

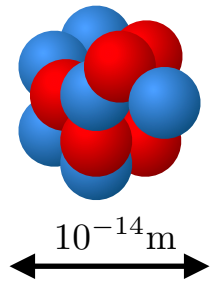
On the sensitivity of a ^{229}Th clock to (quadratic) light dark matter

Konstantin Springmann

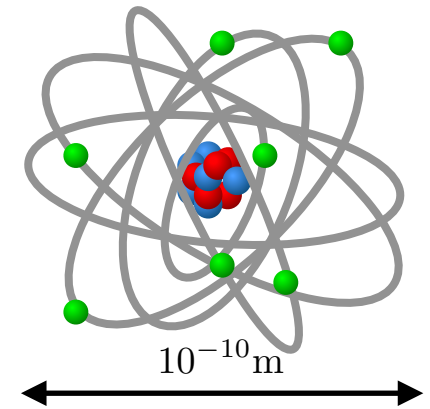
Weizmann Institute



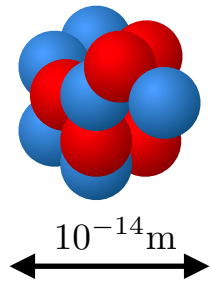
High precision experiments for light dark matter



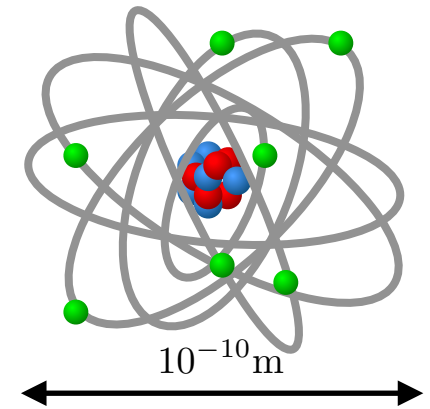
Outline



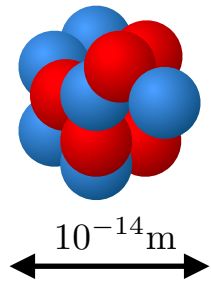
- **Sensitivity of a ^{229}Th nuclear clock to new physics**
 1. Quantum clocks simplified
 2. Atomic clocks
 3. ^{229}Th as a nuclear clock
 4. Approaches to determine K
- **The case of quadratically coupled ULDM**
 1. Natural ULDM: Quadratic twin
 2. Local DM density around Earth with $\phi^2 \bar{N}N$ - coupling



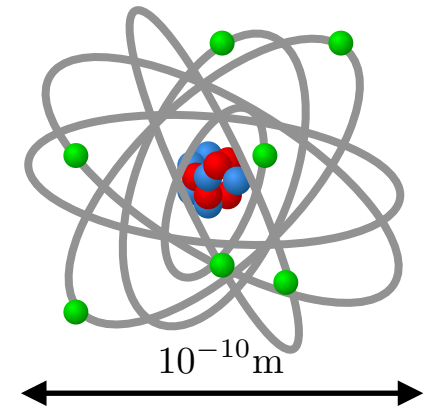
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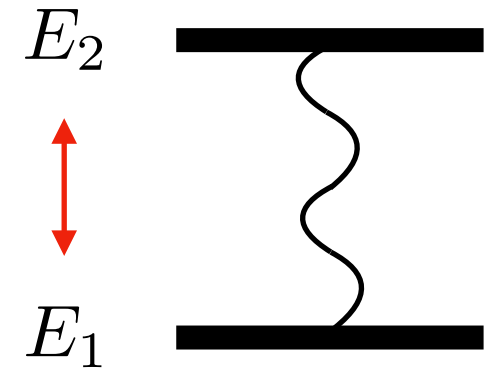


Quantum clocks simplified



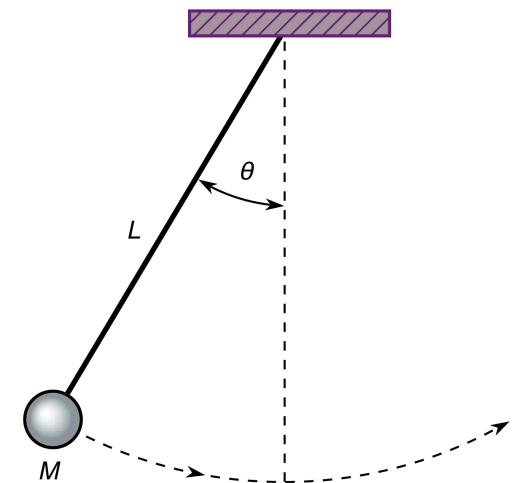
- Prepare 2 level system that is excited with a laser

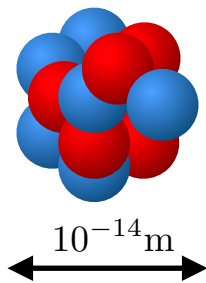
$$\psi_1(0) = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \Rightarrow \psi_1(t) = \frac{|0\rangle + \exp(i\Delta Et) |1\rangle}{\sqrt{2}}$$



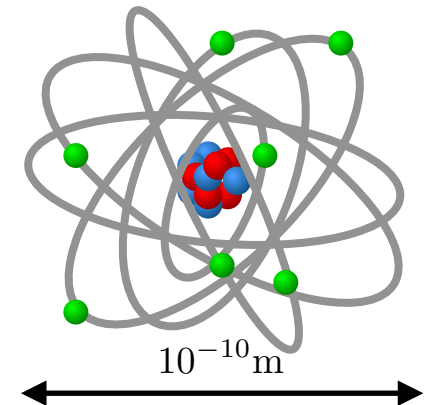
- Creates extremely precise oscillator $\Leftrightarrow$ perfect pendulum

$$|\langle \psi_1(0) | \psi_1(t) \rangle| = \cos \left(\frac{\Delta Et}{2} \right)^2$$

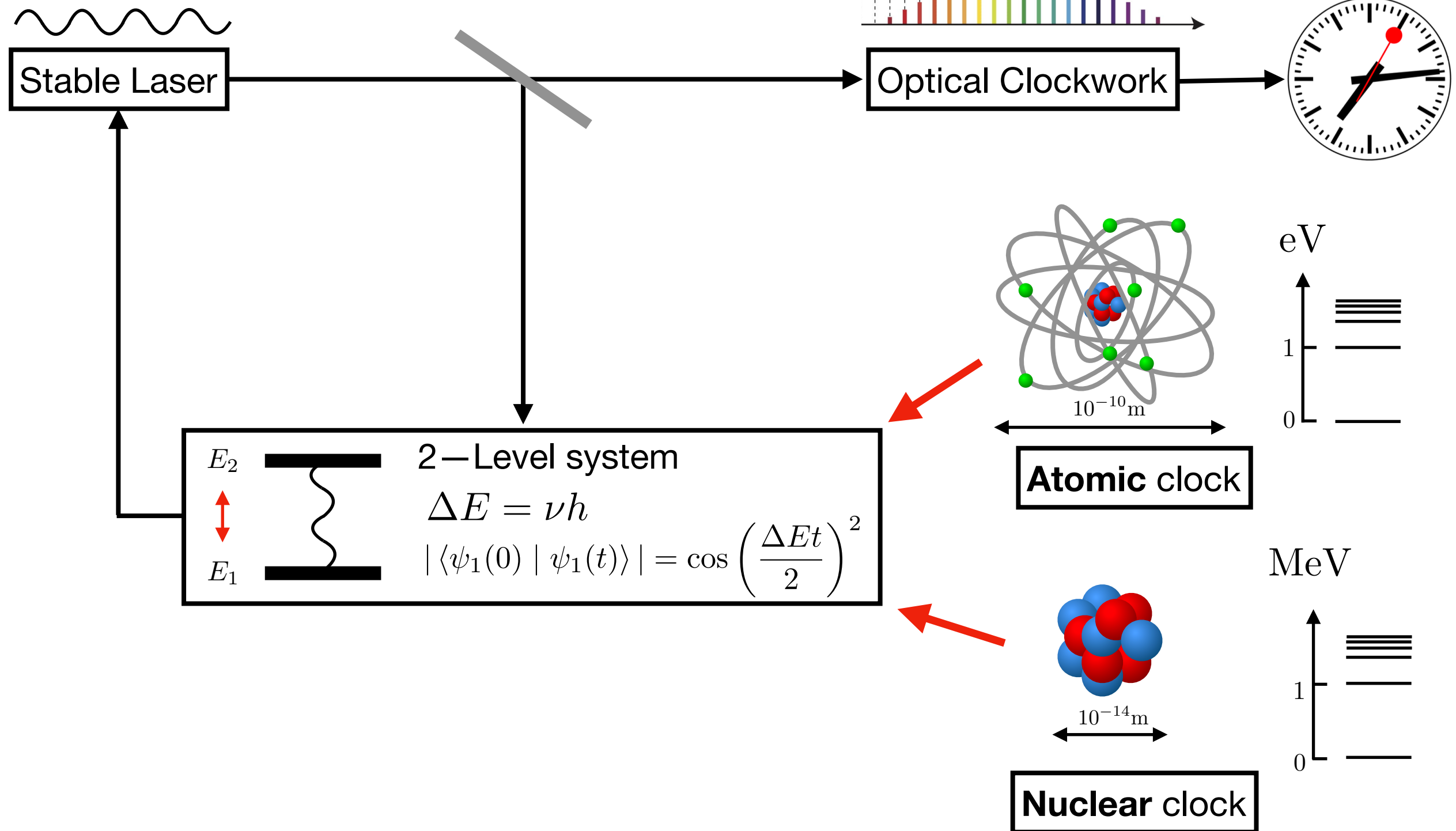




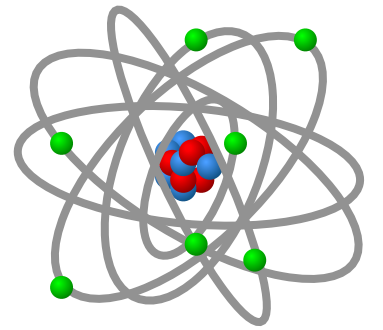
Quantum clocks simplified



Scheme of a clock

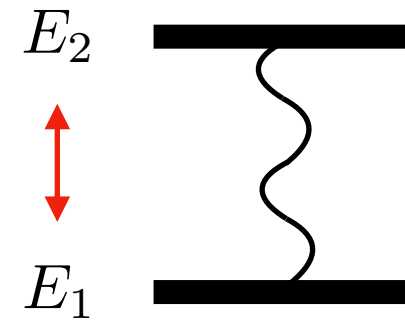


Atomic clocks

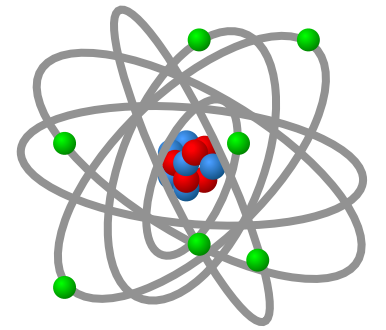


- Electronic transition energy

$$\Delta E \sim m_{\text{red}} \alpha^2 = m_e \left(1 + \frac{m_e}{m_N} \right) \alpha^2$$

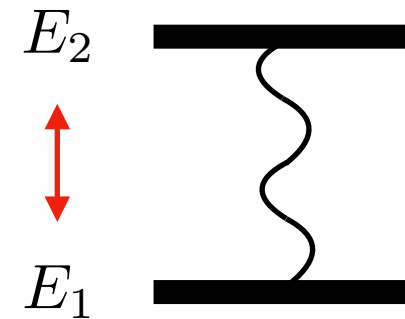


Atomic clocks



- Electronic transition energy

$$\Delta E \sim m_{\text{red}} \alpha^2 = m_e \left(1 + \frac{m_e}{m_N} \right) \alpha^2$$



- How precise? **Best atomic clocks** reach

Ytterbium E3 single-ion clock: $\sim 3.2 \times 10^{-18}$

N. Huntemann et al., PRL 116 (2016)

Strontium optical lattice clock: $\sim 2.1 \times 10^{-18}$

T.L. Nicholson et al. Nature comm. (2015)

Ytterbium optical lattice clock: $\sim 1.4 \times 10^{-18}$

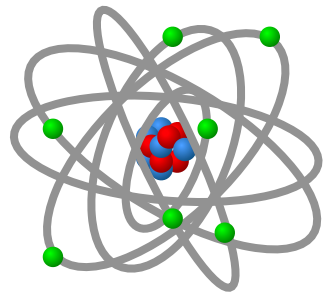
W.F. McGrew et al. Nature 5634 (2018)

Aluminium single-ion clock: $\sim 9.4 \times 10^{-19}$

S.M. Brewer et al. PRL 123 (2019)

Achieved Accuracy: 1s in 30 billion years

Atomic clocks as new physics detector



- QCD coupled ULDM induces variations in $\Delta E \sim m_{\text{red}} \alpha^2 = m_e \left(1 + \frac{m_e}{m_N}\right) \alpha^2$

- Sensitivity factor K_{atom} e.g. for the QCD axion $\frac{\delta f_A}{f_A} = K_{\text{atom}} \theta^2(t)$

System	Transition	K_{atom}
H	Ground state hyperfine	$+1.2 \times 10^{-2}$
Cs	Ground state hyperfine	$+2.6 \times 10^{-3}$
Rb	Ground state hyperfine	$+4.5 \times 10^{-3}$
Si	cavity	-1.5×10^{-5}
Sr	$^1S_0 \rightarrow ^3P_0$	-3.2×10^{-5}
Al ⁺	$^1S_0 \rightarrow ^3P_0$	-3.1×10^{-5}
Hg ⁺	$^2S_{1/2} \rightarrow ^2D_{5/2}$	$+1.4 \times 10^{-5}$
Yb	$^1S_0 \rightarrow ^3P_0$	-3.5×10^{-5}
Yb ⁺ (E2)	$^2S_{1/2} \rightarrow ^2D_{3/2}$	-4.6×10^{-5}
Yb ⁺ (E3)	$^2S_{1/2} \rightarrow ^2F_{7/2}$	$+6.0 \times 10^{-5}$

Suppression typically

$$K_{\text{atom}} \sim 10^{-3} - 10^{-5}$$

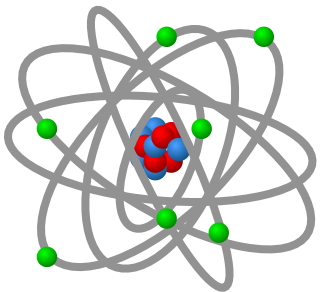
G-factor: $\frac{m_e}{Am_N} \sim 10^{-5}$

Vib. modes $\sqrt{\frac{m_e}{Am_N}} \sim 10^{-3}$

Charge radius $A^{8/3} \alpha^4 \left(\frac{m_e}{m_N}\right)^3$

Kim, Lenoci, Perez, Ratzinger 24'

Atomic clocks as new physics detector

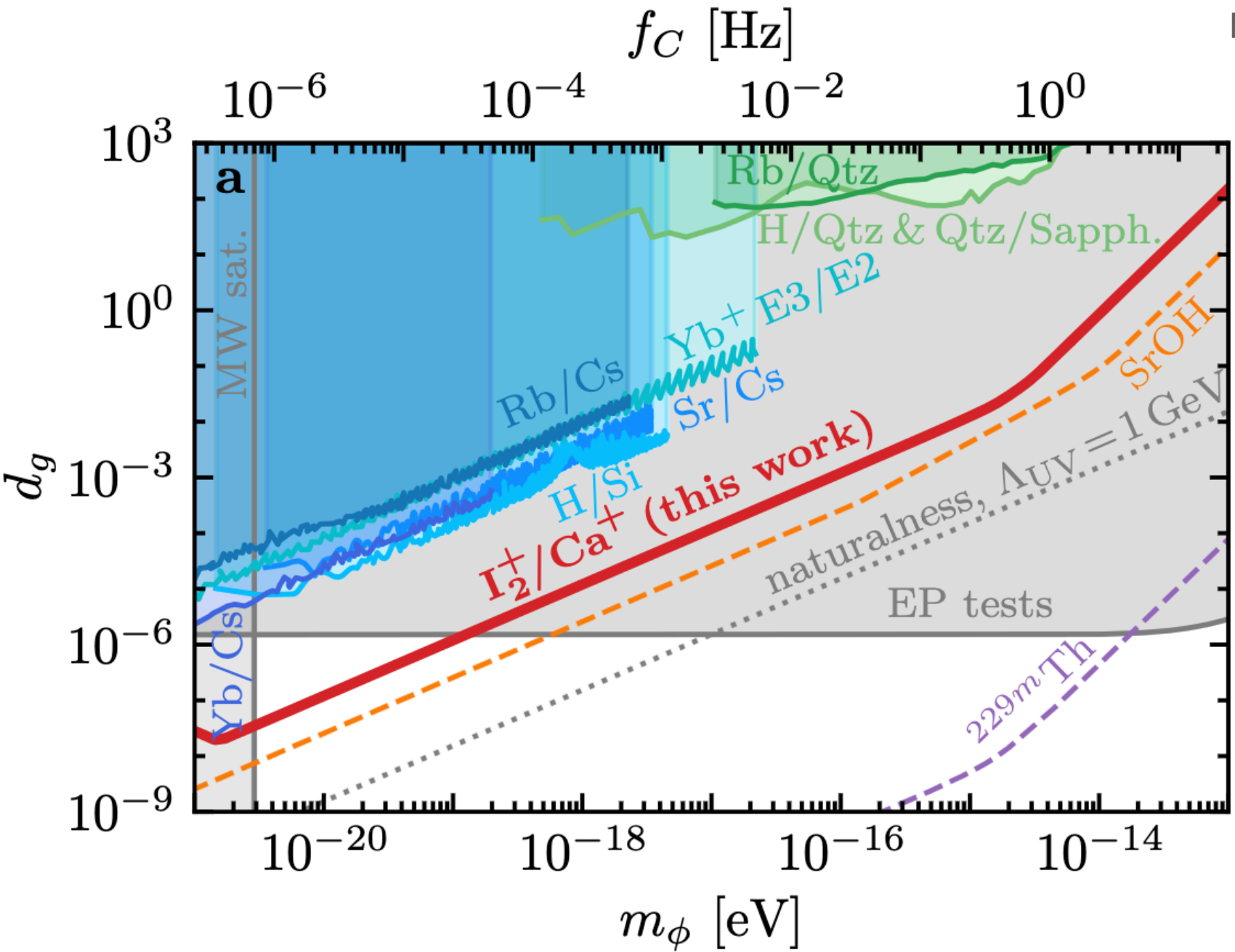


$$\mathcal{L} \supset -\kappa\phi \frac{d_g\beta_s}{2g_s} G_{\mu\nu}^a G^{a\mu\nu}$$

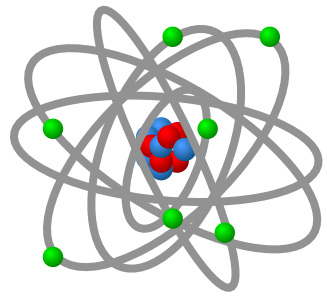
Coupled with vibrational modes

$$\sqrt{\frac{m_e}{Am_N}} \sim 10^{-3}$$

Madge, Perez, Meir 24'



Atomic clocks as new physics detector

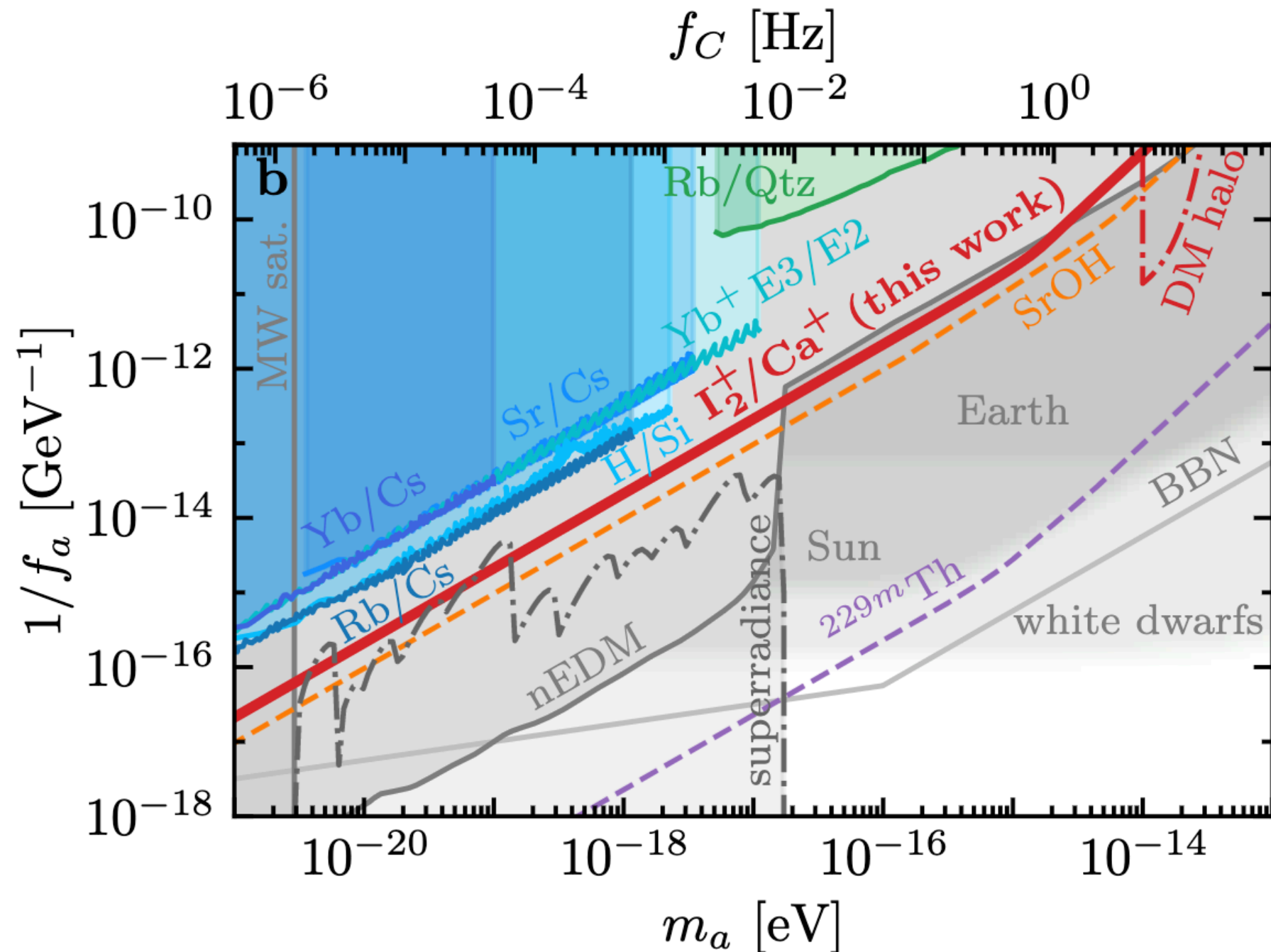


$$\mathcal{L} \supset \frac{g_s^2}{32\pi^2} \frac{a}{f_a} \tilde{G}_{\mu\nu}^a G^{a\mu\nu}$$

