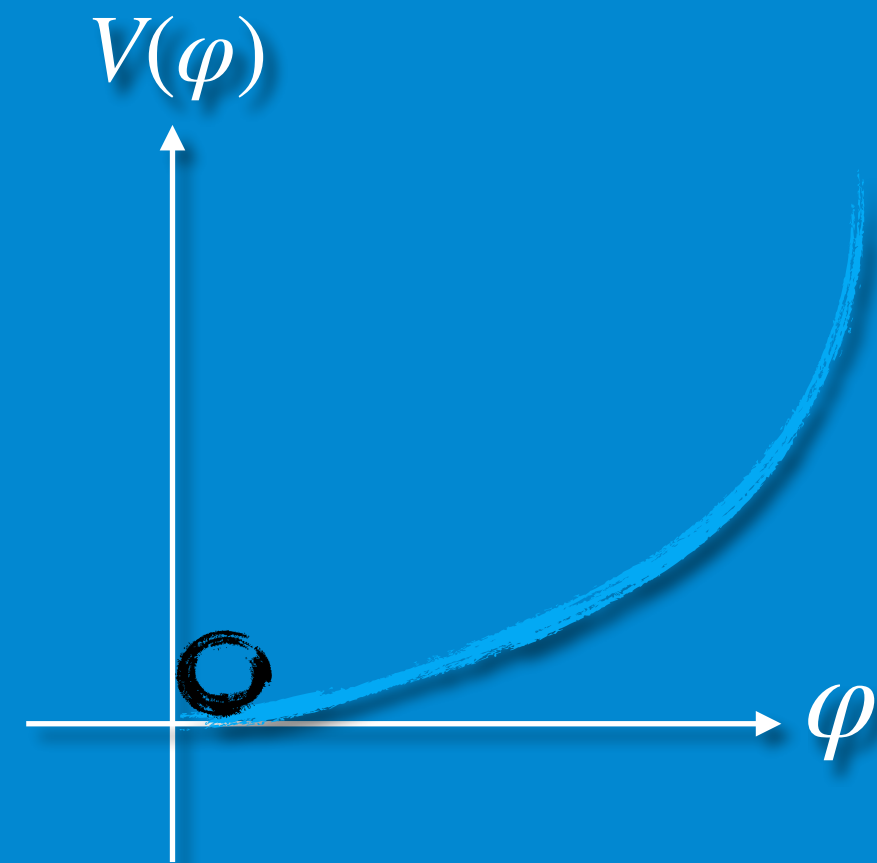


Effective methods for bubble nucleation in cosmological phase transitions (*and some pheno*)

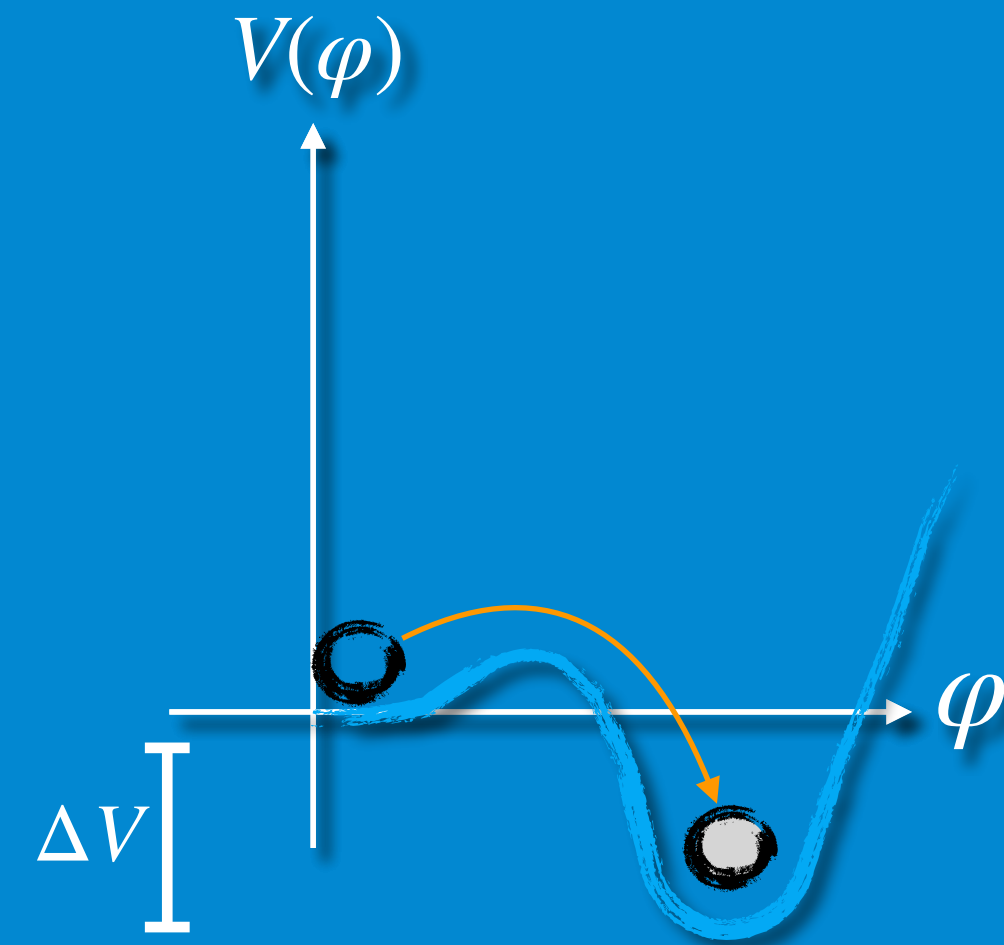
Maciej Kierkla, Uppsala University

Based on: 2312.12413, 2503.13597, 2506.15496, 2607.18233

Field theory
picture

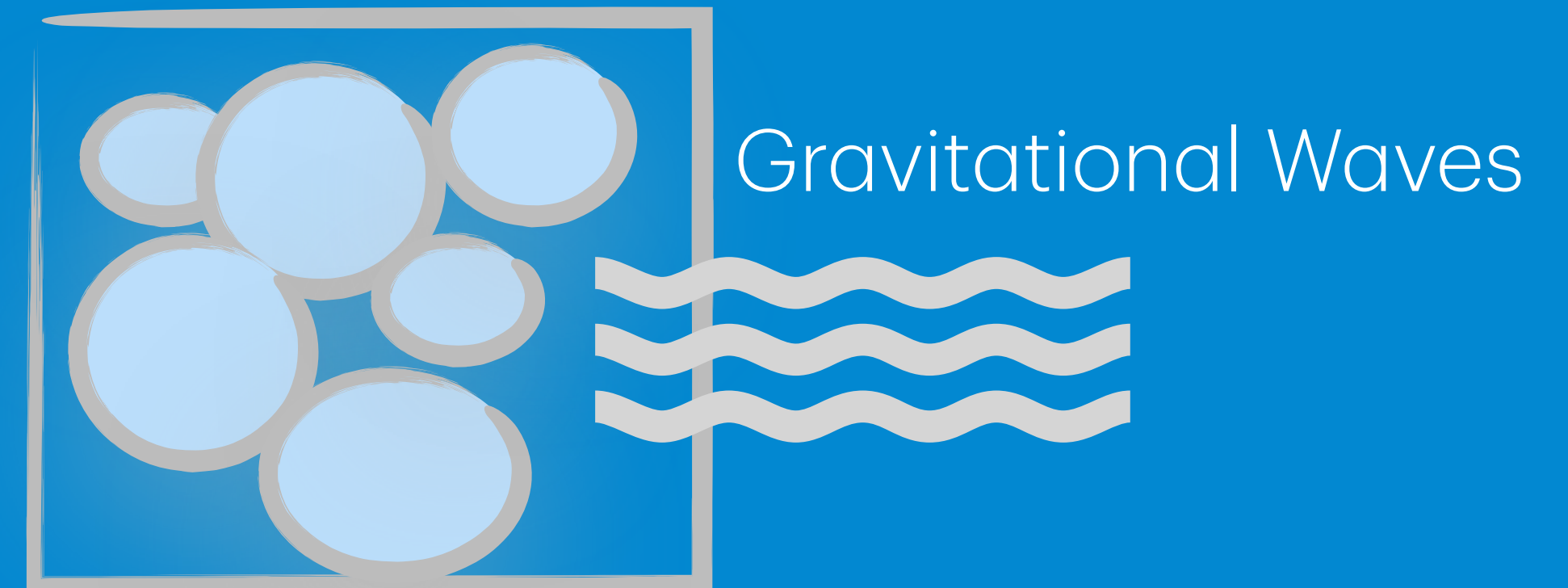
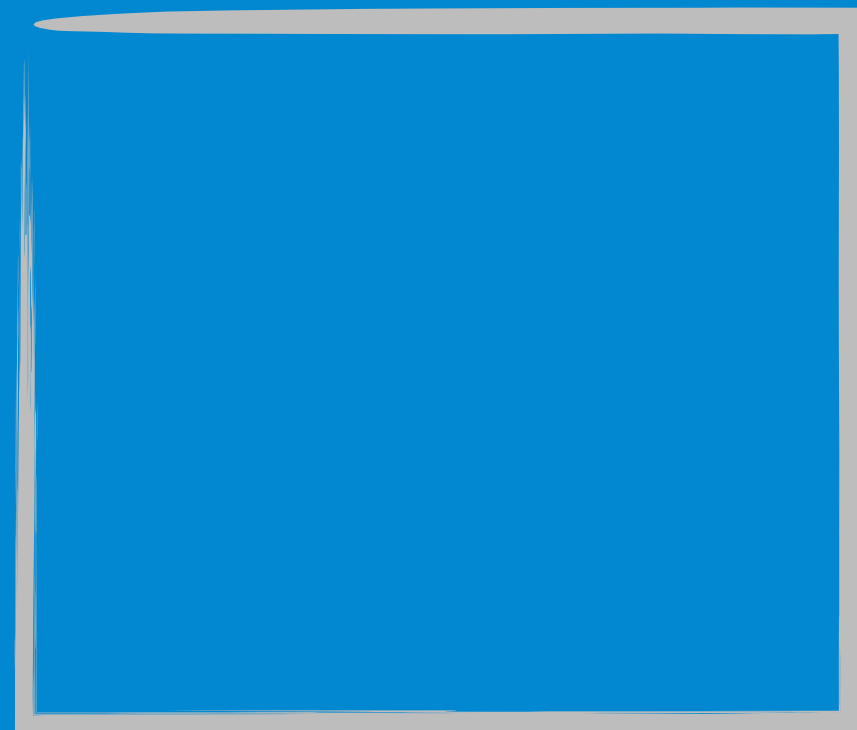


