One instanton, many consequences:

Axion potentials and proton decay, beyond the Standard Model

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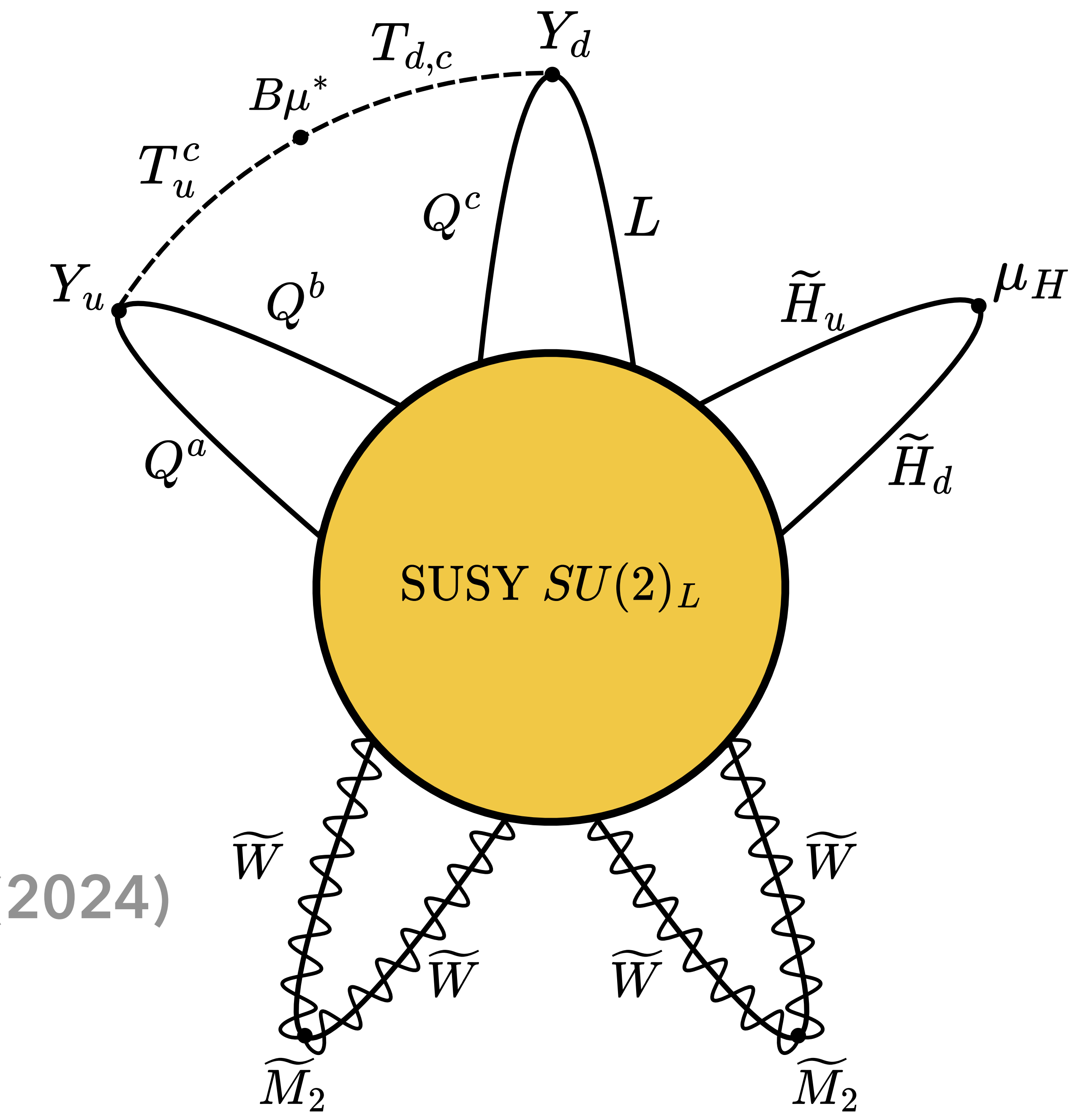
Based on **arXiv:2411.00101** and work in progress with:
Seth Koren and **Adam Martin**