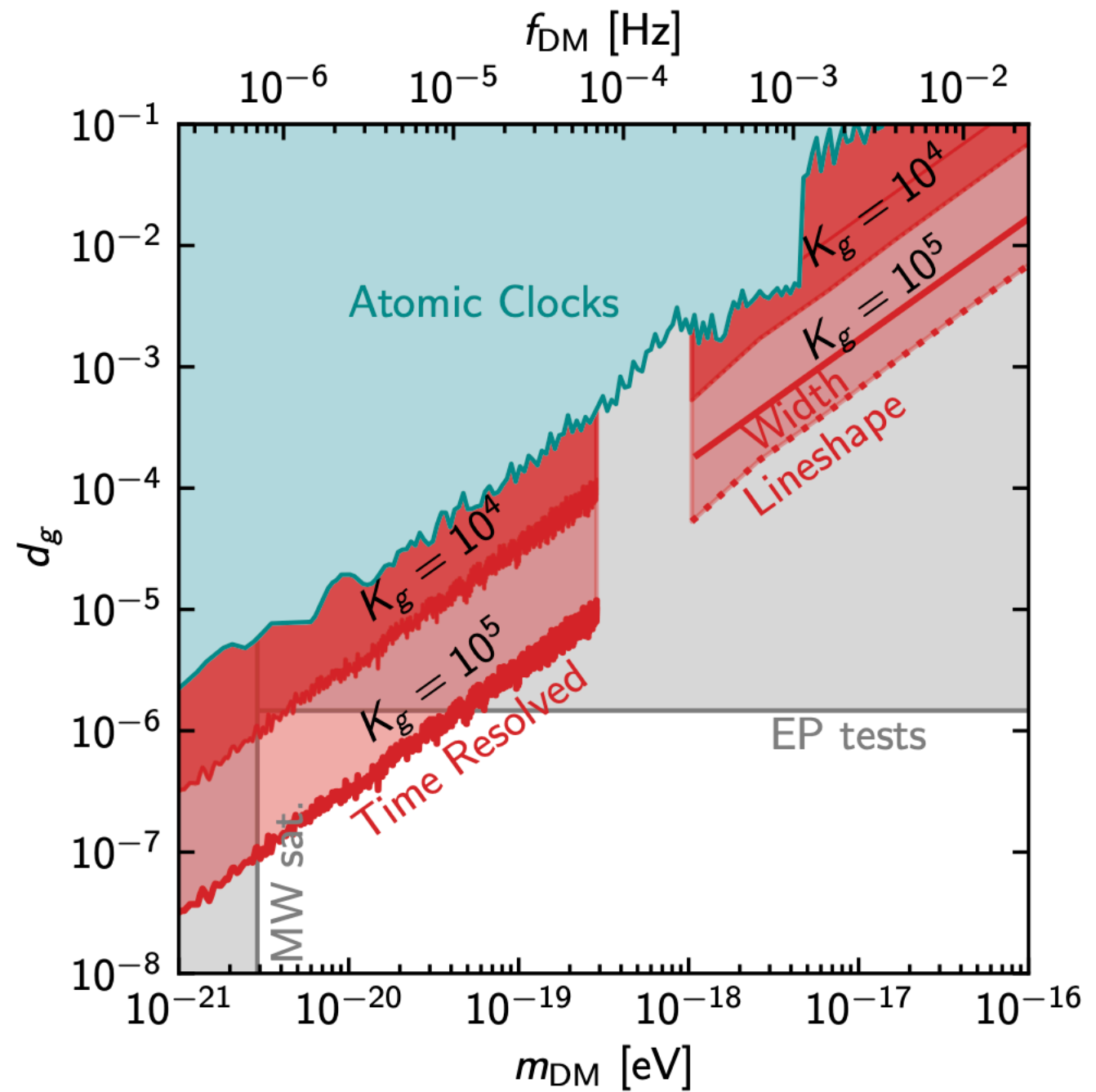
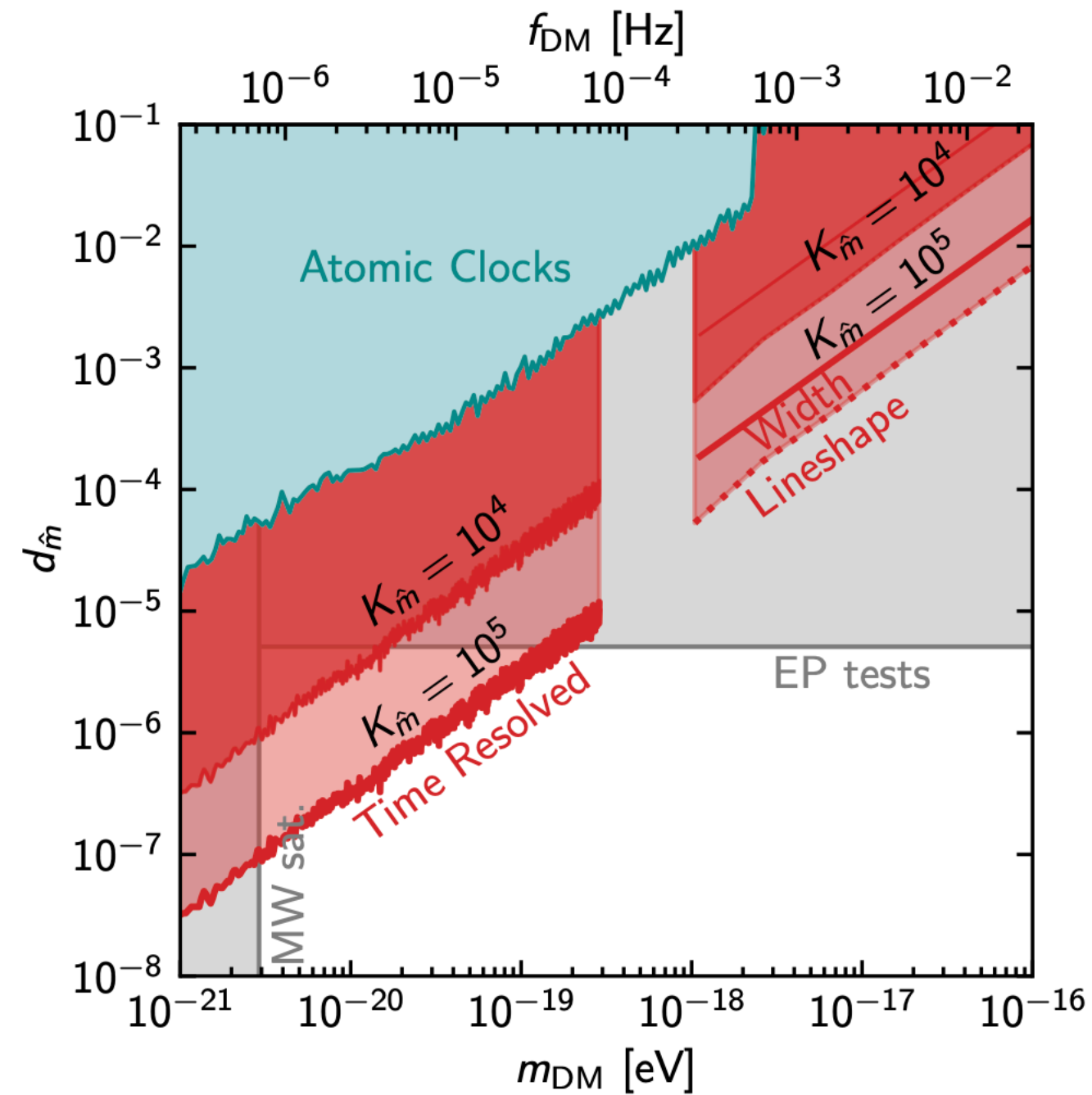
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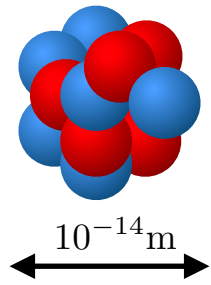
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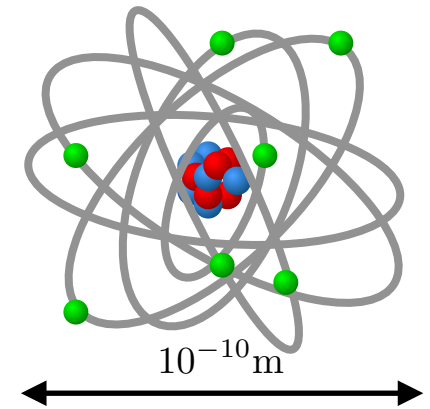


Gilad's talk

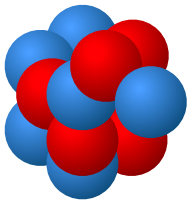




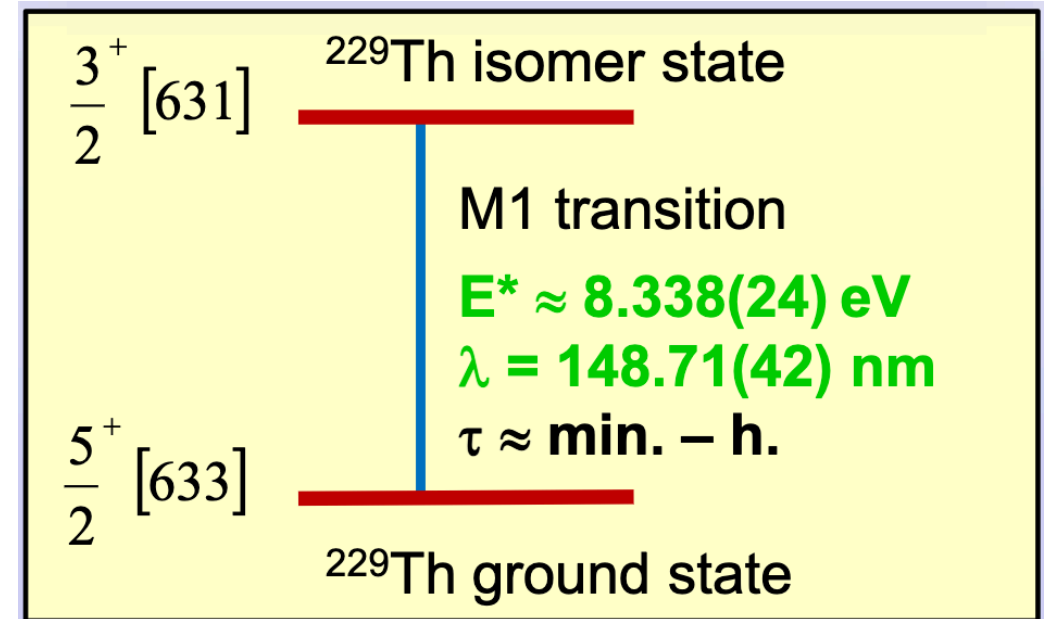
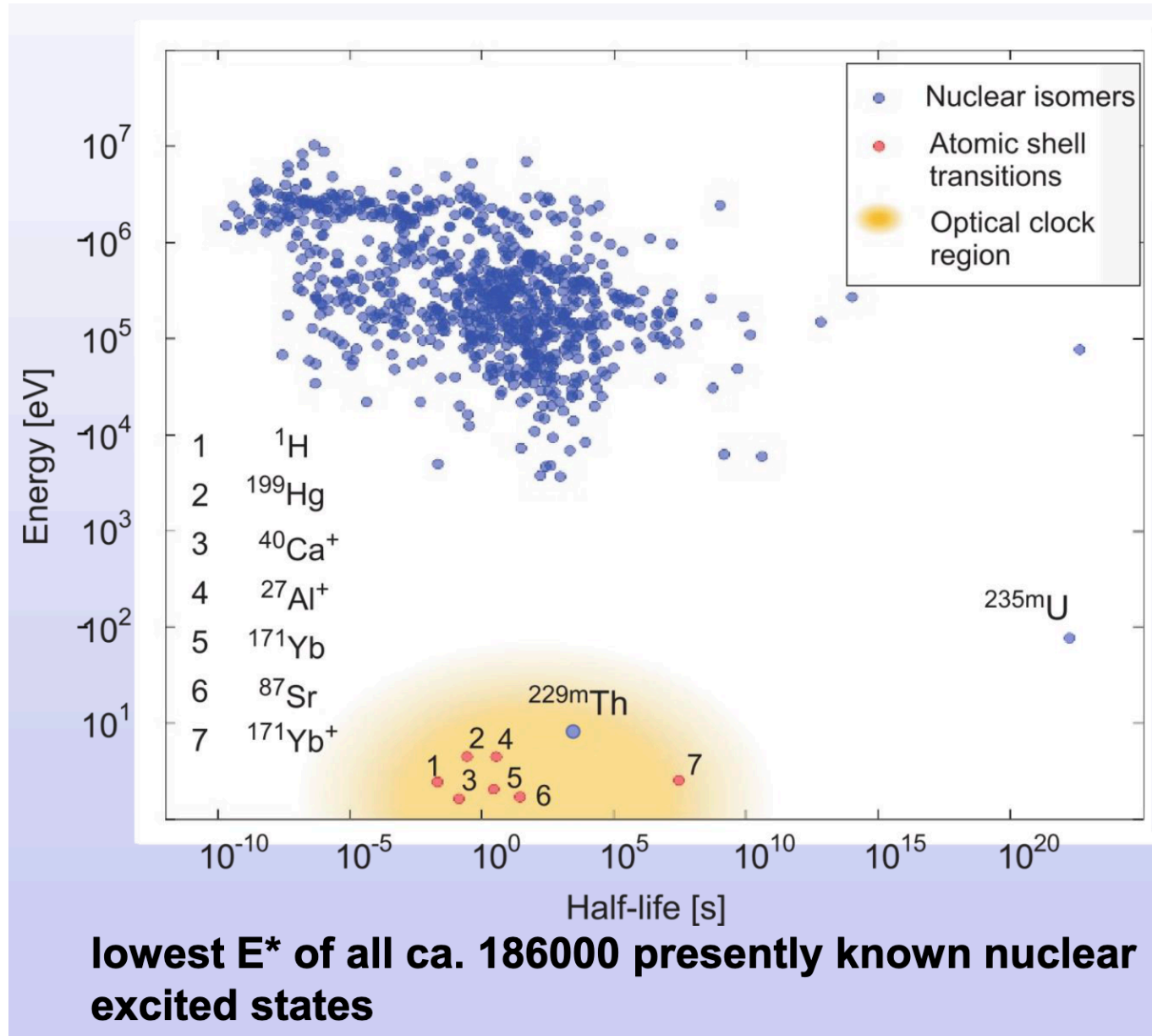
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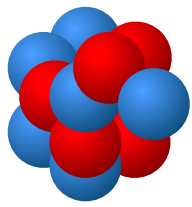
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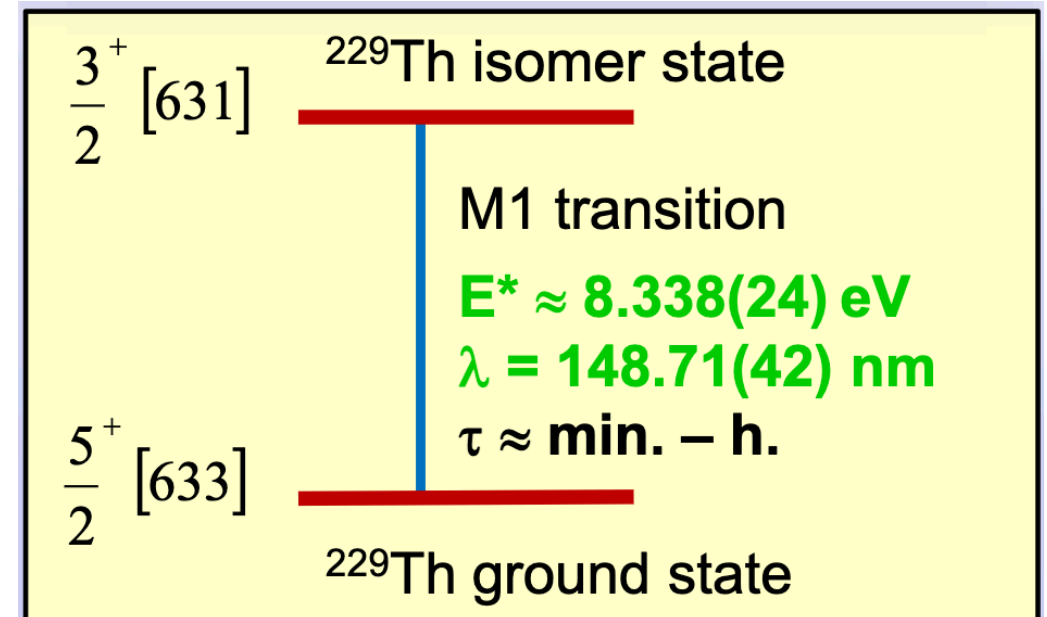
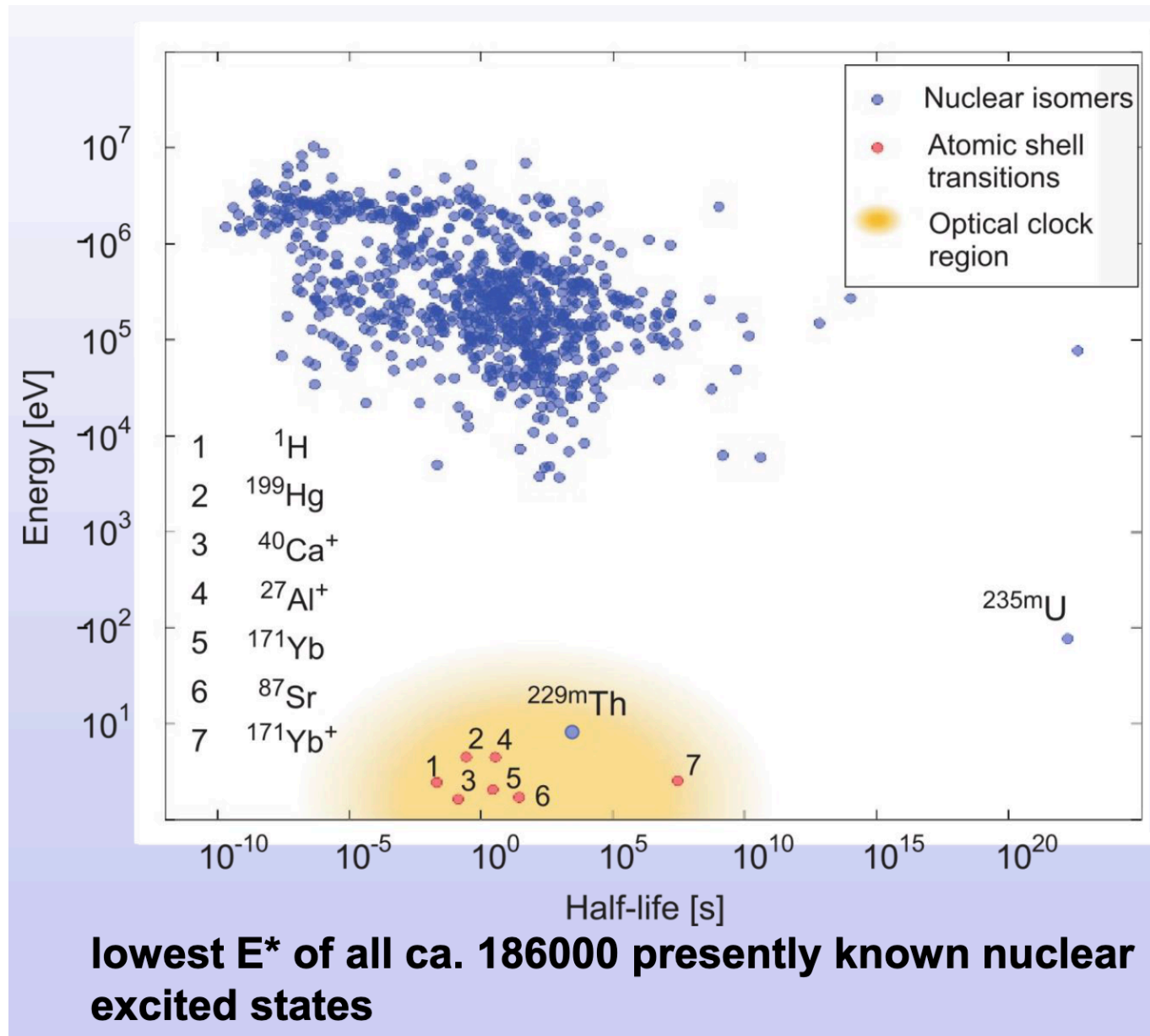
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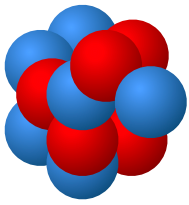
- ^{229m}Th : unusually low excitation energy
 $\Delta E \equiv \nu_0 \sim 8 \text{ eV} \rightarrow$ laser excitable



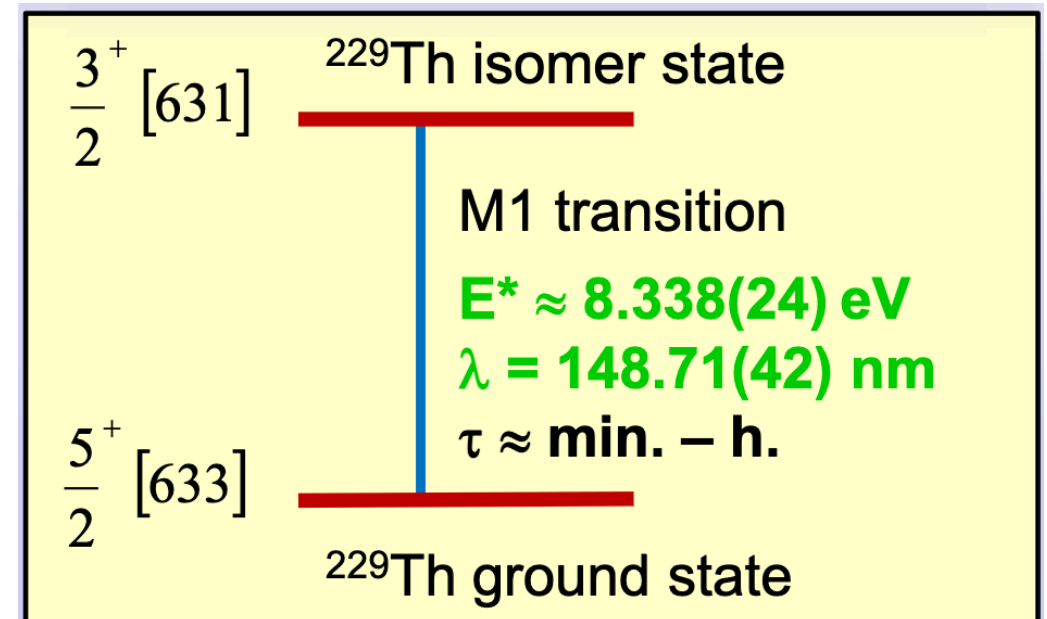
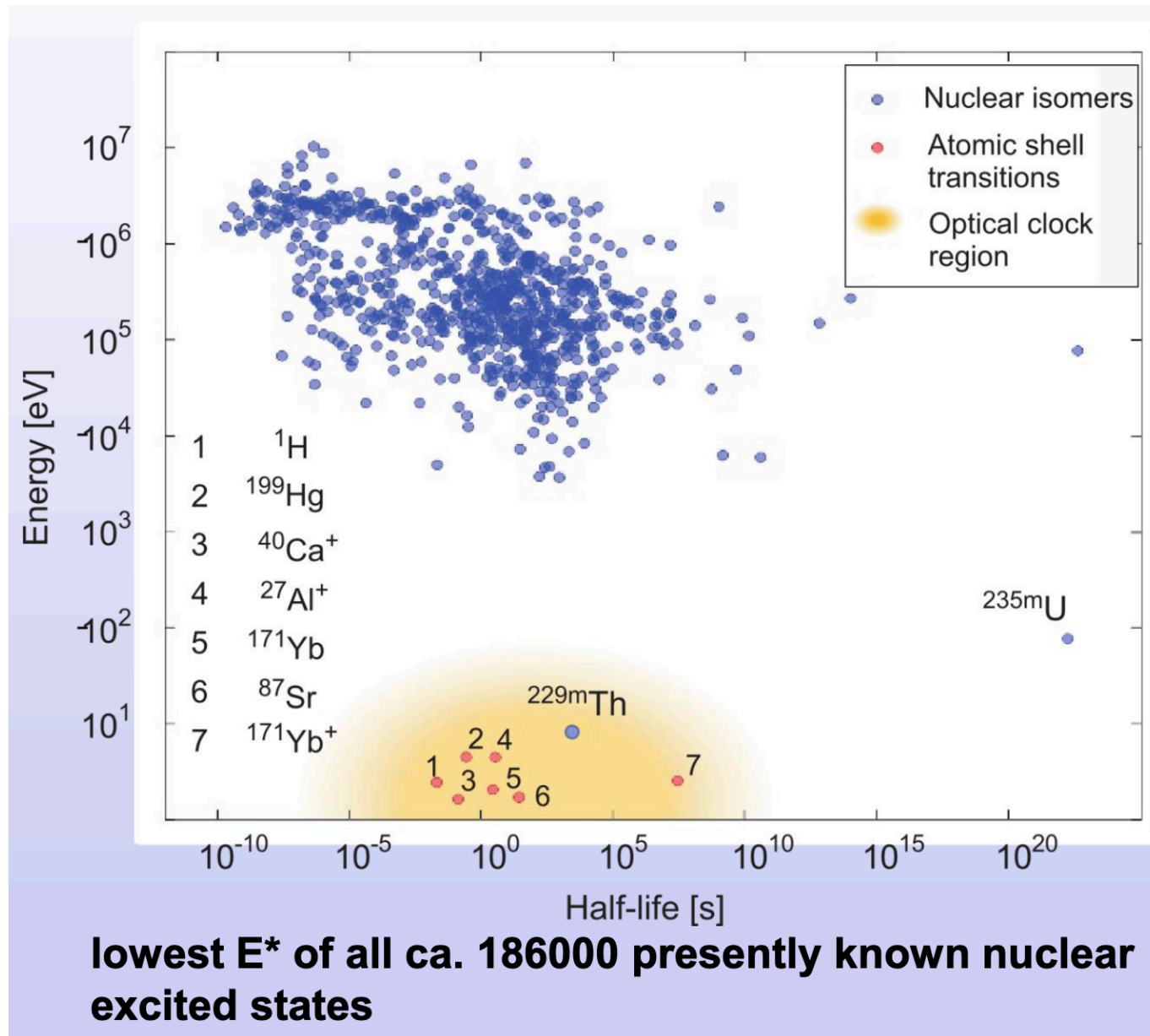
^{229}Th as nuclear clock



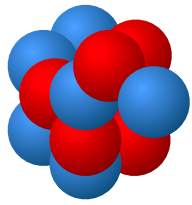
- $^{229\text{m}}\text{Th}$: unusually low excitation energy $\Delta E \equiv \nu_0 \sim 8 \text{ eV} \rightarrow$ laser excitable
- $^{229\text{m}}\text{Th}$: Lifetime $\tau \sim 0.5 \text{ h} \rightarrow$ Quality factor $Q \sim \nu_0 \tau \sim 10^{20}$



^{229}Th as nuclear clock



- ^{229m}Th : unusually low excitation energy $\Delta E \equiv \nu_0 \sim 8 \text{ eV} \rightarrow$ laser excitable
- ^{229m}Th : Lifetime $\tau \sim 0.5 \text{ h} \rightarrow$ Quality factor $Q \sim \nu_0 \tau \sim 10^{20}$
- ^{229m}Th : Less sensitive to external fields



^{229}Th experimental progress in $\delta\nu/\nu$

- **Revolution:** Accuracy in isomeric shift

10^{-1} **(2020)**: indirect γ -ray spectroscopy

T. Sikorsky et al., *Phys. Rev. Lett.* **125**, 142503 (2020)

10^{-3} **(2022, ISOLDE)**: VUV photons from isomer decay in $^{229}\text{Th} : \text{CaF}_2$:

S. Kraemer et al., *Nature* **617**, 706–710 (2023)

10^{-6} **(March/April 2024, PTB)**: First resonant laser excitation in $^{229}\text{Th} : \text{CaF}_2$:

J. Tiedau et al., *Phys. Rev. Lett.* **132**, 182501 (2024)

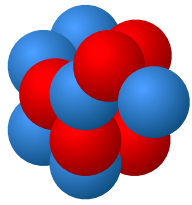
10^{-11} **(June 2024, JILA)**: VUV frequency comb spectroscopy referenced to ^{87}Sr clock

C. Zhang et al., *Nature* **633**, 63–70 (2024)

10^{-13} **(2025, JILA)**: Using two crystals $^{229}\text{Th} : \text{CaF}_2$:

T. Ooi et al., *Nature* **650**, 72–78 (2026)

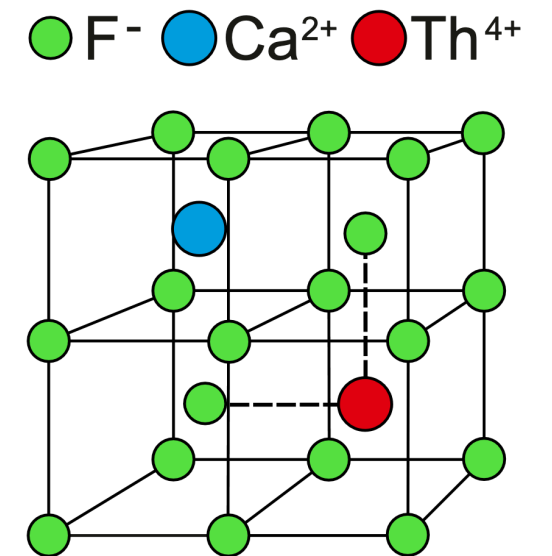
This, and another enhancement factor of $10^8 - 10^{10}$ to new physics probes is extremely promising!