$$T \gg T_c$$



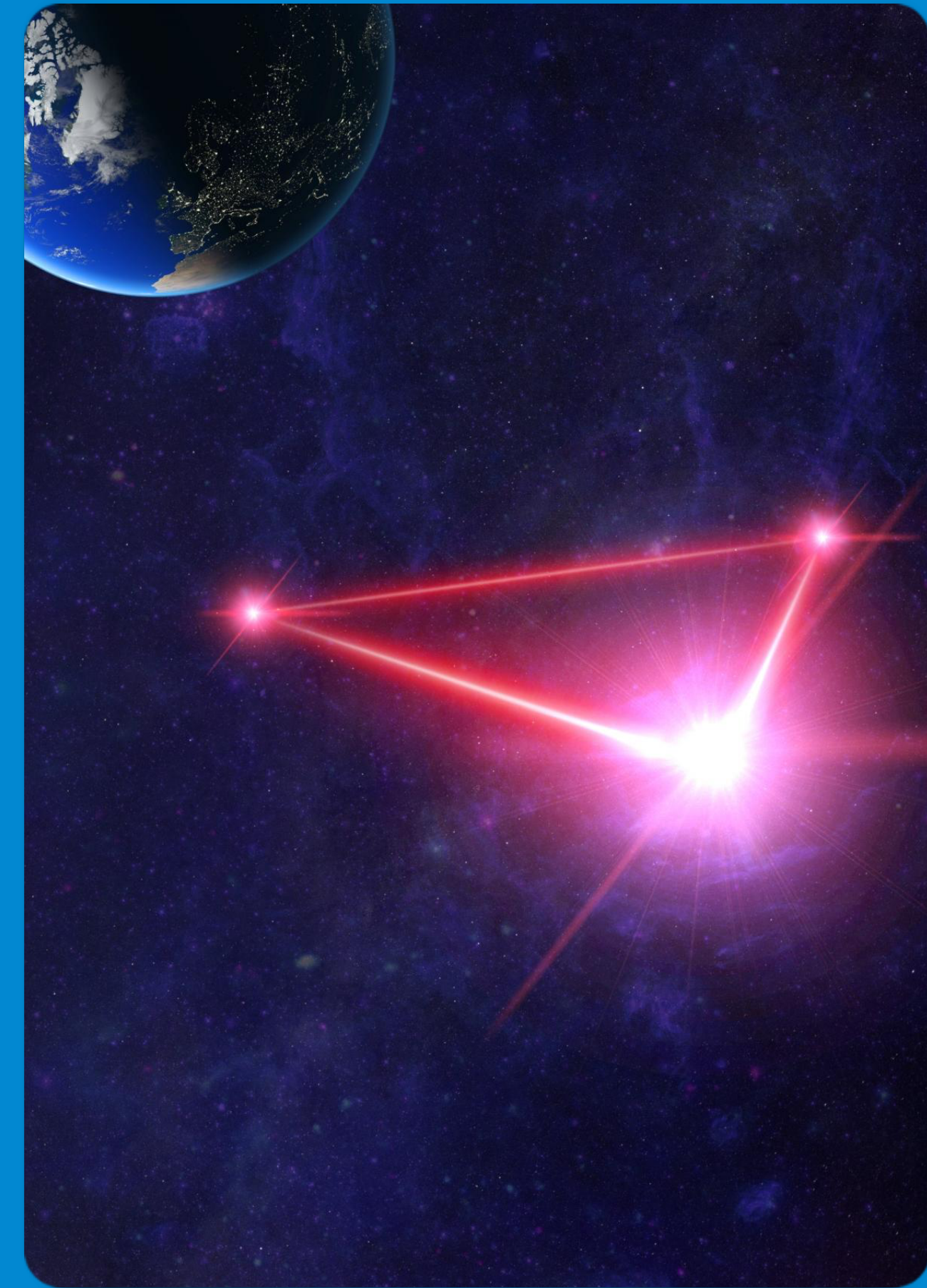
$$T_p$$

Cosmology
picture



Motivation

- LISA detector will start collecting data in the late 2030's
- LISA will be sensitive to the frequencies of order mHz, which includes PT at the EW scale
- Detection of such a signal is a “smoking gun” for new physics!
- There could be associated phenomena, e.g., dark matter, primordial black holes, and baryogenesis
- We need to be able to provide robust predictions

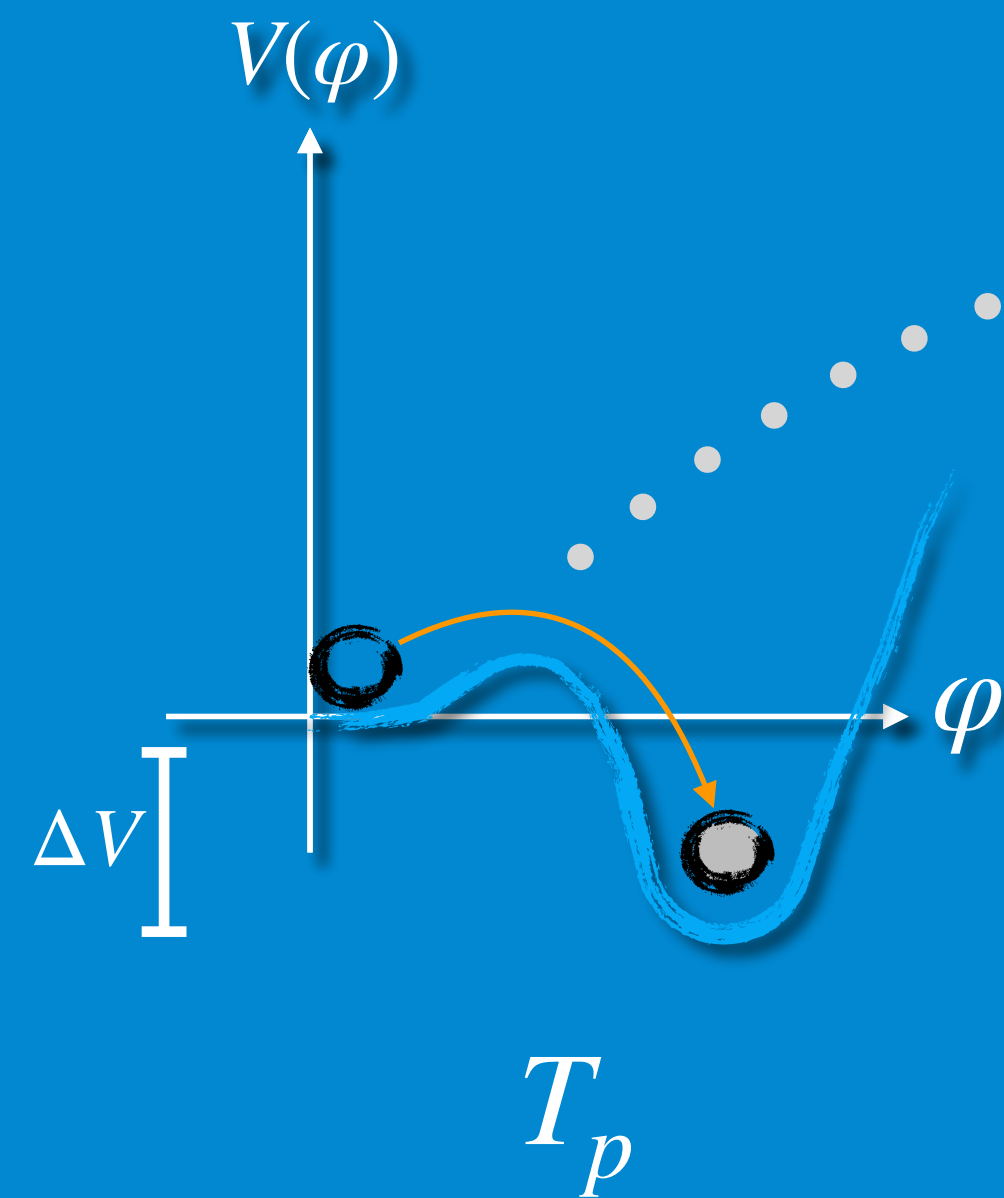


NASA/JPL-Caltech/NASAE/ESA/CXC/STScI/GSFC/VS/S.Barke (CC BY 4.0).

