Cluster of Excellence PRISMA++ | Precision Physics, Fundamental Interactions and Structure of Matter

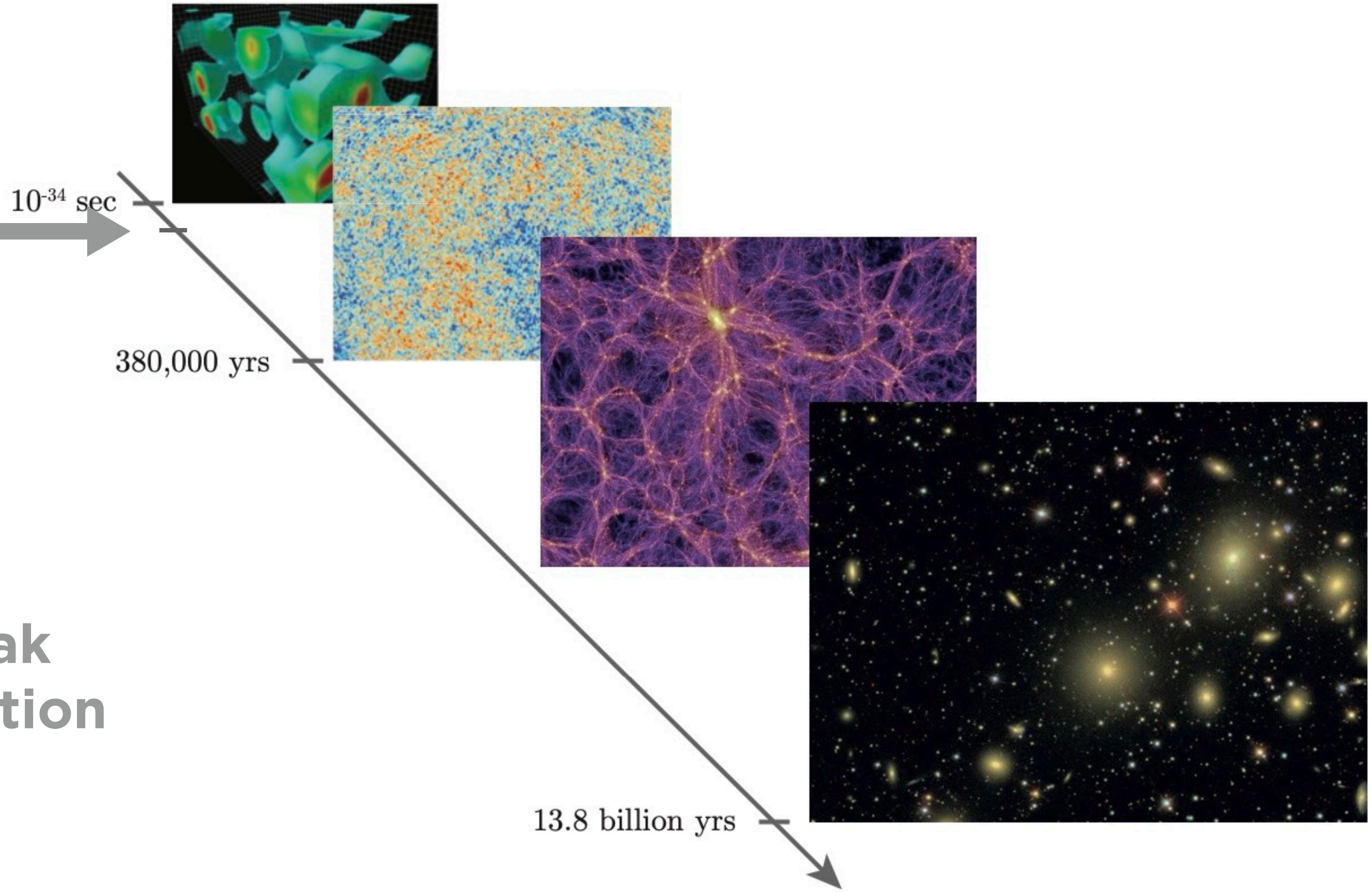
Superdaisy Resummation in Theories with Broken Gauge Symmetries

Martin Münzenberg & Carlos Tamarit

work in progress

1. Background

Universe Evolution



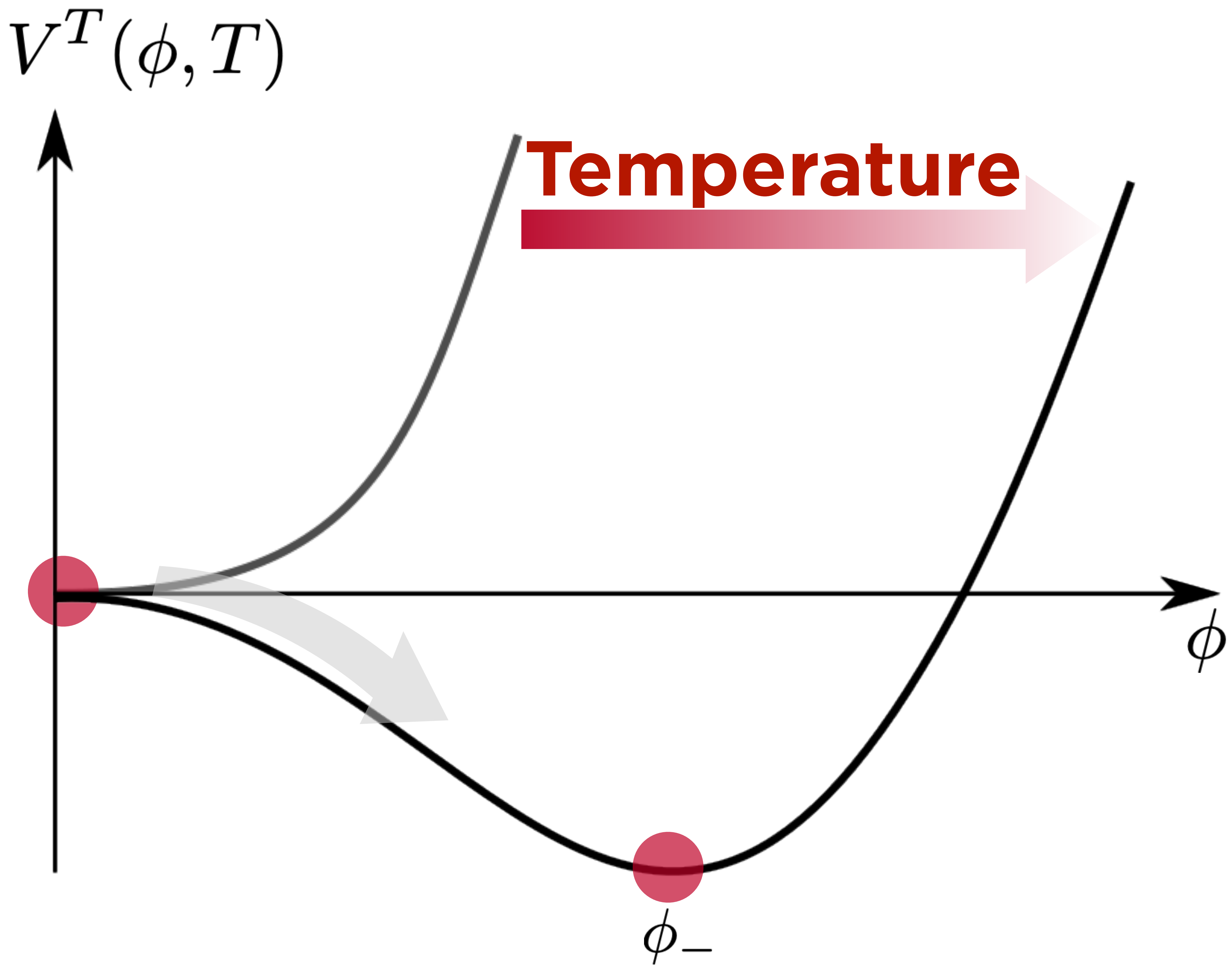
Temperature

Electroweak Phase Transition
$10^{-12} s$

SM Electroweak Transition

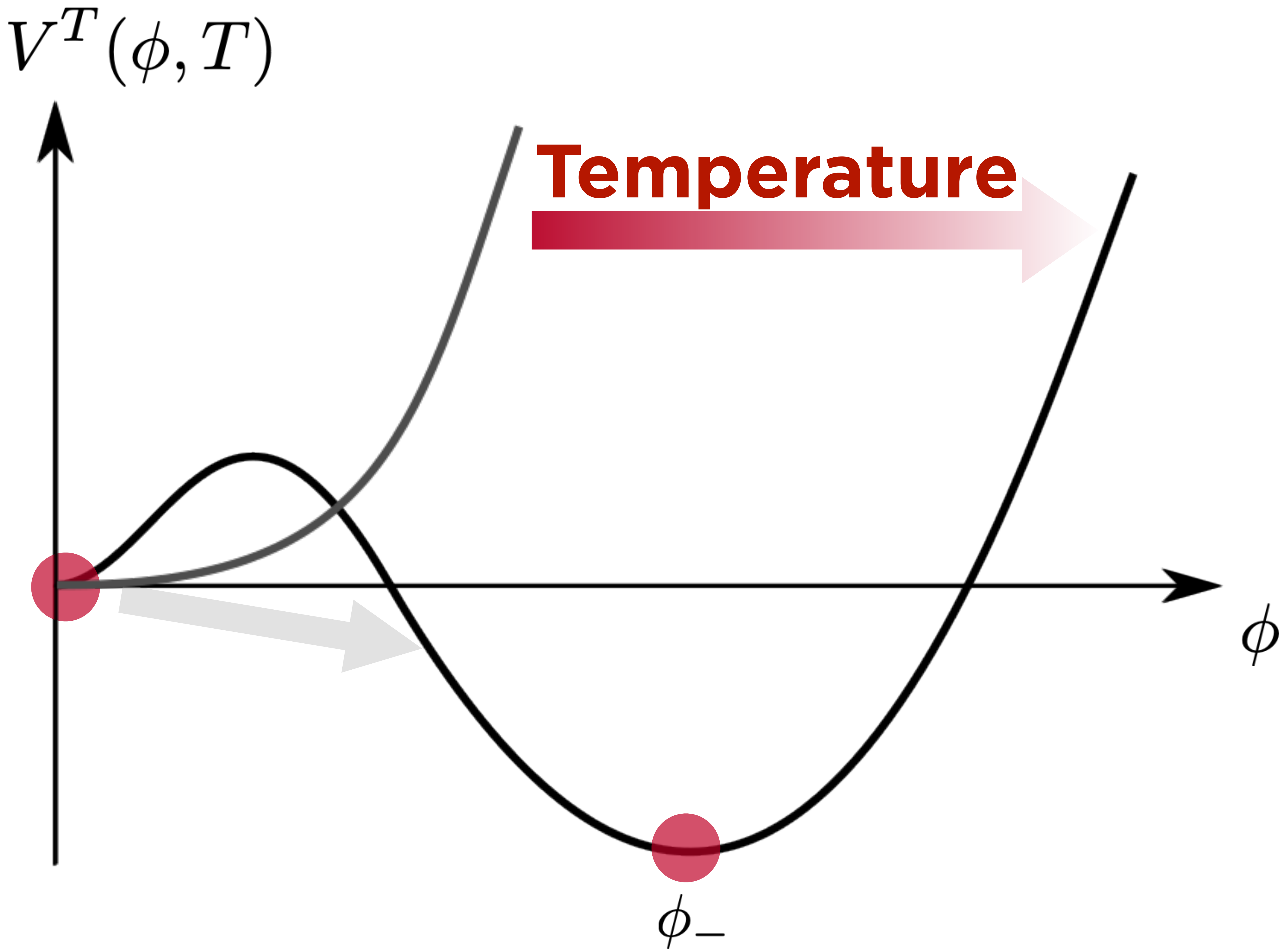
Crossover:

