Supercooling and Conformality in Confining Transitions

Effective Theories for Nonperturbative Physics, Durham

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Thoughts based on various works with
Prateek Agrawal (UCSB), Vazha Loladze (Oxford→ Maryland),
John March-Russell (Oxford) and Mario Reig (CERN)

arXiv: 2608.13557 [hep-th]
arXiv: 2508.10091 [hep-ph]
arXiv: 2504.00199 [hep-ph]

Flow of today's discussion

- Motivation
- Defining supercooling
- $\mathcal{N} = 4$ SYM on $S^3 \times S^1$
- Near-conformal confining theories
- Far-from-conformal theories: confinement in $SU(N)$
- A general holographic argument
- Concluding Remarks

Motivation

Phase transitions in strongly coupled gauge theories has been an active research area — still lack complete understanding of phase transition dynamics in these theories.

- ⋆ One such transition of interest is thermal deconfinement-confinement transition
- ⋆ Complementary approach of Lattice+SUSY+holography+EFTs needed to get the full picture

Motivation

In terms of phenomenology, in beyond the standard model of particle physics, first order phase transitions can act as source of gravitational waves.

- ★ Gravitational Wave detection— LISA— near Electroweak scale
- ★ Chance to study transitions in various Beyond the Standard Model (BSM) physics

LISA Cosmology Working Group, JCAP 10 (2024) 020, C.Grojean, G. Servant, Phys.Rev. D 75 (2207) 043507; ...

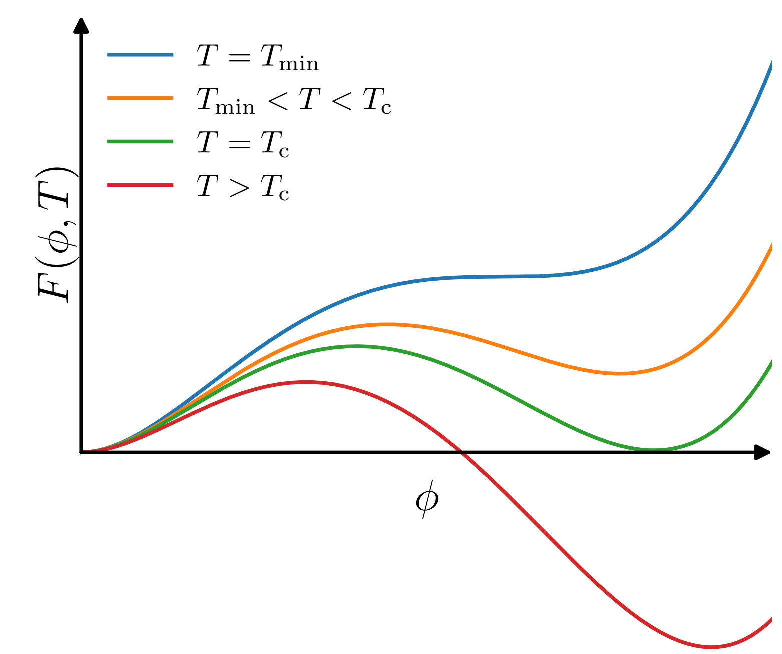
- ★ Many BSM models have strongly coupled theories going under phase transition— Solutions to electroweak hierarchy problem, Randall-Sundrum models and its string-inspired deformations, composite dark matter models etc...

Defining supercooling

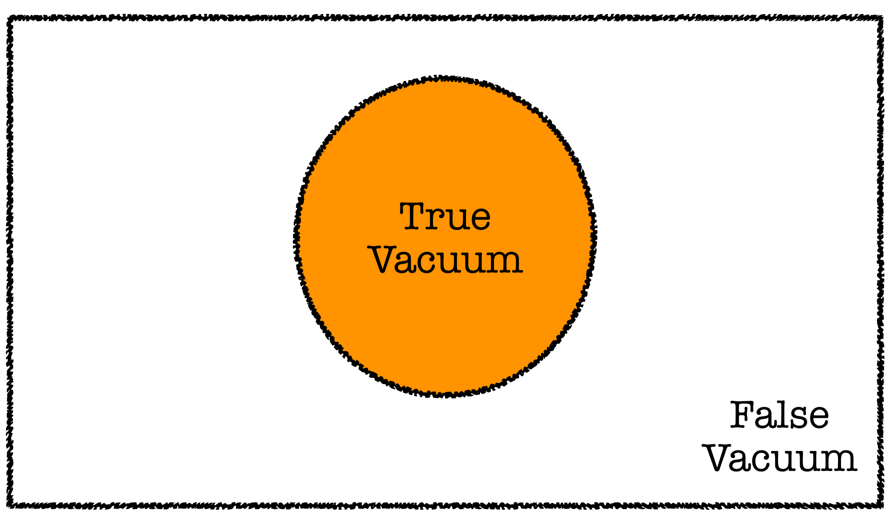
- Nucleation rate of the bubbles is given by

$$\Gamma_{\mathrm{nuc}} \sim e^{-S_{\mathrm{b}}} \longrightarrow \text{Euclidean Bounce}$$