See also: 2403.03723, 2508.09989, 2503.01962

Key quantity for PT dynamics

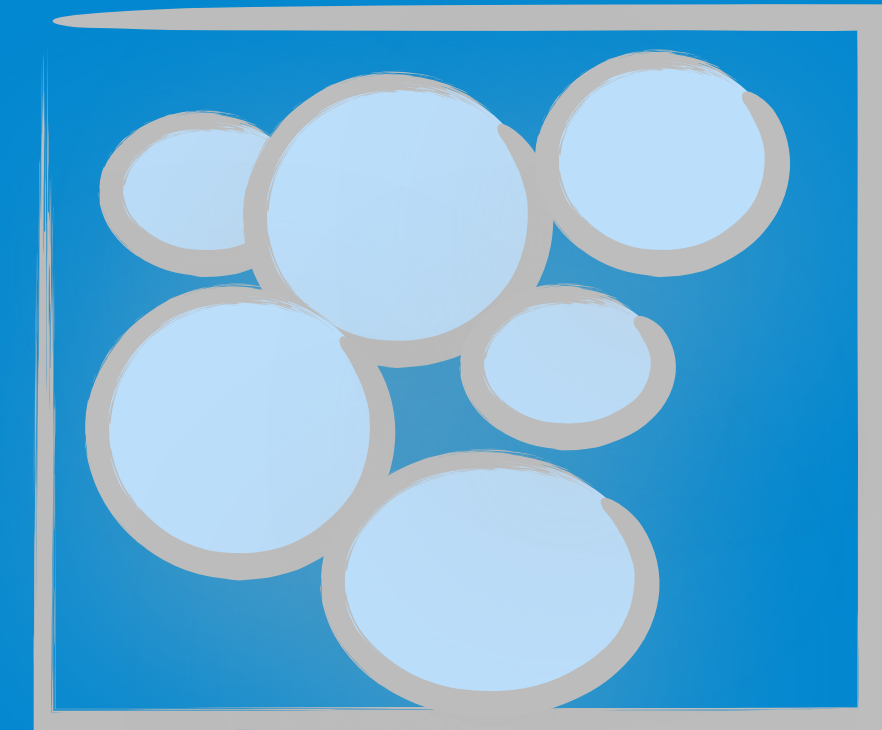
Field theory



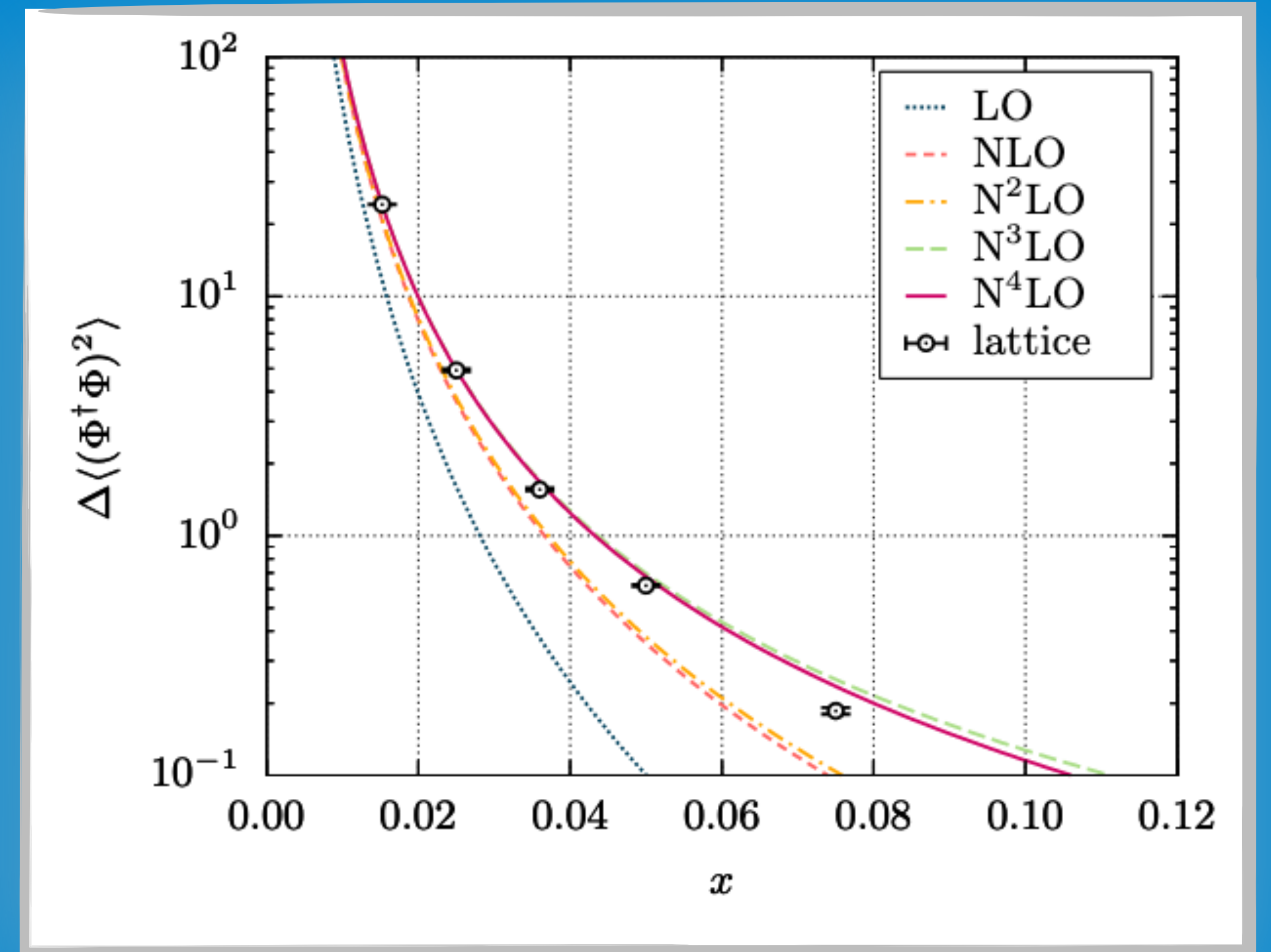
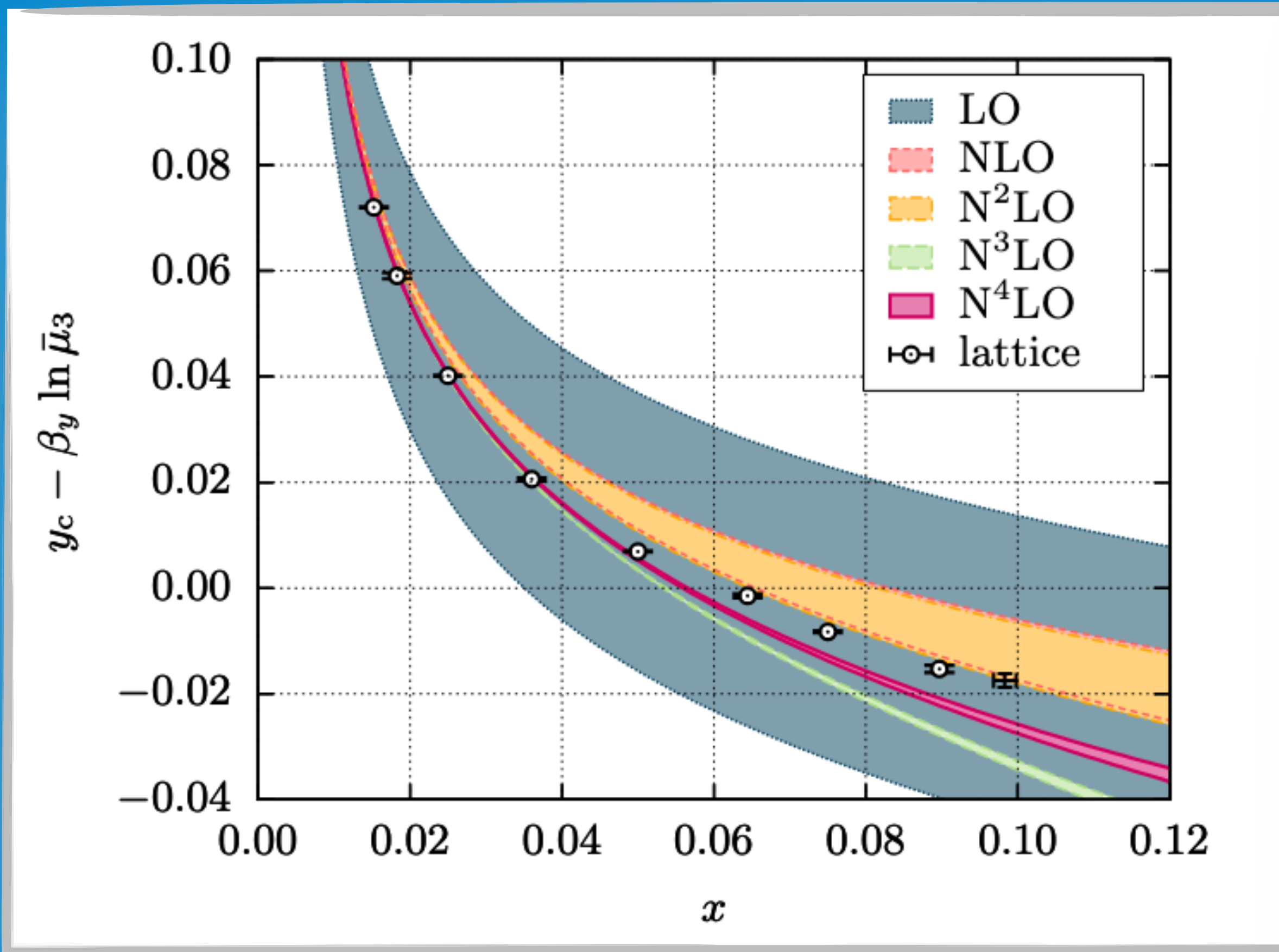
Bubble nucleation rate

