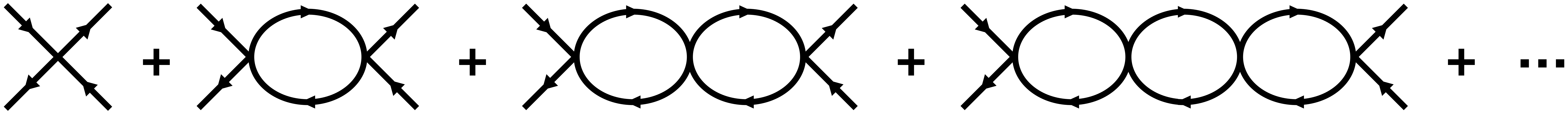
Renormalisable four-fermion interactions in four dimensions



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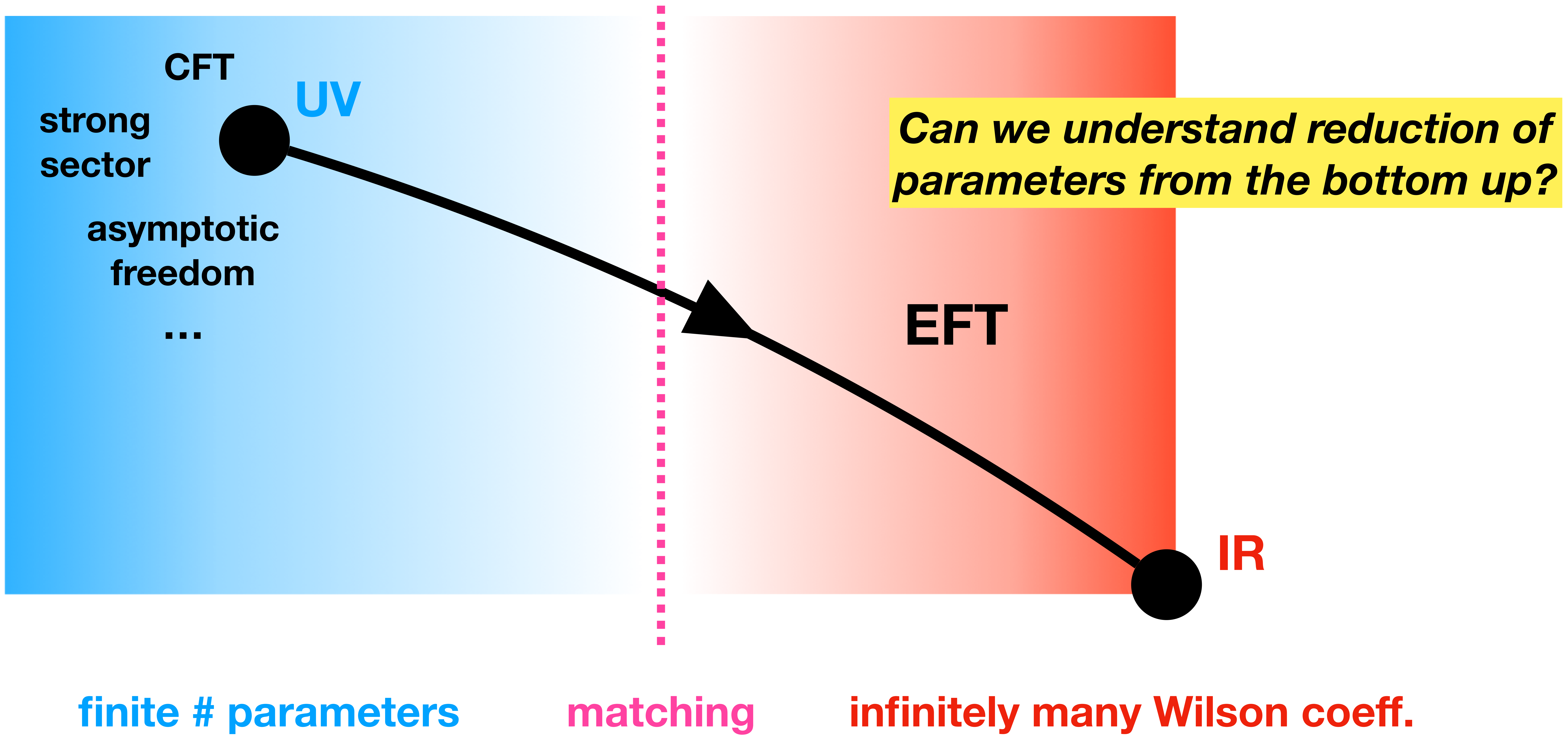
TU Dortmund / University of Sussex

based on ***2603.19357*** *with Daniel Litim*

University of Sussex
technische universität dortmund

EFT for non-perturbative physics @ Durham 2026

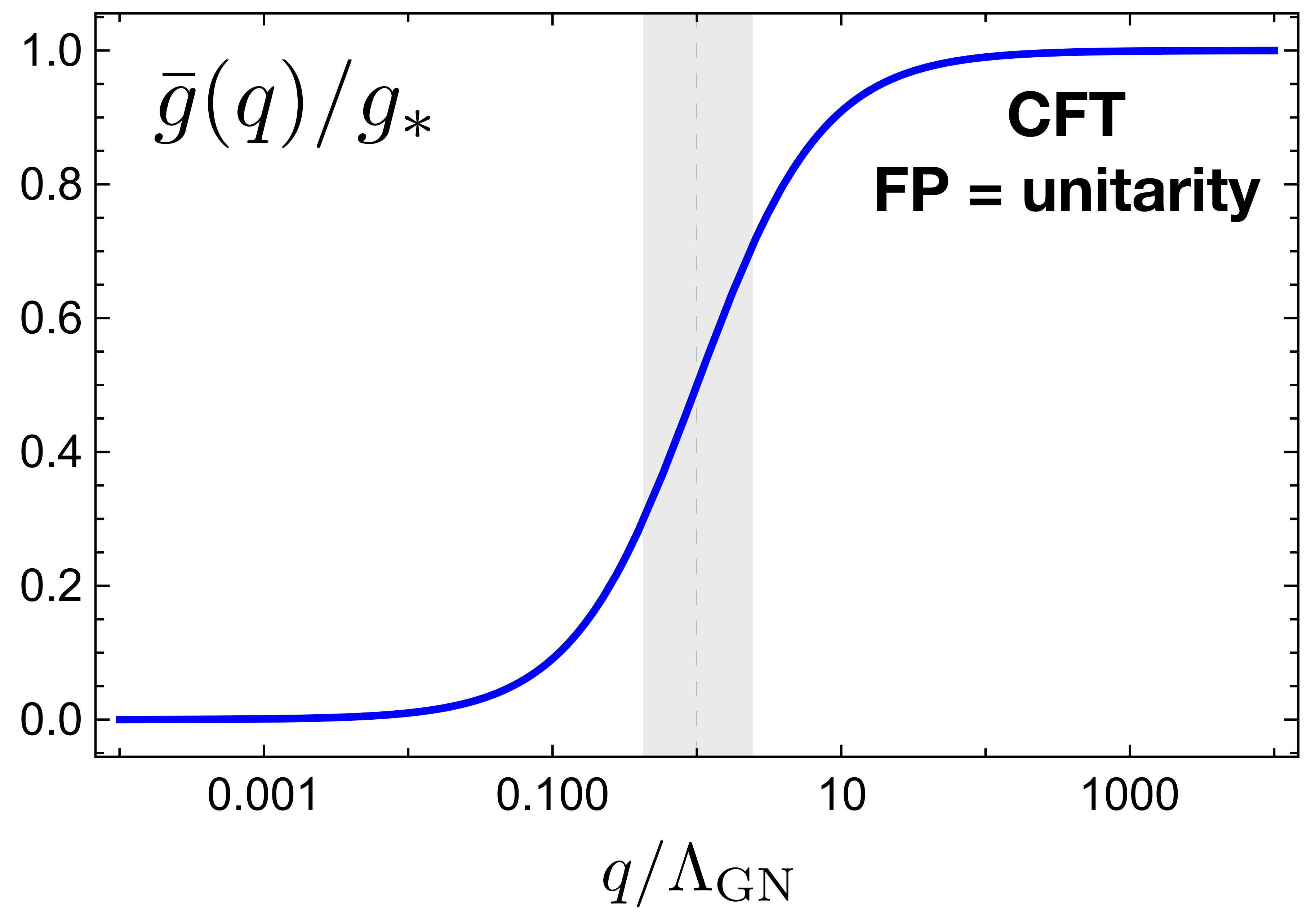

CFT
UV
strong
sector
asymptotic
freedom
...
EFT
IR
Can we understand reduction of parameters from the bottom up?
finite # parameters
matching
infinitely many Wilson coeff.