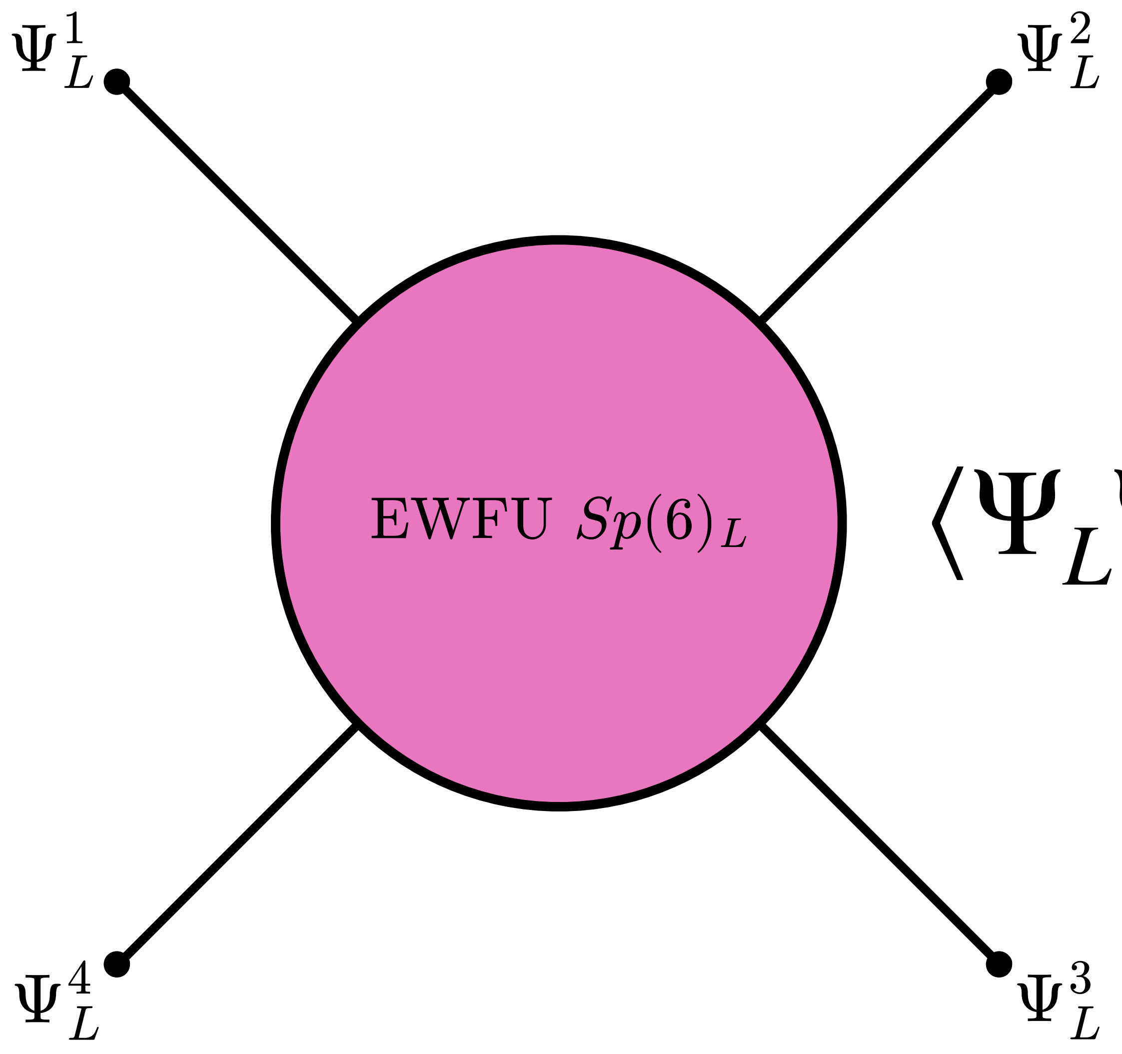
What one instanton can do

A single topological saddle can generate very different physical effects

$$\mathscr{E}(\theta) \Rightarrow V(a)$$



Csáki, D'Agnolo, Kuflik & PS (2024)
PS (2024)



$$\langle \Psi_L \Psi_L \Psi_L \Psi_L \rangle \supset \langle QQQL \rangle$$

Koren, Martin & PS (Soon)

The difference is how fermion zero modes are saturated

The BPST solution and collective coordinates

The self-dual unit charge solution

$$\boxed{F_{\mu\nu} = +\tilde{F}_{\mu\nu} \qquad Q_I = +1 \qquad S_I = \frac{8\pi^2}{g^2}}$$

In regular gauge:
$$A_\mu(x;\gamma) = 2\eta_{a\mu\nu}\frac{(x-x_0)_\nu}{(x-x_0)^2+\rho^2}U(\Omega)T^aU^\dagger(\Omega)$$

characterized by *collective coordinates* $\gamma = (x_0^\mu, \rho, \Omega)$:

x_0 — Position of the instanton center

ρ — Instanton size

Ω — Instanton orientation within the gauge group

Topological sectors and the θ-vacuum

The Yang–Mills θ vacuum is a superposition of topologically distinct vacua

$$|\theta\rangle = \sum_{n=-\infty}^{+\infty} e^{in\theta}|n\rangle = \cdots + e^{-i\theta}|-1\rangle + |0\rangle + e^{+i\theta}|1\rangle + \cdots$$

(anti-)instantons interpolate between different vacua

$\Delta N_{\rm CS} = \pm 1$

Topological charge:

$$Q[A] = \frac{1}{16\pi^2}\int d^4x \; \mathrm{Tr}\, F_{\mu\nu}\tilde{F}_{\mu\nu} = \Delta N_{\rm CS}$$

From tunneling to the axion potential

Tunneling lifts the degeneracy of the $|n\rangle$ vacua and produces a θ-dependent energy

$$|n\rangle \underset{I}{\overset{\bar{I}}{\leftrightarrows}} |n+1\rangle$$

Amplitudes carry topological phases:

$\boxed{I: \; e^{-i\theta}}$ $\boxed{\bar{I}: \; e^{+i\theta}}$ $\Longrightarrow$ $\boxed{\Delta\mathscr{E}_I(\theta) = -\left(\mathscr{A}_I e^{-i\theta} + \mathrm{h.c.}\right)}$

Promoting θ to an axion:

$$\theta \to \theta + \frac{a}{f_a} \quad \Longrightarrow \quad \boxed{V_I(a) = -\left(\mathscr{A}_I e^{-i\left(\theta + \frac{a}{f_a}\right)} + \mathrm{h.c.}\right)}$$

The rest of the calculation is the determination of $\mathscr{A}_I$

Instantons violate anomalous global symmetries

The ABJ anomaly turns topology into charge violation

$$\partial_\mu J^\mu_Q = \frac{\mathscr{A}_Q}{16\pi^2} \mathrm{Tr}\, F_{\mu\nu} \tilde{F}^{\mu\nu}$$