Lattice result for the physical SM



1st-order transition:

Conventional perturbative resummation



Critical Temperature

- Consider simple scalar theory:

$$V_0 = -\frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4$$

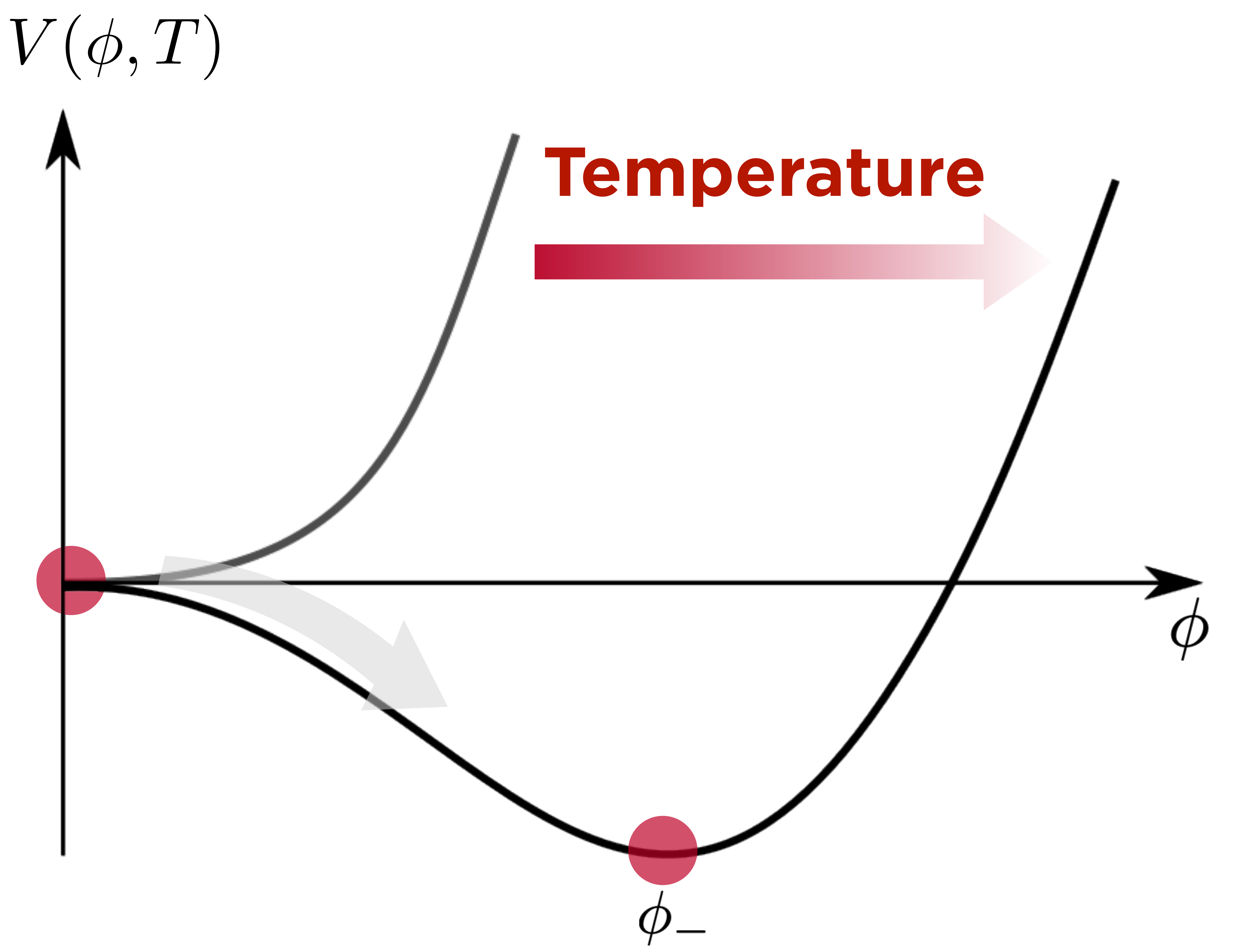
- At high temperature, schematically:

$$m_{\text{eff}}^2 = -\mu^2 + \lambda T^2$$

➡ Symmetry restoration, when

$$T_c = \frac{\mu}{\sqrt{\lambda}}$$

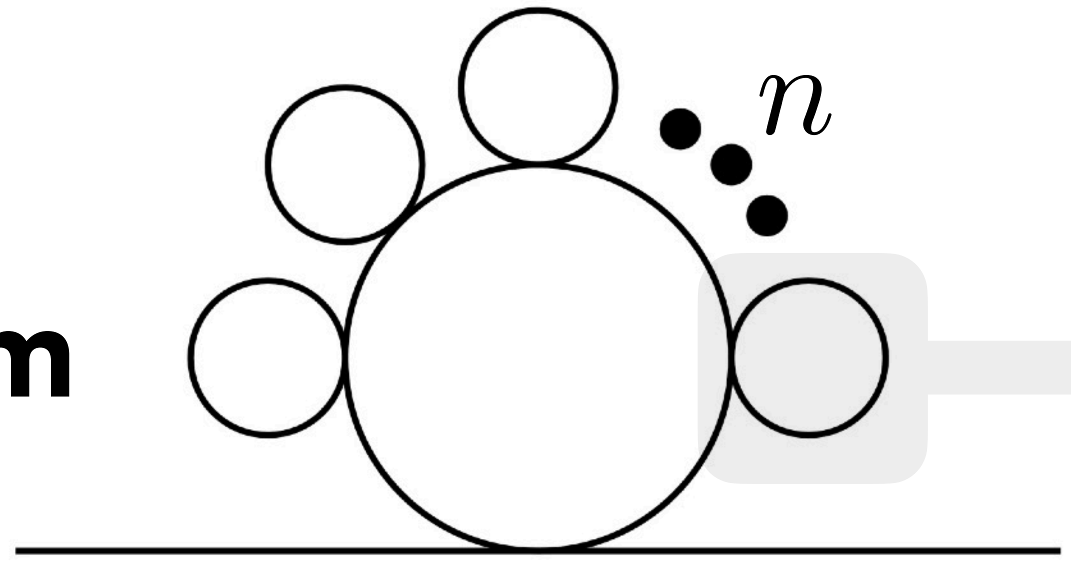
Effective Potential

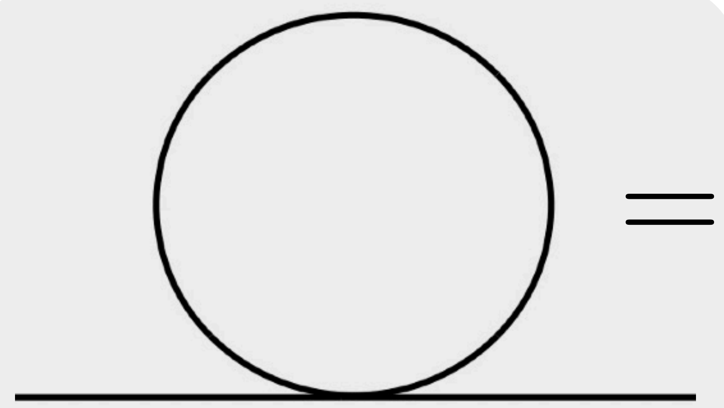


The Problem: Breakdown of Perturbation Theory

- Consider one-loop diagaram with n **insertions** of **self energy diagram**

Daisy Diagram





$$= \lambda T \sum_n \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{1}{p^2 + m^2} \propto \lambda T^2$$

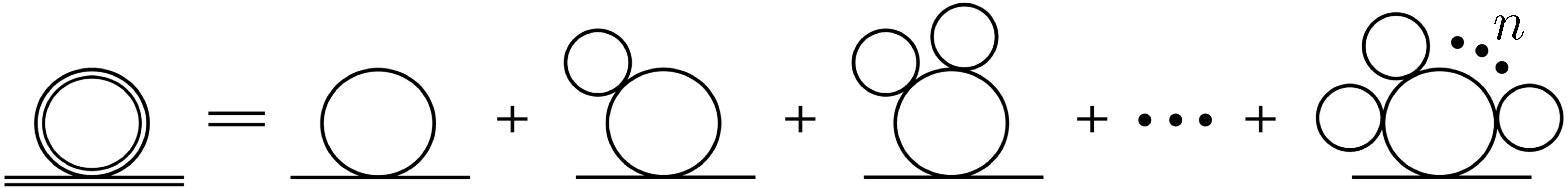
- Each self energy insertion contributes multiplicative factor of $\alpha = \dfrac{\lambda T^2}{m^2}$