A toy EFT

$$\bar{\psi}_a \partial\!\!\!/ \psi_a - \frac{1}{2N} G(\bar{\psi}_a \psi_a)^2 + \cancel{\cdots}$$

2+1 dimensions: UV complete & conformal at high energy

Wilson ’73
Gross, Neveu ’74
Parisi ’75
Gawędzki, Kupiainen ’85
TIP! **Rosenstein, Warr, Park ’88 (PRL)**
Rosenstein, Warr, Park ’91 (Phys.Rept.)
Gat, Kovner, Rosenstein, Warr ’90
de Calan, Faria Da Veiga, Magnen, Seneor ’91
+ many more



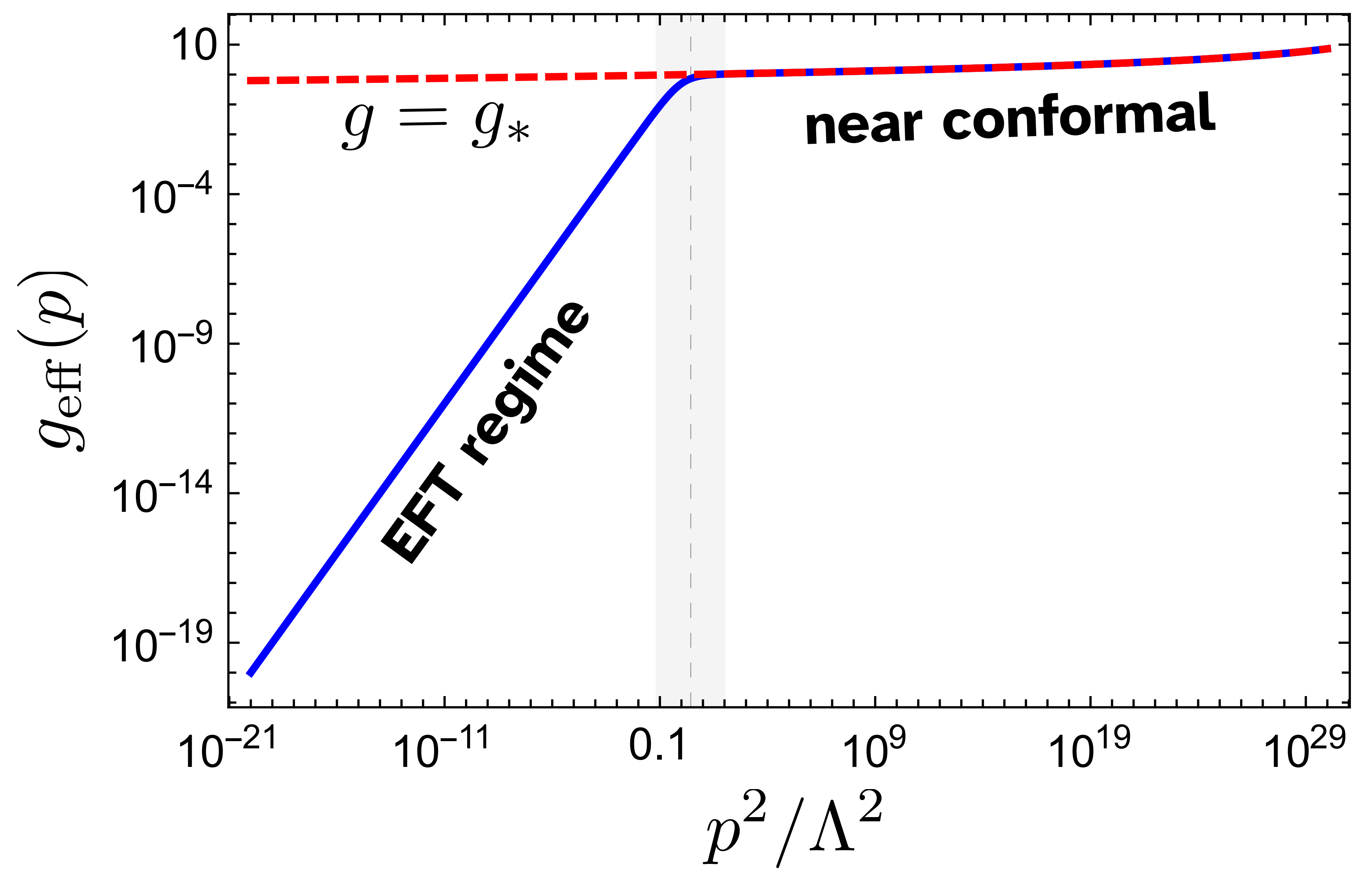
A toy EFT

$$\bar{\psi}_a \not{\partial} \psi_a - \frac{1}{2N} G(\bar{\psi}_a \psi_a)^2 + \cdots$$

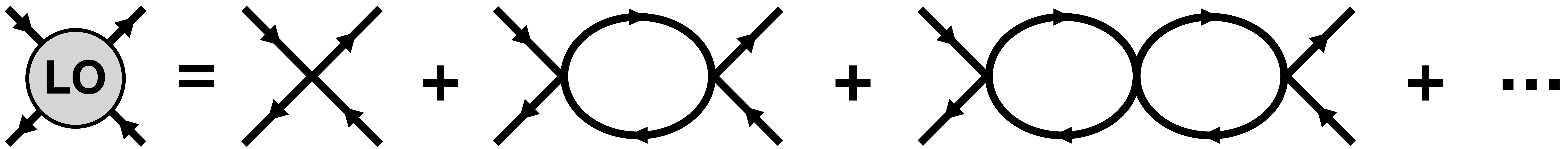
3+1 dimensions: UV complete & *near*-conformal at high energy

Formal renormalisability in 1/N dictates:

- tower of higher-derivative operators
- eight-fermion interactions (but no higher)
- only three fundamental parameters