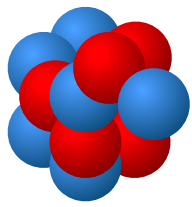


^{229}Th measurements

First resonant laser excitation in ^{229}Th : CaF_2 :

- Shine broad ($\sim \text{few} \times \text{GHz}$) but super powerful laser on ^{229}Th -doped CaF_2 crystal

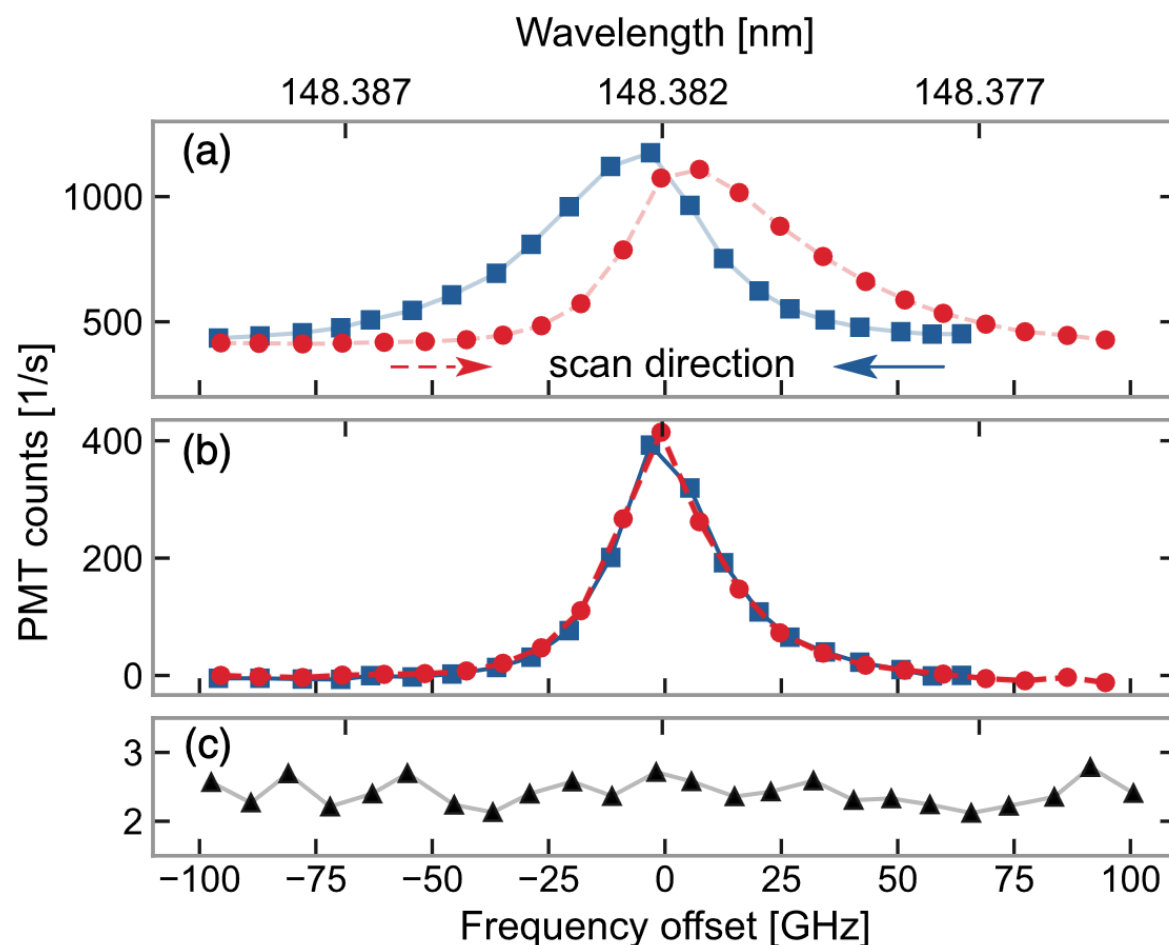
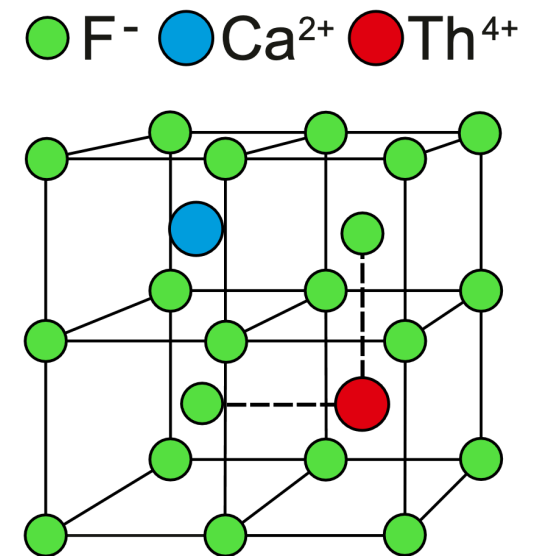




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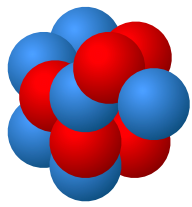
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- Scan frequencies and after $\tau \sim 0.5 \text{ h}$ get back fluorescent light at $\nu = 2020.409(7) \text{ THz}$



$$\frac{\delta\nu}{\nu} \sim 10^{-6}$$

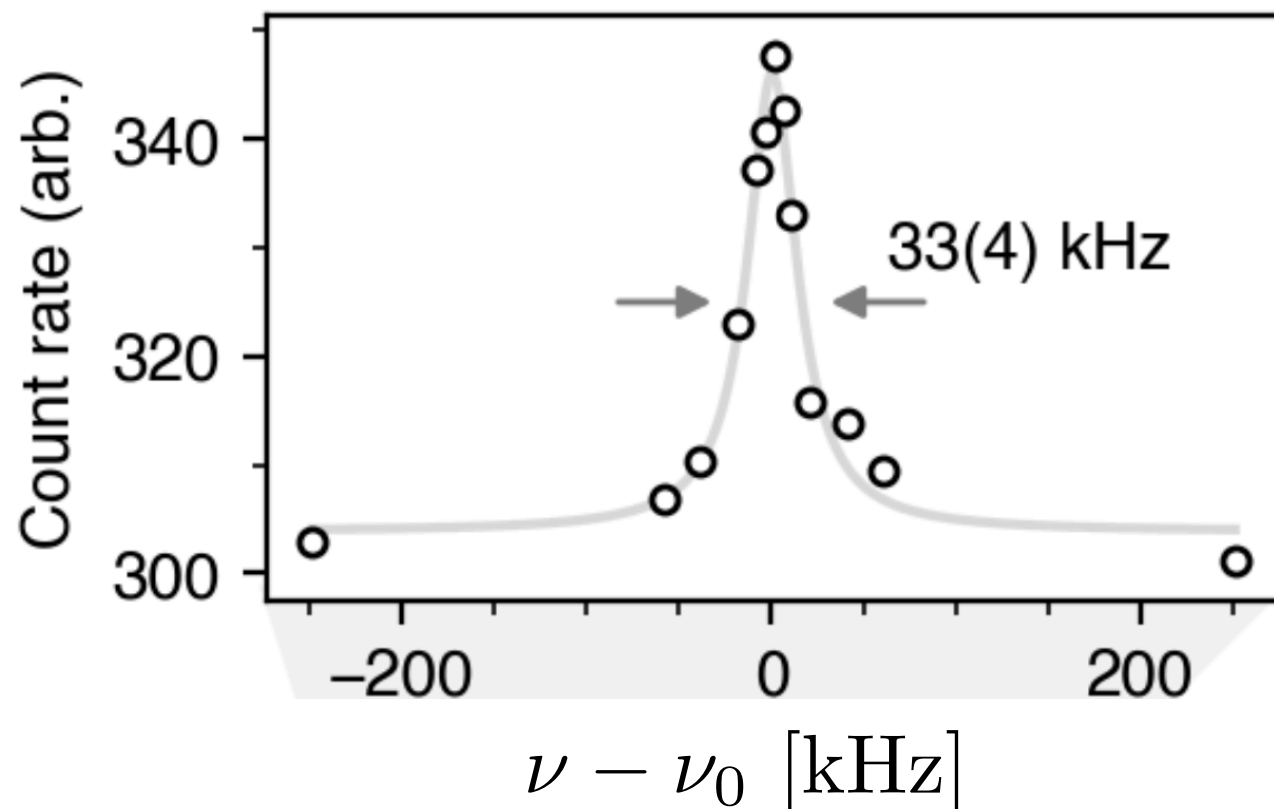
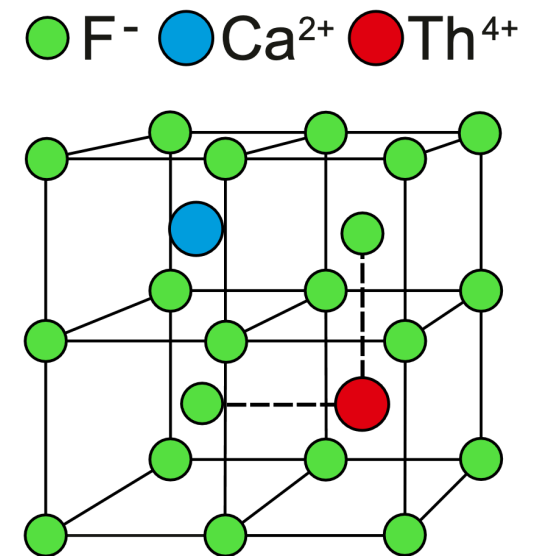
J. Tiedau et al., *Phys. Rev. Lett.* **132**, 182501 (2024)



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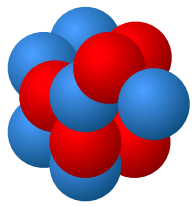
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- Comb spectroscopy: using 2 crystals gives reproducibility



$$\frac{\delta\nu}{\nu} \sim 10^{-13}$$

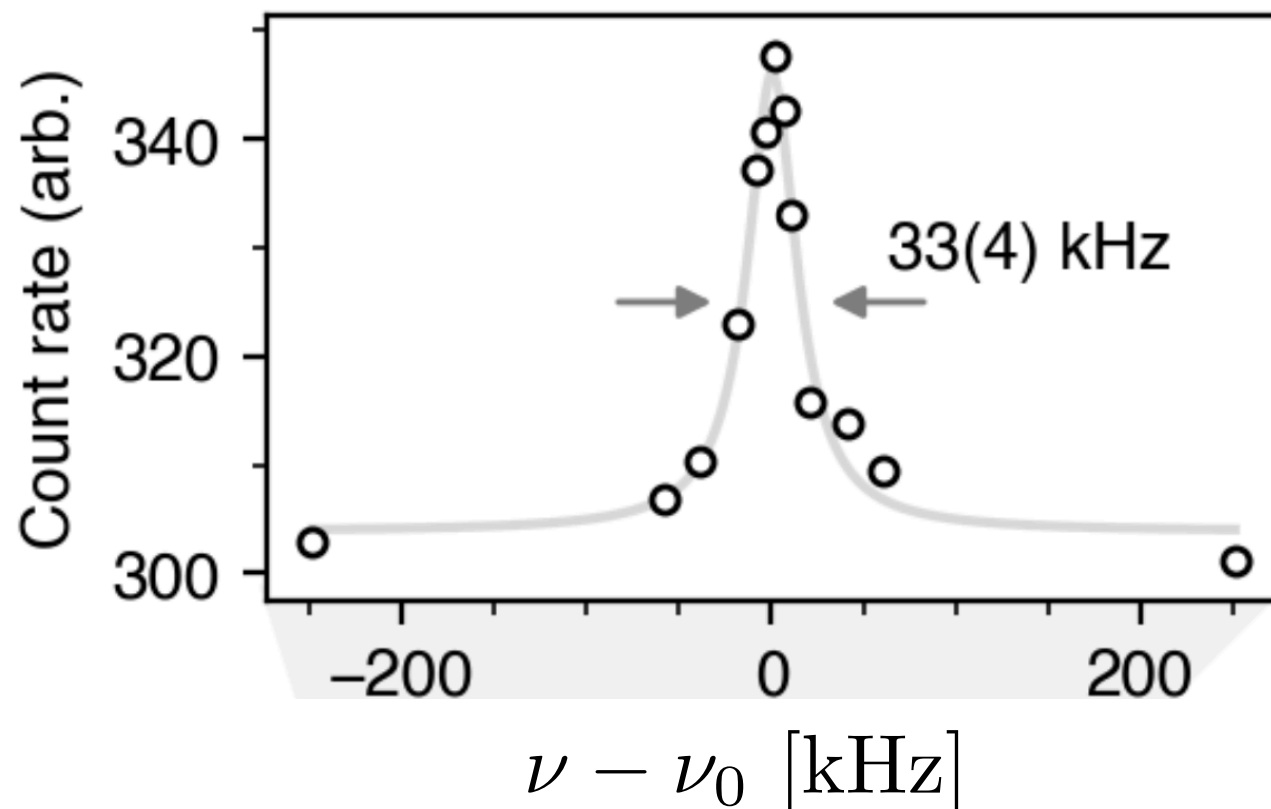
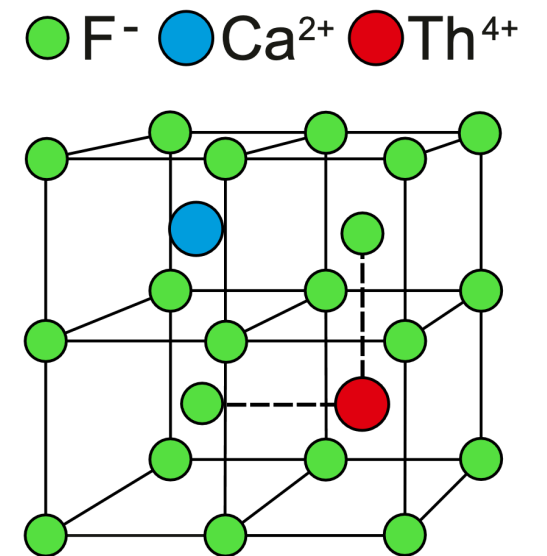
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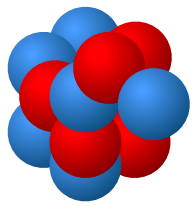
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$$\frac{\delta\nu}{\nu} \sim 10^{-13}$$

T. Ooi et al., *Nature* **650**, 72–78 (2026)

Great, but still worse than atomic clocks...



^{229}Th as nuclear clock candidate

Enhanced Sensitivity?

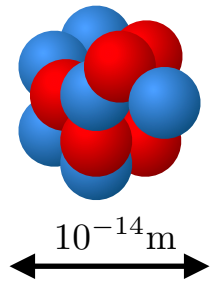
- Transition energy $\Delta E_{is} = \Delta E_{\text{QCD}} - \Delta E_{\text{EM}} \simeq 8 \text{ eV}$
- Naively $\Delta E_{\text{EM}} \simeq \frac{(eZ)^2}{A^{1/3}} \frac{\Lambda_{\text{QCD}}}{A} \sim \text{MeV} \sim \Delta E_{\text{QCD}}$
- ULDM coupled to strong sector

$$\frac{\delta(\Delta E_{is})}{\Delta E_{is}} = \frac{\Delta E_{\text{QCD}}(t) - \Delta E_{\text{EM}}}{\Delta E_{is}} \Rightarrow \frac{\Delta E_{\text{QCD}}}{\Delta E_{is}} \times d_g \frac{m_N}{M_{pl}} \cos(m_\phi t)$$

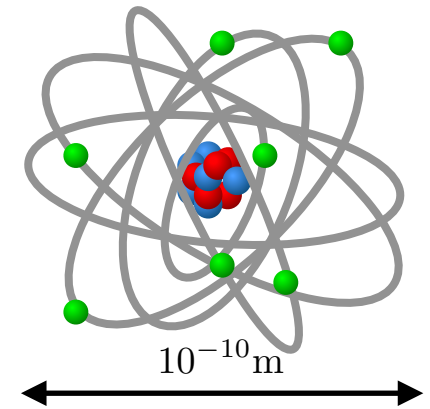
Then the sensitivity factor is indeed

$$K_{\text{nuc}} = \frac{\Delta E_{\text{EM}}}{\Delta E_{is}} \sim 10^5$$

This and no suppression gives $K_{\text{nuc}}/K_{\text{atom}} \sim 10^8 - 10^{10}$ enhancement!



Outline

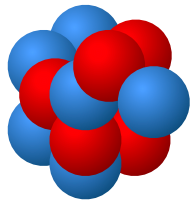


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Approaches to estimate the sensitivity

- Calculation of ΔE_{nuc} is basically impossible, but can try to calculate ΔE_{EM}

Classical approaches

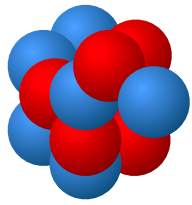
Rotor models

Data driven

Halo model

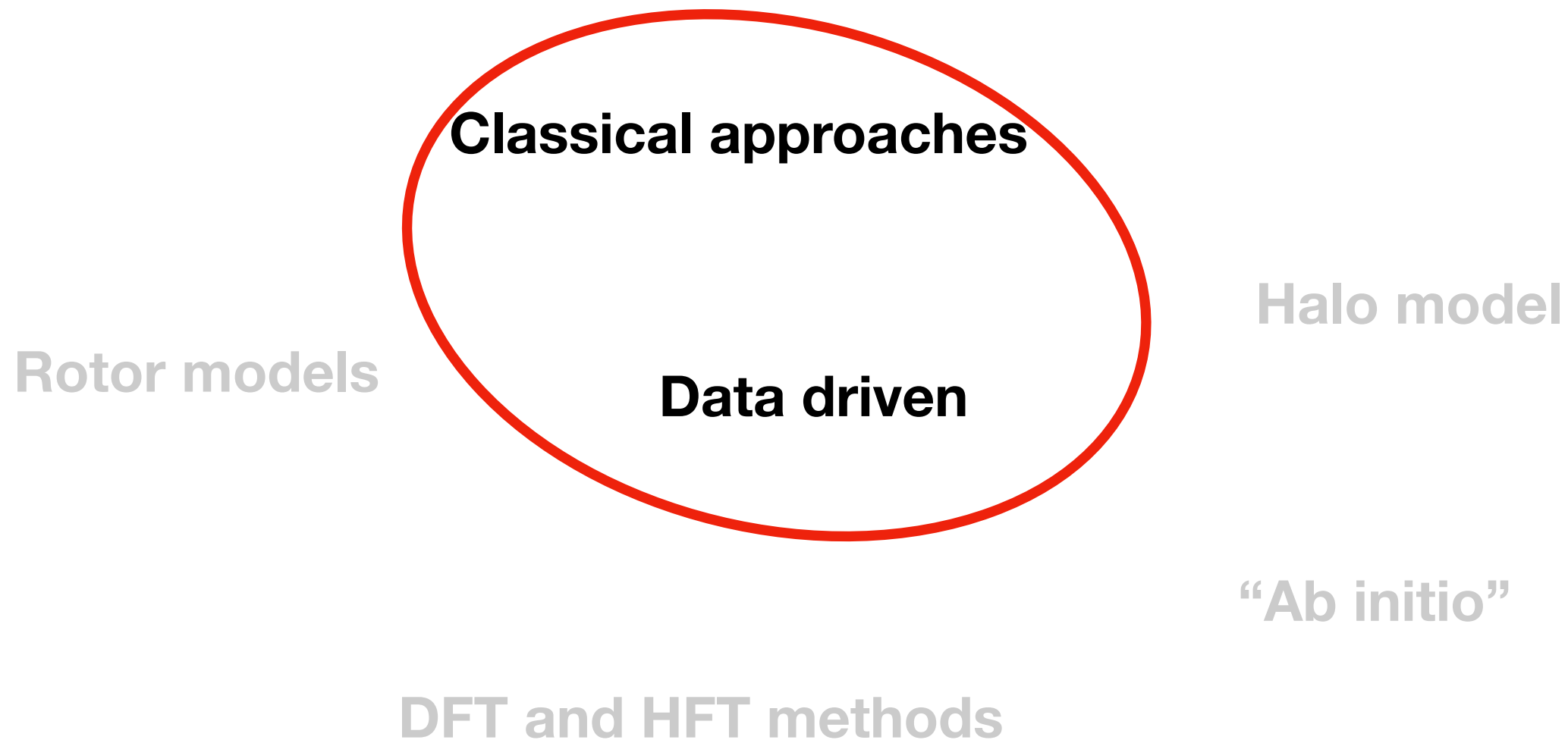
“Ab initio”

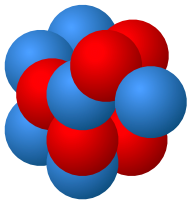
DFT and HFT methods



Approaches to estimate the sensitivity

- Calculation of ΔE_{nuc} is basically impossible, but can try to calculate ΔE_{EM}

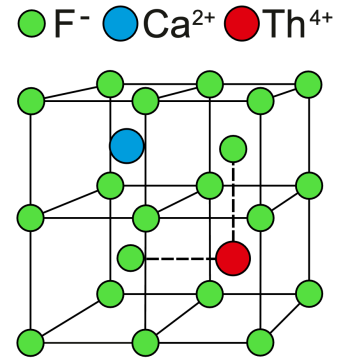
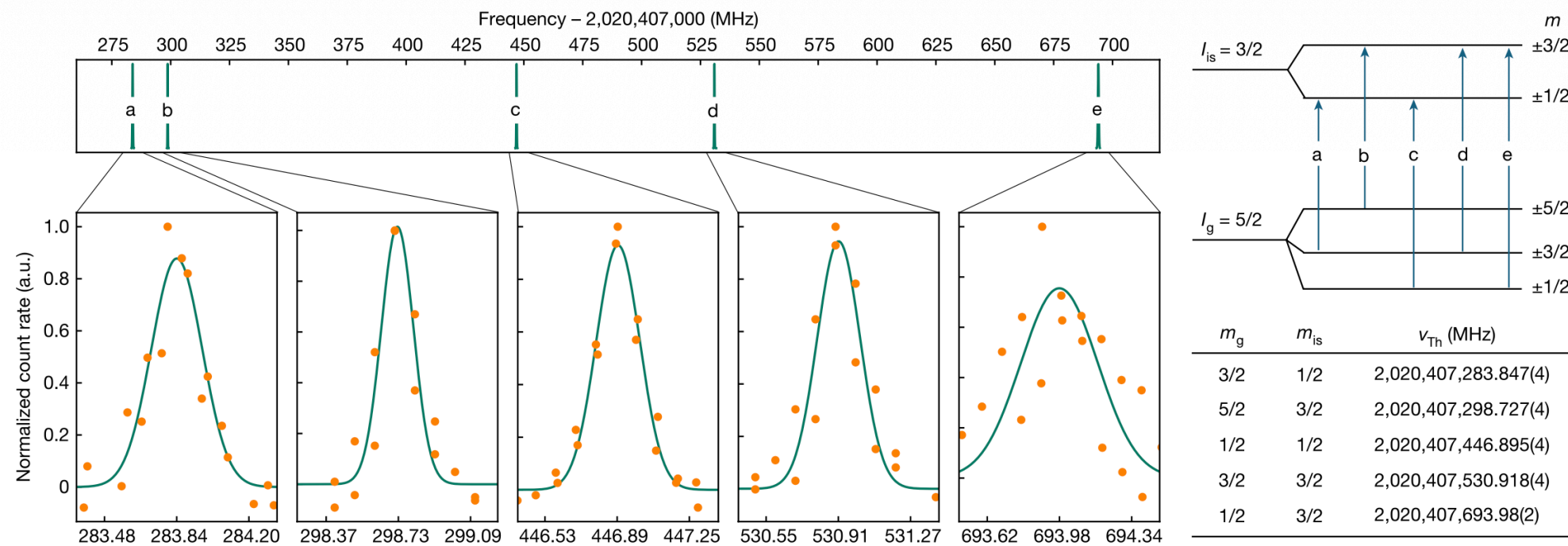




Data driven approaches

C. Zhang et al., *Nature* **633**, 63–70 (2024)

- State-resolved VUV frequency-comb spectroscopy referenced to the JILA ^{87}Sr -clock



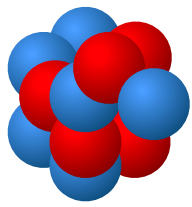
- Crystal structure induces splitting in the isomer shift via hyperfine interactions

$$\mathcal{H} = \sum_k \frac{1}{k} V^{(k)} Q_{\text{spec}}^{(k)}$$

$V^{(k)}$: Electric field gradients (crystal structure)

$Q_{\text{spec}}^{(k)}$: Spectroscopic nuclear multipole moments

- For instance for quadrupole moment ($k = 2$): $\Delta E_{Q(2)} = \partial_z^2 V(\vec{r})|_{\vec{r}=0} Q_{\text{spec}}^{(2)}$



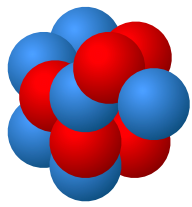
Hyperfine interaction hamiltonian

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$V^{(k)}$: Electric field gradients (crystal structure)

$Q_{\text{spec}}^{(k)}$: Spectroscopic nuclear multipole moments

Rank k	Electric $P = (-1)^k$	Magnetic $P = (-1)^{k+1}$
0	monopole ($Ze, \langle r^2 \rangle$)	— none —
1	dipole (EDM)	dipole (μ)
2	quadrupole (Q)	quadrupole
3	octupole	octupole (Ω)
4	hexadecapole	hexadecapole



Hyperfine interaction hamiltonian

$$\mathcal{H} = \sum_k \frac{1}{k} V^{(k)} Q_{\text{spec}}^{(k)}$$

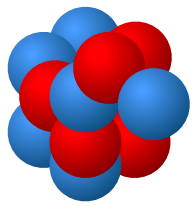
$V^{(k)}$: Electric field gradients (crystal structure)

$Q_{\text{spec}}^{(k)}$: Spectroscopic nuclear multipole moments

Rank k	Electric $P = (-1)^k$	Magnetic $P = (-1)^{k+1}$
0	monopole ($Ze, \langle r^2 \rangle$)	— none —
1	dipole (EDM)	dipole (μ)
2	quadrupole (Q)	quadrupole
3	octupole	octupole (Ω)
4	hexadecapole	hexadecapole

Subject to selection rules

ground state
 $I = \frac{5}{2}: k \leq 5$



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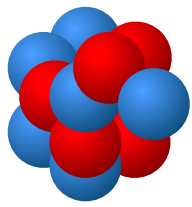
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isomer
 $I = \frac{3}{2}: k \leq 3$

ground state
 $I = \frac{5}{2}: k \leq 5$



Hyperfine interaction hamiltonian

$$\mathcal{H}_{\text{eff}} \simeq \sum_{k=0,2,4} \frac{1}{k} V^{(k)} Q_{\text{spec}}^{(k)}$$

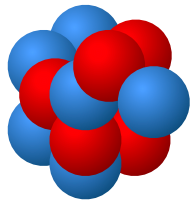
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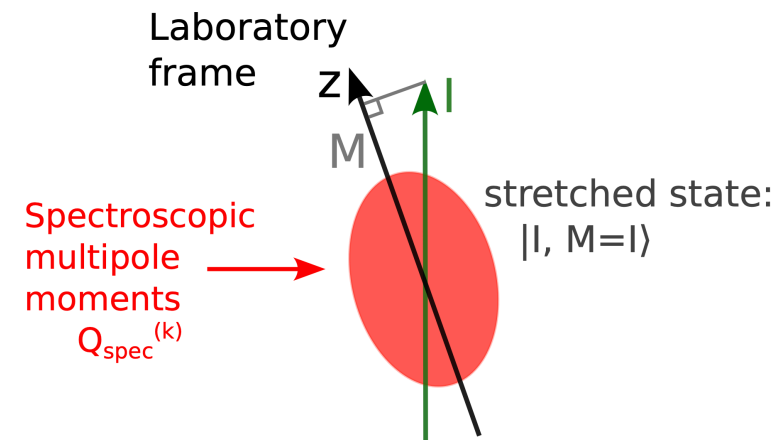
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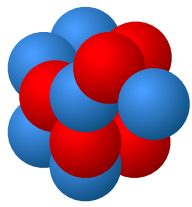
ground state
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Spectroscopic vs. intrinsic moments

