

Generalized Symmetries and Non-invertible Naturalness



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Notre Dame $\rightarrow$ CERN $\rightarrow$ Durham

Effective Theories For
Nonperturbative Physics
2026

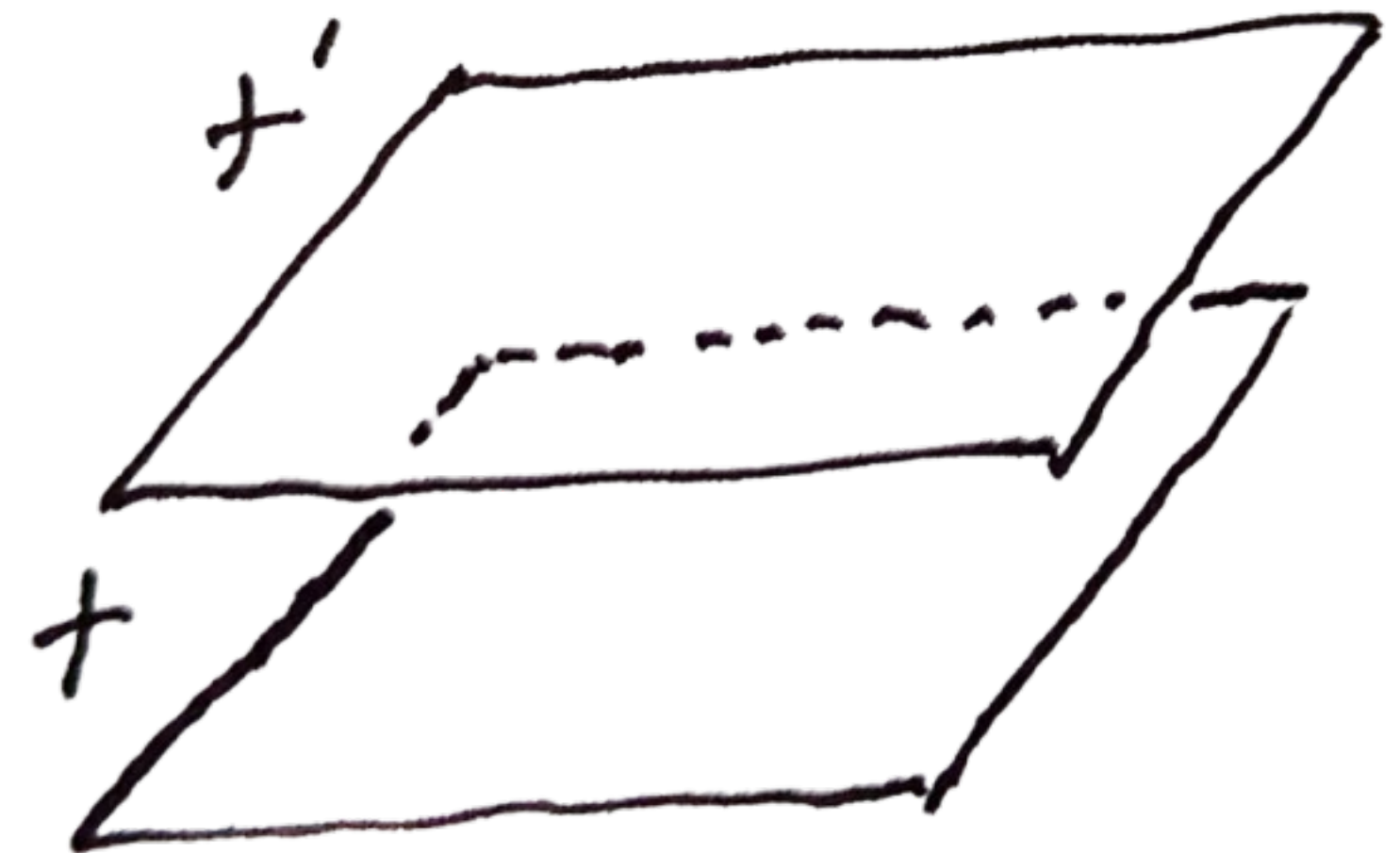
Global Symmetry of Point Operators

Noether: A continuous global symmetry gives a current

J_μ with $\partial_\mu J^\mu = \vec{\nabla} \cdot \vec{J} - \dot{J}_0 = 0$, and

$$Q(\mathcal{M}_{\text{space}}, t) = \int_{\mathcal{M}_{\text{space}}} J^0 dx^1 dx^2 dx^3$$

is conserved under time translations



But why is this so non-covariant?

We can do better with some basic ideas from *topology*.

Recall topology deals with properties that do not change under smooth deformations: ‘global’ rather than ‘local’

GGs
Framework
Gaiotto, Kapustin,
Seiberg, Willett

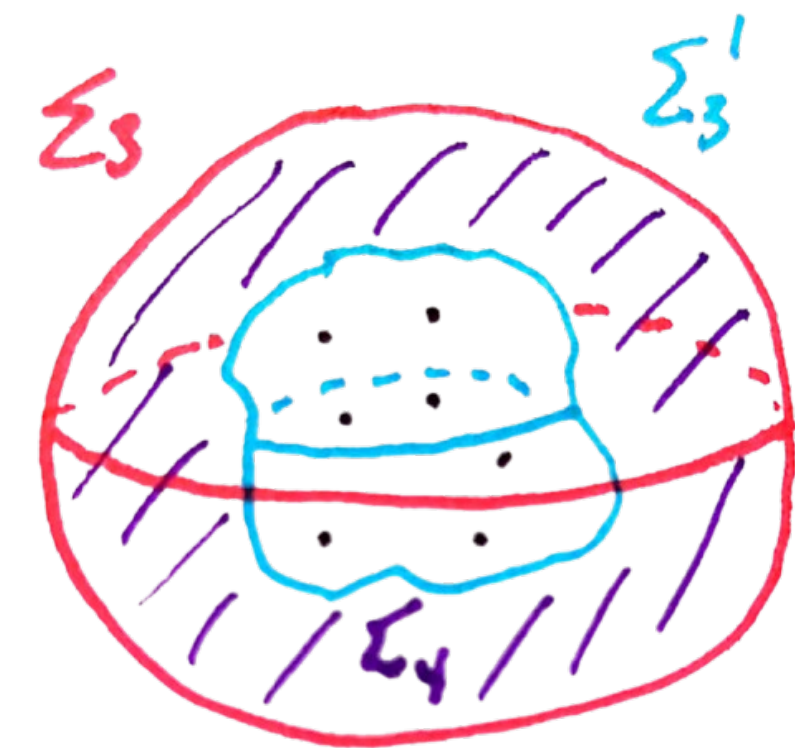
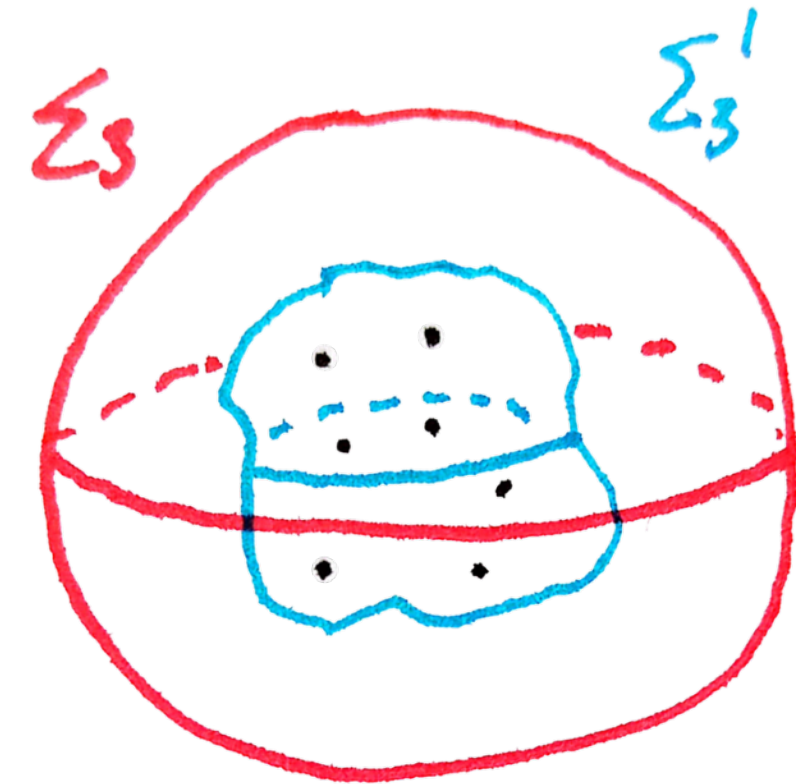
Symmetry detecting operators

Generalized Noether surfaces detect symmetry

charges of local operators $Q(\Sigma_3) = \int_{\Sigma_3} J^\mu \hat{n}_\mu d^3x$

Time translation upgraded to topological invariance

$$Q(\Sigma_3) - Q(\Sigma'_3) = \int_{\Sigma_3} J^\mu \hat{n}_\mu d^3x - \int_{\Sigma'_3} J^\mu \hat{n}_\mu d^3x = \int_{\Sigma_4} \partial_\mu J^\mu d^4x = 0$$

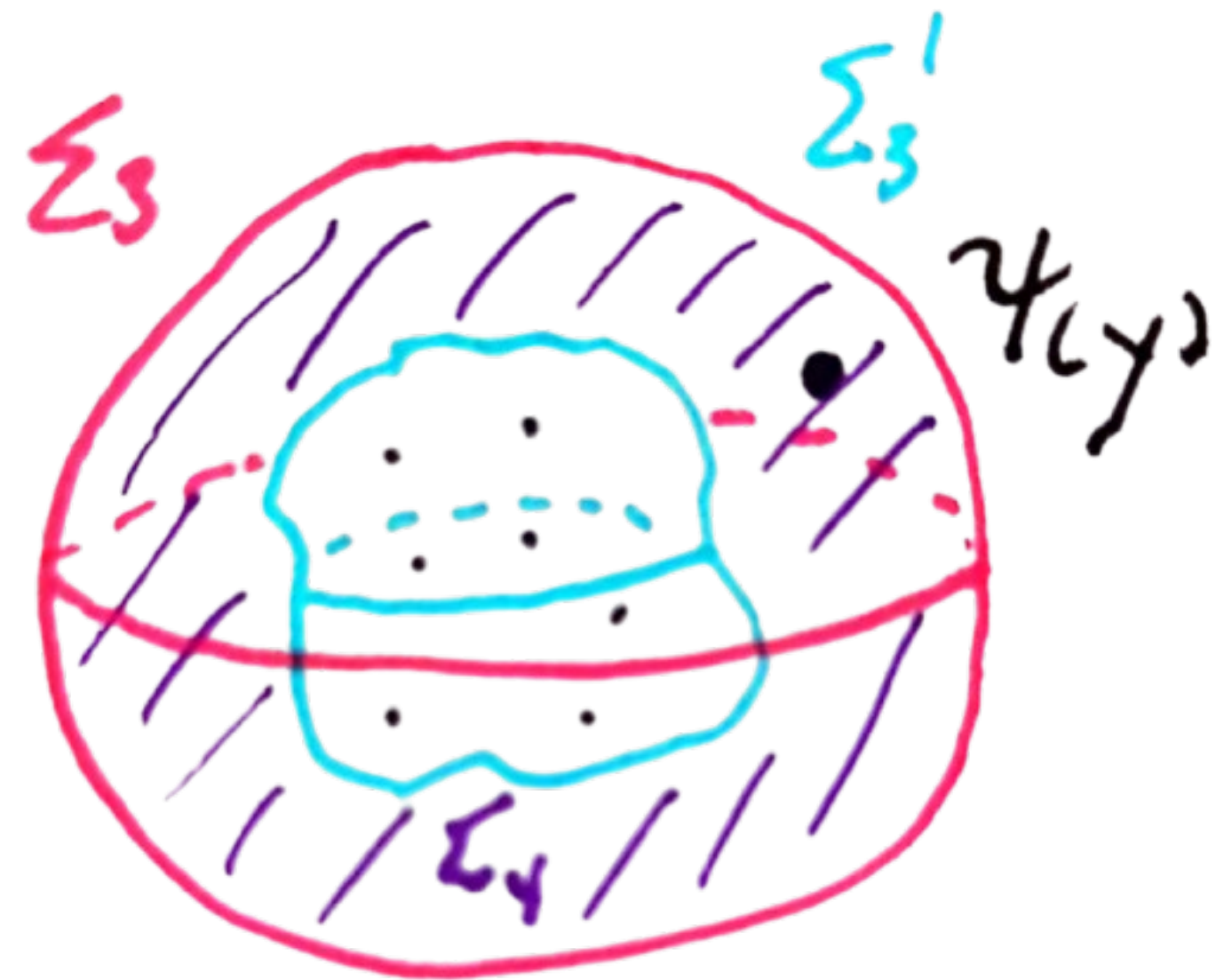


Generalized Noether Charges

With charged objects around, acts by Ward identity $\left(\partial_\mu J^\mu(x)\right)\psi(y) = q_\psi\psi(y)\delta(x-y)$

$$Q(\Sigma_3)\psi(y) - Q(\Sigma'_3)\psi(y) = \left(\int_{\Sigma_4} \partial_\mu J^\mu d^4x\right)\psi(y) = q_\psi\psi(y)$$

