

Fun_(FUNDAMENTAL PHYSICS) with Spin Sensors

Sebastian A. R. Ellis

King's College London

What are we hoping to achieve?

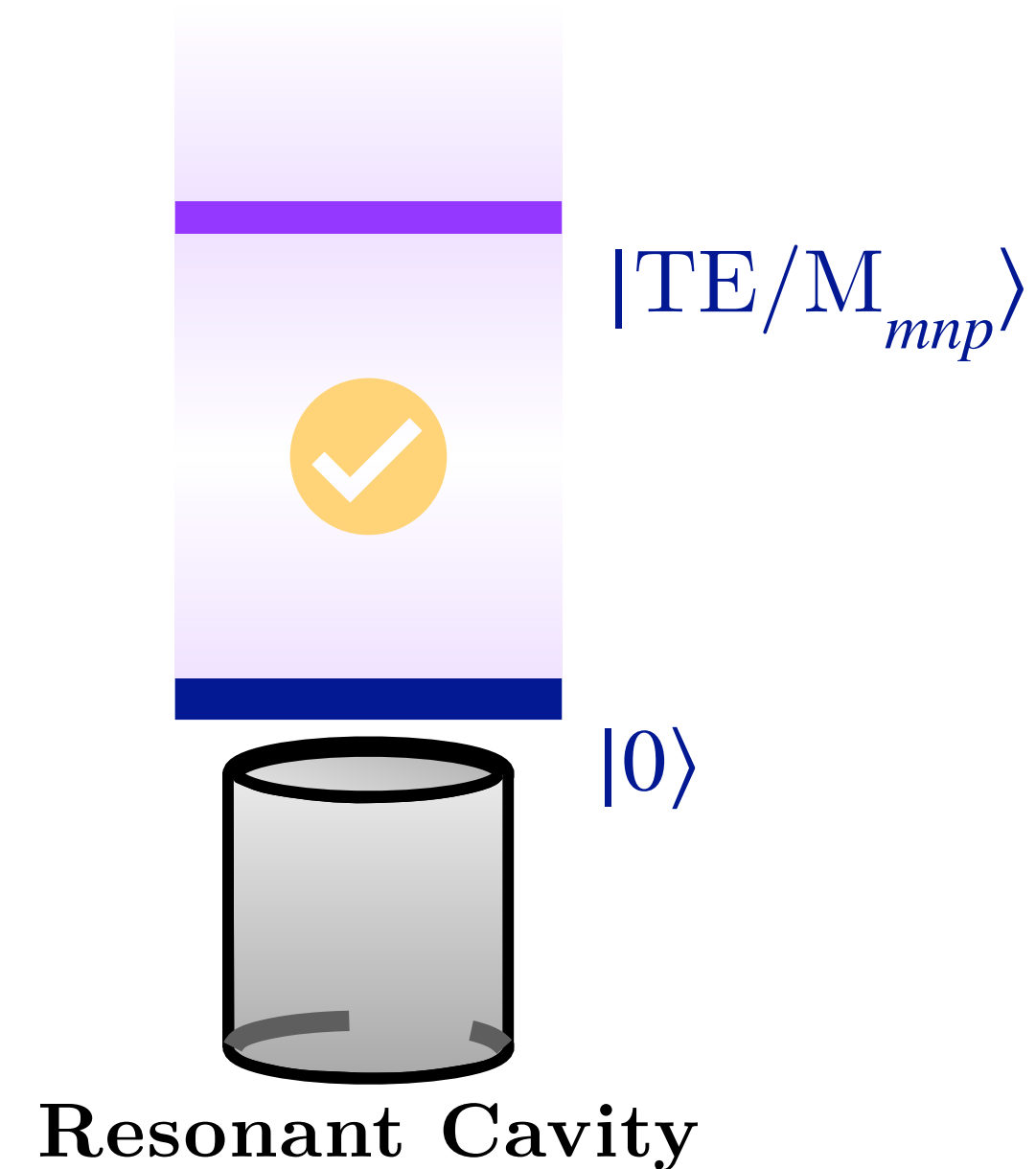
We want to detect new physics with small energy deposition

Quantum sensors characterised by a qubit/bosonic system with manipulable states

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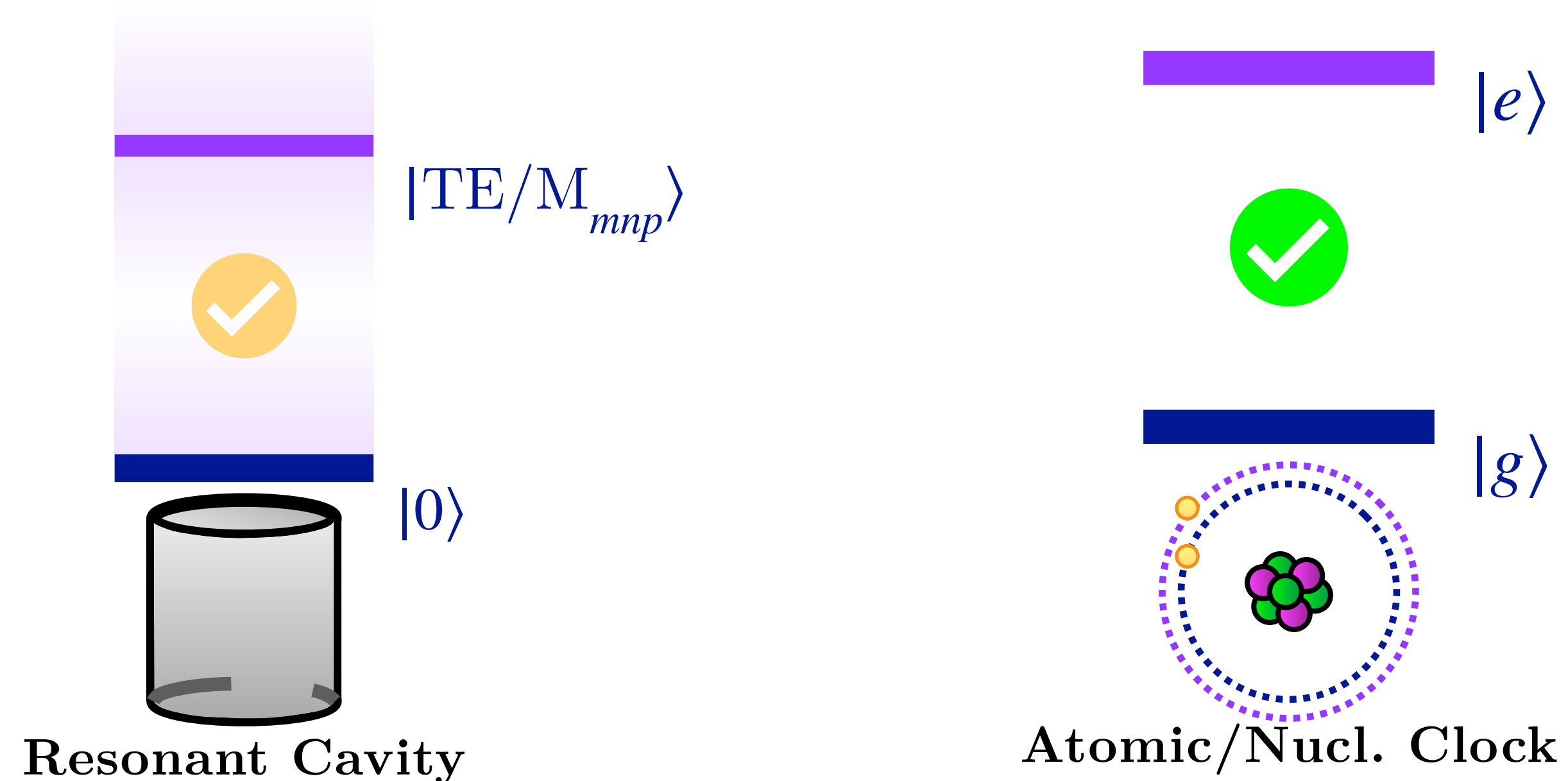
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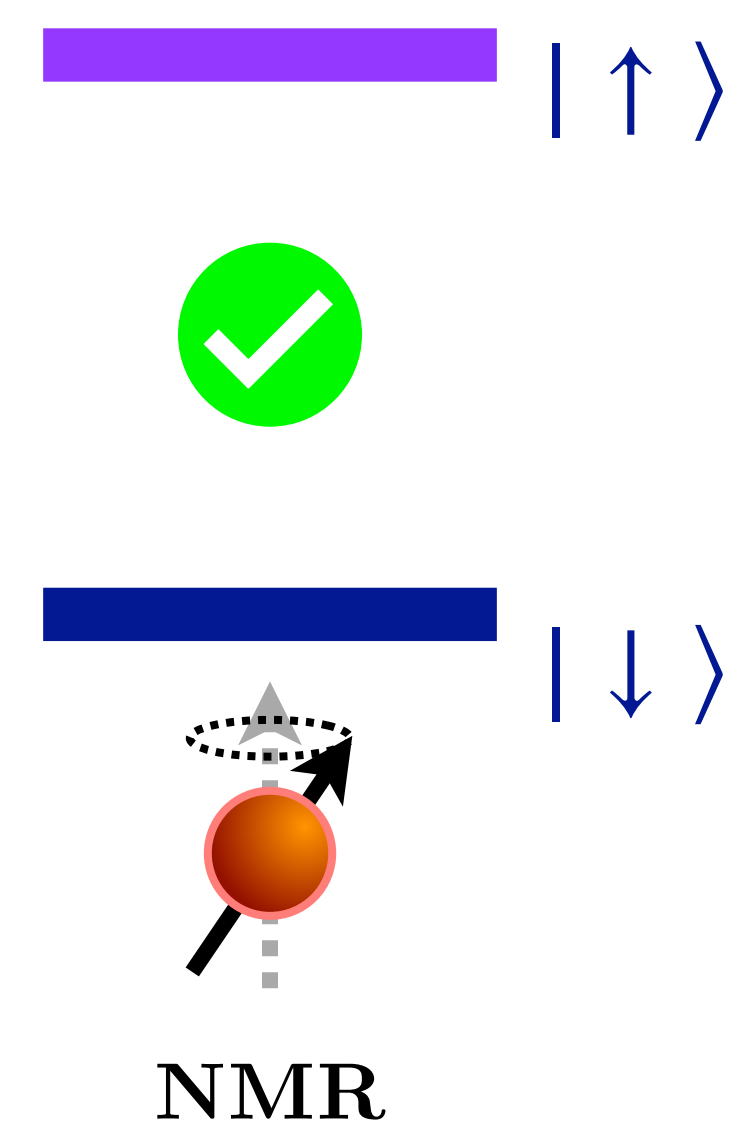
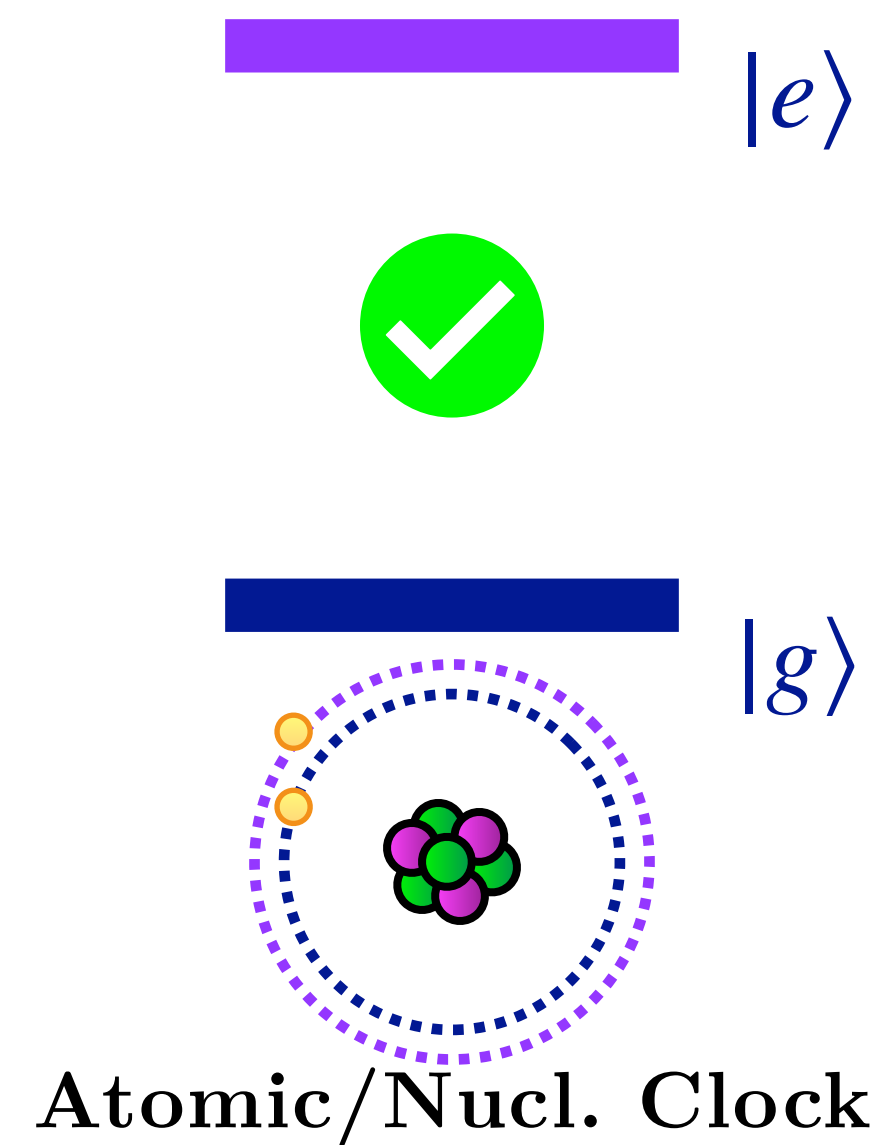
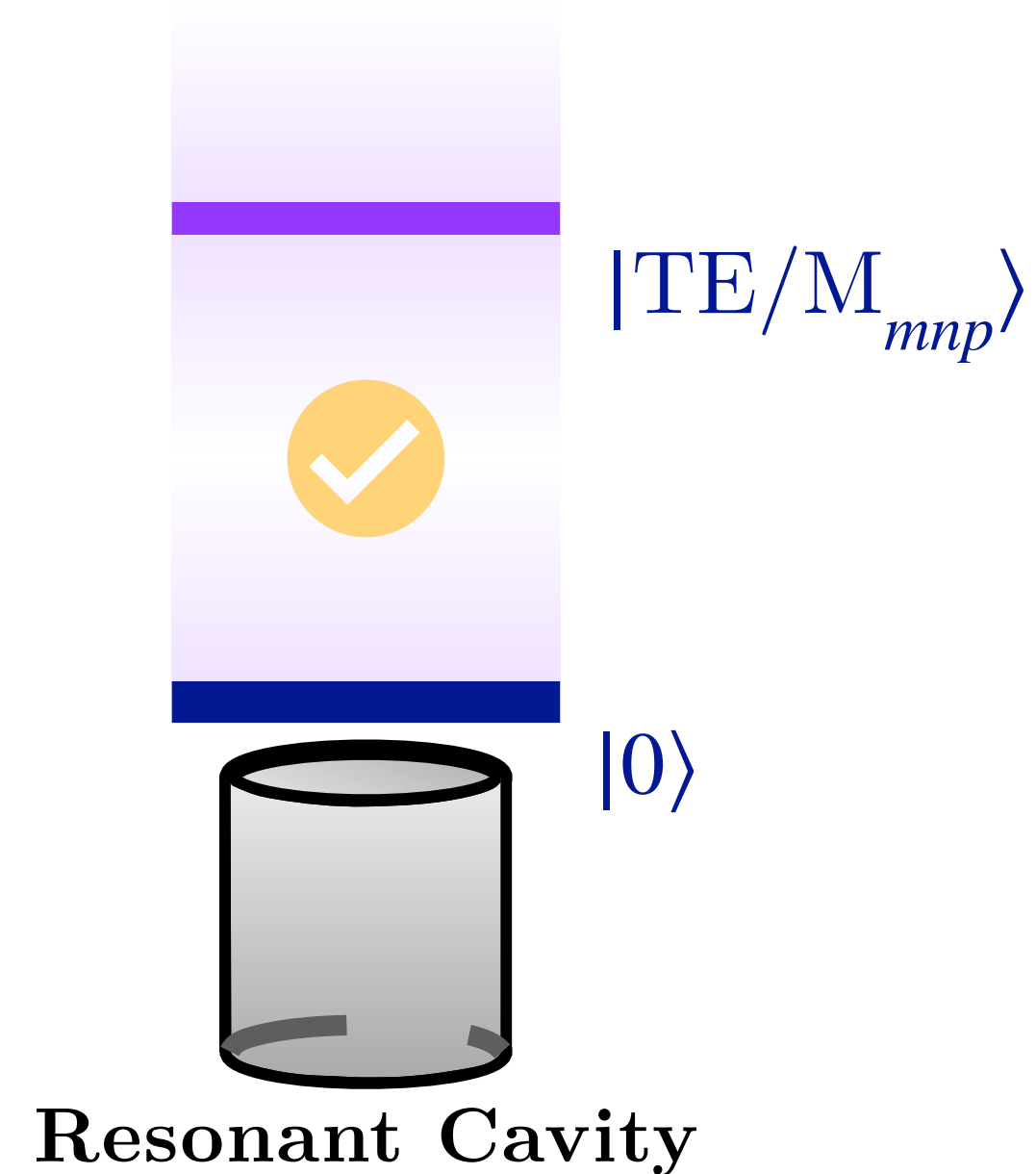
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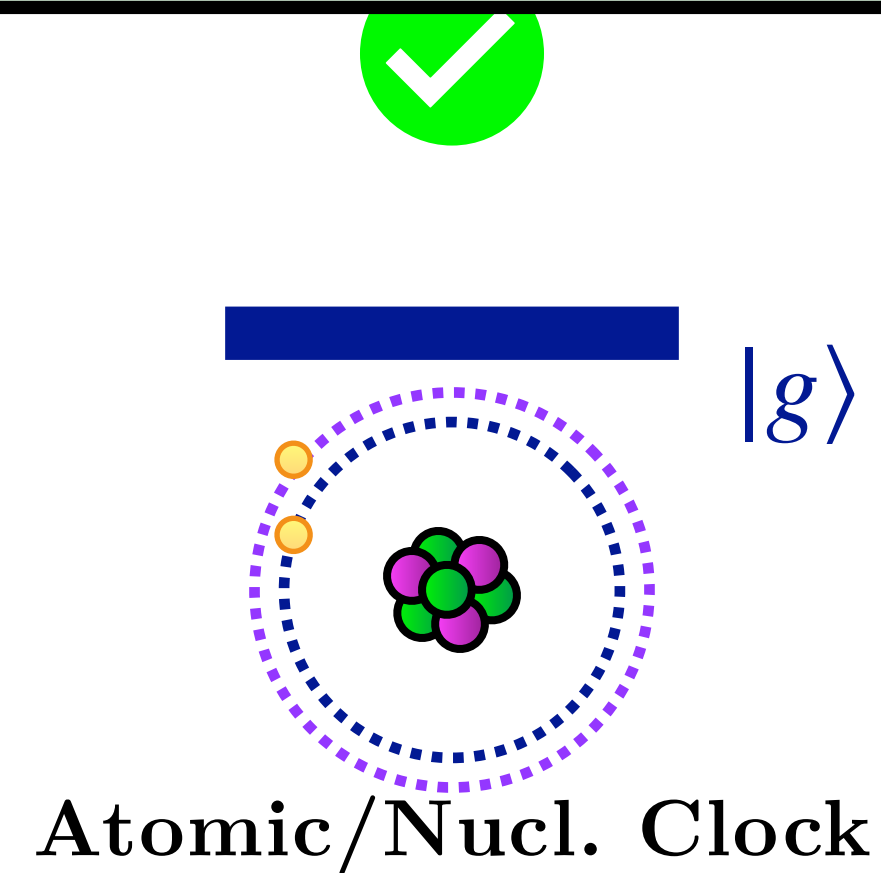
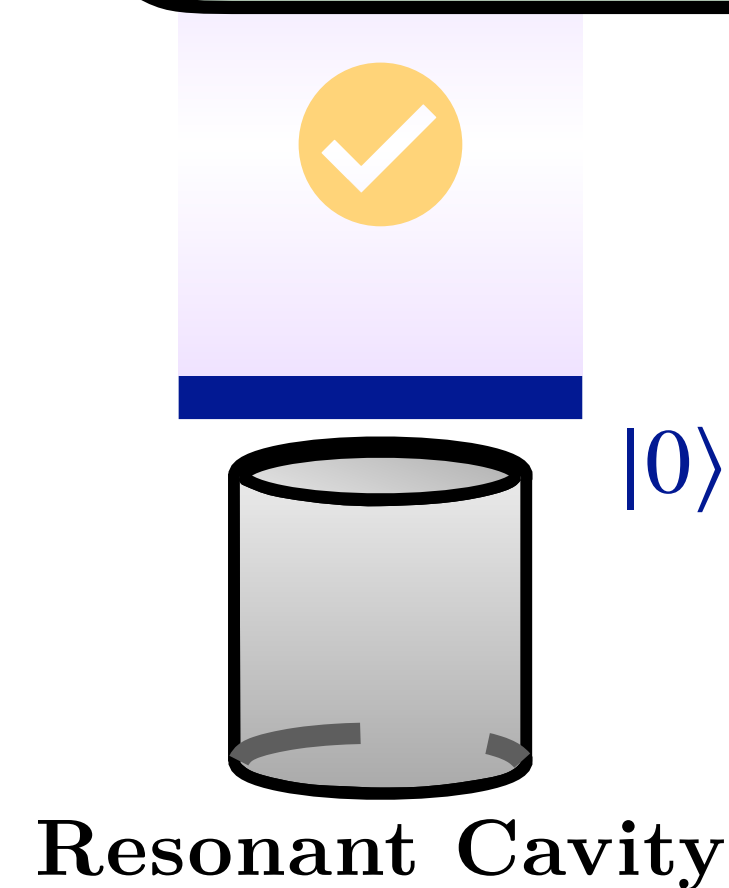


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We want to detect new physics with small energy deposition

Quantum sensors characterised by a qubit/bosonic system with manipulable states

Many more not shown here...



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Many experiments using a variety of quantum-limited sensors operating already

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Can we maximise the use-case of those sensors?

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Many experiments using a variety of quantum-limited sensors operating already

Can we maximise the use-case of those sensors?

Can we kick-start development towards large-scale devices?

Quantum Sensor Zoology

What does your BSM target couple to? $\hat{H}_{\text{int}}$

$$\hat{H}_{a\gamma} = g \int_V a \mathbf{E} \cdot \mathbf{B} \qquad \hat{H}_{AA'} = \epsilon m_{A'}^2 \int_V \mathbf{A} \cdot \mathbf{A}' \qquad \hat{H}_{SB} = -\mu \hat{\mathbf{S}} \cdot \mathbf{B}$$

What is the sensor? $\hat{H}_{\text{sensor}}$

$$\hat{H}_{\text{EM}} = \frac{1}{2} \int_V \mathbf{B} \cdot \mathbf{B} + \mathbf{E} \cdot \mathbf{E} + \mathbf{J} \cdot \mathbf{A} \qquad \hat{H}_{\text{mech}} = \frac{\hat{p}^2}{2M} + \frac{1}{2} M \omega^2 \hat{x}^2 \qquad \hat{H}_{\text{spin}} \supset \hbar \omega_0 \hat{S}_z$$

How do you read out the sensor? $\hat{H}_{\text{readout}}$

$$\hat{H}_{\text{I/O}} \simeq i\hbar \int_{-\infty}^{\infty} d\omega \kappa(\omega) \left[\hat{b}_{\omega}^{\dagger} \hat{a} - \hat{b}_{\omega} \hat{a}^{\dagger} \right] \qquad \hat{H}_{\text{optomech.}} \simeq g \hat{x} (\hat{a}^{\dagger} + \hat{a}) \qquad \hat{H}_{\text{JC}} \simeq g' (\hat{a}^{\dagger} + \hat{a}) (\hat{S}_+ + \hat{S}_-)$$