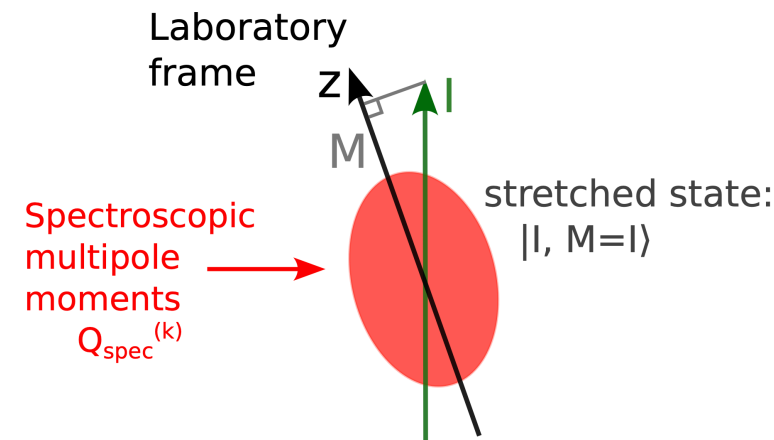
- Given a crystal structure experimentally determine $Q_{\text{spec}}^{(k)}$
(Laboratory fixed frame quantity in stretched state $I = M$)





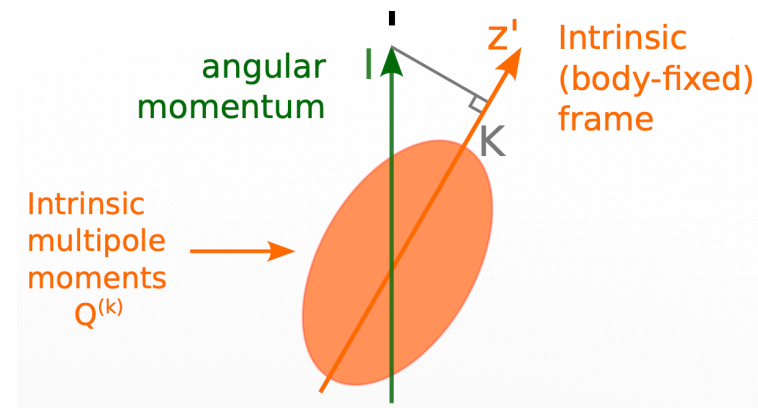
Spectroscopic vs. intrinsic moments

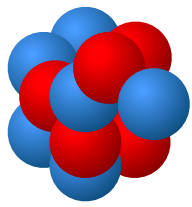
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- Geometrical models calculate body fixed intrinsic quantities

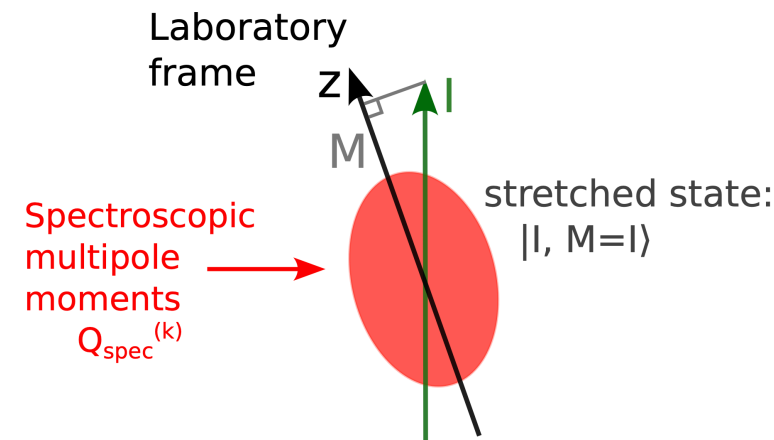
$$Q_{\text{int}}^{(k)}$$





Spectroscopic vs. intrinsic moments

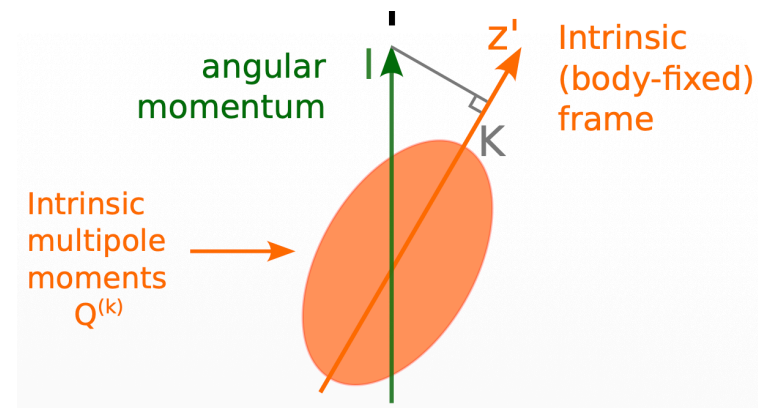
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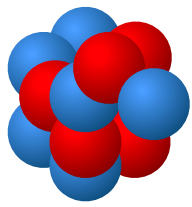


Need to connect them with rotational model

- Geometrical models calculate body fixed intrinsic quantities

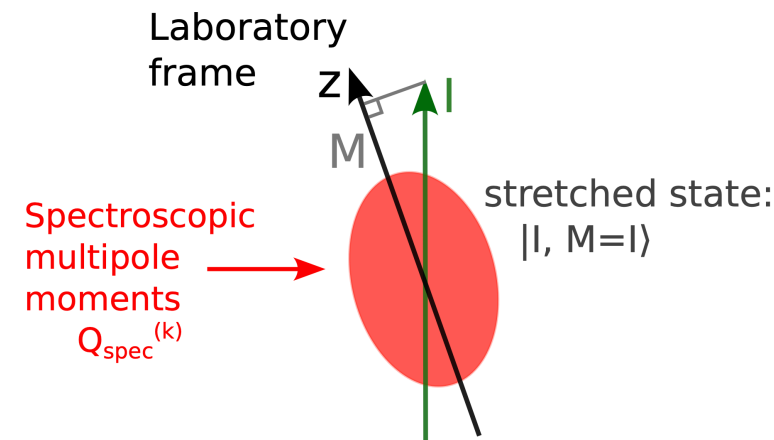
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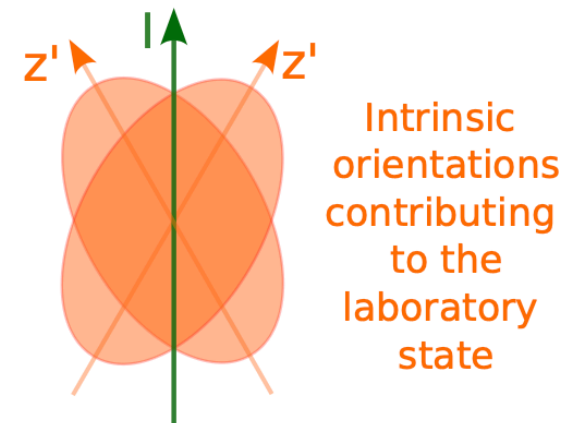


- Laboratory state is a rotating superposition of differently oriented intrinsic states

$$Q_{\text{spec}}^{(k)}(I) = Q_{\text{int}}^{(k)}(2I + 1) \begin{pmatrix} I & k & I \\ -K & 0 & K \end{pmatrix} \begin{pmatrix} I & k & I \\ -I & 0 & I \end{pmatrix}$$

Projects intrinsic K onto I

Projects into stretched state

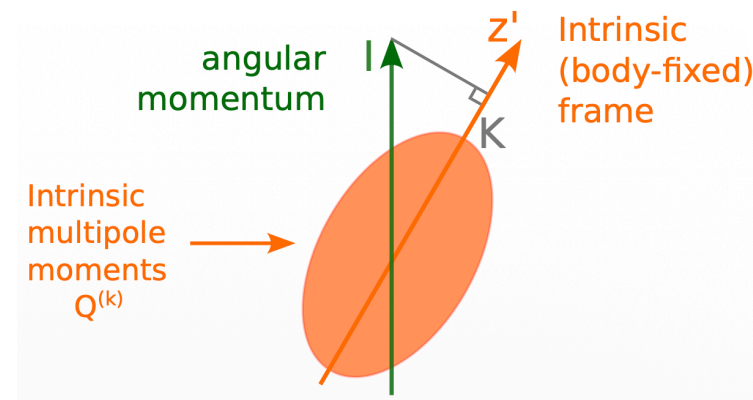


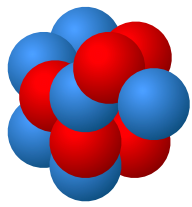
Projection onto nuclear state $|I, M(=I)\rangle$ (Wigner-Eckart theorem)



- Geometrical models calculate body fixed intrinsic quantities

$$Q_{\text{int}}^{(k)}$$

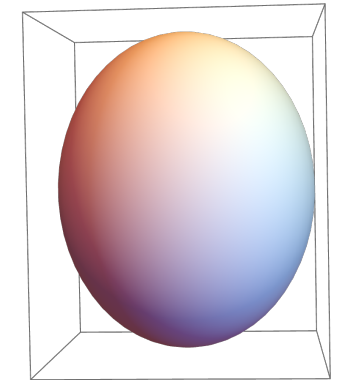




Geometrical classical modelling

Classical approaches

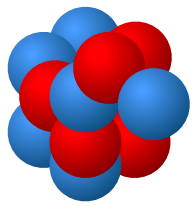
- Classical charge density $\rho(r) \Rightarrow \Delta E_{\text{EM}}$



Constant density prolate spheroid

Berengut, Dzuba, Flambaum, Porsev, PRL 102 (2009)

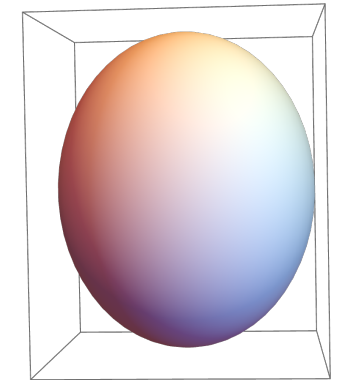
$$E_{\text{C}} = \frac{q_e^2}{2} \int d^3r d^3r' \frac{\rho(\vec{r})\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} = \frac{3}{5} \frac{q_e^2 Z^2}{R_0} \frac{(1 - e^2)^{1/3}}{2e} \ln \left(\frac{1 + e}{1 - e} \right)$$



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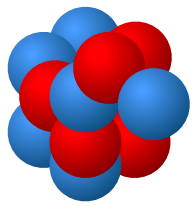
- Compute charge radius and *intrinsic (in body-fixed frame)* quadrupole moment

$$\langle r^2 \rangle = \frac{1}{q_e Z} \int d^3\vec{r} r^2 \rho(r, \theta) = \frac{3}{5} R_0^2 \left(1 + \frac{e^4}{9} + \dots \right)$$

$$Q_0 = \frac{1}{q_e Z} \int d^3\vec{r} r^2 P_2(\cos \theta) \rho(r, \theta) = \frac{3}{5} R_0^2 \left(1 + \frac{e^4}{9} + \dots \right)$$

- Allows to trade $\{R_0, e\} \Leftrightarrow \{\langle r^2 \rangle, Q_0\}$

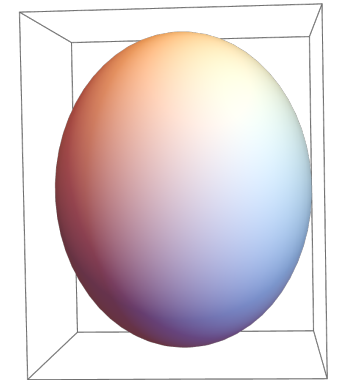
(Which are measured)



Geometrical classical modelling

Classical approaches

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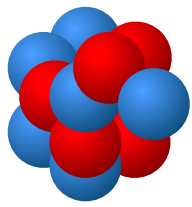
Constant density prolate spheroid

Berengut, Dzuba, Flambaum, Porsev, PRL 102 (2009)

- Then with $\Delta \langle r^2 \rangle = 0.012(2) \text{ fm}^2$ and $\Delta Q_0 = -0.087(332) \text{ fm}^2$

Thielking et al., Nature 556, 321 (2018)

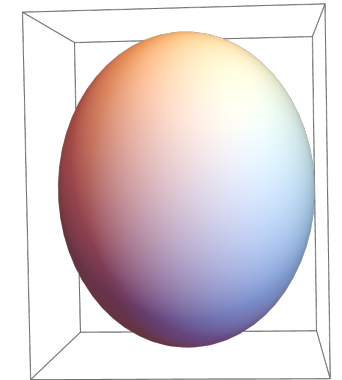
$$\Delta E_{\text{C}} = -485 \text{ MeV} \frac{\Delta \langle r^2 \rangle}{\langle r^2 \rangle} + 11.6 \text{ MeV} \frac{\Delta Q_0}{Q_0} = -0.29(43) \text{ MeV}$$



Geometrical classical modelling

Classical approaches

- Classical charge density $\rho(r) \Rightarrow \Delta E_{\text{EM}}$



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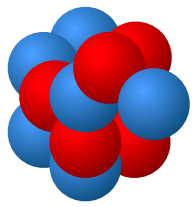
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... which implies $K_{\text{nuc}} = 0 - 10^4$

Dominated by the uncertainty in $\Delta Q_0 / Q_0$

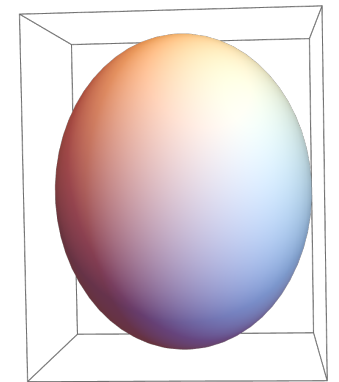


Geometrical classical modelling

Classical approaches

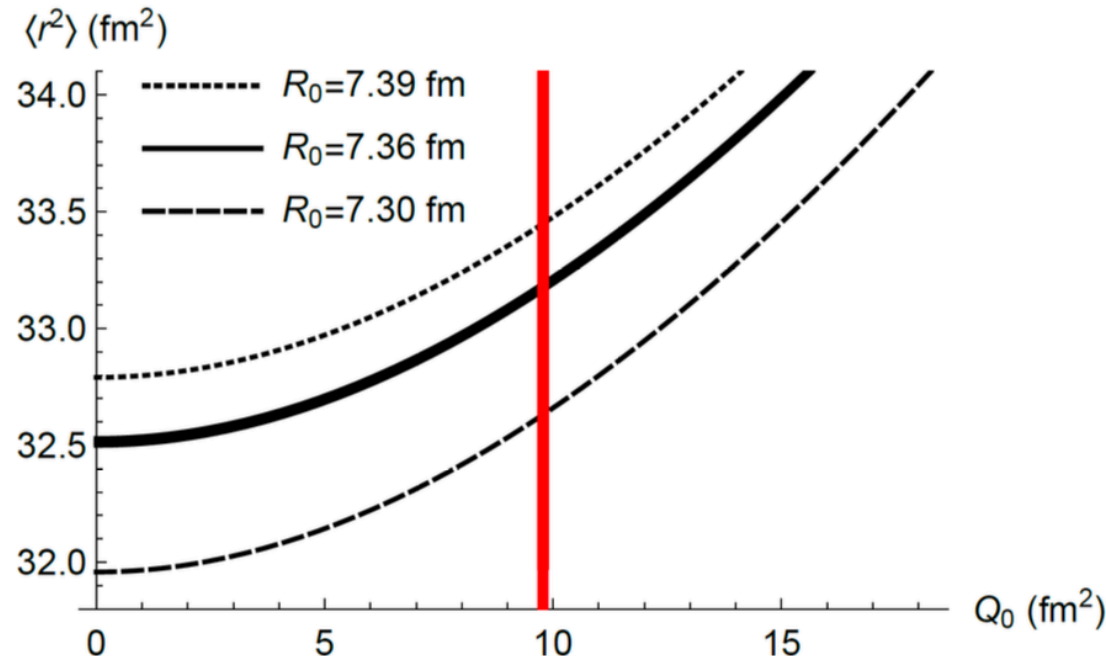
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Thielking et al., Nature 556, 321 (2018)



- Constant volume approximation allows to relate $\langle r^2 \rangle$ And Q_0

Fadeev, Berengut, Flambaum PRL 102 (2020)

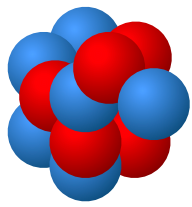


→ $\Delta E_C = -0.067(19) \text{ MeV}$

Is this justified? Questionable..

Hayes, Friar, Möller: PRC 78, 024311 (2008)

Dominated by the uncertainty in $\Delta Q_0 / Q_0$



Geometrical classical modelling

- First laser excitation 2024 reduced ΔQ_0 —uncertainty

J. Tiedau et al., *Phys. Rev. Lett.* **132**, 182501 (2024)

Classical approaches: numerics with Woods-Saxon

Caputo, Gazit, Hammer, Kopp, Paz, Perez, KS, *PRC Lett.* (2024)

See also Beeks et al. (2024)

- Woods-Saxon charge density

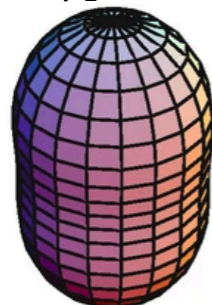
Woods, Saxon (1954)

$$\rho(r, \theta) = \rho_0 \left[1 + \exp \left(\frac{r - R(\theta)}{z} \right) \right]^{-1} \text{ with } R(\theta) = R_0 [1 + \beta_2 Y_{20}(\theta) + \beta_3 Y_{30}(\theta) + \dots]$$

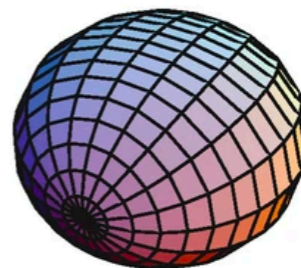
spherical
 $\beta_k=0 \forall k$



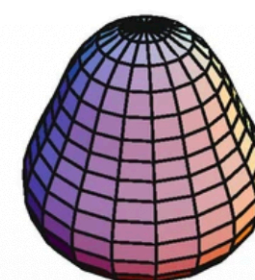
prolate
 $\beta_2 > 0$



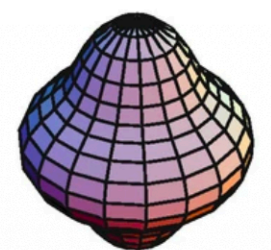
oblate
 $\beta_2 < 0$

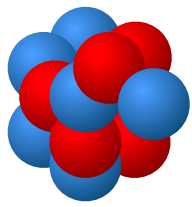


octupole
 $\beta_3 \neq 0$



hexadecapole
 $\beta_4 \neq 0$





Geometrical classical modelling

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J. Tiedau et al., *Phys. Rev. Lett.* **132**, 182501 (2024)

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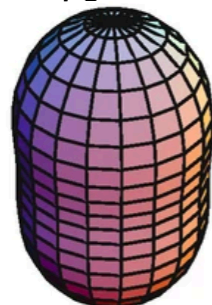
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Unconstrained as of now

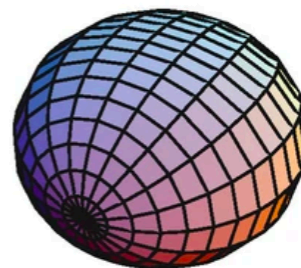
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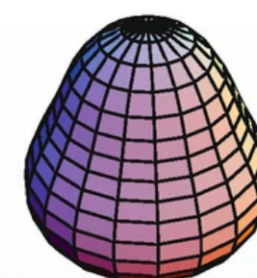
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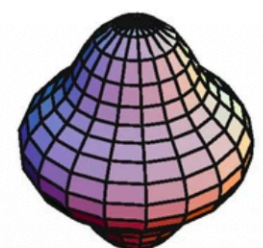
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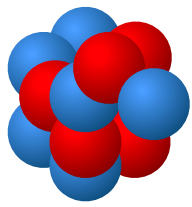


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J. Tiedau et al., *Phys. Rev. Lett.* **132**, 182501 (2024)

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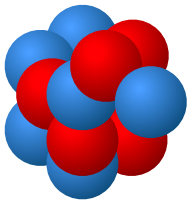
$$\rho(r, \theta) = \rho_0 \left[1 + \exp \left(\frac{r - R(\theta)}{z} \right) \right]^{-1} \quad \text{with} \quad R(\theta) = R_0 [1 + \beta_2 Y_{20}(\theta) + \beta_3 Y_{30}(\theta) + \dots]$$

- Charge radius and quadrupole moment constrain R_0 and β_2

$$\langle r^2 \rangle = \frac{3}{5} R_0^2 \left(1 + \mathcal{O} \left(\frac{z^2}{R_0^2}, \beta_k^2 \right) \right) \quad Q_0 = \frac{3}{\sqrt{5}\pi} R_0^2 \beta_2 \left(1 + \mathcal{O} \left(\frac{z^2}{R_0^2}, \beta_k^2 \right) \right)$$

- For now ignoring $\beta_{3,4}$ and z one finds at 2σ -level

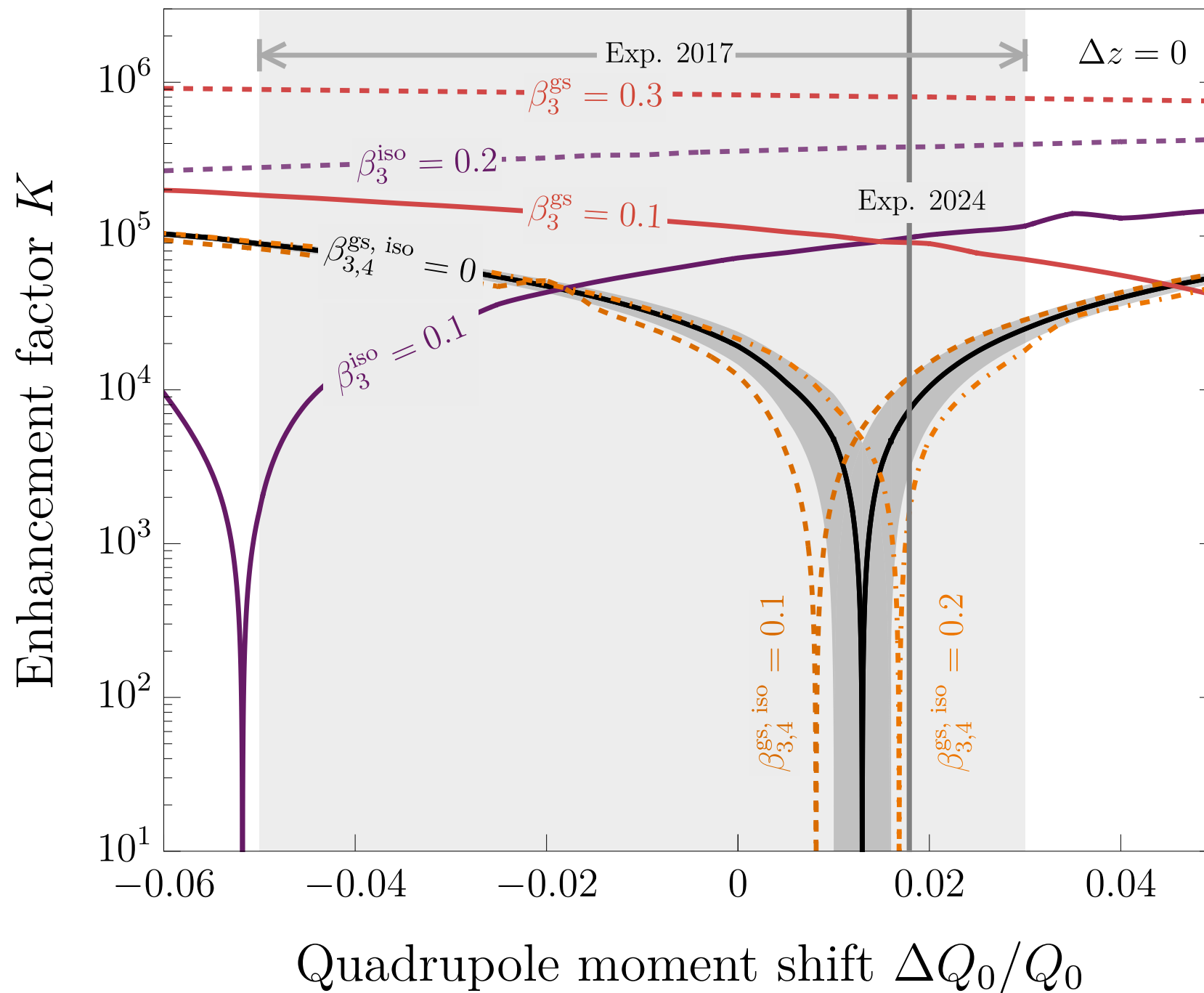
$$K_{\text{nuc}} = 10^3 \left[6 \pm 5 \times \frac{\Delta \langle r^2 \rangle}{0.00105 \text{ fm}^2} \right]$$



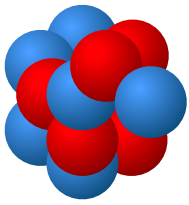
Geometrical classical modelling

- Numerically scan Coulomb energy shift for $\beta_{3,4}$ and z

Caputo, Gazit, Hammer, Kopp, Paz, Perez, KS, PRC Lett. (2024)



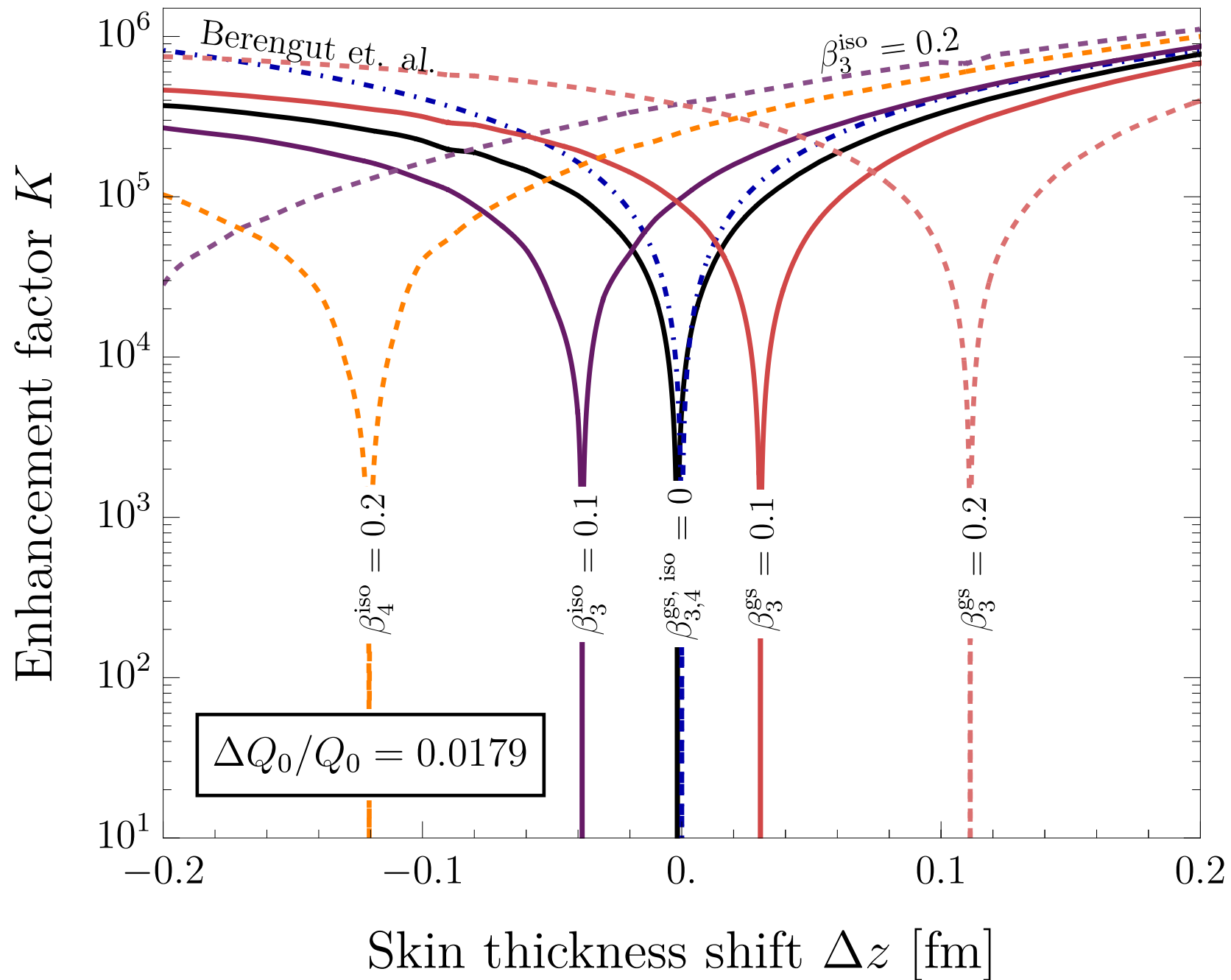
Nightmare scenario seems unlikely



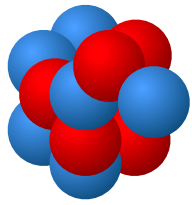
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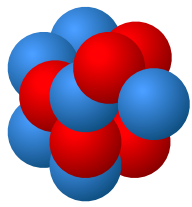
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Going beyond quadratic order

Baruch, Gazit, Hammer, Kirk, Kopp, Paz, Perez, Peik, Soreq and KS, in prep.