Simple: If you move the surface from enclosing $\psi(y)$ to not enclosing $\psi(y)$, the charge changes

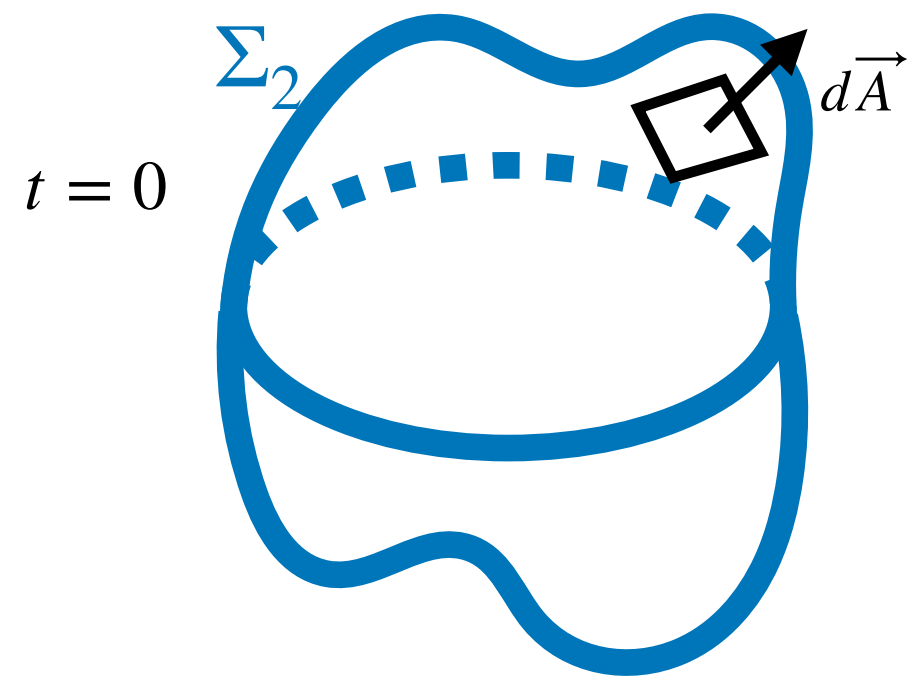


Symmetry $\sim$ Topological surface operator!

Higher-form symmetry

What about topological operators on $d < 3$ -dimensional surfaces?

You're familiar with a topological 2-surface operator in Maxwell theory!



$$Q(\Sigma_2) = \int_{\Sigma_2} \vec{E} \cdot d\vec{A} = \int_{\Sigma_2} F_{\mu\nu} \hat{n}^\mu \hat{n}^\nu d^2x$$

$$Q(\Sigma_2) - Q(\Sigma'_2) = \int_{\Sigma_3} \partial_\mu F^{\mu\nu} \hat{n}_\nu d^3x = 0$$

Gauss' law is the existence of a topological 2-surface $\sim$ global symmetry!

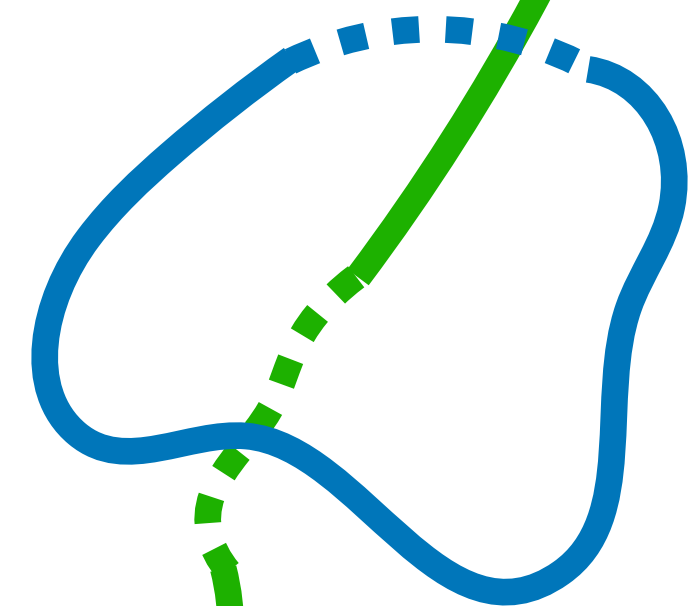
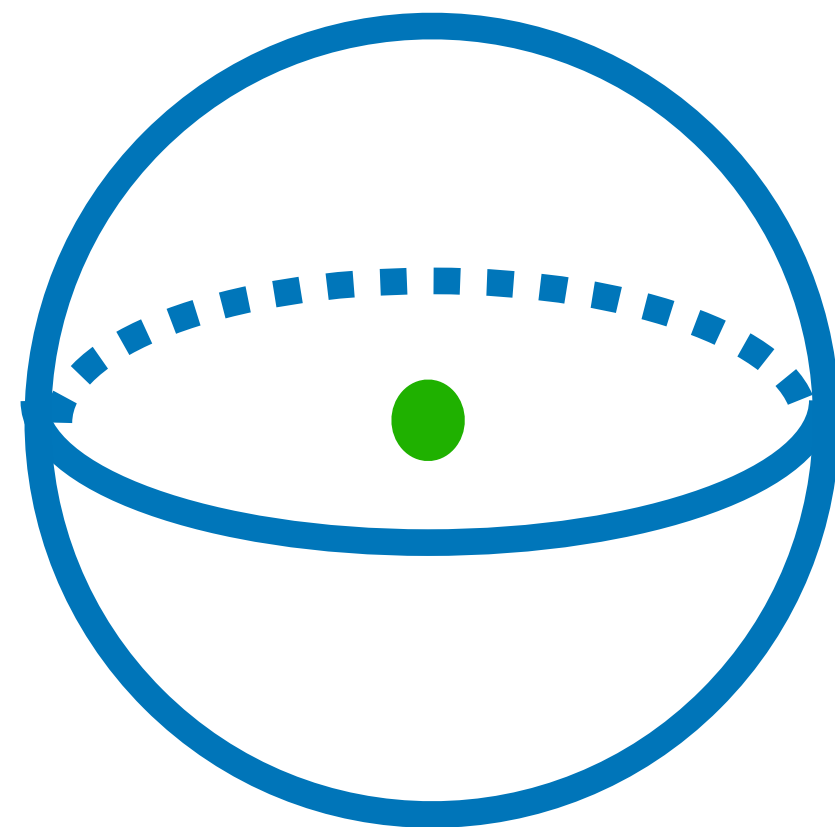
What should the charged object be now?

‘Linking’ is a topological function that generalizes ‘encloses’

$$\text{Link} \left(\Sigma_p, \Sigma_{d-p-1} \right) \in \mathbb{Z}$$

Digression because $d=3$ space is easier for us primates

$$p = 0, d - p - 1 = 2 \quad p = 1, d - p - 1 = 1$$



What should the charged object be now?

What dimensional operators could 'link with' this 2-dim surface?

$$\text{Link} \left(\Sigma_p, \Sigma_{d-p-1} \right) \in \mathbb{Z}$$

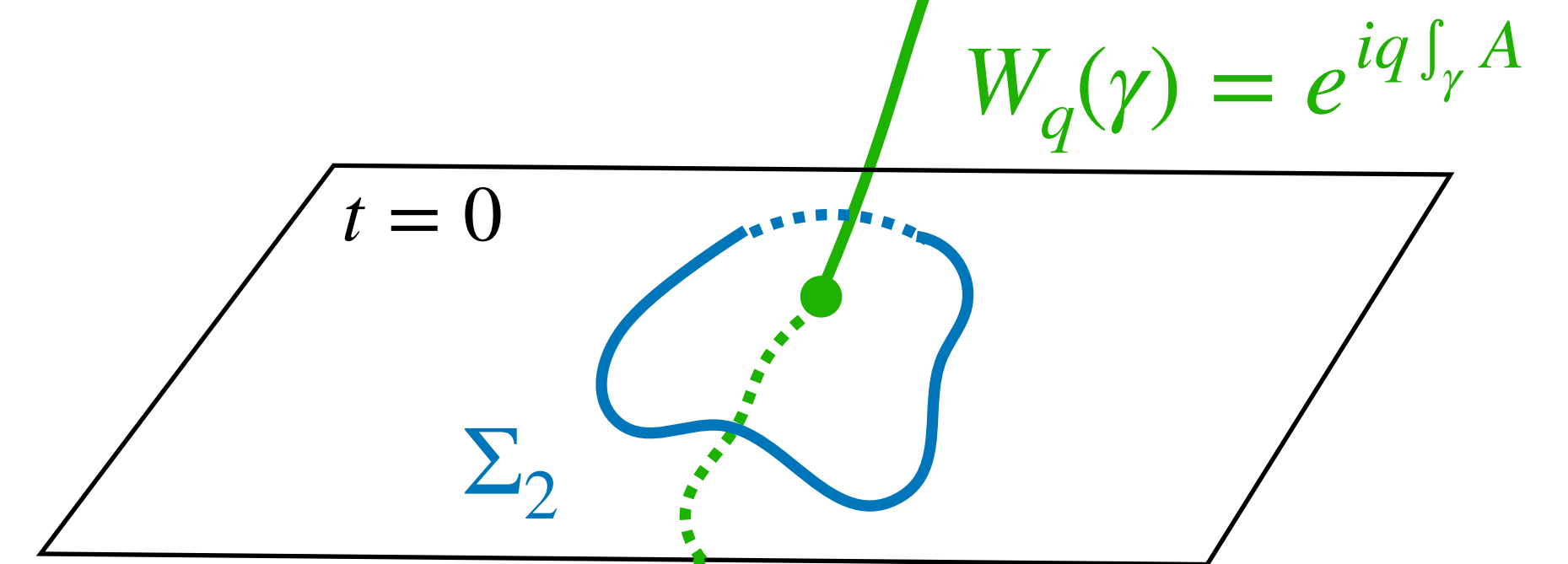
$p = 0$ point links a closed 3-surface

$p = 1$ line links a closed 2-surface

It's a Wilson line!