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This Talk

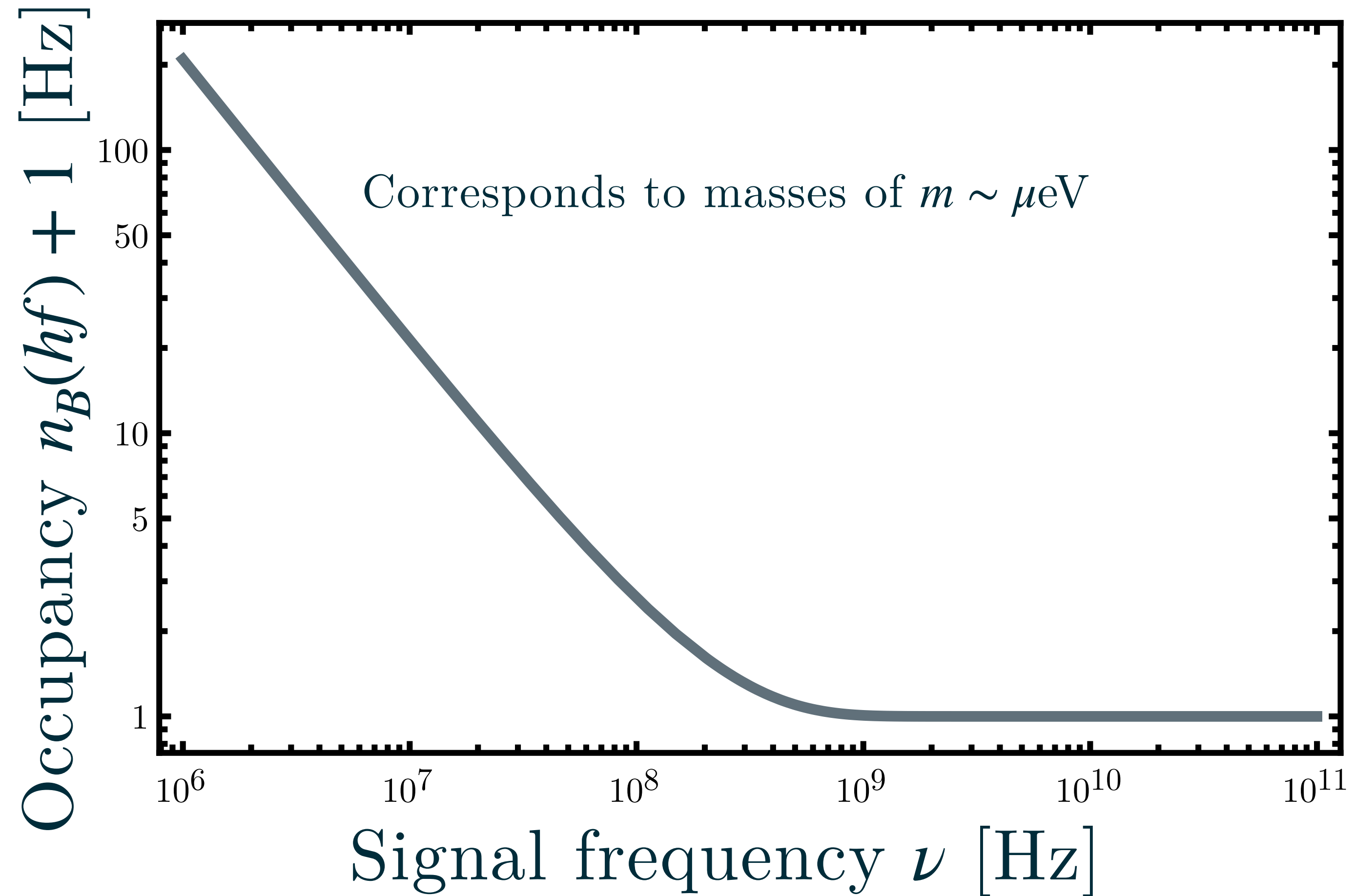
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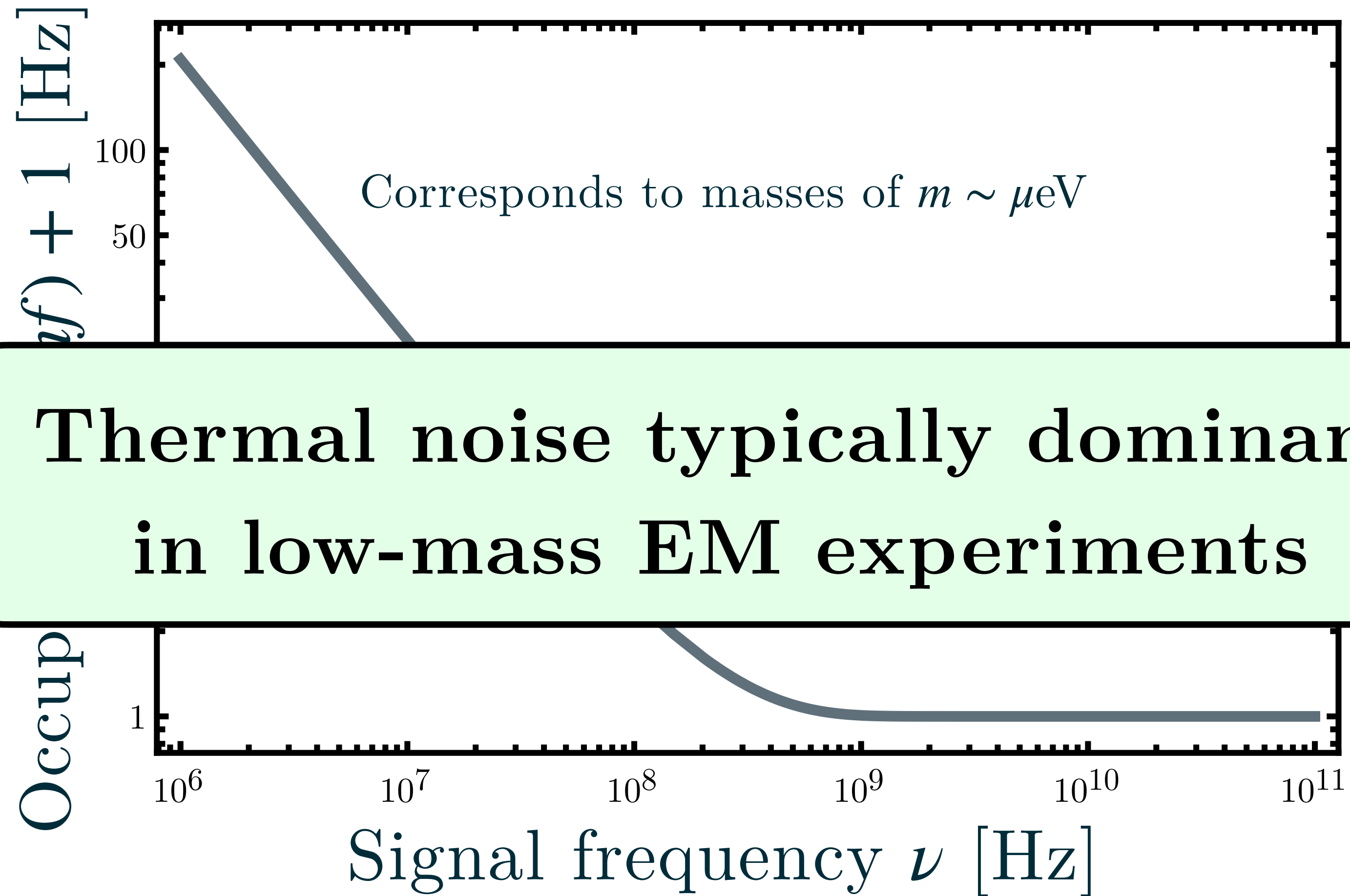
Quantum Noise

Cooling below about $T \sim 0.01\text{K}$ difficult for macroscopic systems



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Cooling below about $T \sim 0.01\text{K}$ difficult for macroscopic systems



Spin Sensor Noise

Exception to the previous statement is spin systems

$$\frac{n_{\uparrow}}{n_{\downarrow}} = e^{\Delta E/T_s}$$

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Macroscopic spin systems with B -field applied:

$$\langle M_x^{\text{s.p.}}(t)^2 \rangle \simeq \frac{J(J+1)n\gamma^2}{6V} \frac{1}{1 + T_2^2(\omega - \omega_0)^2}$$

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Spin projection noise (QM origin) independent of T_s

Spin Signals

Spin signals from an interaction $\hat{H}_{\text{int}} = \gamma \hat{S}_z B(t)$

What is the signal $B(t)$ here?

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PRL 134 (2025) 23, 231401
V. Domcke SARE, N. L. Rodd

Spin Signals — Importance of the Box

Standard conductive shielding used in typical experiments has $\sigma \gtrsim 10^6 \text{ S/m} \sim 100 \text{ eV}$

Skin depth of a signal at frequency ω in terms of

DC conductivity is $\delta_s = \sqrt{\frac{2}{\omega\mu\sigma}}$

Which boundary is relevant in an experiment will

depend on shielding factor $\eta \simeq \frac{|\mathbf{B}_{\text{in}}|}{|\mathbf{B}_0|}$

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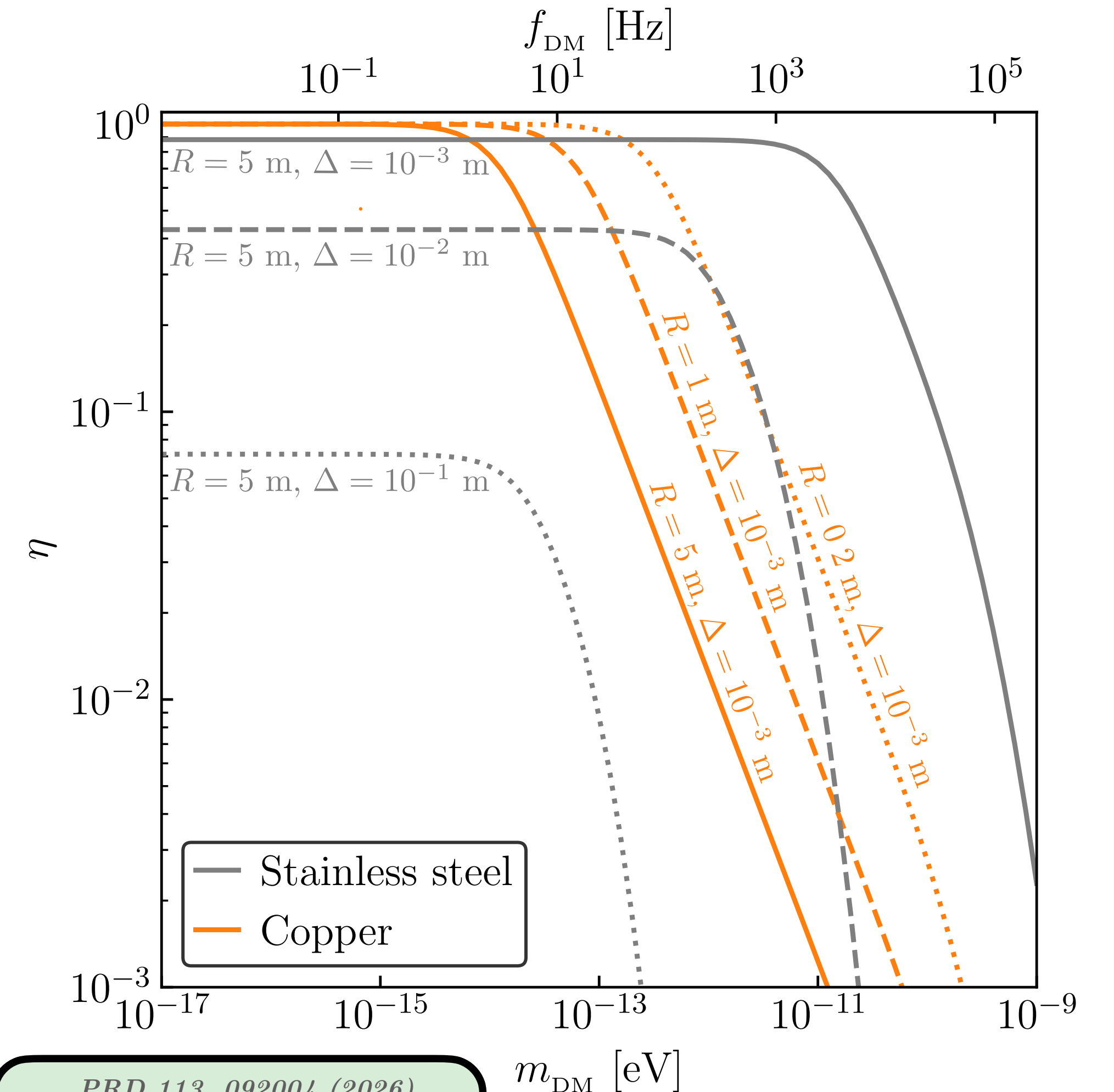
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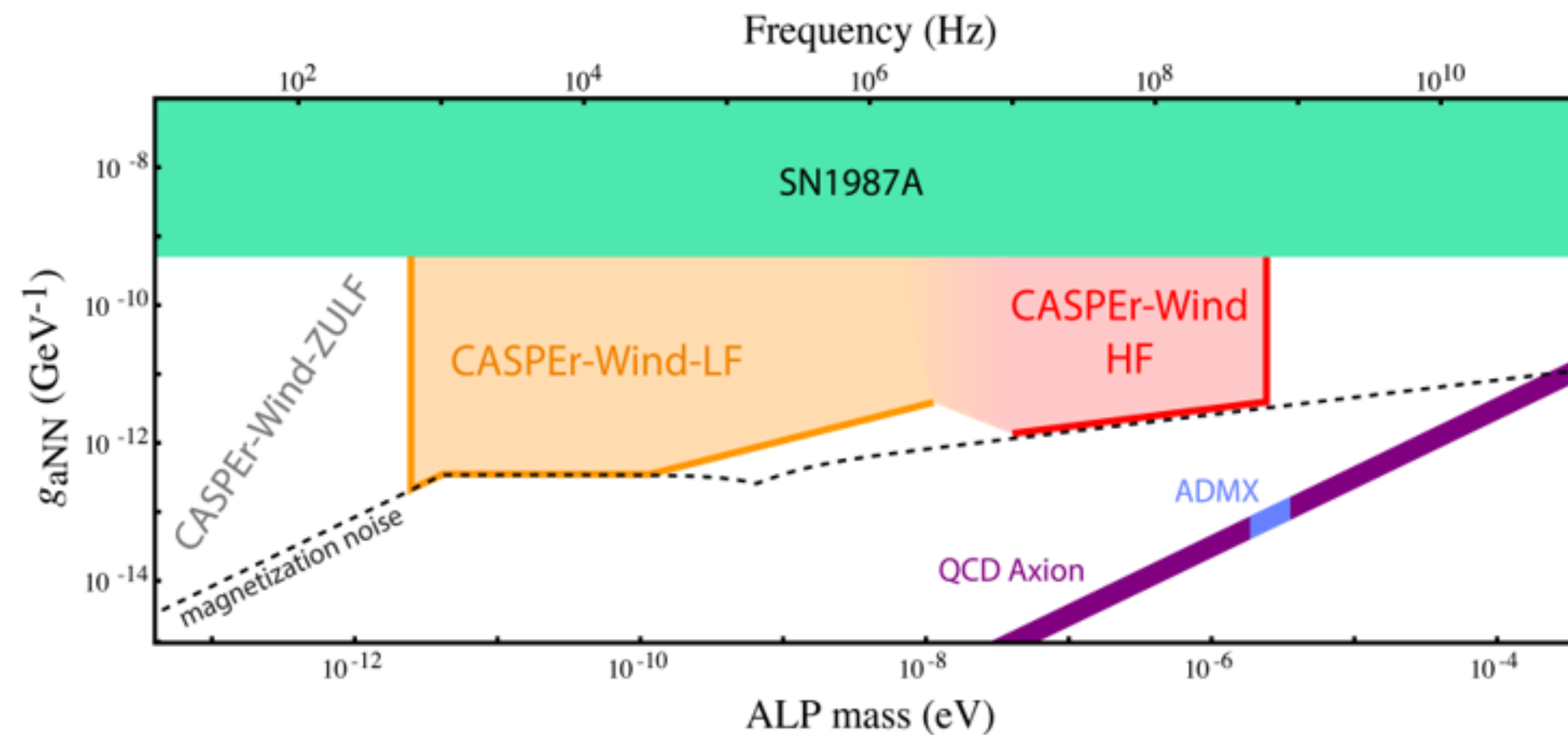
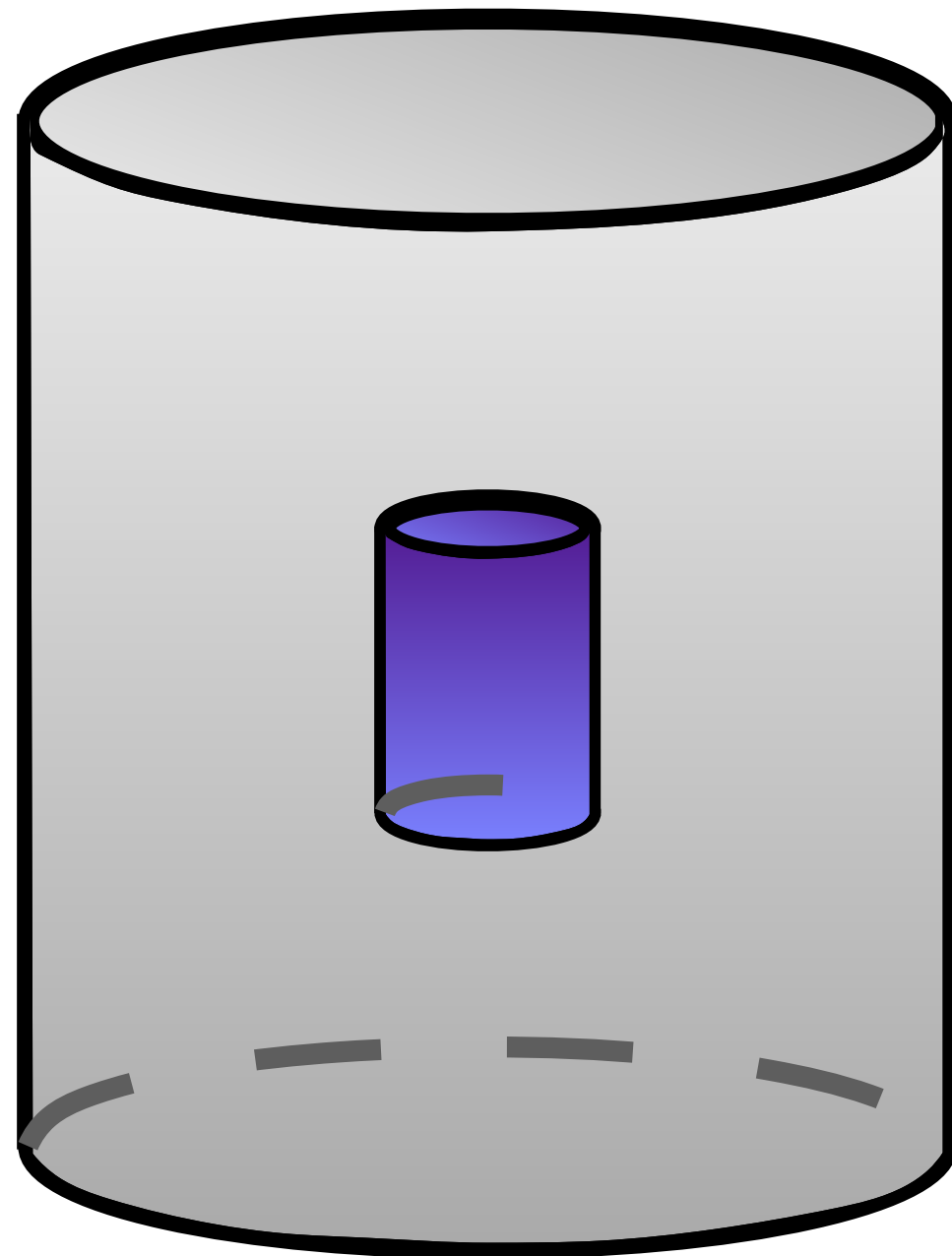
For $m_{\text{DM}} \gtrsim 10^{-10} \text{ eV}$, a copper shield ensures the box provides the relevant BCs.



PRD 113, 092004 (2026)
L. Badurina, D. Blas, J. Ellis, SARE

The box is the Box: CASPEr

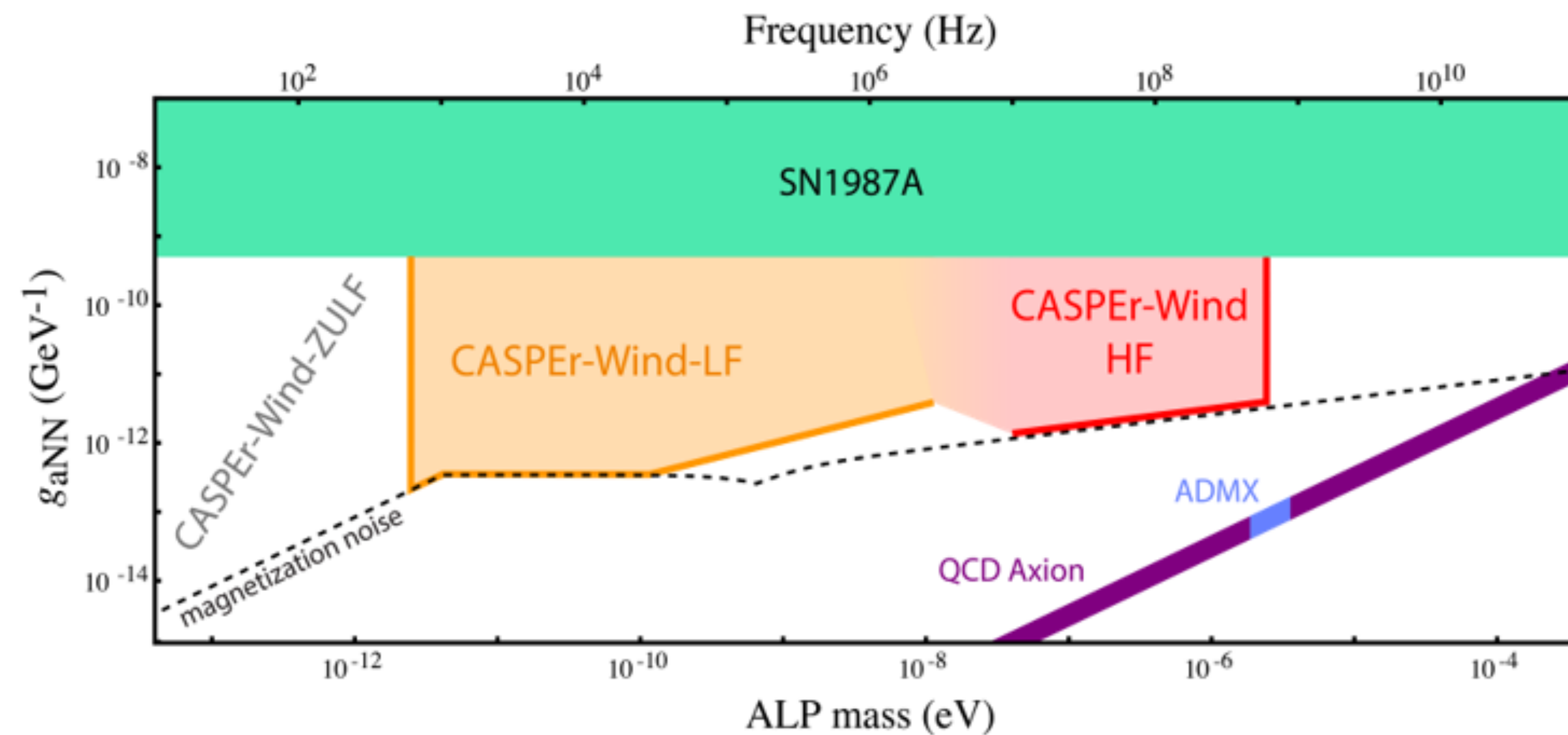
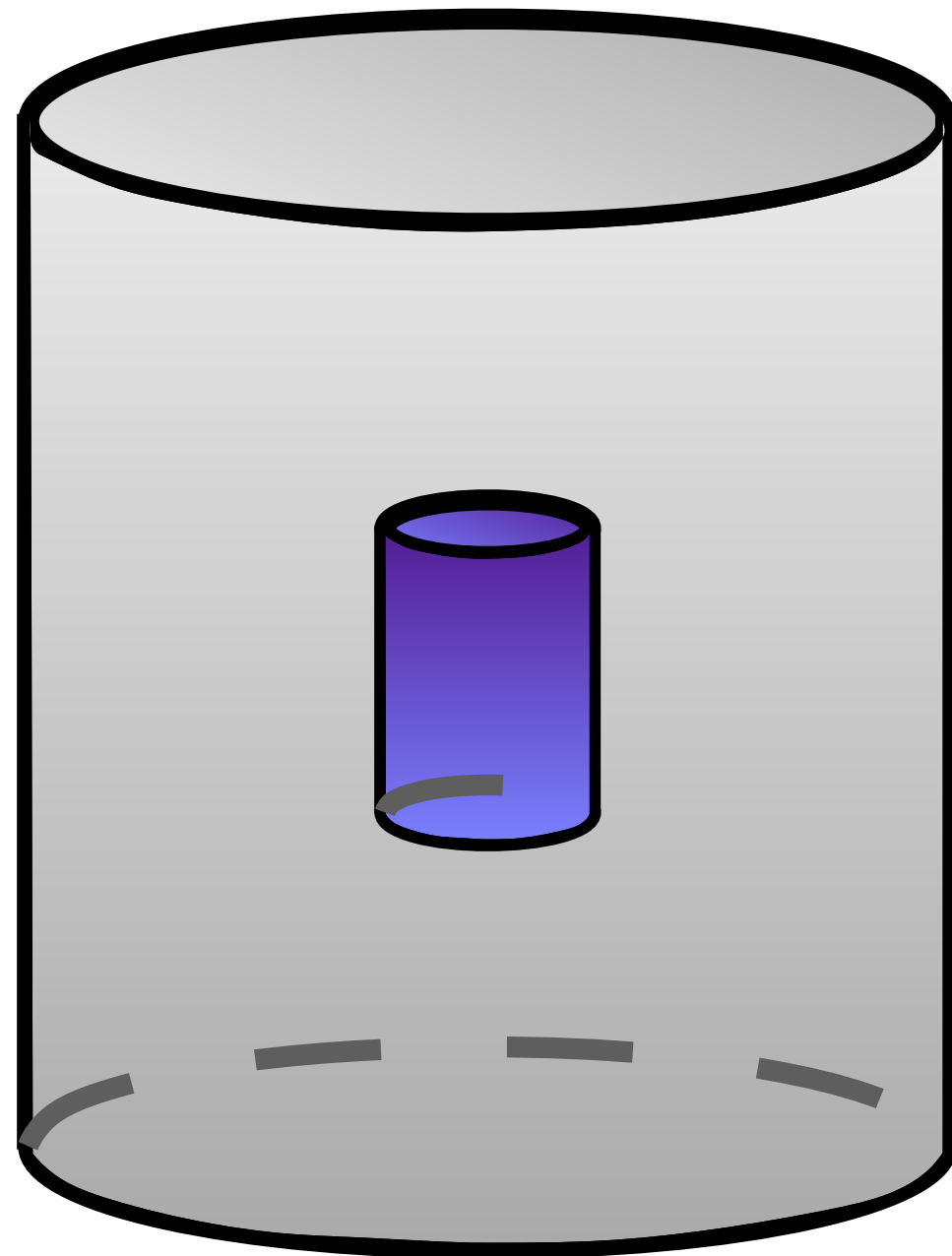
CASPEr-wind searches for $\hat{H}_{\text{int}} = -2 g_N \nabla a \cdot \hat{S}$
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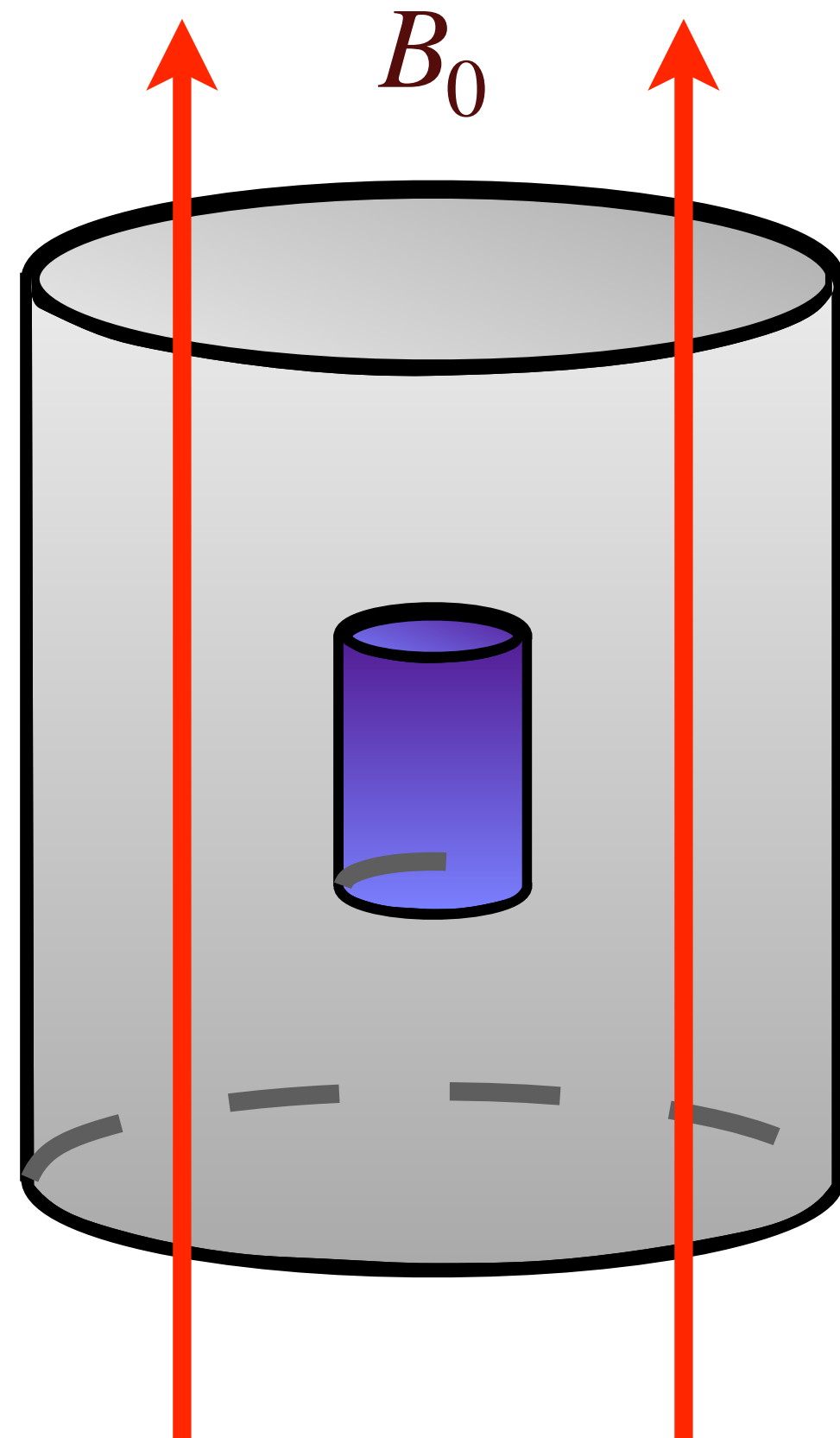
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Hyper-polarisation achieved by applying a big
magnetic field...