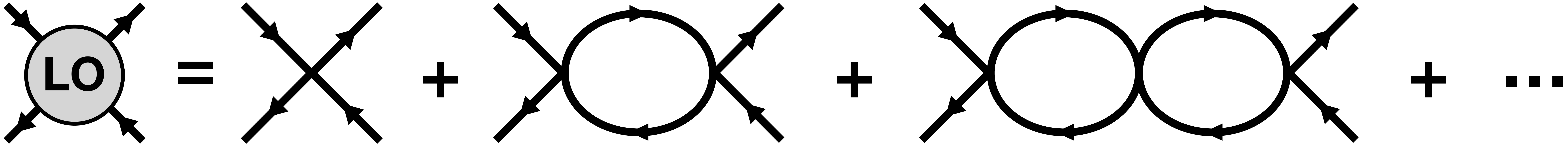


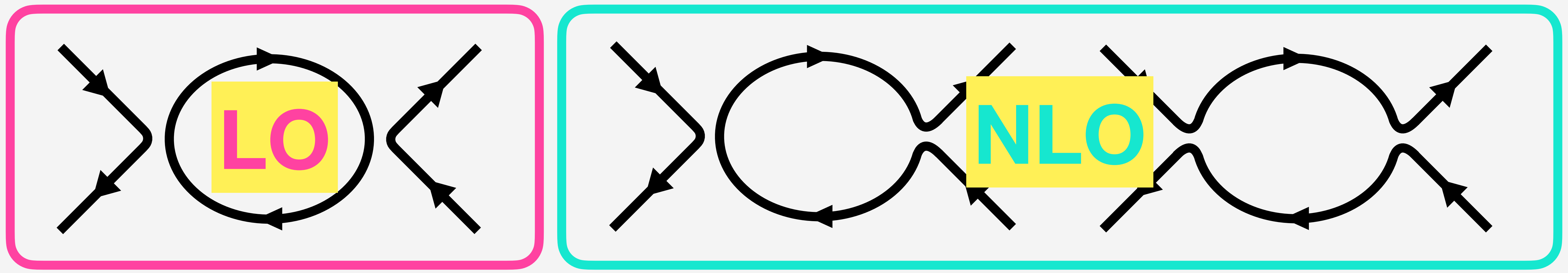
Large-*N* diagrammatics



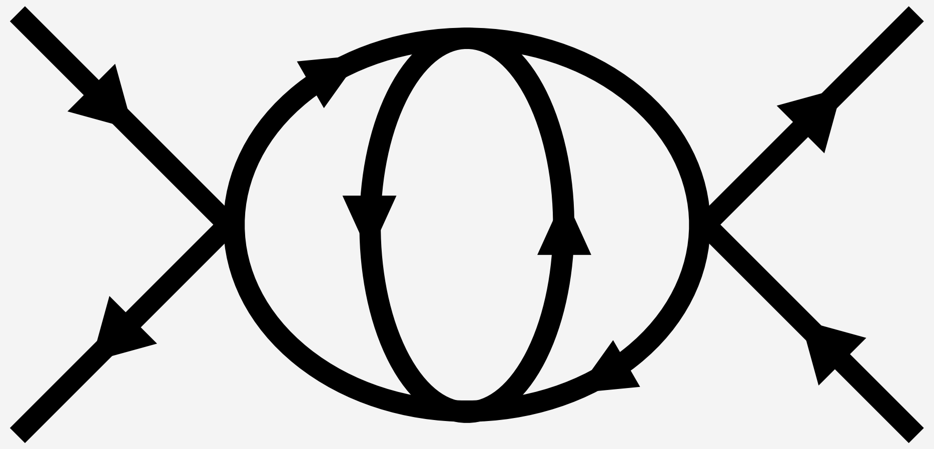
$\mathcal{O}\left(\frac{1}{N}\right)$ $\qquad \frac{G}{N} \qquad \frac{G}{N} N \frac{G}{N} \qquad \frac{G}{N}\, N\, \frac{G}{N}\, N\, \frac{G}{N}$