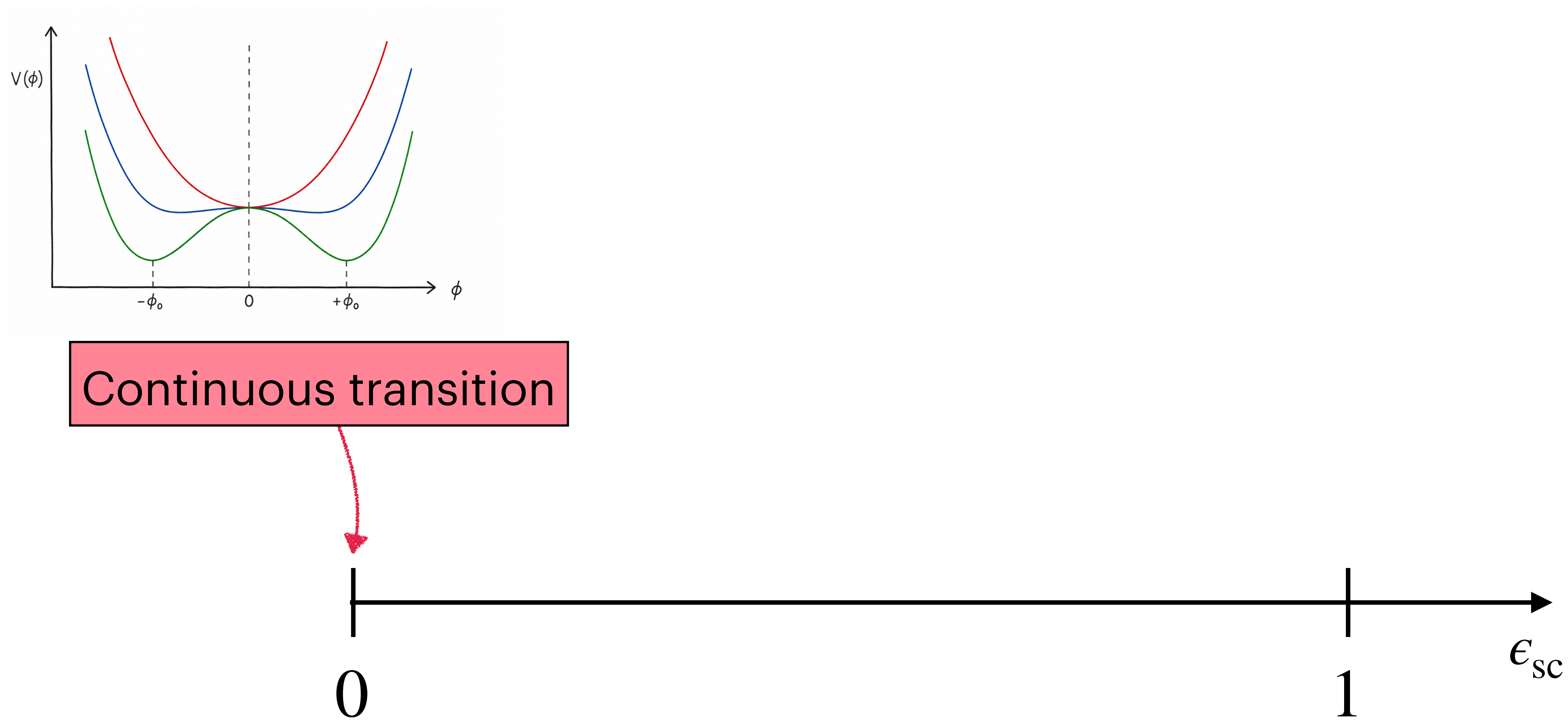
- Thermal channel— $O(3)$ symmetric
- Near critical temperature T_{c}



$$S_{\mathrm{b}} \propto \frac{1}{T_{\mathrm{c}}}\left(1 - \frac{T}{T_{\mathrm{c}}}\right)^{-2}, \quad \epsilon = 1 - \frac{T}{T_{\mathrm{c}}}$$



The limiting cases



Supercooling is bounded above by

$$\epsilon_{sc} = 1 - \frac{T_{min}}{T_c}$$

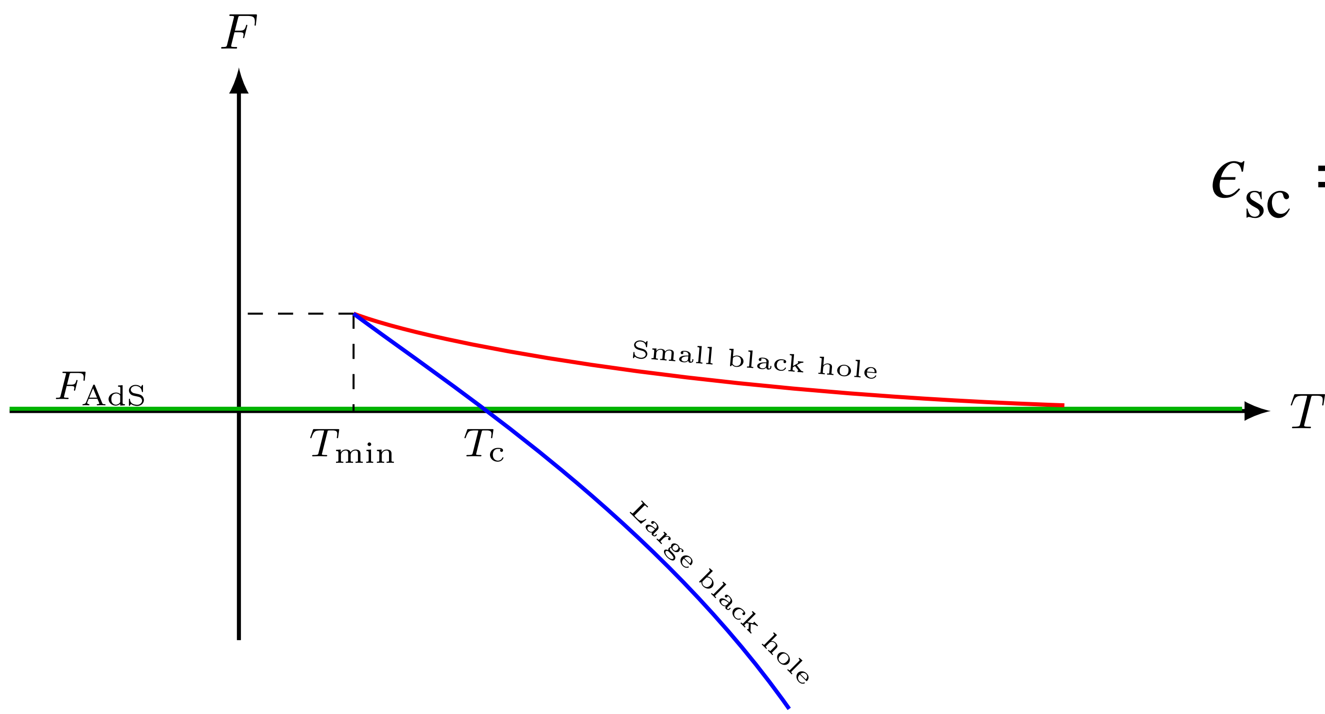
Hawking— Page transition with AdS$_{D+1}$ and $S^{D-1} \times S^1$ boundary

- A well established holographic setup is AdS$_5$ with $\mathcal{N} = 4$ SYM on the boundary which is a CFT.