➡ Fixed order perturbation theory requires $\alpha \ll, \quad \lambda \ll 1$

> Near PT $T_c = \dfrac{\mu}{\sqrt{\lambda}}$ — perturbation theory **breaks down!**

The Solution: Daisy Resummation

- **Resumming geometric series** of daisy diagrams



$$\Rightarrow \quad \mathcal{D}(\mathbf{p},\omega_n) = \mathcal{D}_0(\mathbf{p},\omega_n)\sum_n \big(-\Pi\mathcal{D}_0(\mathbf{p},\omega_n)\big)^n = \frac{1}{\omega_n^2+\mathbf{p}^2+m^2+\Pi}$$

- Propagator mass is replaced by **effective mass:**

$$m^2(\phi) \to m^2_{\text{eff}}(\phi,T) \equiv m^2(\phi) + \Pi(\phi,T)$$ **function of h and T**

Going Beyond High-Temperature Self-Energy

- Standard daisy resummation leading high-temperature self-energy:

$$\Pi = \lambda T^2, \qquad m(\phi)/T \ll 1$$

- However, across the bubble wall, masses can get heavy:

$$m \sim \lambda\phi, \qquad \phi/T \sim \mathcal{O}(1)$$

- Self energies become field dependent: $\Pi(\phi, T) \sim T^2 J_B'\left(\frac{m^2(\phi)}{T^2}\right) + \dots$

➡ To describe **smooth thermal decoupling** of heavy modes, full thermal functions are needed! This can affect

$$V(\phi, T),\ p = -V,\ e = V - T\partial_T V,\ c_s^2,\ v_w \dots$$

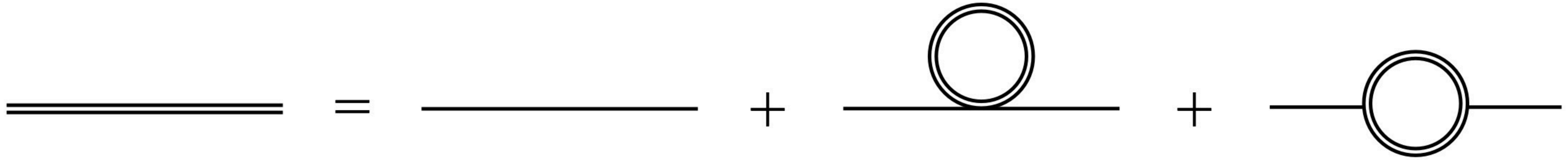
Self-Consistent Thermal Masses: Gap Equation

- Beyond high-T, self-energy depends on mass running inside the loop

$$\Pi = \Pi(m^2(\phi), T)$$

- This motivates a self-consistent determination of the dressed mass:

$$\boxed{\mathcal{M}^2(\phi,T) = m^2(\phi) + \Pi(\mathcal{M}^2(\phi,T),T)}$$ **Gap Equation**

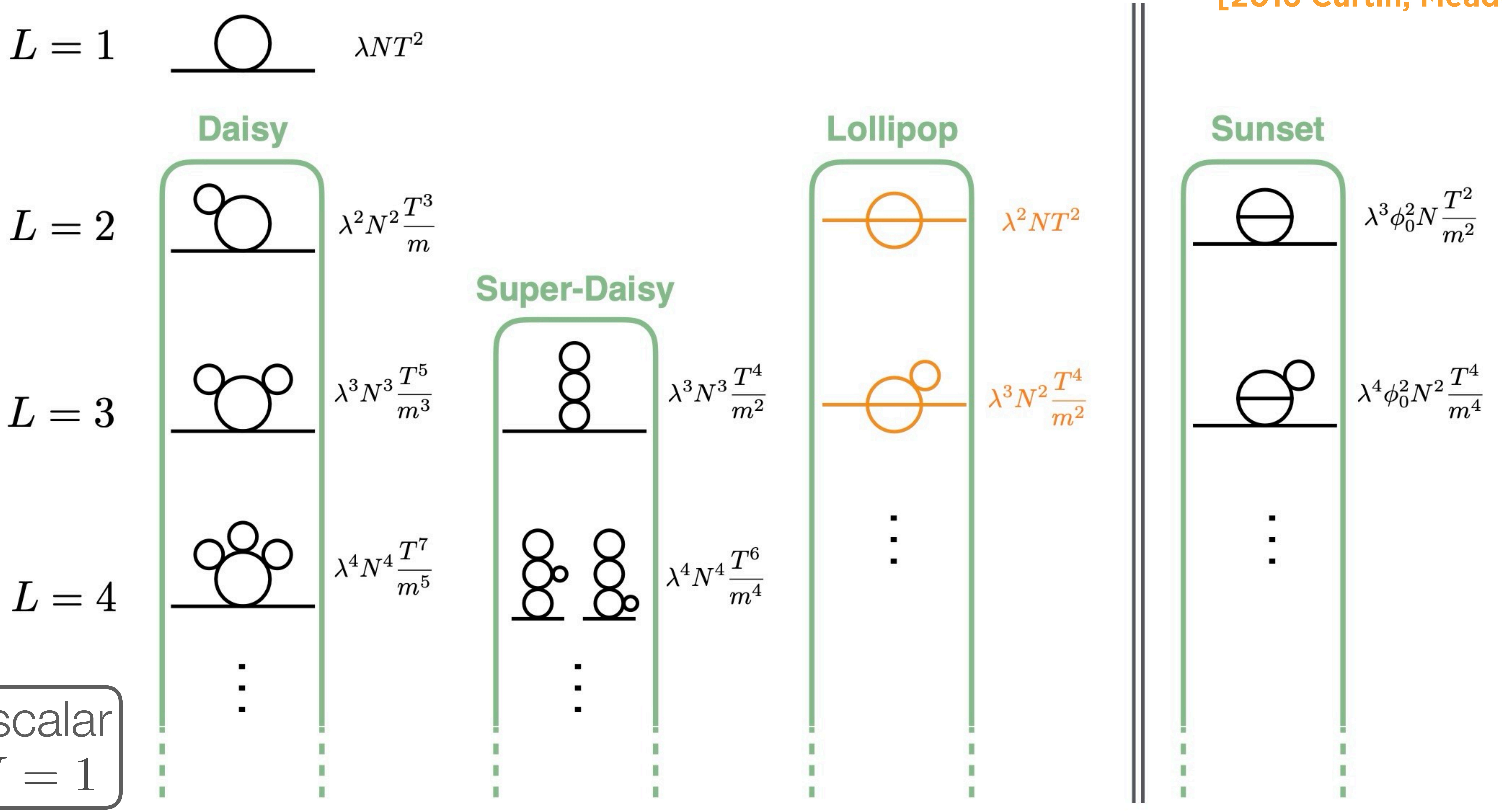


- Iterating the gap equation gives:

$$\mathcal{M}^2_{(0)} = m^2, \qquad \mathcal{M}^2_{(n+1)} = m^2 + \Pi(\mathcal{M}^2_{(n)}, T)$$

Beyond Daisies: The Relevant Topologies

[2016 Curtin, Meade, Ramani]



Single-scalar case: $N = 1$

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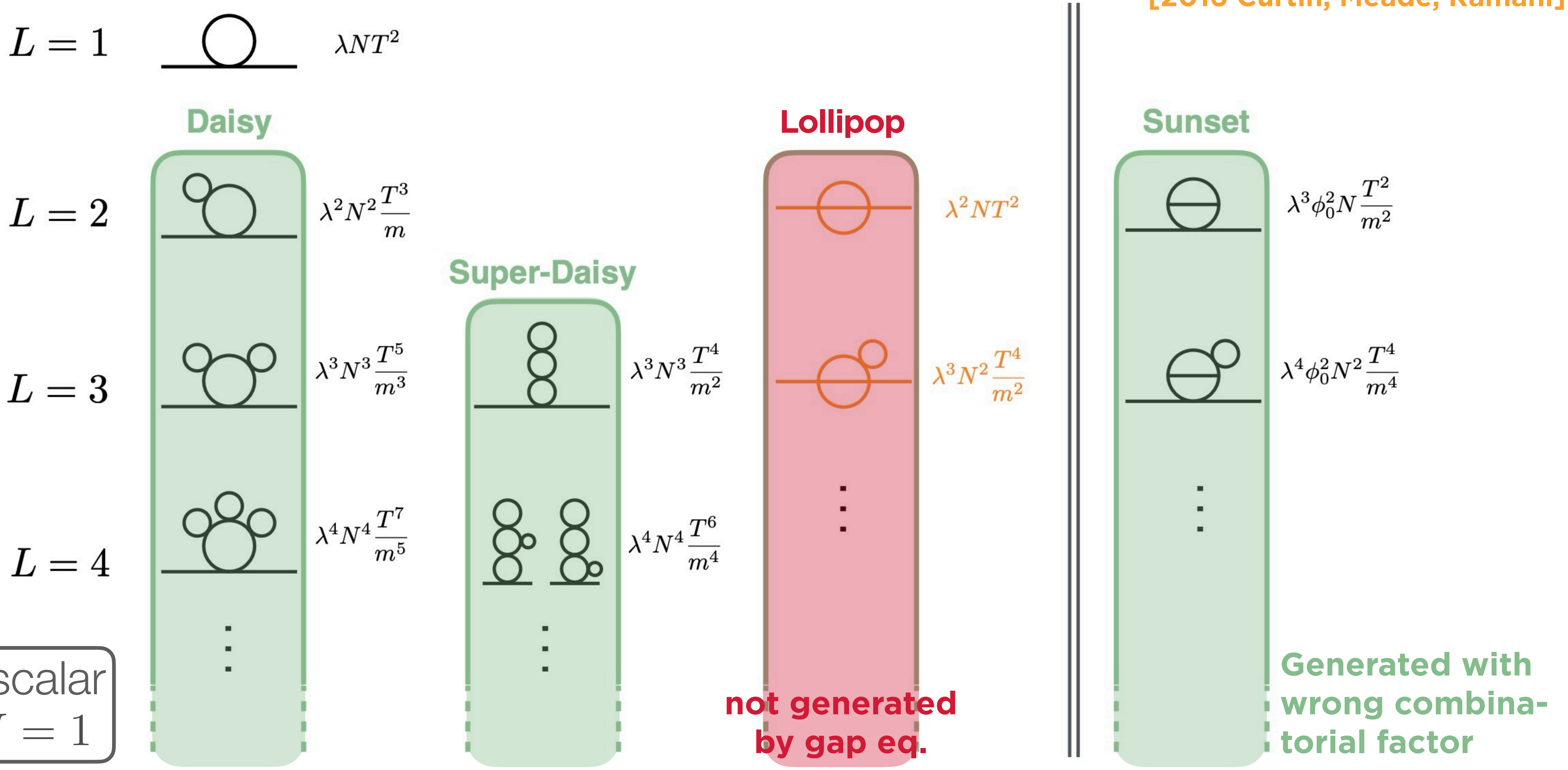