- Zhang. et al. neglect hexadecapole moment ($\sim \beta_4$) C. Zhang et al., *Nature* **633**, 63–70 (2024)
- There is a degeneracy with the crystal symmetry: can fit data with β_2 and β_4

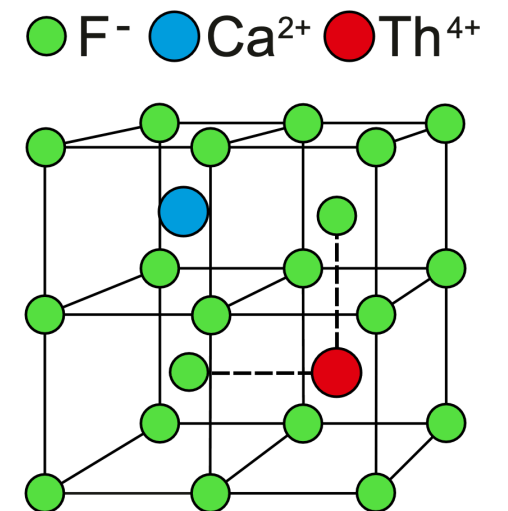
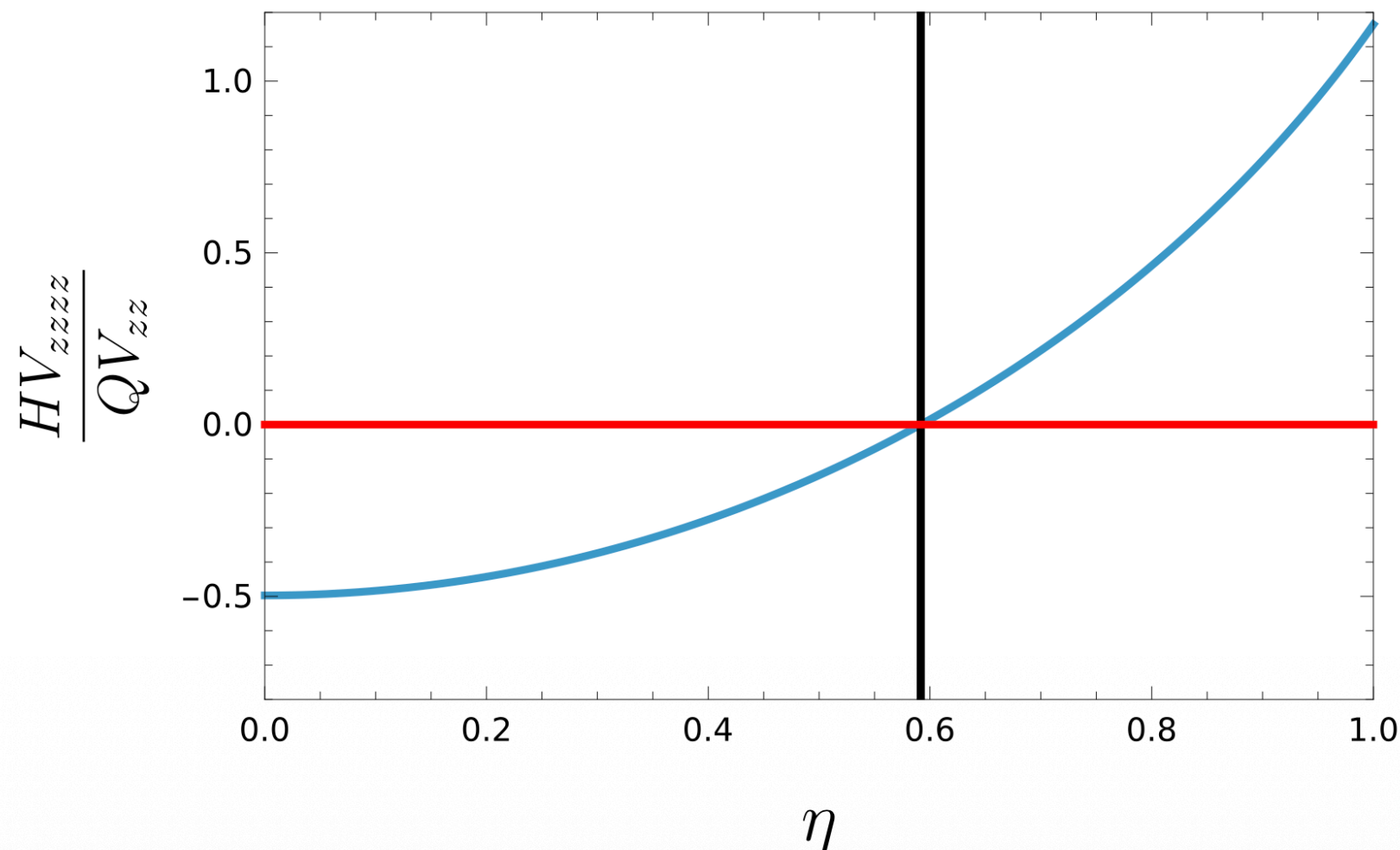


Going beyond quadratic order

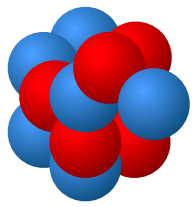
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Hexadecapole splitting
Quadrupole splitting as a function of crystal asymmetry parameter $\eta \equiv \frac{V_{xx} - V_{yy}}{V_{zz}}$



⇒ how does this affect K_{nuc} ?



Going beyond quadratic order

Baruch, Gazit, Hammer, Kirk, Kopp, Paz, Perez, Peik, Soreq and KS, in prep.

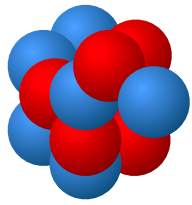
- Zhang. et al. neglect hexadecapole moment ($\sim \beta_4$) C. Zhang et al., *Nature* **633**, 63–70 (2024)
- There is a degeneracy with the crystal symmetry: can fit data with β_2 and β_4
- However, what would we expect for $\frac{\text{Hexadecapole splitting}}{\text{Quadrupole splitting}}$?

$$\mathcal{H}^{(k)} \sim \frac{q_e^2 Z^2}{s} \left(\frac{R_{\text{nuc}}}{s} \right)^k \Rightarrow \frac{\mathcal{H}^{(4)}}{\mathcal{H}^{(2)}} \sim \left(\frac{R_{\text{nuc}}}{s} \right)^2 \sim 10^{-10}$$

$$R_{\text{nuc}} \simeq 5 \text{ fm}$$

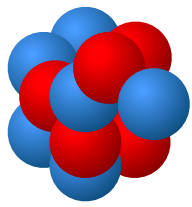
- Up to now however unconstrained...

$$s \simeq 5 \text{ \AA}$$



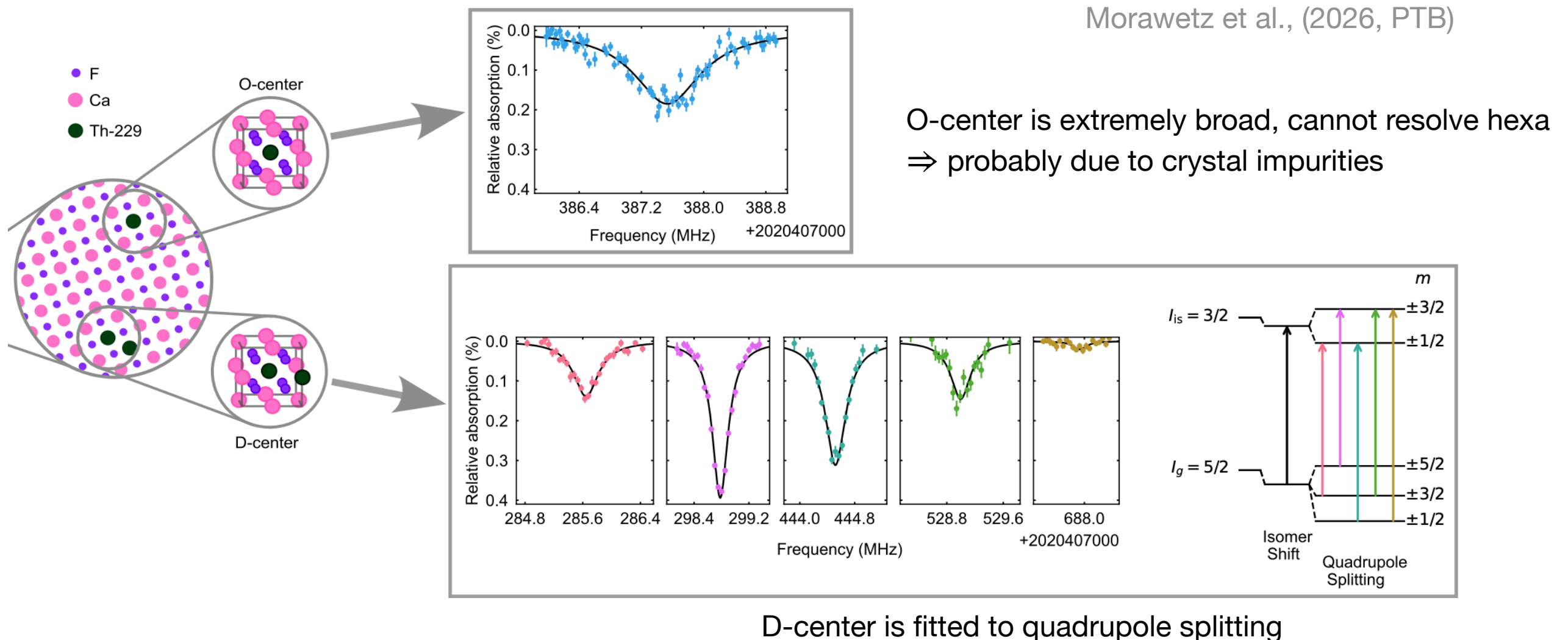
Multipoles from crystal symmetry?

- Crystal symmetries determine which couplings are allowed and how many independent parameters there are: more symmetry $\Rightarrow$ less parameters
- Cubic symmetry: no quadrupole $\Rightarrow$ hexadecapole leading contribution?

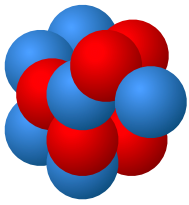


Multipoles from crystal symmetry?

- Crystal symmetries determine which couplings are allowed and how many independent parameters there are: more symmetry $\Rightarrow$ less parameters
- Cubic symmetry: no quadrupole $\Rightarrow$ hexadecapole leading contribution?
- Recent measurements hints at defect site with cubic symmetry



- Degeneracy could be lifted by studying different lattices



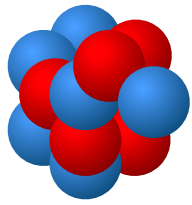
Going beyond quadratic order

- Employing again a geometric model we find for K with ($z = 0.5$ fm and $\Delta z = 0$)

$$\begin{aligned} K \simeq & 7.9 \times 10^3 - 3.8 \times 10^4 \left(\frac{\langle r^2 \rangle}{(5.7557 \text{ fm})^2} - 1 \right) \pm 190 \frac{\sigma_{\langle r^2 \rangle}}{0.16 \text{ fm}^2} \\ & - 1.8 \times 10^4 \left(\frac{\delta \langle r^2 \rangle}{0.0105 \text{ fm}^2} - 1 \right) \pm 2250 \frac{\sigma_{\delta \langle r^2 \rangle}}{0.0013 \text{ fm}^2} \\ & + 5.2 \times 10^4 \left(\frac{Q_2^{(g)}}{3.11 \text{ eb}} - 1 \right) \pm 340 \frac{\sigma_{Q_2^{(g)}}}{0.02 \text{ eb}} \\ & + 1.5 \times 10^6 \left(\frac{\mathcal{R}_{Q_2}}{0.57003} - 1 \right) \pm 26 \frac{\sigma_{\mathcal{R}_{Q_2}}}{10^{-5}} \\ & + 6 \times 10^4 \left(\frac{Q_4^{[G]}}{4 \text{ eb}^2} \right)^2 (\mathcal{R}_{Q_4} - 1) \end{aligned}$$

Baruch, Gazit, Hammer, Kirk, Kopp, Paz, Perez, Peik, Soreq, KS, in prep.

Nightmare scenario seems unlikely



Approaches to estimate the sensitivity

- Calculation of ΔE_{nuc} is impossible, but can try to calculate ΔE_{EM}

Classical approaches

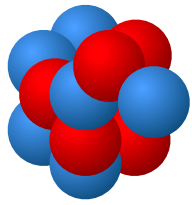
Rotor models

Data driven

Halo model

“Ab initio”

DFT and HFT methods



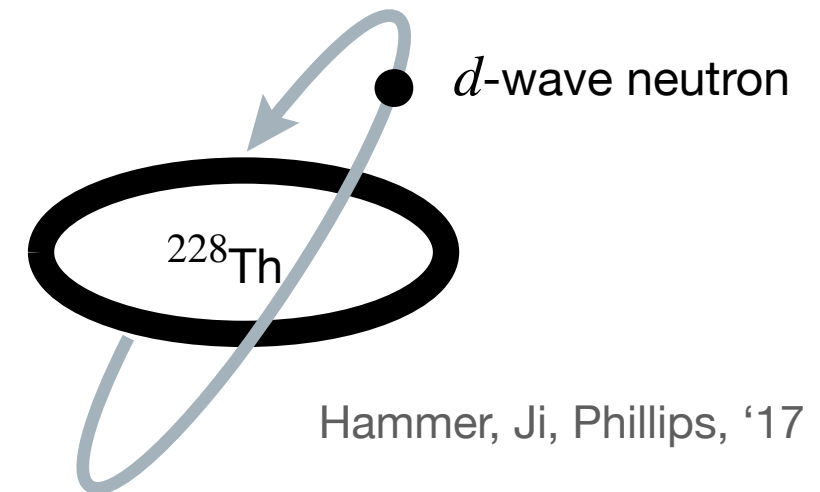
Halo EFT model

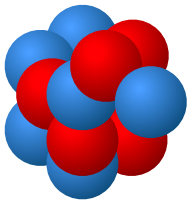
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Beeks et al: Nat. Rev. (21)

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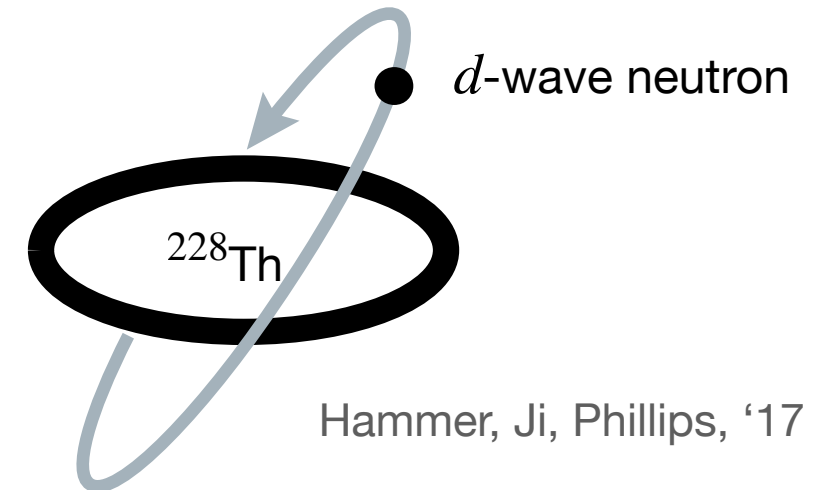


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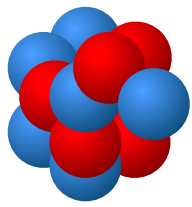
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- Spin—orbit coupling introduces isomer shifts: define spin orbit charge density

In the lab frame:
$$\rho_{\text{SO}} = \frac{\mu_n}{2m_n^2} i\sigma_{\alpha\beta} \cdot [(\nabla\Phi_{\alpha}^{\dagger}) \times (\nabla\Phi_{\beta})]$$

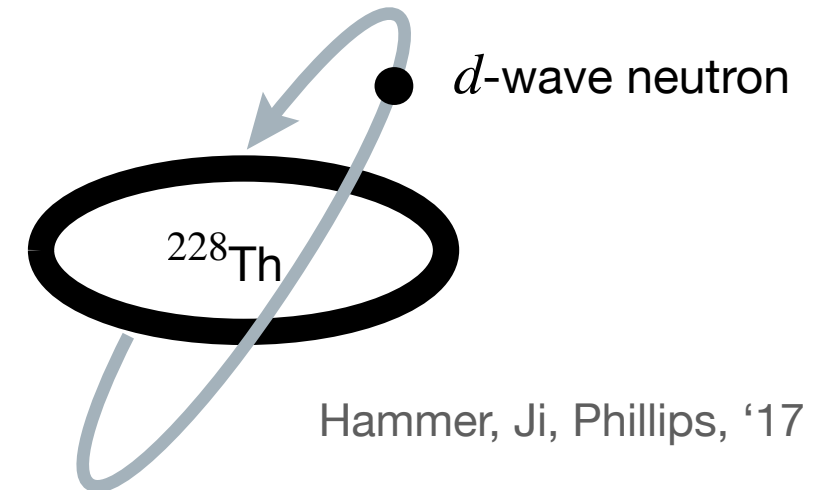


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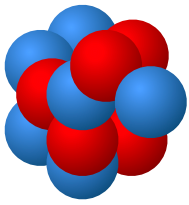
- Charge radius

$$\langle r^2 \rangle = -\frac{\mu_n}{m_n^2 Z} (\kappa + 1)$$

$$\kappa = \begin{cases} l & \text{for } j = l - \frac{1}{2} \\ -(l+1) & \text{for } j = l + \frac{1}{2} \end{cases}$$

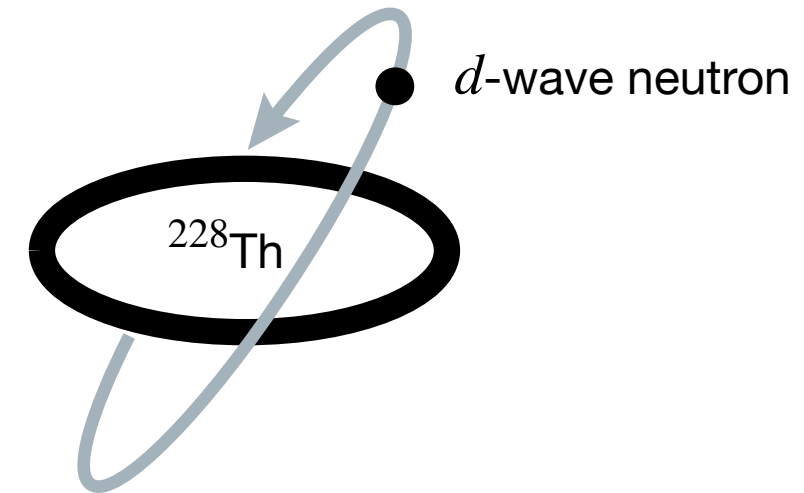
$$\Delta\langle r^2 \rangle_{\text{SO}} \equiv \langle r^2 \rangle_{\text{SO}}^{\text{iso}} - \langle r^2 \rangle_{\text{SO}}^{\text{gs}} = 0.0047 \text{ fm}^2$$

$$\Delta\langle r^2 \rangle_{\text{exp}} = 0.0105(13) \text{ fm}^2$$

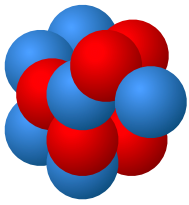


Halo EFT model

- Spectroscopic quadrupole moment in stretched state $m = j$ agrees poorly

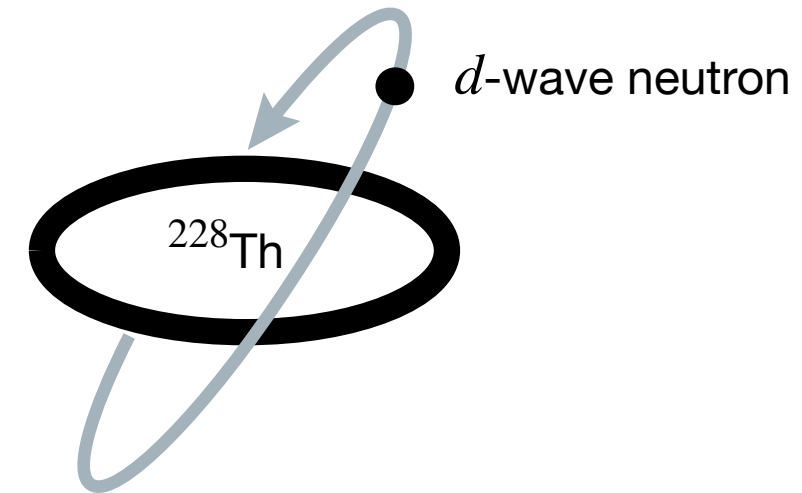


$$\frac{\Delta Q_{2,\text{SO}}}{q_e Z} = \frac{13}{5} \frac{\mu_n}{Z m_n^2} = -0.00243767 \text{ fm}^2 \quad \Rightarrow \quad \frac{\Delta Q_{2,\text{SO}}}{\Delta Q_2^{\text{exp}}} = 1.64 \times 10^{-3}$$



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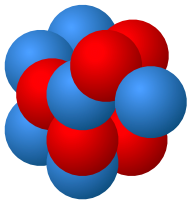
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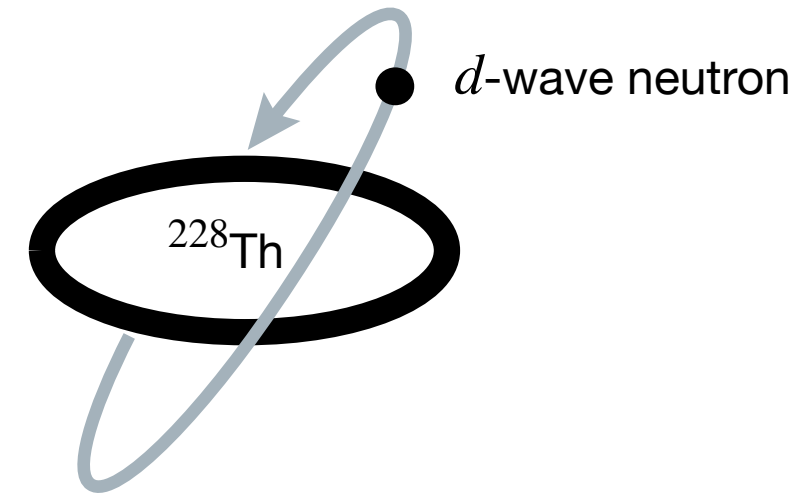
- Prediction for spectroscopic hexadecapole moment in stretched state $m = j$

$$\Delta Q_{4,\text{SO}} = 0.036 p q_e \text{ fm}^4$$



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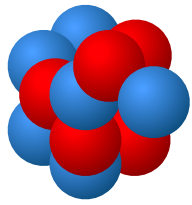
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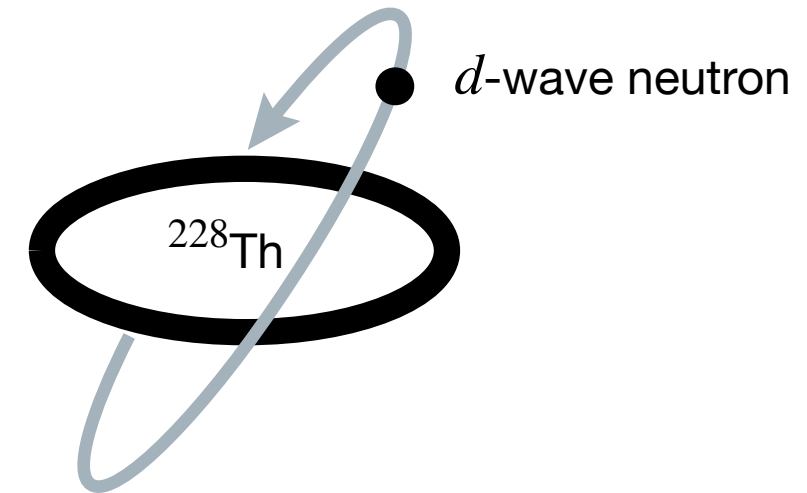
- Calculate contribution to the energy

$$E_{\text{SO}} = \frac{e \mu_n}{2 m_n^2} \frac{1}{2} \left[j(j+1) - l(l+1) - \frac{3}{4} \right] \int_0^\infty \left[\frac{[u(r)]^2}{r} \frac{dV_C}{dr} \right] dr$$

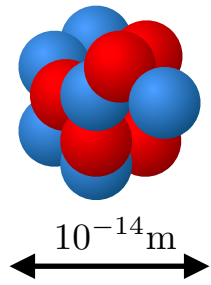
$$\Delta E_{\text{SO}} = E_{\text{SO}}|_{j=5/2} - E_{\text{SO}}|_{j=3/2} \approx 144 p \text{ keV} \quad \Rightarrow \quad K_{\text{nuc}} \sim 10^4$$



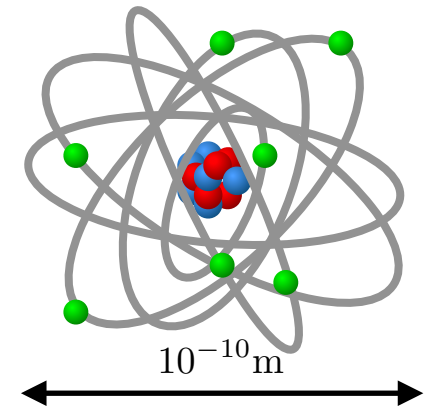
Halo EFT model



Take away: K_{nuc} is model-dependent but nightmare scenario unlikely



Outline



- **Sensitivity of ^{229}Th nuclear clock to new physics**



1. Quantum clocks simplified
2. Atomic clocks
3. ^{229}Th as a nuclear clock
4. Approaches to determine K

- **The case of quadratically coupled ULDM**

1. Natural ULDM: Quadratic twin
2. Local DM density around Earth with $\phi^2 \bar{N}N$ - coupling

Why quadratically coupled?

