\Phi_{M_2}(u) + (M_1 \leftrightarrow M_2)$$
$$+\int_0^1 d\xi du dv T_i^{II}(\xi,u,v)\Phi_B(\xi)\Phi_{M_1}(v)\Phi_{M_2}(u).$$

Dynamical approach

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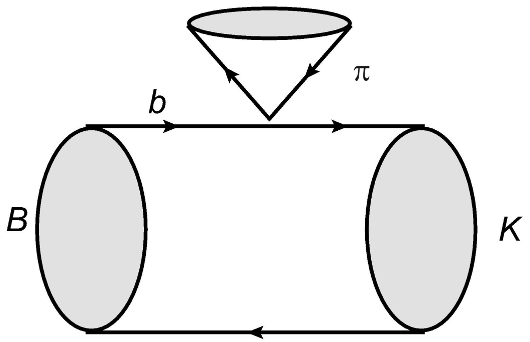

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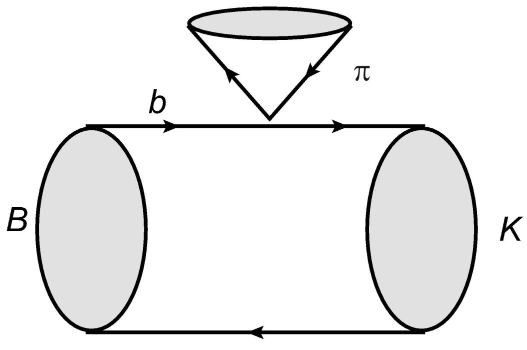

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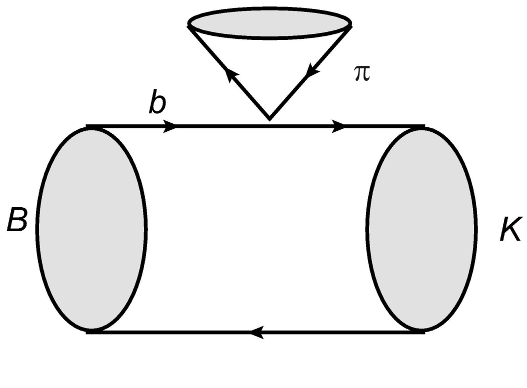
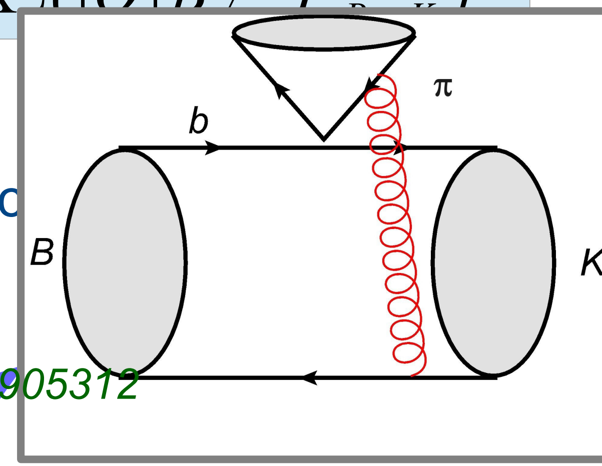
$$+ \int_0^1 d\xi du dv T_i^{II}(\xi, u, v) \Phi_B(\xi) \Phi_{M_1}(v) \Phi_{M_2}(u).$$

Dynamical approach

Consider $B \to K\pi$



$\langle K\pi | Q | B\rangle \sim F_{B\to K} f$



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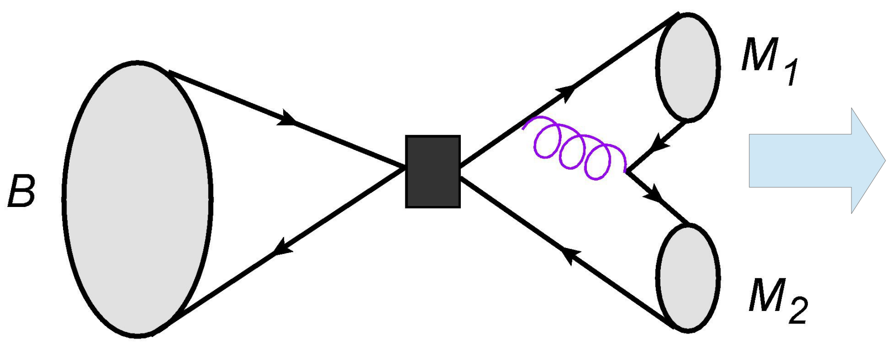
$$\langle M_1 M_2 | \hat{Q}_i | B \rangle = \sum_j F_j^{B\to M_1}(0) \int_0^1 du T_{ij}^I(u) \Phi_{M_2}(u) + (M_1 \leftrightarrow M_2)$$

$$+ \int_0^1 d\xi du dv T_i^{II}(\xi, u, v) \Phi_B(\xi) \Phi_{M_1}(v) \Phi_{M_2}(u).$$

Dynamical approach

Unfortunately, most decay channels involve (power suppressed) contributions which lead to large uncertainties.