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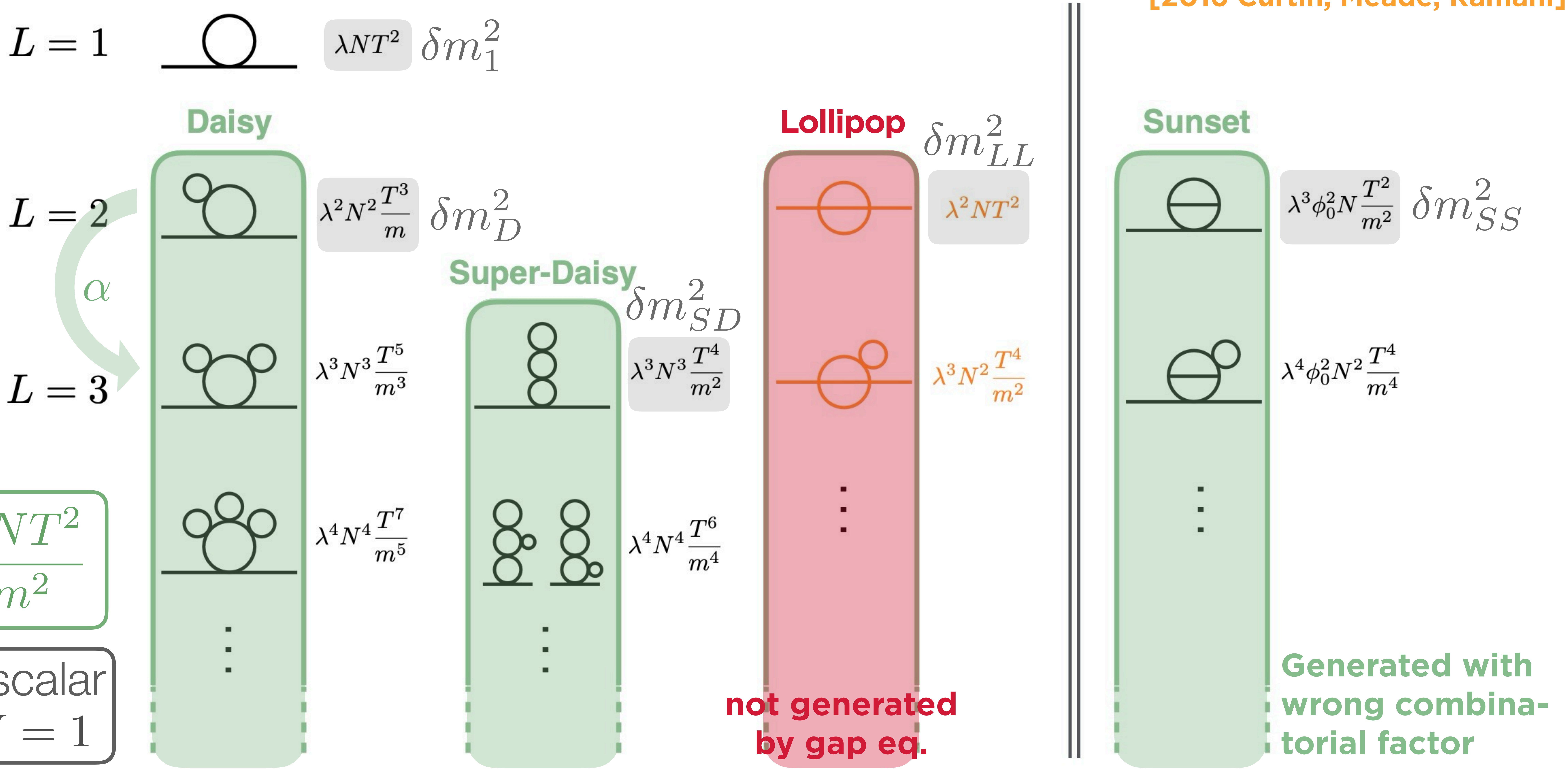


$$\alpha = \frac{\lambda N T^2}{m^2}$$

Single-scalar case: $N = 1$

Convergence After Resummation

- After resumming in α, compare leading contributions from different families:

$$\frac{\delta m_D^2}{\delta m_1^2} \sim \frac{\delta m_{SD}^2}{\delta m_D^2} \sim \boxed{\lambda N \frac{T}{m} \equiv \beta}$$

➡ β controls the hierarchy of the resummed expansion.

$$\alpha \sim 1 \longrightarrow \beta = \alpha \frac{m}{T} \sim \sqrt{N\lambda} \ll 1$$

- Other subleading topologies:

$$\frac{\delta m_{LL}^2}{\delta m_D^2} \sim \beta \frac{1}{N}\,, \qquad \frac{\delta m_{SS}^2}{\delta m_D^2} \sim \beta \frac{\phi^2}{T^2} \frac{1}{N^2}$$

➡ **Superdaisy, lollipop** and **sunset** enter at same next order in β

(up to additional N and ϕ/T suppression)

Dressing Procedure

Full Dressing (FD):

- Direct mass replacement in CW and thermal potential (standard approach)

$$V_{\text{eff}}^{\text{fd}}(\phi, T) = V_{\text{tree}} + V_{\text{CW}}(\mathcal{M}^2) + V_T(\mathcal{M}^2)$$

$$\sim \sum_n \int_{\mathbf{p}} \log(\omega_n^2 + \mathbf{p}^2 + \mathcal{M}^2)$$

➡ Dressed propagator **and** vertex

$$\frac{\mathrm{d}V_{\text{eff}}^{\text{fd}}}{\mathrm{d}\phi} \propto \left(\frac{\mathrm{d}\mathcal{M}^2}{\mathrm{d}\phi}\right) \int_{\mathbf{p}} \frac{1}{w_n^2 + \mathbf{p}^2 + \mathcal{M}^2}$$

Partial Dressing (PD):

- Mass replacement in tadpole, integrating back w.r.t. field

$$V_{\text{eff}}^{\text{pd}} = V_{\text{tree}} + \int \mathrm{d}\phi \, \frac{\mathrm{d}(V_{\text{CW}} + V_T)}{\mathrm{d}\phi}(\mathcal{M}^2)$$

$$\sim \frac{\mathrm{d}}{\mathrm{d}\phi} \sum_n \int_{\mathbf{p}} \log(\omega_n^2 + \mathbf{p}^2 + m^2)\Big|_{m^2 \to \mathcal{M}^2}$$

➡ Dressed propagator

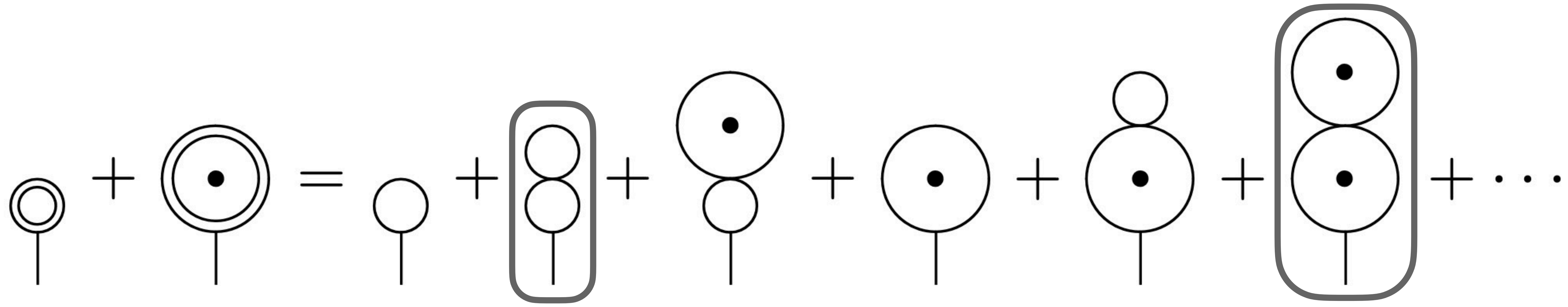
$$\frac{\mathrm{d}V_{\text{eff}}^{\text{pd}}}{\mathrm{d}\phi} \propto \left(\frac{\mathrm{d}m^2}{\mathrm{d}\phi}\right) \int_{\mathbf{p}} \frac{1}{w_n^2 + \mathbf{p}^2 + \mathcal{M}^2}$$

Full Dressing (FD) vs. Partial Dressing (PD)

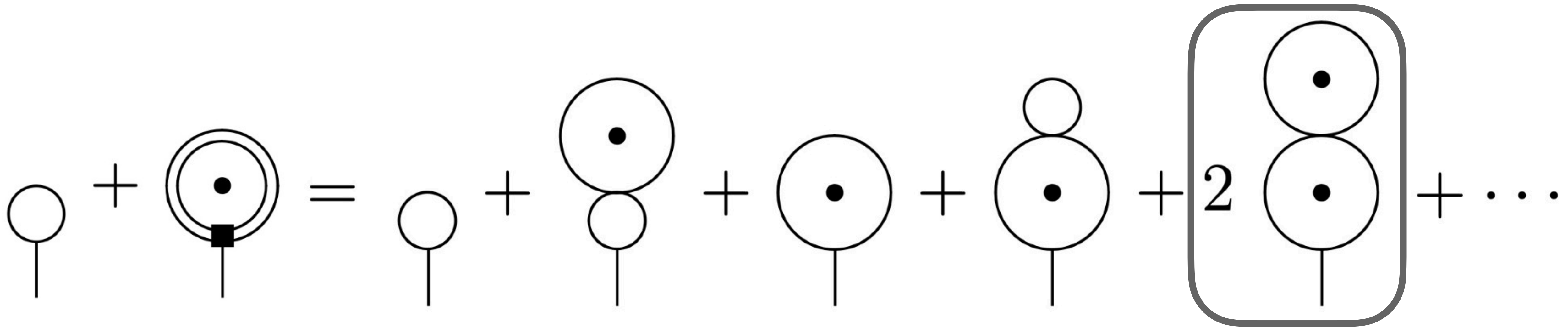
Which one is the correct one?