Large-*N* diagrammatics

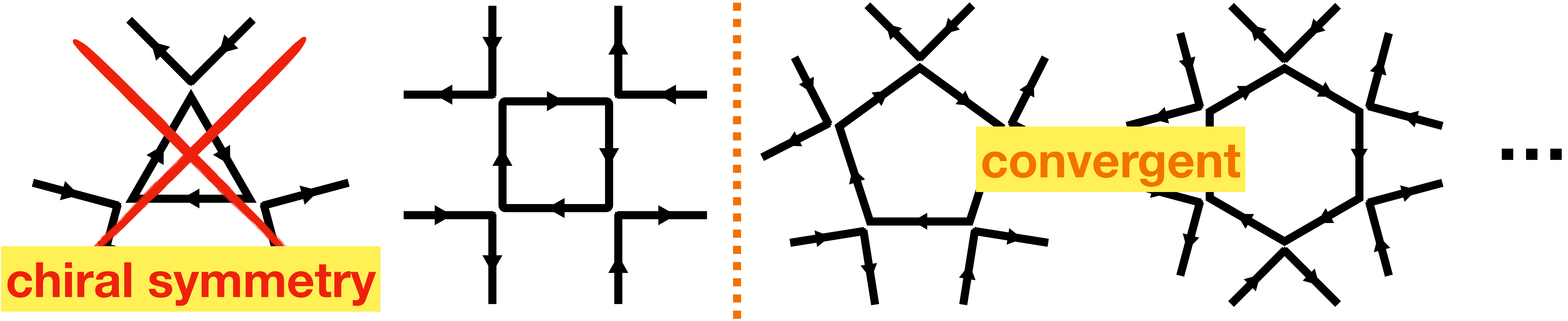




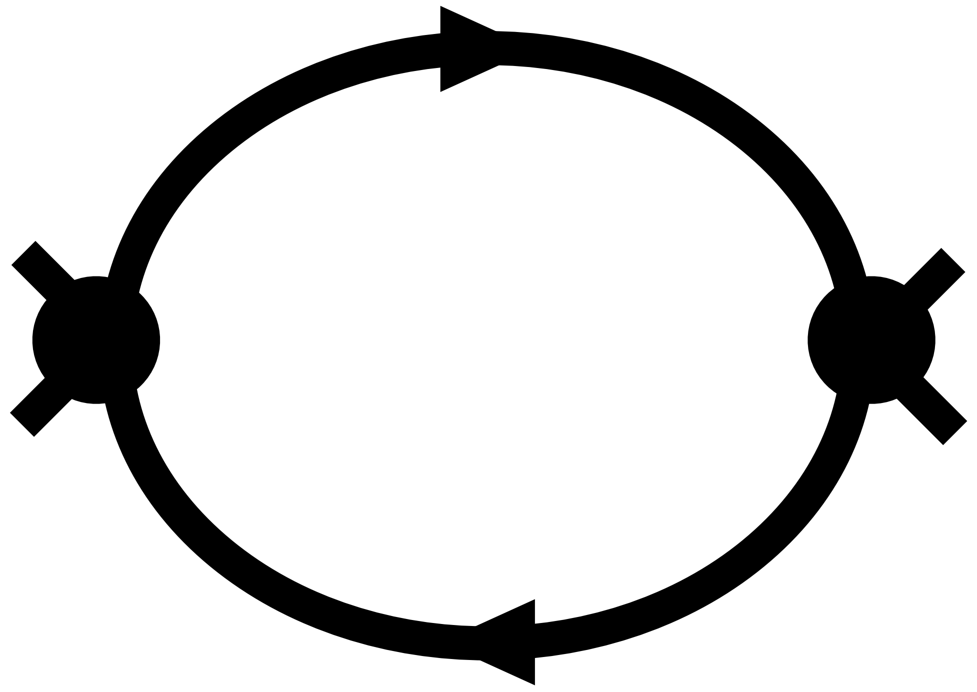
Large-N leading: #traces = #loops



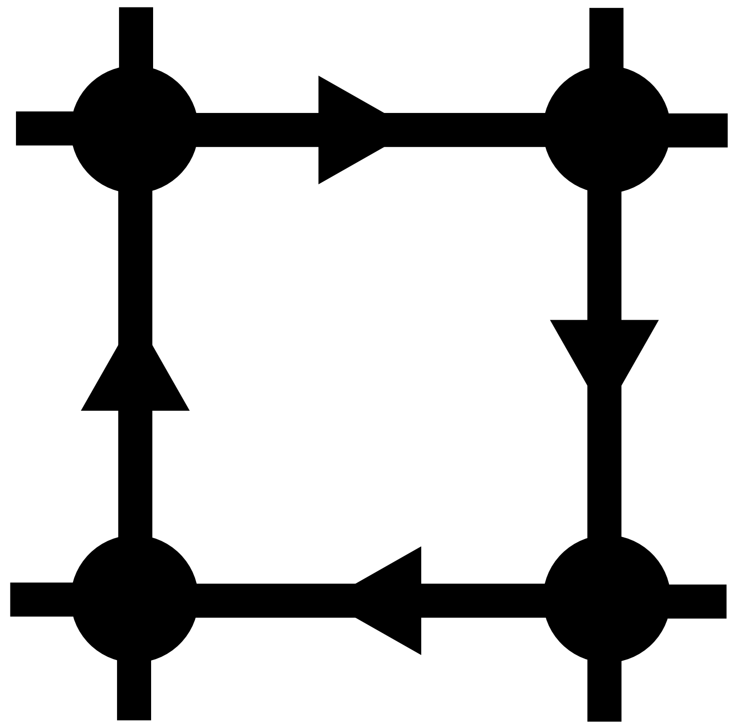
N^{n}LO: #traces = #loops - n



Divergent diagrams



A

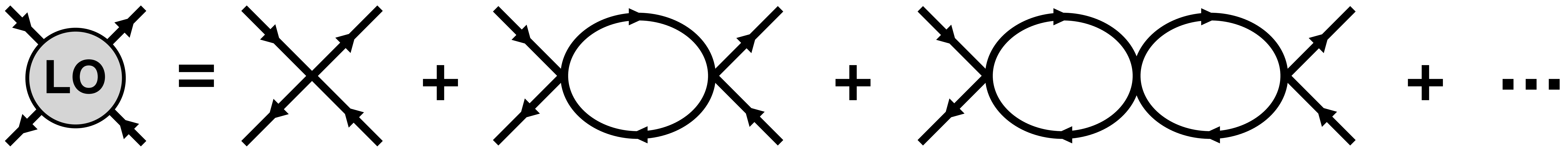


B

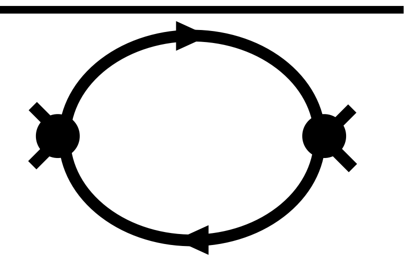
	A	B
3d:	$\sim \Lambda$	$\sim \Lambda^{-1}$
4d:	$\sim \Lambda^2 + p^2 \ln \Lambda$	$\sim \ln \Lambda$

need higher derivative 4F + 8F counterterms

Four-point function



$$\sim \frac{N^{-1}}{G^{-1} + \quad}$$



_3d_: absorb Λ into G[-1]

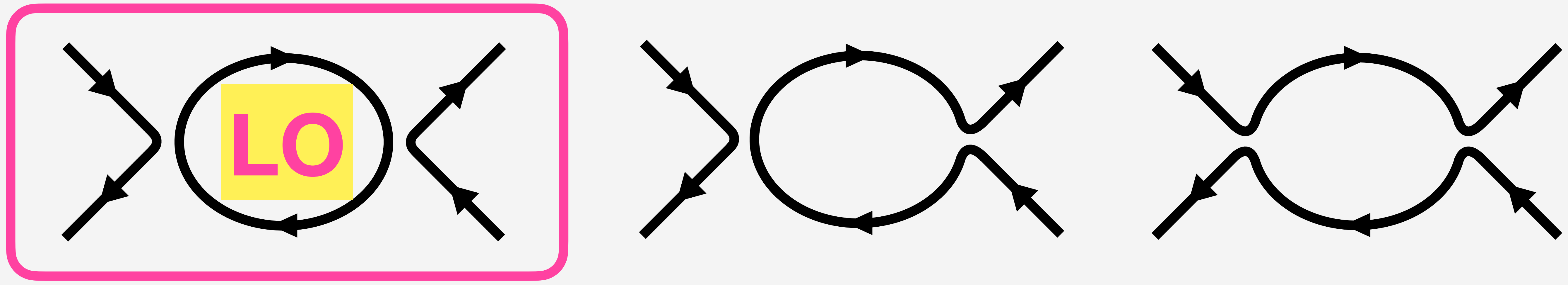
mixed polynomial + non-local divs

$$\frac{G}{N} + \frac{G^2}{N}(\Lambda^2 + p^2 \ln \Lambda) + \frac{G^3}{N}(\Lambda^4 + p^4 \ln^2 \Lambda + \ldots) + \frac{G^4}{N}(\Lambda^6 + p^6 \ln^3 \Lambda + \ldots) + \ldots$$

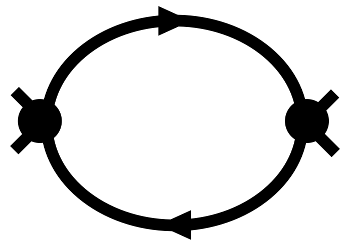
_4d_: need infinitely many higher derivative counterterms

Four-point function

$$\frac{1}{2N}G(\bar{\psi}_a\psi_a)^2 \;\to\; \frac{1}{2N}(\bar{\psi}_a\psi_a)F(-\partial^2)(\bar{\psi}_b\psi_b)$$

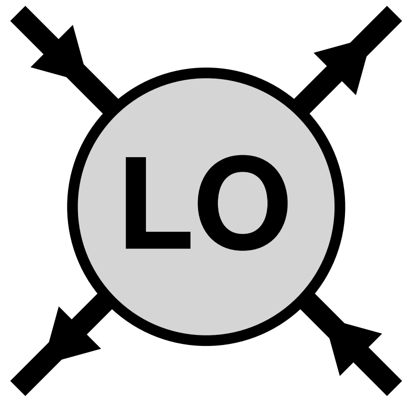


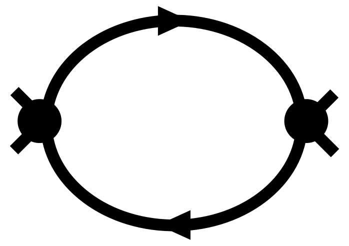
Large-N leading: #traces = #loops

$F(p^2)^2 \times$ 

vertices factorise @ LO

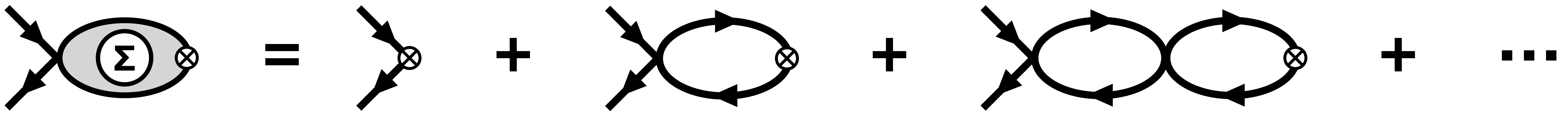
$$F(p^2) = \frac{1}{Zp^2 + G^{-1}} \;\to$$

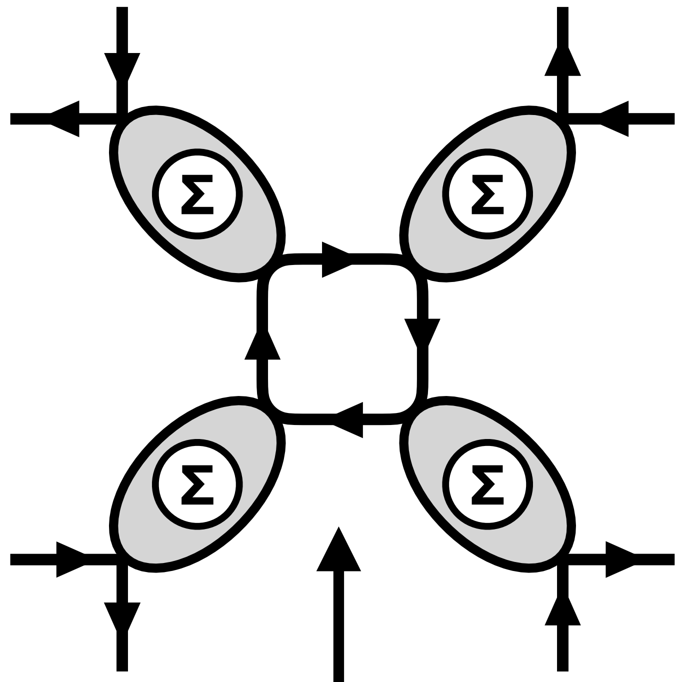


$$\sim \frac{N^{-1}}{Zp^2 + G^{-1} + }$$ 

subtract infinitely many higher-derivative divergences with one additional parameter