A very well known example of this are annihilation topologies which are affected by LCDA end point singularities



β, b $$\int_0^1 \frac{dx}{x} \to \int_{\Lambda_h/m_B}^1 \frac{dx}{x} = \ln\frac{m_B}{\Lambda_h},$$

the annihilation sub-amplitudes are ubiquitous to most of the QCDF amplitudes, consider for instance

$$\mathcal{A}_{\bar{B}_s\to\pi^+\pi^-} = B_{\pi\pi}\left[b_4^p - \tfrac{1}{2}b_{4,\mathrm{EW}}^p\right] + B_{\pi\pi}\left[\delta_{pu}b_1 + b_4^p + b_{4,\mathrm{EW}}^p\right]$$

Uncertainties can be large, of order 100%

$$\mathcal{B}(B_s \to \pi^+\pi^-) = (0.024^{+0.160}_{-0.024}) \times 10^{-6}$$

Dynamical/Topological approach

Decay amplitudes in QCDF can be obtained with the ***magic formula***

$$\begin{aligned}
\mathcal{A}^{\rm QCDF} =& i\frac{G_F}{\sqrt{2}}\sum_{p=u,c} A_{M_1M_2}\Big\{BM_1\left(\alpha_1\delta_{pu}\hat{U}_p+\alpha_4^p\hat{I}+\alpha_{4,\rm EW}^p\hat{Q}\right)M_2\Lambda_p \\
&+BM_1\Lambda_p\cdot{\rm Tr}\left[\left(\alpha_2\delta_{pu}\hat{U}_p+\alpha_3^p\hat{I}+\alpha_{3,\rm EW}^p\hat{Q}\right)M_2\right] \\
&+B\left(\beta_2\delta_{pu}\hat{U}_p+\beta_3^p\hat{I}+\beta_{3,\rm EW}^p\hat{Q}\right)M_1M_2\Lambda_p \\
&+B\Lambda_p\cdot{\rm Tr}\left[\left(\beta_1\delta_{pu}\hat{U}_p+\beta_4^p\hat{I}+\beta_{4,\rm EW}^p\hat{Q}\right)M_1M_2\right] \\
&+B\left(\beta_{S2}\delta_{pu}\hat{U}_p+\beta_{S3}^p\hat{I}+\beta_{S3,\rm EW}^p\hat{Q}\right)M_1\Lambda_p\cdot{\rm Tr}\,M_2 \\
&+B\Lambda_p\cdot{\rm Tr}\left[\left(\beta_{S1}\delta_{pu}\hat{U}_p+\beta_{S4}^p\hat{I}+\beta_{S4,\rm EW}^p\hat{Q}\right)M_1\right]\cdot{\rm Tr}\,M_2\Big\}+(M_1\leftrightarrow M_2)\,,
\end{aligned}$$

$$\Lambda_p=\begin{pmatrix}0\\ \lambda_p^{(d)}\\ \lambda_p^{(s)}\end{pmatrix},\qquad \hat{U}=\begin{pmatrix}1&0&0\\0&0&0\\0&0&0\end{pmatrix},$$

$$\hat{Q}=\frac{3}{2}Q=\begin{pmatrix}1&0&0\\0&-\frac{1}{2}&0\\0&0&-\frac{1}{2}\end{pmatrix},\qquad \hat{I}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix},$$

Dynamical/Topological approach

Decay amplitudes in QCDF can be obtained with the ***magic formula***

$$
\begin{aligned}
\mathcal{A}^{\mathrm{QCDF}} = & i\frac{G_F}{\sqrt{2}} \sum_{p=u,c} A_{M_1 M_2} \Big\{ B M_1 \left(\alpha_1 \delta_{pu} \hat{U}_p + \alpha_4^p \hat{I} + \alpha_{4,\mathrm{EW}}^p \hat{Q} \right) M_2 \Lambda_p \\
& + B M_1 \Lambda_p \cdot \mathrm{Tr}\left[\left(\alpha_2 \delta_{pu} \hat{U}_p + \alpha_3^p \hat{I} + \alpha_{3,\mathrm{EW}}^p \hat{Q} \right) M_2 \right] \\
& + B \left(\beta_2 \delta_{pu} \hat{U}_p + \beta_3^p \hat{I} + \beta_{3,\mathrm{EW}}^p \hat{Q} \right) M_1 M_2 \Lambda_p \\
& + B \Lambda_p \cdot \mathrm{Tr}\left[\left(\beta_1 \delta_{pu} \hat{U}_p + \beta_4^p \hat{I} + \beta_{4,\mathrm{EW}}^p \hat{Q} \right) M_1 M_2 \right] \\
& + B \left(\beta_{S2} \delta_{pu} \hat{U}_p + \beta_{S3}^p \hat{I} + \beta_{S3,\mathrm{EW}}^p \hat{Q} \right) M_1 \Lambda_p \cdot \mathrm{Tr}\, M_2 \\
& + B \Lambda_p \cdot \mathrm{Tr}\left[\left(\beta_{S1} \delta_{pu} \hat{U}_p + \beta_{S4}^p \hat{I} + \beta_{S4,\mathrm{EW}}^p \hat{Q} \right) M_1 \right] \cdot \mathrm{Tr}\, M_2 \Big\} + (M_1 \leftrightarrow M_2)\,,
\end{aligned}
$$

The α_i subamplitudes correspond to factorizable contributions, some of them have been estimated up to NNLO in QCD at leading order in Λ_{QCD}/m_b

The β_i subamplitudes correspond to annihilation contributions which are non-factorizable.

They are normally addressed using educated ansätze.

Dynamical/Topological approach

Due to the current limitations of our first-principles techniques we aim to get bounds on the different QCDF subamplitudes using data

In principle each one of the subamplitudes α_i and β_i is channel dependent, however there is not enough experimental information to fit all of them in a channel dependent fashion.

The idea is to decompose the **magic formula** considering the different **flavour structures**.

This naturally leads to a natural equivalence between the topological decomposition (a parametrization of the physical amplitudes) and the dynamical formalism of QCDF

Even though there is a systematic decomposition using flavour structures this does not imply that the flavour symmetry SU(3) has to be exact.