➡ FD miscounts daisy and superdaisy diagrams beyond one-loop order
[1993 Boyd, Brahm, Hsu]

$V'^{\mathrm{pd}}_{\mathrm{eff}}(\phi, T) =$



$V'^{\mathrm{fd}}_{\mathrm{eff}}(\phi, T) =$



Notice: At leading order in temperature $(\Pi \propto T^2)$, both schemes agree!
This approximation is called Truncated Full Dressing (**TFD**).

Optimised Partial Dressing (OPD)

- Thermal effective potential and self-energies are functions of **thermal bosonic/fermionic functions**.

$$V_T(h,T) \sim \frac{T^4}{2\pi^2} J_{B/F}\left(\frac{m_i^2(h)}{T^2}\right), \quad \Pi_i(h,T) \sim \sum_j c_{ij}\left[T^2 J'_{B/F}\left(\frac{m_j^2(h)}{T^2}\right) + \cdots\right]$$

- Thermal functions admit a high-temperature expansion:

$$J_B(x) = -\frac{\pi^4}{45} + \frac{\pi^2}{12}x - \frac{\pi}{6}x^{3/2} - \frac{x^2}{32}\log\frac{x}{a_b} + \cdots$$

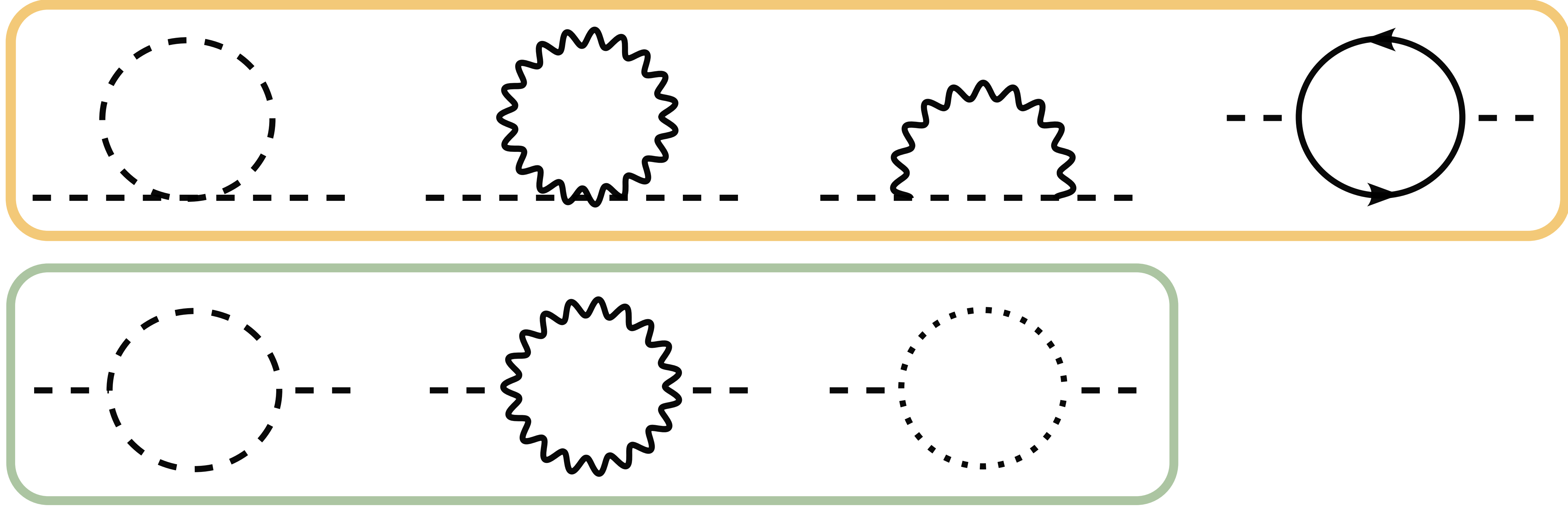
- Optimised Partial Dressing (**OPD**) uses hight-T approximation for effective masses, but keeps the **full** thermal functions in effective potential

➡ High accuracy, only $\mathcal{O}(10\%)$ slower than TFD [2016 Curtin, Meade, Ramani] [2022 Curtin, Roy, White]

2. The Work

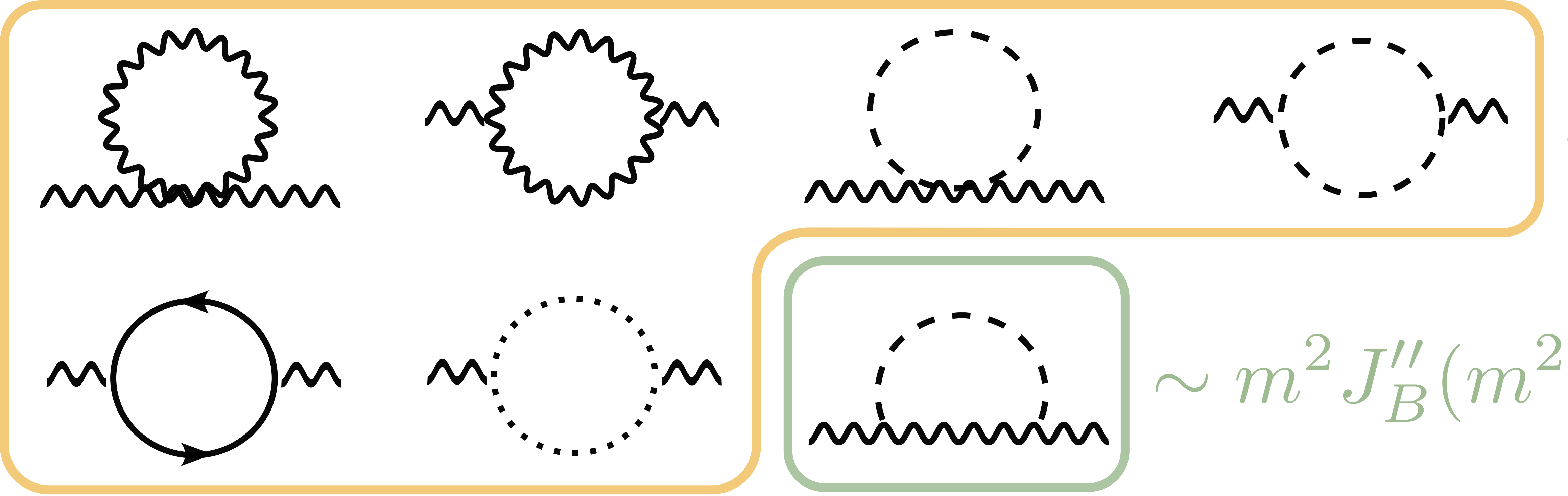
SM Self Energies

- **Scalars**



Work done in [2016 Curtin,Meade, Ramani] using OPD and second derivative of potential