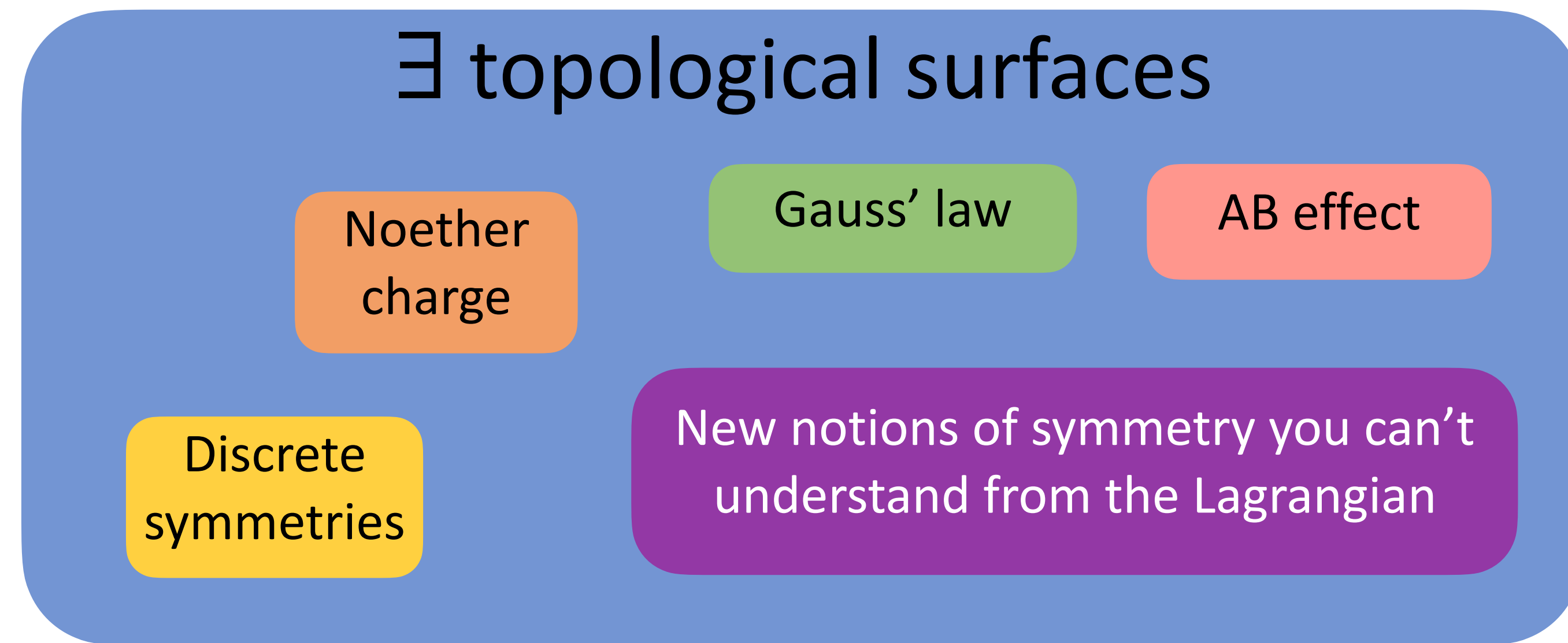
$$W_q(\gamma) = e^{iq \int_\gamma A}$$

(worldline γ of charge q
object in limit $m \rightarrow \infty$)



Core conceptual point of generalized symmetries

All symmetries can be understood as the existence of some surface operators that are topologically invariant.



Some basic intros: Gomes '23, Brennan & Hong '23, Iqbal '24, my papers
More formal: Schäfer-Nameki '23, Bhardwaj et al. '23, Simons Lectures '24

Generalized Symmetry Breaking

Higher-form symmetry breaking is **qualitatively different** from zero-form symmetry breaking!

Zero-form symmetry

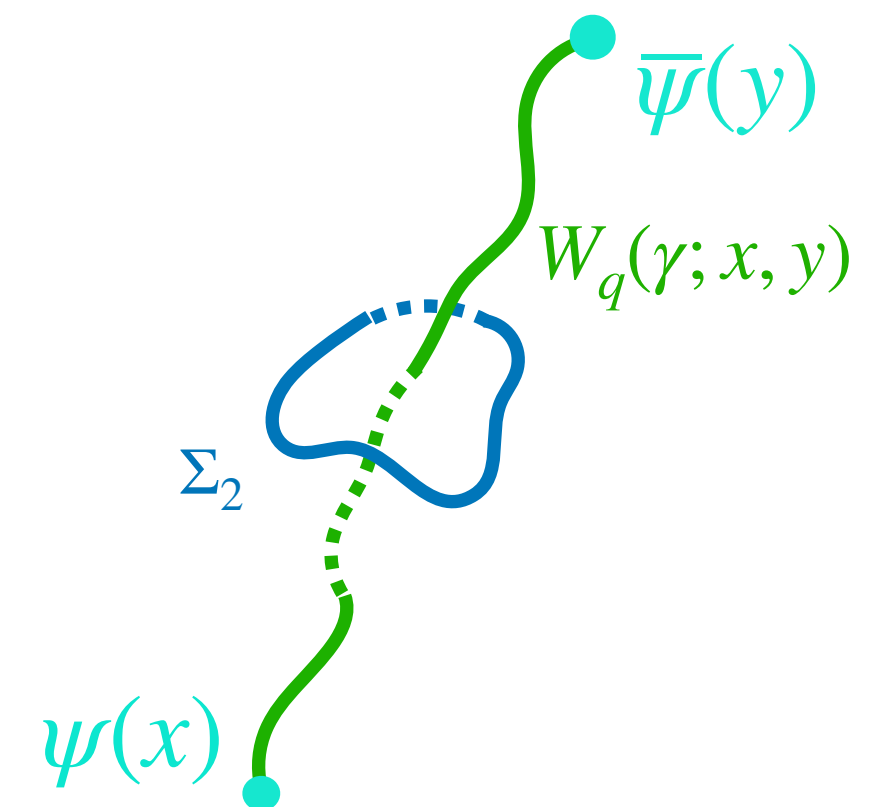
Explicit breaking from charged local operators in $\mathcal{L}$

E.g. $\mathcal{L} \supset c_{ij}(\tilde{H}L_i)(\tilde{H}L_j)$

Gauss' law **breaks** with dynamical electron!

Higher-form symmetry

Explicit breaking when new dynamical degrees of freedom in UV



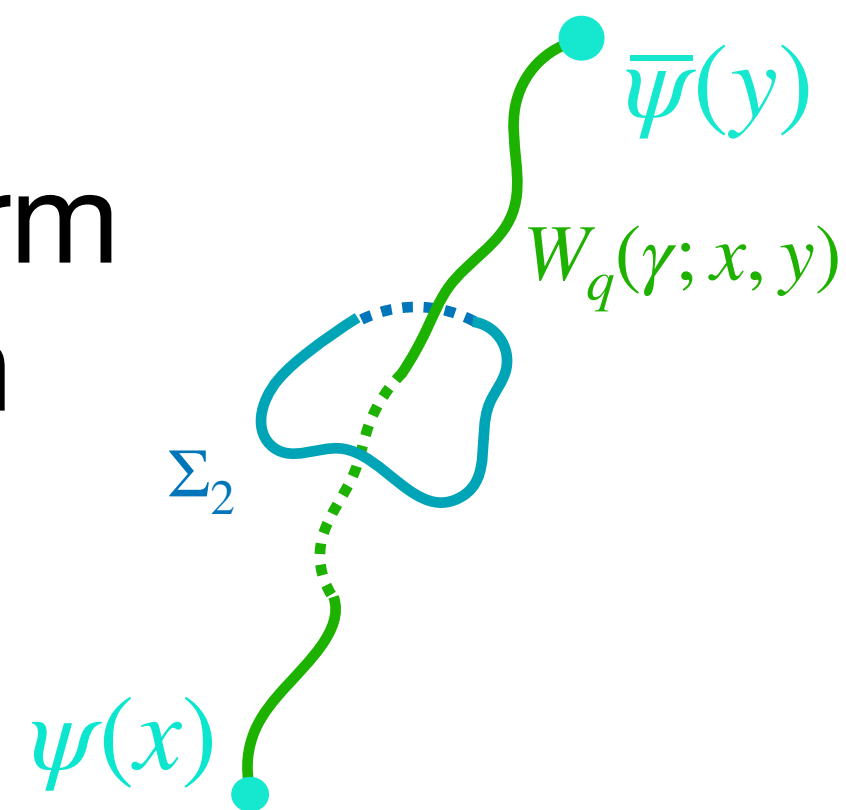
$$V(r) = \frac{-q^2}{4\pi r} \left(1 + \frac{q^2}{16\pi^{3/2}} \frac{e^{-2m_e r}}{(m_e r)^{3/2}} + \dots \right)$$

Uehling, '35 $r \gg m_e$

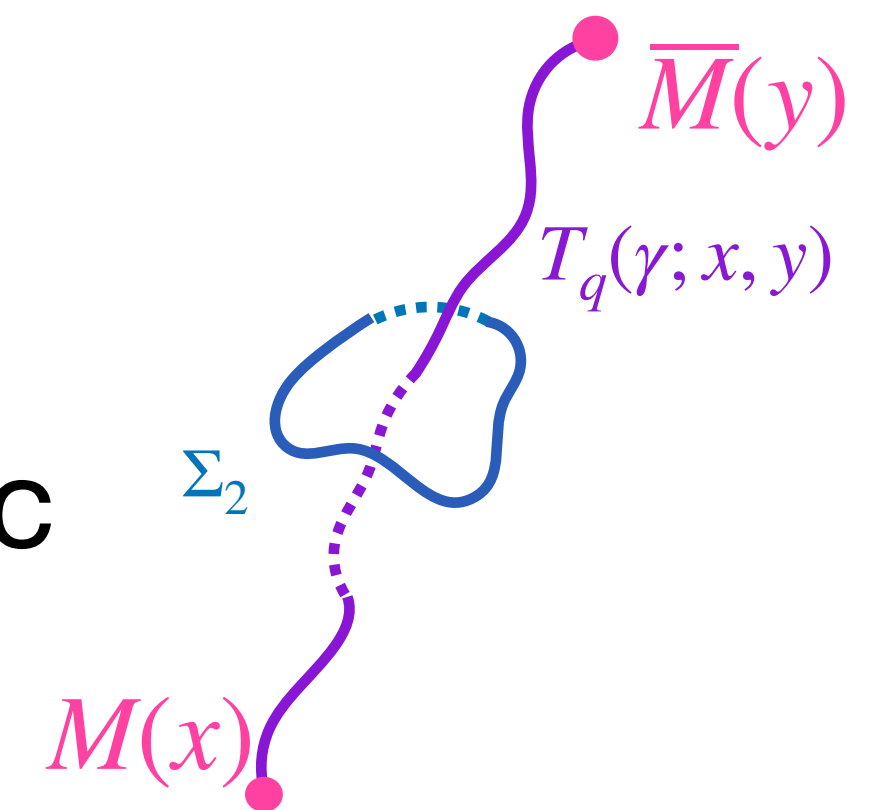
Electric and Magnetic Symmetry Breaking

Recall $U(1)_{\text{EM}}$ at long distances has both Wilson and 't Hooft line operators

$U(1)_e^{(1)}$ electric one-form symmetry breaks when you see electrically charged matter



$U(1)_m^{(1)}$ magnetic one-form symmetry breaks when you see magnetic monopoles!