Integrating over spacetime the anomaly equation gives:

$$\Delta Q = \mathscr{A}_Q Q_I$$

Therefore $|n\rangle \to |n+1\rangle$ induces $Q \to Q + \mathscr{A}_Q$

A unit instanton changes anomalous global charges by $\mathscr{A}_Q$

From topology to the 't Hooft operator

't Hooft operators from instantons

A unit instanton changes anomalous global charges by $\mathscr{A}_Q$:
$|n\rangle \to |n+1\rangle$ induces $Q \to Q + \mathscr{A}_Q$

LH Weyl fermions $\psi_f \in (R_f, S_f)$
under $G_{\text{inst}} \times G_{\text{spec}}$

Zero modes:
$$n_f = 2T_{G_{\text{inst}}}(R_f) \times \dim(S_f)$$

Anomaly coefficient:
$$\mathscr{A}_Q = \sum_f q_f \, 2T(R_f) \times \dim S_f$$

A unit instanton changes the anomalous charge:
$$\Delta Q = \sum_f q_f \, n_f = \mathscr{A}_Q$$

The corresponding 't Hooft operator:

$$\boxed{\mathscr{O}_{\text{tHooft}} = \mathscr{T}^{\{I_{f,r}\}} \prod_f \prod_{r=1}^{n_f} \psi_f^{I_{f,r}}}$$

$I_{f,r} = (\text{Lorentz}, G_{\text{inst}}, G_{\text{spec}})$

Inserts exactly the required fermion zero modes

The one-instanton generating functional

At leading semiclassical order, the relevant nonperturbative information is encoded in:

$$\boxed{Z_I[J_\phi, J_\psi]}$$

Vacuum energy/axion potential

$$Z_I^{\text{int}}[\{J\}] = \exp\left[-\int d^4x\, \mathscr{L}_{\text{int}}\left(\left\{\frac{\delta}{\delta J}\right\}\right)\right] Z_I^{(0)}[\{J\}]$$

$$Z_I^{\text{int}}[\{0\}] \;\Rightarrow \mathscr{E}(\theta) \;\Rightarrow V(a)$$

Correlation functions

$$\left.\frac{\delta^n Z_I}{\delta J_\psi^n}\right|_{J=0} \Longrightarrow \langle \psi \cdots \psi \rangle_I$$

Wilson coefficients

$$\text{Correlators} \xrightarrow{\text{LSZ/matching}} C_{\mathscr{O}}^{(I)}$$

Axion potentials and anomalous amplitudes are different projections of the same object

Embedding of the BPST instanton

A charge-one instanton of G is obtained by embedding $SU(2)_{\text{inst}} \subset G$

$$A_\mu^{(G)}(x;\gamma) = 2\eta_{a\mu\nu}\,\frac{(x-x_0)_\nu}{(x-x_0)^2+\rho^2}\,U(\Omega)T^aU^\dagger(\Omega)$$

$$T^a \in \mathfrak{su}(2) \subset \mathfrak{g}$$
$$U \in G/\mathscr{T}_G$$

$$R\downarrow_{SU(2)_{\text{inst}}} = \bigoplus_{j\in\frac{1}{2}\mathbb{N}} n_j(R)\,[j]$$

$$\text{Adj}_G \to [1] \oplus (2h_G^\vee - 4)[1/2] \oplus \dim\mathscr{T}_G[0]$$
$$\dim\mathscr{T}_G = d_G - 4h_G^\vee + 5$$

Fermion zero modes for any G-representation are obtained from $SU(2)$ isospin-j PS (2024)

$$\boxed{\textit{General } G \textit{ calculation} \Longrightarrow \textit{standard } SU(2) \textit{ isospin-}j \textit{ building blocks}}$$

Semiclassical expansion around the instantons

$$Z_I^{(0)}[J_\psi, J_\phi] = e^{-i\theta}\mathcal{N}\int \mathscr{D}a_\mu\,\mathscr{D}c\,\mathscr{D}\bar{c}\,\prod_f [\mathscr{D}\psi_f][\mathscr{D}\psi_f^\dagger]\,\prod_s [\mathscr{D}\phi_s][\mathscr{D}\phi_s^\dagger]\,e^{-S_E[A^{\mathrm{cl}}+a,c,\bar{c},\psi,\phi]}$$

Computed using background field methods: $A_\mu = \boxed{A_\mu^{\mathrm{cl}}} + \boxed{a_\mu}$

Instanton solution (background)

Quantum fluctuations

<u>Other fields are zero at the background level</u>

Semiclassical expansion around the instantons

$$Z_I^{(0)}[J_\psi, J_\phi] = e^{-i\theta}\mathcal{N}\int \mathscr{D}a_\mu\,\mathscr{D}c\,\mathscr{D}\bar{c}\,\prod_f[\mathscr{D}\psi_f][\mathscr{D}\psi_f^\dagger]\,\prod_s[\mathscr{D}\phi_s][\mathscr{D}\phi_s^\dagger]\,e^{-S_E[A^{\text{cl}}+a,c,\bar{c},\psi,\phi]}$$

Computed using background field methods: $A_\mu = A_\mu^{\text{cl}} + a_\mu$

$$Z_I^{(0)}[J_\psi, J_\phi] = \; e^{-i\theta}\left(\det{}_{(0)}\mathscr{M}_{\text{A}}\right)^{-1/2}\left(\mathbf{D}_A\right)^{-1/2}(\mathbf{D}_{\text{gh}})\left(\det{}_{(0)}\mathscr{M}_\psi\right)\left(\mathbf{D}_\psi\right)\left(\mathbf{D}_\phi\right)^{-1} e^{-8\pi^2/g^2(\mu)} \times e^{\text{sources}}$$

$$\left(\mathbf{D}_X\right) \equiv \left(\frac{\det'\mathscr{M}_X}{\det(\mathscr{M}_X+\mu^2)}\frac{\det(\mathscr{M}_X^0+\mu^2)}{\det\mathscr{M}_X^0}\right)$$