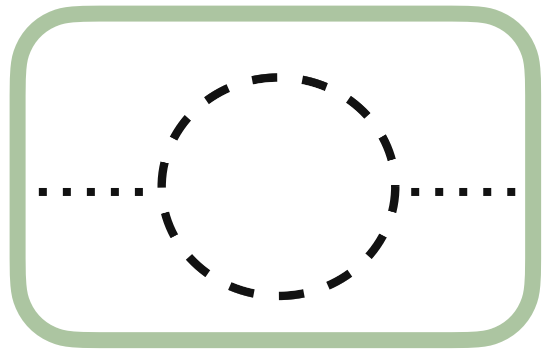
- **Gauge bosons**



$\sim T^2 J_B'(m^2/T^2)$

$\sim m^2 J_B''(m^2/T^2)$

- **Ghosts**



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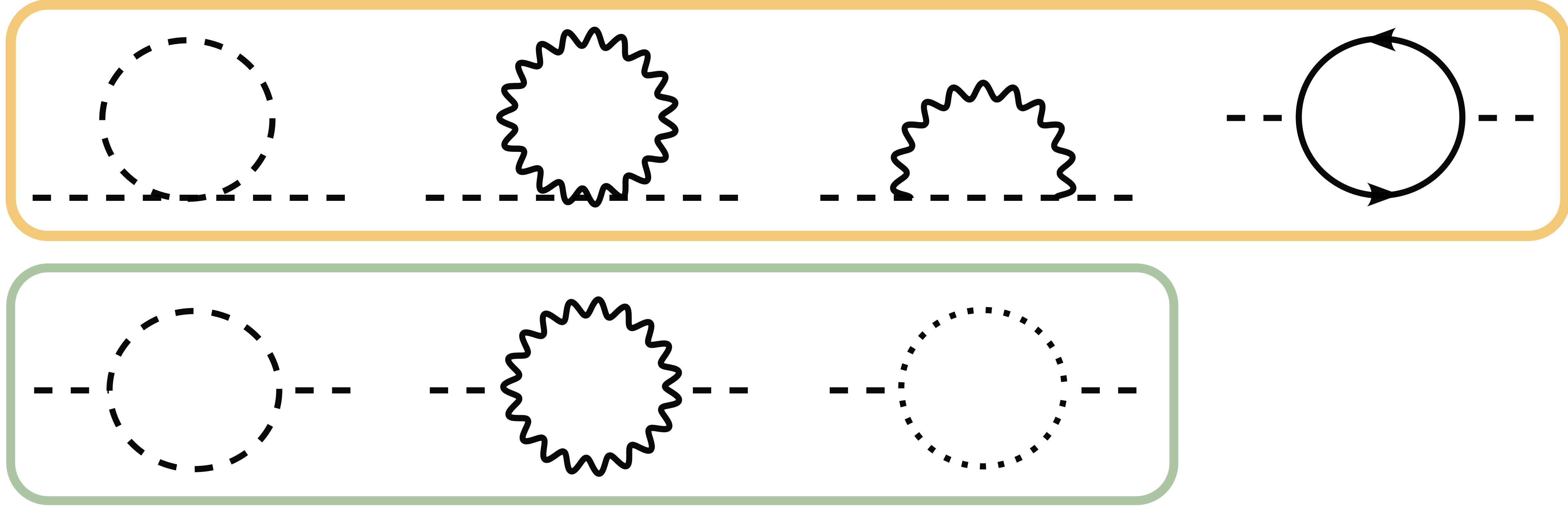
SM Self Energies

- **Scalars**



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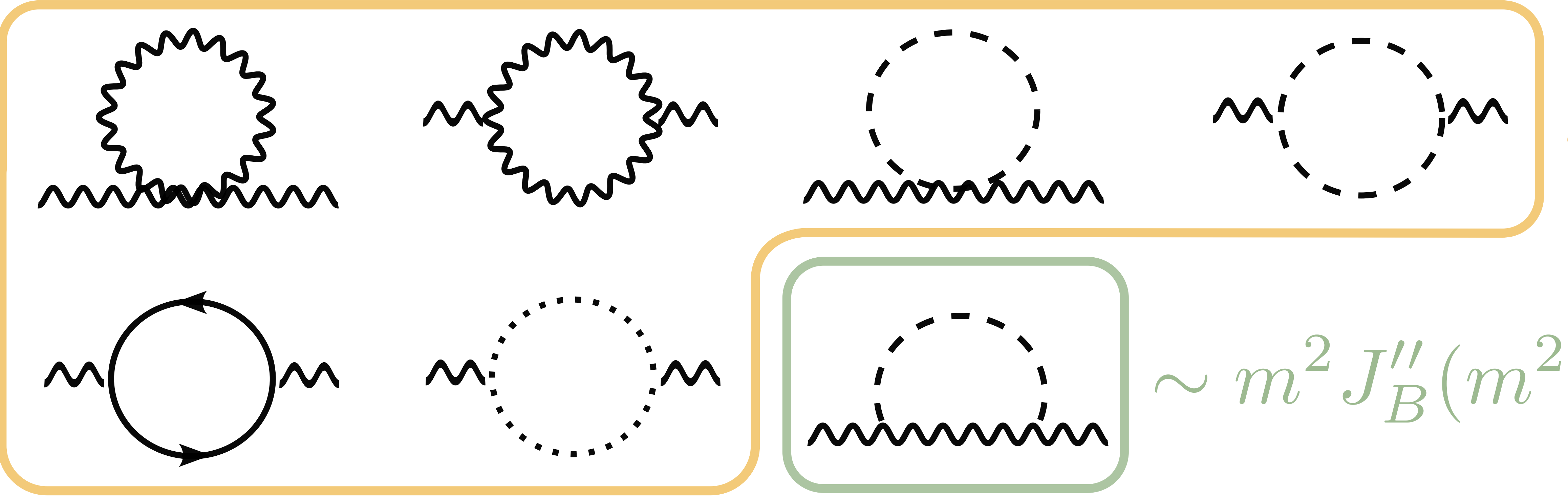

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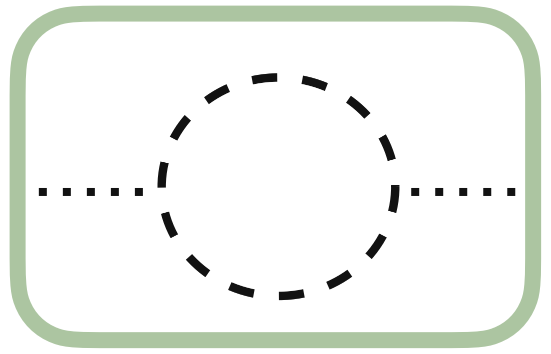


$\sim m^2 J_B''(m^2/T^2)$

Include Contributions Beyond Leading Order in T

Scalar PD Resummation

- Start from the **scalar** tadpole:

$$\Gamma_1^{\text{scalar}} \sim \frac{1}{2}\mathrm{Tr}\log\mathcal{D}_0^{-1}(\mathbf{p},\omega_n) \Longrightarrow V'_{\text{scalar}}(h,T) \sim T\sum_n \int_{\mathbf{p}} \mathrm{tr}\left[\mathcal{D}_0(\mathbf{p},\omega_n)\partial_h m_\chi^2(h,\xi)\right]$$

with scalar propagator:

$$\mathcal{D}_0(\mathbf{p},\omega_n)_{ij} = \left(\omega_n^2 + \mathbf{p}^2 + m_\chi^2(h,\xi)\right)_{ij}^{-1}.$$

$$= \mathrm{diag}(m_{G^\pm}^2, m_{G^\pm}^2, m_h^2, m_{G^0}^2)$$

- Resum self-energy insertions into propagator:

$$\mathcal{D}_0(\mathbf{p},\omega_n)_{ij} \longrightarrow \mathcal{D}(\mathbf{p},\omega_n)_{ij} = \left(\omega_n^2 + \mathbf{p}^2 + m_\chi^2(h,\xi) + \Pi(0)\right)_{ij}^{-1}$$

with $\Pi_{ij}(0) = \mathrm{diag}\left(\Pi_{G^\pm}(0), \Pi_{G^\pm}(0), \Pi_h(0), \Pi_{G^0}(0)\right)$

Scalar PD Resummation

- Trace reduces to a sum over scalar modes:

$$\mathrm{tr}\left[\mathcal{D}(\mathbf{p},\omega_n)\,\partial_h m_\chi^2(h,\xi)\right] = \sum_s n_s \frac{\partial_h m_s^2(h,\xi)}{\omega_n^2 + \mathbf{p}^2 + \mathcal{M}_s^2(h,T)}\,, \qquad s \in \{G^\pm, h, G^0\}$$

with $n_{G^\pm} = 2,\ n_h = n_{G^0} = 1$ and effective mass $\mathcal{M}_s^2(h,T) = m_s^2(h,\xi) + \Pi_s(0)$

➡ Final PD-resummed scalar tadpole:

$$\boxed{V'^{\,\mathrm{PD}}_{\mathrm{scalar}}(h) \propto \sum_s n_s \frac{\mathrm{d}}{\mathrm{d}h}\left[\frac{m_s^4(h,\xi)}{64\pi^2}\left(\log\frac{m_s^2(h,\xi)}{\mu^2} - \frac{3}{2}\right) + \frac{T^4}{2\pi^2} J_B\!\left(\frac{m_s^2(h,\xi)}{T^2}\right)\right]_{m_s^2 \to \mathcal{M}_s^2}}$$

Important: Replace **only** explicit mass arguments; do **not** dress derivatives!

Gauge-Boson PD Resummation

- Start from the **gauge boson** tadpole:

$$\mathrm{diag}\left(m_W^2,\, m_W^2,\, m_Z^2,\, 0\right)$$

$$\Gamma_1^{\text{gauge}} \sim \frac{1}{2}\,\mathrm{Tr}\log \mathcal{D}_0^{-1}(\mathbf{p},\omega_n) \Longrightarrow V'_{\text{gauge}}(h,T) \sim T\sum_n \int_{\mathbf{p}} \mathrm{tr}\left[\mathcal{D}_0(\mathbf{p},\omega_n)\,\partial_h m_A^2(h)\right]$$

with gauge boson propagator:

$$\boxed{P_i P_j = \delta_{ij} P_i\,,\ \sum_i P_i = \mathbf{1}}$$

projectors

$$\mathcal{D}_0(\mathbf{p},\omega_n)^{\mu\nu}_{ab} \equiv \underbrace{(P_T + P_L)^{\mu\nu}}_{=\ \delta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}}(p^2 + m_A^2)^{-1}_{ab} + \xi\, \underbrace{P_D^{\mu\nu}}_{=\ \frac{k^\mu k^\nu}{k^2}}(p^2 + \xi m_A^2)^{-1}_{ab}$$

➡ Lives in $\text{Lorentz space} \otimes \text{internal gauge space}$

Gauge-Boson PD Resummation

- Thermal medium introduces preferred four-velocity u^μ — **general tensor decomposition:** [1997 Weldon]

$$\Pi^{\mu\nu}_{ab}(k) = P^{\mu\nu}_T \Pi_T(k)_{ab} + P^{\mu\nu}_L \Pi_L(k)_{ab} + P^{\mu\nu}_C \Pi_C(k)_{ab} + P^{\mu\nu}_D \Pi_D(k)_{ab}$$

spatially trans. $\perp \mathbf{k}$

thermal long. $\perp k^\mu$ **within** (u, k)**-plane**

four longitudinal $\parallel k^\mu$

- In **static infrared limit** $(k^0 = 0, |\mathbf{k}| \to 0)$:

$$\boxed{\begin{gathered}\Pi_T(0)_{ab} = \Pi_D(0)_{ab},\ \Pi_C(0)_{ab} = 0 \\ \Pi^{\mu\nu}_{ab}(0) = (P^{\mu\nu}_T + P^{\mu\nu}_D)\Pi_T(0)_{ab} + P^{\mu\nu}_L \Pi_L(0)_{ab}\end{gathered}}$$