Dynamical/Topological approach

We notice that the magic formula can be written as

$$\mathcal{A}^{\rm QCDF} = i\frac{G_F}{\sqrt{2}}A_{M_1M_2}\Big\{B_iM^i_j(\tilde{\hat{C}}_1)^{jl}_kM^k_l + B_iM^i_j(\tilde{\hat{C}}_2)^{lj}_kM^k_l + B_i(\tilde{\hat{C}}_3)^{ij}_kM^k_lM^l_j$$

$$+B_i(\tilde{\hat{C}}_4)^{li}_kM^k_rM^r_l + B_i(\tilde{\hat{C}}_5)^{ij}_kM^k_iM^l_l + B_i(\tilde{\hat{C}}_6)^{ji}_k)M^k_jM^l_l\Big\},$$

where

$$(\tilde{\hat{C}}_r)^{ij}_k = \sum_{p=u,c}\Big[(\tilde{T}_r\delta_{pu}\hat{U} + \tilde{P}^{(1),p}_r\hat{I} + \tilde{P}^{(2),p}_r\hat{Q})^i_k\Lambda^j_p\Big]$$

and

$$\tilde{T}_k \in \{\alpha_1, \alpha_2, \beta_2, \beta_1, \beta_{S2}, \beta_{S1}\},$$

$$\tilde{P}^{(1),p}_k \in \{\alpha^p_4, \alpha^p_3, \beta^p_3, \beta^p_4, \beta^p_{S3}, \beta^p_{S4}\},$$

$$\tilde{P}^{(2),p}_k \in \{\alpha^p_{4,EW}, \alpha^p_{3,EW}, \beta^p_{3,EW}, b^p_{4,EW}, \beta^p_{S3,EW}, b^p_{S4,EW}\}$$

Dynamical/Topological approach

Now $\hat{Q} = \frac{3}{2}\hat{U} - \frac{1}{2}\hat{I}$ ⇨ all the terms can be written in terms of $\hat{U}$ and $\hat{I}$

The CKM unitarity allows two possibilities $\Lambda_t = -\Lambda_u - \Lambda_c.$

1) Λ_u and Λ_t decomposition

$$\tilde{\hat{C}}_r = \left[\tilde{T} + \frac{3}{2}\tilde{P}_2^u - \frac{3}{2}\tilde{P}_2^c\right]\hat{U}\otimes\Lambda_u + \left[\tilde{P}_1^u - \tilde{P}_1^c - \frac{1}{2}\left\{\tilde{P}_2^u - \tilde{P}_2^c\right\}\right]\hat{I}\otimes\Lambda_u$$
$$-\frac{3}{2}\tilde{P}_2^c\hat{U}\otimes\Lambda_t - \left[\tilde{P}_1^c - \frac{\tilde{P}_2^c}{2}\right]\hat{I}\otimes\Lambda_t,$$

Direct connection with the topological parametrization

$$U_k^i(\Lambda_u)^j = \bar{H}_k^{ij}, \qquad U_k^i(\Lambda_t)^j = \tilde{H}_k^{ij}, \qquad (\Lambda_t)^i = \tilde{H}^i.$$

2) Λ_u and Λ_c decomposition

$$\tilde{\hat{C}}_r = \left[\tilde{T}_r + \frac{3}{2}\tilde{P}_r^{(2),u}\right]\hat{U}\otimes\Lambda_u + \left[\tilde{P}_r^{(1),u} - \frac{\tilde{P}_r^{(2),u}}{2}\right]\hat{I}\otimes\Lambda_u$$
$$+\frac{3}{2}\tilde{P}_r^{(2),c}\hat{U}\otimes\Lambda_c + \left[\tilde{P}_r^{(1),c} - \frac{\tilde{P}_r^{(2),c}}{2}\right]\hat{I}\otimes\Lambda_c.$$

Λ_u and Λ_t parameterization

$$T = A_{M_1M_2}\left[\alpha_1 + \frac{3}{2}\alpha^u_{4,\mathrm{EW}} - \frac{3}{2}\alpha^c_{4,\mathrm{EW}}\right], \qquad C = A_{M_1M_2}\left[\alpha_2 + \frac{3}{2}\alpha^u_{3,\mathrm{EW}} - \frac{3}{2}\alpha^c_{3,\mathrm{EW}}\right],$$

$$E = A_{M_1M_2}\left[\beta_1 + \frac{3}{2}\beta^u_{4,\mathrm{EW}} - \frac{3}{2}\beta^c_{4,\mathrm{EW}}\right], \qquad A = A_{M_1M_2}\left[\beta_2 + \frac{3}{2}\beta^u_{3,\mathrm{EW}} - \frac{3}{2}\beta^c_{3,\mathrm{EW}}\right],$$

$$T_{AS} = A_{M_1M_2}\left[\beta_{S1} + \frac{3}{2}\beta^u_{S4,\mathrm{EW}} - \frac{3}{2}\beta^c_{S4,\mathrm{EW}}\right],$$

$$T_{ES} = A_{M_1M_2}\left[\beta_{S2} + \frac{3}{2}\beta^u_{S3,\mathrm{EW}} - \frac{3}{2}\beta^c_{S3,\mathrm{EW}}\right],$$

$$T_{PA} = A_{M_1M_2}\left[\beta^u_4 - \beta^c_4 - \left(\frac{\beta^u_{4,\mathrm{EW}}}{2} - \frac{\beta^c_{4,\mathrm{EW}}}{2}\right)\right],$$

$$T_{SS} = A_{M_1M_2}\left[\beta^u_{S4} - \beta^c_{S4} - \left(\frac{\beta^u_{S4,\mathrm{EW}}}{2} - \frac{\beta^c_{S4,\mathrm{EW}}}{2}\right)\right],$$