Semiclassical expansion around the instantons

$$Z_I^{(0)}[J_\psi, J_\phi] = e^{-i\theta}\mathcal{N}\int \mathscr{D}a_\mu\,\mathscr{D}c\,\mathscr{D}\bar{c}\,\prod_f [\mathscr{D}\psi_f][\mathscr{D}\psi_f^\dagger]\,\prod_s [\mathscr{D}\phi_s][\mathscr{D}\phi_s^\dagger]\,e^{-S_E[A^{\rm cl}+a,c,\bar{c},\psi,\phi]}$$

Computed using background field methods: $A_\mu = A_\mu^{\rm cl} + a_\mu$

$$Z_I^{(0)}[J_\psi, J_\phi] = \; e^{-i\theta}\left(\det{}_{(0)}\mathscr{M}_{\rm A}\right)^{-1/2}\left(\mathbf{D}_A\right)^{-1/2}(\mathbf{D}_{\rm gh})\left(\det{}_{(0)}\mathscr{M}_\psi\right)\left(\mathbf{D}_\psi\right)\left(\mathbf{D}_\phi\right)^{-1} e^{-8\pi^2/g^2(\mu)} \times e^{\rm sources}$$

$$\left(\prod_{i=1}^{4h_G^\vee}\frac{d\gamma^i}{\sqrt{2\pi}}\right)(\det U)^{1/2} = d^4x_0\,d\rho\,d\mu_{G/\mathscr{T}_G}(\Omega)\,\frac{2^{7-6h_G^\vee}}{\pi^{2h_G^\vee}}\,\rho^{4h_G^\vee-5}\left(\frac{8\pi^2}{g^2}\right)^{2h_G^\vee}$$

Semiclassical expansion around the instantons

$$Z_I^{(0)}[J_\psi, J_\phi] = e^{-i\theta}\mathcal{N}\int \mathscr{D}a_\mu\, \mathscr{D}c\, \mathscr{D}\bar{c}\, \prod_f [\mathscr{D}\psi_f][\mathscr{D}\psi_f^\dagger] \prod_s [\mathscr{D}\phi_s][\mathscr{D}\phi_s^\dagger]\, e^{-S_E[A^{\mathrm{cl}}+a,c,\bar{c},\psi,\phi]}$$

Computed using background field methods: $A_\mu = A_\mu^{\mathrm{cl}} + a_\mu$

$$Z_I^{(0)}[J_\psi, J_\phi] = e^{-i\theta}\left(\det{}_{(0)}\mathscr{M}_{\mathrm{A}}\right)^{-1/2}\left(\mathbf{D}_A\right)^{-1/2}(\mathbf{D}_{\mathrm{gh}})\left(\det{}_{(0)}\mathscr{M}_\psi\right)\left(\mathbf{D}_\psi\right)\left(\mathbf{D}_\phi\right)^{-1} e^{-8\pi^2/g^2(\mu)} \times e^{\mathrm{sources}}$$

$$\left(\prod_{i=1}^{4h_G^\vee}\frac{d\gamma^i}{\sqrt{2\pi}}\right)(\det U)^{1/2} = d^4x_0\, d\rho\, d\mu_{G/\mathscr{T}_G}(\Omega)\,\frac{2^{7-6h_G^\vee}}{\pi^{2h_G^\vee}}\,\rho^{4h_G^\vee-5}\left(\frac{8\pi^2}{g^2}\right)^{2h_G^\vee}$$

$$\left(\mathbf{D}_A\right)^{-1/2}(\mathbf{D}_{\mathrm{gh}})\left(\mathbf{D}_\psi\right)\left(\mathbf{D}_\phi\right)^{-1} = \rho^{-4h_G^\vee+\sum_f T_G(R_f)}\exp\left[b_0^G\ln(\mu\rho) + A_{\mathrm{tot}}\right]$$

The one-instanton generating functional

Putting the pieces together

$$Z_I^{(0)}[\{J\}] = \int d\mu_I \prod_f \left[\prod_{a=1}^{n_f} \int \frac{d\bar{\eta}_{f,a}}{\sqrt{\nu_{f,a}}} \exp\left(-\sum_{a=1}^{n_f} \bar{\eta}_{f,a} \int d^4x\, J_f^\dagger(x) \psi^\dagger_{0,f,a}(x;\Omega) \right) \right]$$

$$\times \prod_s \exp\left[-\int d^4x\, d^4y\, J_s^\dagger(x)\, D_s(x,y;\Omega)\, J_s(y) \right]$$

$$d\mu_I = e^{-i\theta} d^4x_0 \frac{d\rho}{\rho^5}\, d\mu_{G/\mathcal{T}_G}(\Omega) \frac{2^{7-6h_G^\vee}}{\pi^{2h_G^\vee}} \left(\frac{8\pi^2}{g^2(1/\rho)} \right)^{2h_G^\vee} \rho^{\sum_f T_G(R_f)} \exp\left[-\frac{8\pi^2}{g^2(1/\rho)} + A_{\text{tot}} \right]$$

Small instanton-induced axion potentials

Pablo Sesma – IPPP Durham – 25 August 2026

From one instanton to the axion potential

Why one instanton is enough?

$$e^{-\mathscr{E}(\theta)V_4} = \lim_{V_4\to\infty} \langle\theta|\, e^{-\mathscr{H}V_4}\,|\theta\rangle$$

Dilute instanton gaz approximation Callan, Dashen & Gross (1976) and 't Hooft (1976)

From one instanton to the axion potential

Why one instanton is enough?