Interactions with the SM

- E.g. axions are **parity odd** $\longrightarrow$ **Quadratic** (non-derivative) couplings to SM

$$\mathcal{L} \supset C_\gamma \frac{\phi^2}{f^2} F_{\mu\nu} F^{\mu\nu} - C_N \frac{\phi^2}{f^2} m_N \bar{N} N - C_e \frac{\phi^2}{f^2} m_e \bar{e} e + \dots$$

shift-symm. breaking spurion

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- If QCD axion:

shift-symm. breaking spurion

$$C_\gamma = \frac{\alpha}{16\pi^2} \frac{z}{(1+z)^2} \frac{\pi}{3}$$

$$C_e = \frac{3\alpha}{4\pi} C_\gamma \ln \frac{m_\pi^2}{m_e^2}$$

$$C_N = \frac{1}{2} \frac{z}{(1+z)^2} \frac{\sigma_N}{m_N}$$

Beadle, Ellis, Quevillon, Vuong ('23)
Kim, Lenoci, Perez, Ratzinger ('23)

Hook, Huang ('17)
Balkin, Serra, KS, Weiler ('20)

Variations in fundamental constants

$$\frac{\delta\alpha}{\alpha} = C_\gamma \frac{\phi^2}{f^2}$$

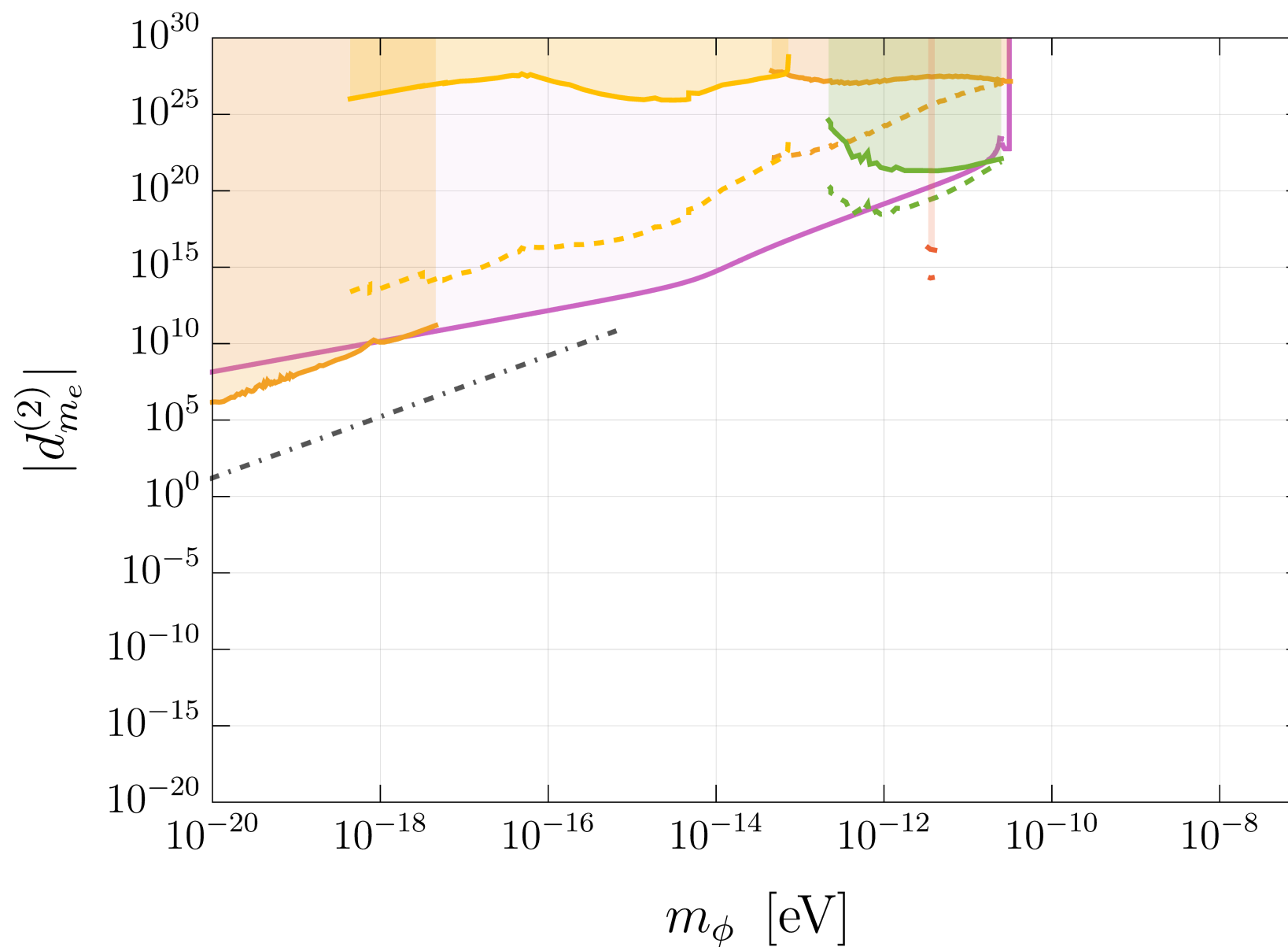
$$\frac{\delta m_N}{m_N} = C_N \frac{\phi^2}{f^2}$$

$$\frac{\delta m_e}{m_e} = C_e \frac{\phi^2}{f^2}$$

Parameter space

Coupling to electrons

$$\mathcal{L} \supset -\frac{d_{m_e}^{(2)} m_e}{2M_{\text{Pl}}^2} \phi^2 \bar{e}e$$



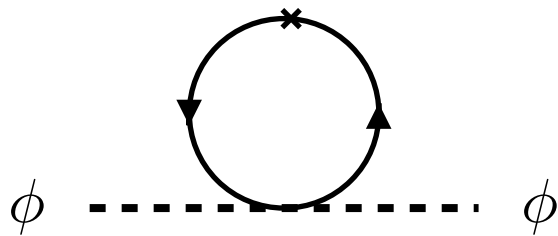
Goldstone Naturalness

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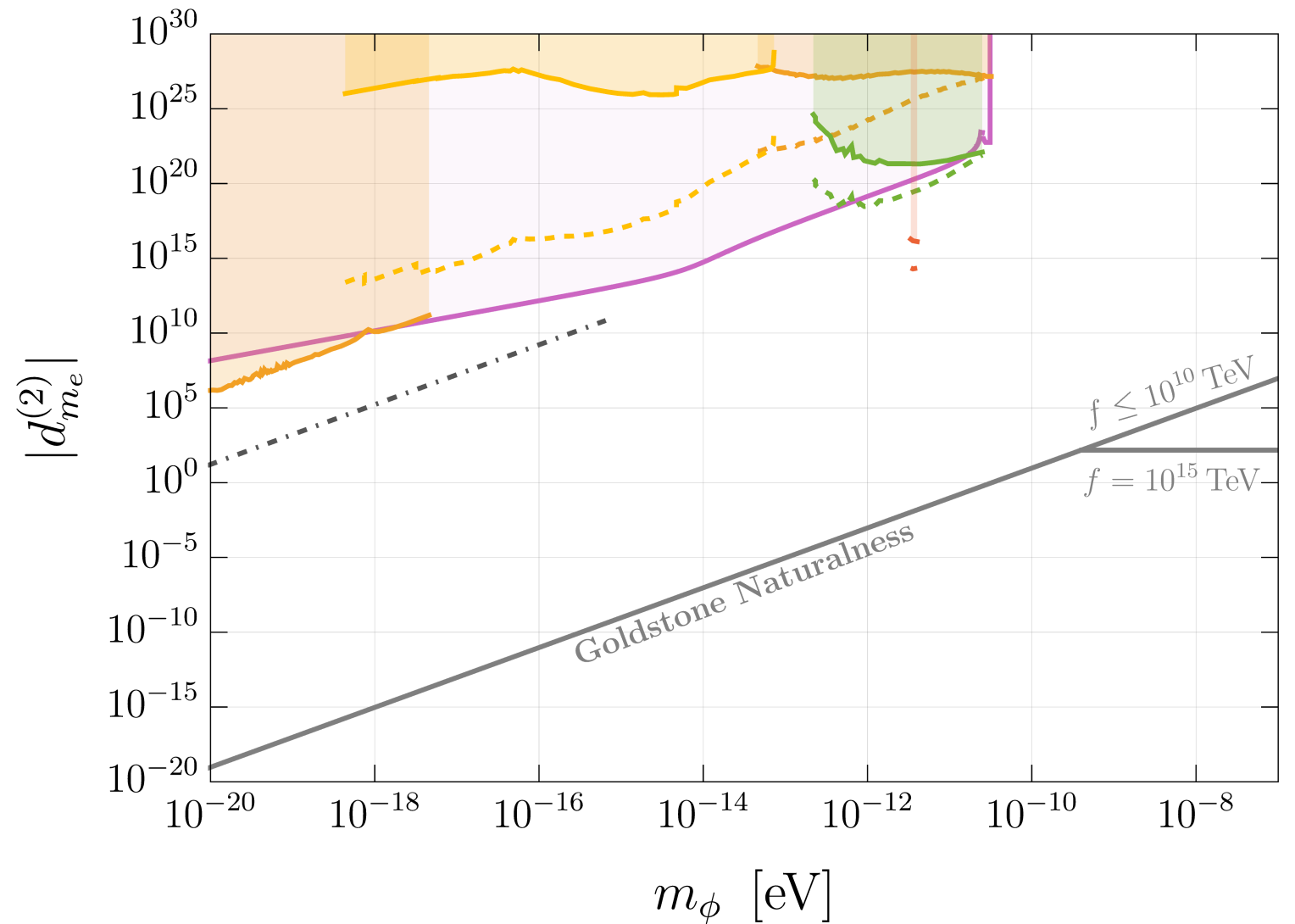
$$\mathcal{L} \supset -\frac{d_{m_e}^{(2)} m_e}{2M_{\text{Pl}}^2} \phi^2 \bar{e}e$$

- At 1-loop get mass correction

$$m_\phi^2 \gtrsim \delta m_\phi^2 \simeq \frac{d_{m_\psi}^{(2)} m_\psi^2 \Lambda^2}{32\pi^2 M_{\text{Pl}}^2}$$



$$\Lambda = 10\text{TeV}$$



Goldstone Naturalness

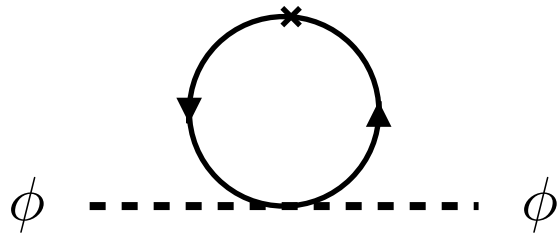
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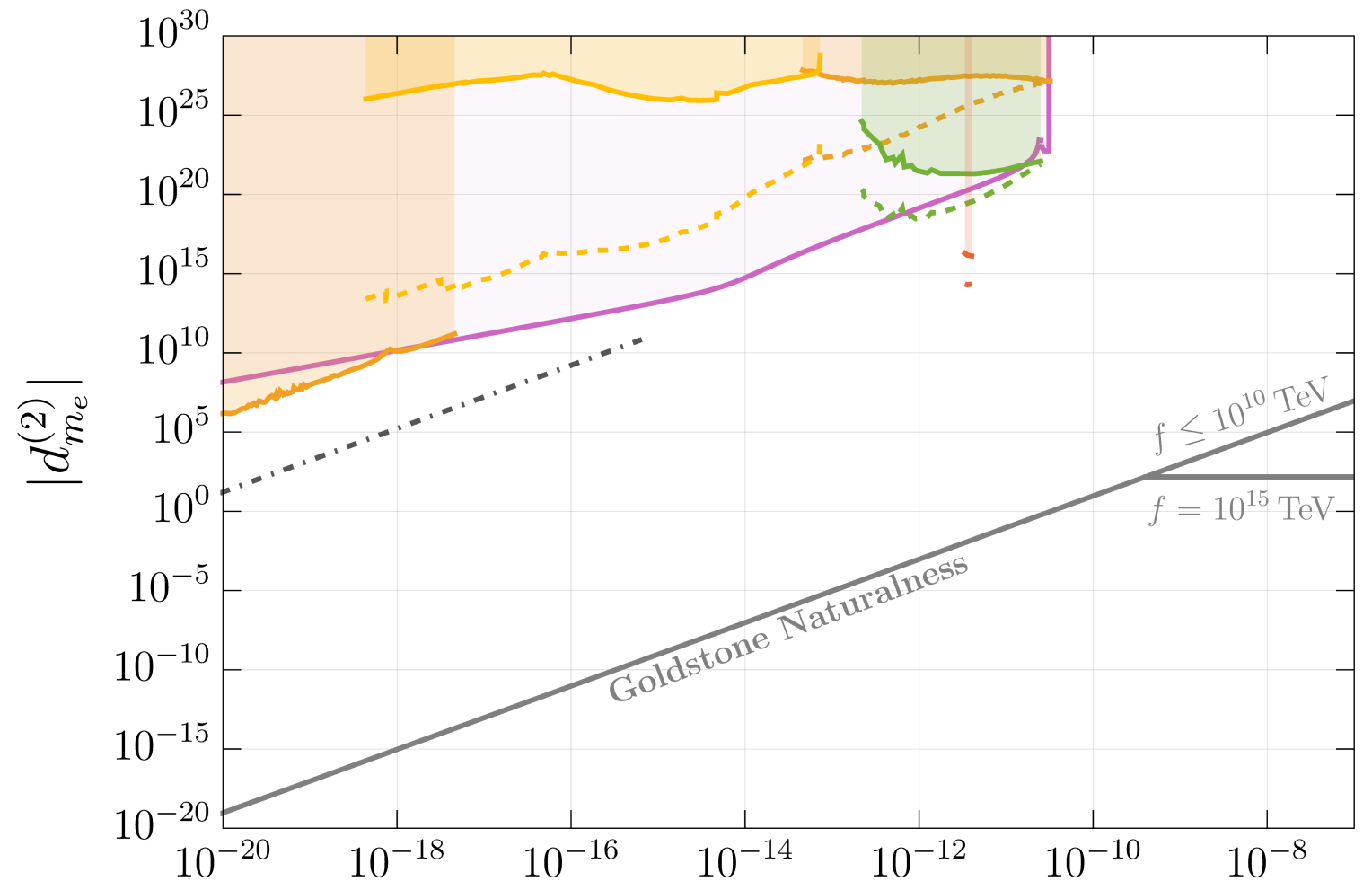
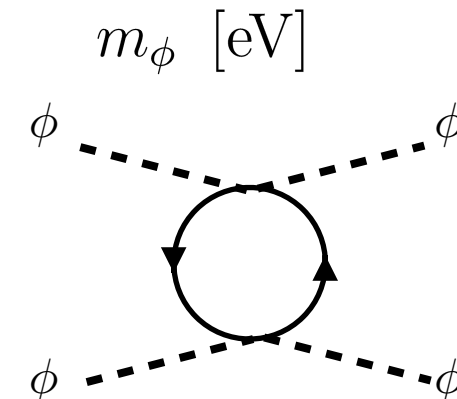
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- At 1-loop get quartic correction

$$\delta\lambda \simeq \frac{(d_{m_\psi}^{(2)})^2 m_\psi^2 \Lambda^2}{64\pi^2 M_{\text{Pl}}^4} \lesssim \frac{\delta m_\phi^2}{f^2} \implies f \lesssim \sqrt{\frac{2}{d_{m_\psi}^{(2)}}} M_{\text{Pl}}$$



Quadratic twin

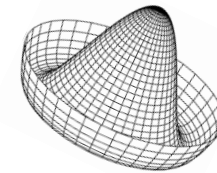


$U(1)$ —realization

Delaunay, Geller, Heller-Algazi, Perez, KS, '25

- Consider complex scalar Σ charged under $U(1)$ which is spont. broken

$$V(\Sigma) = \lambda (|\Sigma|^2 - f^2)^2$$



- Σ couples to the SM and a twin copy thereof $\Sigma = \begin{pmatrix} \Sigma_{\text{SM}} \\ \Sigma_{\text{twin}} \end{pmatrix} = f \begin{pmatrix} \sin(\theta) \\ \cos(\theta) \end{pmatrix}$

$$\mathcal{L}_{\text{explicit}} \supset c (|\Sigma_{\text{SM}}|^2 \mathcal{O}_{\text{SM}} + |\Sigma_{\text{twin}}|^2 \mathcal{O}_{\text{twin}})$$

which explicitly breaks $U(1) \rightarrow (\mathbb{Z})_{\text{SM}} \times (\mathbb{Z})_{\text{twin}}$

Quadratic twin

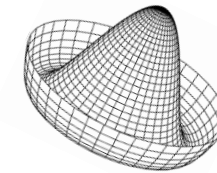


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$$\Delta V = \frac{d_{m_e}^{(2)} m_e^2 \Lambda^2}{16\pi^2 M_{\text{Pl}}^2} (|\Sigma_{\text{SM}}|^2 + |\Sigma_{\text{twin}}|^2) = \text{const.}$$

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Quadratic twin



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- Effective mass protection $\sum_i d_i^{(2)} \left(\frac{f}{M_{\text{Pl}}} \right)^2$

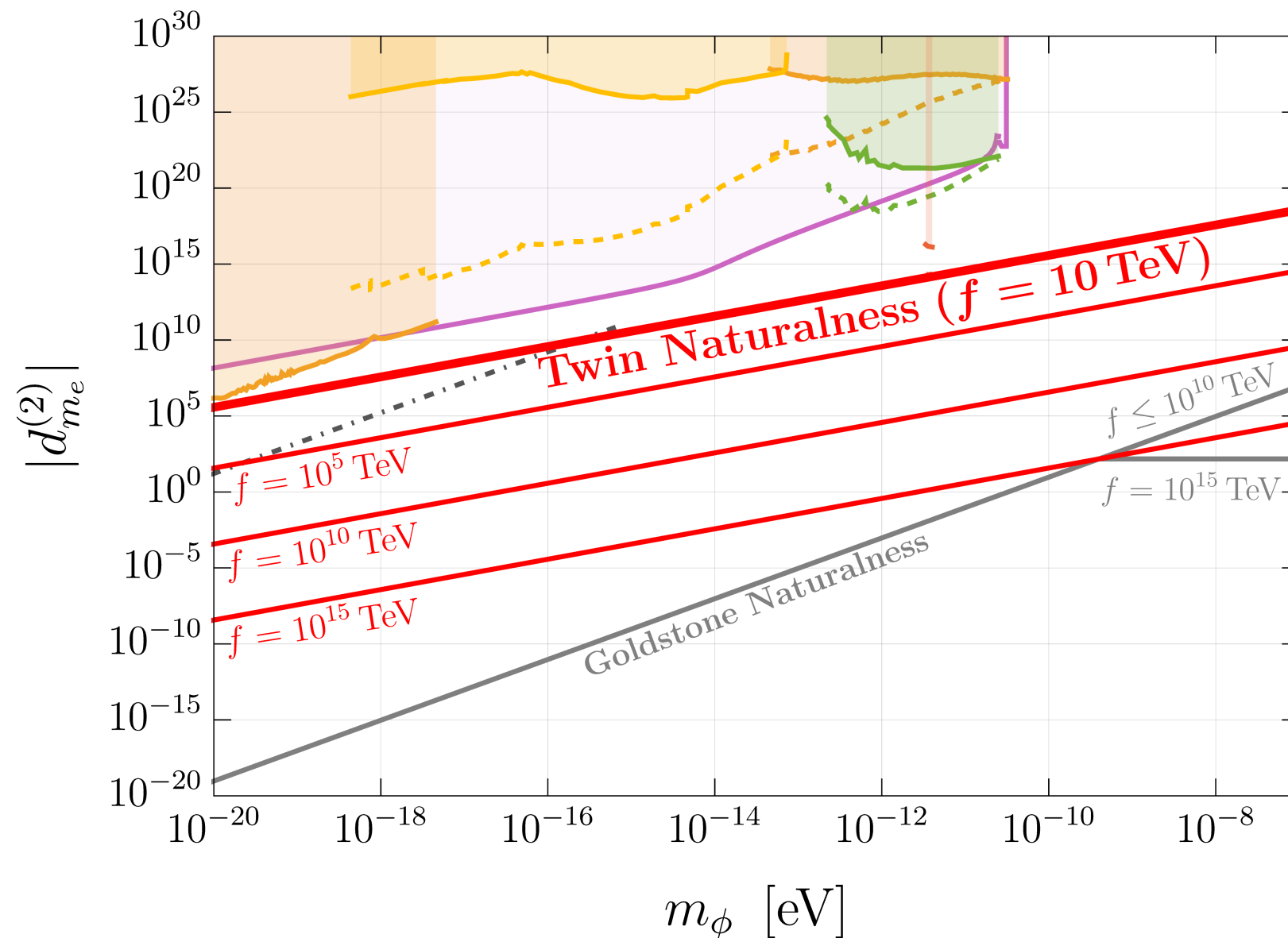
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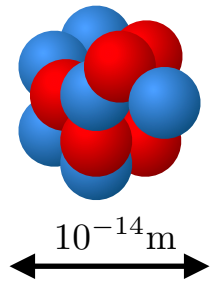


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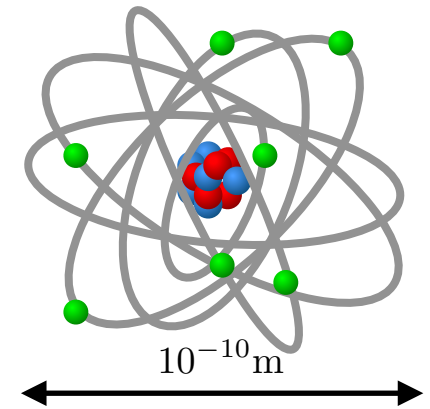
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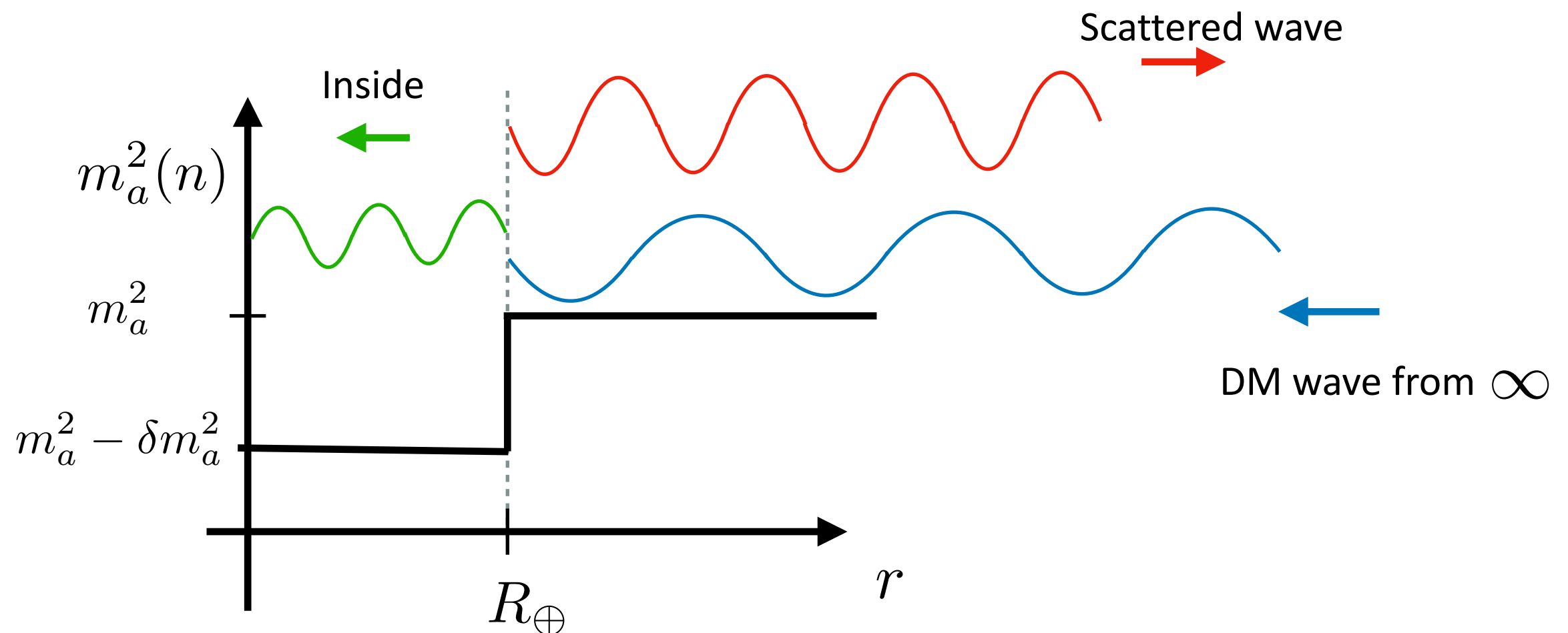
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Local DM density around Earth with $\phi^2 \bar{N}N$ - coupling

- Axion mass/potential is density dependent $m_a^2(n) = m_a^2 + \frac{4z}{(1+z)^2} \frac{c_1 n m_\pi^2}{f_a^2}$
- In Earth the reduction is $\delta m_a^2(n_\oplus) \sim 10^{-18} \text{ eV}^2 \left(\frac{10^9}{f_a}\right)^2$ $n \simeq \langle N^\dagger N \rangle$
- If axion is DM it scatters in potential (neglecting gravity)



Solving the (static) EOM

Banerjee, Bloch, Bonnefoy, Ellis, Perez, Savoray, KS, Stadnik, '25

- We solve in steady state limit with BC $a(\vec{r})|_{r \gg R_\oplus} \rightarrow a_0 e^{-i\vec{k} \cdot \vec{r}}$ (Assume monochromatic for now)

$$r > R_\oplus, \quad (\nabla^2 - m_a^2) a_\omega = 0$$

$$r < R_\oplus, \quad (\nabla^2 - m_a^2 + \delta m_a^2) a_\omega = 0$$

- The solutions outside is $r > R_\oplus$ (Spherically symmetric $\rightarrow m = 0$)

$$a_\omega^{\text{out}}(r, \theta) = a_0 \sum_{\ell=0}^{\infty} (2\ell + 1) i^\ell j_\ell(kr) P_\ell(\cos \theta) + \sum_{\ell=0}^{\infty} c_\ell^{\text{out}} h_\ell^{(1)}(kr) P_\ell(\cos \theta)$$