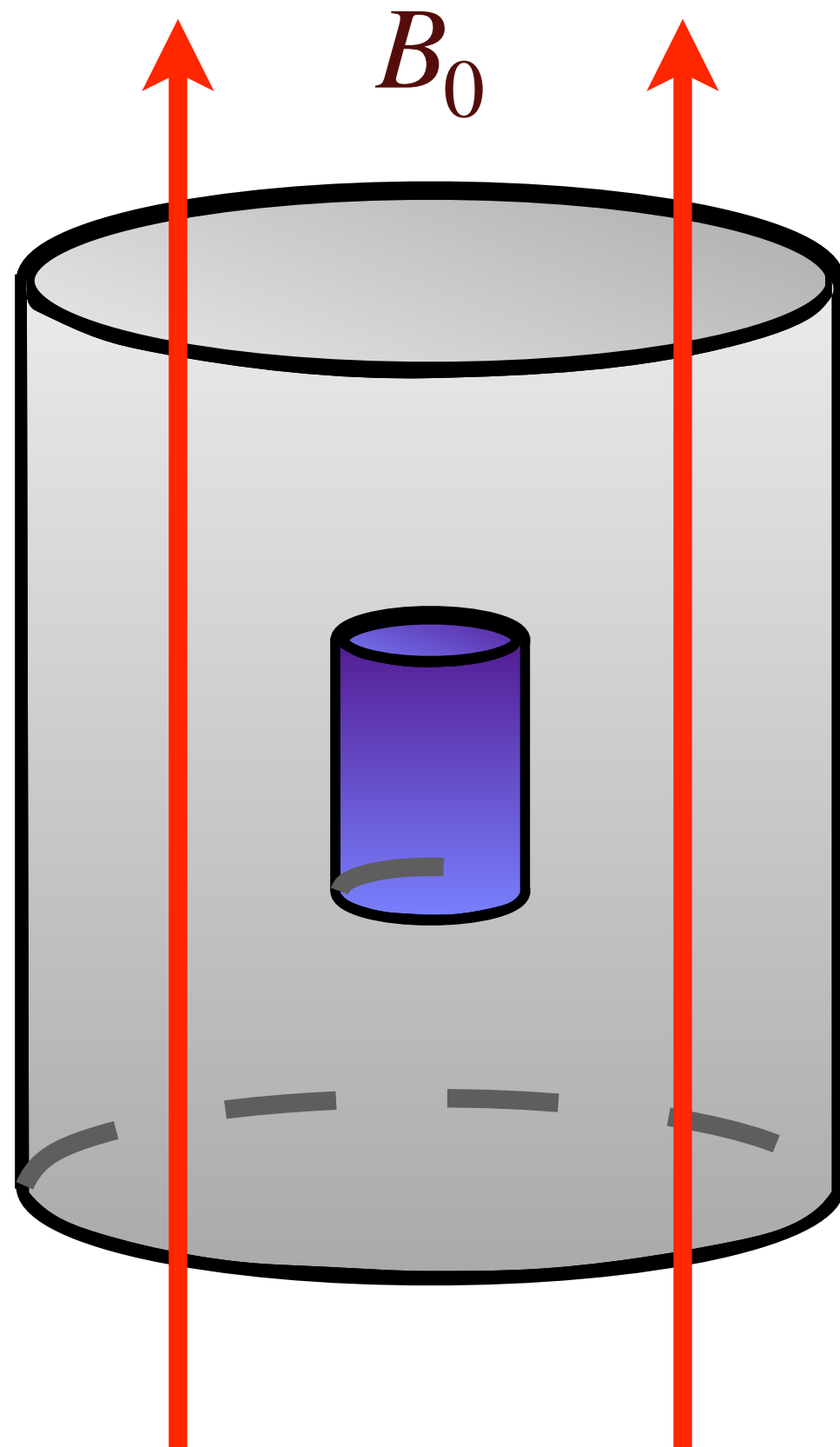
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Box acts as a driven cavity, whose EoMs are

$$\left(\nabla^2 - \varepsilon\mu \partial_t^2\right) \begin{Bmatrix} \mathbf{E}(\mathbf{x}, t) \\ \mathbf{H}(\mathbf{x}, t) \end{Bmatrix} = \begin{Bmatrix} \frac{1}{\varepsilon} (\nabla \rho_{\text{eff}} + \varepsilon\mu \partial_t \mathbf{J}_{\text{eff}}) \\ -\nabla \times \mathbf{J}_{\text{eff}} \end{Bmatrix}.$$

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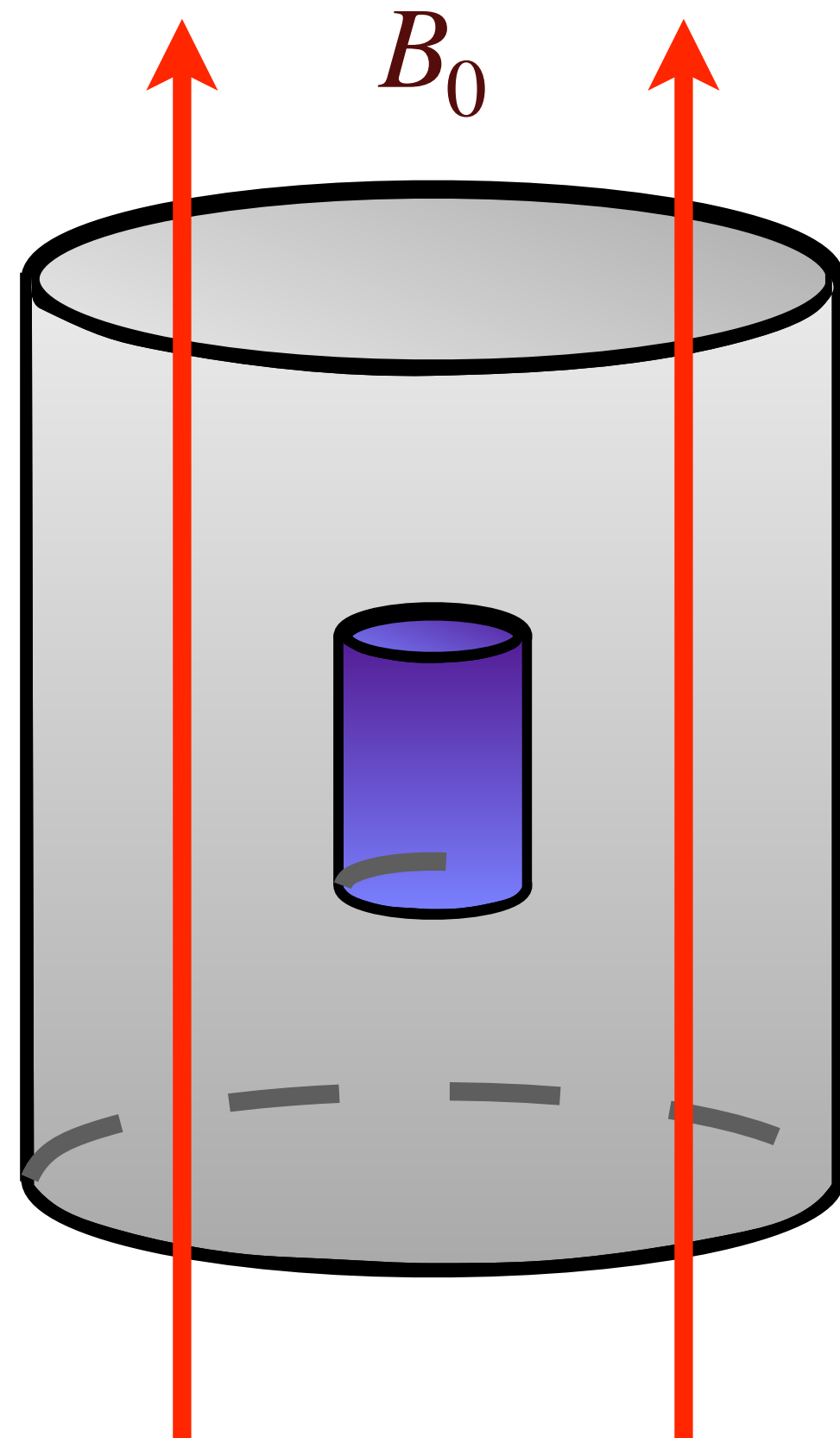
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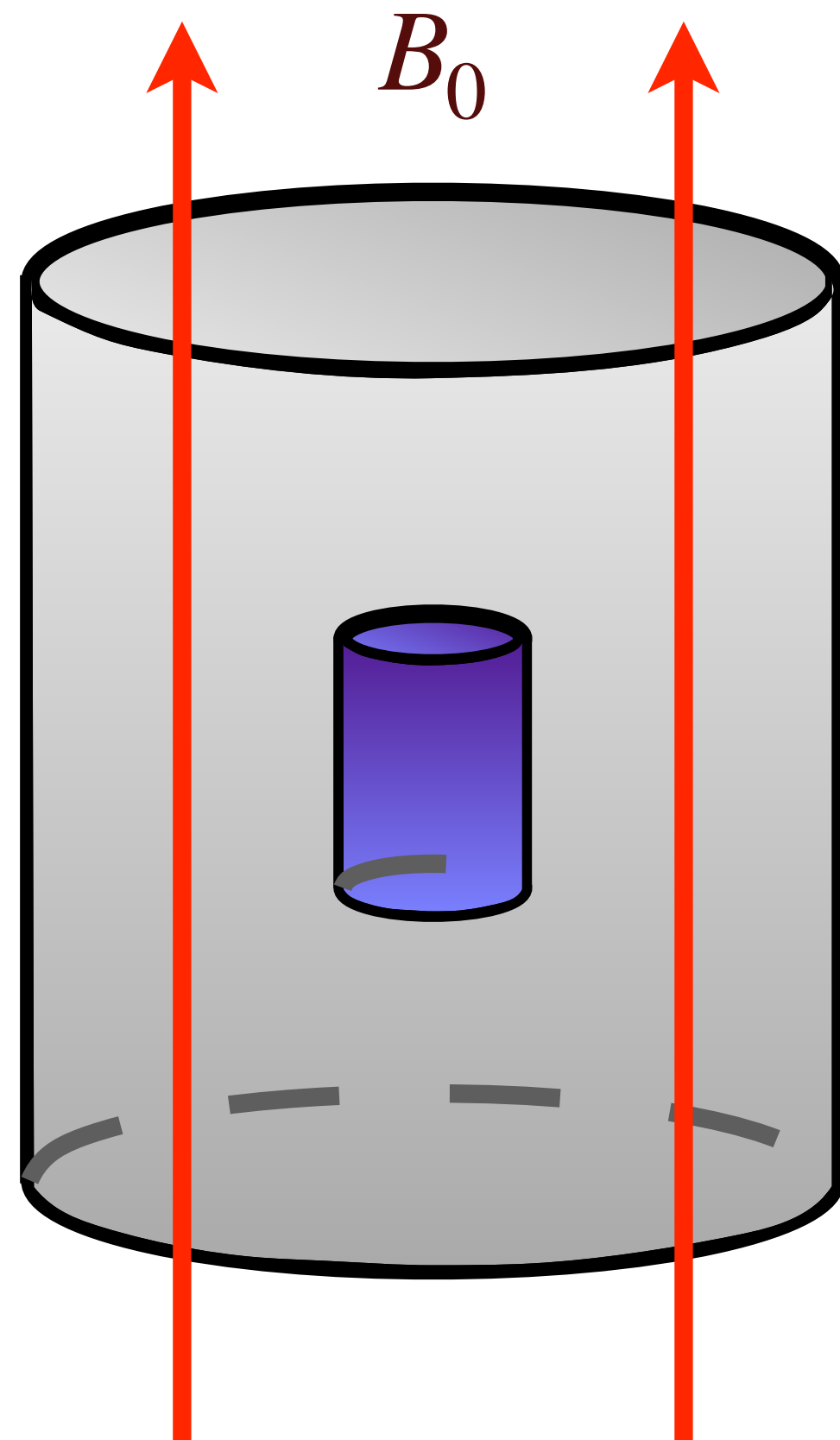
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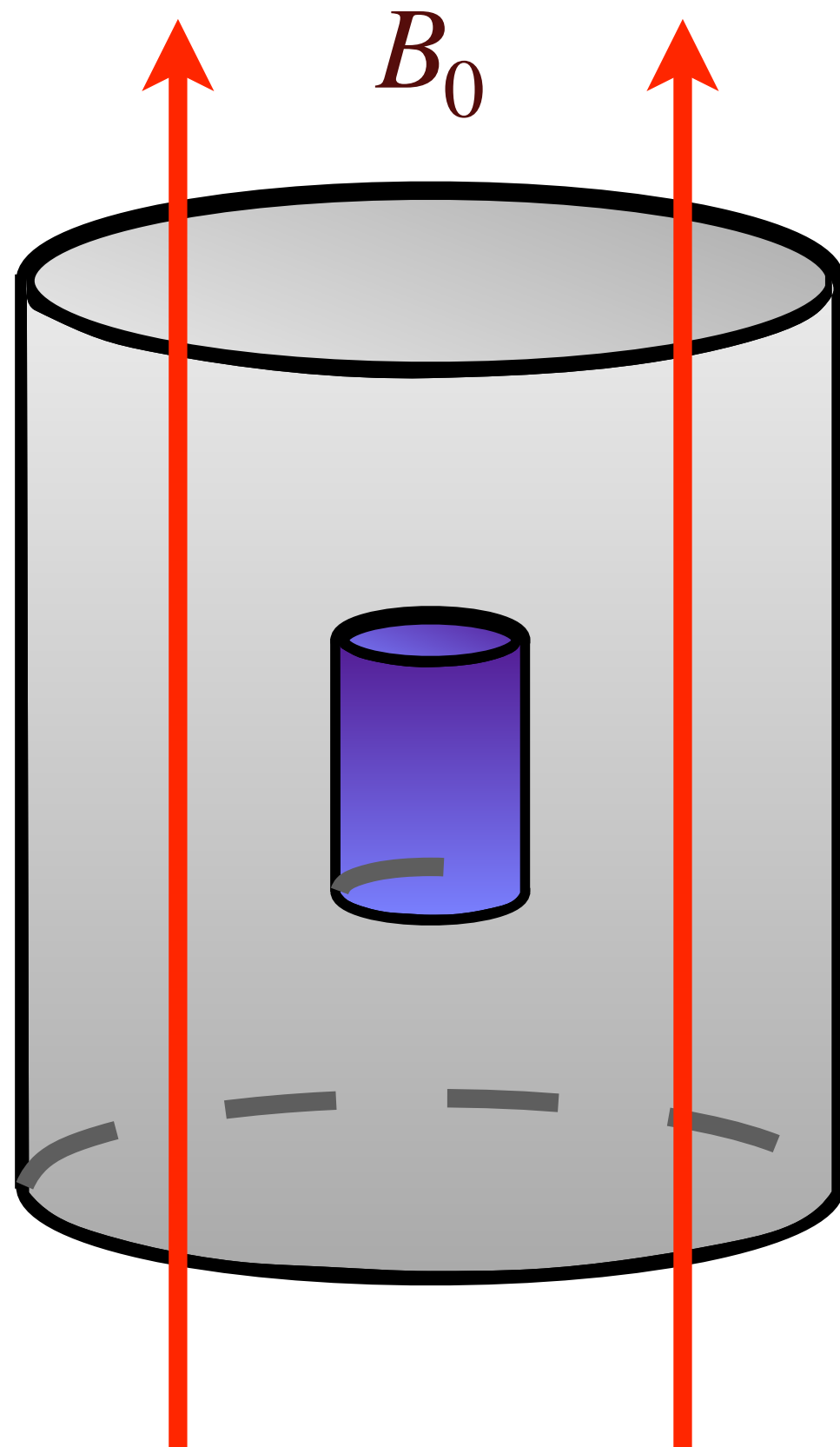


General solution for **electric** fields:

$$\mathbf{E}(\mathbf{x}, t) = \int \frac{d\omega}{2\pi} e^{-i\omega t} \sum_{\ell} c_{\ell}(\omega) \mathbf{E}_{\ell}(\mathbf{x}).$$

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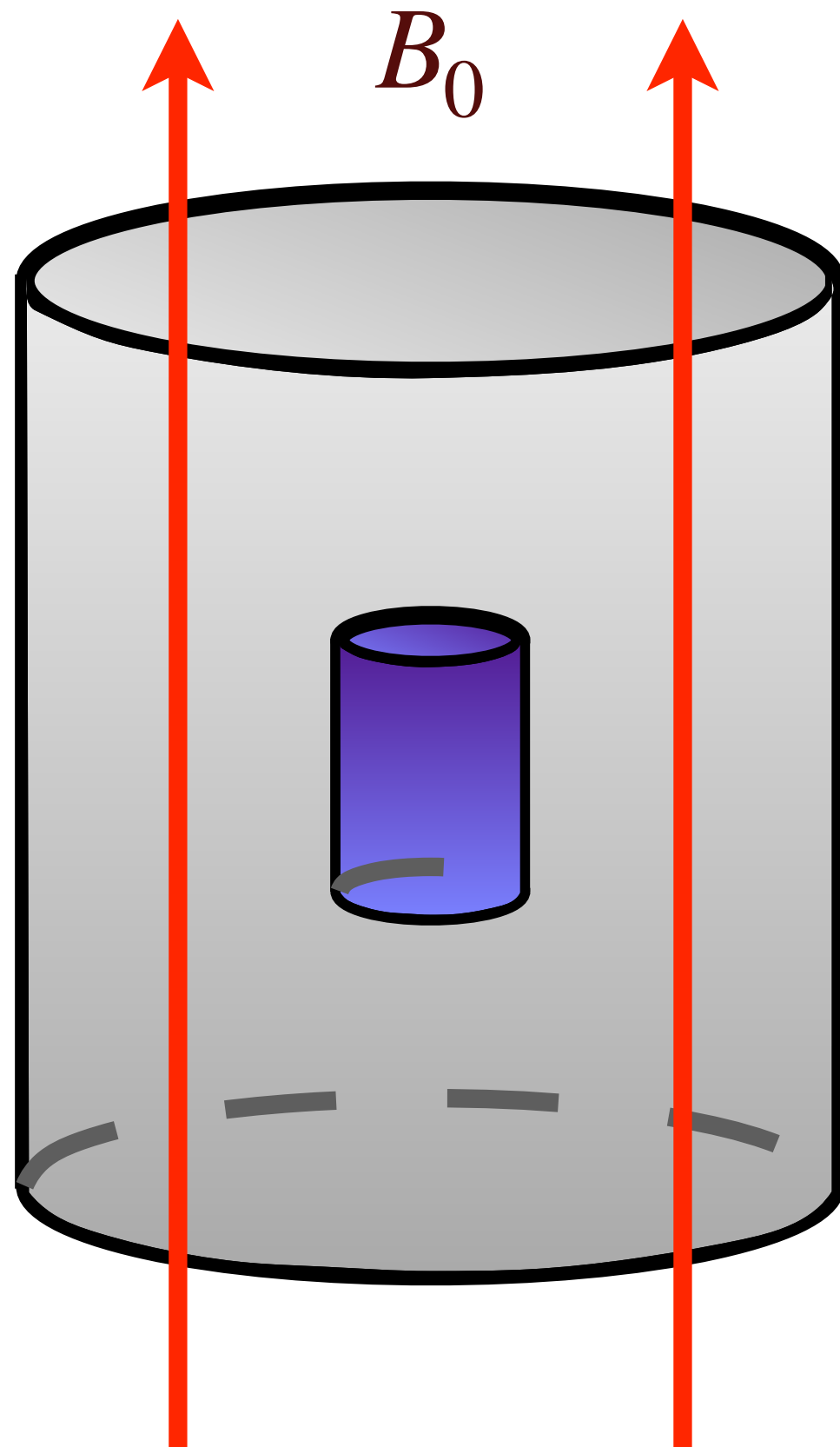
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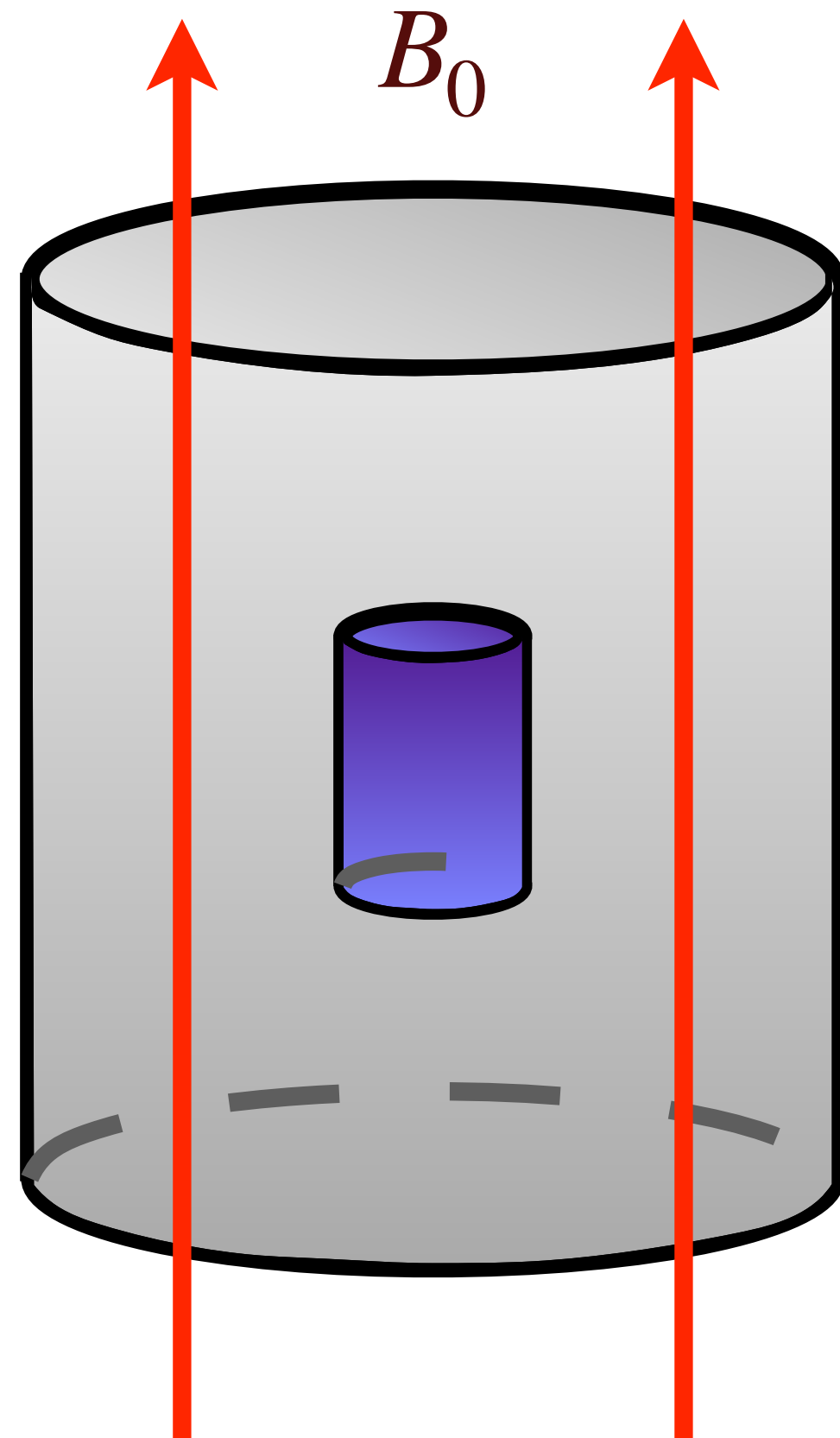
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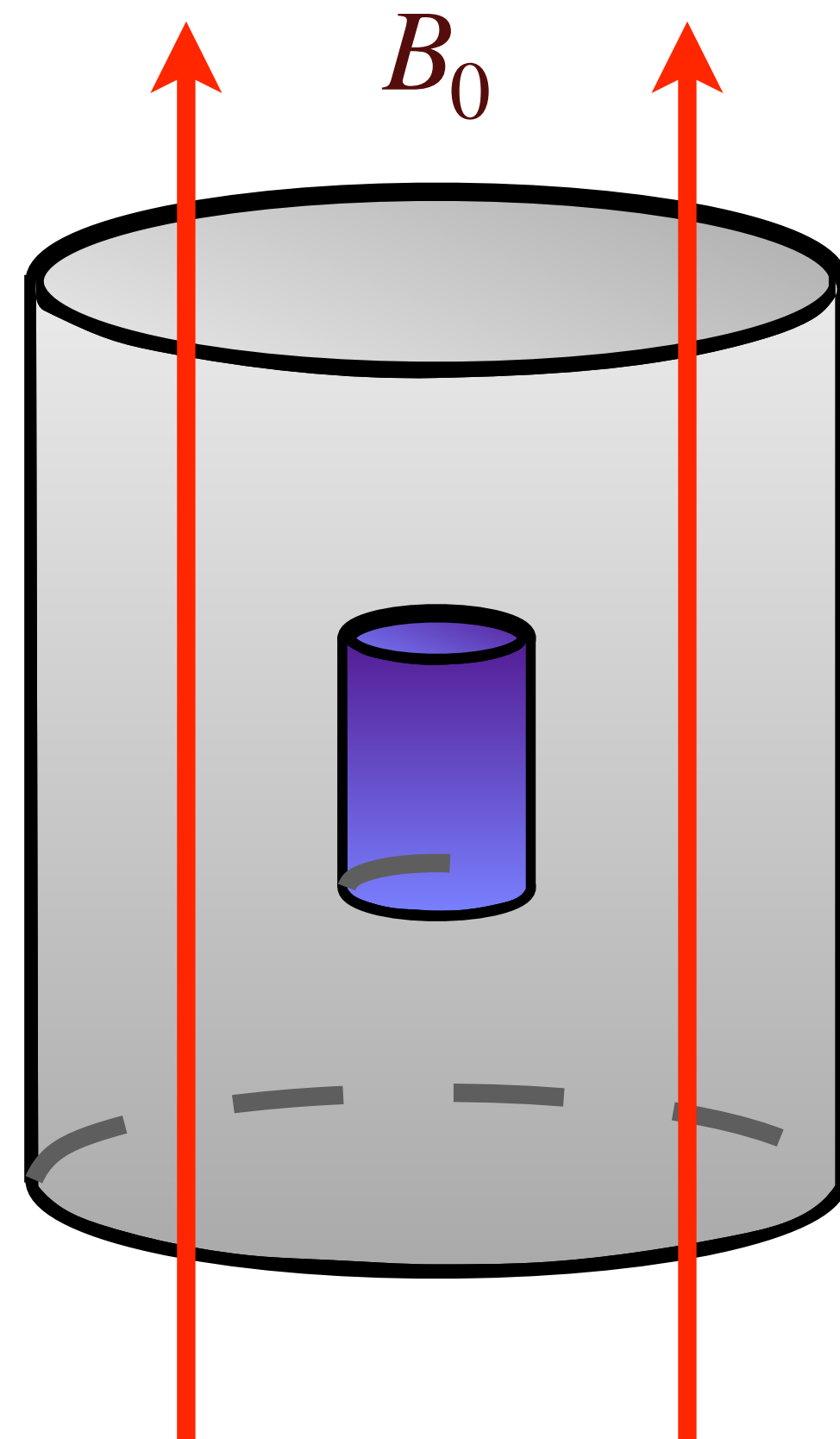
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Magnetic field larger
by $\omega_{\ell}/\omega \gg 1$