$$e^{-\mathscr{E}(\theta)V_4} = \lim_{V_4\to\infty} \langle\theta| e^{-\mathscr{H}V_4} |\theta\rangle \xrightarrow{\text{DIGA}} \sum_{n_I,n_{\bar{I}}} \frac{(W_G^I)^{n_I}}{n_I!}\frac{(W_G^{\bar{I}})^{n_{\bar{I}}}}{n_{\bar{I}}!} = \exp\left[W_G^I + \text{h.c.}\right]$$

$$W_G^I = V_4 \mathscr{A}_I e^{-i\theta} \qquad\qquad V(a) = -\left[\mathscr{A}_I e^{-i(\theta+a/f_a)} + \text{h.c.}\right]$$

Dilute instanton gaz approximation Callan, Dashen & Gross (1976) and 't Hooft (1976)

Interactions and zero-mode saturation

Interactions are incorporated by acting with source derivatives on $Z_I^{(0)}[\{J\}]$

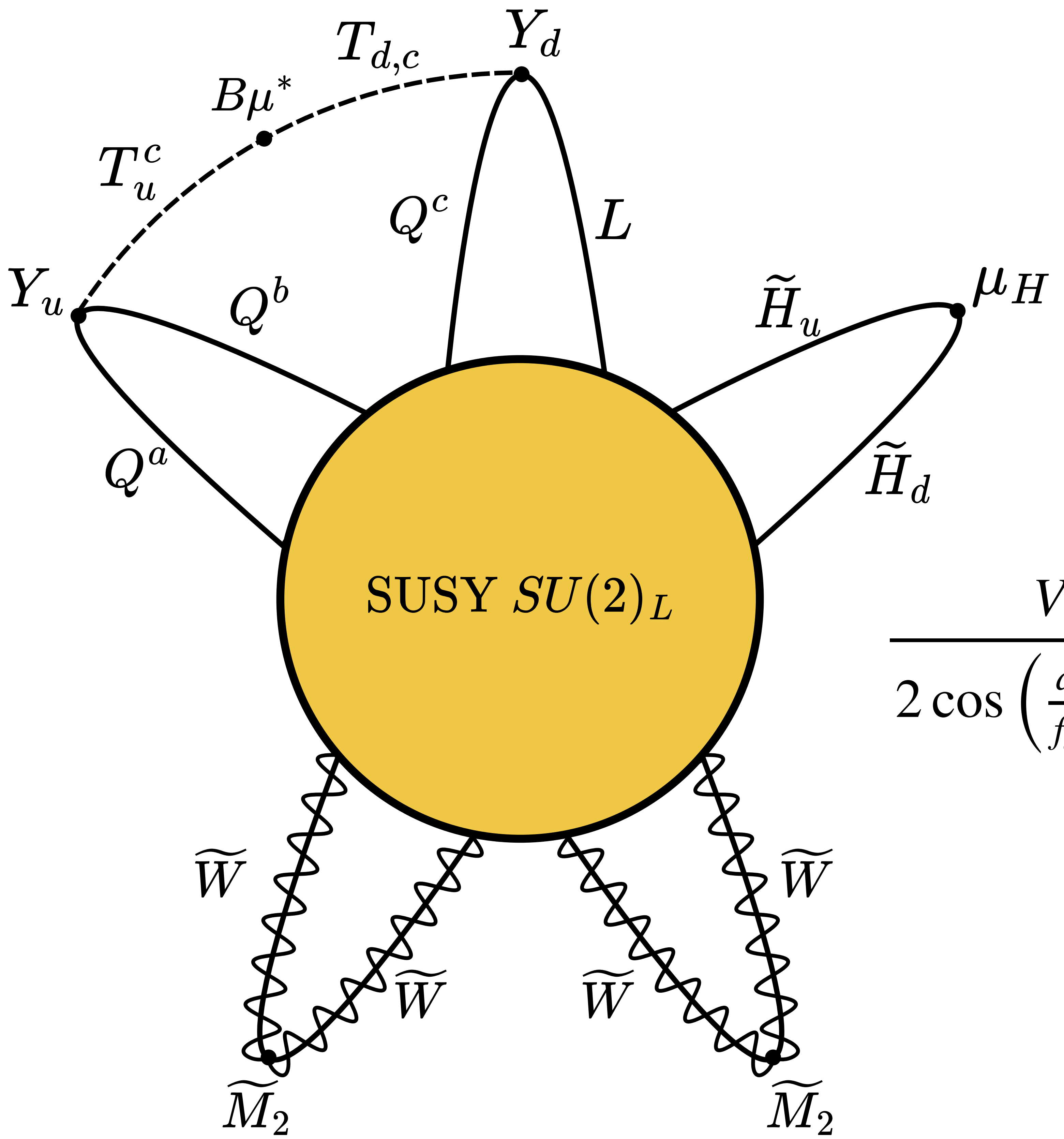
The generating functional of the interacting theory is obtained as:

$$Z_I^{\text{int}}[\{J\}] = \exp\left[-\int d^4x\ \mathscr{L}_{\text{int}}\left(\left\{\frac{\delta}{\delta J}\right\}\right)\right] Z_I^{(0)}[\{J\}]$$

$$Z_I^{(0)}[J] \propto \prod_f \left[\int \prod_{a=1}^{n_f} d\bar{\eta}_{f,a} \exp\left(-\sum_{a=1}^{n_f} \bar{\eta}_{f,a} \int d^4x\, J_f^\dagger(x) \psi_{0,f,a}^\dagger(x;\Omega)\right)\right]$$

Only interaction terms that soak up every fermion zero mode give a non-zero result