But does a discrete 1-form symmetry remain?
Test at LHC!
SK & A. Martin

$$U_\alpha[\Sigma_2] = e^{i\alpha \int_{\Sigma_2} F_{\mu\nu} \hat{n}^\mu \hat{n}^\nu d^2x}$$

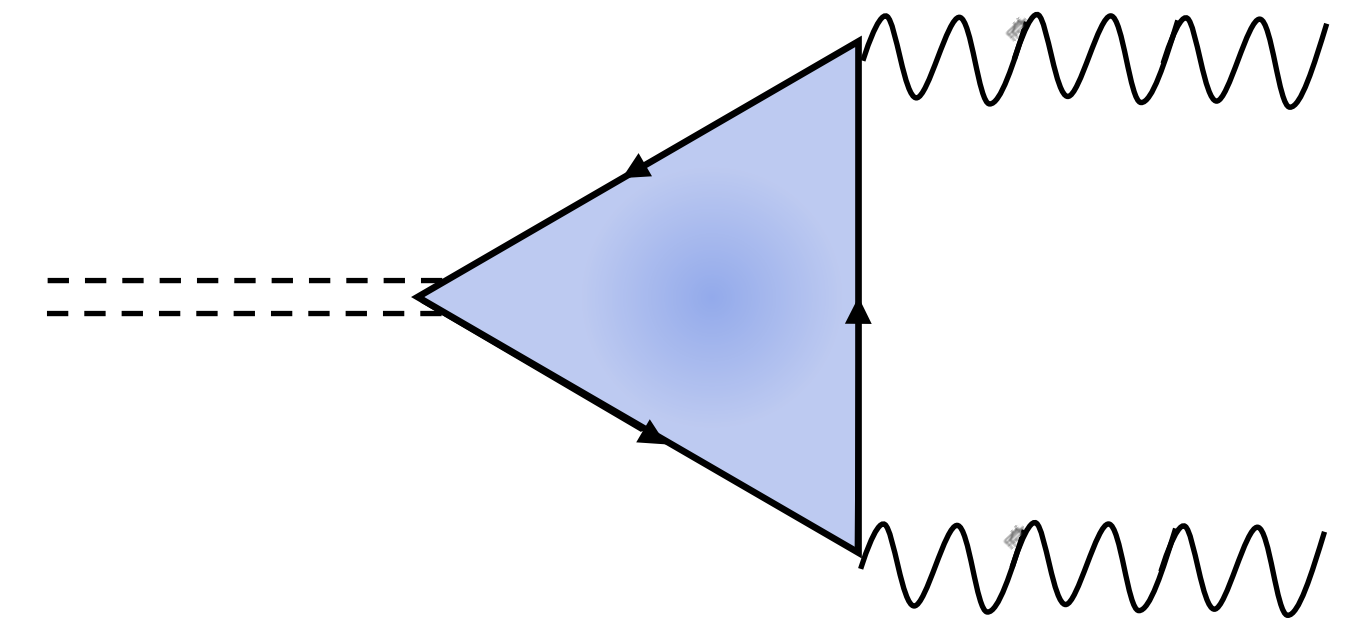
$$U_\alpha[\Sigma_2] = e^{i\alpha \int_{\Sigma_2} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} \hat{n}^\mu \hat{n}^\nu d^2x}$$

Dynamical electric charges are easy, but magnetic monopoles only arise from ultraviolet theory which includes new topology. A drastic change!

Motivating non-invertible symmetries: The mystery of the missing instantons

Recall a classical zero-form global symmetry $U(1)_X$ can be **anomalous** in quantum theory with F gauge group

$$\partial_\mu J_X^\mu = 0 \quad \longrightarrow \quad \partial_\mu J_X^\mu = \frac{\mathcal{A}_X}{8\pi^2} F^{\mu\nu} \tilde{F}_{\mu\nu}$$



Instanton configurations have $\int_{\mathcal{M}} F\tilde{F} \neq 0$ so ‘activate’ the anomaly

But what about when they don’t?

E.g. famously $\pi_3(U(1)) = 0$ and *there are no Abelian instantons in $\mathbb{R}^4$* , so $\int_{\mathbb{R}^4} F\tilde{F} = 0$

Old lesson: X is anomalous but S -matrix preserves X anyway

EFT philosophy: If there is ever a zero, there should be a symmetry!

Somehow despite X being anomalous **there must remain a subtle sort of symmetry** that demands the S -matrix preserves X

A hint: X *can* be violated around magnetic monopoles

c.f. Callan-Rubakov

Dirac '31
Callan, Rubakov '80s
Ongoing...

A hand-drawn diagram on a piece of paper. At the top, a wavy vertical line is labeled "M worldline" in blue. To the left of the line, a horizontal arrow points right towards the line, labeled "p+" in green. To the right of the line, a horizontal arrow points right away from the line, labeled "e+" in green. At the bottom, the formula $\Delta X \propto \int F \tilde{F} = \int \vec{E} \cdot \vec{B} \neq 0$ is written in red.

Fig. 1: A confused effective field theorist



There's a subtler notion of symmetry!

Choi, Lam, Shao
2205.05086
Córdova, Ohmori
2205.06243

Can't do $U_\alpha[\Sigma_3] = e^{i\alpha Q[\Sigma_3]} = e^{i\alpha \int_{\Sigma_3} J_X^\mu \hat{n}_\mu d^3x}$, not conserved

Can't do $\hat{J}_X^\mu = J_X^\mu - \frac{\mathcal{A}}{4\pi^2} \epsilon^{\mu\nu\rho\sigma} A_\nu \partial_\rho A_\sigma$, not gauge invariant

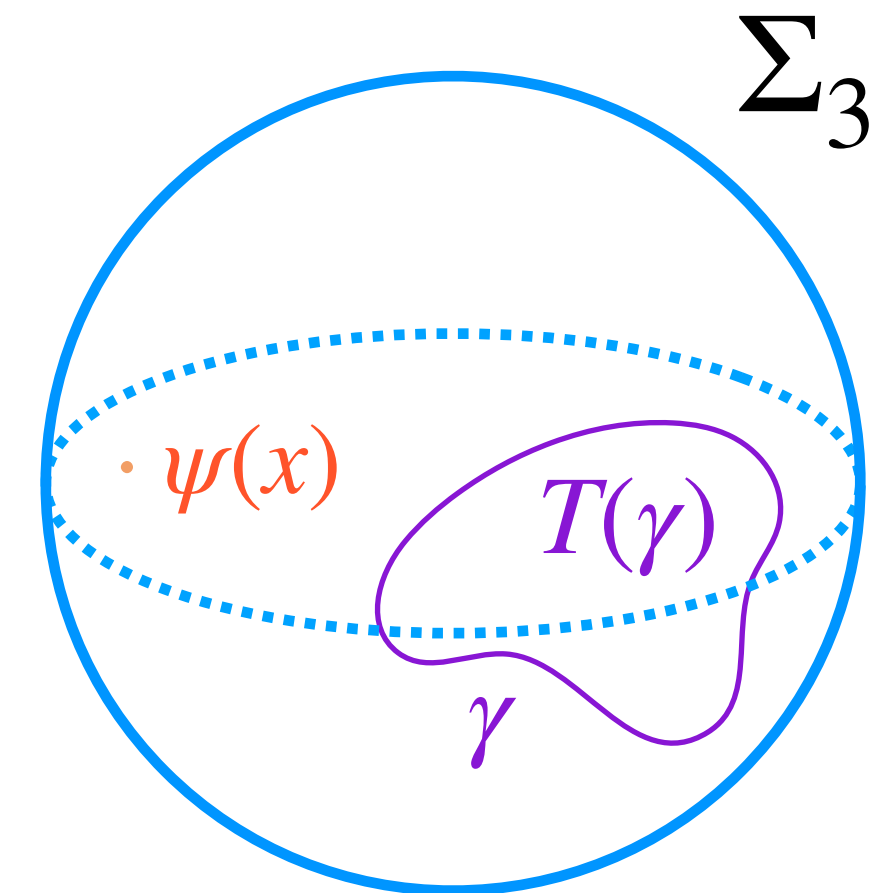
Fig. 2: Another
victory for
naturalness

Can construct a topological, gauge invariant operator by including a Chern-Simons theory which talks to the bulk magnetic current.

$$\begin{aligned}
 & \Sigma_3 \quad D_{\frac{2\pi}{N}}(\Sigma_3) = \int Dc \, e^{i \int_{\Sigma_3} \frac{2\pi}{N} \mathbf{J}_\mu \hat{n}^\mu + \frac{1}{2\pi} \epsilon^{\mu\nu\sigma} c_\mu \partial_\nu A_\sigma + \frac{N}{4\pi} \epsilon^{\mu\nu\sigma} c_\mu \partial_\nu c_\sigma} \\
 & = \cdot \psi(x) e^{2\pi i q/N} \int_{T(\gamma)} W(\gamma)^{1/N}
 \end{aligned}$$



Non-invertible Naturalness



$$D_{\frac{2\pi}{N}}(\Sigma_3) = \int Dc \, e^{i \int_{\Sigma_3} \frac{2\pi}{N} \star J_{\text{chiral}}^{(1)} + \frac{1}{2\pi} c \wedge \star J_{\text{mag}}^{(2)} + \frac{N}{4\pi} c \wedge dc}$$

$$= \cdot \psi(x) e^{2\pi i q / N} \text{ (purple blob) }$$

$$T(\gamma) W(\gamma)^{1/N}$$

This is a symmetry structure which acts on *both* local **operators** and 't Hooft lines.

Then *both* controls the form of the **Lagrangian** and breaks when **magnetic monopoles** appear! Operators protected by this symmetry must be generated!

Find a spurion for a **non-invertible** symmetry of your IR theory

$\Rightarrow$

Coupling can be *generated by nonperturbative gauge theory effects* in a UV theory with the right magnetic monopoles.

Symmetry guidance *past* technical naturalness!

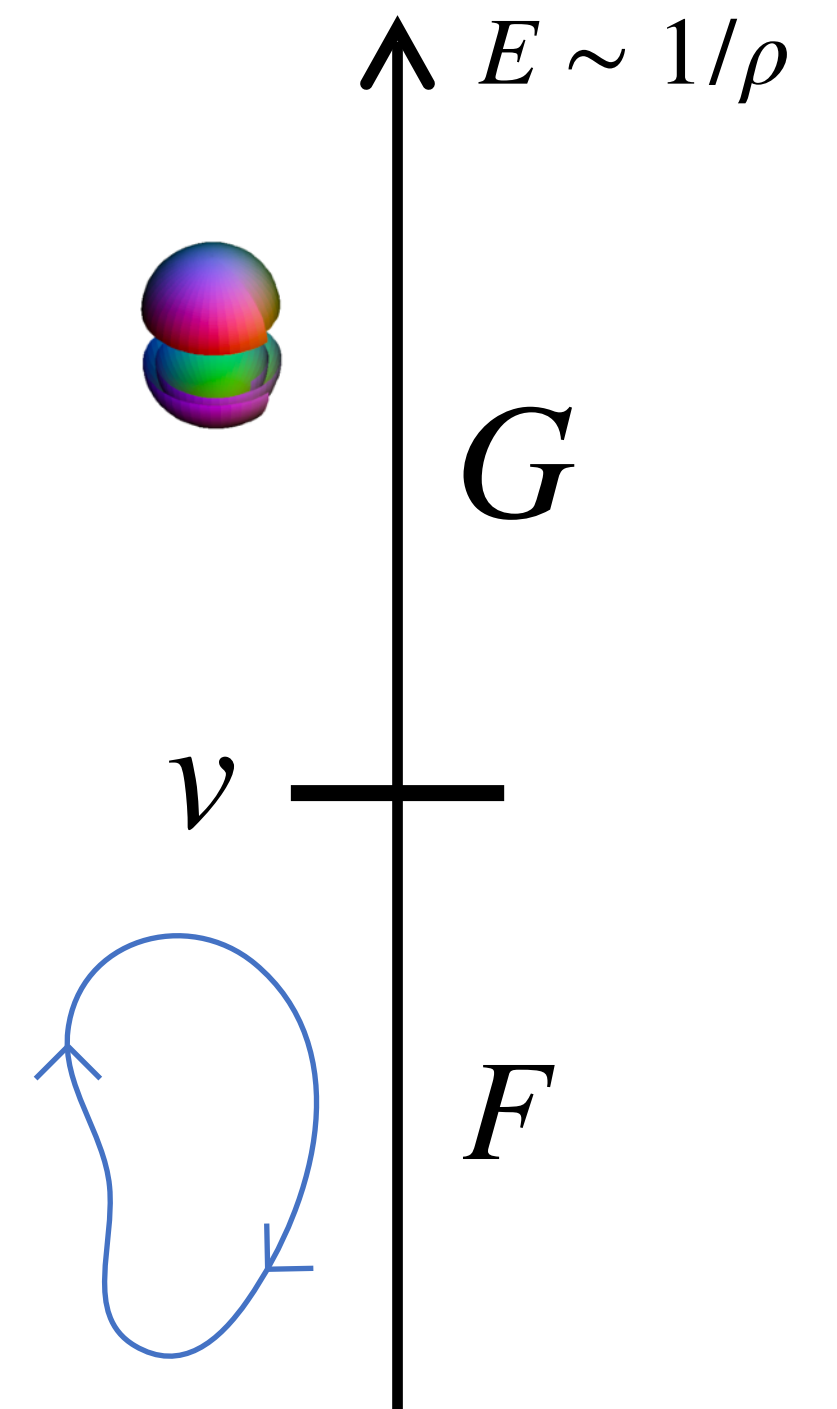
Model-building strategy

A classical global symmetry X protects some operator $\mathcal{O}$ and has an F anomaly

$$\partial_\mu J_X^\mu = \frac{\mathcal{A}}{8\pi^2} F^{\mu\nu} \tilde{F}_{\mu\nu}$$