$$T_S = A_{M_1M_2}\left[\alpha^u_3 - \alpha^c_3 - \left(\frac{\alpha^u_{3,\mathrm{EW}}}{2} - \frac{\alpha^c_{3,\mathrm{EW}}}{2}\right) + (\beta^u_{S3} - \beta^c_{S3}) - \left(\frac{\beta^u_{S3,\mathrm{EW}}}{2} - \frac{\beta^c_{S3,\mathrm{EW}}}{2}\right)\right],$$

$$T_P = A_{M_1M_2}\left[\alpha^u_4 - \alpha^c_4 - \left(\frac{\alpha^u_{4,\mathrm{EW}}}{2} - \frac{\alpha^c_{4,\mathrm{EW}}}{2}\right) + (\beta^u_3 - \beta^c_3) - \left(\frac{\beta^u_{3,\mathrm{EW}}}{2} - \frac{\beta^c_{3,\mathrm{EW}}}{2}\right)\right],$$

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Λ_u and Λ_t parameterization

$$S = A_{M_1M_2}\left[\alpha_3^c + \beta_{S3}^c - \frac{\alpha_{3,\mathrm{EW}}^c}{2} - \frac{\beta_{S3,\mathrm{EW}}^c}{2}\right],$$

$$P = A_{M_1M_2}\left[\alpha_4^c + \beta_3^c - \frac{\alpha_{4,\mathrm{EW}}^c}{2} - \frac{\beta_{3,\mathrm{EW}}^c}{2}\right],$$

$$P_A = A_{M_1M_2}\left[\beta_4^c - \frac{\beta_{4,\mathrm{EW}}^c}{2}\right], \qquad P_{SS} = A_{M_1M_2}\left[\beta_{S4}^c - \frac{\beta_{S4,\mathrm{EW}}^c}{2}\right],$$

$$P_C = \frac{3}{2}A_{M_1M_2}\alpha_{3,\mathrm{EW}}^c, \qquad P_T = \frac{3}{2}A_{M_1M_2}\alpha_{4,\mathrm{EW}}^c,$$

$$P_{TA} = \frac{3}{2}A_{M_1M_2}\beta_{3,\mathrm{EW}}^c, \qquad P_{TE} = \frac{3}{2}A_{M_1M_2}\beta_{4,\mathrm{EW}}^c,$$

$$P_{AS} = \frac{3}{2}A_{M_1M_2}\beta_{S4,\mathrm{EW}}^c, \qquad P_{ES} = \frac{3}{2}A_{M_1M_2}\beta_{S3,\mathrm{EW}}^c,$$

These relationships are exact

To disentangle individual QCDF subamplitudes further assumptions are required.

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Λ_u and Λ_t parameterization

Assumptions based on QCDF at NLO and NNLO

$$\alpha_3^u = \alpha_3^c = \alpha_3, \qquad \alpha_{3,\mathrm{EW}}^u = \alpha_{3,\mathrm{EW}}^c = \alpha_{3,\mathrm{EW}},$$

$$\beta_i^u = \beta_i^c = \beta_i, \qquad b_i^u = b_i^c = b_i,$$

The following differences are small

$$\left|\alpha_{4,\mathrm{EW}}^c - \alpha_{4,\mathrm{EW}}^u\right|$$

NLO

$$\left|a_4^c - a_4^u\right|$$

NNLO

$$T = A_{M_1 M_2}\,\alpha_1, \qquad C = A_{M_1 M_2}\,\alpha_2, \qquad E = B_{M_1 M_2}\,b_1,$$

$$A = B_{M_1 M_2}\,b_2, \qquad T_{AS} = B_{M_1 M_2}\,b_{S1}, \qquad T_{ES} = B_{M_1 M_2}\,b_{S2},$$

$$T_{PA} = 0, \qquad T_{SS} = 0, \qquad T_S = 0, \qquad |T_P| < 0.02,$$

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Λ_u and Λ_t parameterization

Assumptions based on QCDF at NLO and NNLO

$$\alpha_3^u = \alpha_3^c = \alpha_3, \qquad \alpha_{3,\mathrm{EW}}^u = \alpha_{3,\mathrm{EW}}^c = \alpha_{3,\mathrm{EW}},$$

$$\beta_i^u = \beta_i^c = \beta_i, \qquad b_i^u = b_i^c = b_i,$$

The following differences are small

$$\left|\alpha_{4,\mathrm{EW}}^c - \alpha_{4,\mathrm{EW}}^u\right|$$

NLO

$$\left|a_4^c - a_4^u\right|$$

NNLO

$$S = A_{M_1M_2}\left[\alpha_3 + \beta_{S3} - \frac{\alpha_{3,\mathrm{EW}}}{2} - \frac{\beta_{S3,\mathrm{EW}}}{2}\right], \quad P = A_{M_1M_2}\left[\alpha_4^c + \beta_3 - \frac{\alpha_{4,\mathrm{EW}}^c}{2} - \frac{\beta_{3,\mathrm{EW}}}{2}\right],$$

$$P_A = B_{M_1M_2}\left(b_4 - \frac{b_{4,\mathrm{EW}}}{2}\right), \qquad P_{SS} = B_{M_1M_2}\left(b_{S4} - \frac{b_{S4,\mathrm{EW}}}{2}\right),$$

$$P_C = \frac{3}{2}A_{M_1M_2}\,\alpha_{3,\mathrm{EW}}, \qquad P_T = \frac{3}{2}A_{M_1M_2}\,\alpha_{4,\mathrm{EW}}^c,$$

$$P_{TA} = \frac{3}{2}B_{M_1M_2}\,b_{3,\mathrm{EW}}, \qquad P_{TE} = \frac{3}{2}B_{M_1M_2}\,b_{4,\mathrm{EW}},$$

$$P_{AS} = \frac{3}{2}B_{M_1M_2}\,b_{S4,\mathrm{EW}}, \qquad P_{ES} = \frac{3}{2}B_{M_1M_2}\,b_{S3,\mathrm{EW}},$$