Instanton contribution from the Electroweak sector of the MSSM



$$W_G^{\text{int}} = \exp\left[-\int d^4x \, \mathscr{L}_{\text{int}}\left(\left\{\frac{\delta}{\delta J}\right\}\right)\right] Z[\{J\}]\Bigg|_{\{J\}=0}$$

$$\frac{V(a)}{2\cos\left(\frac{a}{f_a}+\theta_{\text{EW}}\right)} \supset 3^{n_g}(n_g!)\det(Y_uY_d)_{2n_g}\int_{M_{\text{GUT}}^{-1}}^{\widetilde{M}_S^{-1}}\frac{d\rho}{\rho^5}\,\delta_2(\rho)(\mu\rho)(\tilde{M}_2\rho)^2(\rho^2 B\mu)^{n_g}\left[\frac{4\rho^4}{\pi^4}\int_{x_1,x_2,x_3}\frac{D_{T_d}(x_1-x_3)D_{T_u}(x_3-x_2)}{(x_1^2+\rho^2)^3(x_2^2+\rho^2)^3}\right]^{n_g}$$

Include any interactions in a consistent way and we can work out all the $O(1)$ numbers!

Csáki, D'Agnolo, Kuflik & PS (2024)
PS (2024)

Instanton-induced proton decay

Electroweak-Flavor unification

A minimal unified theory with gauge family symmetry

Davighi & Tooby-Smith (2022)

$$G_{\text{EWFU}} = SU(4) \times Sp(6)_L \times Sp(6)_R$$

Left-handed fermion

$$\boxed{\Psi_L \sim (\mathbf{4}, \mathbf{6}, \mathbf{1})}$$

Right-handed fermion

$$\boxed{\Psi_R \sim (\mathbf{4}, \mathbf{1}, \mathbf{6})}$$

All SM fermions + 3 RH neutrinos unify into a single multiplet per chirality

Proton decay: a non-perturbative window into EWFU

The structure that forbids $QQQL$ perturbatively does not protect it from instantons

Davighi & Tooby-Smith (2022)

UV interactions contract $SU(4)$ indices through inner products

$\Downarrow$

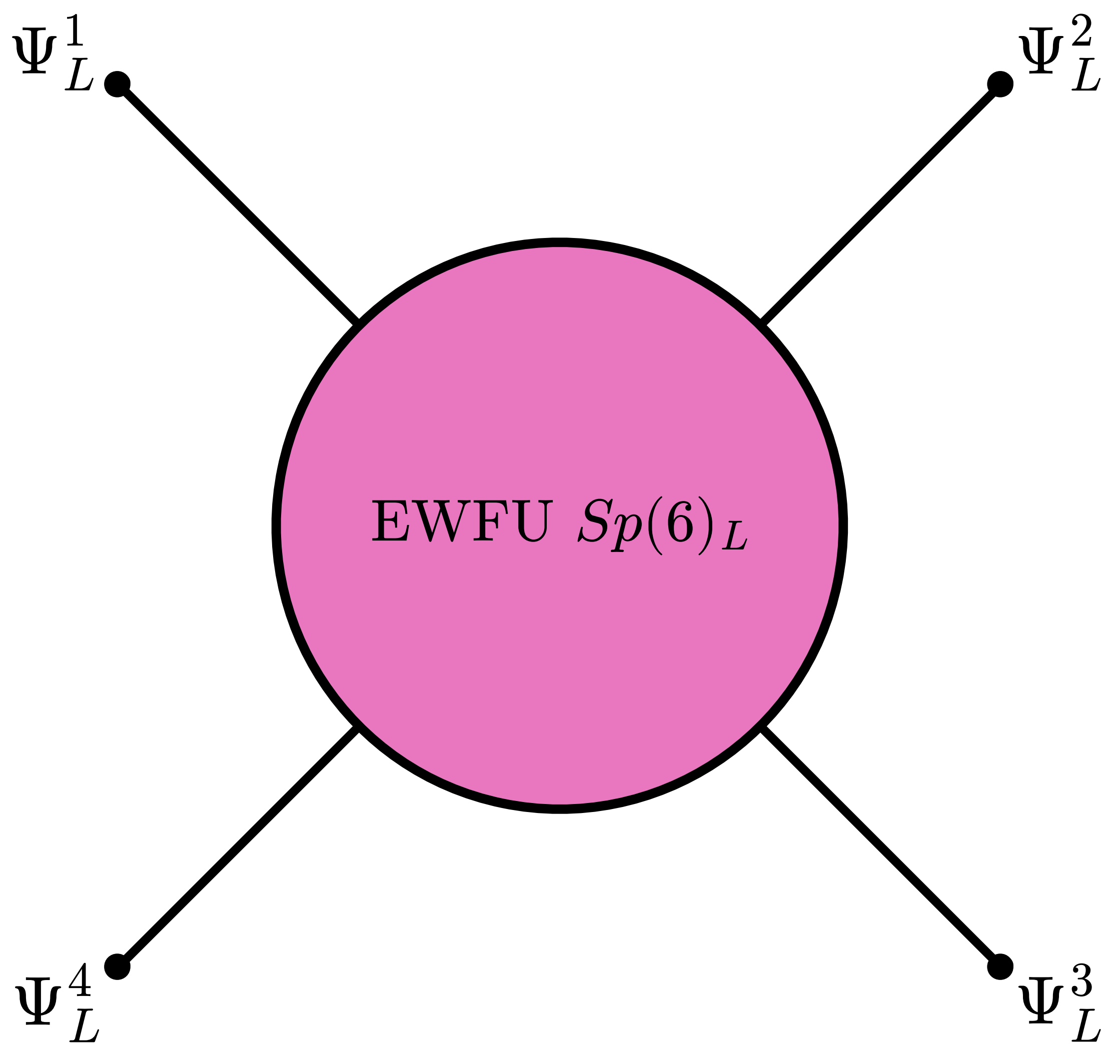
ϵ_{ABCD} never appears
and when $SU(4) \to SU(3)_c$