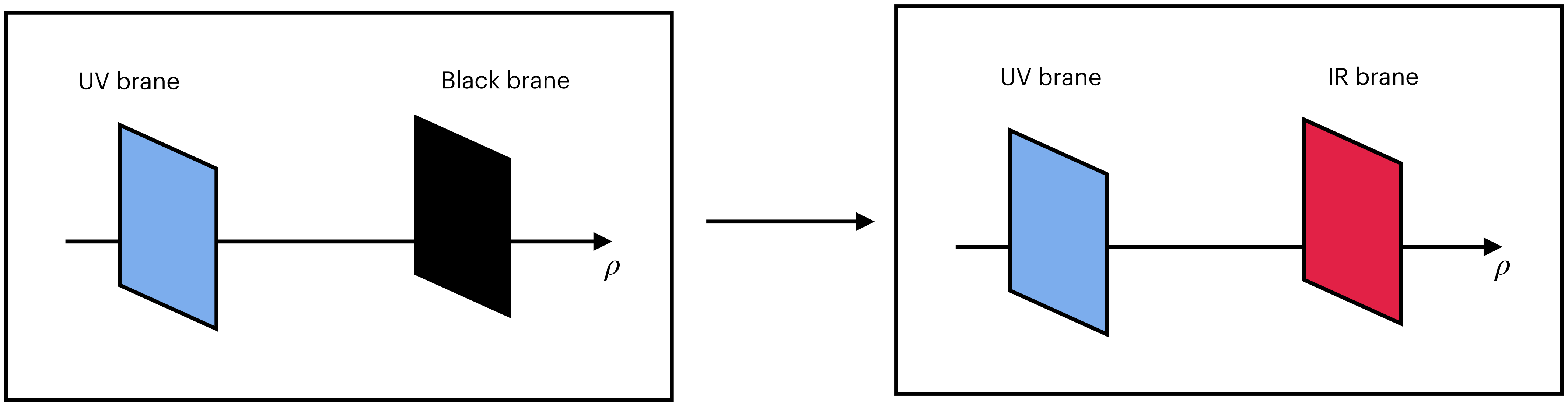
- When on the boundary $\mathbb{R}^3 \to S^3$ a scale is generated which is ratio of the radii of the two sphere that explicitly breaks CFT geometrically— allows for phase transitions. (Witten hep-th/9803131)

$$T_c = \frac{D-1}{2\pi L}\,, \; T_{\min} = \frac{\sqrt{D(D-2)}}{2\pi L}$$

$$\epsilon_{sc} = 1 - \frac{T_{\min}}{T_c} = 1 - \frac{\sqrt{D(D-2)}}{D-1} \approx \frac{1}{2D^2}$$



Near-conformal confining theories

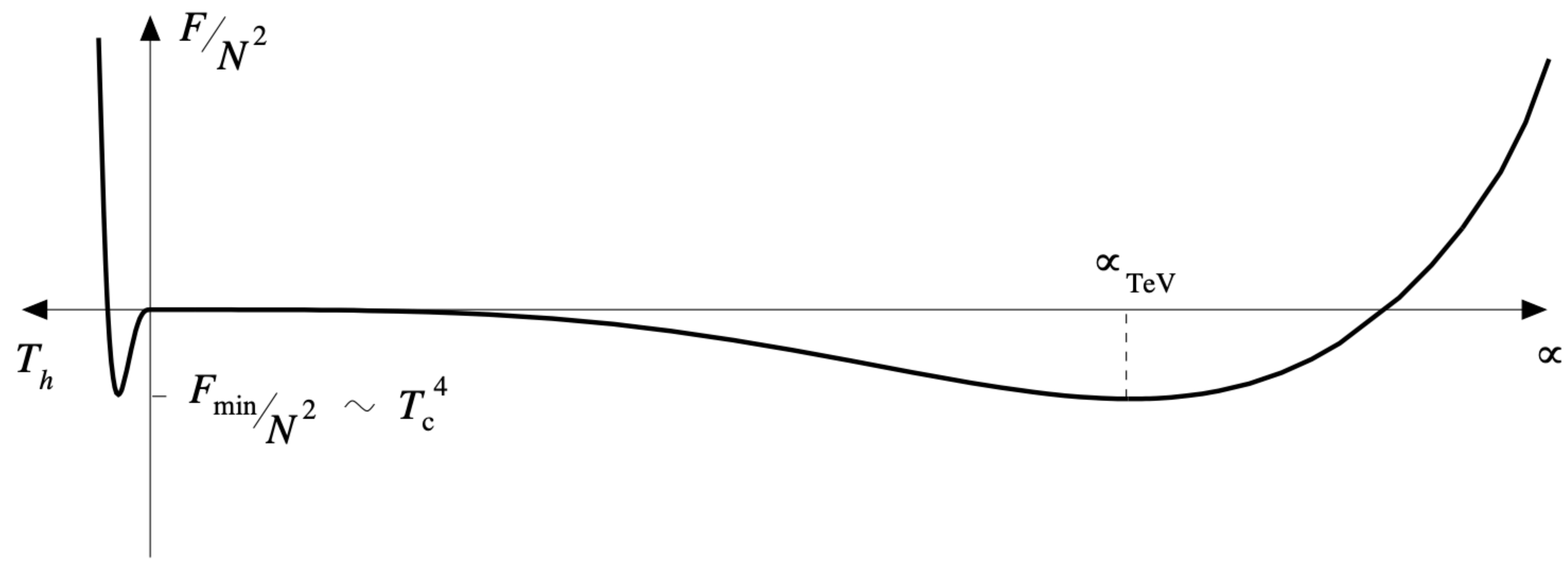


- Presence of a pseudo-Nambu-Goldstone mode
- Can be studied using Holographic setup of Randall-Sundrum model
- Conformal symmetry breaking:
    - Fine-tuned IR brane: spontaneous (no radion potential)
    - Stabilisation: weak explicit ($\lambda(\phi)\phi^4$ type potential)