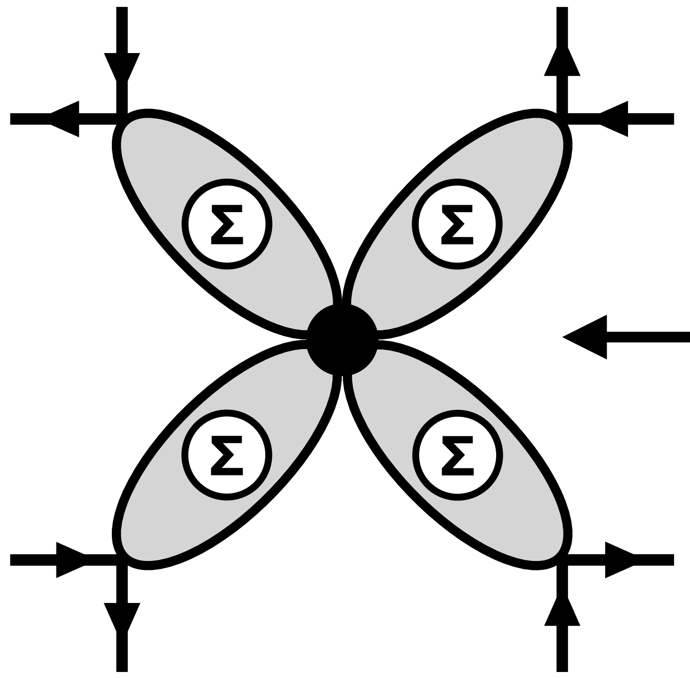
Eight-point function





C

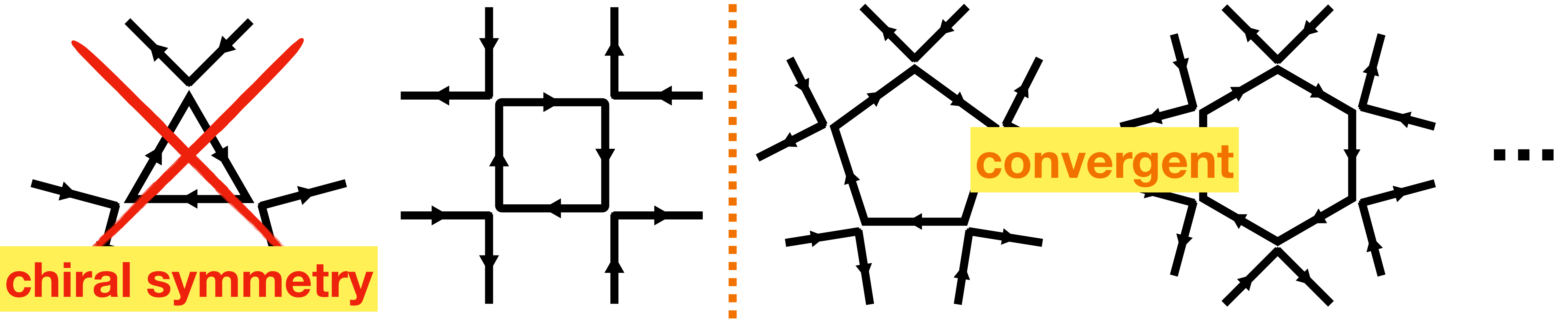
$\sim \ln \Lambda$



D

$$\frac{\lambda}{4!N^3}\left[F(-\partial^2)(\bar{\psi}_a\psi_a)\right]^4$$

Everything @ LO = products of bubble sums with:



Renormalisable four-fermion theory

$$\bar{\psi}_a \not{\partial} \psi_a - \frac{1}{2N}(\bar{\psi}_a \psi_a) F(-\partial^2)(\bar{\psi}_b \psi_b) + \frac{\lambda}{4! N^3}\left[F(-\partial^2)(\bar{\psi}_a \psi_a)\right]^4$$

$$F(p^2) = \frac{1}{Zp^2 + G^{-1}} \approx G \quad \text{for} \quad Zp^2 \ll G^{-1}$$

- **Predictive:** 3 parameters Z, G, λ
- **Tower** of higher derivative 4F + 8F
- **NO higher** (10F, 12F, …)

Renormalised couplings:

$$g(\mu) = \mu^2 \, G(\mu)$$

$$Z(\mu) \qquad \lambda(\mu)$$

Running couplings

non-universal:

$$\beta_g(g) = 2g\left(1 - \frac{g}{g_*}\right)$$

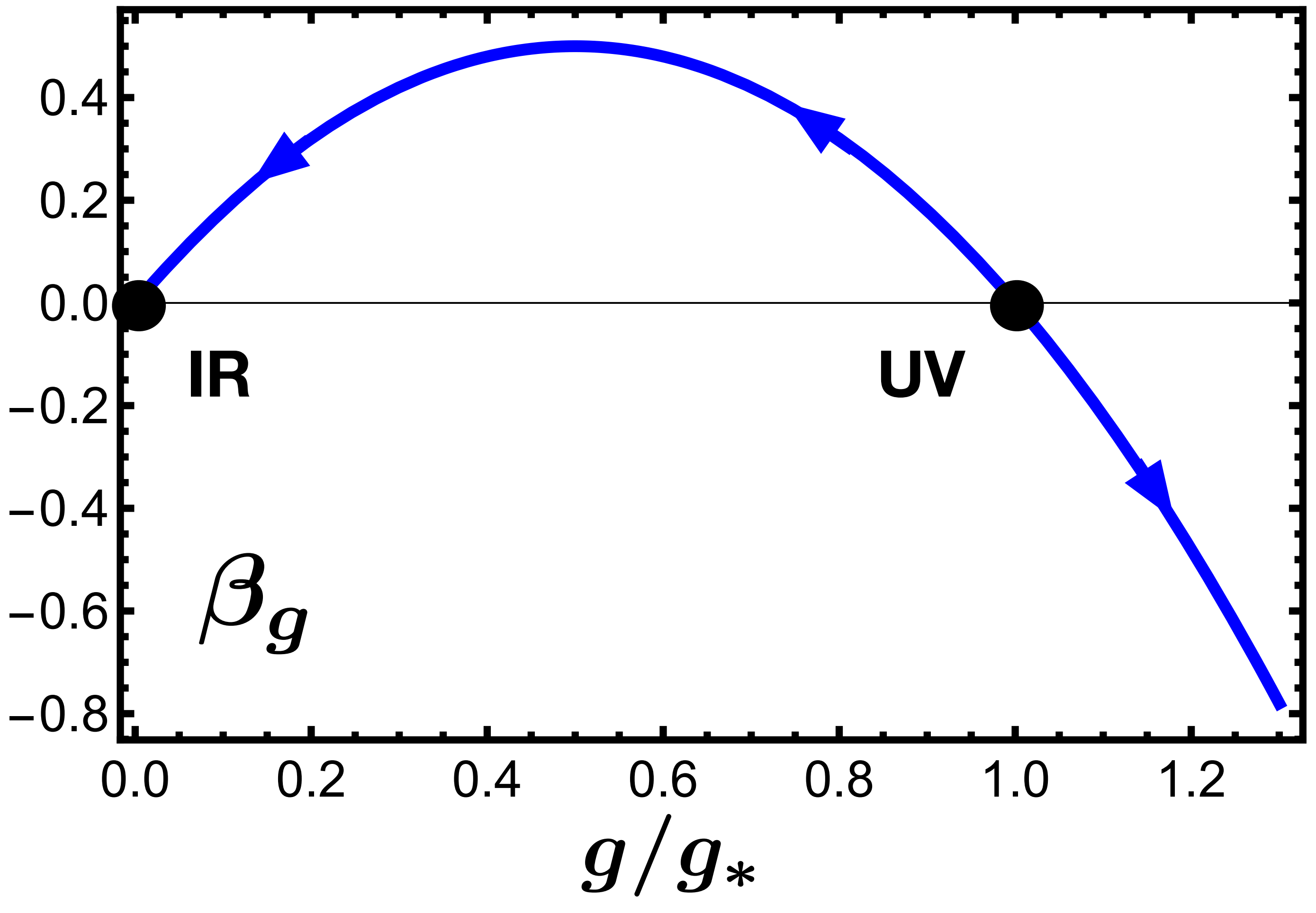
UV fixed point

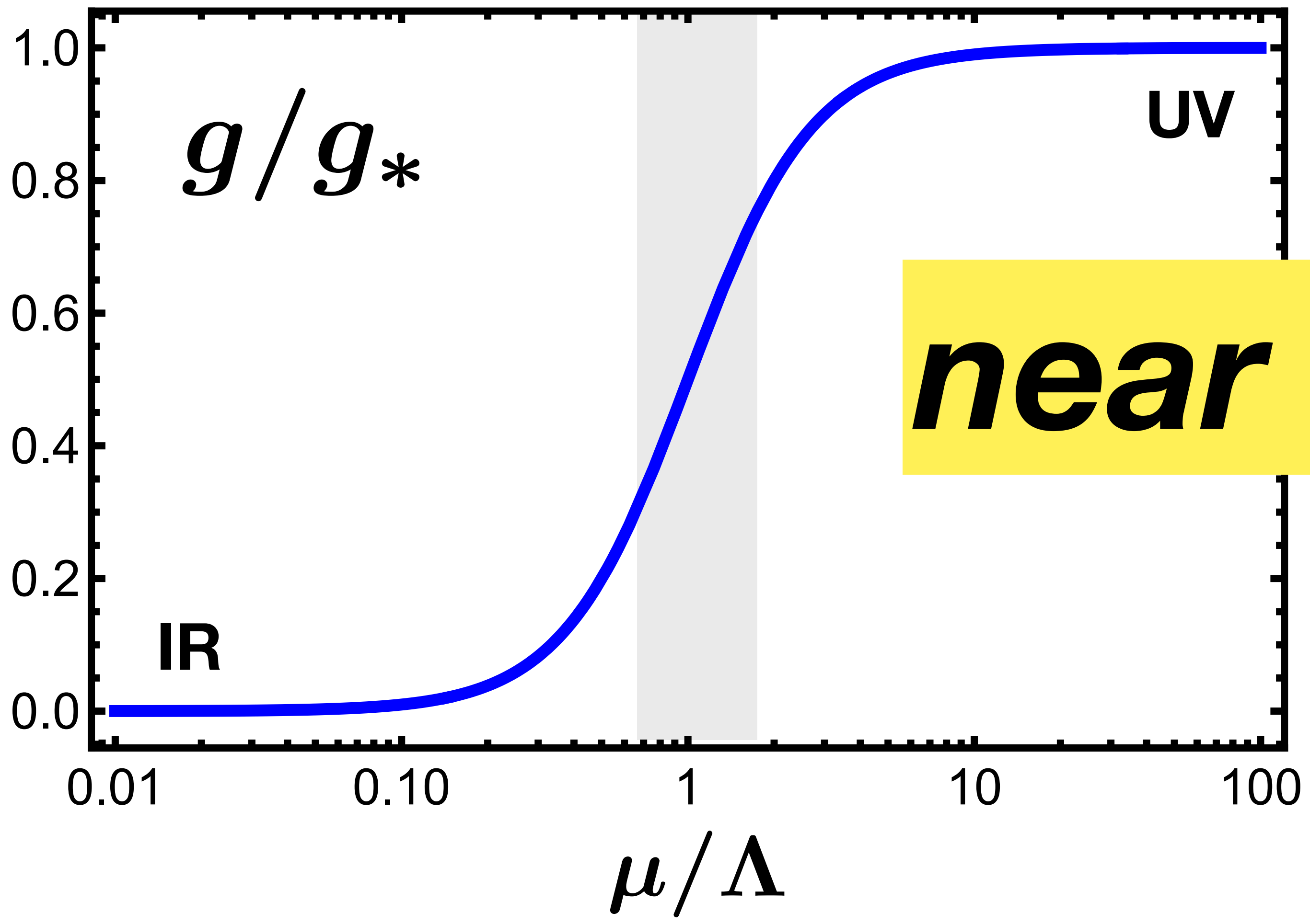
e.g. MOM scheme: $g_* = 8\pi^2$

universal:

$$\beta_Z = -\frac{1}{4\pi^2} \qquad \beta_\lambda = -\frac{3}{\pi^2}$$

logarithmic running





***near* conformal in UV**

Running couplings

non-universal:

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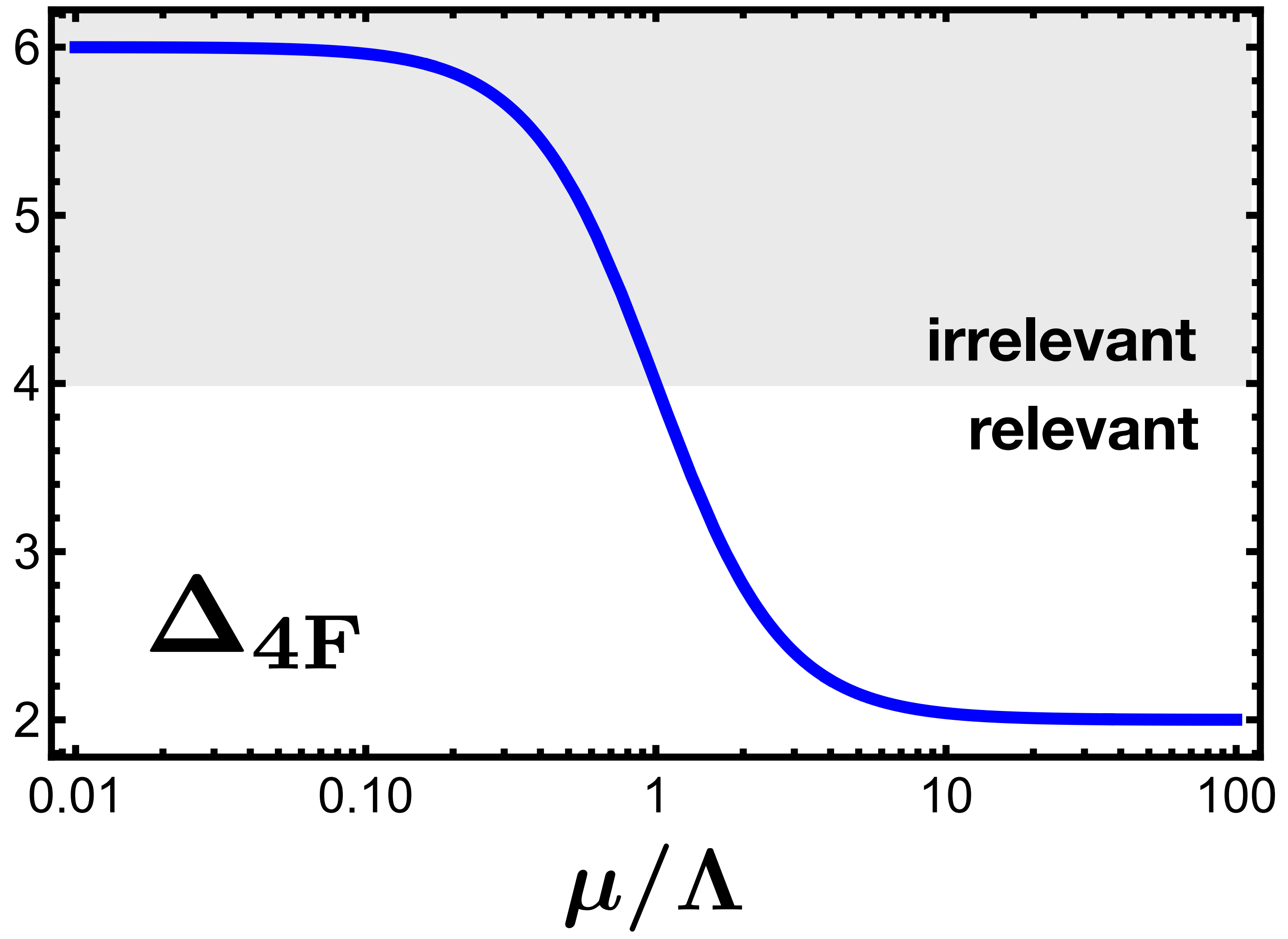
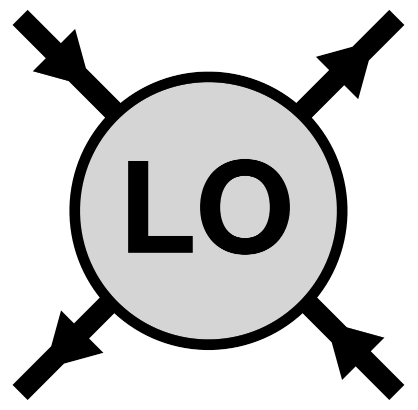