$\Downarrow$

ϵ_{abc} is not generated

$\Rightarrow$ no perturbative $QQQL$

Koren, Martin & PS (Soon)



$Sp(6)_L$ instanton

$\Downarrow$

$\epsilon_{ABCD}\Psi_L^A\Psi_L^B\Psi_L^C\Psi_L^D$

$\Downarrow$

$\epsilon_{abc}Q^aQ^bQ^cL$

No perturbative decay $\Rightarrow$ Λ_{EWFU} can be low $\Rightarrow$ instantons becomes a sensitive probe

Electroweak-Flavor unification

Where do instantons appear in EWFU?

Davighi & Tooby-Smith (2022)

$$SU(4) \times Sp(6)_L \times Sp(6)_R$$

$\Lambda_L,\ \Lambda_R$ — $\langle S_L \rangle \sim (\mathbf{1}, \mathbf{14}, \mathbf{1})$ — $\langle S_R \rangle \sim (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{6})$

Λ_H — [Heavy Higgses integrated out]

$$SU(3) \times \prod_i SU(2)_{L,i} \times Sp(4)_{R,12} \times U(1)_R$$

$\epsilon\Lambda_H$ — $\langle \Phi_L \rangle \sim (\mathbf{1}, \mathbf{14}, \mathbf{1})$ — $\langle \Phi_R \rangle \sim (\mathbf{1}, \mathbf{1}, \mathbf{14})$

$$SU(3) \times SU(2)_L \times U(1)_Y$$

v — $\langle H_1 \rangle \sim (\mathbf{1}, \mathbf{6}, \mathbf{6})$ — $\langle H_{15} \rangle \sim (\mathbf{15}, \mathbf{6}, \mathbf{6})$

$$SU(3) \times U(1)_{\text{em}}$$

Electroweak-Flavor unification

Where do instantons appear in EWFU?

Davighi & Tooby-Smith (2022)

$SU(4) \times Sp(6)_L \times Sp(6)_R$

$\Lambda_L,\ \Lambda_R$ — $\langle S_L \rangle \sim (\mathbf{1}, \mathbf{14}, \mathbf{1})$ — $\langle S_R \rangle \sim (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{6})$

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$SU(3) \times \prod_i SU(2)_{L,i} \times Sp(4)_{R,12} \times U(1)_R$

$\epsilon\Lambda_H$ — $\langle \Phi_L \rangle \sim (\mathbf{1}, \mathbf{14}, \mathbf{1})$ — $\langle \Phi_R \rangle \sim (\mathbf{1}, \mathbf{1}, \mathbf{14})$

$SU(3) \times SU(2)_L \times U(1)_Y$

v — $\langle H_1 \rangle \sim (\mathbf{1}, \mathbf{6}, \mathbf{6})$ — $\langle H_{15} \rangle \sim (\mathbf{15}, \mathbf{6}, \mathbf{6})$

$SU(3) \times U(1)_{\text{em}}$

$$Sp(6)_L \to SU(2)_{L,1} \times SU(2)_{L,2} \times SU(2)_{L,3} \to SU(2)_L$$

Ordinary instantons:

$\pi_3(G) \neq 1$, for every compact simple factor here $\pi_3(G) = \mathbb{Z}$

Instantons in the broken directions:

for $G \to H$, $\quad \pi_3(G/H) \cong \dfrac{\pi_3(G)}{\mathrm{Im}\left[\pi_3(H) \to \pi_3(G)\right]}$ (when $\pi_2(H) = 1$)

Csáki & Murayama (1998)

*If $\pi_3(G/H) \neq 1$ parent instantons exist that cannot be rotated entirely into H: **constrained instantons***

Electroweak-Flavor unification

Where do instantons appear in EWFU?

$$Sp(6)_L \to SU(2)_{L,1} \times SU(2)_{L,2} \times SU(2)_{L,3} \to SU(2)_L$$

Instanton sectors relevant for proton decay

- $Sp(6)_L$ instantons: $\pi_3(Sp(6)) = \mathbb{Z}$ and $\Psi_L \sim (\mathbf{4}, \mathbf{6}, \mathbf{1})$ has four zero modes
- $SU(2)_{L,i}$ instantons in the family-resolved phase: for $i = 1,2,3$ $\pi_3\left(SU(2)_{L,i}\right) = \mathbb{Z}$
- $SU(2)^3/SU(2)_D$ constrained instantons: $\pi_3\left(SU(2)^3/SU(2)_D\right) = \mathbb{Z}^3/\mathbb{Z}_{\text{diag}} \cong \mathbb{Z}^2$
  Can be parametrically enhanced (just as for the axion mass of **Csáki, Ruhdorfer & Shirman (2019)**)

Primitive fermionic correlator

Any anomalous amplitude is obtained by saturating the fermion zero modes

For each LH Weyl fermion $\psi_f \in (R_f, S_f)$ under $G_{\text{inst}} \times G_{\text{spec}}$ there are

$n_f = 2T_{G_{\text{inst}}}(R_f) \times \dim(S_f)$ zero modes

$$\mathcal{G}_I\left[\{x_{f,r}, A_{f,r}\}\right] \equiv \left\langle \prod_f \prod_{r=1}^{n_f} \psi_f^{\dagger A_{f,r}}(x_{f,r}) \right\rangle_I = \int d\mu_I \prod_f \det_{1\leq r,a\leq n_f}\left[\psi_{0,f,a}^{\dagger A_{r,f}}(x_{f,r};\rho,\Omega)\right]$$