P.Creminelli, A.Nicolis, and R..Rattazzi, hep-th/0107141

F.Coradeschi, P.Lodone, D.Pappadopulo, R.Rattazzi, L.Vitale arXiv: 1306.4601

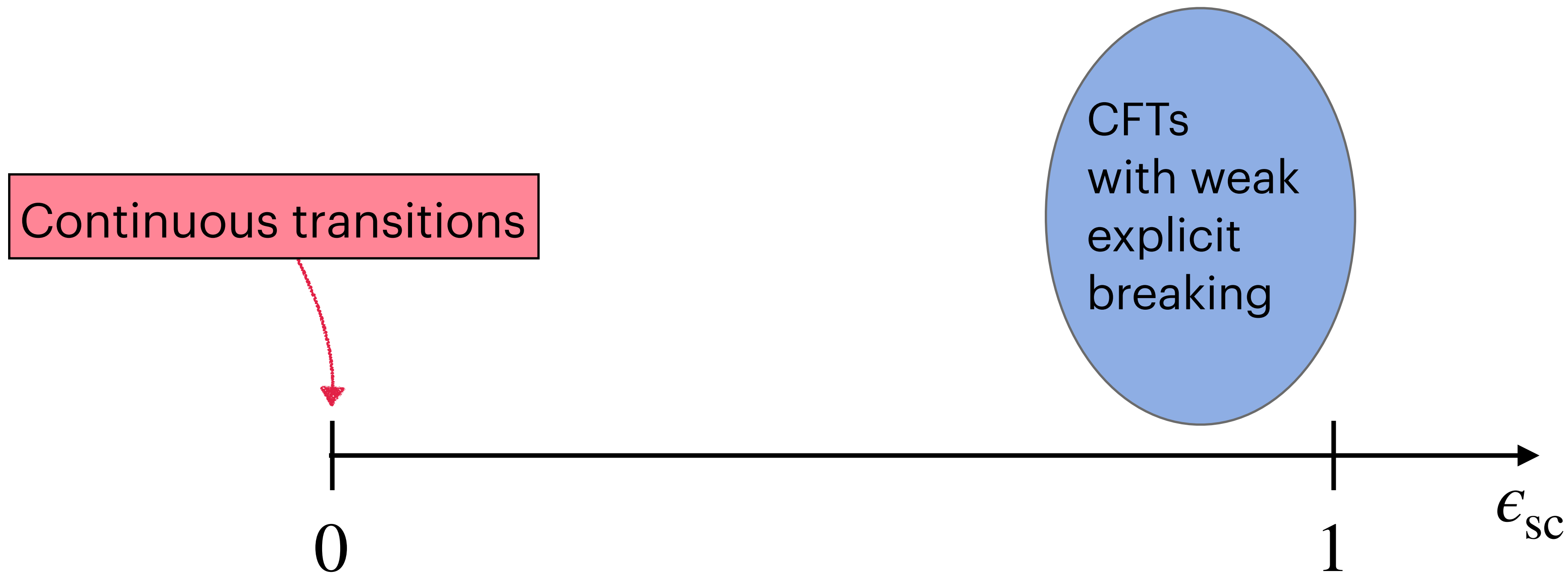
Near-conformal theories



- No back-reaction of the scalar field in the black brane phase
- Large maximum supercooling (close to $\epsilon \approx 1$) possible

[1] P.Creminelli, A.Nicolis, and R..Rattazzi, hep-th/0107141
[2] L.Randall and G.Servant, hep-ph/0607158
[3] B.Hassanain, J. March-Russell, and M.Schvellinger, 0708.2060
[5] P.Baratella, A.Pomarol, and F.Rompineve, 1812.06996
[6] K.Agashe, P.Du, M. Ekhterachian et.al., 1910.06238
[7] K.Agashe, P.Du, M. Ekhterachian et.al., 2010.04083

[1] P.Agrawal, GRK, V.Loladze, and M.Reig, 2504.00199
[2] K.Fujikura, S.Nakagawa, Y.Nakai, et.al., 2503.22293


Continuous transitions
CFTs
with weak
explicit
breaking
0
1
$\epsilon_{\rm sc}$

$SU(N)$ Thermal Yang Mills Theory

Based on PRL 136, 041902, with PA, VL, and JMR

For $N \geq 3$ it is first order phase transition with Polyakov loop Ω as the order parameter

$$\Omega(x) = \frac{1}{N}\mathrm{Tr}\left(\mathscr{P}\exp\left(i\oint A_4(x,x_4)\,dx_4\right)\right) = \frac{1}{N}\mathrm{Tr}\exp\left(iA_4(x)L\right)$$

Effective $\mathbb{Z}_N^{(0)}$ symmetry

Deconfined $\langle\Omega\rangle \neq 0$

Confined $\langle\Omega\rangle = 0$

Lattice Data

Calculate thermodynamic quantities near $T = T_{\mathrm{cr}}$

[1] B. Lucini, M. Teper, and U. Wenger, JHEP 02 (2005) 033,
[2] A. Salami, T. Rindlisbacher, and K. Rummukainen, 2502.01396 [hep-lat]

Latent Heat: $Q_{\mathrm{h}} = \left(0.7731 - 0.959(58)/N^2\right)^4 N^2 T_{\mathrm{cr}}^4$, Wall tension: $\sigma_{\mathrm{BW}} = \left(0.0189(11) - 0.190(19)/N^2\right) N^2 T_{\mathrm{cr}}^3$

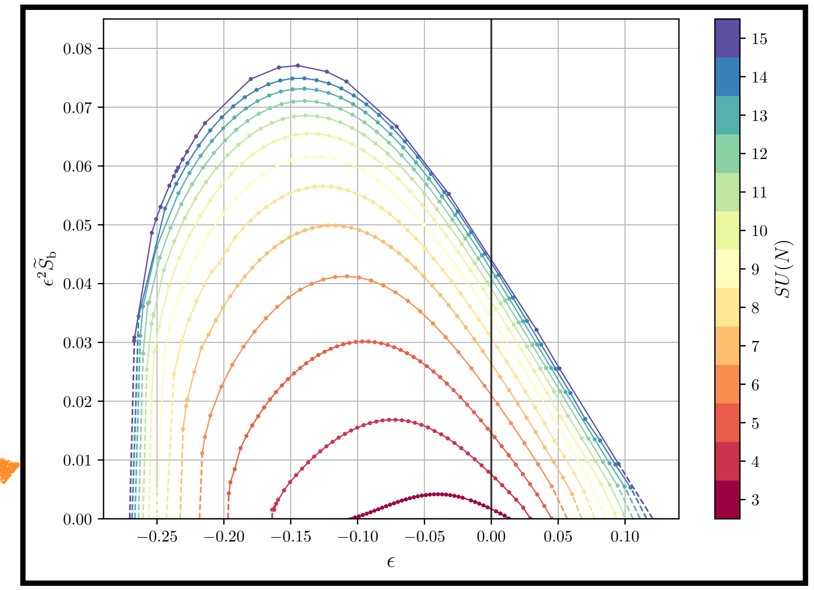
I.Garcia Garcia, R. Lasenby, and J.March-Russell, 1505.07109

$SU(N)$ thermal Yang—Mills theory

Go to supersymmetric avatar of the theory on $\mathbb{R}^3 \times S^1$ to study the transition analytically and get qualitative insights