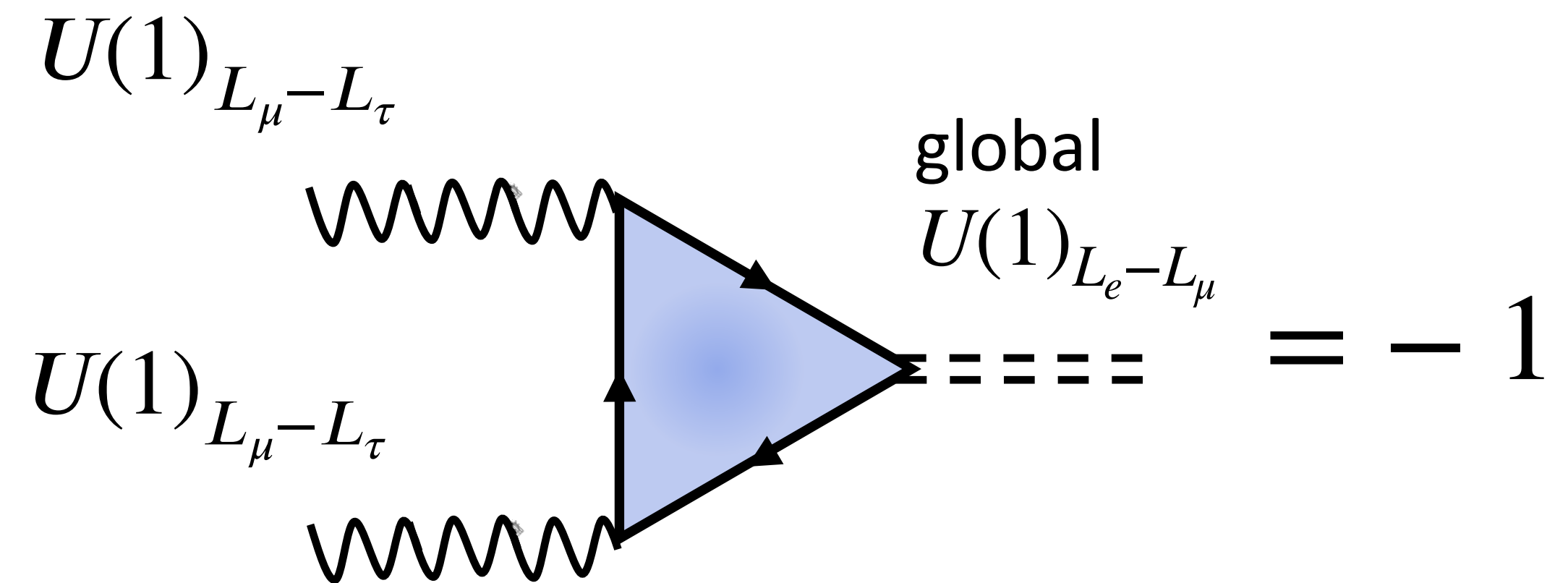
But some values of $\int_{\mathcal{M}} F\tilde{F}$ not realized for $\mathcal{M} = \mathbb{R}^4$

Non-invertible X symmetry tells us $\mathcal{O}$ could be generated by instantons in the theory $G \supset F$ which has G/F -monopoles



Go beyond the SM and gauge $U(1)_{L_\mu-L_\tau}$

There's a new ABJ anomaly diagram to consider



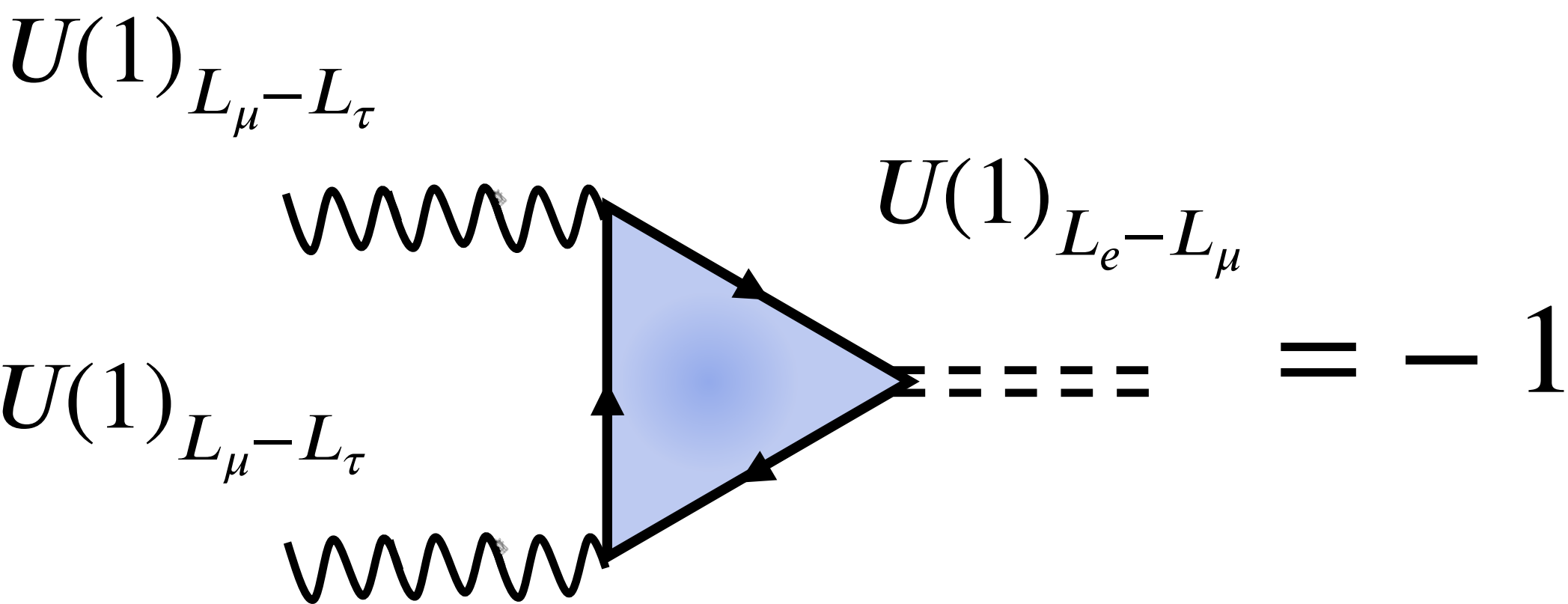
So the global $U(1)_{L_e-L_\mu}$ current is no longer conserved

$$\partial_\mu J^\mu_{L_e-L_\mu} = \frac{-1}{8\pi^2} F_{L_\mu-L_\tau} \tilde{F}_{L_\mu-L_\tau}$$

But the $U(1)_{L_\mu-L_\tau}$ gauge theory cannot saturate this anomaly

Non-invertible
symmetry!

Beyond with $Z'_{L_\mu-L_\tau}$



Non-invertible symmetry protects neutrino masses, focus on $\mathbb{Z}_3^L \subset U(1)_{L_e-L_\mu}$

	L_i	$\bar{e}_i$
$\mathbb{Z}_3^L$	$+1$	-1

Disallows $(\tilde{H}L)^2$

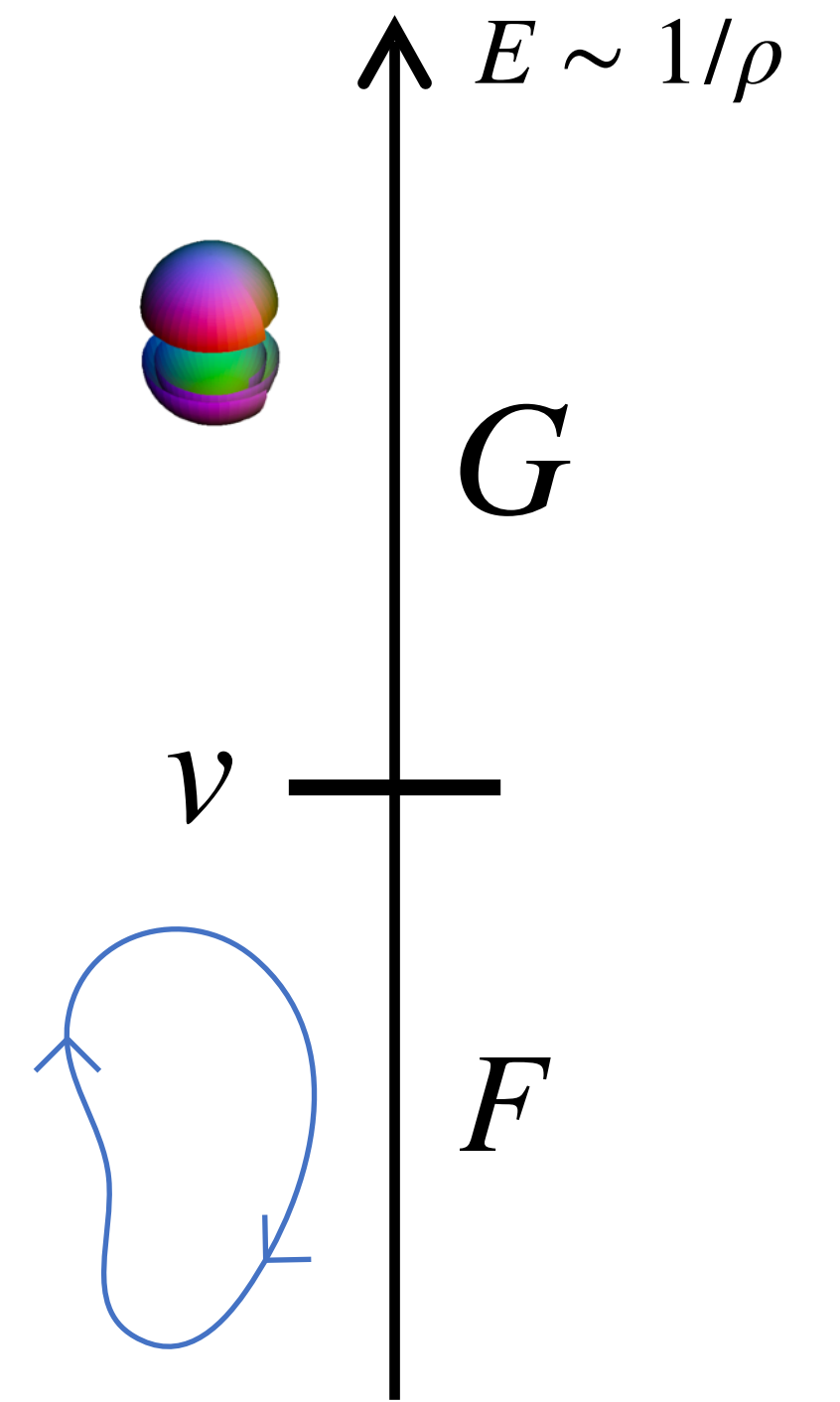
$$L = (L_e - L_\mu) - (L_\mu - L_\tau) \pmod{3}$$