Gauge-Boson PD Resummation

$$\boxed{\Pi^{\mu\nu}_{ab}(0) = (P^{\mu\nu}_T + P^{\mu\nu}_D)\Pi_T(0)_{ab} + P^{\mu\nu}_L \Pi_L(0)_{ab}}$$

- Independent self-energies from temporal and spatial components:

$$\Pi_L(0)_{ab} = \Pi^{00}_{ab}(0) \xrightarrow{T \gg m} \Pi^{00}_{ab}(0)\Big|_{\mathrm{LOHT}} \sim g^2T^2 \neq 0,$$

$$\Pi_T(0)_{ab} = \Pi^{ii}_{ab}(0) \xrightarrow{T \gg m} \Pi^{ii}_{ab}(0)\Big|_{\mathrm{LOHT}} = 0 \quad (\text{no sum over } i)$$

Leading high-T

$$M_L^2 = m_A^2 + \Pi^{00}(0)$$

$$M_T^2 = M_D^2 = m_A^2$$

➡ **Only L-sector** receives thermal mass correction

Beyond leading high-T

$$M_L^2 = m_A^2 + \Pi^{00}(0)$$

$$M_T^2 = M_D^2 = m_A^2 + \Pi^{ii}(0)$$

➡ **T- and D-sectors** receive corrections

Gauge-Boson PD Resummation

How about **internal gauge space?**

- Working in tree-level mass basis $m_A^2(h)_{ab} = \mathrm{diag}\left(m_W^2,\, m_W^2,\, m_Z^2,\, 0\right)$
- Unlike scalar sector, gauge boson sector features mixing:

diagonal

$$\Pi_X(0)_{ab} = \begin{pmatrix} \Pi_{W,X}(0) & 0 & 0 & 0 \\ 0 & \Pi_{W,X}(0) & 0 & 0 \\ 0 & 0 & \Pi_{Z,X}(0) & \Pi_{Z\gamma,X}(0) \\ 0 & 0 & \Pi_{\gamma Z,X}(0) & \Pi_{\gamma,X}(0) \end{pmatrix}$$

off-diagonal

➡ **Charged sector** gets simple mass shift $m_W^2(h) \to \mathcal{M}_{W,X}^2 \equiv m_W^2(h) + \Pi_{W,X}(0)$
but **neutral sector** needs more treatment!

Gauge-Boson PD Resummation

- Diagonalise neutral 2x2 mass matrix $(\mathcal{M}_X^2)_{Z\gamma} = \begin{pmatrix} m_Z^2 + \Pi_{Z,X}(0) & \Pi_{Z\gamma,X}(0) \\ \Pi_{\gamma Z,X}(0) & \Pi_{\gamma,X}(0) \end{pmatrix}$ using orthogonal transformations:

$$U_X^T (\mathcal{M}_X^2)_{Z\gamma} U_X = \mathrm{diag}(\mathcal{M}_{1,X}^2, \mathcal{M}_{2,X}^2)$$

- Propagator becomes diagonal:

$$\mathcal{D}_X(\mathbf{p}, \omega_n)_{(Z\gamma)} = U_X \ \mathrm{diag}\left(\frac{1}{\omega_n^2 + \mathbf{p}^2 + \mathcal{M}_{1,X}^2}, \frac{1}{\omega_n^2 + \mathbf{p}^2 + \mathcal{M}_{2,X}^2} \right) U_X^T.$$

- Evaluate the trace:

Eigenmodes weighted by overlap with tree-level Z

$$\mathrm{tr}\left[\mathcal{D}_X(\mathbf{p}, \omega_n)_{(Z\gamma)} \left(\partial_h m_A^2\right)_{(Z\gamma)} \right] = \sum_{i=1}^{2} \frac{U_{Zi,X}^2 \, m_Z^{2\prime}(h)}{\omega_n^2 + \mathbf{p}^2 + \mathcal{M}_{i,X}^2(h,T)}$$

Gauge-Boson PD Resummation

- Each Lorentz $X \in \{L, T, D\}$ sector has its own mass matrix:

$$(\mathcal{M}_T^2)_{ab} = (\mathcal{M}_D^2)_{ab} = m_A^2(h)_{ab} + \Pi_{ab}^{ii}(0),$$

$$(\mathcal{M}_L^2)_{ab} = m_A^2(h)_{ab} + \Pi_{ab}^{00}(0).$$

➡ Final PD-resummed gauge-boson tadpole:

$$\boxed{\begin{aligned} V'^{\,\mathrm{PD}}_{\mathrm{gauge}}(h) \propto \sum_X n_X \Bigg\{ & 2\,\frac{\mathrm{d}}{\mathrm{d}h}\left[\frac{m_{W,X}^4(h)}{64\pi^2}\left(\log\frac{m_{W,X}^2(h)}{\mu^2} - \frac{3}{2}\right) + \frac{T^4}{2\pi^2} J_B\!\left(\frac{m_{W,X}^2(h)}{T^2}\right)\right]_{m_W^2 \to \mathcal{M}_{W,X}^2} \\ & + \sum_{i=1}^{2} U_{Zi,X}^2\,\frac{\mathrm{d}}{\mathrm{d}h}\left[\frac{m_{Z,X}^4(h)}{64\pi^2}\left(\log\frac{m_{Z,X}^2(h)}{\mu^2} - \frac{3}{2}\right) + \frac{T^4}{2\pi^2} J_B\!\left(\frac{m_{Z,X}^2(h)}{T^2}\right)\right]_{m_Z^2 \to \mathcal{M}_{i,X}^2} \Bigg\} \end{aligned}}$$

with $n_T = 2,\ n_L = n_D = 1$ and $m_{W/Z,X}^2(h) = \left[1 + (\xi - 1)\delta_{XD}\right] m_{W/Z}^2(h)$

Ghosts PD Resummation

- In the infrared limit, **ghost** self-energies have no leading $\mathcal{O}(T^2)$ contribution

  ➡ Only relevant beyond leading high-T

- Start from the ghost tadpole:

$$\Gamma_1^{\text{ghost}} \sim -\operatorname{Tr}\log \mathcal{D}_0^{-1} \Longrightarrow V'_{\text{ghost}}(h,T) \sim -T\sum_n \int_{\mathbf{p}} \operatorname{tr}\left[\mathcal{D}_0(\mathbf{p},\omega_n)\,\partial_h\left(\xi m_A^2(h)\right)\right]$$

with ghost propagator:

$$\left(\mathcal{D}_0(\mathbf{p},\omega_n)\right)_{ab} = \left[\omega_n^2 + \mathbf{p}^2 + \underbrace{\xi m_A^2(h)}_{}\right]^{-1}_{ab}.$$

$$m_A^2(h) = \operatorname{diag}\left(m_W^2,\, m_W^2,\, m_Z^2,\, 0\right)$$

- Resum daisy insertions into the propagator:

$$\mathcal{D}_0(\mathbf{p},\omega_n)_{ab} \longrightarrow \mathcal{D}(\mathbf{p},\omega_n)_{ab} = \left(\omega_n^2 + \mathbf{p}^2 + \xi m_A^2 + \Pi_c(0)\right)^{-1}_{ab}$$

Ghosts PD Resummation

- Effective mass matrix is given by:

Only these entries receive mass corrections

$$\mathcal{M}_c^2(h,T) = \begin{pmatrix} \underbrace{\xi m_W^2 + \Pi_{c^\pm}}_{= \mathcal{M}_{c^\pm}^2} & 0 & 0 & 0 \\ 0 & \xi m_W^2 + \Pi_{c^\pm} & 0 & 0 \\ 0 & 0 & \underbrace{\xi m_Z^2 + \Pi_{c^Z}}_{= \mathcal{M}_{c^Z}^2} & \Pi_{\bar{c}^Z c^\gamma} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

➡ Final PD-resummed ghost tadpole:

$$\boxed{\begin{aligned} V'^{\,\mathrm{PD}}_{\mathrm{ghost}}(h) \propto & -2\frac{\mathrm{d}}{\mathrm{d}h}\left[\frac{(\xi m_W^2(h))^2}{64\pi^2}\left(\log\frac{\xi m_W^2(h)}{\mu^2} - \frac{3}{2}\right) + \frac{T^4}{2\pi^2} J_B\!\left(\frac{\xi m_W^2(h)}{T^2}\right)\right]_{\xi m_W^2 \to \mathcal{M}_{c^\pm}^2} \\ & - \frac{\mathrm{d}}{\mathrm{d}h}\left[\frac{(\xi m_Z^2(h))^2}{64\pi^2}\left(\log\frac{\xi m_Z^2(h)}{\mu^2} - \frac{3}{2}\right) + \frac{T^4}{2\pi^2} J_B\!\left(\frac{\xi m_Z^2(h)}{T^2}\right)\right]_{\xi m_Z^2 \to \mathcal{M}_{c^Z}^2} \end{aligned}}$$

Self-Consistent Gap Equations

- Beyond leading high-T, self-energies depend on the masses running inside the loops through thermal functions $J_B(m^2/T^2)/J_F(m^2/T^2)$

➡ Solve self-consistent gap equation:

$$\mathcal{M}_i^2(h,T) = m_i^2(h) + \Pi_i\big(\{\mathcal{M}_j^2(h,T)\};h,T\big)$$