Poppitz, Schafer, Unsal, 1205.0290, 1212.1238

$$S_{\rm b} = -\frac{\Lambda^3 N^2}{\sigma_{\rm str}^2 L_{\rm cr}} \underbrace{h(\epsilon, N) \frac{(\epsilon - \epsilon_{\rm sc})(\epsilon - \epsilon_{\rm sh})}{\epsilon^2}}_{\widetilde{S}_{\rm b}}$$

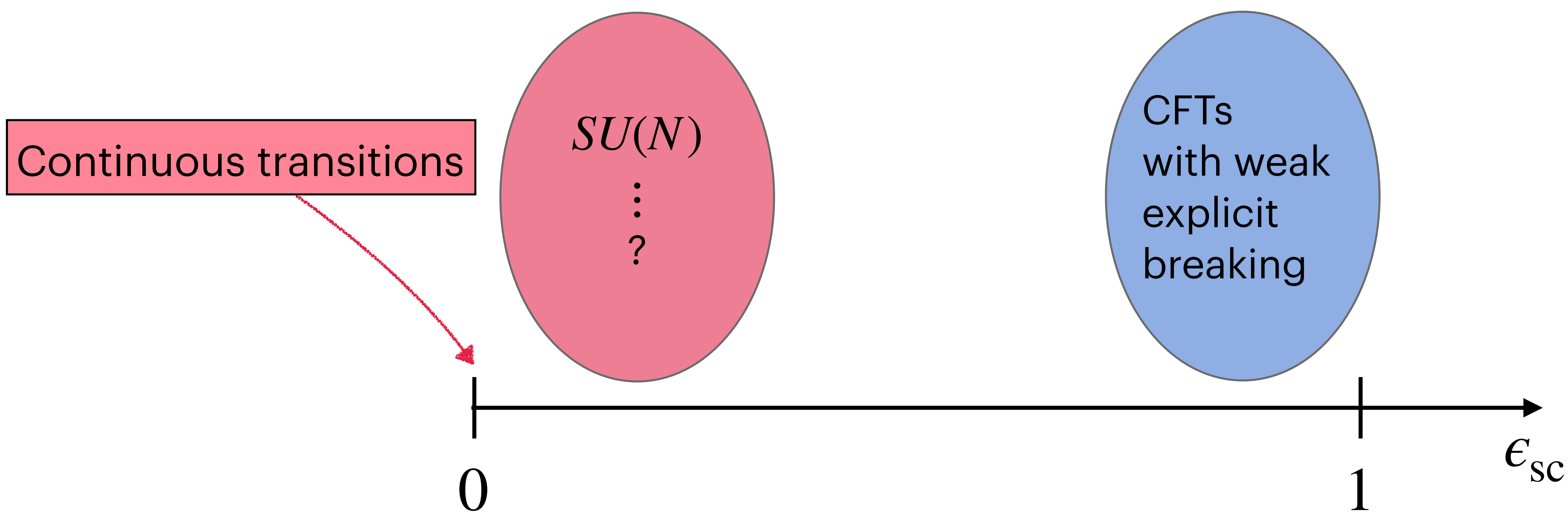


Lattice gives us data near the critical temperature— this corresponds to $\epsilon \to 0$ limit.

$$\lim_{\epsilon \to 0} \epsilon^2 S_{\rm b}^{\rm YM} \sim -\frac{T_{\rm cr}^4 N^2}{\sigma_{\rm str}^2} \epsilon_{\rm sc}^{\rm YM} \epsilon_{\rm sh}^{\rm YM} \sim \frac{16\pi}{3} \frac{\sigma_{\rm BW}^3}{Q_{\rm h}^2 T_{\rm cr}}$$

$$\epsilon_{\rm sc}^{\rm YM} \lesssim 0.08$$

Recent progress in lattice simulations show encouraging progress in this direction
K. Rummukainen, R. Seppa, D. Weir, 2603.22088 [hep-lat]



Question: Can we generalise this?

Answer: Yes!

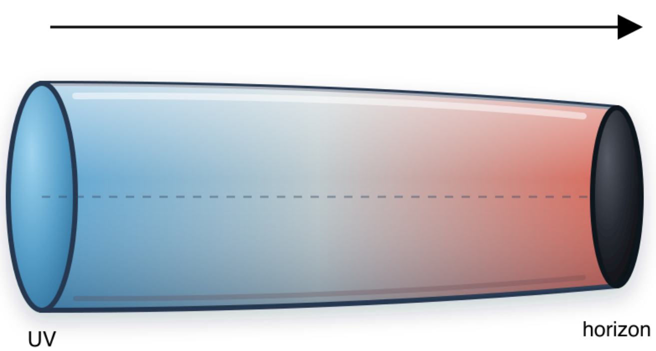
General holographic argument

Based on 2608.13557 [hep-th]
with PA, VL

Einstein-Scalar gravity $\longleftrightarrow$ Confining Gauge Theories

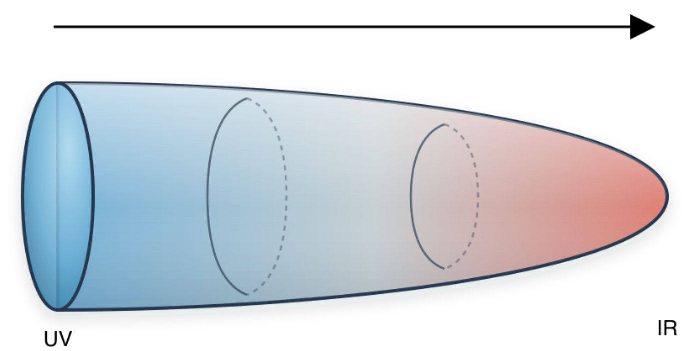
(Large-N, ’t Hooft limit)

1. Asymptotically AdS
2. Deformations of AdS in the bulk



Hawking-Page transition

$\longleftrightarrow$



Black brane phase

Dual to deconfined phase

Deformed AdS phase

Dual to confined phase

The system

Action

$$S_{\text{bulk}} = -\frac{M_{D+1}^{D-1}}{2}\int d^{D+1}x\sqrt{g}\left(R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right)$$

Metric Ansatz

$$ds^2 = e^{A(u)}\left(f(u)dt^2 + \delta_{ij}dx^i dx^j\right) + \frac{du^2}{f(u)}$$