Huber, Tetlalmatzi-Xolocotzi 2111.06418

Λ_u and Λ_c parameterization

$$c_1^{\mathrm{QCDF},u} = A_{M_1M_2}\left(\alpha_1 + \frac{3}{2}\alpha_{4,EW}^u\right),$$

$$c_2^{\mathrm{QCDF},u} = A_{M_1M_2}\left(\beta_2 + \frac{3}{2}\beta_{3,EW}^u\right),$$

$$c_3^{\mathrm{QCDF},u} = A_{M_1M_2}\left(\alpha_4^u - \frac{\alpha_{4,EW}^u}{2} + \beta_3^u - \frac{\beta_{3,EW}^u}{2}\right),$$

$$c_4^{\mathrm{QCDF},u} = A_{M_1M_2}\left(\alpha_2 + \frac{3}{2}\alpha_{3,EW}^u\right),$$

$$c_5^{\mathrm{QCDF},u} = A_{M_1M_2}\left(\beta_{S2} + \frac{3}{2}\beta_{S3,EW}^u\right),$$

$$c_6^{\mathrm{QCDF},u} = A_{M_1M_2}\left(\alpha_3^u - \frac{\alpha_{3,EW}^u}{2} + \beta_{S3}^u - \frac{\beta_{S3,EW}^u}{2}\right),$$

$$c_7^{\mathrm{QCDF},u} = A_{M_1M_2}\left(\beta_1 + \frac{3}{2}b_{4,EW}^u\right),$$

$$c_8^{\mathrm{QCDF},u} = A_{M_1M_2}\left(\beta_{S1} + \frac{3}{2}b_{S4,EW}^u\right),$$

$$c_9^{\mathrm{QCDF},u} = A_{M_1M_2}\left(\beta_4^u - \frac{b_{4,EW}^u}{2}\right),$$

$$c_{10}^{\mathrm{QCDF},u} = A_{M_1M_2}\left(\beta_{S4}^u - \frac{b_{S4,EW}^u}{2}\right),$$

$$c_1^{\mathrm{QCDF},c} = \frac{3}{2}A_{M_1M_2}\alpha_{4,EW}^c,$$

$$c_2^{\mathrm{QCDF},c} = \frac{3}{2}A_{M_1M_2}\beta_{3,EW}^c,$$

$$c_3^{\mathrm{QCDF},c} = A_{M_1M_2}\left(\alpha_4^c - \frac{\alpha_{4,EW}^c}{2} + \beta_3 - \frac{\beta_{3,EW}}{2}\right),$$

$$c_4^{\mathrm{QCDF},c} = \frac{3}{2}A_{M_1M_2}\alpha_{3,EW}^c,$$

$$c_5^{\mathrm{QCDF},c} = \frac{3}{2}A_{M_1M_2}\beta_{S3,EW}^c,$$

$$c_6^{\mathrm{QCDF},c} = A_{M_1M_2}\left(\alpha_3^c - \frac{\alpha_{3,EW}^c}{2} + \beta_{S3}^c - \frac{\beta_{S3,EW}^c}{2}\right),$$

$$c_7^{\mathrm{QCDF},c} = \frac{3}{2}A_{M_1M_2}b_{4,EW}^c,$$

$$c_8^{\mathrm{QCDF},c} = \frac{3}{2}A_{M_1M_2}b_{S4,EW}^c,$$

$$c_9^{\mathrm{QCDF},c} = A_{M_1M_2}\left(\beta_4^c - \frac{b_{4,EW}^c}{2}\right),$$

$$c_{10}^{\mathrm{QCDF},c} = A_{M_1M_2}\left(\beta_{S4}^c - \frac{b_{S4,EW}^c}{2}\right).$$

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Introducing SU(3) flavour breaking

Two sources of SU(3) flavour breaking

$$T = A_{M_1 M_2}\left[\alpha_1 + \frac{3}{2}\alpha^u_{4,\mathrm{EW}} - \frac{3}{2}\alpha^c_{4,\mathrm{EW}}\right]$$

$$A_{M_1 M_2} = (m_B^2 - m_{M_1}^2) F_0^{B\to M_1}(m_{M_2}^2) f_{M_2}$$

Largest source of breaking

Gegenbauer moments (breaking is small)

Introduce bare quantities which will be assumed to be SU(3) exact

$$T = A_{M_1M_2}\tilde{T}, \quad C = A_{M_1M_2}\tilde{C}, \quad E = B_{M_1M_2}\tilde{E},$$

$$A = B_{M_1M_2}\tilde{A}, \quad T_{AS} = B_{M_1M_2}\tilde{T}_{AS}, \quad T_{ES} = B_{M_1M_2}\tilde{T}_{ES},$$

$$S = A_{M_1M_2}\tilde{S}, \quad P = A_{M_1M_2}\tilde{P}, \quad P_A = B_{M_1M_2}\tilde{P}_A,$$

$$P_{SS} = B_{M_1M_2}\tilde{P}_{SS}, \quad P_C = A_{M_1M_2}\tilde{P}_C, \quad P_T = A_{M_1M_2}\tilde{P}_T,$$

$$P_{TA} = B_{M_1M_2}\tilde{P}_{TA}, \quad P_{TE} = B_{M_1M_2}\tilde{P}_{TE}, \quad P_{AS} = B_{M_1M_2}\tilde{P}_{AS},$$

$$P_{ES} = B_{M_1M_2}\tilde{P}_{ES}.$$

χ^2-*fit*

To find the minimum of the χ^2 function we use

random sampling together with the minimization algorithm

Sequential-Least-Squares (Python SciPy optimize library)