$$\frac{\Gamma}{H^4}$$

Cosmology

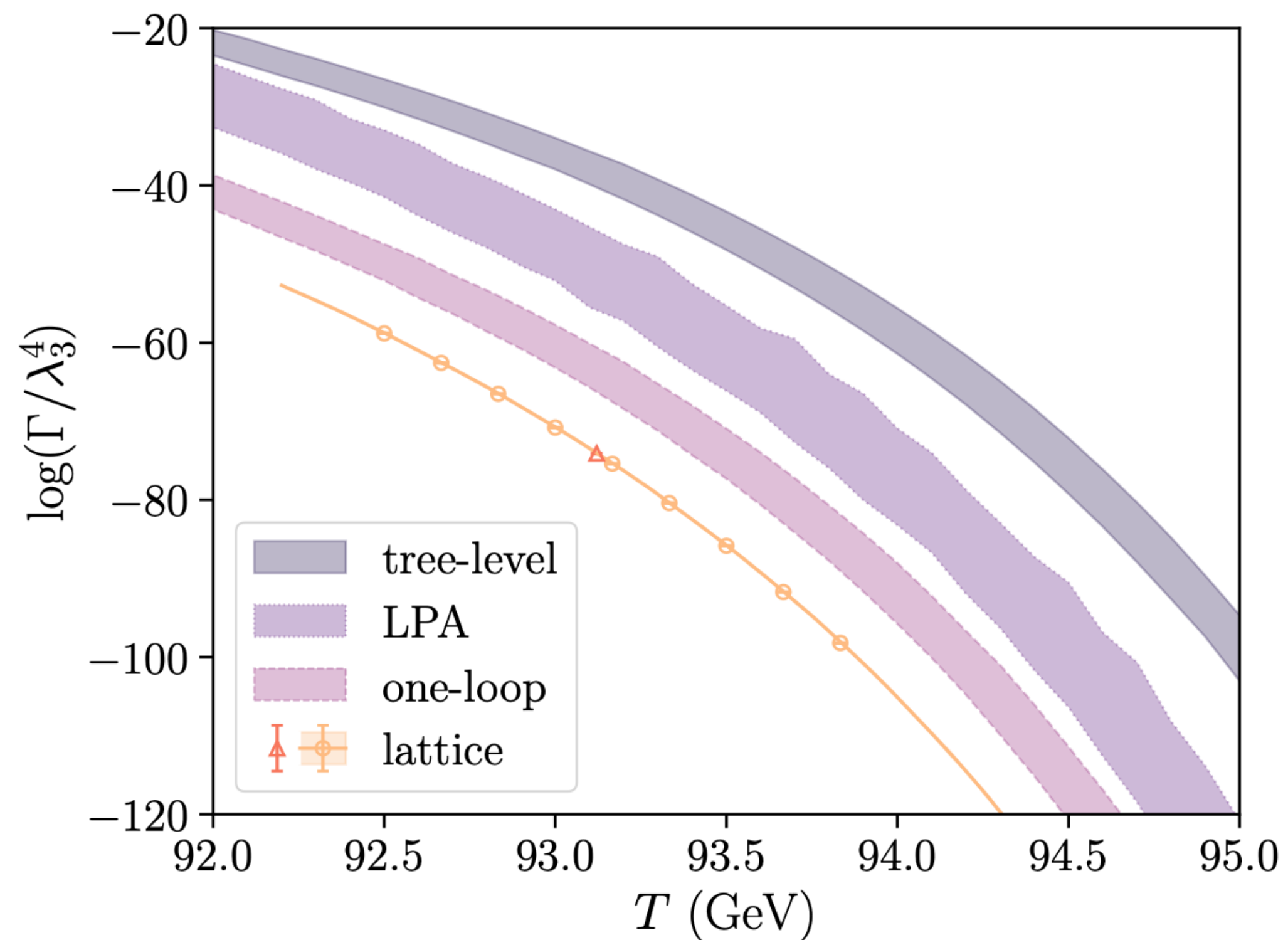


Equilibrium thermodynamics (pert. th.) done



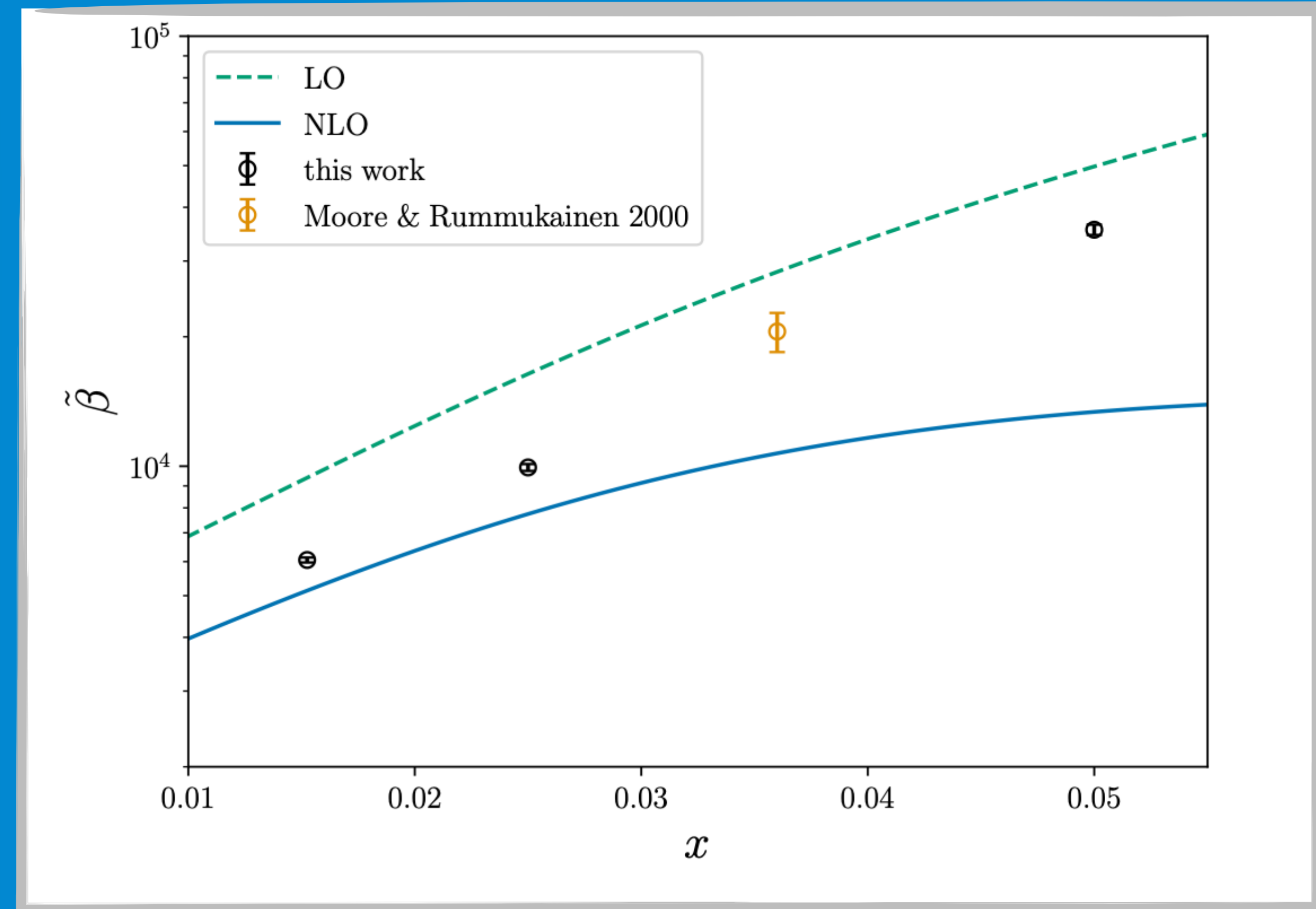
But for nucleation there's still a lot of stuff to do!

Real-scalar model



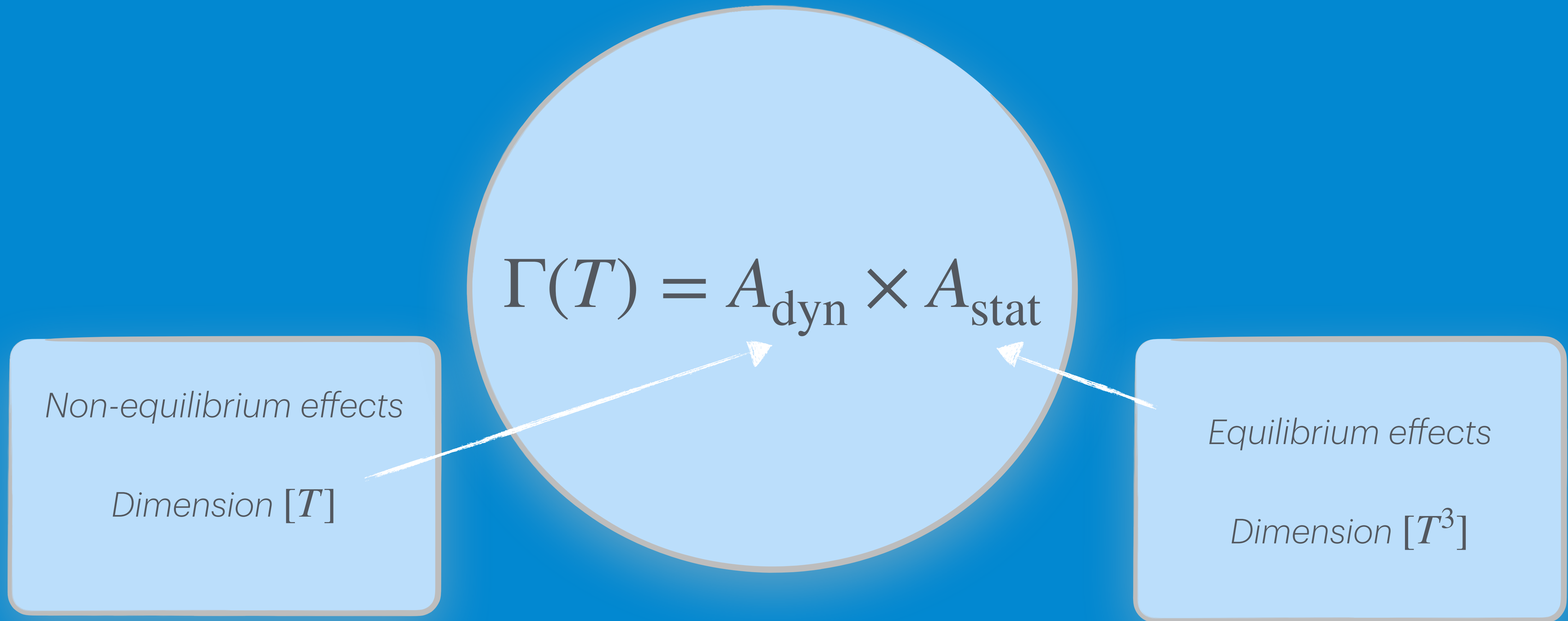
O. Gould, A. Kormu, D. Weir, Phys.Rev.D 111 (2025) 5, 5

SU(2)+Higgs 3d-EFT

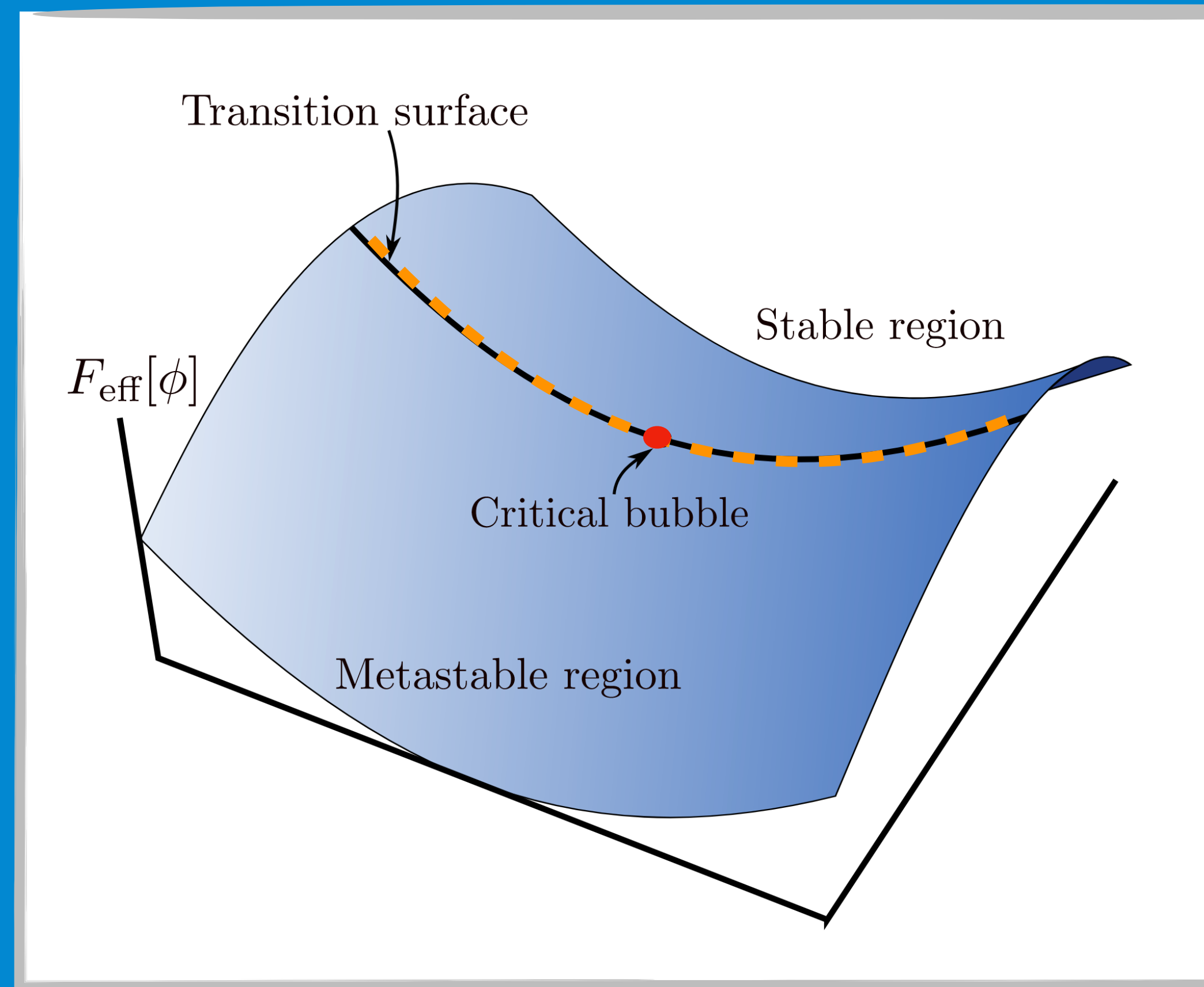


O. Gould, S. Güyer, K. Rummukainen, Phys.Rev.D 106 (2022) 11

Langer's classical nucleation theory



We use a saddle-point of free energy



Nucleation rate can be obtained as the probability “flux” over the *transition surface*.