Koren, Martin & PS (Soon)

Electroweak flavor unification

The $Sp(6)_L$ instanton generates a $QQQL$ operator

$$\Delta\mathscr{L}^{(I)}_{\text{eff}} = C_I \mathcal{O}_{Sp(6)} + C_{\bar{I}} \mathcal{O}^{\dagger}_{Sp(6)}$$

$$\mathcal{O}^{\dagger}_{Sp(6)} = \frac{1}{2}\epsilon_{ijk}\left(\sum_{f=1}^{3}\epsilon_{\alpha\beta}\epsilon_{rs}Q_f^{ir\alpha}Q_f^{js\beta}\right)\left(\sum_{g=1}^{3}\epsilon_{\gamma\delta}\epsilon_{tu}Q_g^{kt\gamma}L_g^{u\delta}\right)$$

$$\equiv \sum_{f,g=1}^{3}\mathcal{O}_{fg} \supset QQQL$$

Wilson coefficient

$$C_I = e^{-i\theta-\alpha(1)}\frac{\pi^2}{630}\int d\rho\ \rho\left(\frac{8\pi^2}{g^2_{Sp(6)}(1/\rho)}\right)^8 \exp\left(-\frac{8\pi^2}{g^2_{Sp(6)}(1/\rho)}\right)$$

Primitive correlator + LSZ reduction + matching determines C_I

Koren, Martin & PS (Soon)

Electroweak flavor unification

Why are those constrained instantons enhanced?

$$SU(2)_{L,1} \times SU(2)_{L,2} \times SU(2)_{L,3} \to SU(2)_L$$

At diagonal breaking matching scale: $\frac{1}{g_L^2} = \frac{1}{g_1^2} + \frac{1}{g_2^2} + \frac{1}{g_3^2}$, $\quad g_1 = g_2 = g_3 \equiv g \;\Rightarrow\; g^2 = 3g_L^2$

Ordinary diagonal $SU(2)_L$ instantons $(1,1,1)$

$$S_D(\rho) = \frac{8\pi^2}{g_L^2(1/\rho)}$$

Amplitude $\propto \exp\left[-\frac{8\pi^2}{g_L^2(1/\rho)}\right]$

One-factor constrained sector

$(1,0,0)$, $(0,1,0)$, $(0,0,1)$

$$S_i(\rho) = \frac{8\pi^2}{g_i^2(1/\rho)} + \pi^2 V_i^2 \rho^2$$

$V_1^2 = v_{12}^2$, $\quad V_2^2 = v_{12}^2 + v_{23}^2$, $\quad V_3^2 = v_{23}^2$

Smaller gauge action $\Rightarrow$ less exponential suppression!

Koren, Martin & PS (Soon)

Electroweak flavor unification

$SU(2)^3/SU(2)$ ***constrained instanton: Wilson coefficient***

$$\Delta\mathscr{L}_{\text{eff}}^{(I)} = \sum_{i=1}^{3}\left[C_{\bar{I},i}\mathcal{O}_{ii} + C_{I,i}\mathcal{O}_{ii}^{\dagger}\right]\,, \qquad \mathcal{O}_{ii} \supset Q_iQ_iQ_iL_i$$

$$\boxed{C_{\bar{I},i} = 4\pi^2 e^{i\theta_i} K_{\alpha,i}\int d\rho\;\rho\left(\frac{8\pi^2}{g_i^2(1/\rho)}\right)^4 \exp\left[-\frac{8\pi^2}{g_i^2(1/\rho)} - \pi^2 V_i^2\rho^2\right]}$$

Higgs scales

$$V_1^2 = v_{12}^2$$
$$V_2^2 = v_{12}^2 + v_{23}^2$$
$$V_3^2 = v_{23}^2$$

Finite-pieces

$$K_{\alpha,1} = K_{\alpha,2} = \exp\left[-\alpha(1) + 2\alpha(1/2)\right] \simeq 0.86$$

$$K_{\alpha,3} = \exp\left[-\alpha(1) - 2\alpha(1/2)\right] \simeq 0.48$$

Koren, Martin & PS (Soon)

Electroweak flavor unification

$SU(2)^3/SU(2)$ *constrained instanton: Wilson coefficient*

Operator structure

Constrained sectors generate the same dimension-six structure

$\mathcal{O}_{ii} \supset Q_iQ_iQ_iL_i$

Primitive correlator + LSZ + matching $\Rightarrow C_{I,i}$

Semiclassical weight

Compared with the ordinary diagonal instanton

$S_i^{\text{gauge}} < S_D^{\text{gauge}}$

Smaller action $\Rightarrow$ less exponential suppression

Phenomenological message

- EWFU is perturbatively proton-stable
- Instantons reintroduce proton decay nonperturbatively
- Proton decay potentially becomes a sensitive probe of the EWFU symmetry-breaking sector

Koren, Martin & PS (Soon)

Conclusion

What we have achieved

- A general one-instanton framework for physics based on $G_{\text{inst}} \times G_{\text{spec}}$ with arbitrary matter representations
- Axion potentials, anomalous correlators, and Wilson coefficients are different projections of the same object $Z_I[\{J\}]$
- In the MSSM electroweak sector, the formalism reproduces small instanton-induced axion potentials with all $O(1)$ factors under control
- In EWFU, proton decay is absent perturbatively but generated nonperturbatively by $Sp(6)_L$ and enhanced $SU(2)^3/SU(2)$ constrained instantons

Thank you :)