Equations of motion

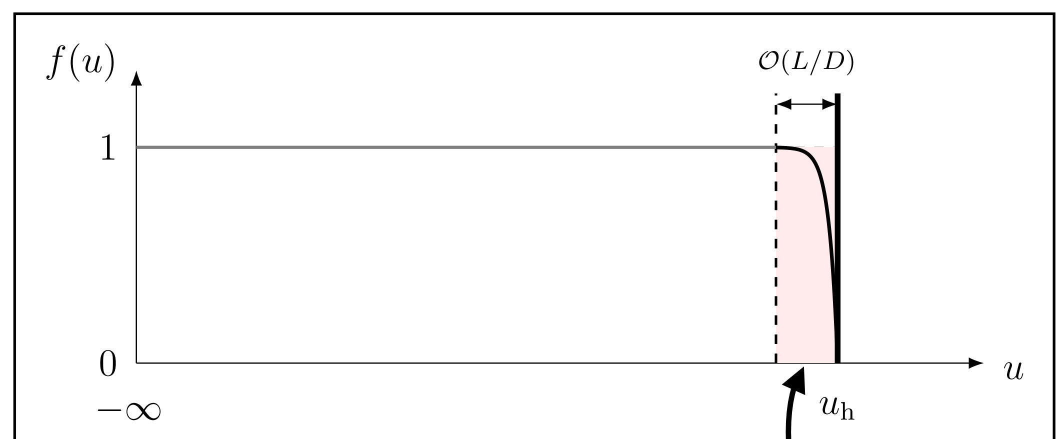
$$\ddot{A} + \frac{1}{2(D-1)}\dot{\phi}^2 = 0$$

$$\frac{\ddot{f}}{\dot{f}} + D\dot{A} = 0$$

$$f\ddot{\phi} + \dot{f}\dot{\phi} + Df\dot{A}\dot{\phi} = \frac{dV}{d\phi}$$

Black brane in *absence* of the scalar field

$$A(u) = -\frac{u}{L}\ ,\ f(u) = 1 - \exp\left(\frac{D}{L}(u - u_{\text{h}})\right)$$



Boundary layer for $D \gg 1$

The superpotential in vacuum

For a single scalar field, one can express the potential $V(\phi)$ in terms of an *auxiliary function* $W(\phi)$ (also known as super-potential). In this case, the vacuum solution is

P.K.Townsend
Phys.Lett.B 148,1984

DeWolfe, Freedman,Gubser,
and Karch hep-th/9909134

$$\boxed{\begin{gathered} \dot{A} = -W \,, \; \dot{\phi} = 2(D-1)W' \\ V = 2(D-1)^2\left(W'^2 - \frac{D}{2(D-1)}W^2\right) \end{gathered}}$$

Presence of black brane makes it harder to solve for geometry and superpotential is not effective.

Localising the horizon

- Define effective curvature scale $L_{\rm eff} = -\dot{A}^{-1}$
- Scale separation between $L_{\rm eff}$ and $L_{\rm eff}/D$ such that

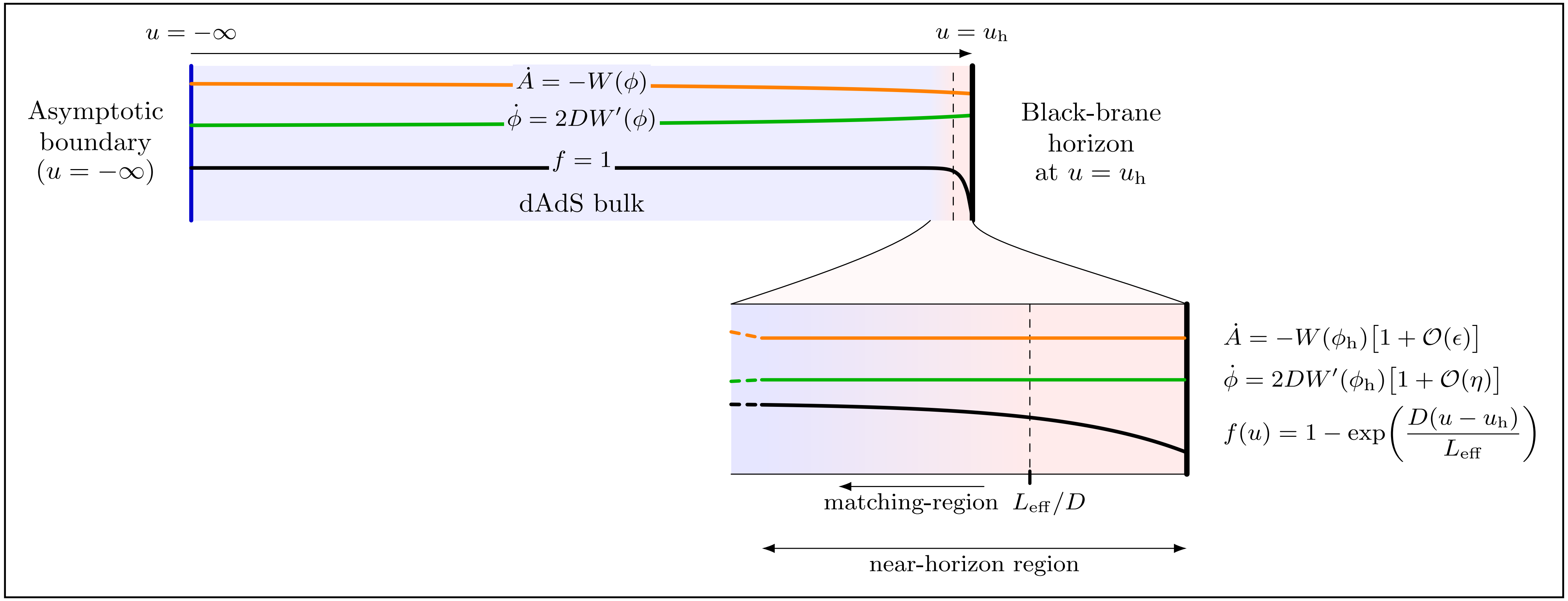
$$\frac{\ddot{f}}{\dot{f}} + D\dot{A} = 0$$

$$f(u) = 1 - \exp\left(\frac{D}{L_{\rm eff}}(u - u_{\rm h})\right)$$

$$\left.\begin{aligned} \frac{dL_{\rm eff}}{du} \ll D &\Rightarrow \left(\frac{W'}{W}\right)^2 \ll \frac{D}{2(D-1)} \\ \left|\frac{\ddot{\phi}L_{\rm eff}}{D}\right| \ll \left|\dot{\phi}\right| &\Rightarrow \frac{W''}{W} \ll \frac{D}{2(D-1)} \end{aligned}\right\} \text{Slow-roll Conditions}$$

Controlled setup for adiabatic approximation

Gubser, Nellore, 0804.0434



Thermodynamics— Temperature

Temperature can be expressed as a function of value of scalar field at the horizon.

$$T = \frac{De^{A(u_\mathrm{h})}}{4\pi}\,|\,\dot{f}(u_\mathrm{h})\,| = \frac{D}{4\pi}\exp\left(\left(-\frac{1}{2D}\int^{\phi_\mathrm{h}} d\phi \frac{W}{W'}\right)\right)|\,W(\phi_\mathrm{h})\,|$$

$$\left(\frac{W'(\phi_\mathrm{min})}{W(\phi_\mathrm{min})}\right)^2 = \frac{1}{2D} \qquad\qquad \frac{W''(\phi_\mathrm{min})}{W(\phi_\mathrm{min})} = \frac{1+\delta}{2D}\,,\ \frac{1}{D} \ll \delta \ll D$$

Thermodynamics— Free Energy

Require free energy difference with respect to dAdS spacetime to calculate critical temperature.