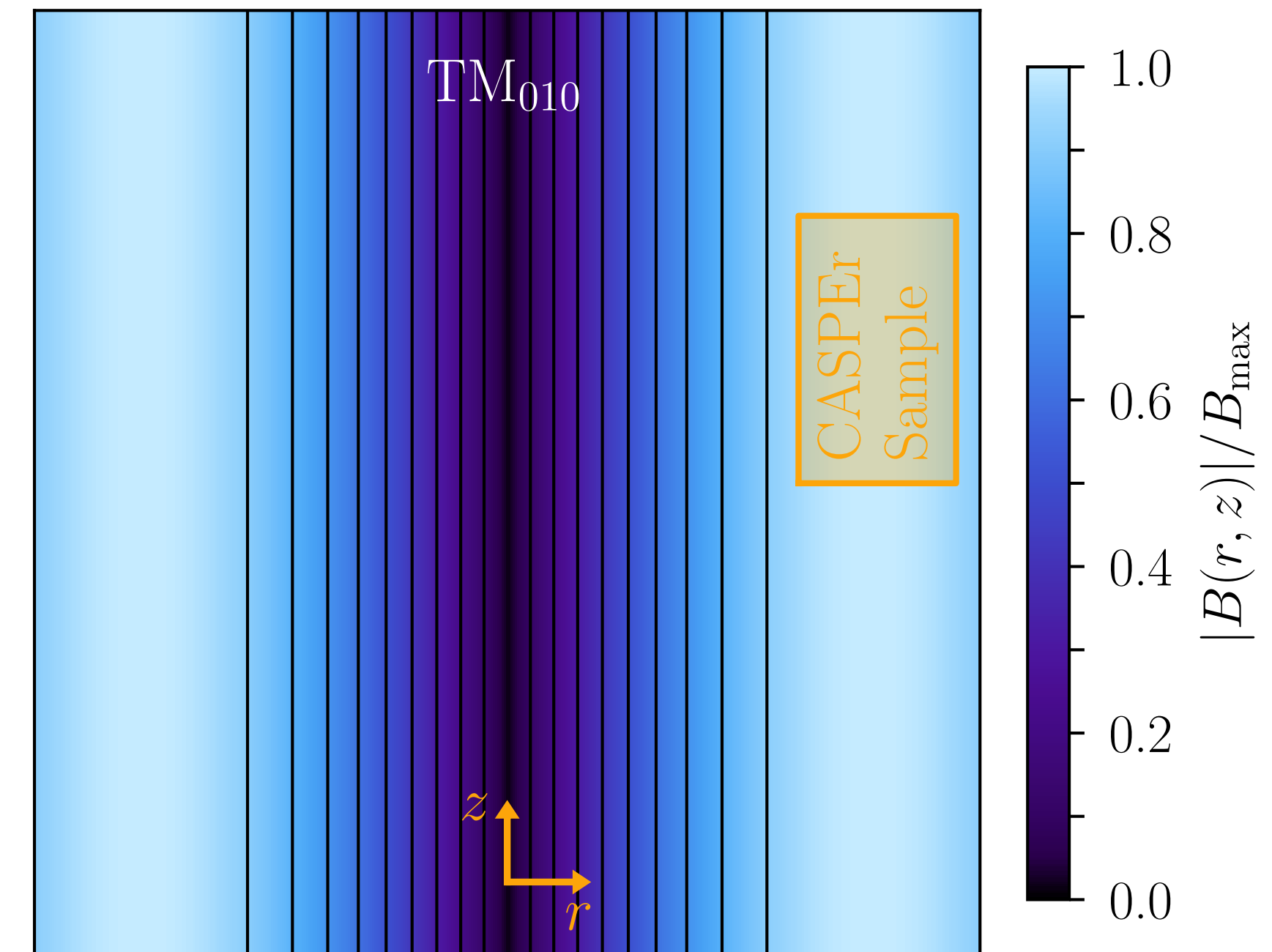
P. Graham, J. Mardon, S. Rajendran,
Y. Zhao (2014)
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Spin Signals — the box is the Box: CASPEr

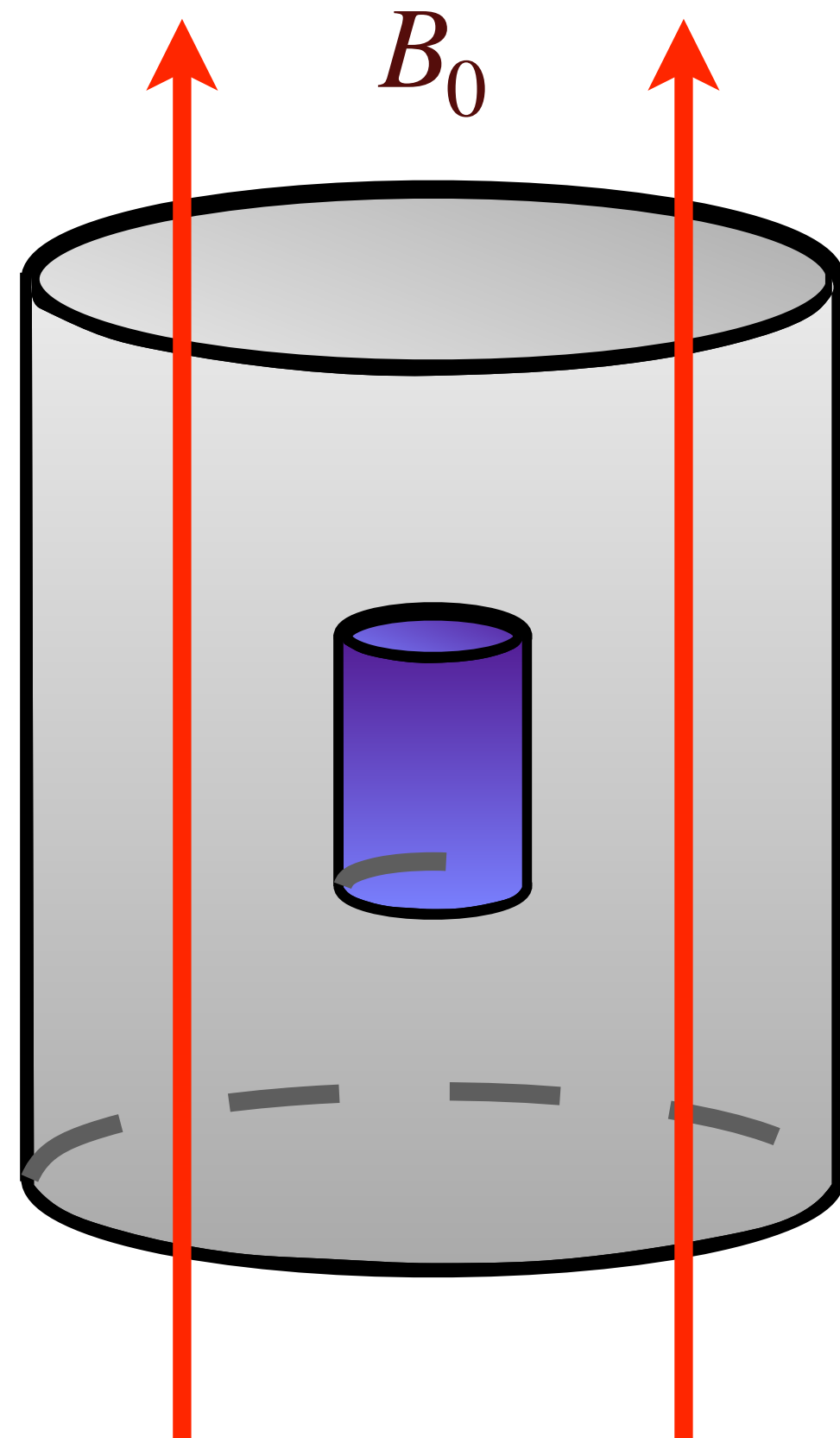


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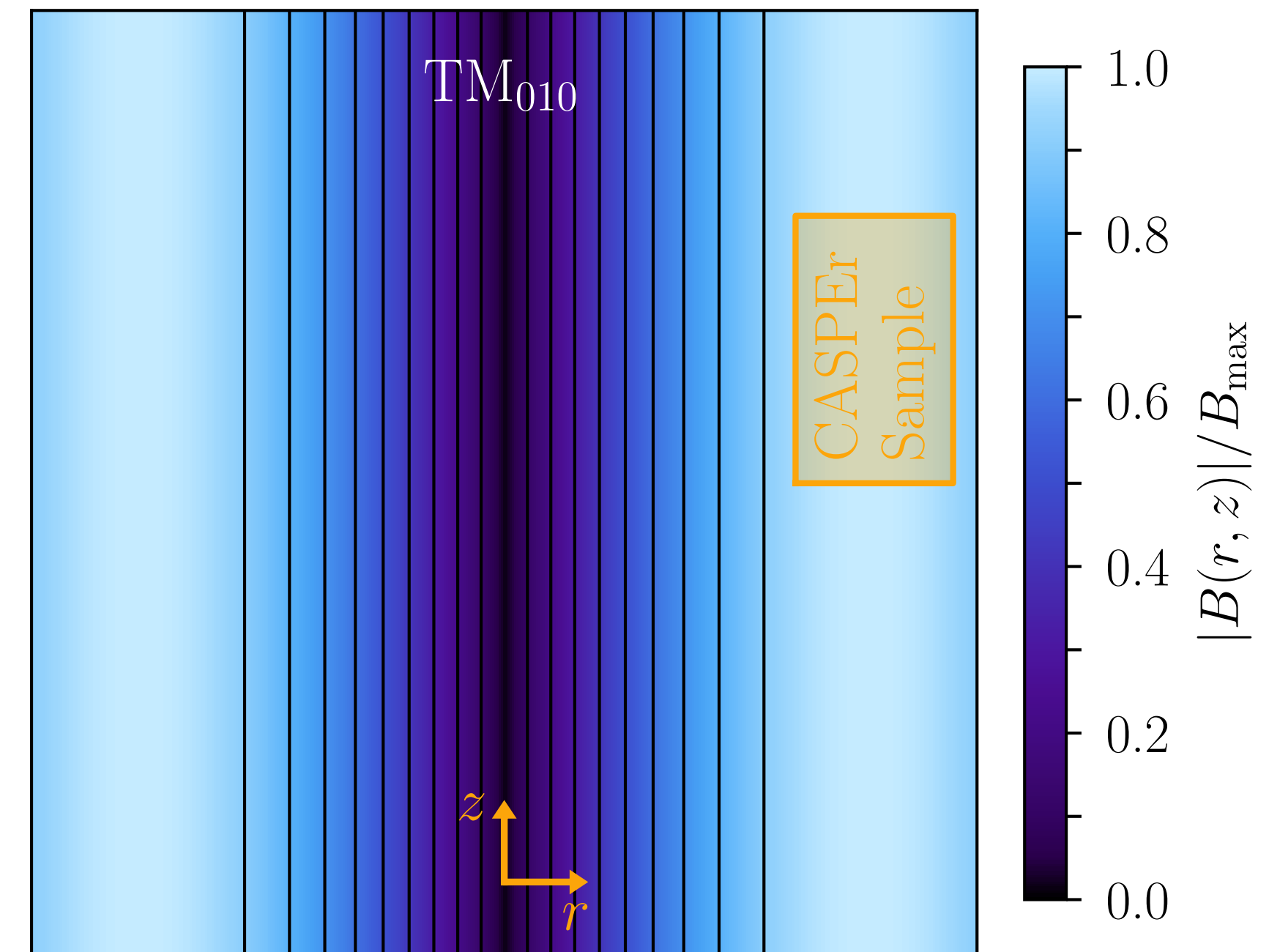


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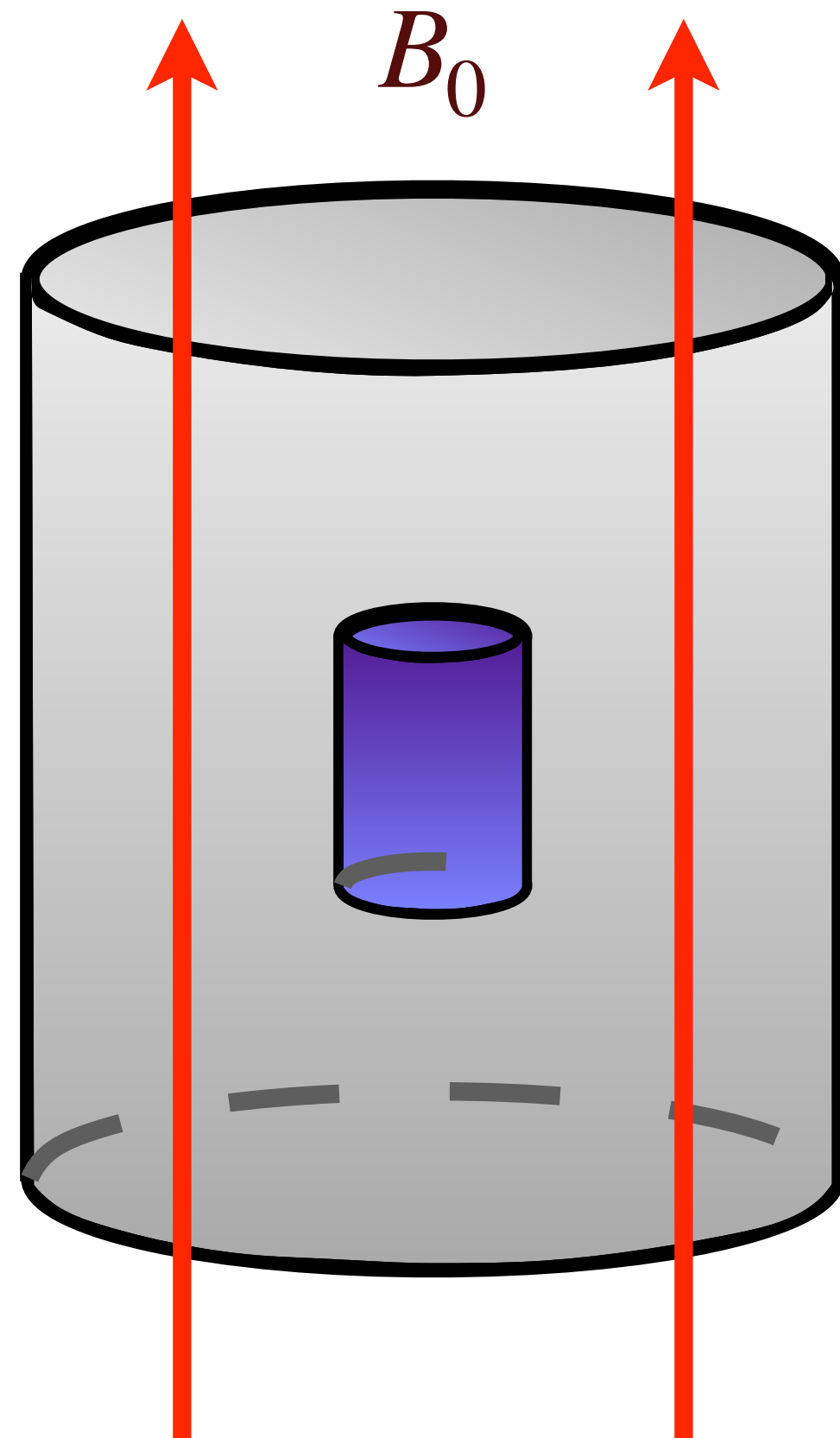
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Dark matter is $\sim$ uniform across the box

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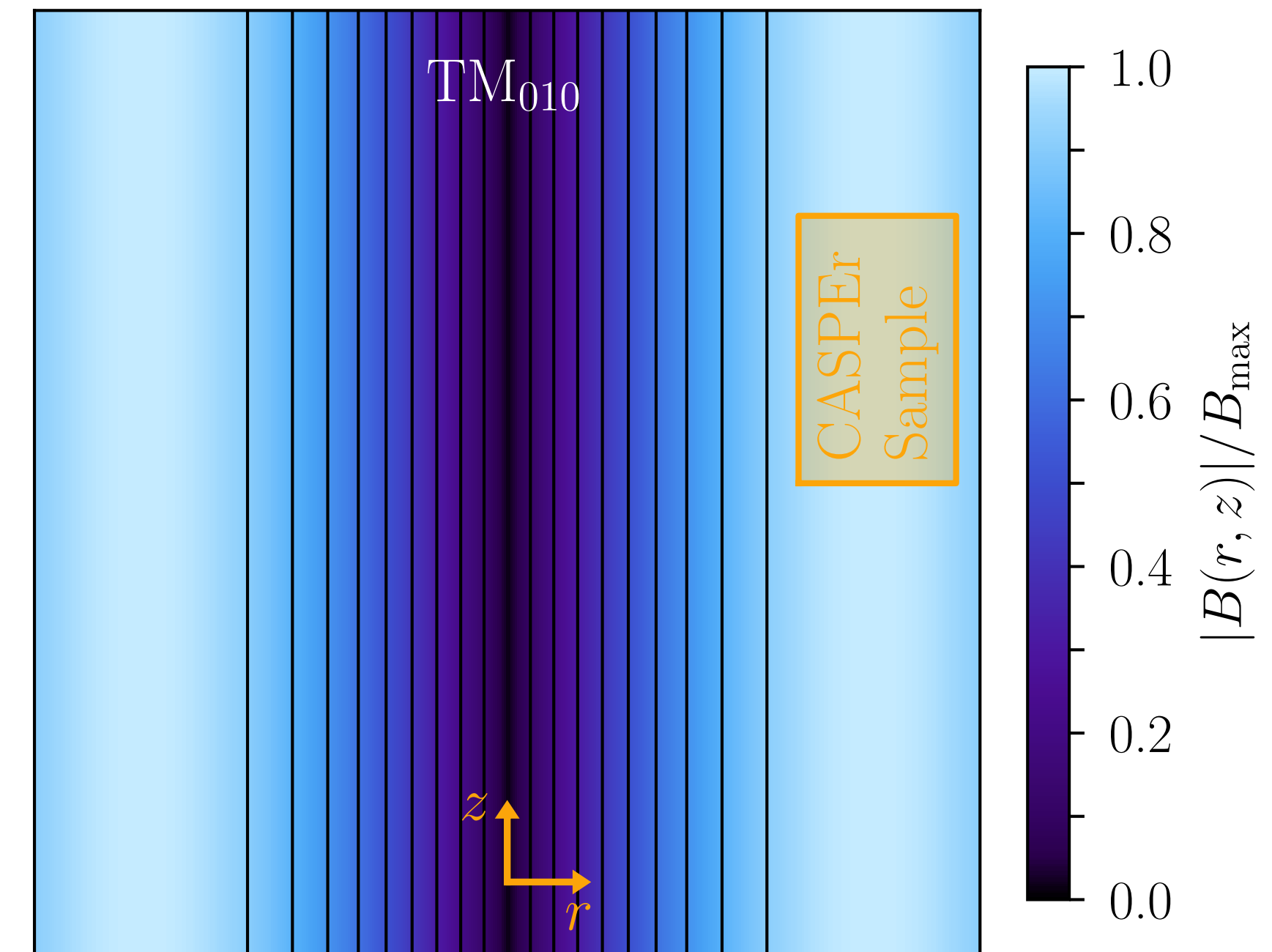
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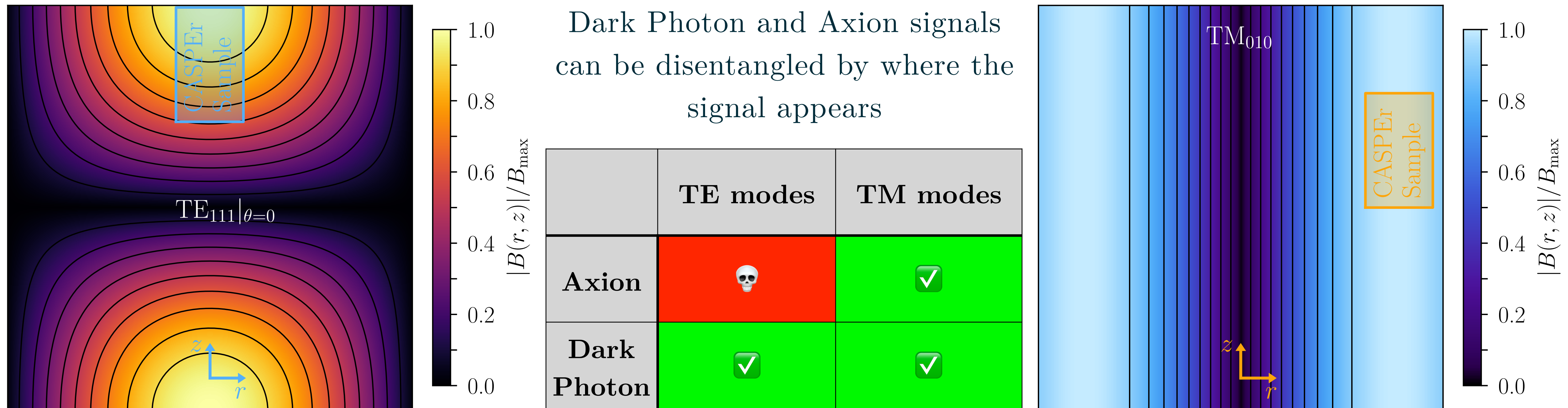
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For Transverse Magnetic modes:

$$\int dV \mathbf{E}_{0n0,M}^* = (-1)^{n+1} \frac{2}{j_{0n}} \sqrt{V}$$



The box is the Box: CASPEr



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Sensitivity for CASPEr-Wind is easily estimated from axion gradient sensitivity

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$$\mathbf{B}_{\text{DP}} \sim \epsilon m_A^2 L \mathbf{A}' \quad \text{or} \quad \mathbf{B}_{a\gamma} \sim g_{a\gamma\gamma} \mathbf{B}_0 L (\partial_t a)$$

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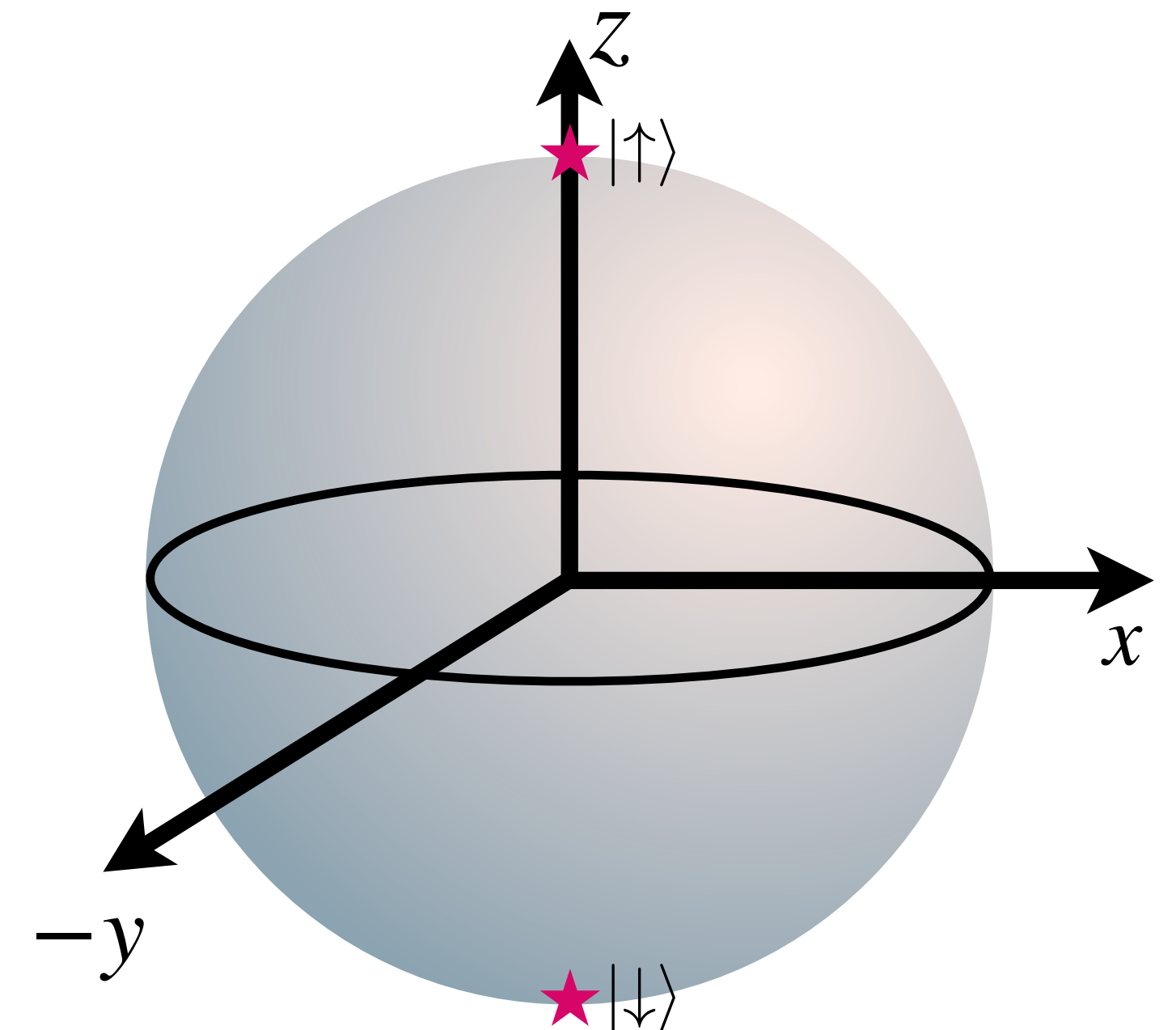
$$\epsilon \sim 10^{-16} c_N \left(\frac{\gamma_{\text{He}}}{\gamma} \right) \left(\frac{10 \text{ cm}}{L} \right) \quad c_\gamma \sim c_N \left(\frac{10 \text{ T}}{B_0} \right) \left(\frac{\gamma_{\text{He}}}{\gamma} \right) \left(\frac{10 \text{ cm}}{L} \right)$$

$$g_{a\gamma\gamma} \equiv c_\gamma \frac{\alpha_{\text{EM}} m_{\text{DM}}}{2\pi m_\pi f_\pi}$$