In the *saddle-point* approximation:

$$A_{\text{stat}} \simeq \mathcal{V} \left| \frac{\det F''_{\text{eff}}[\phi_{\text{CB}}]}{\det F''_{\text{eff}}[\phi_{\text{meta}}]} \right|^{-\frac{1}{2}} \times e^{-\beta \Delta F_{\text{eff}}[\phi_{\text{CB}}]}$$

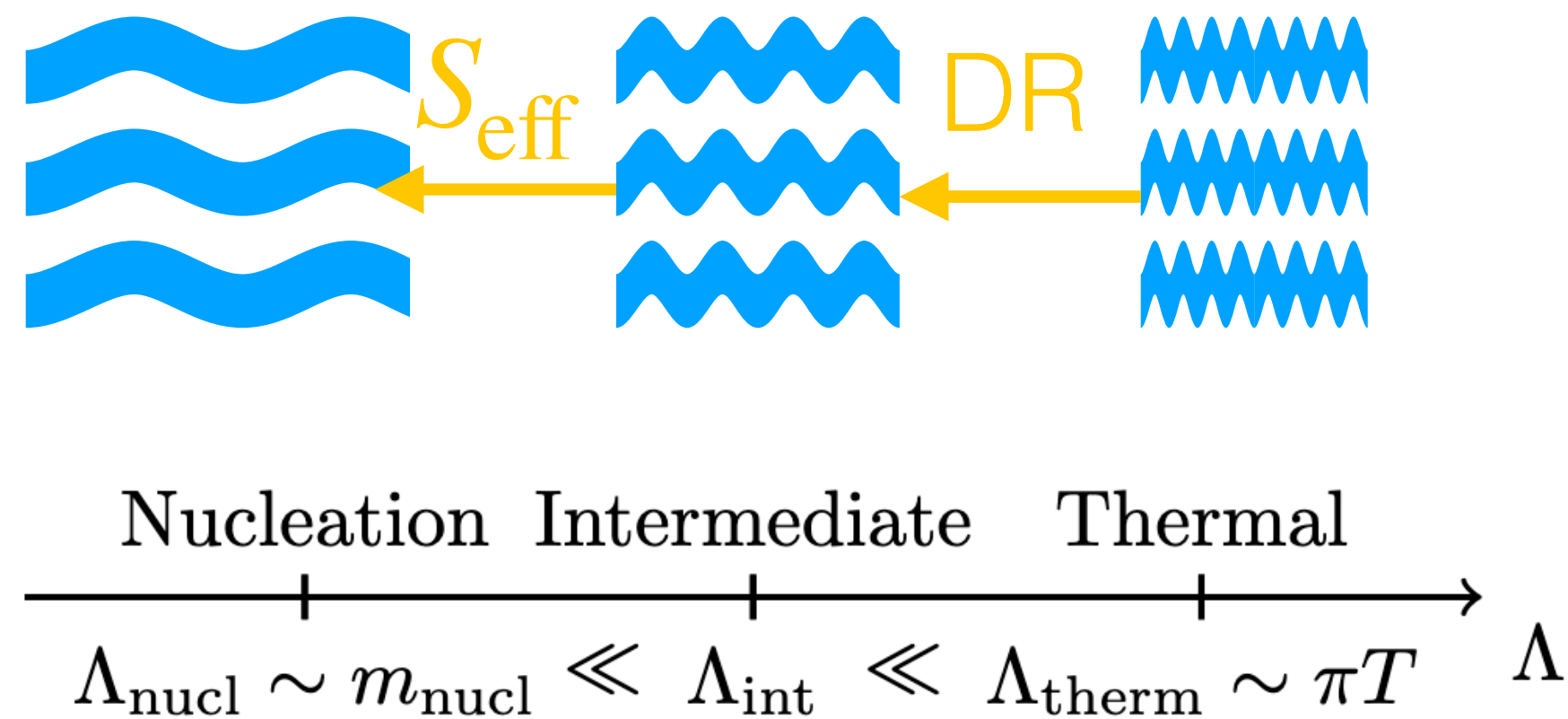
$$A_{\text{dyn}} \simeq \sqrt{|\lambda_-|}$$

J. Hirvonen, O. Gould, 2108.04377

See also: *J. Hirvonen, O. Gould, 2108.04377, J. Hirvonen, 2403.07987, A. Ekstedt 2201.07331*

Idea: use EFT for long-range modes

Building blocks of field ϕ

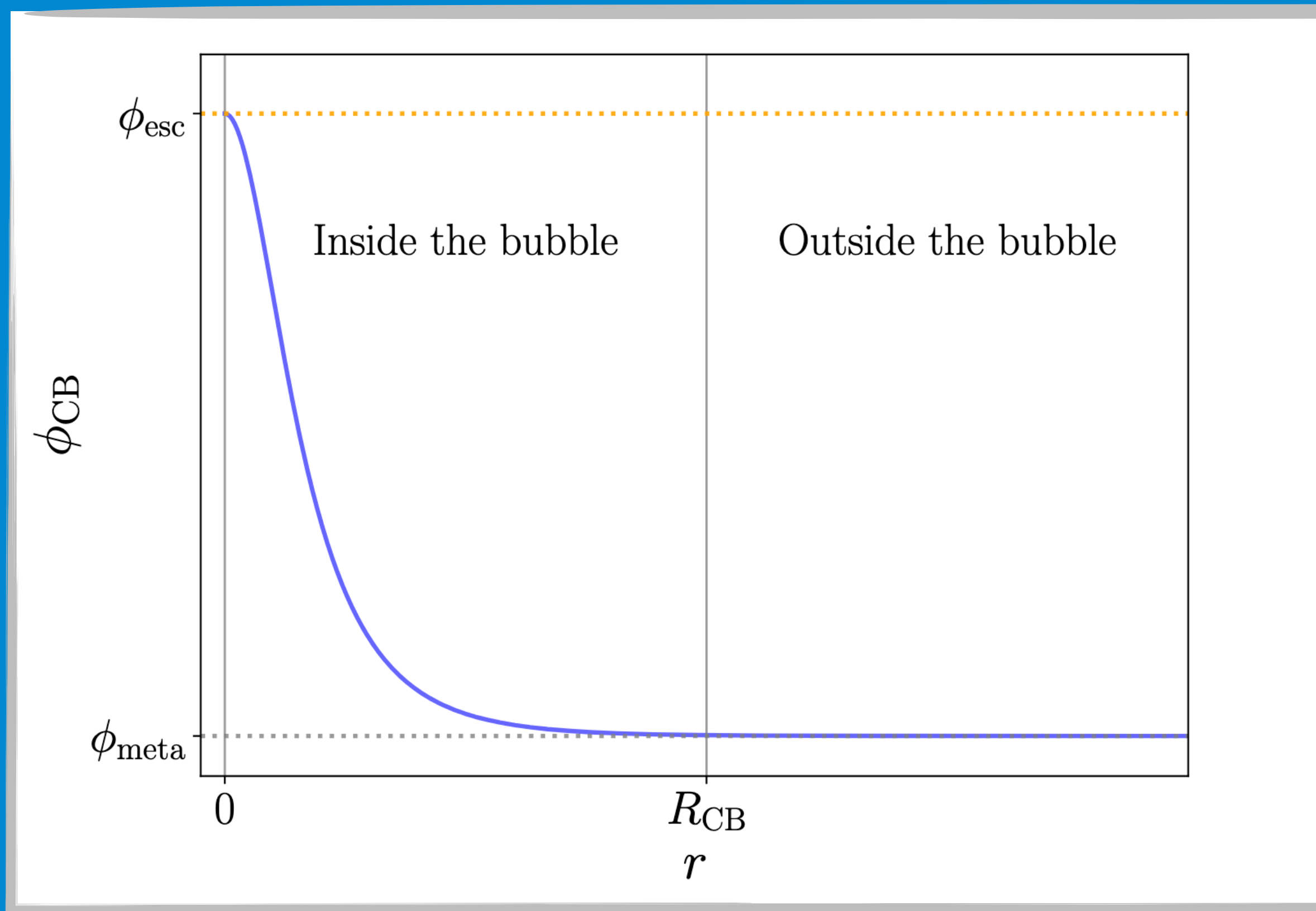


Take quantum fields, integrate out UV modes, and obtain nucleation EFT

$$F_{\text{eff}} \longleftrightarrow S_{\text{nucl}}$$

$$F''_{\text{eff}} \longleftrightarrow S''_{\text{nucl}}$$

Idea: use EFT for long-range modes



We expand around the $O(3)$ -symmetric stationary point of the action

$$A_{\text{stat}} = \underbrace{\mathcal{V} \left| \frac{\det' S''_{\text{nucl}}[\phi_{CB}]}{\det S''_{\text{nucl}}[\phi_{\text{meta}}]} \right|^{-\frac{1}{2}}}_{\text{modes } E < \Lambda_{\text{nucl}}} \underbrace{e^{-\Delta S_{\text{nucl}}[\phi_{CB}]}_{\text{modes } E > \Lambda_{\text{nucl}}}}$$