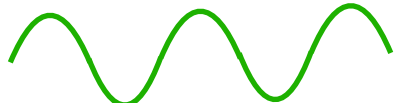



DM wave from ∞ Scattered wave

$k = \sqrt{\omega^2 + m_a^2}$

- The solutions inside is $r < R_\oplus$

$$a_\omega^{\text{in}}(r, \theta) = \sum_{\ell=0}^{\infty} c_\ell^{\text{in}} j_\ell(qr) P_\ell(\cos \theta)$$



Inside

$q = \sqrt{\omega^2 + m_a^2 - \delta m^2}$

- Imposing continuity fixes $c_\ell^{\text{in/out}}$

Axion Amplitude on Earth

Banerjee, Bloch, Bonnefoy, Ellis, Perez, Savoray, KS, Stadnik, '25

- Consider $k_{\text{vir}} R_{\oplus} < 1$, $k_{\text{vir}} \simeq 10^{-3} m_a$

$\Rightarrow$ Dominated by monopole and dipole contributions

Monopole: $|a_{\omega,0}|^2 = \frac{1}{(kR)^2 + (qR)^2 \cot(qR)} \rightarrow \begin{cases} (kR)^{-2}, & qR \rightarrow \pi n + \pi/2 \\ 0, & qR \rightarrow \pi n \end{cases}$

Dipole: $|a_{\omega,1}|^2 \rightarrow \begin{cases} (kR)^{-4}, & qR \rightarrow \pi n \\ 0, & qR \rightarrow \pi n + \pi/2 - \delta(\ell) \end{cases}$

- Resonances that with height $\sim (kR)^{-2\ell-2}$ and width $\sim (kR)^{2\ell}$

No divergences

Axion Amplitude on Earth

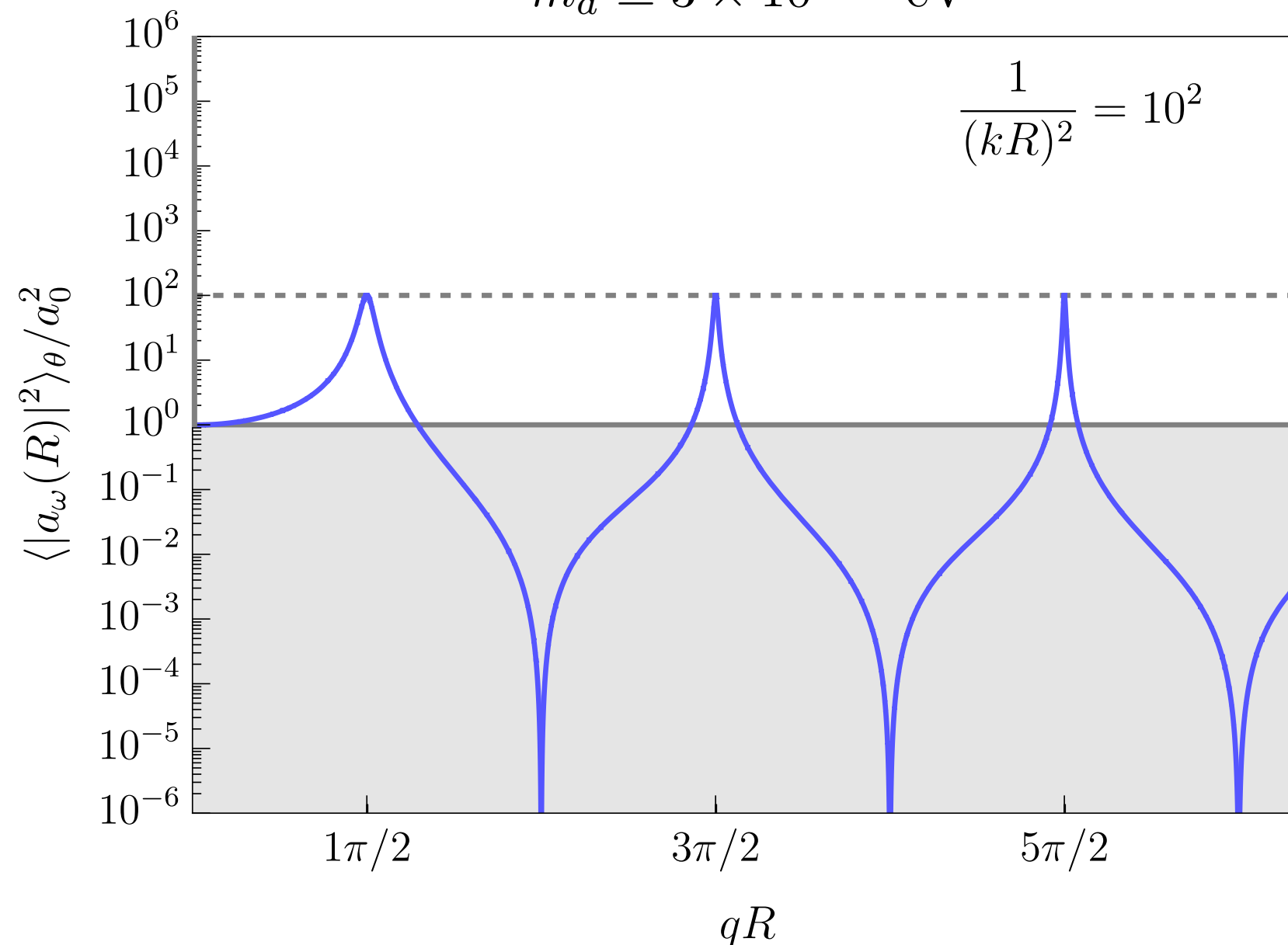
Banerjee, Bloch, Bonnefoy, Ellis, Perez, Savoray, KS, Stadnik, '25

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Dominated by monopole and dipole contributions

$$m_a \simeq 3 \times 10^{-12} \text{ eV}$$



Axion Amplitude on Earth

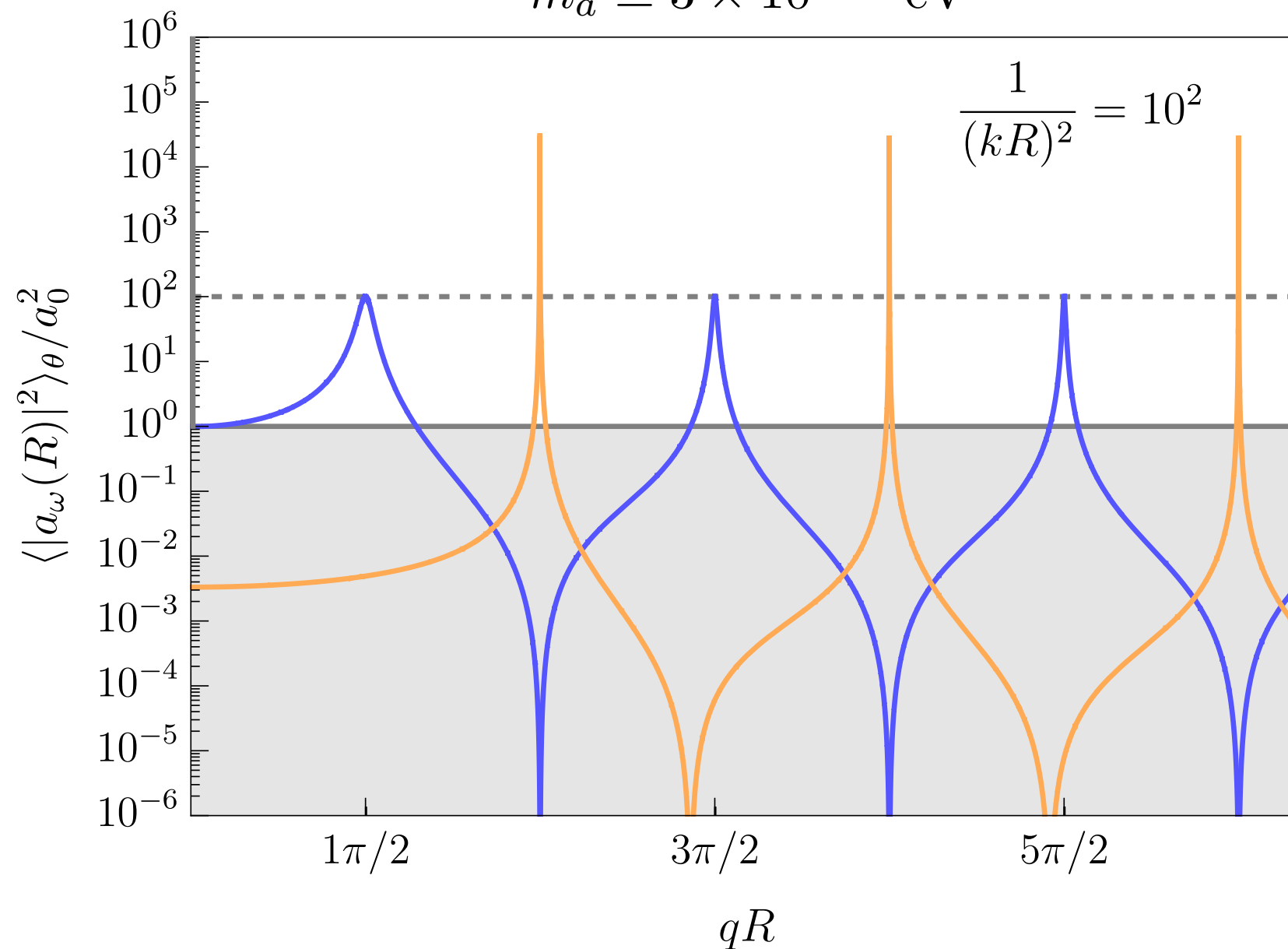
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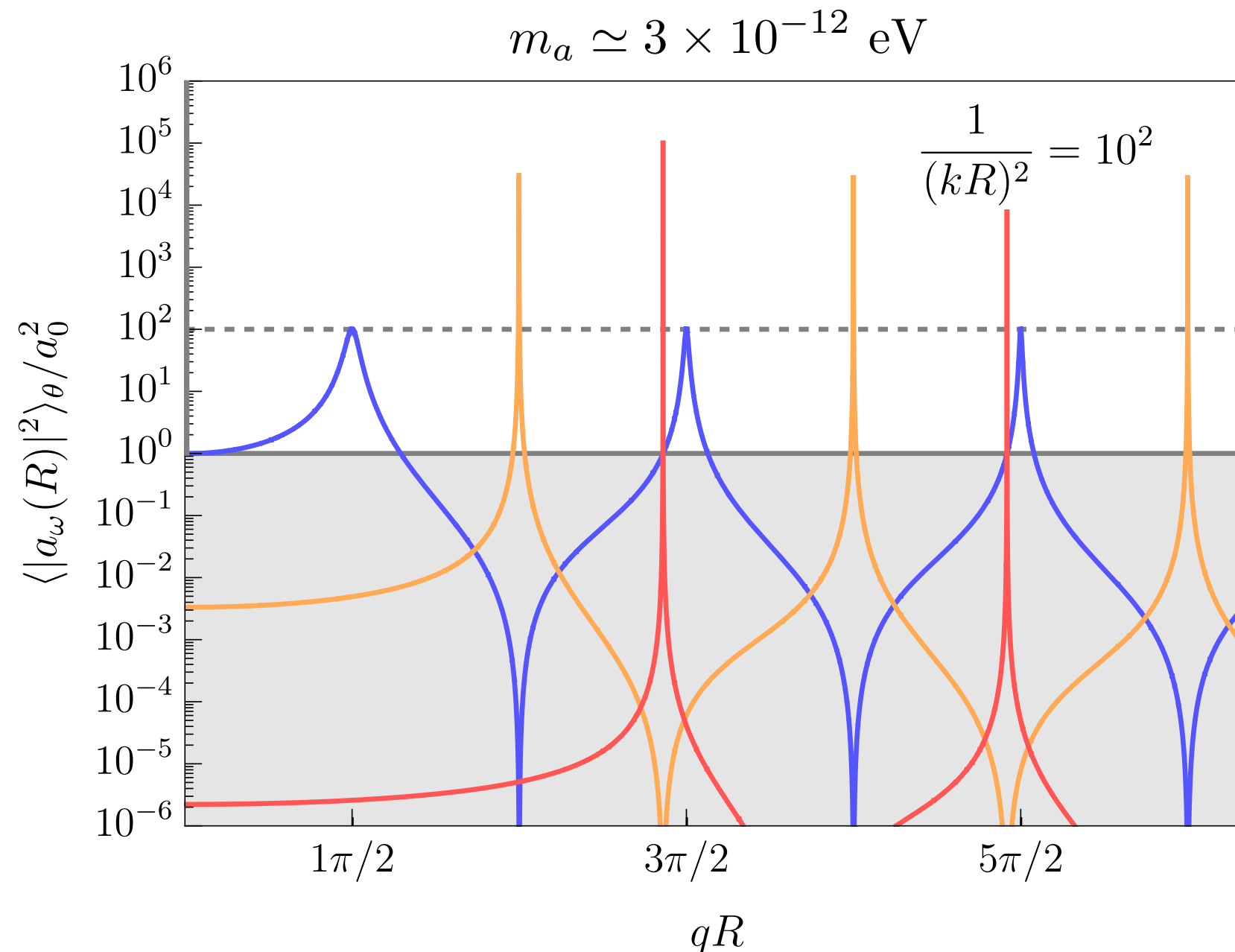


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- Consider $k_{\text{vir}} R_{\oplus} < 1$, $k_{\text{vir}} \simeq 10^{-3} m_a$

$\Rightarrow$ Dominated by monopole and dipole contributions

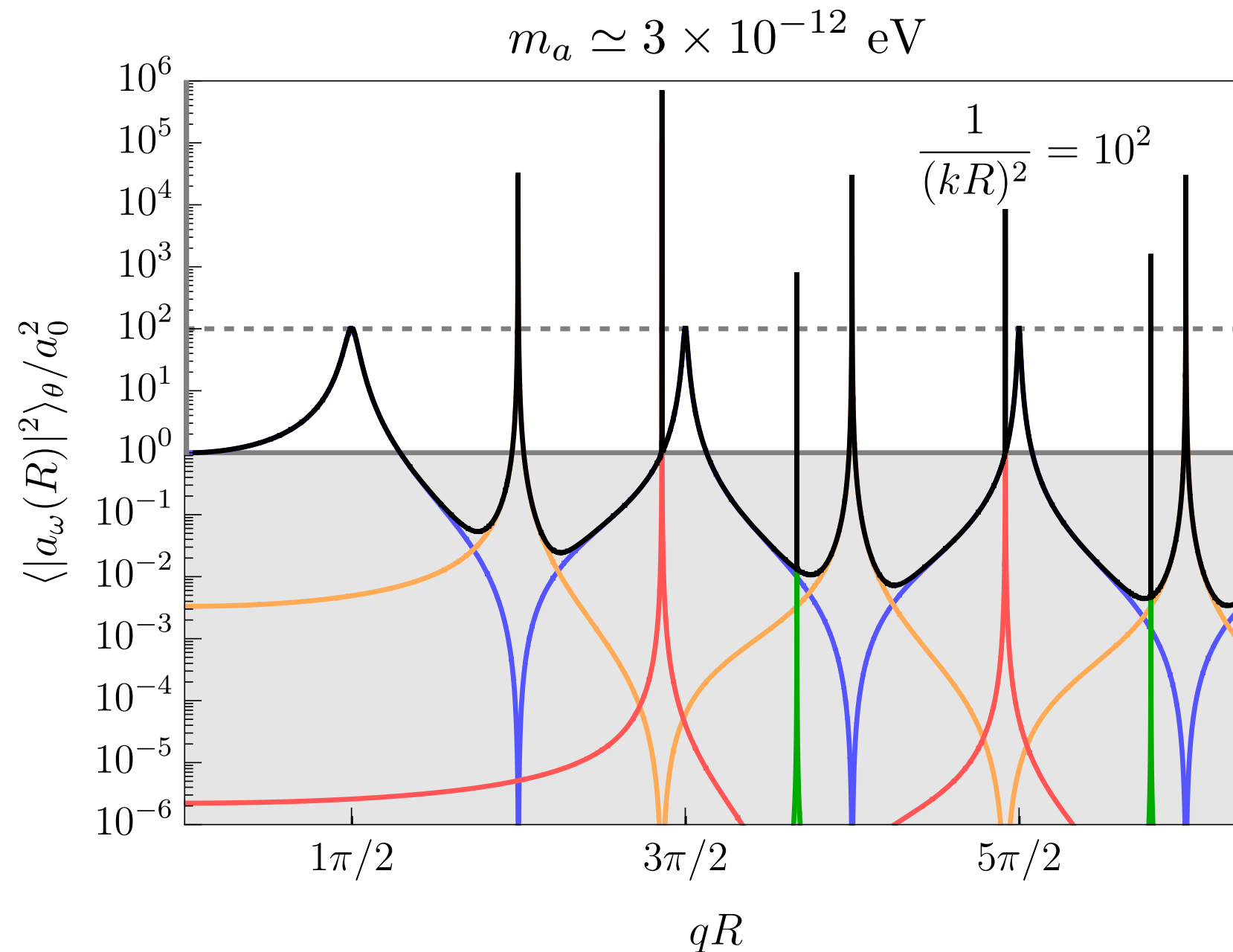


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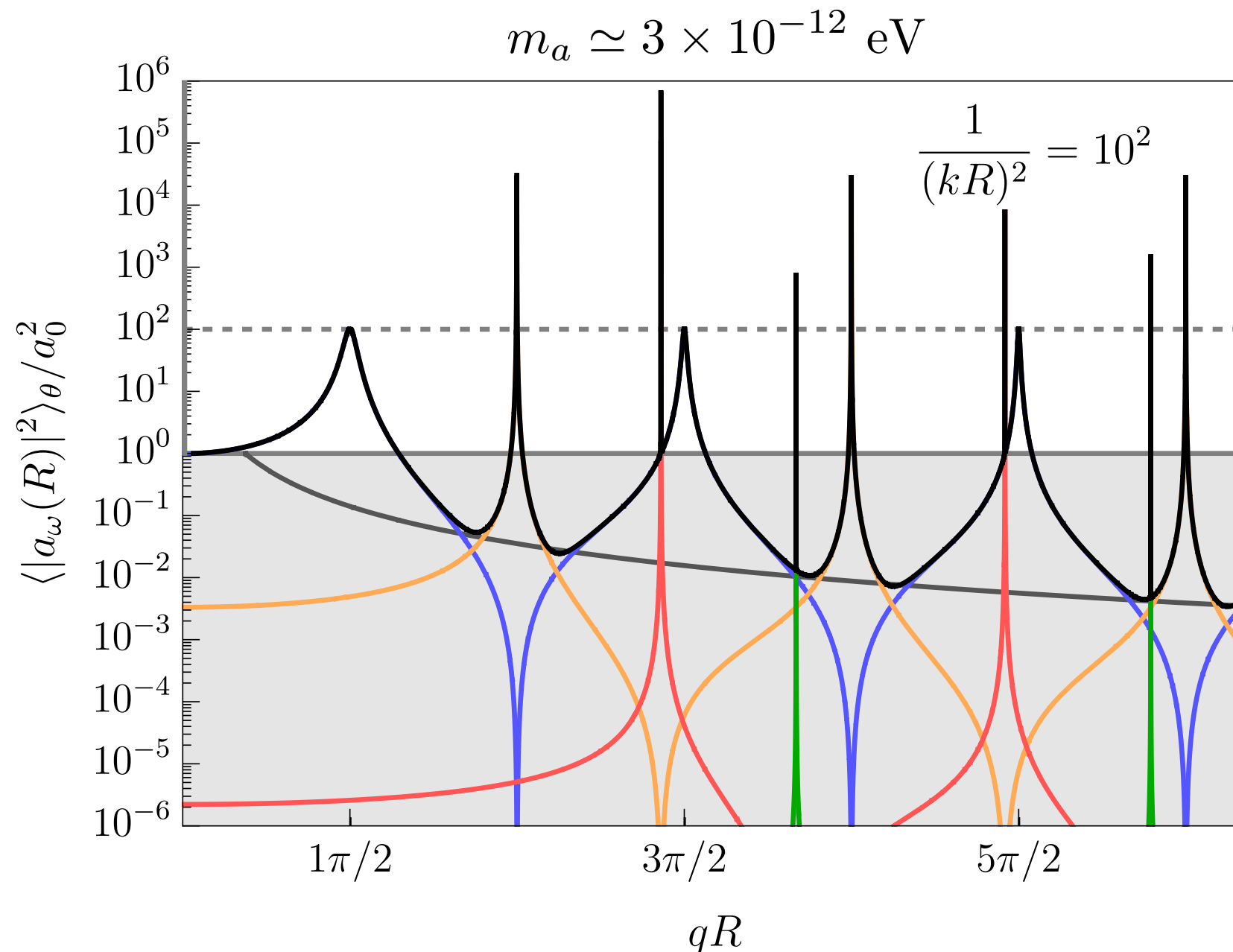


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Off-resonance
suppression

$$\propto kR / (qR)^2$$

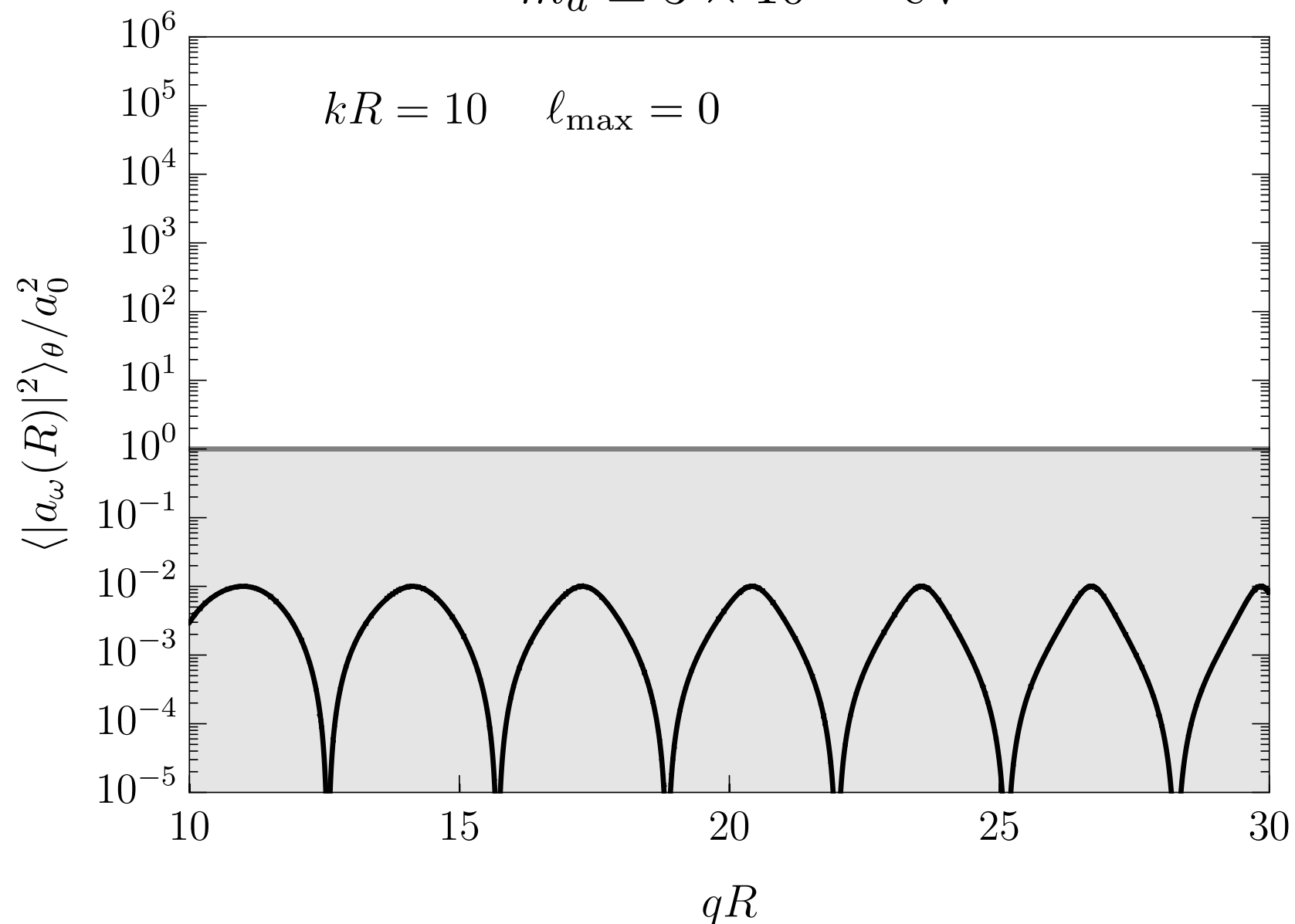
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- Consider $k_{\text{vir}} R_{\oplus} > 1$, $k_{\text{vir}} \simeq 10^{-3} m_a$