Sequential Least Squares allows us to account for bounds on the observables

Our analysis includes:

Branching fractions, direct CP asymmetries and mixing induced CP asymmetries

χ^2-fit

Best fit point

$\tilde{T}$	$1.073 + 0.044\,i$
$\tilde{C}$	$0.334 - 0.689\,i$
$\tilde{A}$	$4.782 + 8.524\,i$
$\tilde{E}$	$-7.655 + 12.782\,i$
$\tilde{T}_{ES}$	$-80.703 + 8.656\,i$
$\tilde{T}_{AS}$	$-8.747 + 20.809\,i$
$\tilde{T}_P$	$-7.421 \times 10^{-5} - 0.0200\,i$

$\tilde{P}_T$	$-0.245 - 0.133\,i$
$\tilde{P}_C$	$0.179 + 0.154\,i$
$\tilde{P}_{TA}$	$-57.273 - 40.248\,i$
P	$0.081 + 0.186\,i$
$\tilde{P}_{TE}$	$35.058 + 28.547\,i$
$\tilde{P}_A$	$-18.150 - 10.960\,i$
$\tilde{P}_{AS}$	$-24.555 - 41.077\,i$
$\tilde{P}_{ES}$	$77.683 + 54.308\,i$
$\tilde{P}_{SS}$	$19.785 + 18.715\,i$
$\tilde{S}$	$-0.199 - 0.112\,i$

$$\alpha_1 = \tilde{T}, \qquad \alpha_1(\pi\pi) = 1.000^{+0.029}_{-0.069} + (0.011^{+0.023}_{-0.050})i\,,$$

$$\alpha_2 = \tilde{C}, \qquad \alpha_2(\pi\pi) = 0.240^{+0.217}_{-0.125} + (-0.077^{+0.115}_{-0.078})i\,.$$

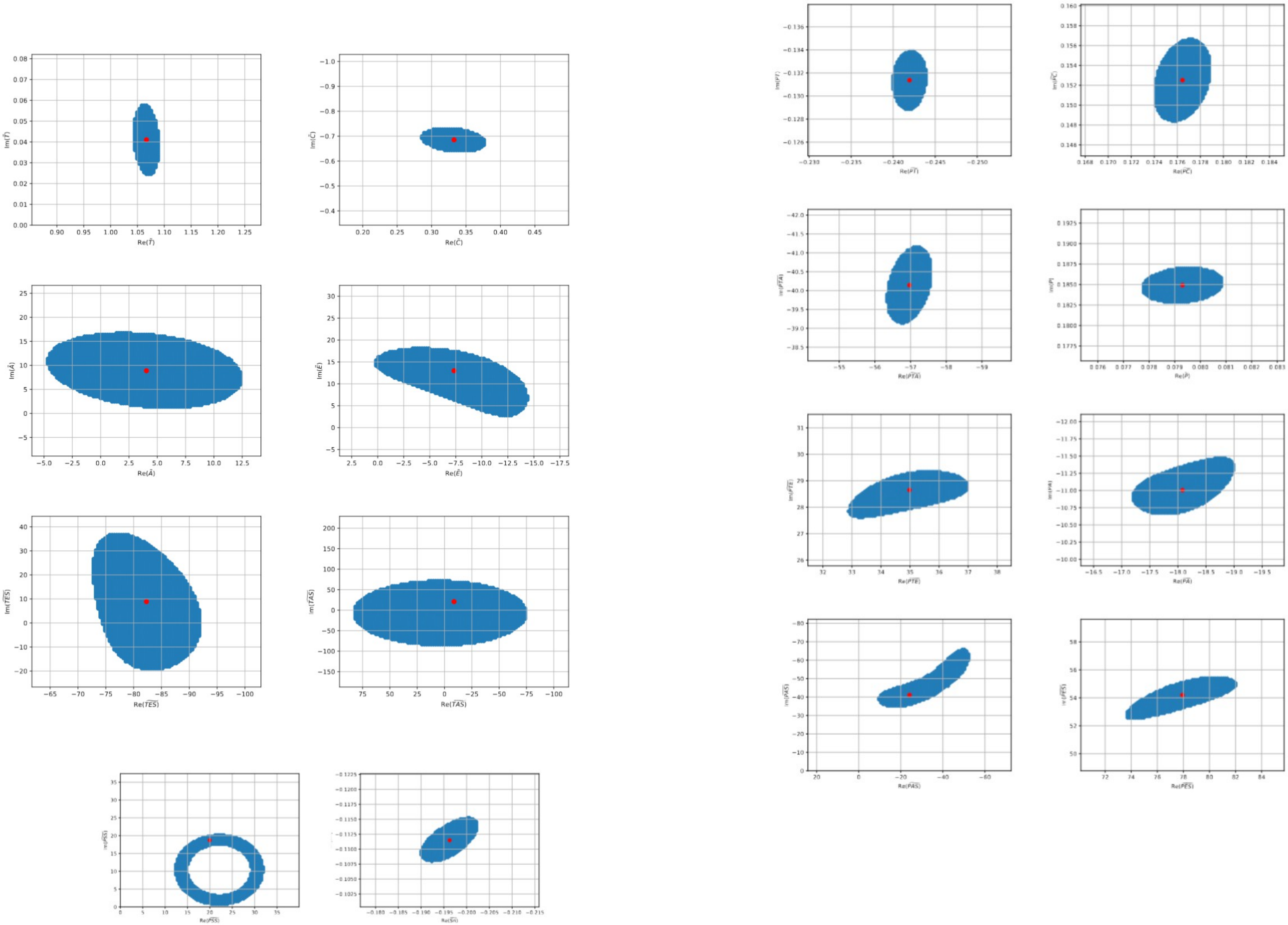
NNLO QCDF calculation

Beneke, Hubert, Xin-Qiang Li, 0911.3655

$\chi^2/dof = 1.67$ $\qquad$ $p_{value} = 0.072$

Underlying amplitudes (annihilation) have to be multiplied by $B_{M_1M_2} = f_{B_q} f_{M_1} f_{M_2}\,,$