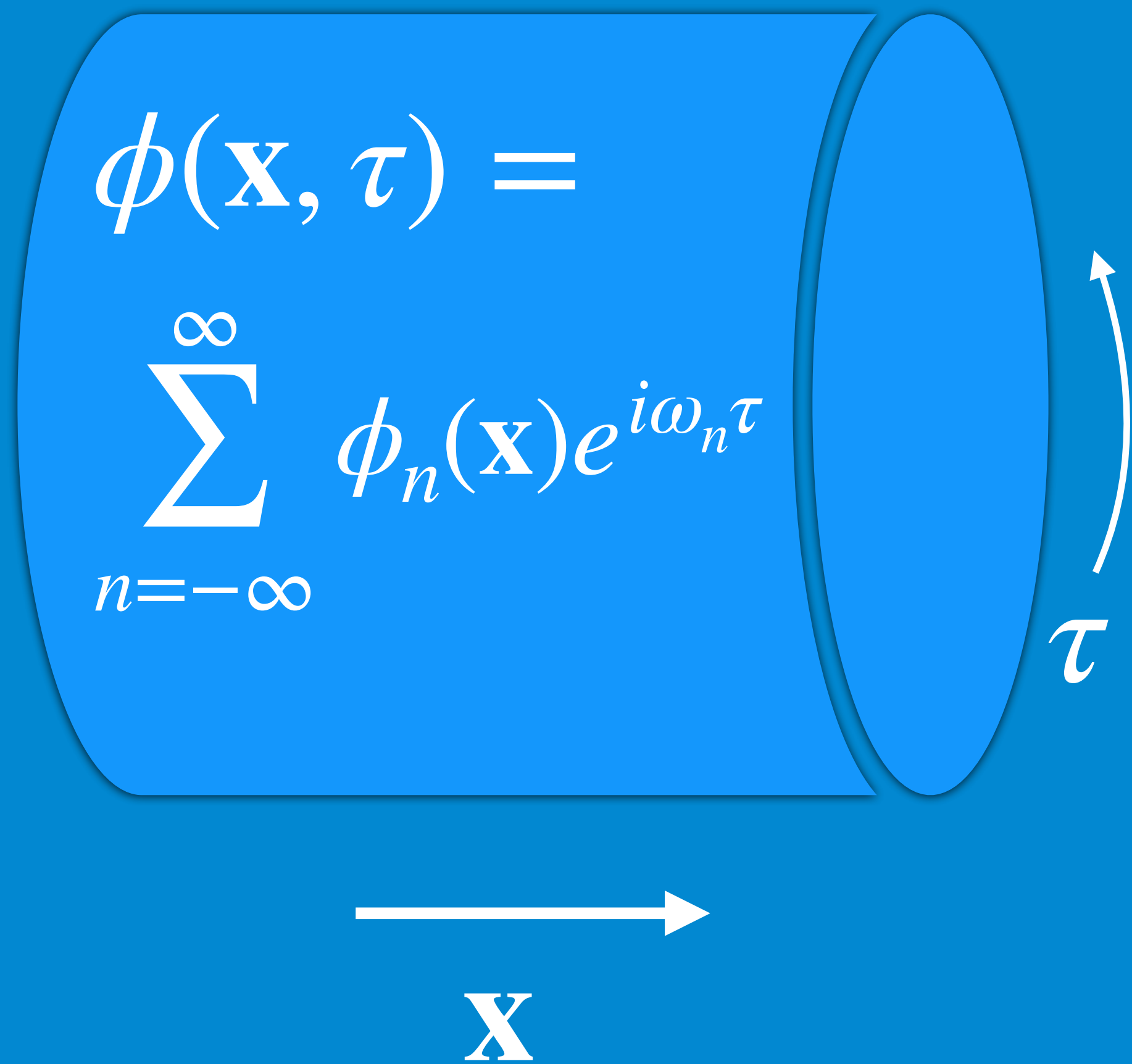
Nucleation rate in EFT language

$$\Gamma(T) = \sqrt{\lambda_-} \times \mathcal{V} \left| \frac{\det' S''_{\text{nuc1}}[\phi_{\text{CB}}]}{\det S''_{\text{nuc1}}[\phi_{\text{meta}}]} \right|^{-\frac{1}{2}} e^{-\Delta S_{\text{nuc1}}[\phi_{\text{CB}}]}$$

Non-equilibrium effects
Dimension $[T]$

Equilibrium effects
Dimension $[T^3]$

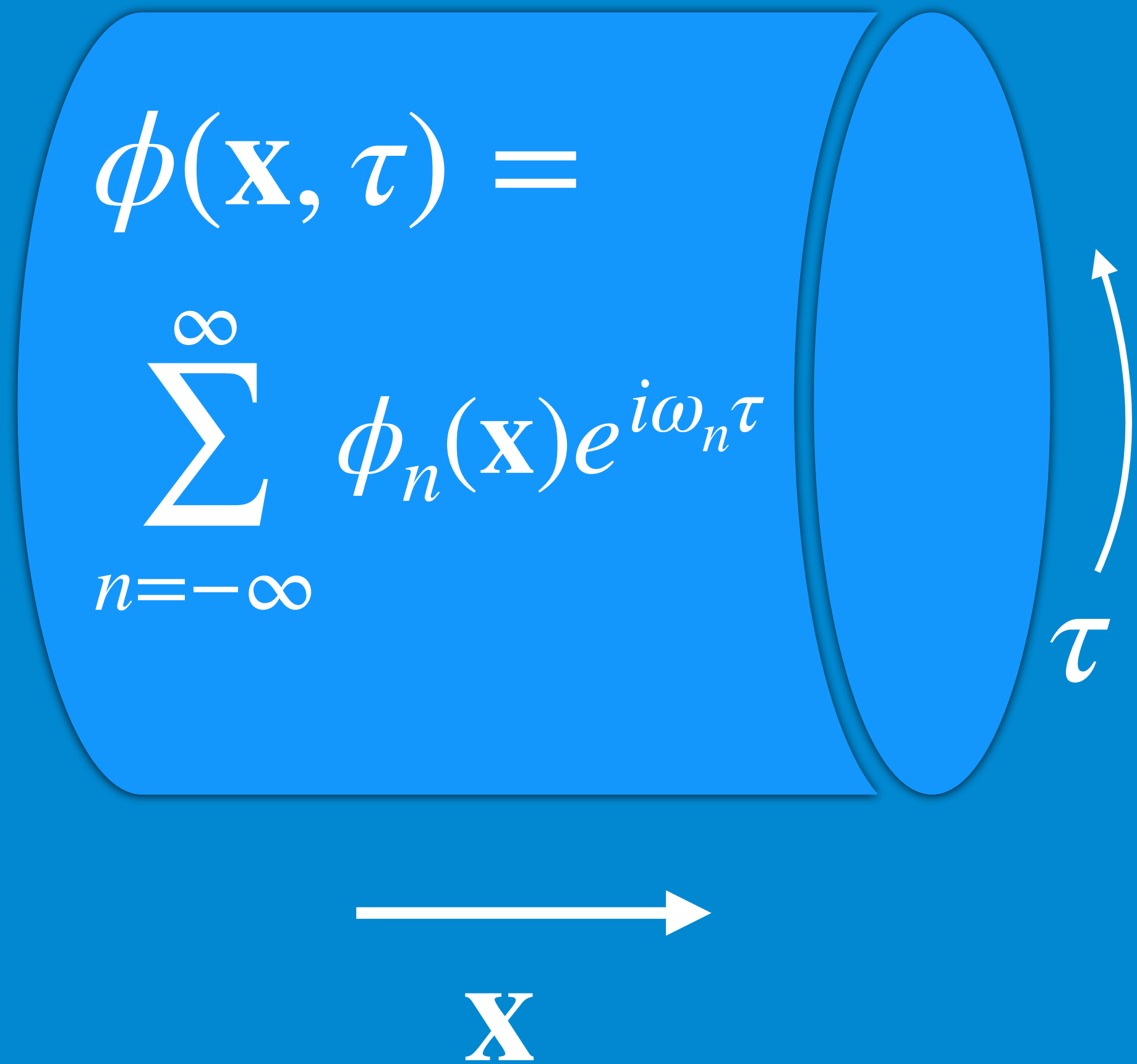
Thermal field theory in imaginary time formalism



$$\omega_n^{\text{bosons}} = 2\pi T n$$

$$S = \frac{1}{T} \int d^3x \sum_{n=0}^{\infty} \left[\frac{1}{2} (\partial_i \phi_n)^2 + \frac{1}{2} (2\pi n T)^2 \phi_n^2 + \frac{1}{2} m^2 \phi_n^2 \right]$$

Thermal field theory in imaginary time formalism


$$\phi(\mathbf{x}, \tau) = \sum_{n=-\infty}^{\infty} \phi_n(\mathbf{x}) e^{i\omega_n \tau}$$