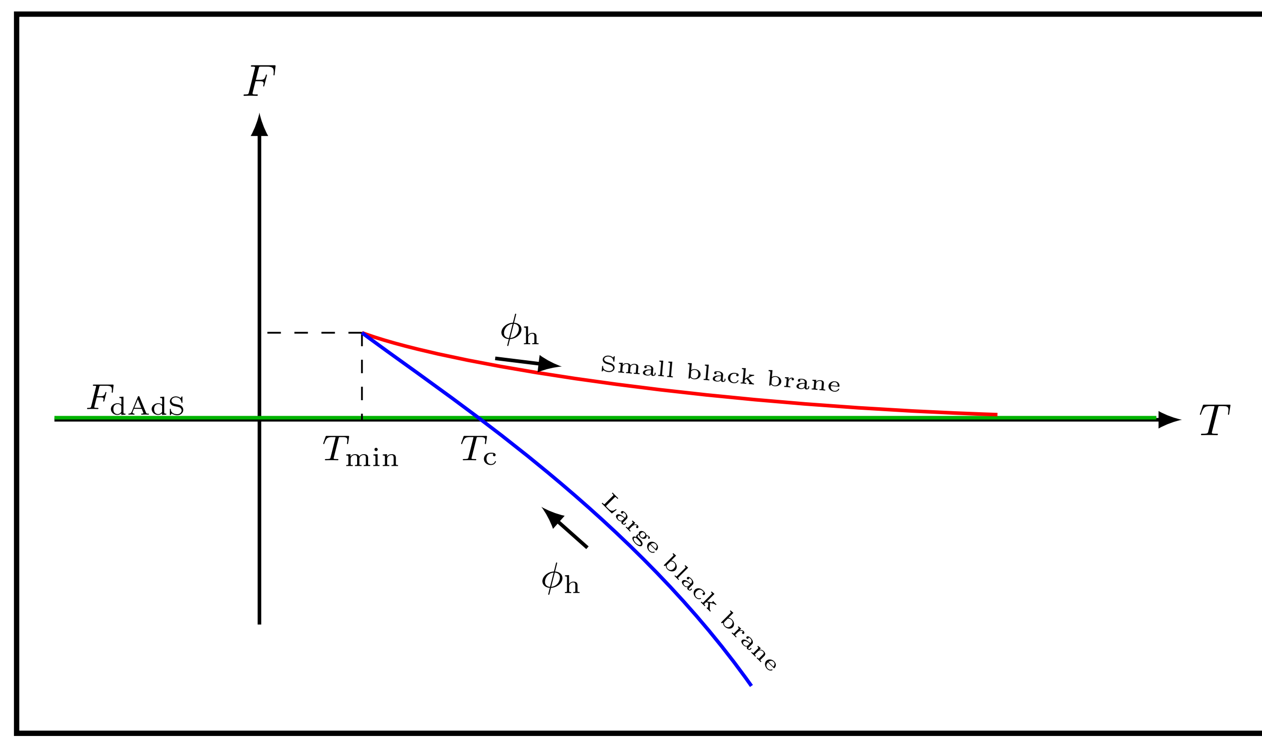
$$\frac{\Delta F}{2\pi M_{\mathrm{pl}}^{D-1} V_{D-1}} = -\int_{\infty}^{\phi_{\mathrm{h}}} e^{(D-1)A(\widetilde{\phi}_{\mathrm{h}})} \frac{dT}{d\widetilde{\phi}_{\mathrm{h}}}\, d\widetilde{\phi}_{\mathrm{h}} \quad .$$

$\Delta F(\phi_{\mathrm{h}} = \phi_{\mathrm{c}}) = 0$ defines critical temperature

In the large-D limit, the exponential in the integral localises the integral near ϕ_{min}

$$\frac{\phi_{\mathrm{min}} - \phi_{\mathrm{c}}}{\sqrt{2D}} = \frac{1}{D}$$

$$\epsilon_{\mathrm{sc}} = 1 - \frac{T_{\mathrm{min}}}{T_{\mathrm{c}}} = \frac{\delta}{D^2}\left(1 + \mathcal{O}\left(\frac{1}{D}, \frac{\delta}{D}\right)\right)$$



Maximum Supercooling

- The speed of sound at critical temperature in the deconfined phase is given by

$$c_s^2(T_c) = \left.\frac{d\log T}{d\log s}\right|_{T=T_c} = \frac{2\delta}{D^2}\left(1 + \mathcal{O}\left(\frac{1}{D}, \frac{\delta}{D}\right)\right)$$