large anomalous dimensions:

dim-6 becomes dim-2

EFT power counting breaks

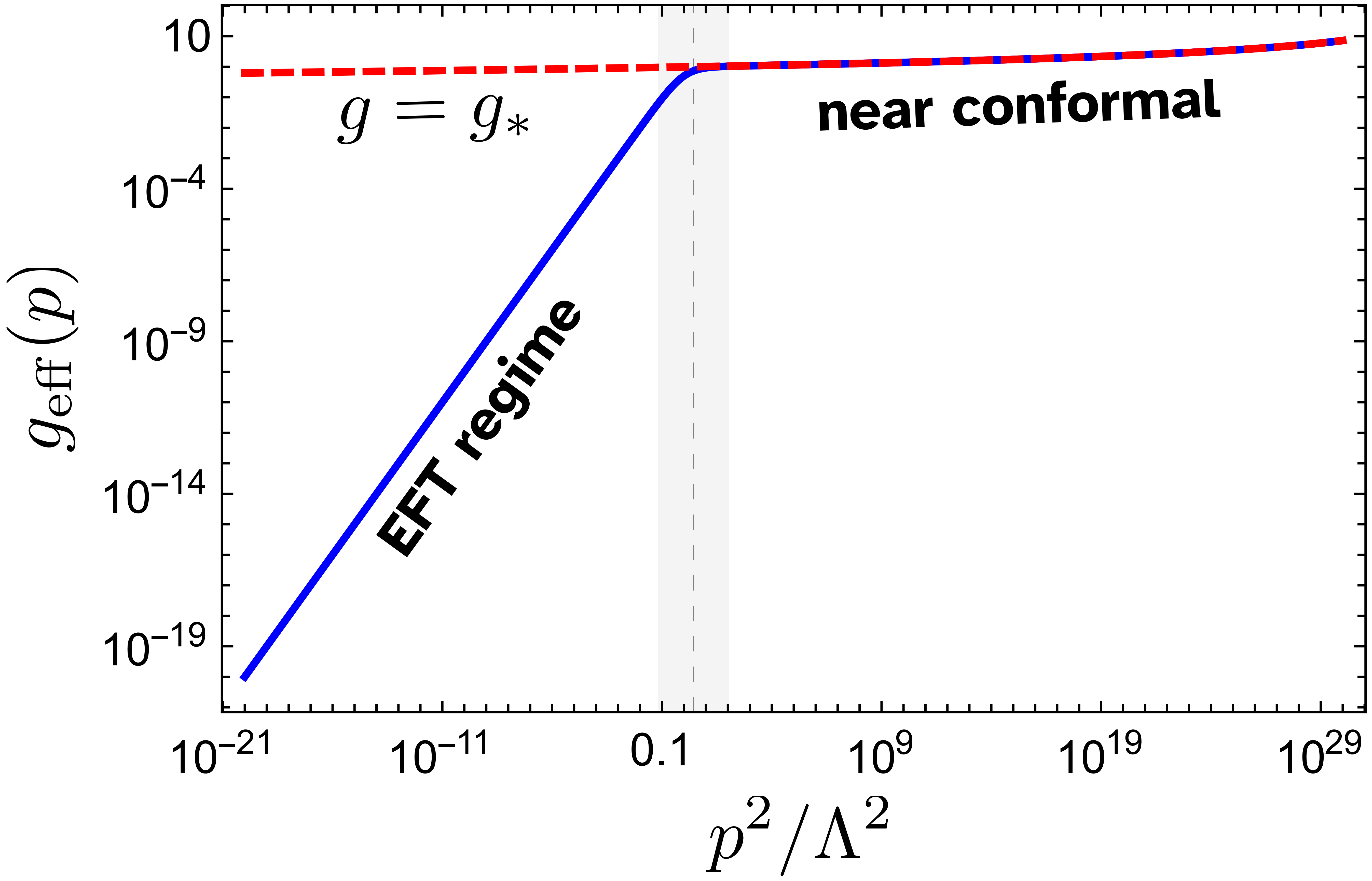
Near conformality & the EFT crossover



$$\sim \frac{N^{-1}}{p^2}\, g_{\text{eff}}(p)$$

$$\boxed{g_{\text{eff}}(p) = \frac{1}{Z(p) + g^{-1}(p)}}$$

EFT scale marks crossover:
onset of strong coupling, higher derivative terms become important



Perturbative dual description

Functional Legendre transform maps to Yukawa theory at infinite N:

$$\bar{\psi}_a(i\partial\!\!\!/ - y\phi)\psi_a + \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda_\phi\phi^4$$

e.g. Weinberg hep-th/9706042

Parameter relations preserved under RG flow:

$$G = \frac{y^2}{m^2} \qquad Z = \frac{1}{y^2} \qquad \lambda = \frac{\lambda_\phi}{y^4}$$

CCH, Litim, *in prep*.

Four-fermion fixed point = massless separatrix in Yukawa theory.