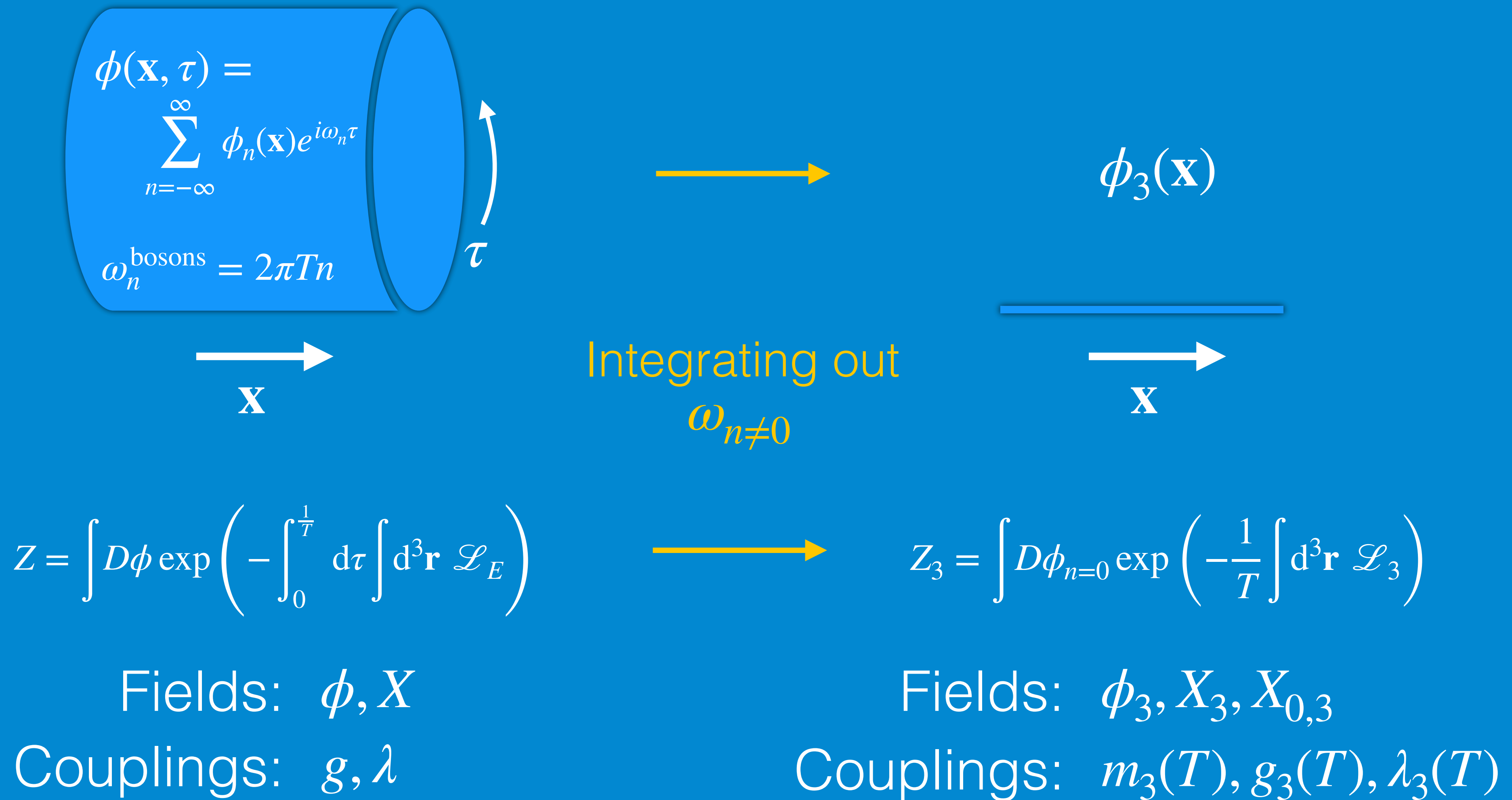
$$\omega_n^{\text{bosons}} = 2\pi T n$$

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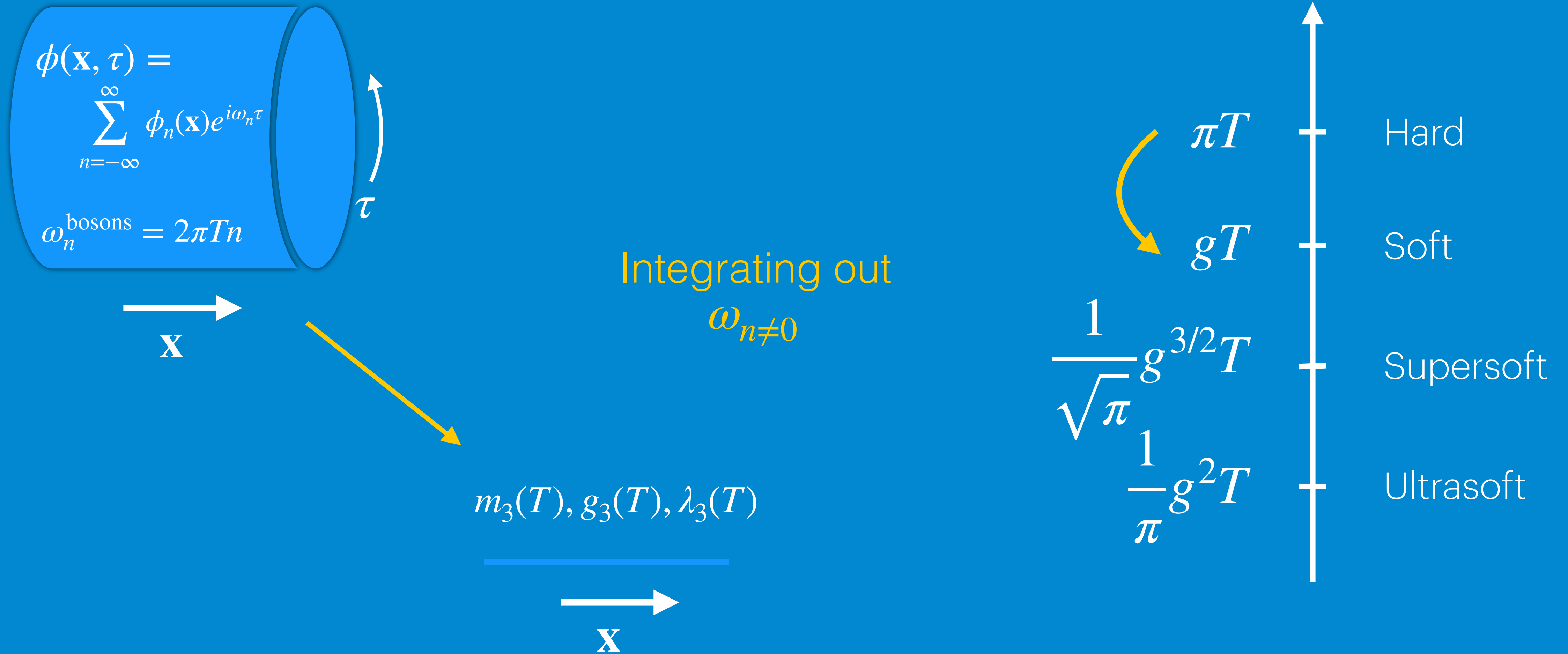
1. When $T \gg m$, the effective mass is
 $M_n^2 \simeq (2\pi n T)^2$

2. There is a hierarchy between the modes
 $M_0 \ll M_1 \ll M_2 \ll \dots$

High-temperature Dimensional Reduction (DR)



High-temperature Dimensional Reduction (DR)



Soft scale 3d EFT

The action of the 3d theory becomes

$$S_3 = \int d^3\mathbf{x} \left\{ \frac{1}{4} F_{ij}^a F_{ij}^a + (D_i \phi)^\dagger (D_i \phi) + \frac{1}{2} (D_i X_0^a)^2 + V_3(\phi, X_0^a) \right\}$$

The potential of scalar field is simple

$$V_3 = \frac{1}{2} m_3^2 \phi_3^2 + \frac{1}{4} \lambda_3 \phi_3^4$$

$$m_3 \sim m_D = gT \neq 0$$

$$m_{X,3}^2 = \frac{1}{4} g_{X,3}^2 \phi_3^2, \quad m_{X_0,3}^2 = m_{D,X}^2 + \frac{1}{2} h_3 \phi_3^2.$$

Soft scale 3d EFT

The action of the 3d theory becomes

$$S_3 = \int d^3\mathbf{x} \left\{ \frac{1}{4} F_{ij}^a F_{ij}^a + (D_i \phi)^\dagger (D_i \phi) + \frac{1}{2} (D_i X_0^a)^2 + V_3(\phi, X_0^a) \right\}$$

The potential of scalar field is simple

$$V_3 = \frac{1}{2} m_3^2 \phi_3^2 + \frac{1}{4} \lambda_3 \phi_3^4$$

Thermal mass

$$m_3 \sim m_D = gT \neq 0$$
$$m_{X,3}^2 = \frac{1}{4} g_{X,3}^2 \phi_3^2, \quad m_{X_0,3}^2 = m_{D,X}^2 + \frac{1}{2} h_3 \phi_3^2.$$

Effective action perturbatively

$$S_{\text{eff}}[\phi_b] = \underbrace{S^{\text{LO}}[\phi_b^{\text{LO}}]}_{\mathcal{O}(g^3)} + \underbrace{S^{\text{NLO}}[\phi_b^{\text{LO}}]}_{\mathcal{O}(g^4)} + \dots$$

I will refer to this as soft expansion

One can get to NLO just with the LO
bounce solution!

Computing effective action - 3D toy model

$$\mathcal{L}_3 = \frac{1}{2}(\partial_i \Phi)^2 + \frac{1}{2}(\partial_i \chi)^2 + \frac{1}{2}m^2 \Phi^2 + \frac{\lambda}{4}\Phi^4 + \kappa \chi^2 \Phi^2 + \frac{1}{2}M^2 \chi^2$$

Expand field on its background

$$\Phi \rightarrow \phi_b(r) + H(x)$$

At 1-loop we need quadratic terms only

$$\mathcal{L} \supset \underbrace{\chi \left(-\nabla^2 + M^2 + \kappa \phi_b^2 \right) \chi}_{\mathcal{M}_\chi}$$

$$e^{iS_{\text{eff}}[\phi_b]} \sim \left[\det \left(-\nabla^2 + M^2 + \kappa \phi_b^2 \right) \right]^{-\frac{1}{2}} e^{-S_3[\phi_b]}$$

Performing the $\int \mathcal{D}\chi$

Computing 1PI effective action - 3D toy model

$$\mathcal{L}_3 = \frac{1}{2}(\partial_i \Phi)^2 + \frac{1}{2}(\partial_i \chi)^2 + \frac{1}{2}m^2 \Phi^2 + \frac{\lambda}{4}\Phi^4 + \kappa \chi^2 \Phi^2 + \frac{1}{2}M^2 \chi^2$$

Integrating out both
 χ - and H -fluctuations

$$e^{i(S_{\text{eff}}[\phi_b] - S_{\text{eff}}[\phi_F])} \sim \underbrace{\left| \frac{\det'(\partial^2 + V''[\phi_b])}{\det(\partial^2 + V''[\phi_F])} \right|^{-\frac{1}{2}}}_{E < \Lambda \sim m} \underbrace{\left[\frac{\det(-\nabla^2 + M^2 + \kappa \phi_b^2)}{\det(-\nabla^2 + M^2 + \kappa \phi_F^2)} \right]^{-\frac{1}{2}}}_{E > \Lambda \sim m} e^{-(S_3[\phi_b] - S_3[\phi_F])}$$

1-loop nucleation rate in gauge-scalar 3D EFT

$$\Gamma = A_{\text{dyn}} \underbrace{\text{Det}_S}_{E < \Lambda} \underbrace{\text{Det}_{X_0} \text{Det}_{XG}}_{E > \Lambda} e^{-\Delta S_3[\phi_b]}$$

Derivative expansion of gauge-Goldstone det

$$\frac{m_3}{m_X} \ll 1$$

$$\delta Z = -\frac{11}{16\pi} \frac{g_3}{\phi_b}$$

$$-\log \text{Det}_{XG} \simeq \underbrace{\int_x \frac{-3}{48\pi} g^3 \phi_b^3}_{\mathcal{O}(g^3) \text{ LO}} + \underbrace{\int_x \frac{1}{2} \delta Z[\phi_b] (\partial \phi_b)^2}_{\mathcal{O}(g^4) \text{ NLO}} + \dots$$

Barrier generating term

Terms with higher powers in
derivatives

Reminder:

$$S_{\text{eff}}[\phi_b] = \underbrace{S^{\text{LO}}[\phi_b^{\text{LO}}]}_{\mathcal{O}(g^3)} + \underbrace{S^{\text{NLO}}[\phi_b^{\text{LO}}]}_{\mathcal{O}(g^4)} + \dots$$

$$S^{\text{LO}}[\phi_b^{\text{LO}}] = 4\pi \int dr \, r^2 \left[\frac{1}{2} (\partial_i \phi_b^{\text{LO}})^2 + V^{\text{LO}}[\phi_b^{\text{LO}}] \right]$$

$$S^{\text{NLO}}[\phi_b^{\text{LO}}] = 4\pi \int dr \, r^2 \left[\frac{1}{2} \delta Z[\phi_b] (\partial \phi_b)^2 + V^{\text{NLO}}[\phi_b^{\text{LO}}] \right]$$

$$V^{\text{LO}}[\phi_b^{\text{LO}}] = \frac{1}{2} m_3^2 \phi_3^2 + \frac{1}{4} \lambda_3 \phi_3^4 - \frac{1}{12\pi} \left(6(m_{X,3}^2)^{\frac{3}{2}} + 3(m_{X_0,3}^2)^{\frac{3}{2}} \right)$$

$\mathcal{O}(\partial^0)$ -terms in deriv..
expansion of gauge
dets

Reminder:

$$S_{\text{eff}}[\phi_b] = \underbrace{S^{\text{LO}}[\phi_b^{\text{LO}}]}_{\mathcal{O}(g^3)} + \underbrace{S^{\text{NLO}}[\phi_b^{\text{LO}}]}_{\mathcal{O}(g^4)} + \dots$$

$$S^{\text{LO}}[\phi_b^{\text{LO}}] = 4\pi \int dr \, r^2 \left[\frac{1}{2} (\partial_i \phi_b^{\text{LO}})^2 + V^{\text{LO}}[\phi_b^{\text{LO}}] \right]$$

$$V^{\text{LO}}[\phi_b^{\text{LO}}] = \frac{1}{2} m_3^2 \phi_3^2 + \frac{1}{4} \lambda_3 \phi_3^4 - \frac{1}{12\pi} \left(6(m_{X,3}^2)^{\frac{3}{2}} + 3(m_{X_0,3}^2)^{\frac{3}{2}} \right)$$