$\Rightarrow$ Sensitive to UV scales: need to sum $\ell_{\text{max}} = kR + \mathcal{O}(1)(kR)^{1/3}$

$$m_a \simeq 3 \times 10^{-10} \text{ eV}$$



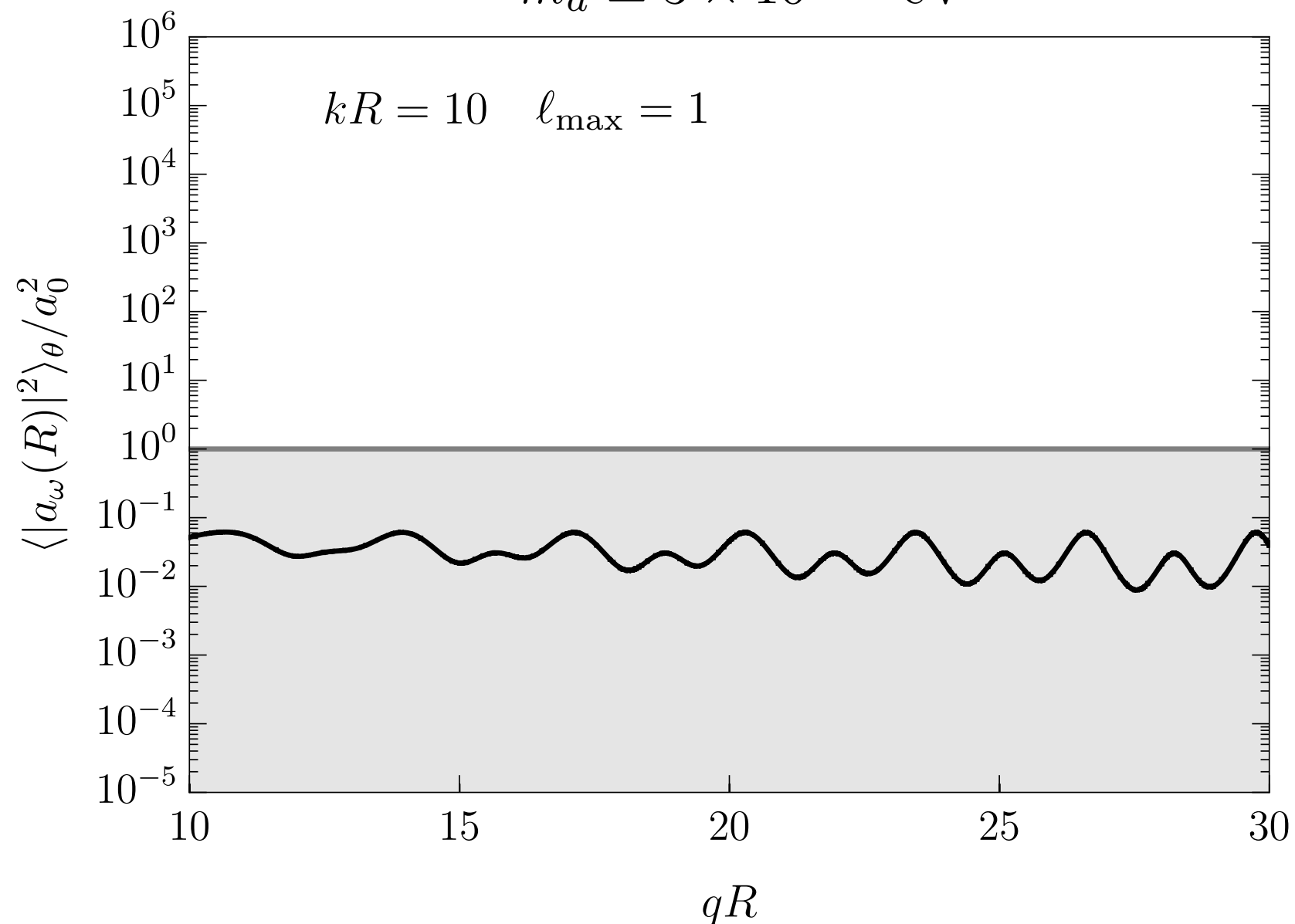
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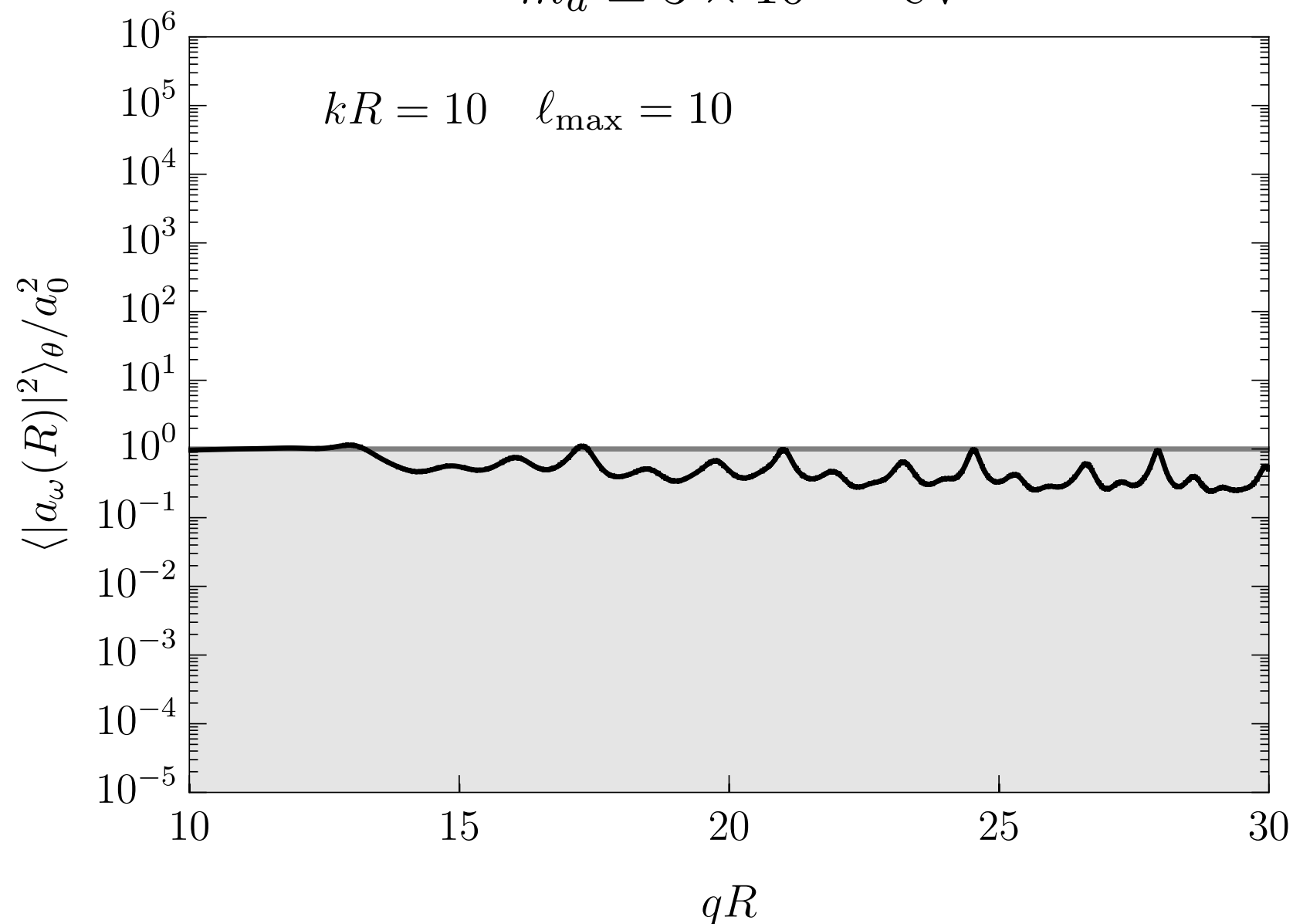
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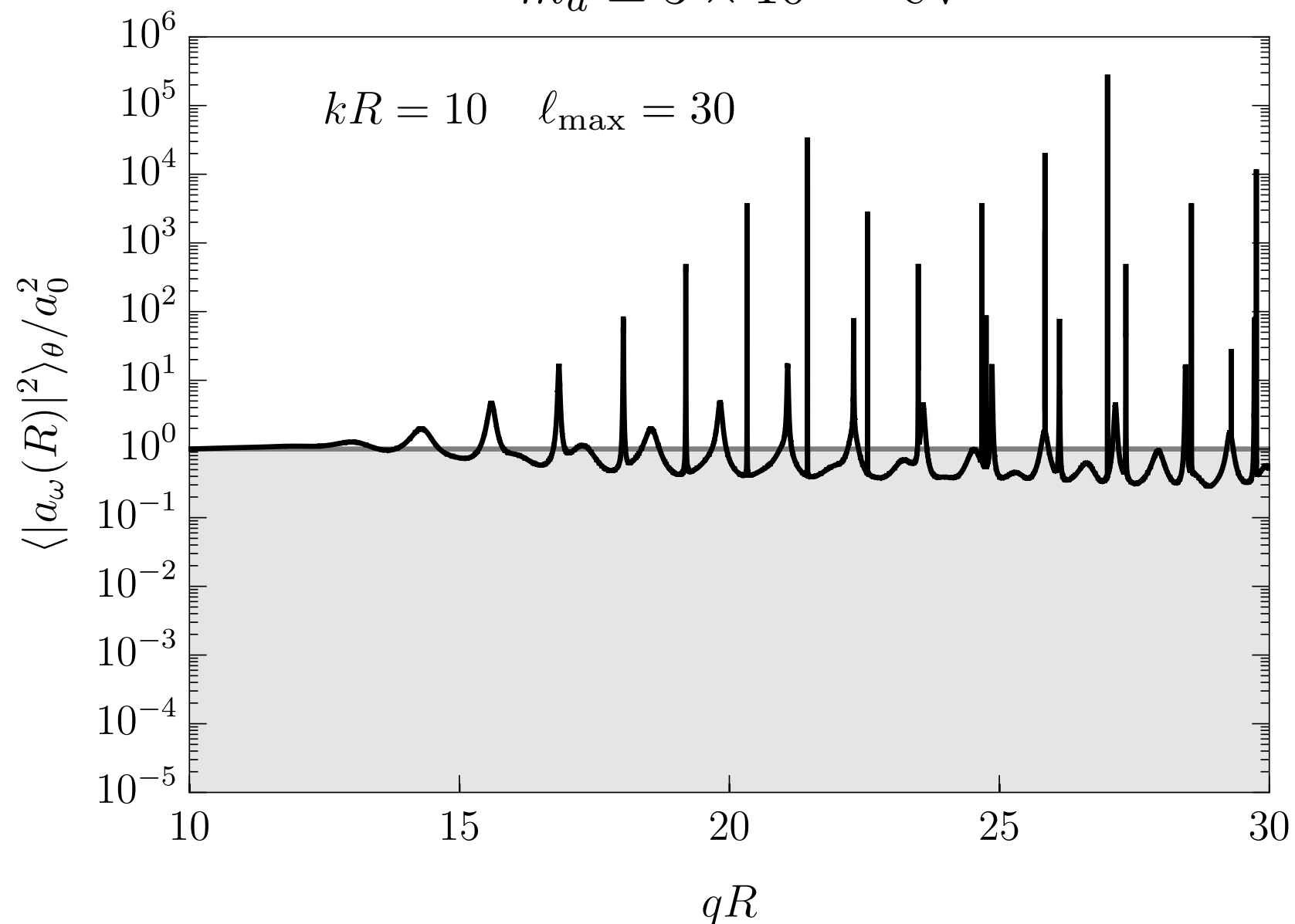
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Axion Gradient on Earth

Banerjee, Bloch, Bonnefoy, Ellis, Perez, Savoray, KS, Stadnik, '25

- Consider $k_{\text{vir}} R_{\oplus} < 1$, $k_{\text{vir}} \simeq 10^{-3} m_a$

$$\frac{1}{k^2 a_0^2} \left| \hat{r} \cdot \vec{\nabla} a_\omega \right|^2 = \frac{(\sin(qR) - qR \cos(qR))^2}{kR^2 (kR^2 \sin(qR) + qR^2 \cos(qR))} \rightarrow \begin{cases} (kR)^{-4}, & qR = \pi n + \pi/2 \\ (kR)^{-2}, & qR = \pi n \end{cases}$$

- Gradients are enhanced by $(kR)^{-4\ell-4}$ on resonance
- Away from resonance the generic expectation is still $(kR)^{-2}$

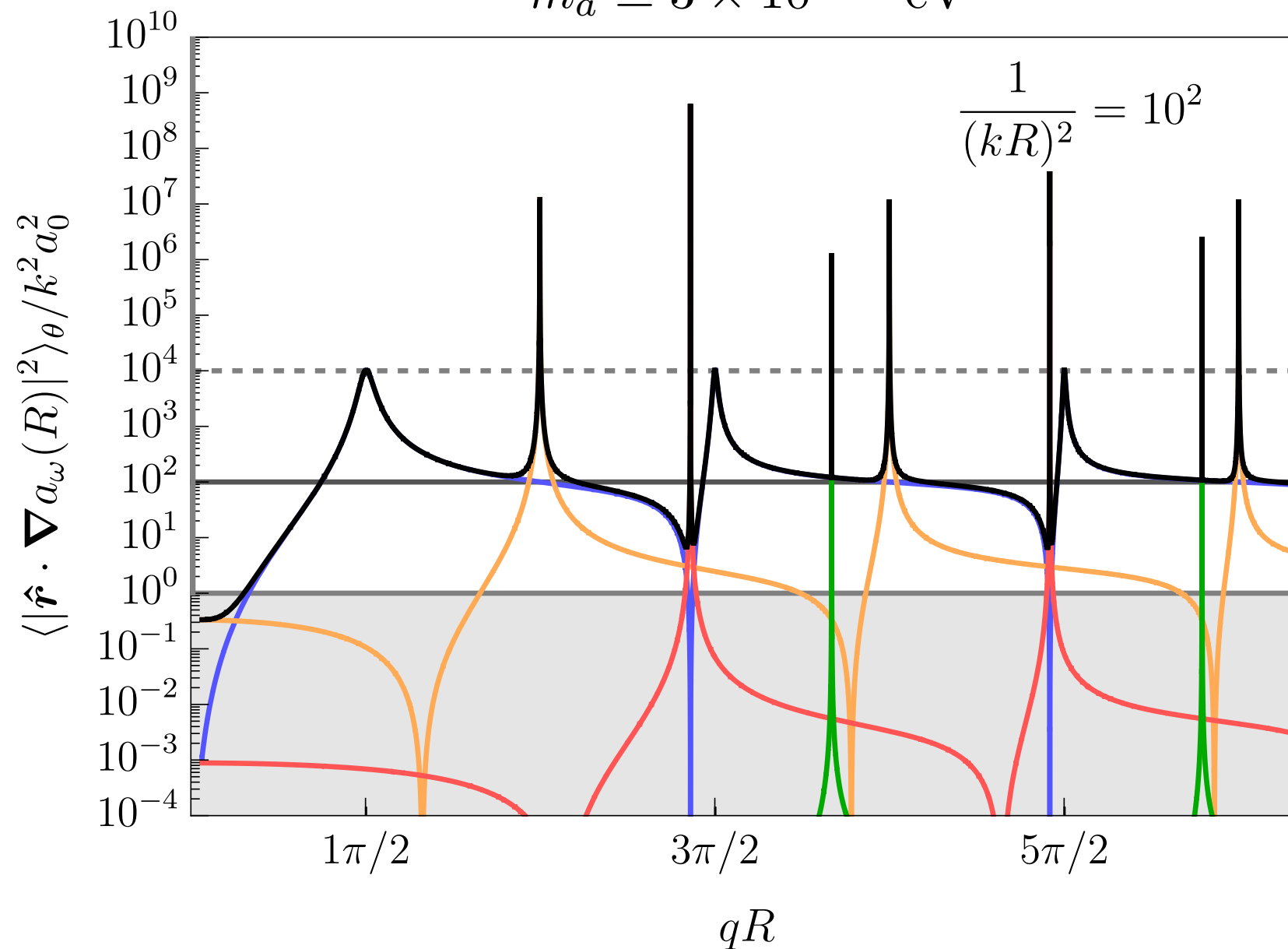
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Away from resonance the generic expectation is still $(kR)^{-2}$

$$m_a \simeq 3 \times 10^{-12} \text{ eV}$$



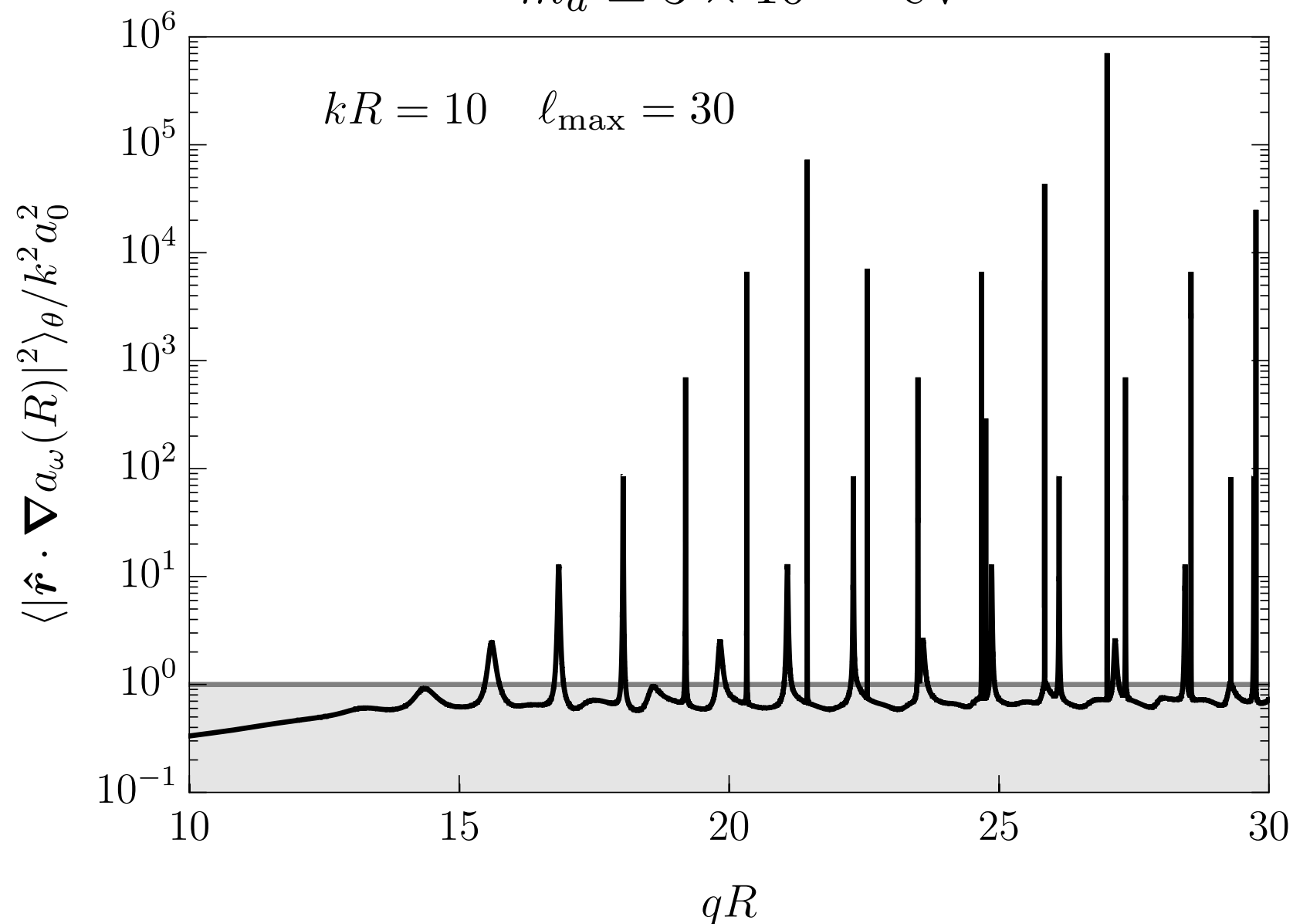
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$$m_a \simeq 3 \times 10^{-10} \text{ eV}$$



Parameter Space

Banerjee, Bloch, Bonnefoy, Ellis, Perez, Savoray, KS, Stadnik, '25

Where is the effect important?

Dense enough: $q > \mathcal{O}(1)k \Rightarrow \delta m > k$

For Earth this implies $\epsilon < 2 \times 10^{-8}$ $m_a = \sqrt{\epsilon} \frac{\sqrt{z}}{1+z} \frac{m_\pi f_\pi}{f_a}$

Sourcing on Earth starts for $\delta m > m_a$ $\epsilon < 2 \times 10^{-15}$

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For Earth this implies $f_a \lesssim 3 \times 10^{13} \text{ GeV}$

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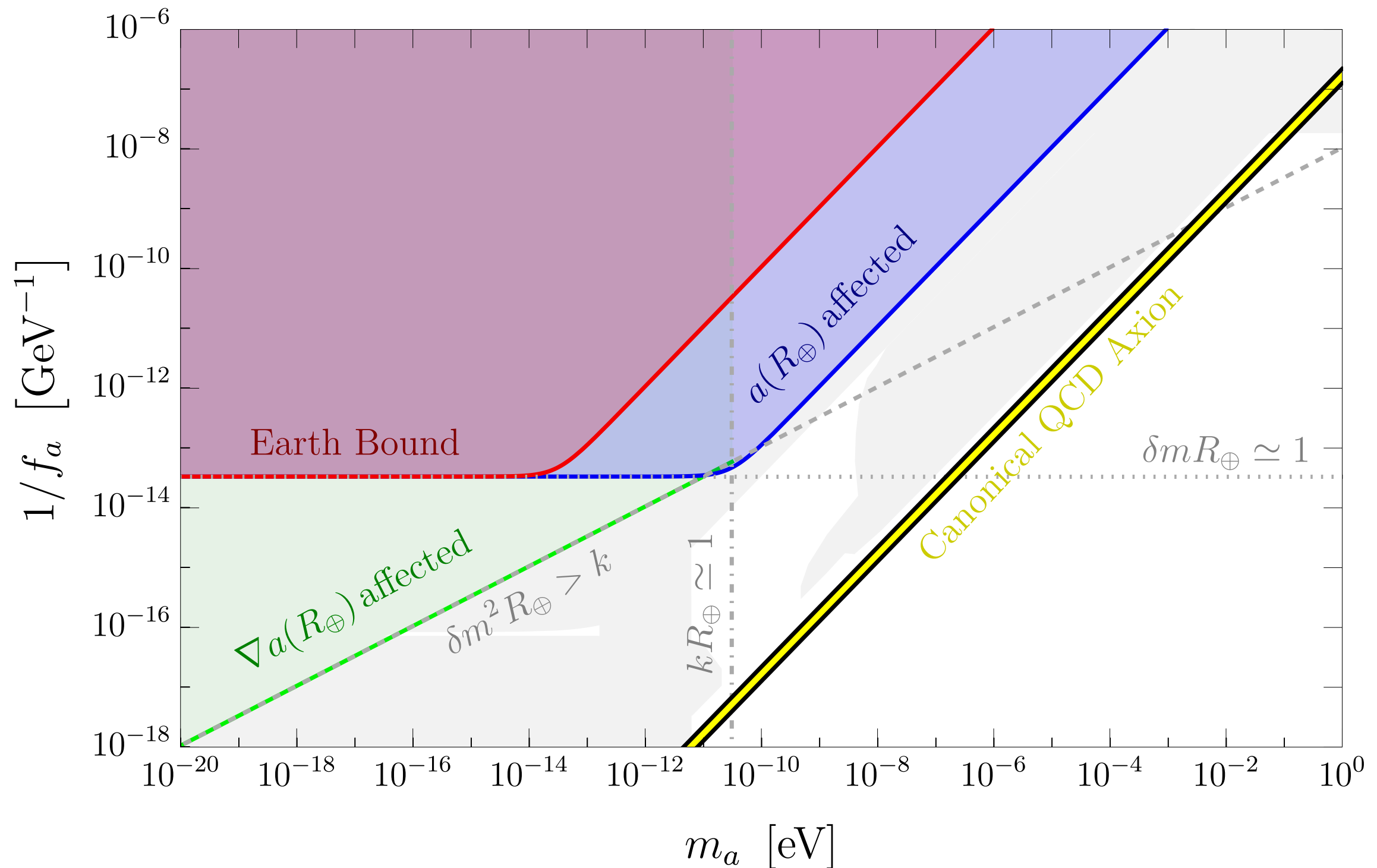
Gradients for $kR_\oplus < qR_\oplus < 1$

$$\ell = 0 \text{ -mode} \quad \frac{1}{ka_0} \hat{r} \cdot \vec{\nabla} a \simeq -\frac{(qR_\oplus)^2}{3kR_\oplus} \simeq -\frac{(\delta m R_\oplus)^2}{3kR_\oplus}$$

Parameter Space

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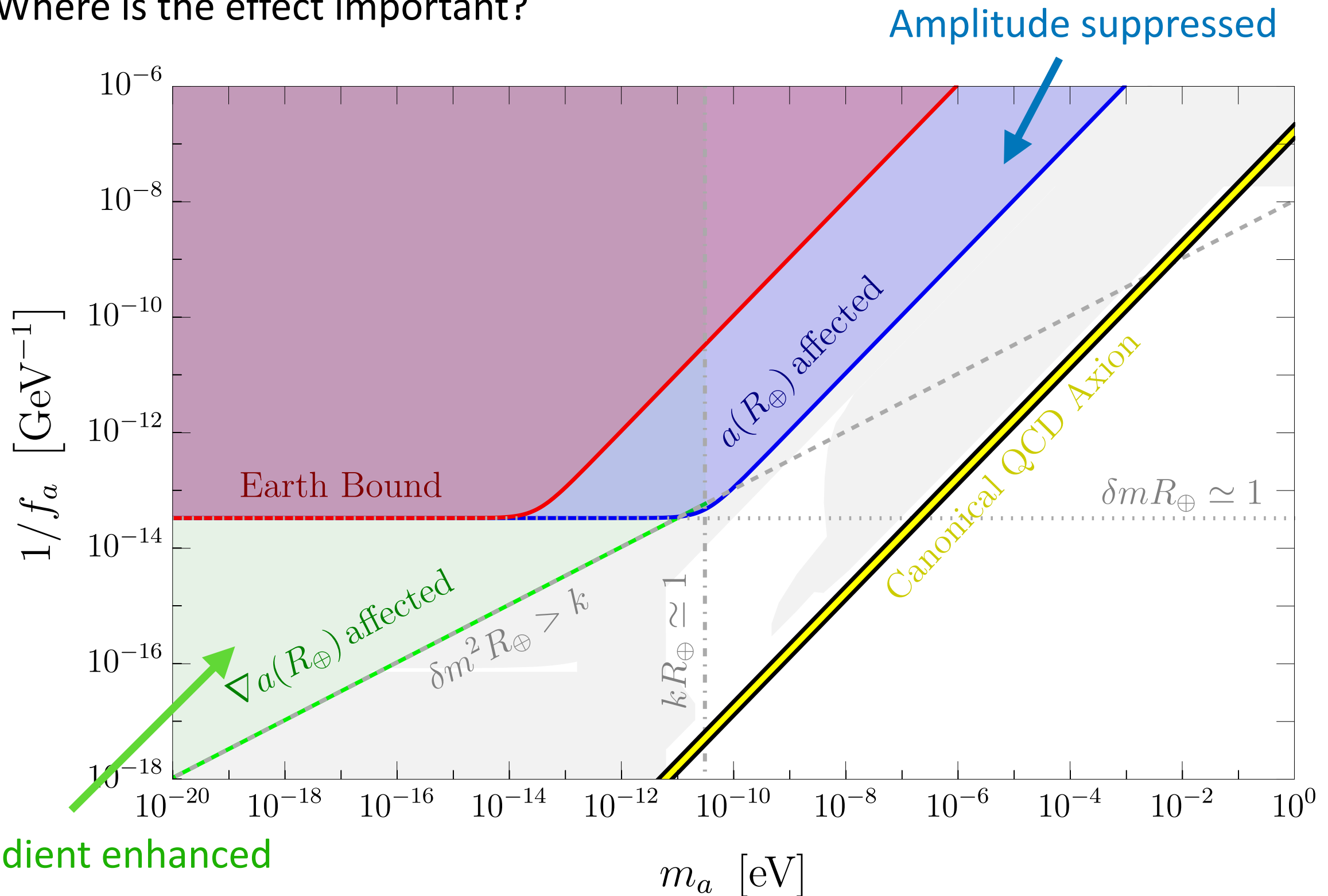
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Where is the effect important?



Conclusions

Nuclear clocks

- provide a unique probe of ULDM coupling to the strong sector and may exhibit a enhanced sensitivity of $10^8 - 10^{10}$ compared to atomic clocks
- Looked at **geometric classical** and **halo EFT** for ^{229}Th , nightmare scenario seems unlikely
- **Hyperfine spectroscopy** allows to test nuclear multipole moments via

$$\mathcal{H} = \sum_k \frac{1}{k} V^{(k)} Q_{\text{spec}}^{(k)}$$

If crystal structure is known $\Rightarrow$ varying k structures allow to probe different multipole

Quadratically coupled ULDM

- Experimentally accessible parameter usually fine tuned: can protect the mass with **twin—Higgs** inspired model
- SM—matter backgrounds distort the local ULDM density
 - $\Rightarrow$ suppresses field amplitudes
 - $\Rightarrow$ enhances field gradients