- Numerically, we solve iteratively:

$$\mathcal{M}^2_{i,(n+1)} = m_i^2 + \Pi_i\left(\{\mathcal{M}^2_{j,(n)}\};h,T\right), \qquad \mathcal{M}^2_{i,(0)} = m_i^2$$

$$n=0:\ \text{daisy} \qquad \longrightarrow \qquad n>0:\ \text{superdaisies}$$

Self-Consistent Gap Equations

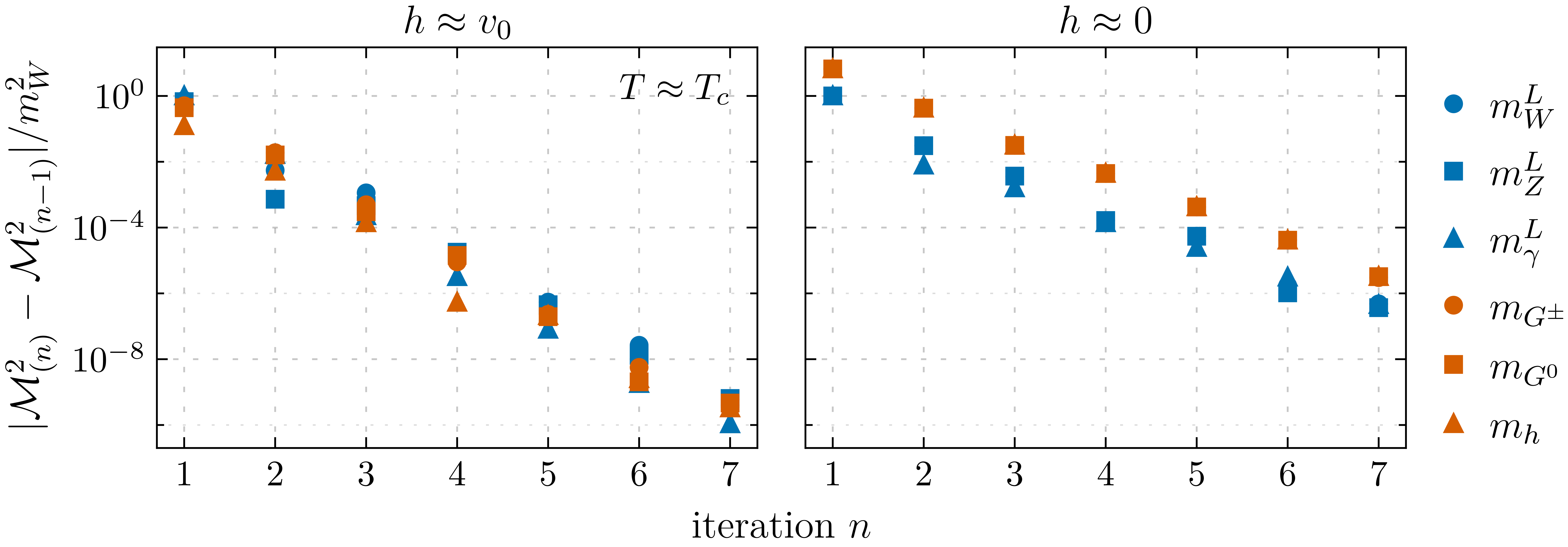
- All sectors are coupled through the masses appearing inside Π_i

Scalars	$\mathcal{M}_s^2 = m_s^2(h,\xi) + \Pi_s\big(\{\mathcal{M}_j^2\}; h, T\big)\,, \qquad s \in \{G^\pm, h, G^0\}.$
Ghosts	$\mathcal{M}_{c_i}^2 = \xi m_i^2(h) + \Pi_{c_i}\big(\{\mathcal{M}_j^2\}; h, T\big)\,, \qquad c_i \in \{c^\pm, c^Z\}.$
Longitudinal GB	$\big(\mathcal{M}_L^2\big)_{ab} = \big(m_A^2\big)_{ab} + \Pi_{ab}^{00}\big(\{\mathcal{M}_j^2\}; h, T\big)\,,$
Transverse GB	$\big(\mathcal{M}_T^2\big)_{ab} = \big(\mathcal{M}_D^2\big)_{ab} = \big(m_A^2\big)_{ab} + \Pi_{ab}^{ii}\big(\{\mathcal{M}_j^2\}; h, T\big)\,.$

➡ At each iteration, evaluate all self-energies using the complete mass spectrum from previous iteration, then update all masses simultaneously.

3. Numerical Results

Gap Equation: Convergence



➡ All scalar and longitudinal gauge masses **converge rapidly** and **simultaneously!**

(Sufficient to use n=4 in further numerical results)

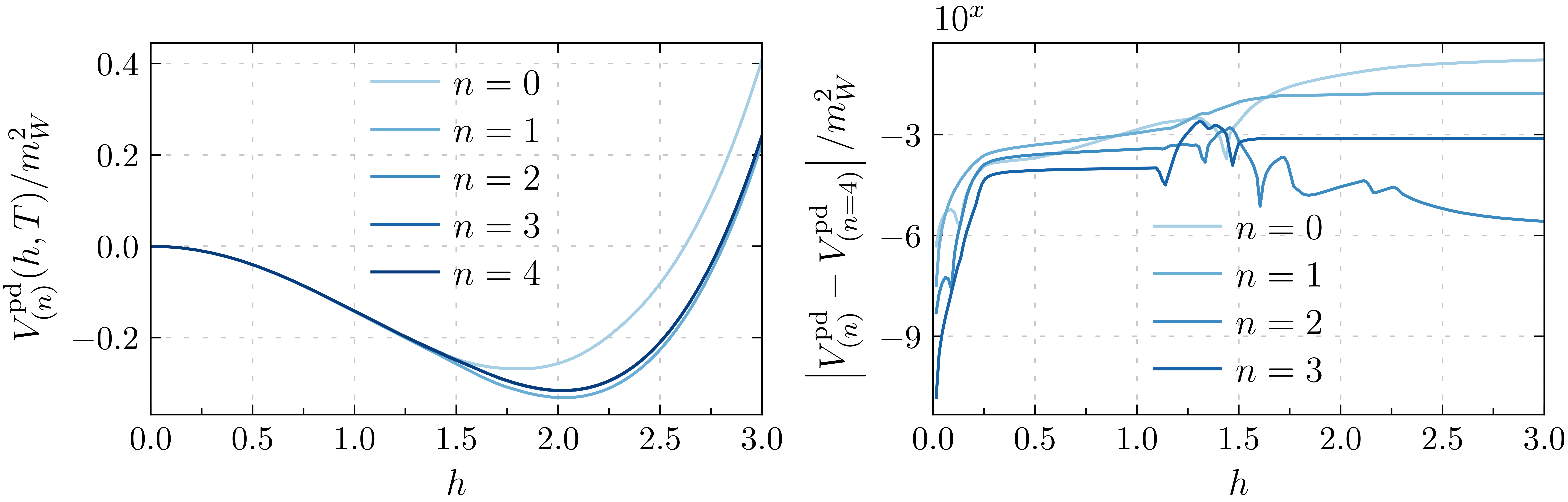
Gap Equation: Effective Masses

Gap Equation: Effective Masses

OPD ≃ PD

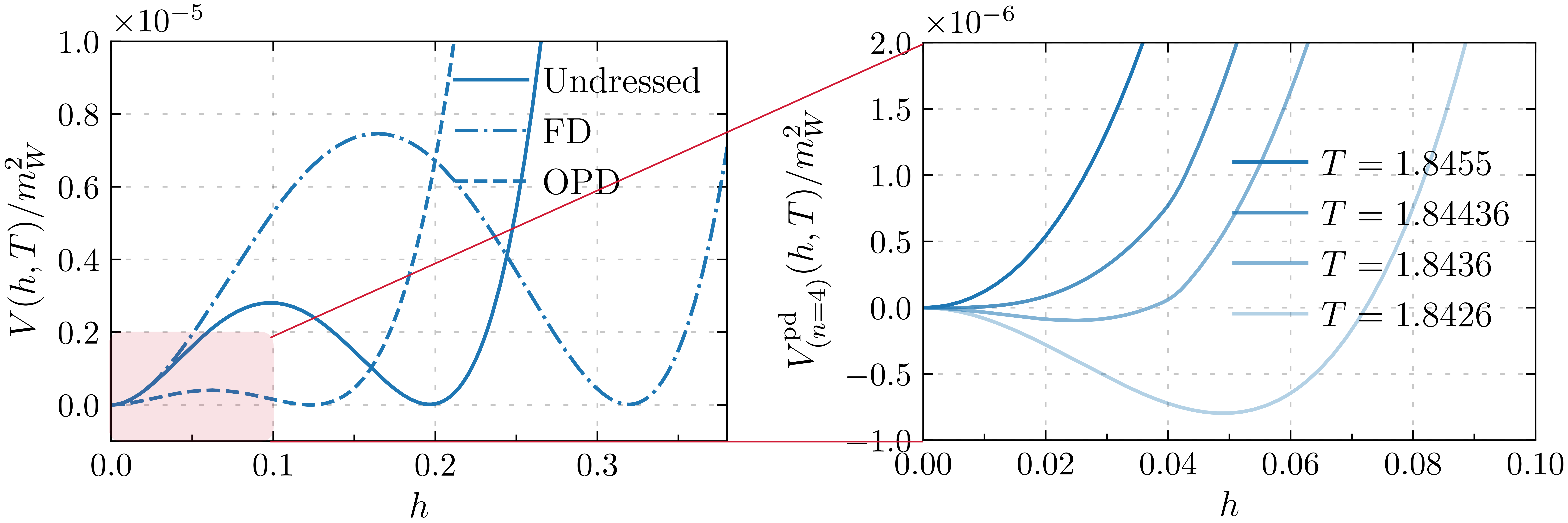
Gap Equation: Effective Masses

Effective Potential: Convergence



➡ The effective potential rapidly stabilizes under gap-equation iteration; **n=4** is used for subsequent numerical analysis.

The Barrier Disappears



➡ Including gauge sector self-consistently **removes weak first-order barrier** and drives perturbative result toward known SM crossover.

Future Work

1. Include lollipop diagram and correct sunset combinatorical factor:

Analytic and numerical study of scale dependence

2. Thermal decoupling and degrees of freedom:

compare with standard dressing and extend to first-order PTs / bubble-wall velocities

3. Gauge-parameter dependence:

Compare $V_{\mathrm{eff}}(\xi)$ and thermodynamic quantities with standard high-T resummation **[2016 Patel, Ramsey-Musolf]**