$$\mu^2 \frac{y^2(\mu)}{m^2(\mu)} = g_*$$

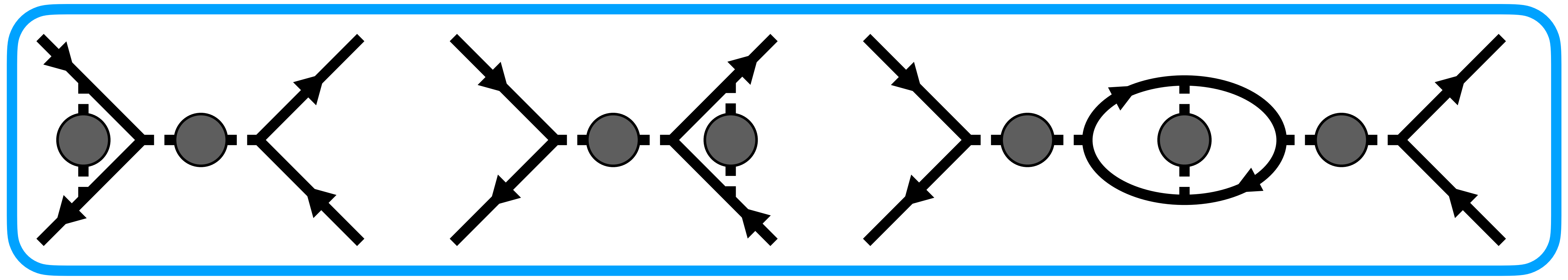
Beyond leading order

Renormalisable order-by-order in 1/N

$$\omega = 4 - \frac{3}{2}E_{\psi} - E_{\bar{\psi}\psi}$$

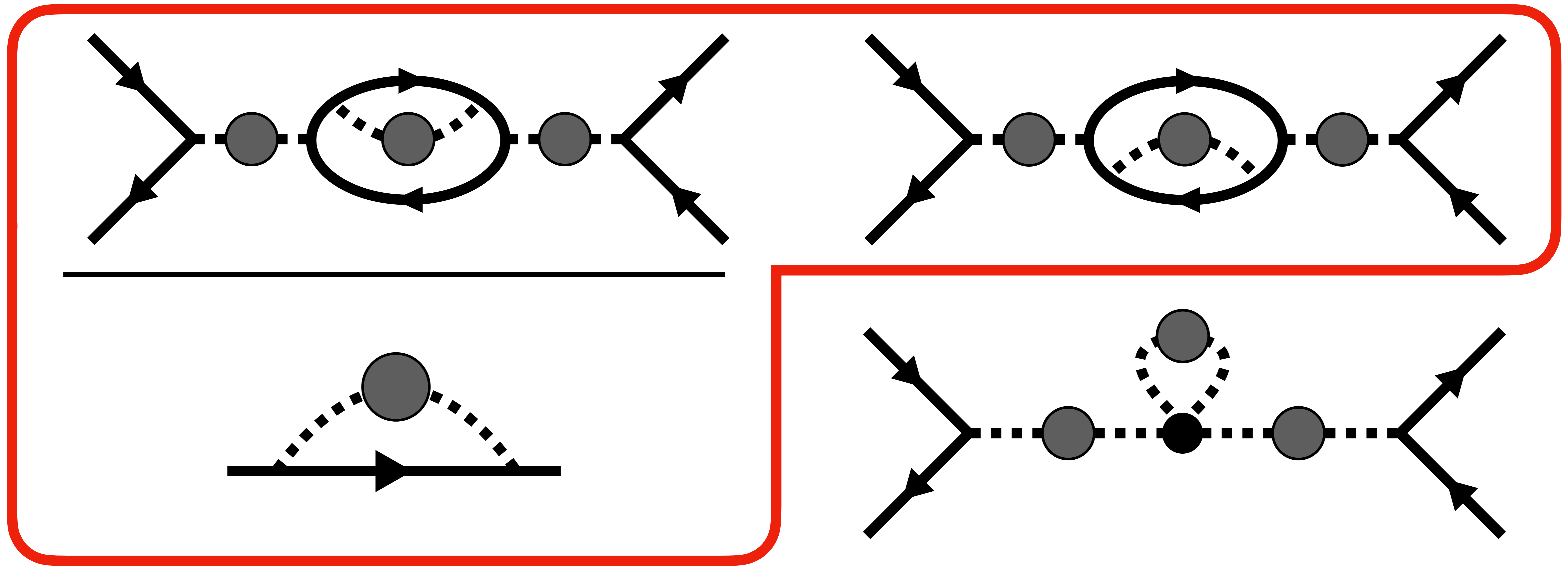
Proof follows similar to 3d:

- power counting with bubble chains
  cf. Park, Rosenstein, Warr, Phys.Rept. 1991
- wavefunction renormalisation at NLO

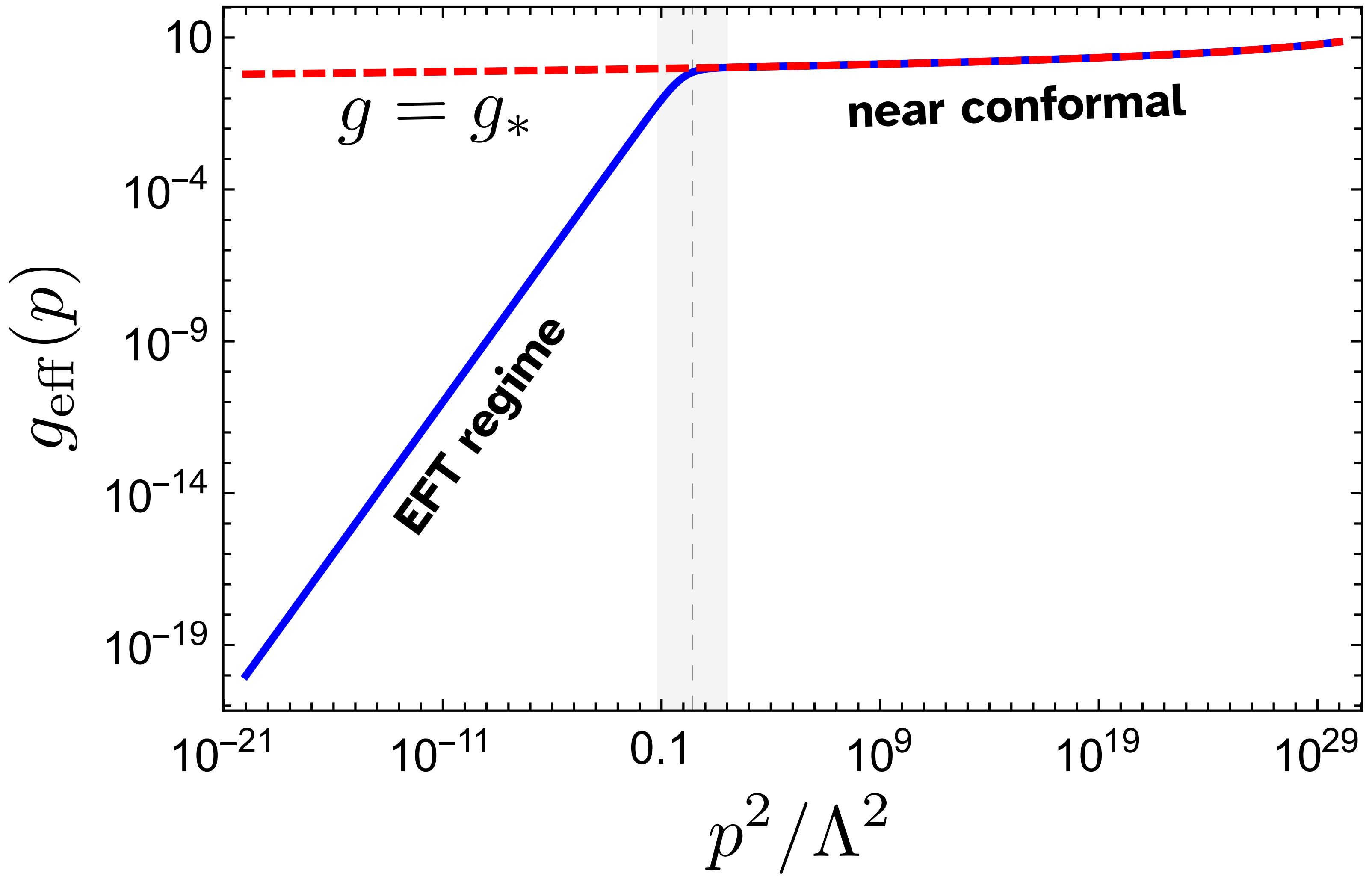


Explicit computation:

- cancellation of subdivergences
- methods developed for large-N_F QED
  Palanques-Mestre, Pascual 1984



Importance of RG scheme



$$\beta_g(g) = 2g\left(1 - \frac{g}{g_*}\right)$$

Physics is universal, e.g. exact amplitudes, but the beta function is not.

- *g** is a **scheme dependent** constant
- **MS** corresponds to limit ***g** to infinity**
- Not a good choice for **approximate calculations** at intermediate scales
- Convergence improved by **non-minimal schemes** (WIP + Yannick Kluth @ Toronto)

Summary

$$\bar{\psi}_a \not{\partial} \psi_a - \frac{1}{2N}(\bar{\psi}_a\psi_a)F(-\partial^2)(\bar{\psi}_b\psi_b) + \frac{\lambda}{4!N^3}\left[F(-\partial^2)(\bar{\psi}_a\psi_a)\right]^4$$

$$F(p^2) = \frac{1}{Zp^2 + G^{-1}} \approx G \quad \text{for} \quad Zp^2 \ll G^{-1}$$

Four-fermion interactions can UV-complete themselves

- Tower of higher derivative 4F + 8F operators, but only **3 parameters**
- **Near scale invariance** in UV with relevant 4F + marginal 8F
- Strong coupling sets in near EFT scale — scheme choice important