The box is the Box: CASPEr

More careful analysis requires solving the Bloch equations

$$\dot{\mathbf{M}} = \mathbf{M} \times \gamma \mathbf{H} - \frac{M_x \hat{\mathbf{x}} + M_y \hat{\mathbf{y}}}{T_2} - \frac{(M_z - M_0) \hat{\mathbf{z}}}{T_1}$$

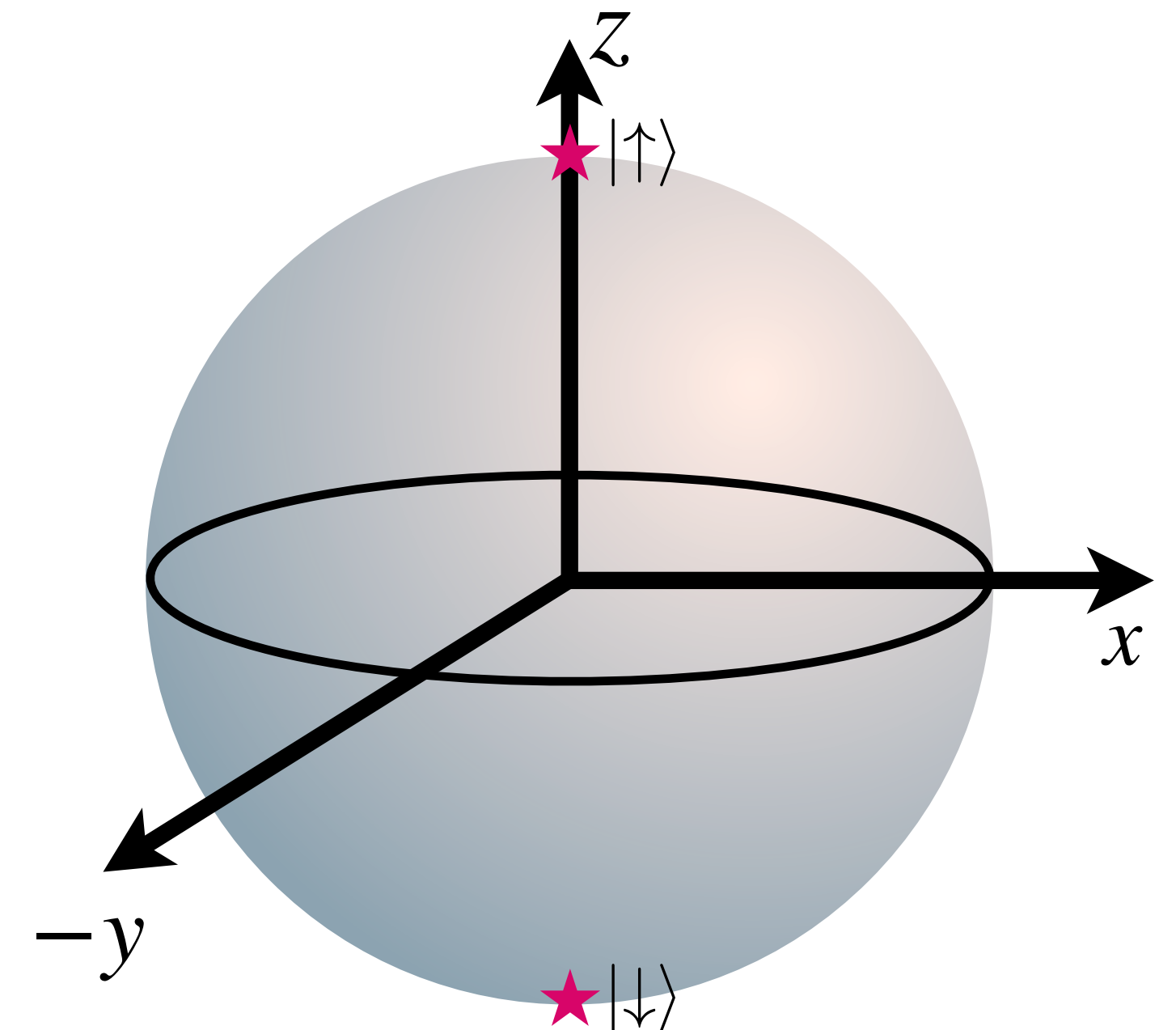


The box is the Box: CASPEr

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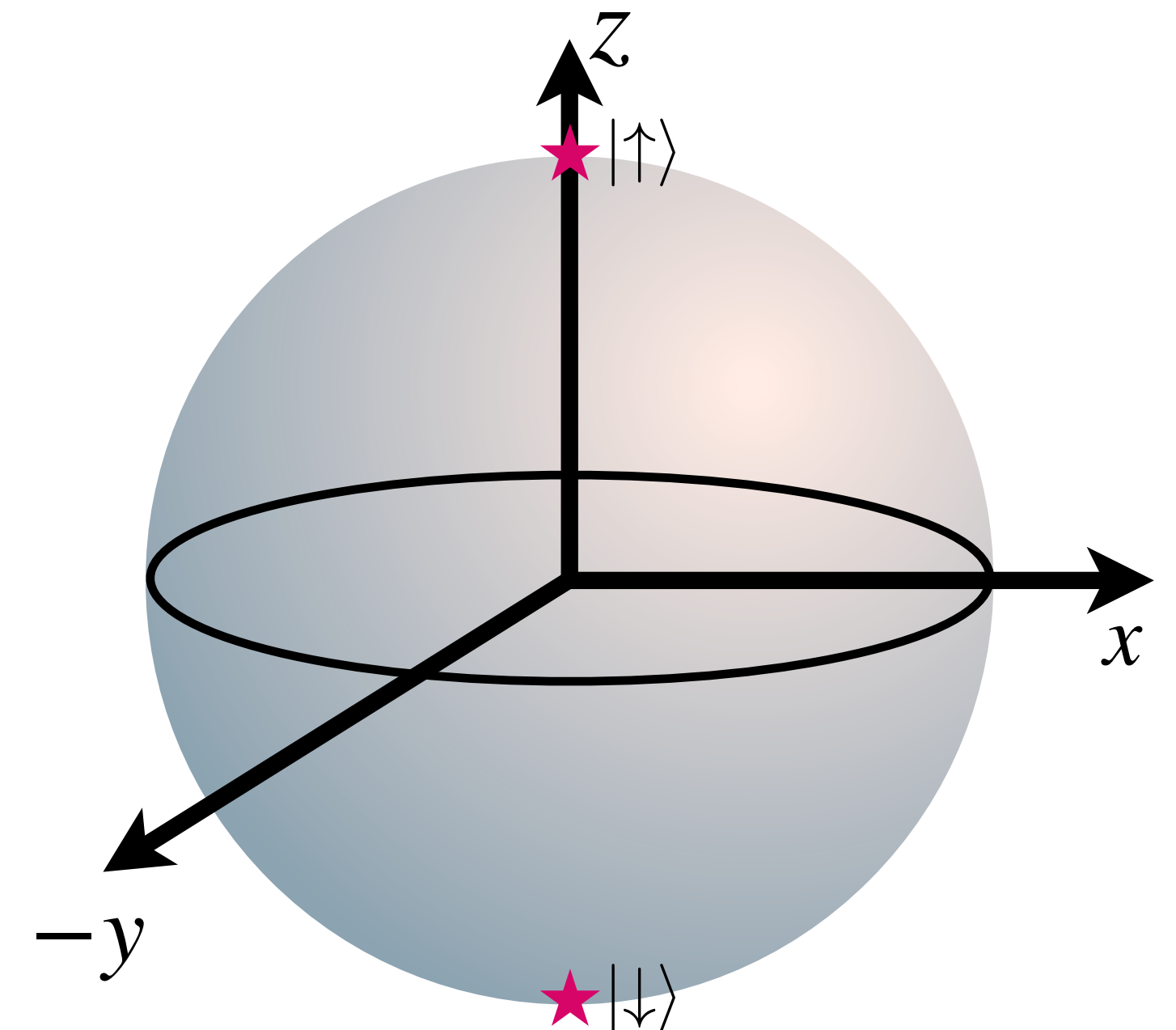
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Time domain solution is simple: $M_+(t) = i\gamma M_0 \int_0^t dt' e^{-(t-t')/T_2} e^{-i\omega_0(t-t')} H_{\text{DM}}^+(t')$



The box is the Box: CASPEr

Signal 2-pt function is non-zero for DM signals

$$\langle M_+(t)^2 \rangle \propto (\gamma M_0 T_2)^2 \langle H_{\text{DM}}^+(t)^2 \rangle$$

SNR comes from comparing this signal to spin-projection noise

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Usual spin state is $|\uparrow\rangle^{\otimes N}$, held fixed by an applied magnetic field

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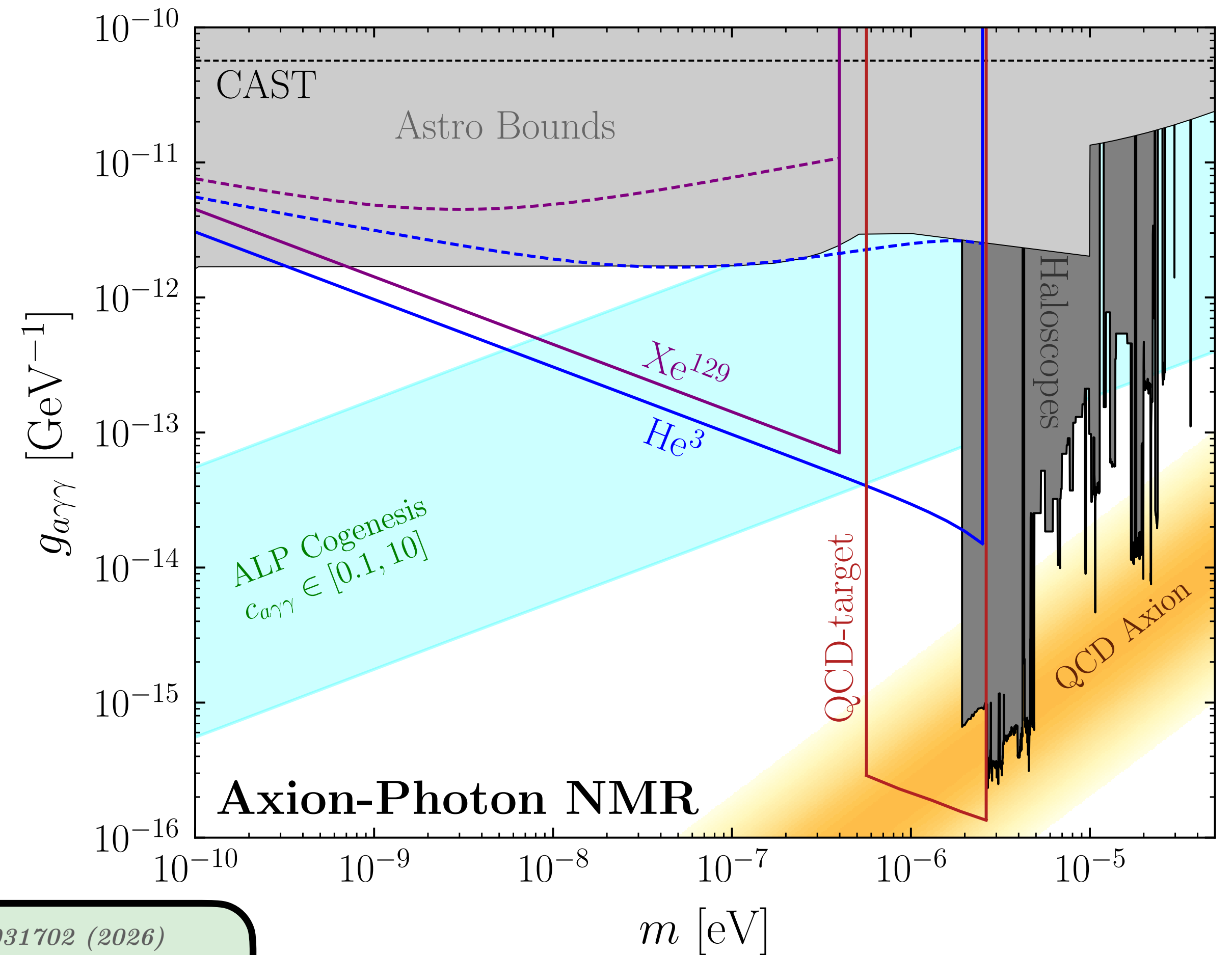
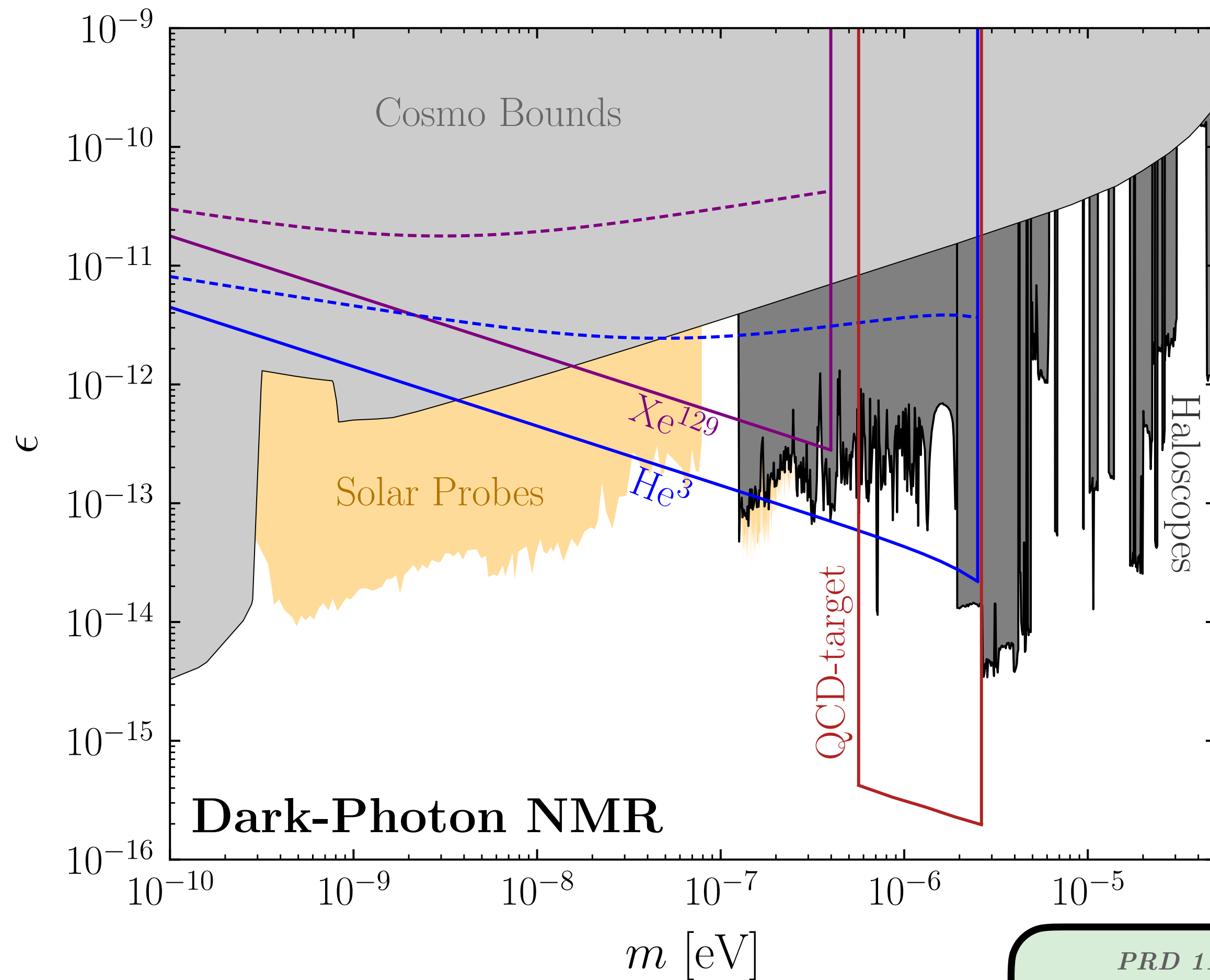
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Quantum Cramér-Rao bound: $\sigma_\theta^2 \geq \frac{1}{\nu F_Q}$ Recall $MV = \sum_i \gamma s_i \sim N\gamma$

Compare the above with the previous slide: $H_{\text{DM}} \propto n^{-1/2} \gamma^{-1}$ — **Standard Quantum Limit** scaling

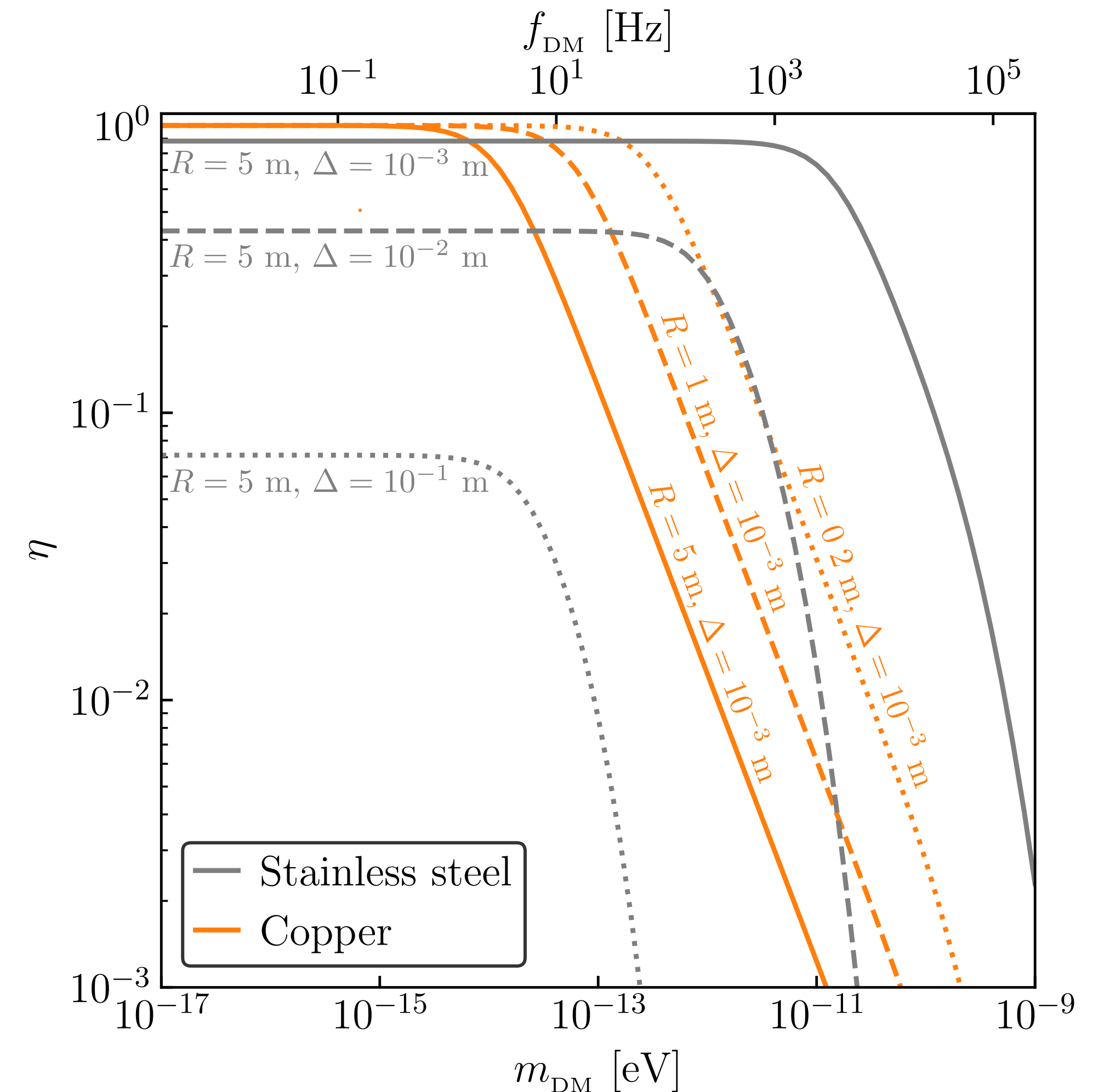
The box is the Box: CASPEr projections



PRD 113, L031702 (2026)
C. Beadle, SARE, J. Leedom, N. L. Rodd

The box is NOT the Box: Ion Interferometry

Below $\sim 10^{-13}$ eV, unless you use a material with high magnetic permeability, the box is not the Box

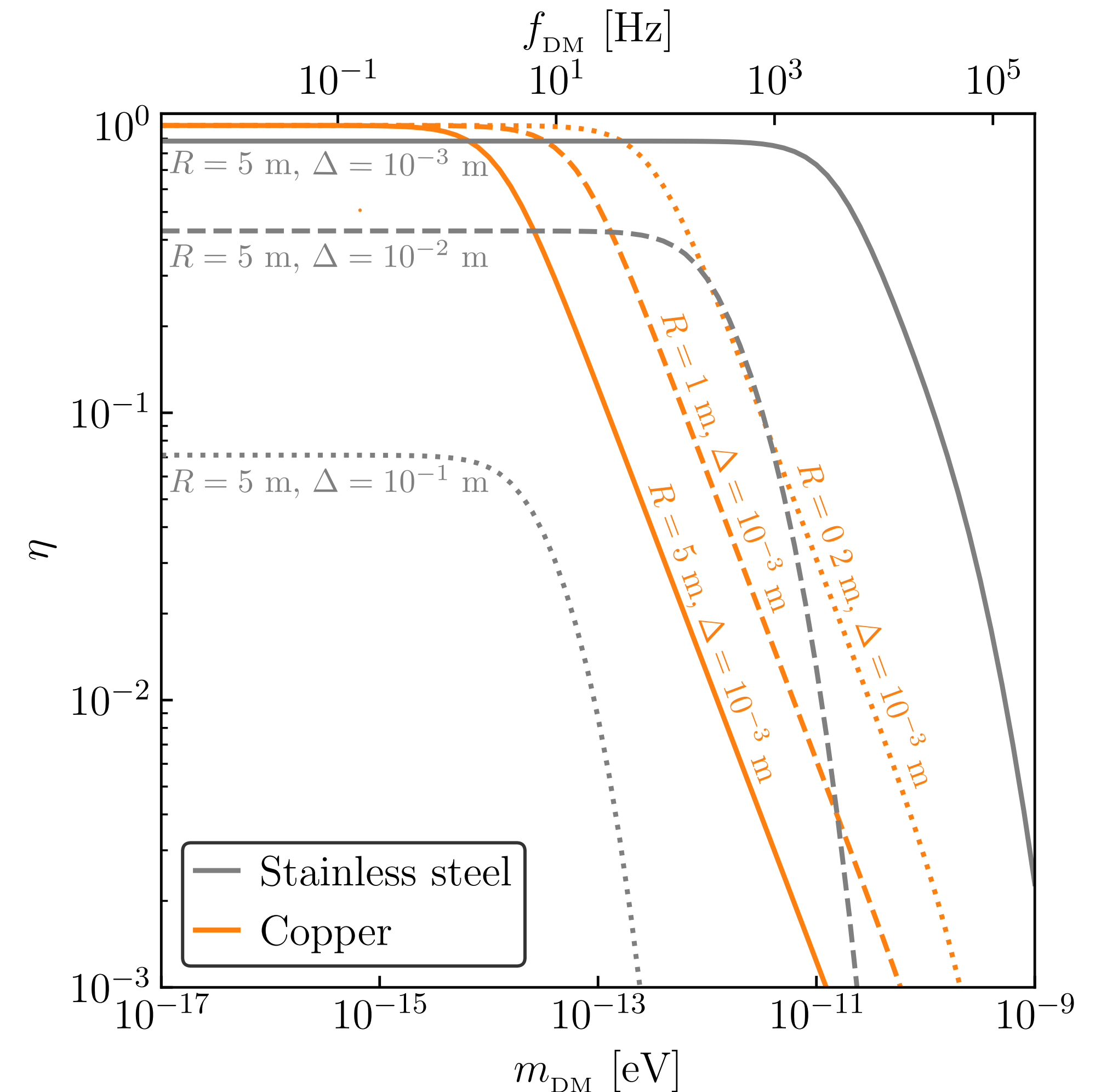


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Earth's ionosphere is the Box

M. Fedderke, P. Graham, D. Jackson
Kimball, S. Kalia (2021)
A. Arza, M. Fedderke, P. Graham, D.
Jackson Kimball, S. Kalia (2022)