χ^2-fit



χ^2-fit

Branching Fractions I			
Channel	Theory 10^{-6}	Experiment 10^{-6}	Pull
$B^- \to \pi^0\pi^-$	5.3847	5.31 ± 0.26	0.287
$B^- \to K^0K^-$	1.3012	1.32 ± 0.17	0.111
$\bar{B}^0 \to \pi^+\pi^-$	5.3480	5.37 ± 0.20	0.110
$\bar{B}^0 \to \pi^0\pi^0$	1.5017	1.55 ± 0.17	0.284
$\bar{B}^0 \to K^+K^-$	0.0747	0.078 ± 0.015	0.218
$\bar{B}^0 \to K^0\bar{K}^0$	1.2164	1.21 ± 0.16	0.040
$\bar{B}_s \to \pi^0K^0$	1.3049	–	–
$\bar{B}_s \to \pi^-K^+$	5.7129	5.90 ± 0.70	0.267
$B^- \to \pi^0K^-$	13.2476	13.20 ± 0.40	0.119
$B^- \to \pi^-K^0$	23.8697	23.90 ± 0.60	0.051
$\bar{B}^0 \to \pi^+K^-$	20.0021	20.00 ± 0.40	0.005
$\bar{B}^0 \to \pi^0\bar{K}^0$	10.1652	10.10 ± 0.40	0.163
$\bar{B}_s \to \pi^+\pi^-$	0.6962	0.72 ± 0.10	0.238
$\bar{B}_s \to \pi^0\pi^0$	0.3481	< 7.70	–
$\bar{B}_s \to K^+K^-$	27.5639	27.20 ± 2.30	0.158
$\bar{B}_s \to K^0\bar{K}^0$	17.9013	17.60 ± 3.10	0.097

Branching Fractions II			
Channel	Theory 10^{-6}	Experiment 10^{-6}	Pull
$B^- \to \eta\pi^-$	3.9570	4.02 ± 0.27	0.233
$\bar{B}^0 \to \eta\pi^0$	0.2851	0.41 ± 0.17	0.735
$\bar{B}_s \to \eta K^0$	0.8314	–	–
$B^- \to \eta K^-$	2.4756	2.40 ± 0.40	0.189
$\bar{B}^0 \to \eta K^0$	1.2586	1.23 ± 0.27	0.106
$\bar{B}_s \to \eta\pi^0$	0.2338	< 1000	–
$B^- \to \eta'\pi^-$	2.2563	2.70 ± 0.90	0.493
$\bar{B}^0 \to \eta'\pi^0$	1.3138	1.20 ± 0.60	0.190
$\bar{B}_s \to \eta' K^0$	2.0411	< 8.16	–
$B^- \to \eta' K^-$	70.4245	70.40 ± 2.50	0.010
$\bar{B}^0 \to \eta' K^0$	65.8420	66.00 ± 4.00	0.039
$\bar{B}_s \to \eta'\pi^0$	7.4706	–	–

Branching Fractions III			
Channel	Theory 10^{-6}	Experiment 10^{-6}	Pull
$\bar{B}^0 \to \eta\eta$	0.5244	< 1.0	–
$\bar{B}_s \to \eta\eta$	5.5637	< 143	–
$\bar{B}^0 \to \eta'\eta'$	0.3722	< 1.70	–
$\bar{B}_s \to \eta'\eta'$	33.0001	33.0 ± 7.0	0.00002
$\bar{B}^0 \to \eta'\eta$	0.2752	< 1.2	–
$\bar{B}_s \to \eta'\eta$	62.6704	< 65	–

χ^2-fit

Direct CP Asymmetries I			
Channel	Theory 10^{-2}	Experiment 10^{-2}	Pull
$B^- \to \pi^0\pi^-$	-2.4411	-1.20 ± 3.10	0.400
$B^- \to K^0K^-$	9.9676	4.00 ± 14.0	0.426
$\bar{B}^0 \to \pi^+\pi^-$	35.9990	31.4 ± 3.0	1.533
$\bar{B}^0 \to \pi^0\pi^0$	33.7471	23.0 ± 18.0	0.597
$\bar{B}^0 \to K^+K^-$	-35.5120	–	–
$\bar{B}^0 \to K^0\bar{K}^0$	-4.4926	-60.0 ± 70.0	0.793
$\bar{B}_s \to \pi^0K^0$	42.2366	–	–
$\bar{B}_s \to \pi^-K^+$	21.5439	22.4 ± 1.2	0.713
$B^- \to \pi^0K^-$	2.8001	2.70 ± 1.20	0.083
$B^- \to \pi^-K^0$	-0.4014	-2.30 ± 1.40	1.356
$\bar{B}^0 \to \pi^+K^-$	-8.3644	-8.31 ± 0.31	0.176
$\bar{B}^0 \to \pi^0\bar{K}^0$	-20.8832	-1.00 ± 13.0	1.529
$\bar{B}_s \to \pi^+\pi^-$	2.7321	–	–
$\bar{B}_s \to \pi^0\pi^0$	2.7321	–	–
$\bar{B}_s \to K^+K^-$	-11.8296	-16.2 ± 3.5	1.249
$\bar{B}_s \to K^0\bar{K}^0$	0.4583	–	–