Model-building logic

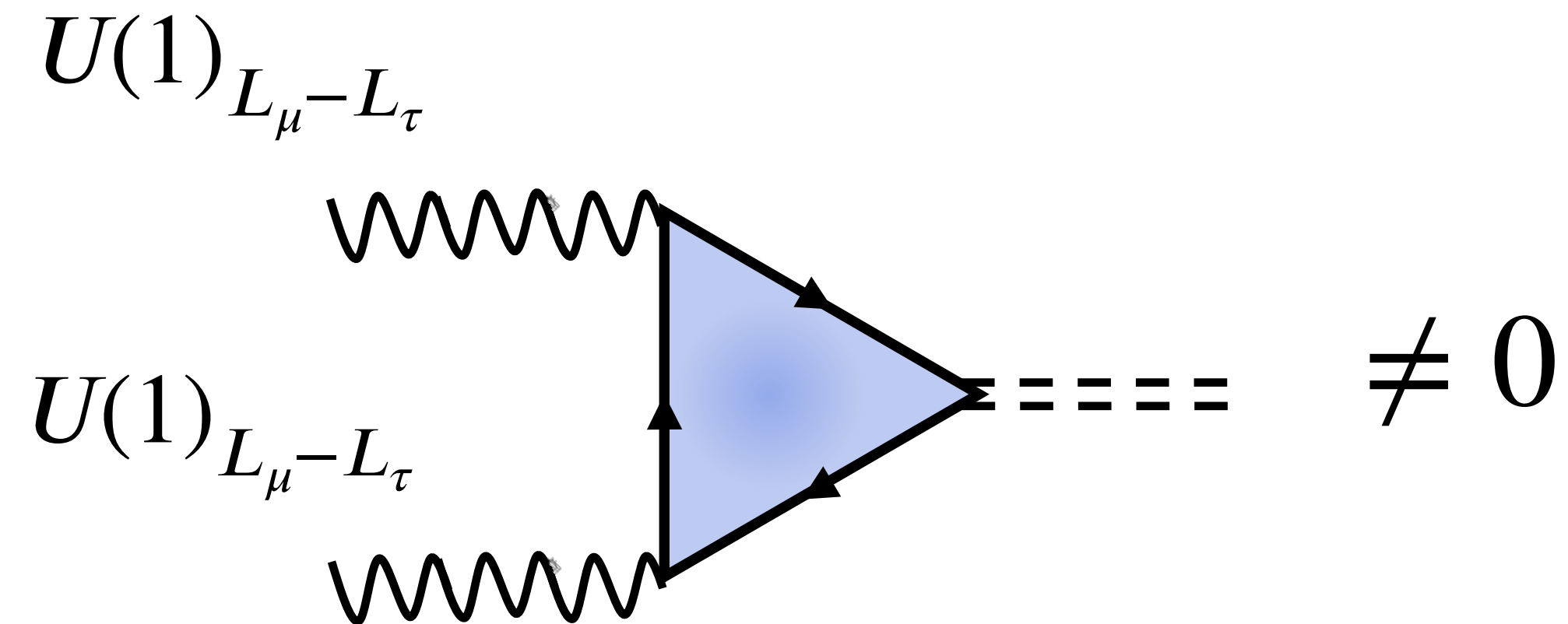
A classical global symmetry $X = \mathbb{Z}_3^L$ protects the operators $\mathcal{O}_{ij} = (\tilde{H}L_i)(\tilde{H}L_j)$ and has an $F = U(1)_{L_\mu - L_\tau}$ anomaly

But while $\int_{\mathcal{M}} F\tilde{F} \in \mathbb{Z}$ generally, $\int_{\mathbb{R}^4} F\tilde{F} = 0$

X is a non-invertible symmetry! In a theory $G \supset F$ with lepton flavor monopoles, $\mathcal{O}_{ij}$ could be classically absent and generated only by G -instantons.



Beyond with $Z'_{L_\mu-L_\tau}$ and N !



Non-invertible symmetry protects neutrino masses
either with or without right-handed neutrinos

	L_i	$\bar{e}_i$
$\mathbb{Z}_3^L$	+1	-1

Disallows $(\tilde{H}L)^2$

	L_i	$\bar{e}_i$	N_i
$\mathbb{Z}_3^{\tilde{L}+N}$	+1	-1	+1

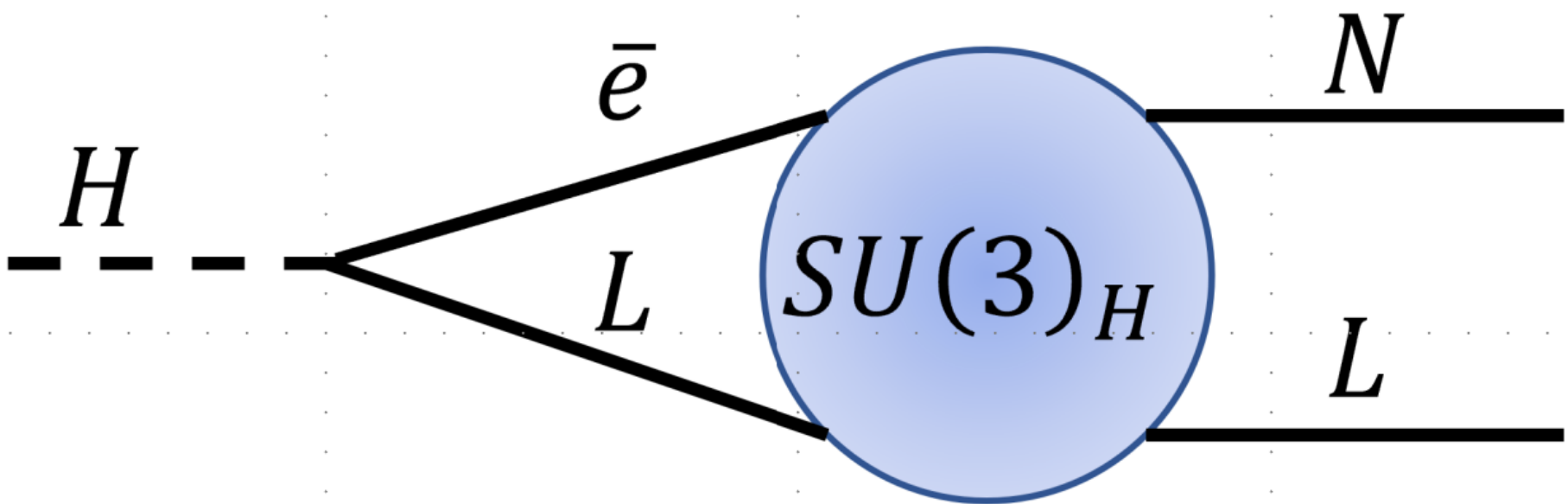
Disallows $\tilde{H}LN$

Dirac masses:

Write down charged lepton mass

$$\mathcal{L} \sim y_\tau H \mathbf{L} \bar{\mathbf{e}}$$

Classical $U(1)_N$ symmetry
protects the Dirac neutrino
mass $\tilde{H} \mathbf{L} \mathbf{N}$

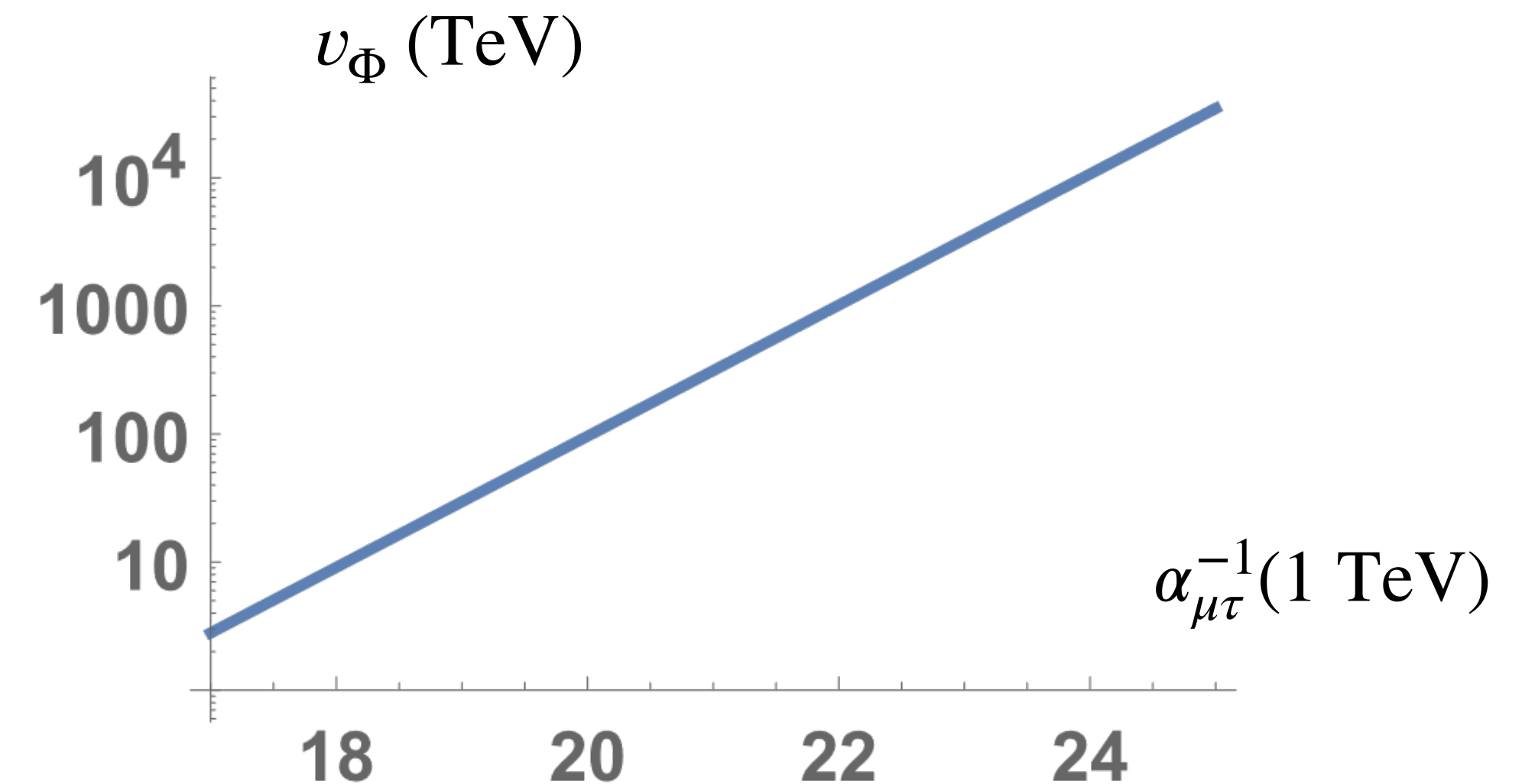


$$\mathcal{L} \sim y_\tau^\star e^{-\frac{8\pi^2}{g_H^2}} \tilde{H} \mathbf{L} \mathbf{N}$$

Economical and predictive

Given the discovery of such a Z' for $U(1)_{L_\mu-L_\tau}$, learn the scale at which $SU(3)_H \rightarrow U(1)_{L_\mu-L_\tau}$

$$v_\Phi^2 \sim M_{Z'}^2 \left(\frac{m_\nu}{m_\tau} \right)^{3/2} \exp \frac{3\pi}{4\alpha_{\mu\tau}(M_{Z'}^2)}$$