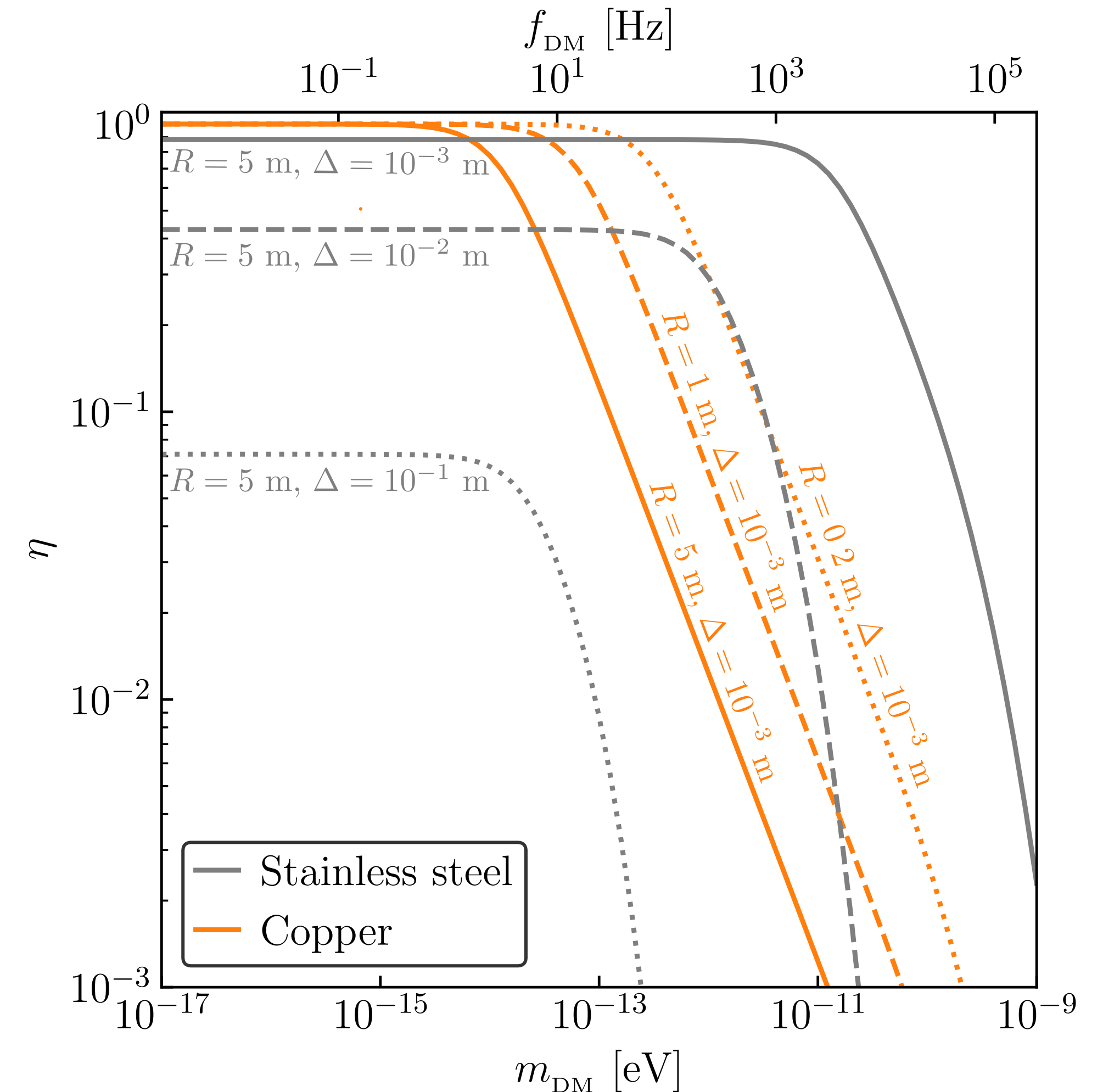
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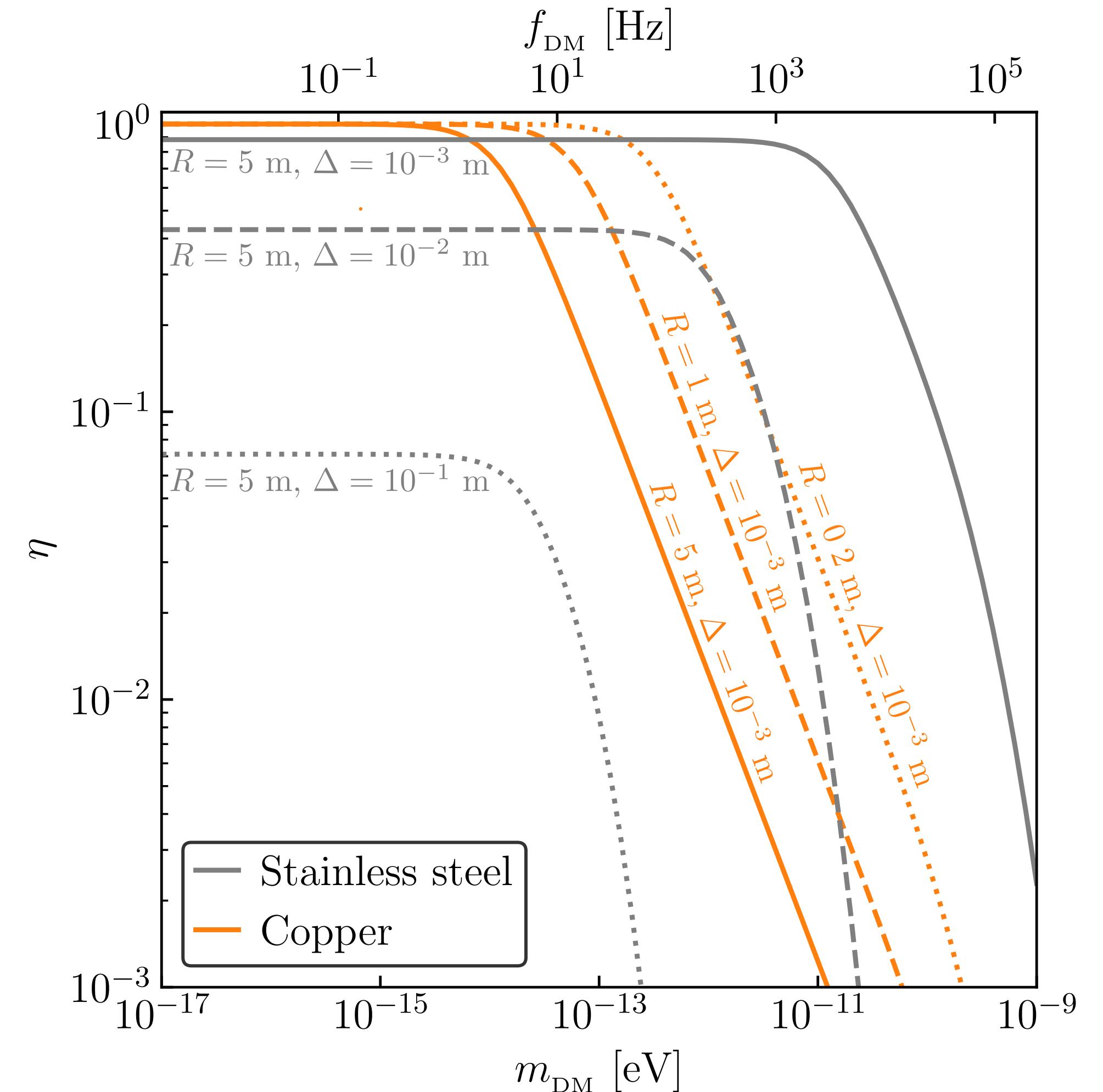
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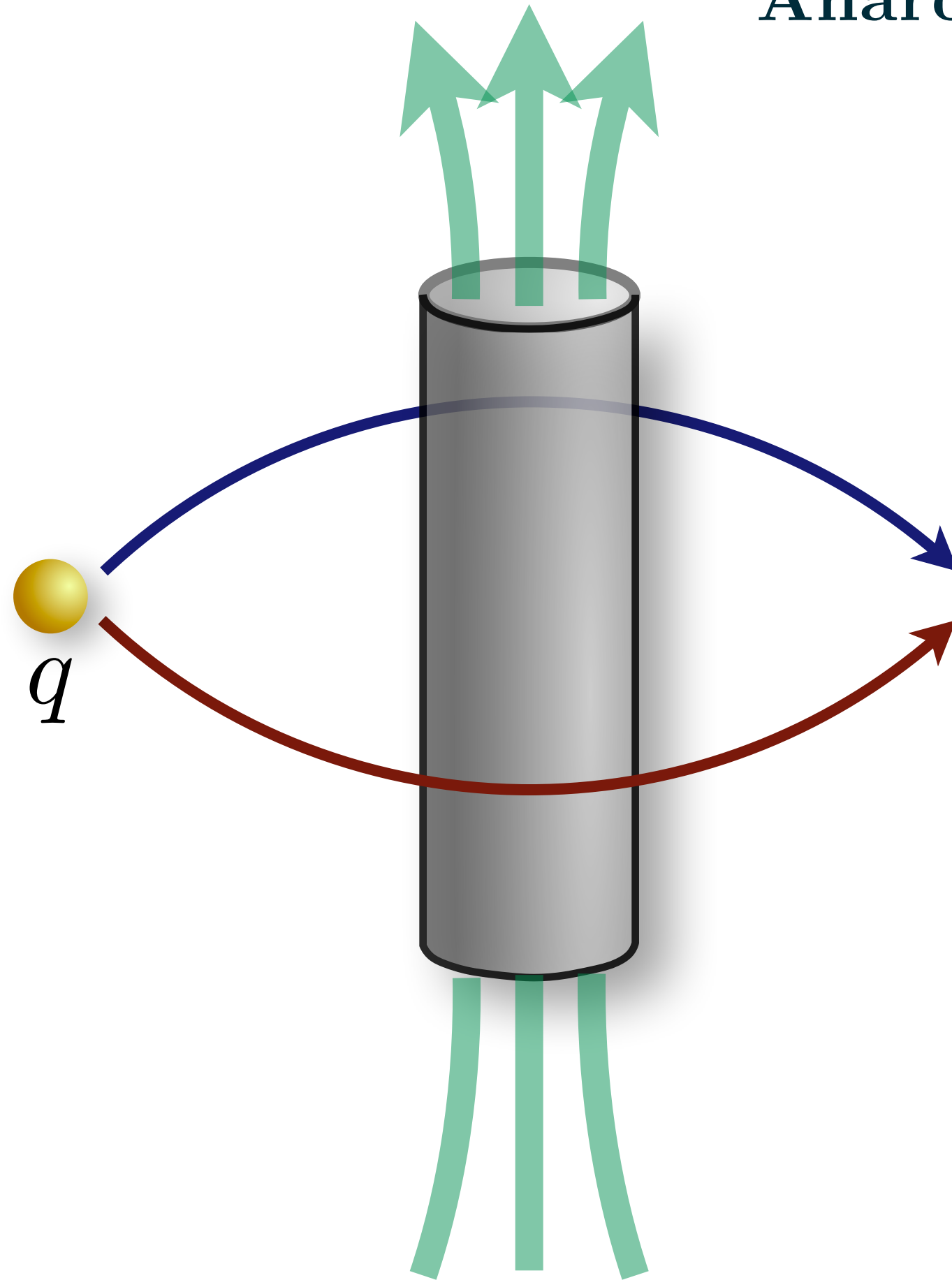
Schumann resonances appear at $\omega_l \simeq \sqrt{l(l+1)} \frac{1}{R_\oplus} \gtrsim 10$ Hz

Below 10 Hz, we can treat the Earth-ionosphere Box as concentric spherical conductive boundaries



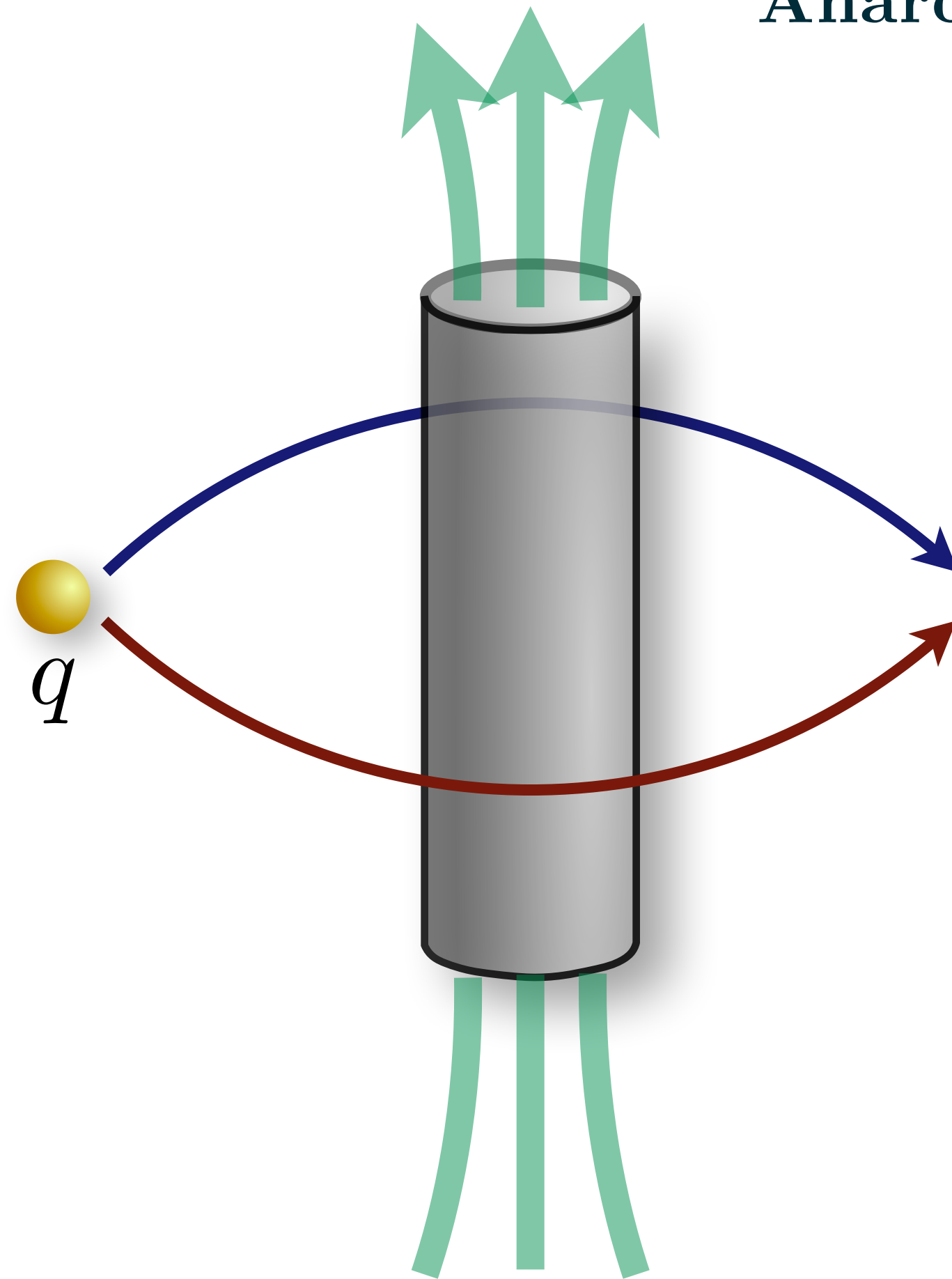
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Aharonov-Bohm: a specific version of a Berry phase



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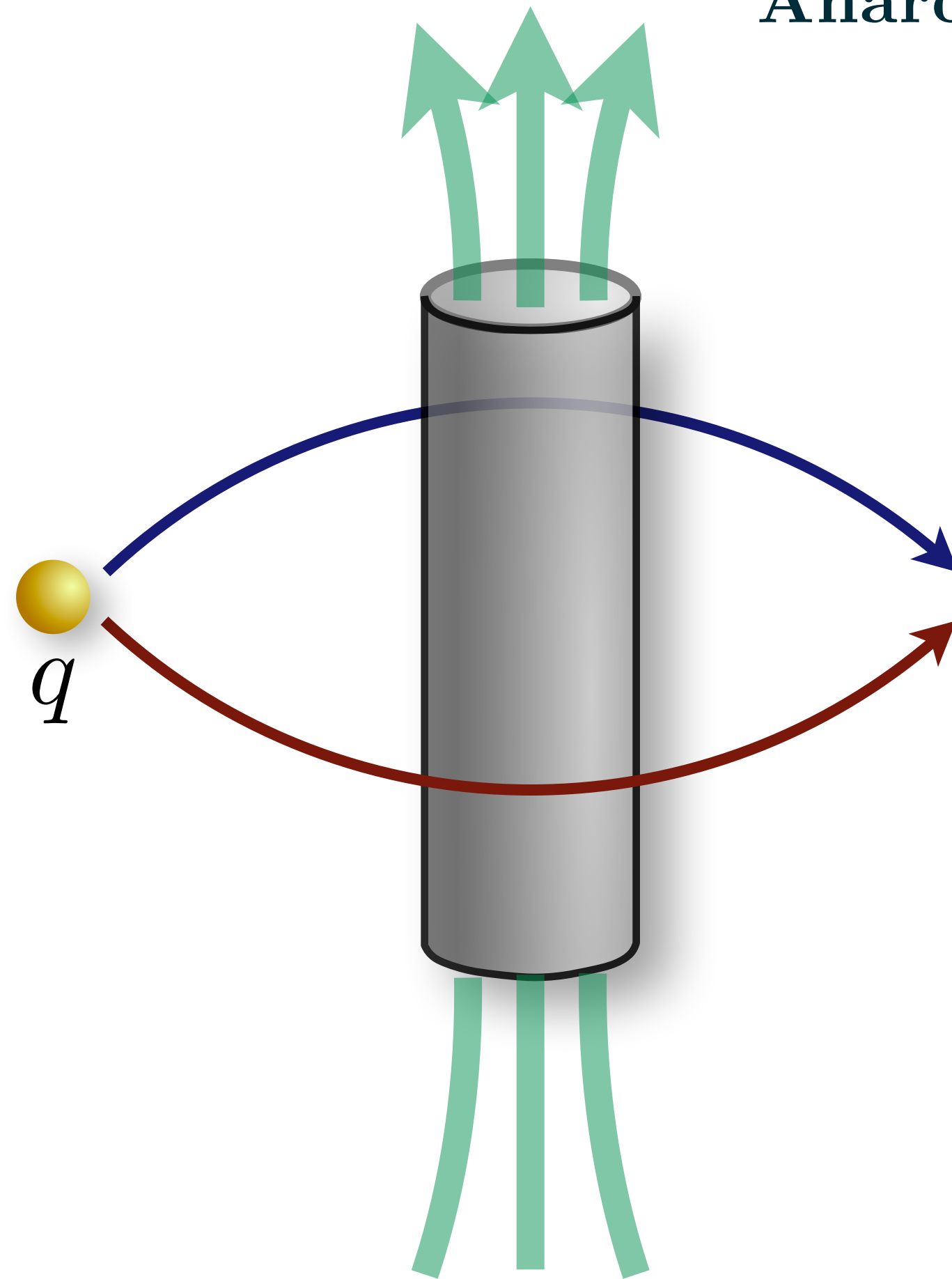
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Accumulated phase is $\Delta\phi = 2e \oint \mathbf{A} \cdot d\mathbf{x}$

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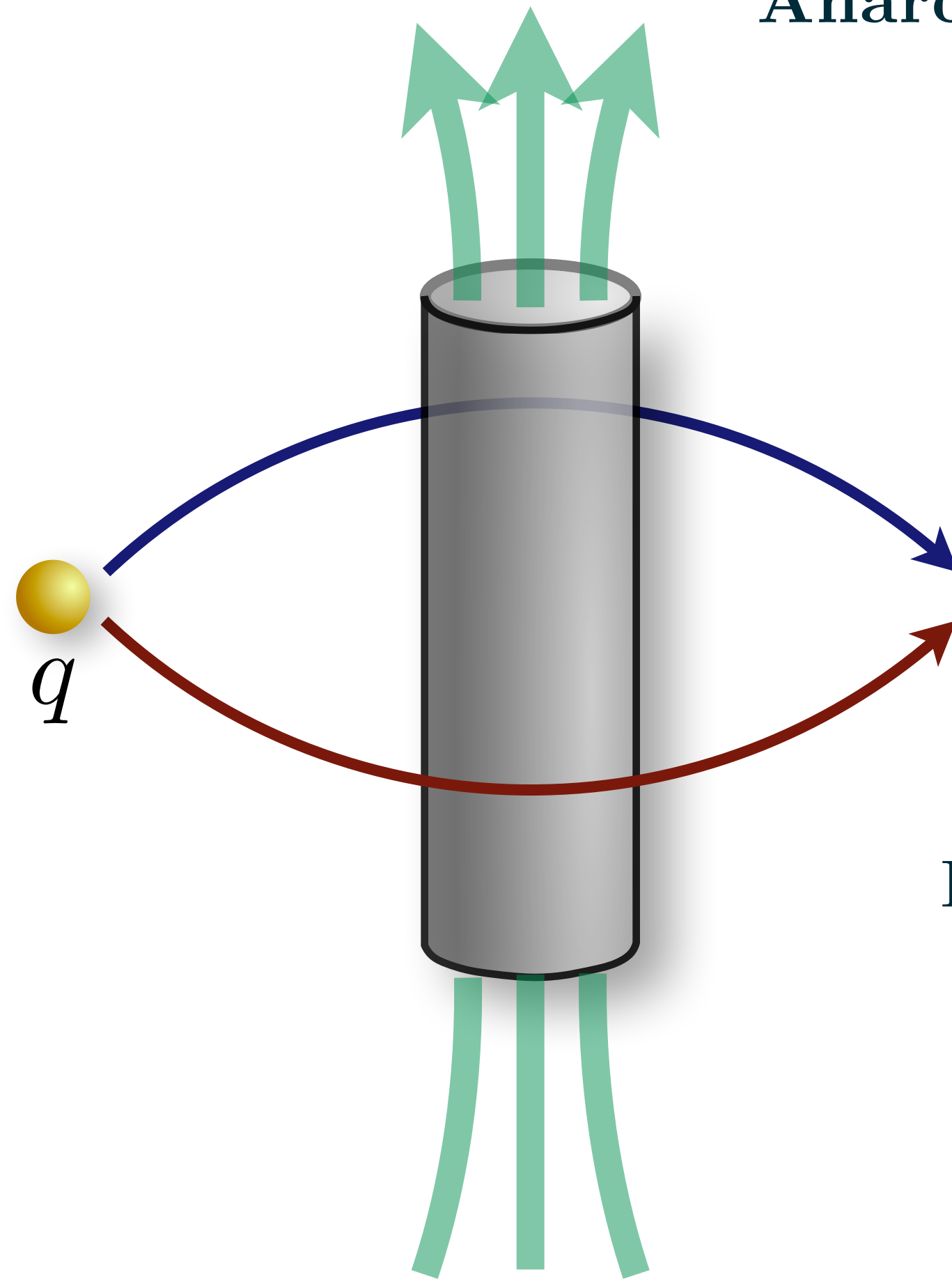


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By Stokes' theorem, relate to area as $\Delta\phi = 2e \int_0^T \mathbf{B} \cdot \frac{d\mathbf{S}}{dt} dt$

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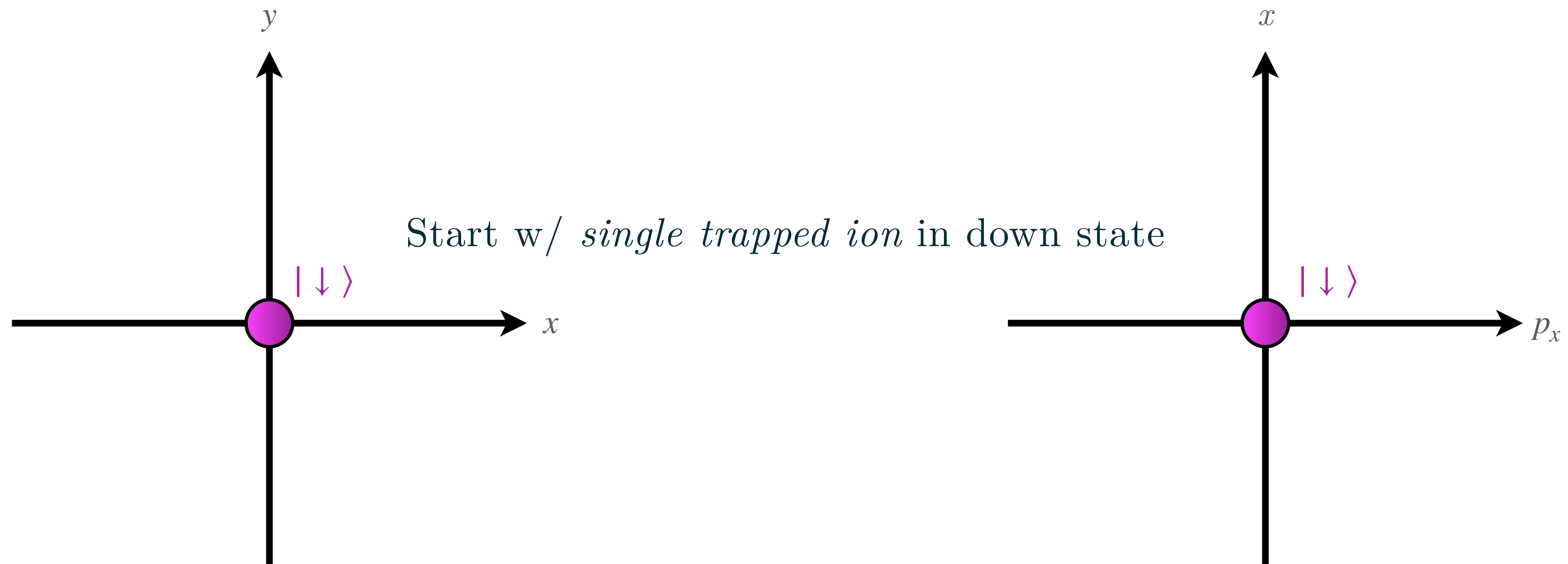
Engineer a system where $\frac{d\mathbf{S}}{dt}$ is large using quantum techniques?