Direct CP Asymmetries II			
Channel	Theory 10^{-2}	Experiment 10^{-2}	Pull
$B^- \to \eta\pi^-$	-11.5156	-14.0 ± 7.0	0.355
$\bar{B}^0 \to \eta\pi^0$	-30.7591	–	–
$\bar{B}_s \to \eta K^0$	-5.7454	< 0.10	–
$B^- \to \eta K^-$	-39.6924	-37.0 ± 8.0	0.337
$\bar{B}^0 \to \eta K^0$	6.8422	–	–
$\bar{B}_s \to \eta\pi^0$	21.8419	–	–
$B^- \to \eta'\pi^-$	9.4288	6.0 ± 16.0	0.214
$\bar{B}^0 \to \eta'\pi^0$	-31.9204	–	–
$\bar{B}_s \to \eta'K^0$	-35.8757	–	–
$B^- \to \eta'K^-$	0.7152	0.40 ± 1.10	0.287
$\bar{B}^0 \to \eta'K^0$	3.9965	8.0 ± 4.0	1.001
$\bar{B}_s \to \eta'\pi^0$	-2.1063	–	–

Direct CP Asymmetries III			
Channel	Theory 10^{-2}	Experiment 10^{-2}	Pull
$\bar{B}^0 \to \eta\eta$	0.1862	–	–
$\bar{B}_s \to \eta\eta$	-12.3429	–	–
$\bar{B}^0 \to \eta'\eta'$	41.0373	–	–
$\bar{B}_s \to \eta'\eta'$	-0.9322	–	–
$\bar{B}^0 \to \eta'\eta$	-10.4983	–	–
$\bar{B}_s \to \eta'\eta$	0.6843	–	–

χ^2-fit

Mixing-Induced CP Asymmetries I			
Channel	Theory 10^{-2}	Experiment 10^{-2}	Pull
$\bar{B}^0 \to \pi^+\pi^-$	-65.5476	-66.6 ± 2.9	0.363
$\bar{B}^0 \to \pi^0\pi^0$	48.4995	–	–
$\bar{B}^0 \to K^+K^-$	93.3888	–	–
$\bar{B}^0 \to K^0\bar{K}^0$	-13.2200	-80.0 ± 50.0	1.336
$\bar{B}_s \to \pi^0K^0$	36.0666	–	–
$\bar{B}_s \to \pi^-K^+$	-97.6517	–	–
$\bar{B}^0 \to \pi^+K^-$	-56.4682	–	–
$\bar{B}^0 \to \pi^0\bar{K}^0$	77.5009	64.0 ± 13.0	1.039
$\bar{B}_s \to \pi^+\pi^-$	-7.5341	–	–
$\bar{B}_s \to \pi^0\pi^0$	-7.5341	–	–
$\bar{B}_s \to K^+K^-$	16.1861	14.0 ± 3.0	0.729
$\bar{B}_s \to K^0\bar{K}^0$	0.1588	–	–

Mixing-Induced CP Asymmetries II			
Channel	Theory 10^{-2}	Experiment 10^{-2}	Pull
$\bar{B}^0 \to \eta\pi^0$	-59.2586	–	–
$\bar{B}_s \to \eta K^0$	56.1498	57.0 ± 10.0	0.085
$\bar{B}^0 \to \eta K^0$	97.5998	–	–
$\bar{B}_s \to \eta\pi^0$	82.9555	–	–
$\bar{B}^0 \to \eta'\pi^0$	-59.1249	–	–
$\bar{B}_s \to \eta' K^0$	32.0111	–	–
$\bar{B}^0 \to \eta' K^0$	70.1952	64.0 ± 5.0	1.239
$\bar{B}_s \to \eta'\pi^0$	-10.1935	–	–

Mixing-Induced CP Asymmetries III			
Channel	Theory 10^{-2}	Experiment 10^{-2}	Pull
$\bar{B}^0 \to \eta\eta$	-83.3659	–	–
$\bar{B}_s \to \eta\eta$	-17.4784	–	–
$\bar{B}^0 \to \eta'\eta'$	-37.1800	–	–
$\bar{B}_s \to \eta'\eta'$	5.3694	–	–
$\bar{B}^0 \to \eta'\eta$	-24.3972	–	–
$\bar{B}_s \to \eta'\eta$	0.6815	–	–

χ^2 *-fit*

In

M. B. Marcos, M. Reboud and K.K. Vos, 2504.05209

a similar analysis is done in the

Λ_u and Λ_c basis

considering channels without singlets

The conclusions are similar concerning the fact that SU(3) breaking fits the data.

In addition we also find that the electroweak penguins deviate in comparison with the QCDF calculation, a similar conclusion can be derived from the results presented in the reference above based on values for the fitted amplitudes

Conclusions

- We have performed a data-driven analysis to extract bounds on QCDF amplitudes for B mesons decaying into pairs of pseudoscalar light mesons.

- We find that it is possible to explain data (including final states with singlets) incorporating SU(3) flavour breaking effects.

- Our analysis finds that data is compatible with the QCDF values of the amplitudes α_1 and the real part of α_2.

- We also find the annihilation amplitudes are bounded to values of at most 10%.

- Nevertheless we find that data requires large strong phases and sizeable electroweak penguin values in comparison with QCDF.