$$\boxed{\Rightarrow \epsilon_{sc} = \frac{c_s^2(T_c)}{2}}$$

With error of $\mathcal{O}(1/D)$ and $\mathcal{O}(c_s^2(T_c)/c_{s,CFT}^2)$

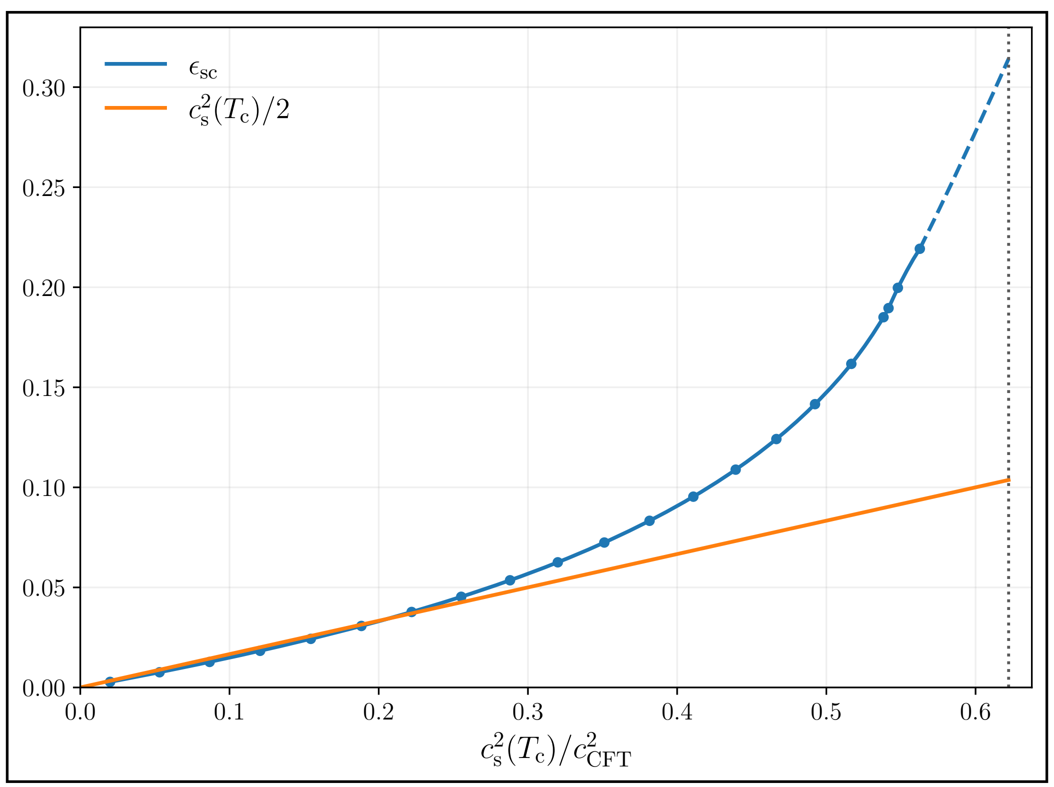
When the $1/D$ is valid— the conformality is broken badly and the speed of sound at critical temperature is small compared to its conformal value. The maximum possible supercooling is set by speed of sound in the deconfined phase at T_c .

Examples

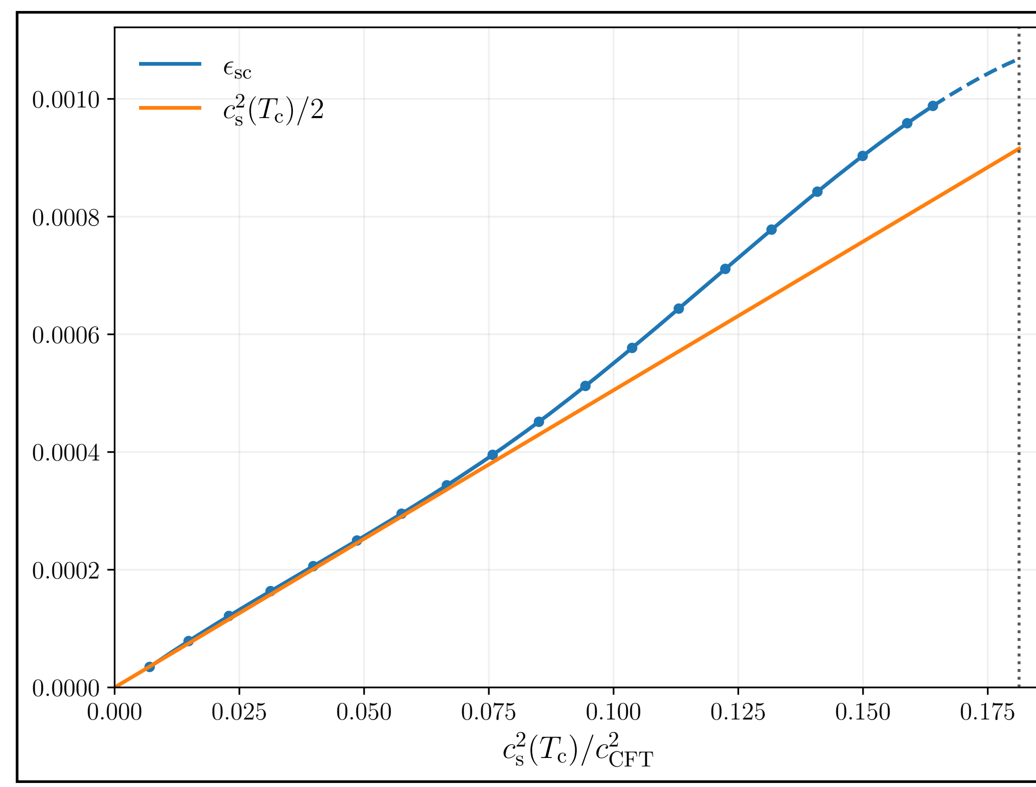
- <u>Exponential Superpotential:</u>

$$W = \frac{1}{L}(1 + \exp(\gamma\phi)) \quad , \quad \underbrace{\frac{1}{\sqrt{2(D-1)}} \leq \gamma < \sqrt{\frac{D}{2(D-1)}}}_{\text{Gubser bound}}$$

$D = 4$



$D = 100$



Examples

- Improved Holography:
  - ⋆ Speed of sound (Gursoy,Kiritsis et.al., 1006.5461): $c_s^2(T_c) = 0.11$
  - ⋆ Maximum supercooling (Missoni, Morgante, and Ramberg, 2607.18800) : $\epsilon_{sc} = 0.04$

- $\mathcal{N} = 4$ SYM on $S^3 \times S^1$ (Witten hep-th/9803131; PA,GRK, VL, MR, 2504.00199)

(generically dual of AdS$_{D+1}$ with $\partial(\text{AdS}_{D+1}) = S^{D-1} \times S^1$)

  - ⋆ Loosely define $c_s^2 = d \log T / d \log S$
  - ⋆ $\epsilon_{sc} = 1 - \sqrt{1 - c_s^2(T_{sc})} \Rightarrow \epsilon_{sc} \approx \dfrac{c_s^2(T_{sc})}{2}$, $c_s^2(T_c) = \dfrac{1}{(D-1)^2}$

Concluding Remarks

- Strongly coupled gauge theories — ubiquitous in BSM models— still a lot to explore about their phase transition dynamics.
- Close relation of supercooling to the conformality of the theory at critical temperature
- $1/D$ expansion in Einstein-Scalar gravity — used for analytical study — possibly extract universal behaviours of dual strongly coupled gauge theories.


That's all Folks!