Ion Interferometry with Cat States

Leverage proposal for rotation sensing made by W. Campbell & P. Hamilton (2017)

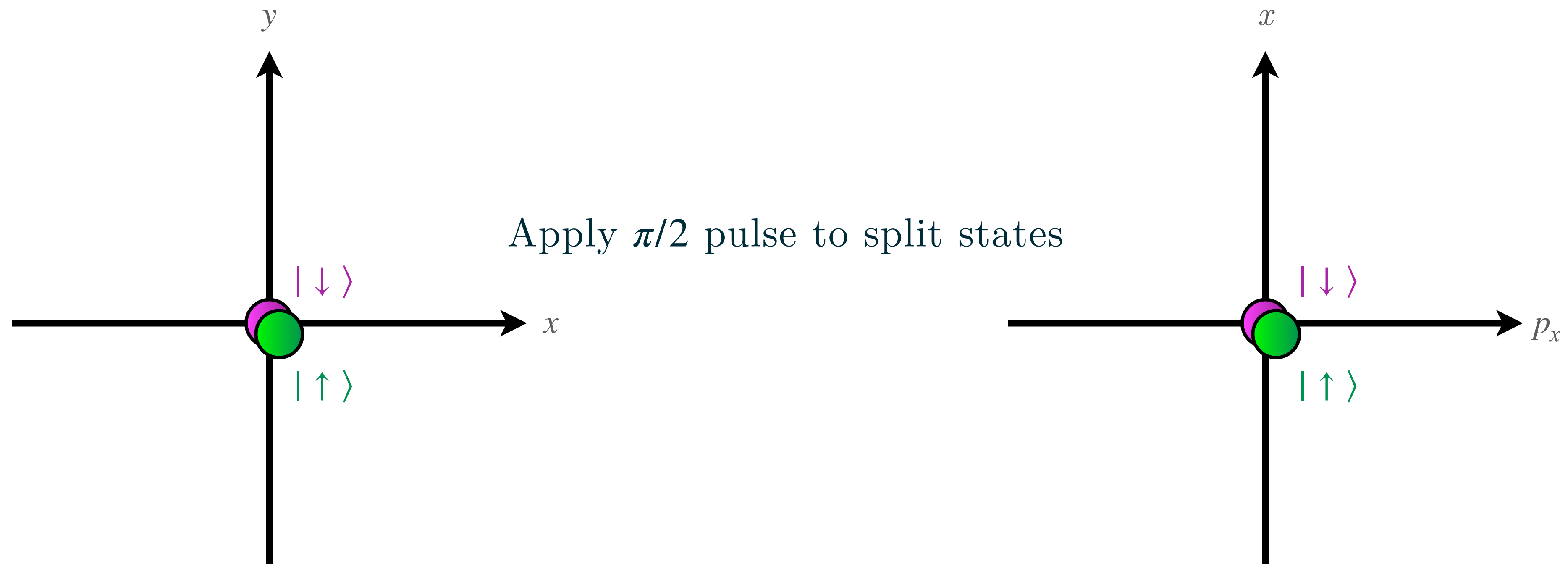
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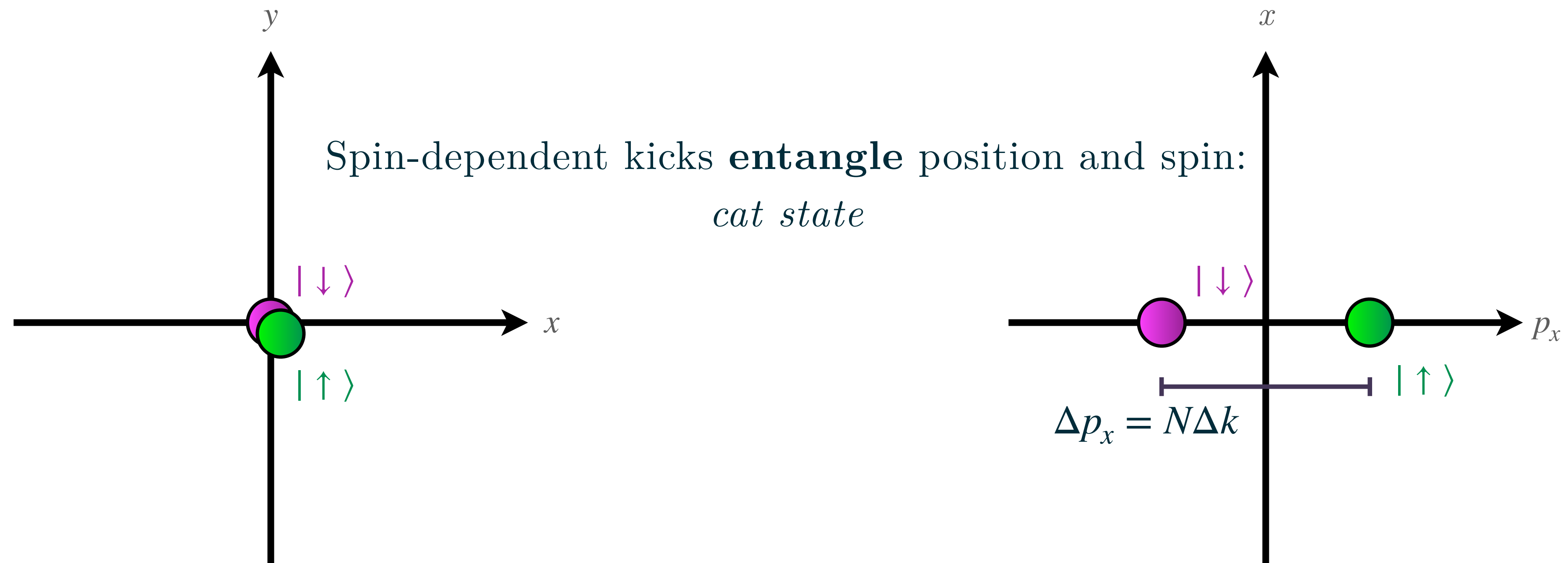
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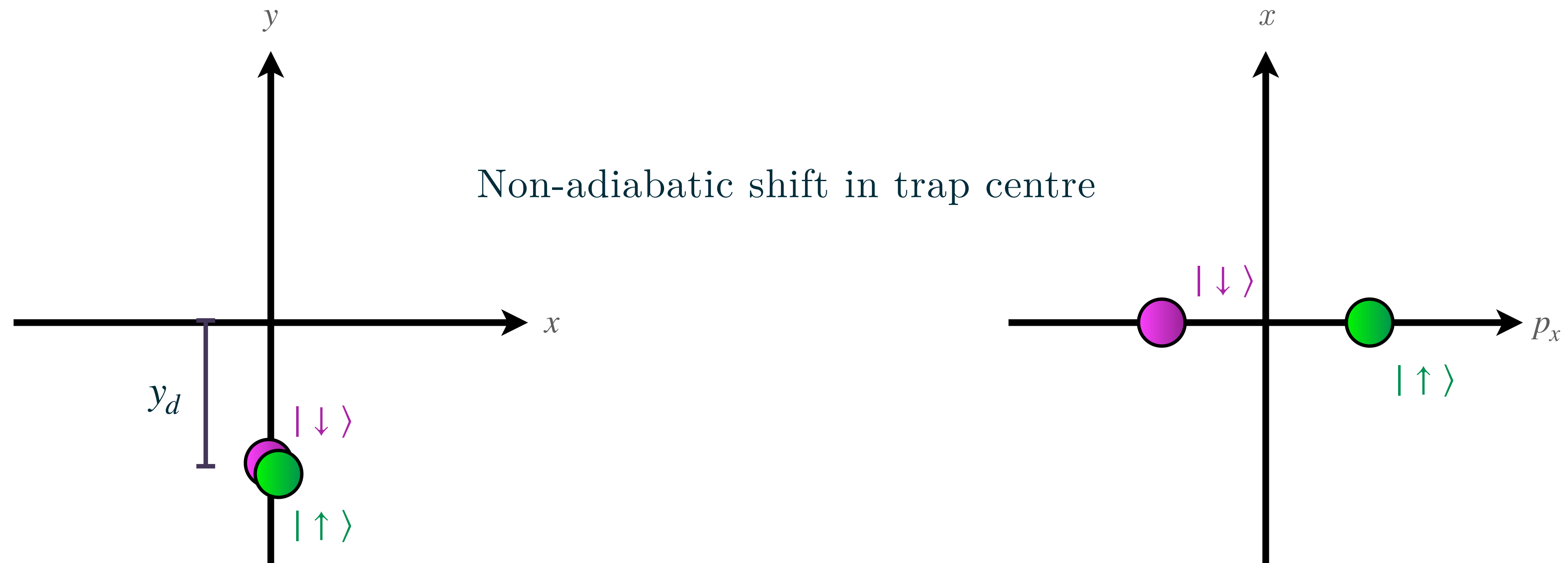
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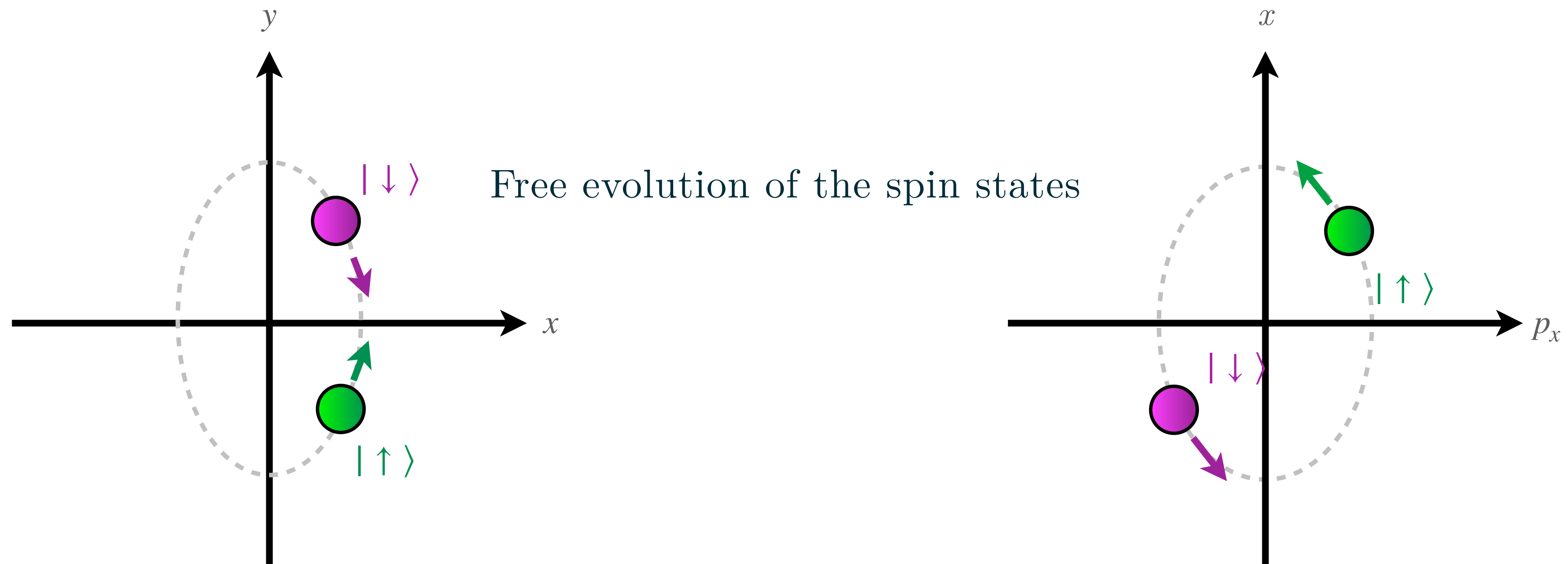
A different spin system

Leverage proposal for rotation sensing made by W. Campbell & P. Hamilton (2017)



Ion Interferometry with Cat States

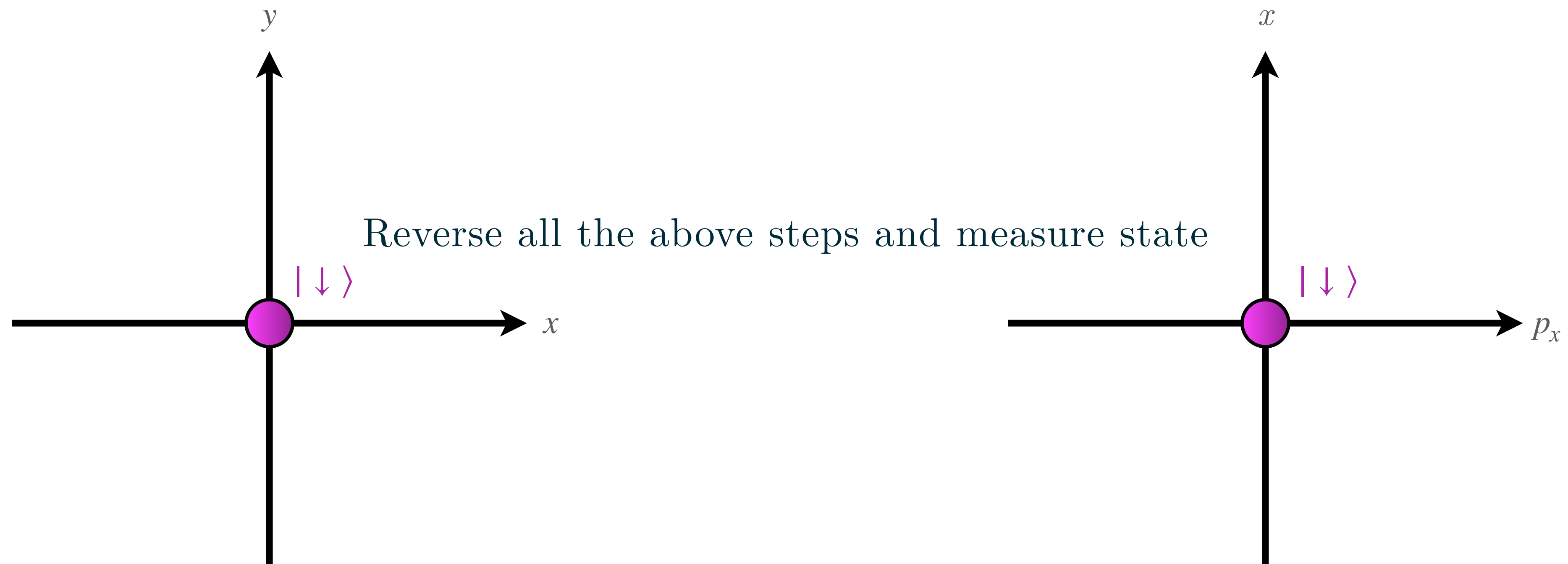
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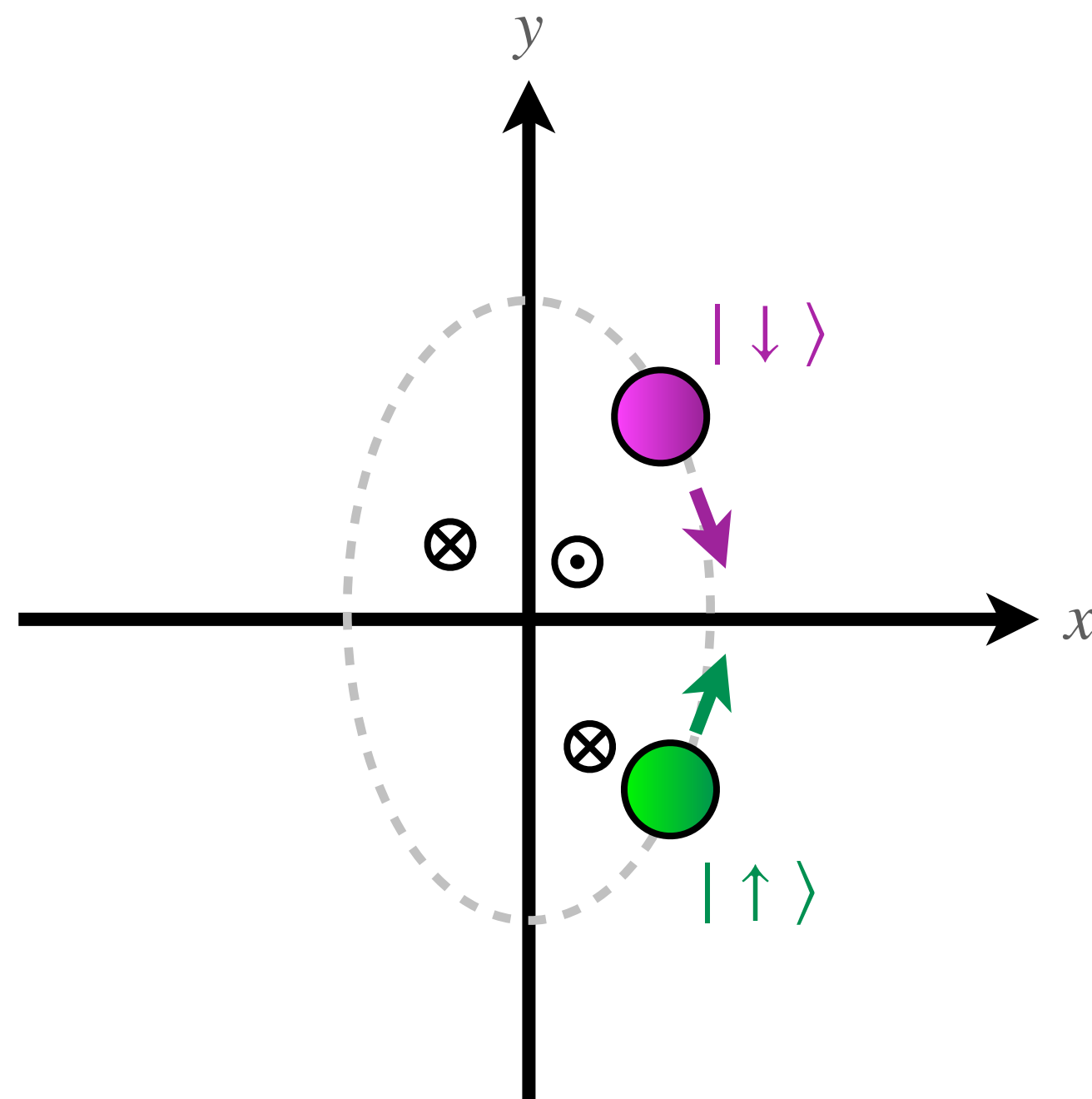
Coherence of states in trap dictates T , and $N_{\text{turns}} \sim \omega_{\text{trap}} T$

Ion Interferometry with Cat States

Leverage proposal for rotation sensing made by W. Campbell & P. Hamilton (2017)



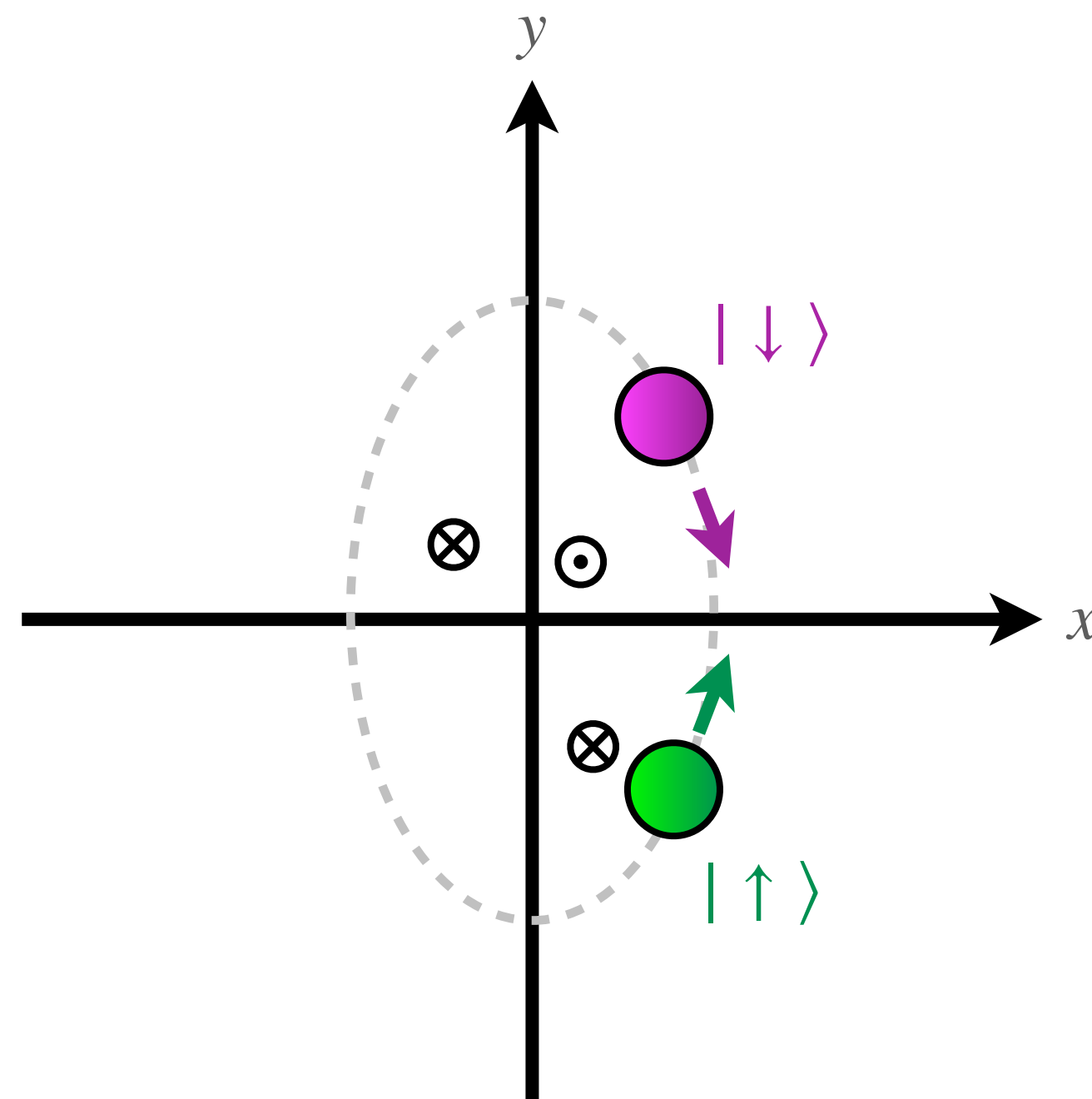
Ion Interferometry with Cat States



PRD 113, 092004 (2026)
L. Badurina, D. Blas, J. Ellis, SARE

With one ion, $y_d = 100\,\mu\text{m}$, $N = 100$ and $\Delta k = 4\pi/355\,\text{nm}$, $\mu \sim \mu_B$

Ion Interferometry with Cat States



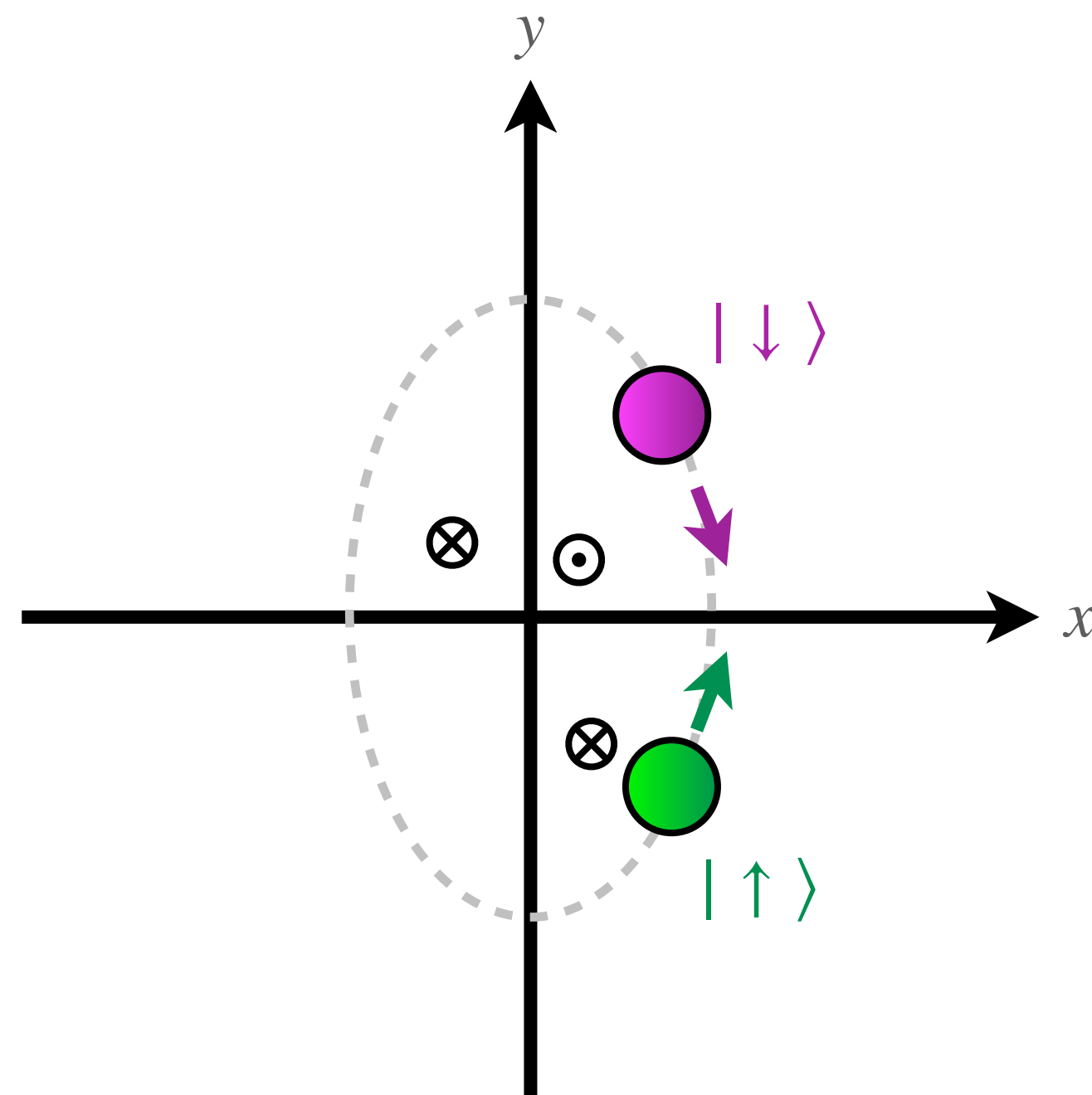
PRD 113, 092004 (2026)
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Counter-propagation of entangled spin & position state
means relative phase of $\varphi_{\uparrow} - \varphi_{\downarrow}$ measurable

$$\Delta\phi = 2 \int_0^T \frac{d\mathbf{S}}{dt} \cdot \mathbf{B} dt \qquad \frac{d\mathbf{S}}{dt} \simeq \frac{y_d N \Delta k}{m_{\text{ion}}}$$

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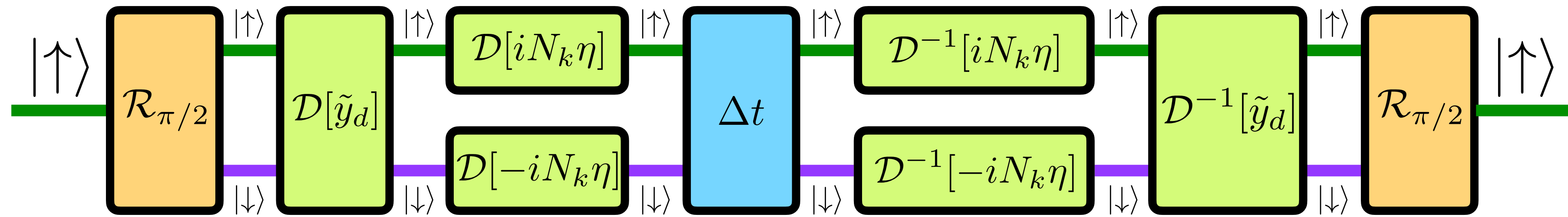
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Morally equivalent to a motional magnetic moment

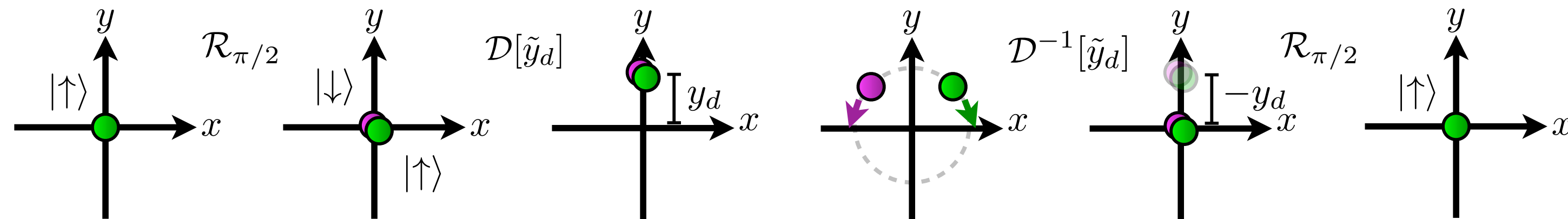
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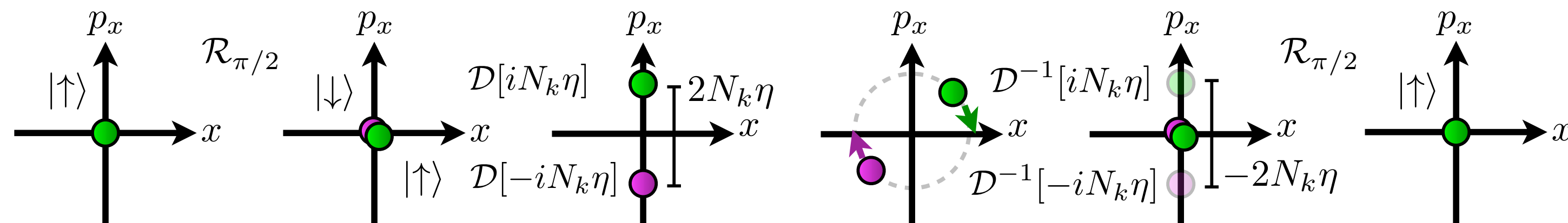
Ion Interferometry with Cat States