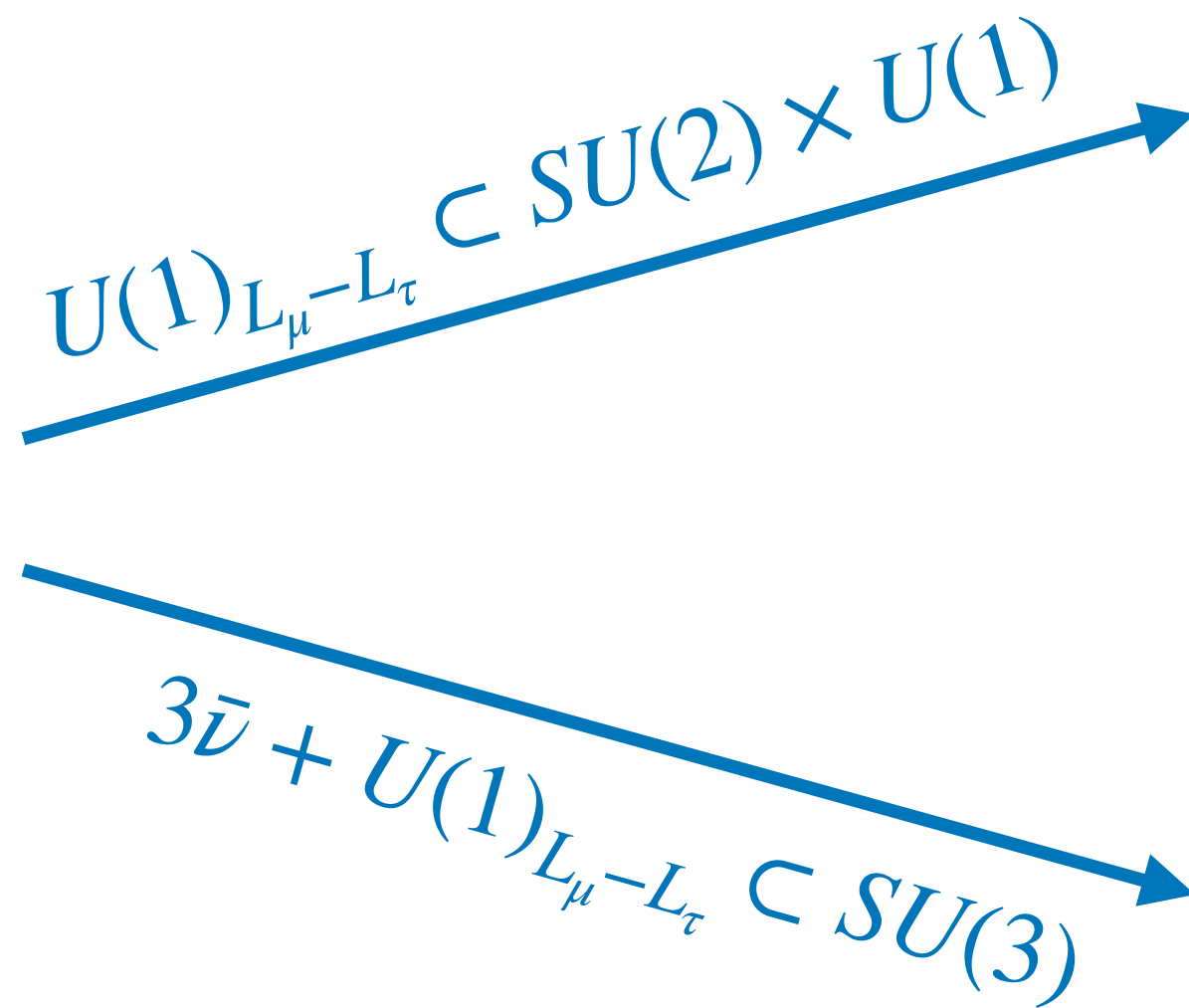


Texture from Higgses implementing $SU(3)_H \rightarrow U(1)_{L_\mu-L_\tau} \rightarrow \emptyset$

Non-invertible symmetry in lepton flavor gauge theory

2211.07639/**PRX** Clay Córdova, Sungwoo Hong, SK, Kantaro Ohmori

Gauged $U(1)_{L_\mu-L_\tau}$ gives
non-invertible symmetry
protecting neutrino masses!



	$SU(3)$
$\mathbf{L}$	$\square$
$\bar{\mathbf{e}}$	$\bar{\square}$
$\bar{\nu}$	$\bar{\square}$

Instantons produce
Majorana neutrinos

$$\mathcal{L} \sim \frac{y_\mu y_\tau}{v_\Phi} e^{-\frac{8\pi^2}{g_H^2}} (\tilde{H}L)(\tilde{H}L)$$

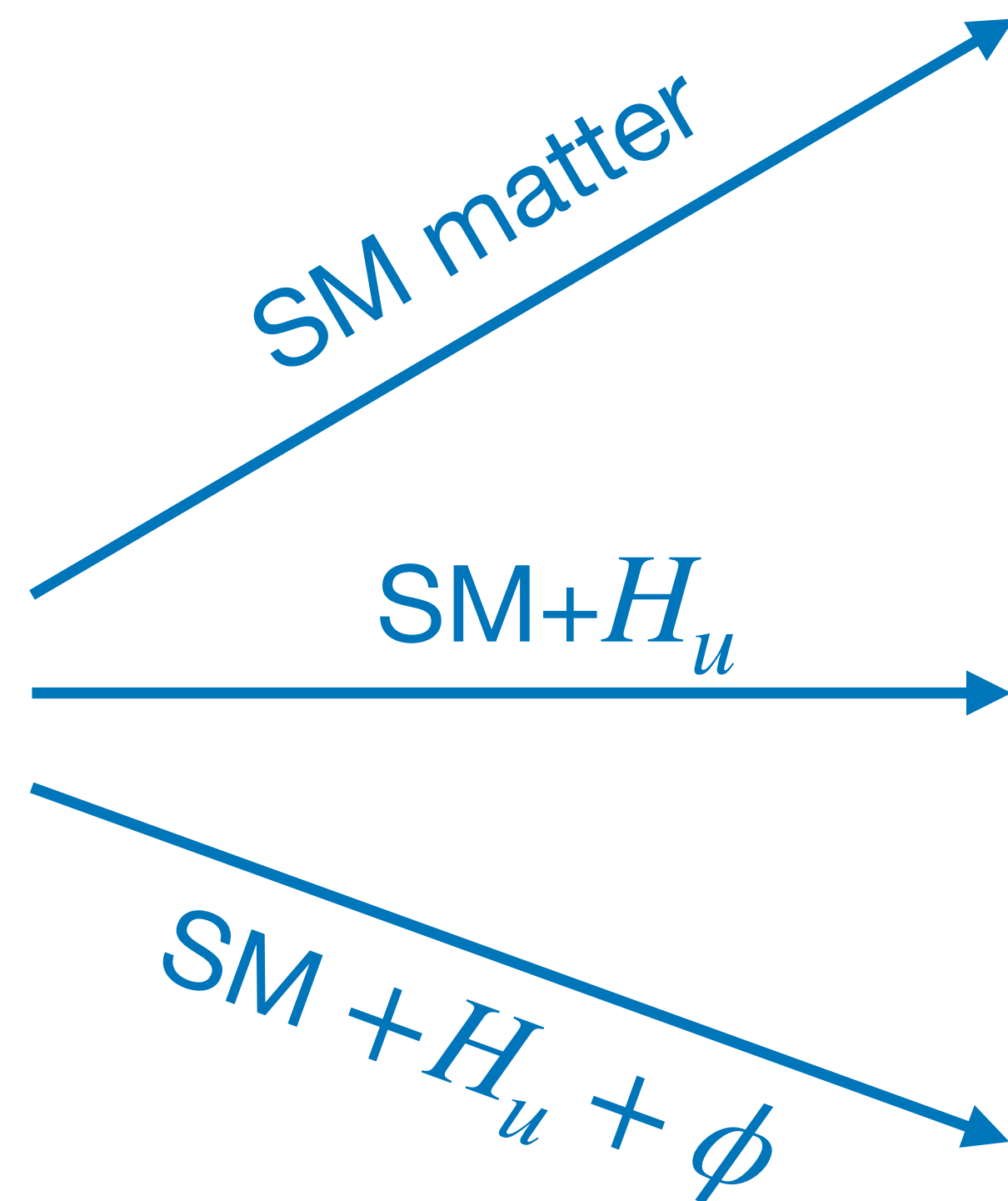
Instantons produce
Dirac neutrinos

$$\mathcal{L} \sim y_\tau e^{-\frac{8\pi^2}{g_H^2}} \tilde{H}L\bar{\nu}$$

Non-invertible PQ Symmetry in quark flavor gauge theory

Since $N_c = N_g$, can gauge $\left(SU(3)_C \times U(1)_{B_1+B_2-2B_3} \right) / \mathbb{Z}_3$ and get non-invertible symmetry!
 Breaking in e.g. $SU(9)$ quark color-flavor unification.

	$SU(9)$
Q	9
$\bar{\mathbf{u}}$	$\bar{9}$
$\bar{\mathbf{d}}$	$\bar{9}$



Massless quark solution
 from $y_b \sim y_t e^{-8\pi^2/g_9^2}$

2402.12453/**PRX** Córdova, Hong, SK

2HDM Alignment with
 Visible Axion

$$m_{12}^2 \sim y_t y_b v_9^2 e^{-8\pi^2/g_9^2}$$

2412.05362/**JHEP** Antonio Delgado, SK

Solve DFSZ DW prob

$$\delta V(a) \sim f_a v_9^3 e^{-8\pi^2/g_9^2} \cos a$$

2603.26867 Gongjun Choi, Hong, SK

Non-invertible symmetries in quark-lepton color-flavor unification

Unify uses of non-invertible symmetry in both sectors!

2605.30413 Antonio Delgado, SK

	$SU(12)$	$SU(2)_L$	$U(1)_R$
Ψ	12	2	0
ψ_u	$\overline{12}$	—	+1
ψ_d	$\overline{12}$	—	−1

$$\begin{aligned}
 G_{UV} &= (SU(12) \times SU(2)_L \times U(1)_R)/\Gamma \\
 &\quad \frac{SU(9) \times U(1)_{Q-N_c L}}{\mathbb{Z}_3} \times SU(3)_\ell \\
 &\quad \frac{(SU(3)_1 \times SU(3)_2 \times SU(3)_3 \times U(1)_{B_i-B_j} \times U(1)_{B_i+B_j-2B_k}) \rtimes S_3}{\mathbb{Z}_3 \times \mathbb{Z}_3} \\
 &\quad \downarrow \\
 G_{SM} &= \frac{SU(3)_C \times SU(2)_L \times U(1)_Y \times \mathbb{Z}_{18}^{B-3(L_i+L_j-L_k)}}{\mathbb{Z}_3 \times \Gamma \times \mathbb{Z}_3}
 \end{aligned}$$

$\Gamma \in \{1, \mathbb{Z}_2\}$

- Yukawa unification
- Rich generalized symmetries and topological defects
- Absolute proton stability

Seth's conclusions

The simple data of various **integer invariants** associated to the SM fermions has already lead us to **new unified theories** to **solve SM naturalness issues** based on the rigid data of the SM.

Generalized global symmetries are a new tool for understanding **nonperturbative aspects** of theories of particle physics.