Position Space

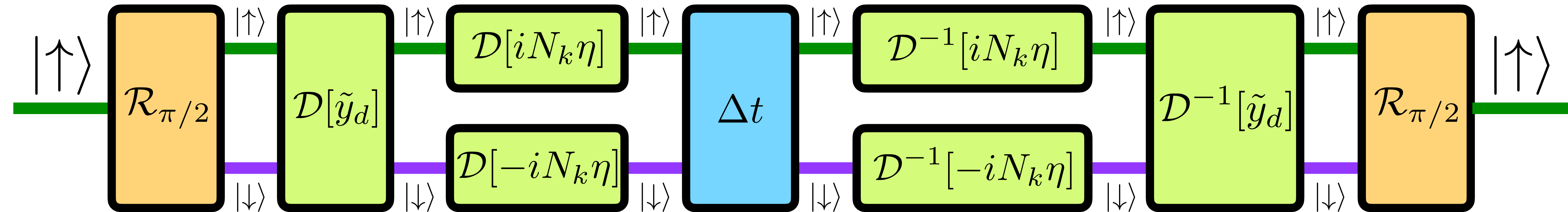


Momentum Space



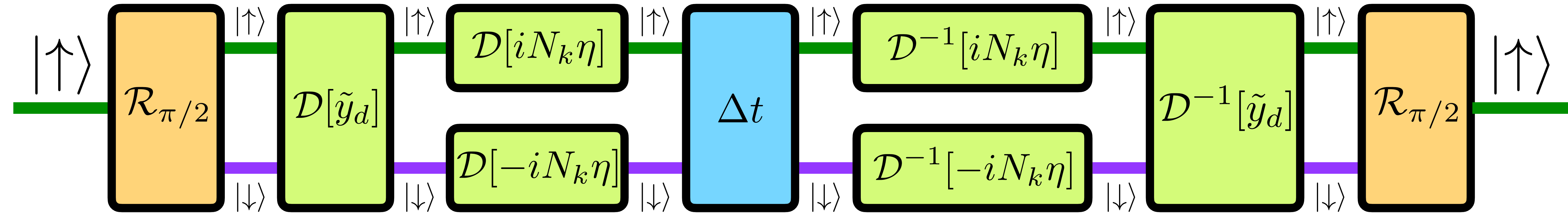
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Ion Interferometry with Cat States



During sensing phase Δt , any real (or effective) magnetic fields will displace spin states

Ion Interferometry with Cat States



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For a real magnetic field due to e.g. Dark Photon or Axion,

$$\mathcal{D}_{\text{AB}} = \exp \left[\frac{i}{\hbar} \beta_z(t) \hat{J}_z \right] \quad \beta_z(t) = \frac{q}{2M} \int_0^t dt' B_{\text{ext}}(t')$$

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A different way in which the Box Matters

For an effective magnetic field, can have same form of displacement operator

$$\mathcal{D}_{\text{AB}} = \exp \left[\frac{i}{\hbar} \beta_z(t) \hat{J}_z \right]$$

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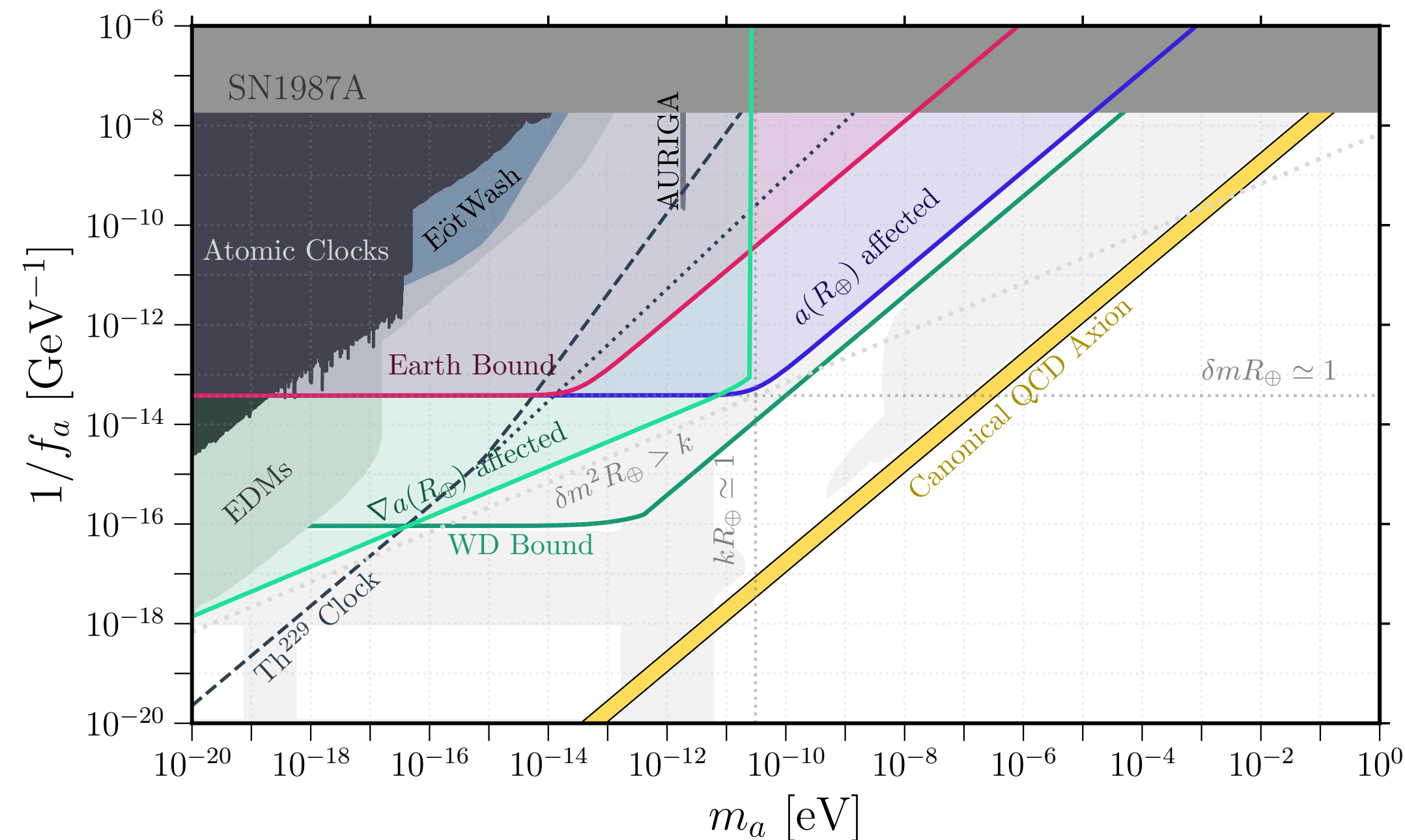
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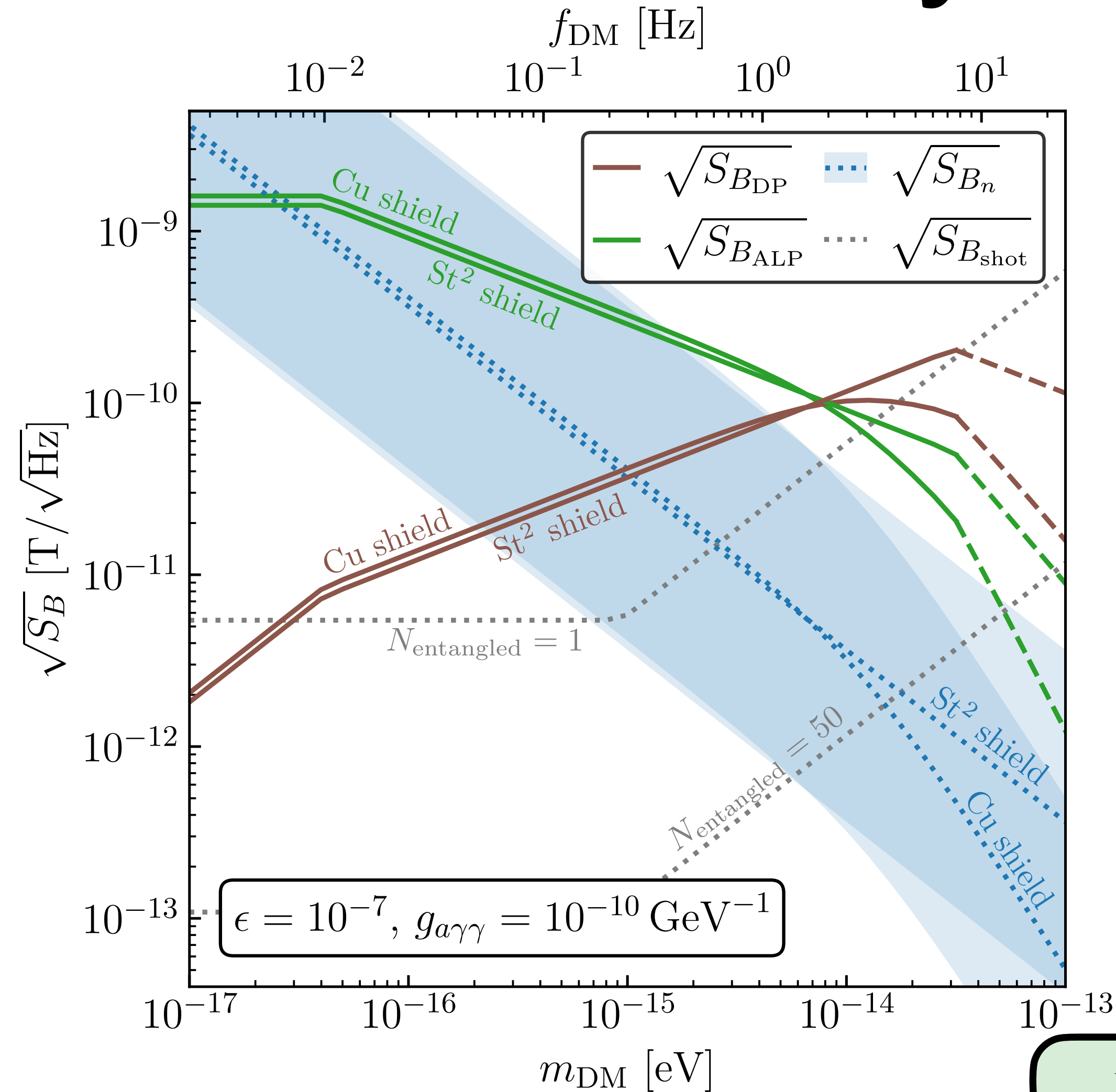
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Tuned QCD axions, or ALPs with quadratic couplings to matter, need to account carefully for effect on axion gradient

JHEP 06 (2026) 039

A. Banerjee, I. Bloch, Q. Bonnefoy, SARE, G. Perez, I. Savoray, K. Springmann, Y. Stadnik

Ion Interferometry with Cat States



Shot noise dominant at high frequency $\delta\phi_{\text{shot}} \sim 1/\sqrt{\text{Hz}}$

Ambient EM noise dominant at low frequency

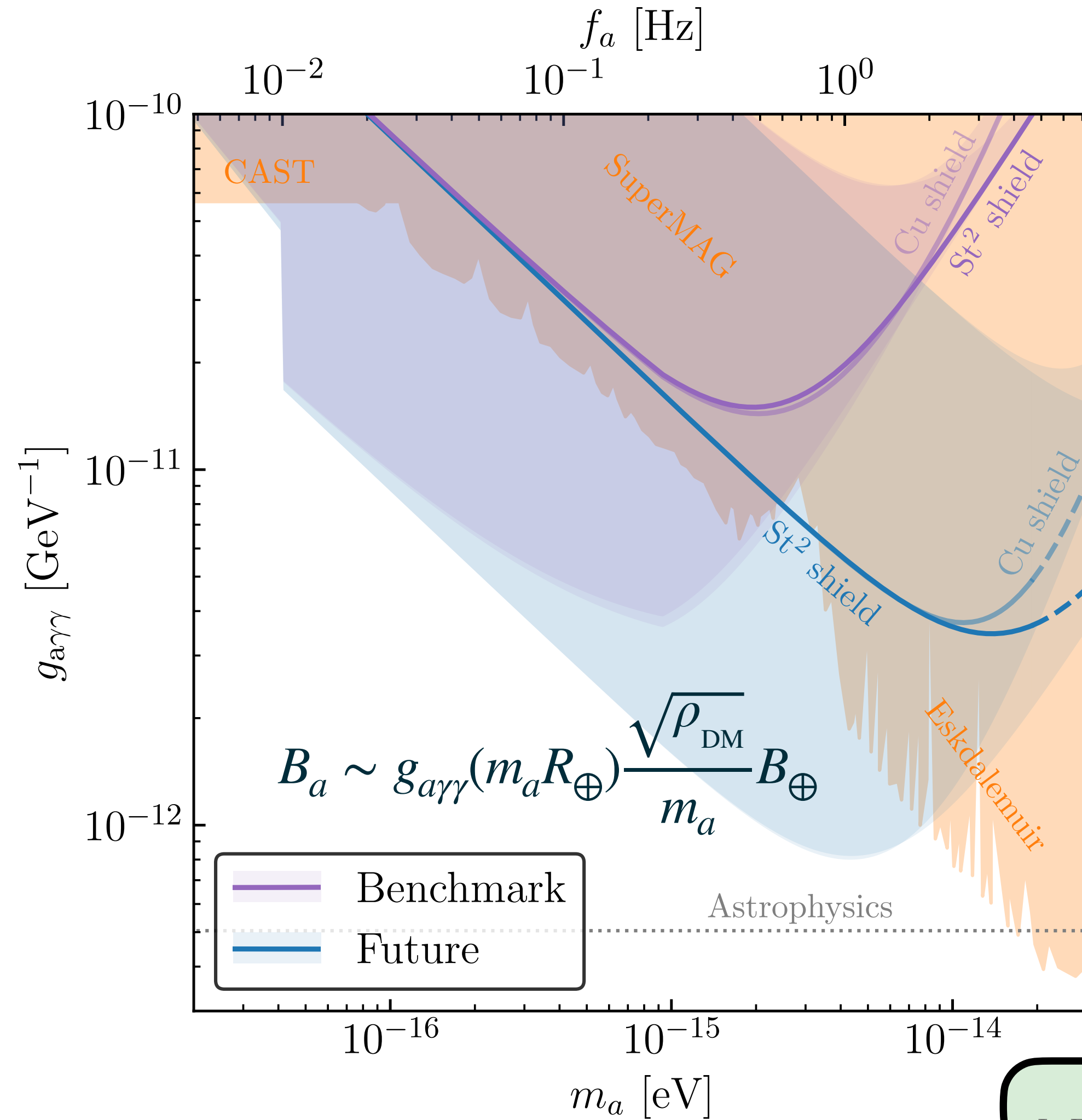
$$\sqrt{S_B} \sim 10^{-11\pm 1} \times \left(\frac{1 \text{ Hz}}{f} \right) \text{ T}/\sqrt{\text{Hz}}$$

Entangling ions can decrease shot noise like $1/N_{\text{ion}}$

PRD 113, 092004 (2026)

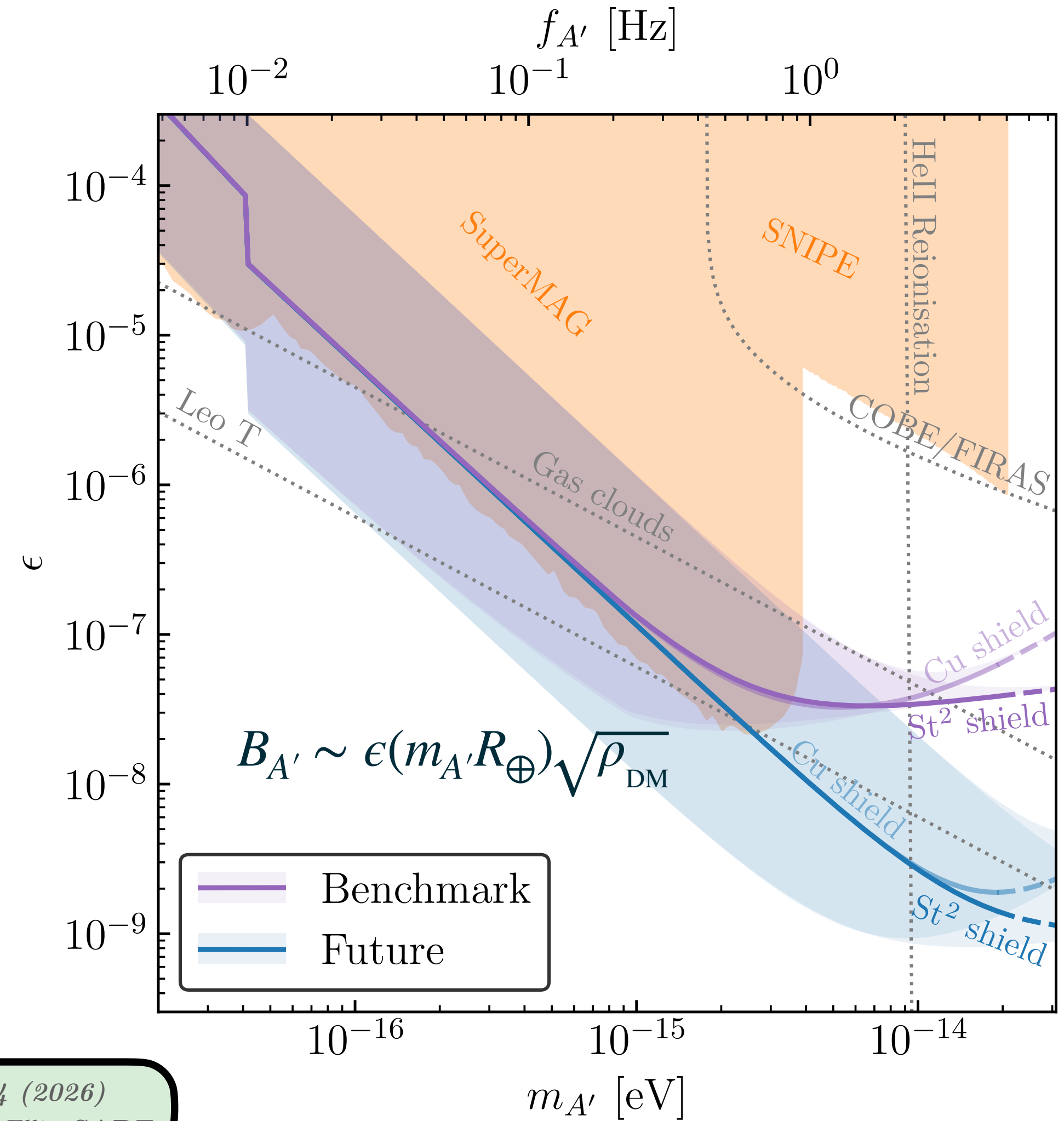
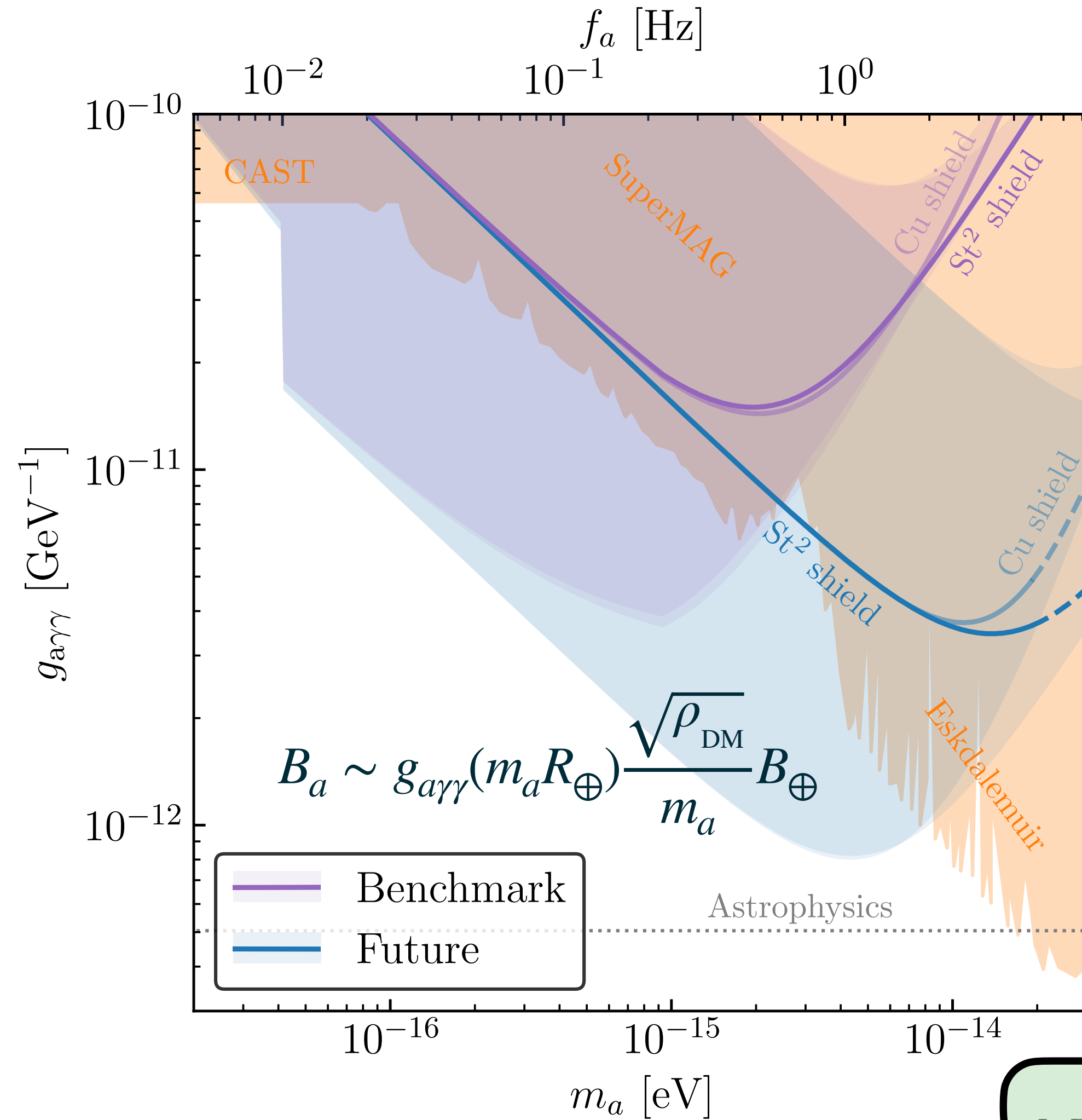
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Ion Interferometry with Cat States



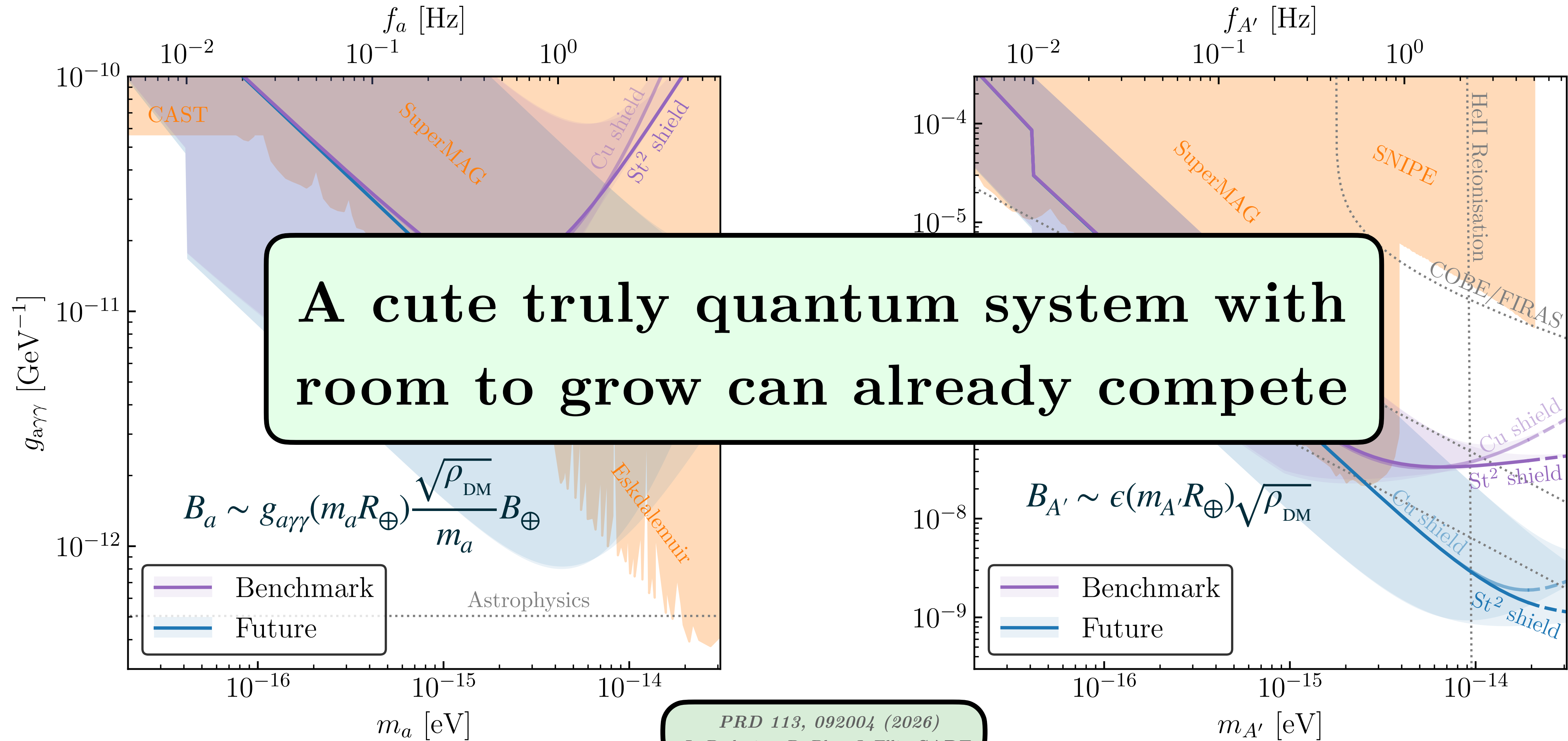
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Non-Standard Spin States

Various spin states can be constructed that can be exploited for $\hat{S}_x$ or $\hat{S}_z$ sensing:

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$$\textit{Half-filled Dicke states: } |D_{N/2}\rangle \simeq \frac{(\pi N)^{1/4}}{2^{N/2}} |N/2, \uparrow; N/2, \downarrow\rangle$$

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$\text{Var}(\hat{S}_x)$ **Engineering and maintaining special quantum states is not easy** sensing

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Conclusions

Spin sensors are nice, manipulable quantum sensors

The **Box** in which the spins are placed **matters**

Most promising avenue for now is large classical ensemble à la CASPEr

Sensitive to more than just the axion wind coupling

At low masses, truly quantum states can compete with classical devices,
with potentially great upside