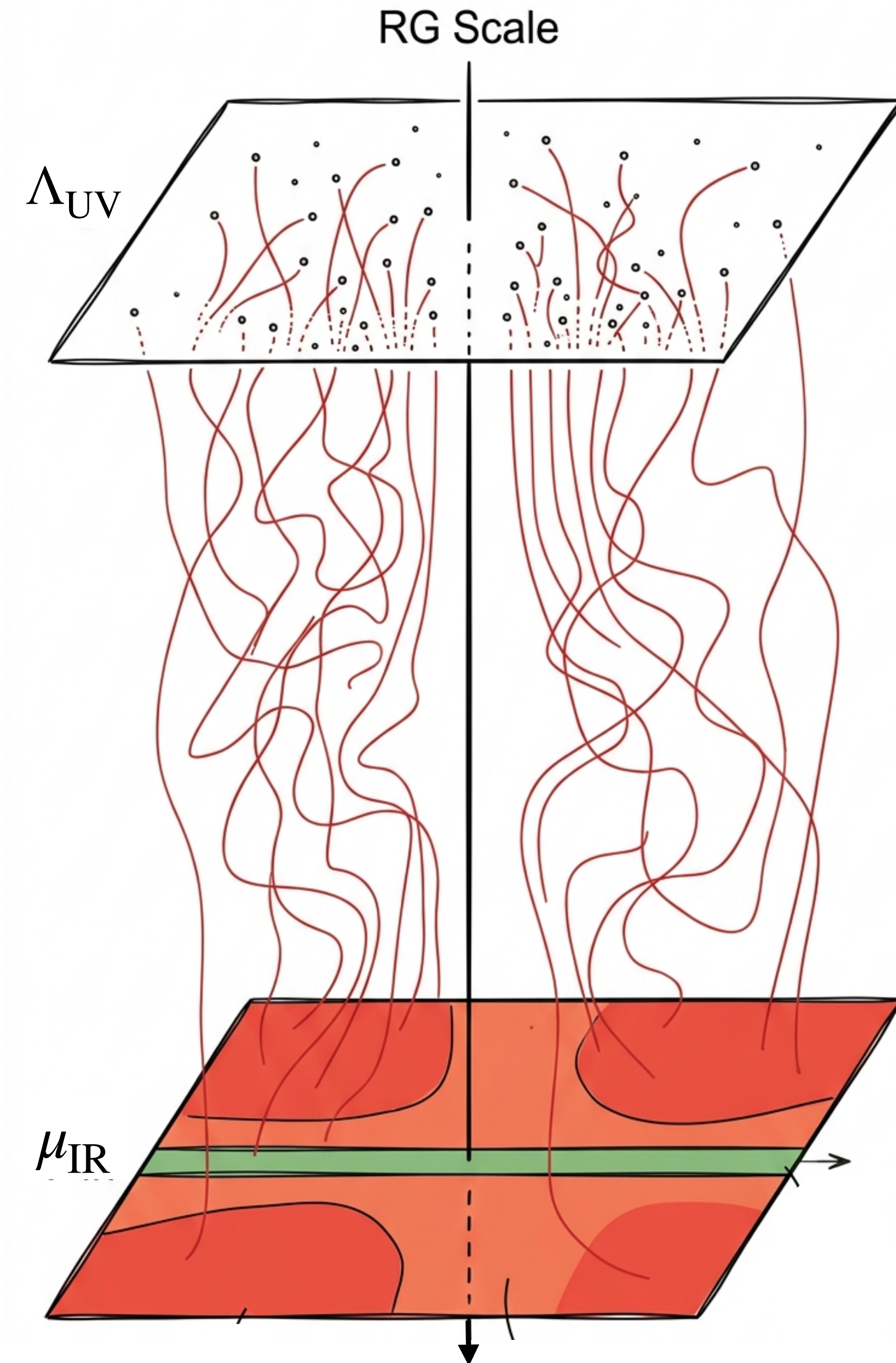
As particle physicists we are not yet done learning about the role of symmetries!

Fig. 3: A primate pleased they newly uncovered some simple, reductionist BSM models



Model building applications of non-invertible symmetry

	Technically Natural	Unnatural
Dim. 0	y_ν (CC,SH,SK,KO 2211.)	$\bar{\theta}$ (CC,SH,SK 2402.)
Dim. 2	m_{12}^2 (AD,SK 2412.)	<i>The</i> Hierarchy Problem
Dim. 4	$\delta V(a)$ (GC,SH,SK 2603.)	<i>The</i> CC Problem



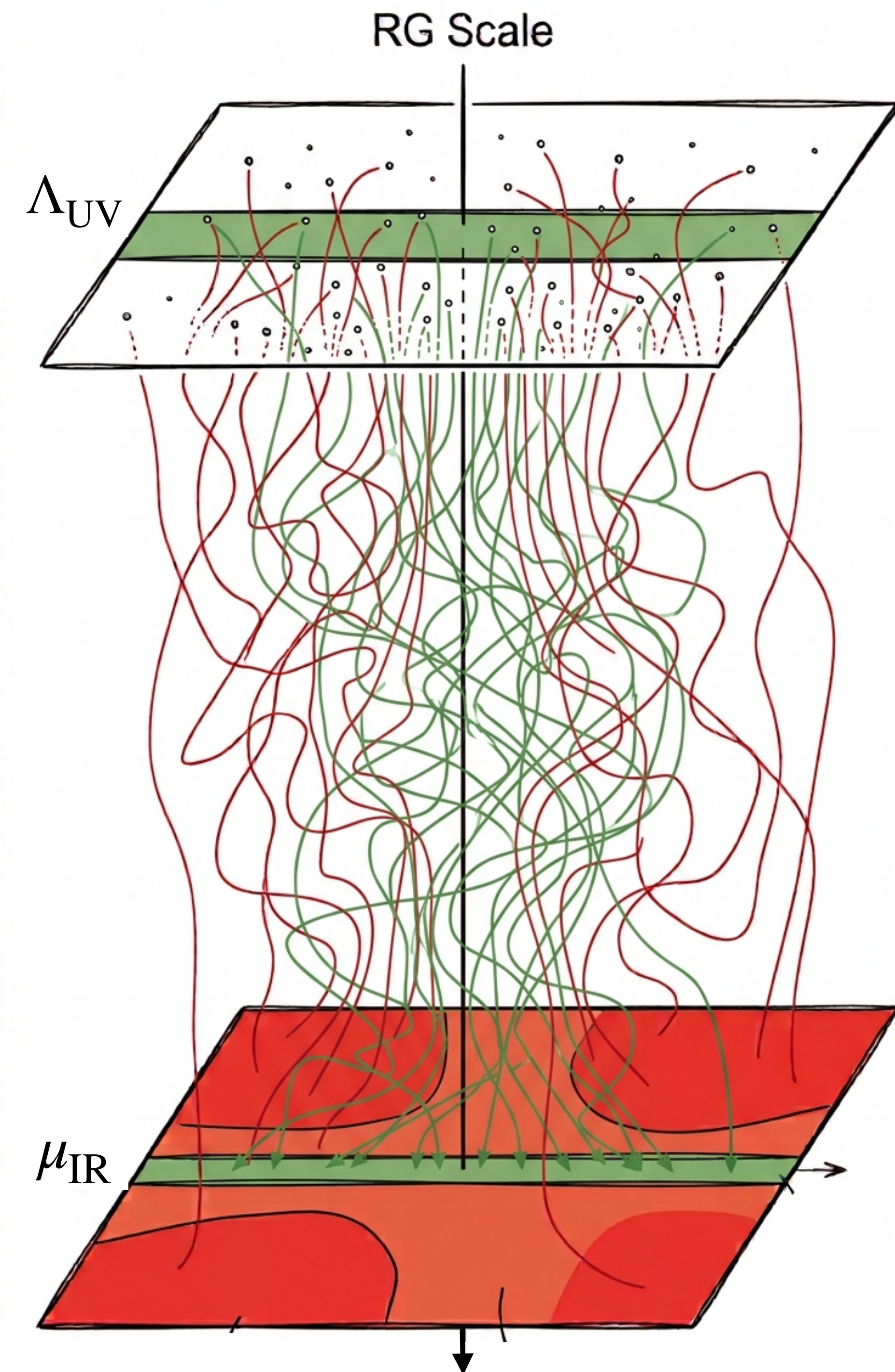
Wilsonian Naturalness:

- Our field theory is fundamentally defined at some high scale and evolves to low energies.
- RG flows in QFTs generically lead to parameters $\mathcal{O}(1)$ in units of the UV cutoff.

$$c_i(\mu_{IR}) \sim \mathcal{O}(1)\Lambda_{UV}^{4-\Delta_i}$$

If we see small parameters in the IR there is a naturalness puzzle: We should find a mechanism that produces the small parameters at nearby energies.

E.g. Neutrino yukawas ridiculously small $y_\nu \sim 10^{-13}$



‘Technical’ Naturalness:

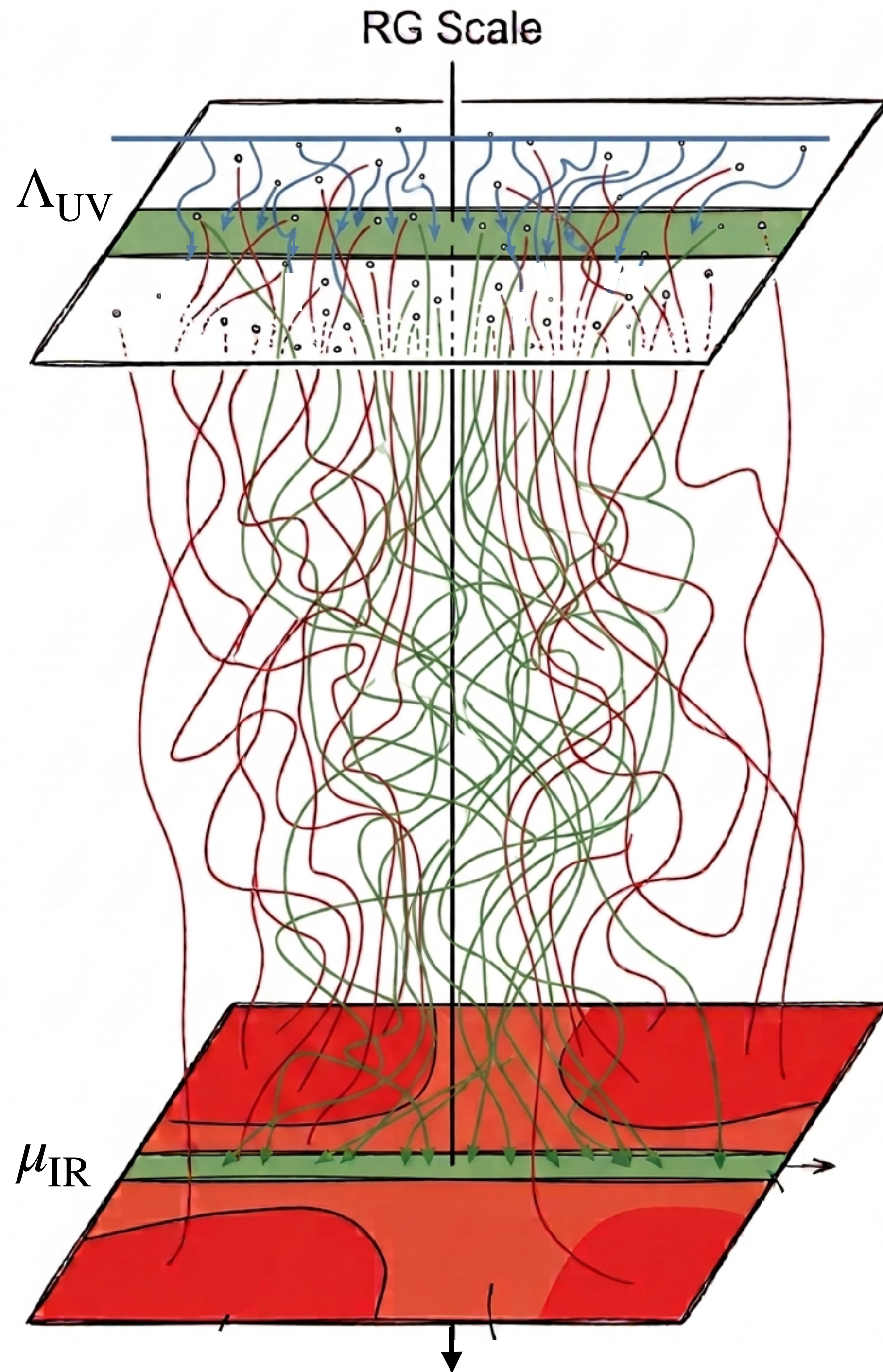
If there is an enhanced symmetry when

$$c_i \rightarrow 0, \text{ then } \frac{dc_i}{d \ln \mu} \propto c_i$$

Then small c_i at a high scale stays small down to the low scale.

So not every small number is equally worrisome; some naturalness puzzles can be postponed for the far UV.
 But this doesn’t actually provide an explanation for the naturalness puzzle.

E.g. $\delta y_\nu \propto y_\nu$ so can solve in UV



‘Non-invertible’ Naturalness:

If there is an enhanced *non-invertible* symmetry when $c_i \rightarrow 0$, then small c_i can be *produced* at a high scale in an expanded gauge theory from nonperturbative effects.

There are some cases where the structure of the IR theory itself points to a UV theory that fully resolves the naturalness puzzle.

$$\text{E.g. } y_\nu = 0 \longrightarrow y_\nu \sim y \exp \left(-\frac{8\pi^2}{g^2} \right)$$