$\mathcal{O}(\partial^0)$ -terms in
expansion of gauge
determinants

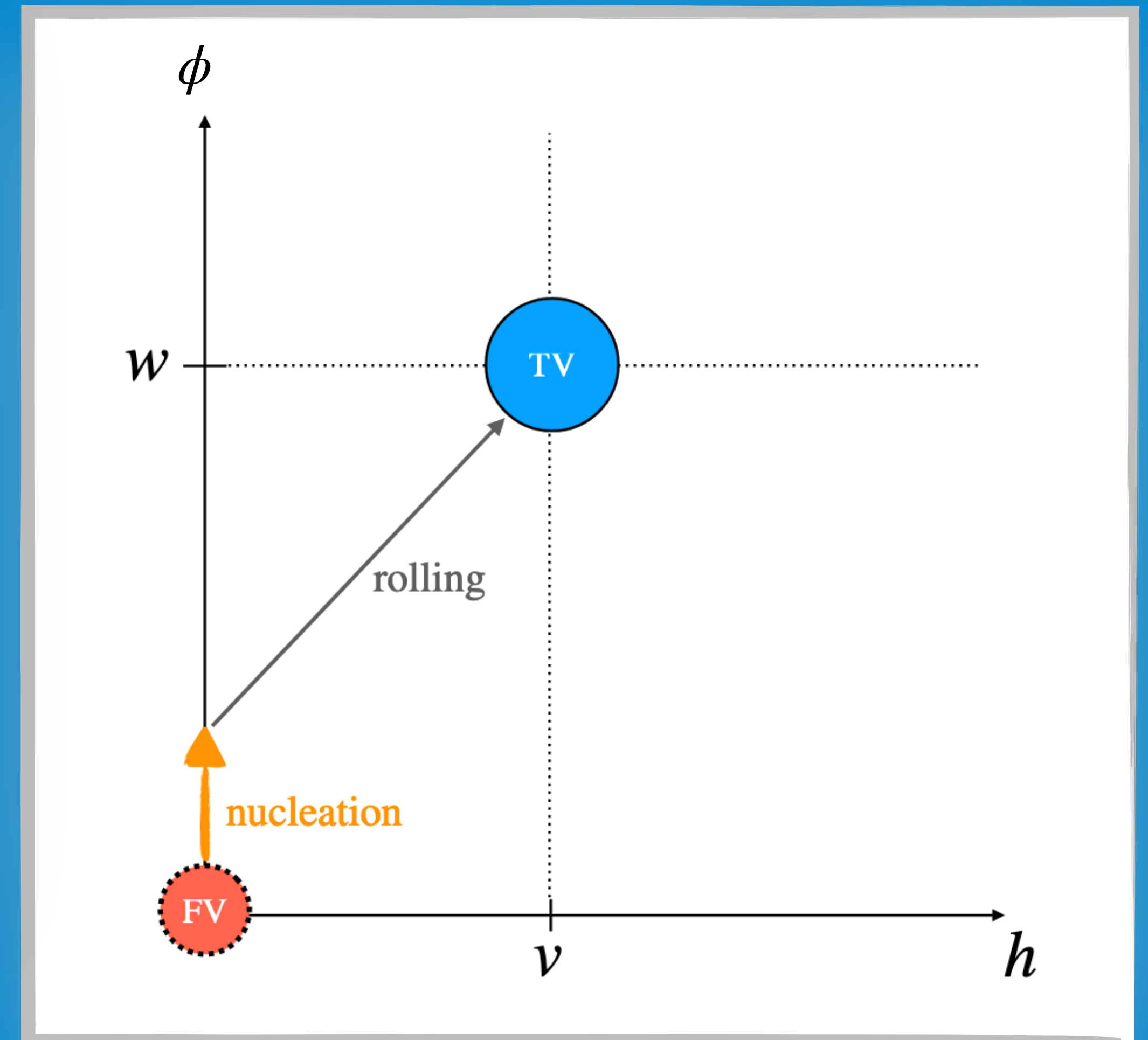
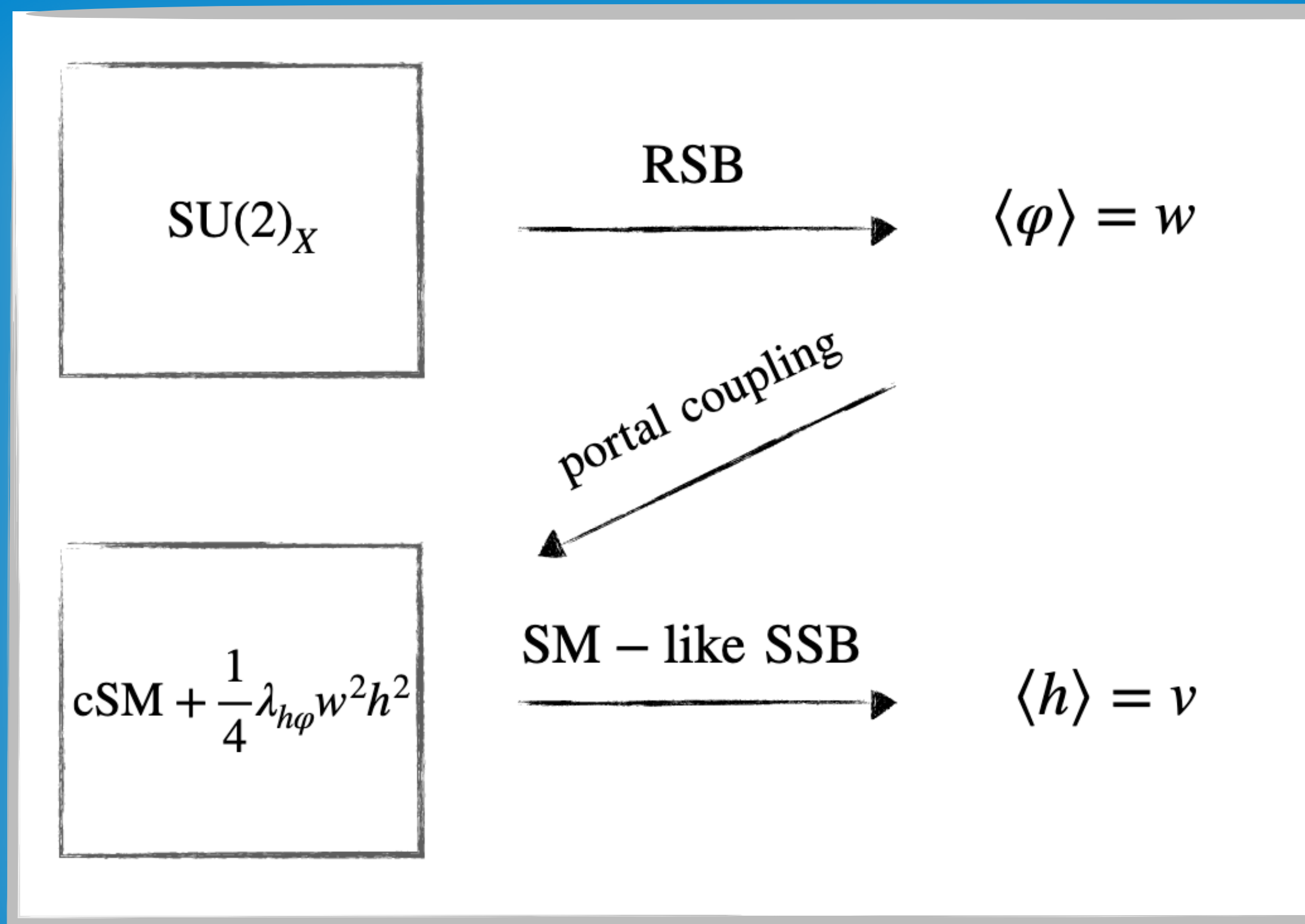
$$S^{\text{NLO}}[\phi_b^{\text{LO}}] = 4\pi \int dr \, r^2 \left[\frac{1}{2} \delta Z[\phi_b] (\partial \phi_b)^2 + V^{\text{NLO}}[\phi_b^{\text{LO}}] \right]$$

$\mathcal{O}(\partial^2)$ -terms in the
expansion of gauge
determinants

2-loop gauge
contributions to the
effective potential

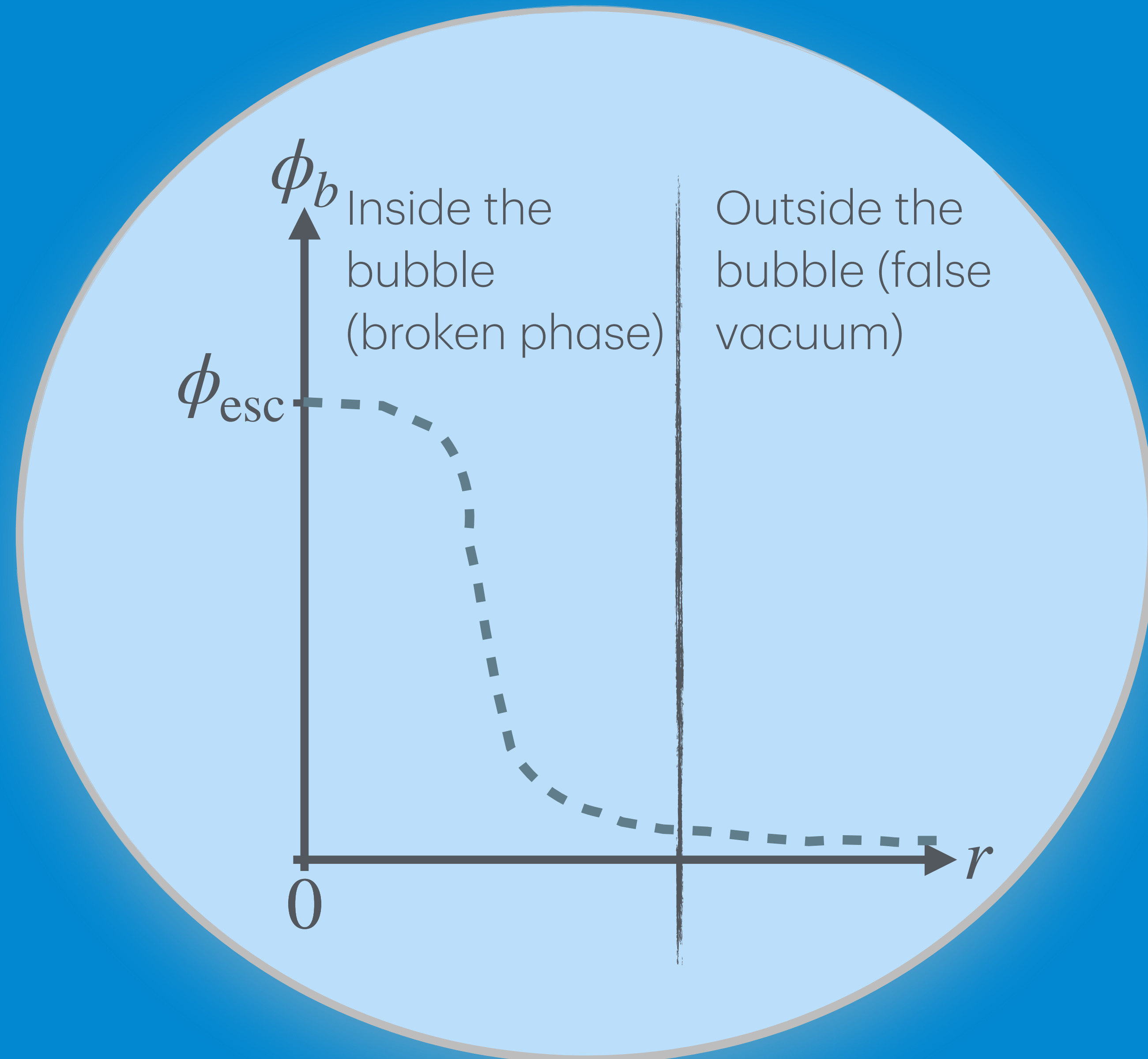
Q: Is derivative expansion always valid? What are the errors? 🤔

Benchmark BSM model: SU(2)cSM

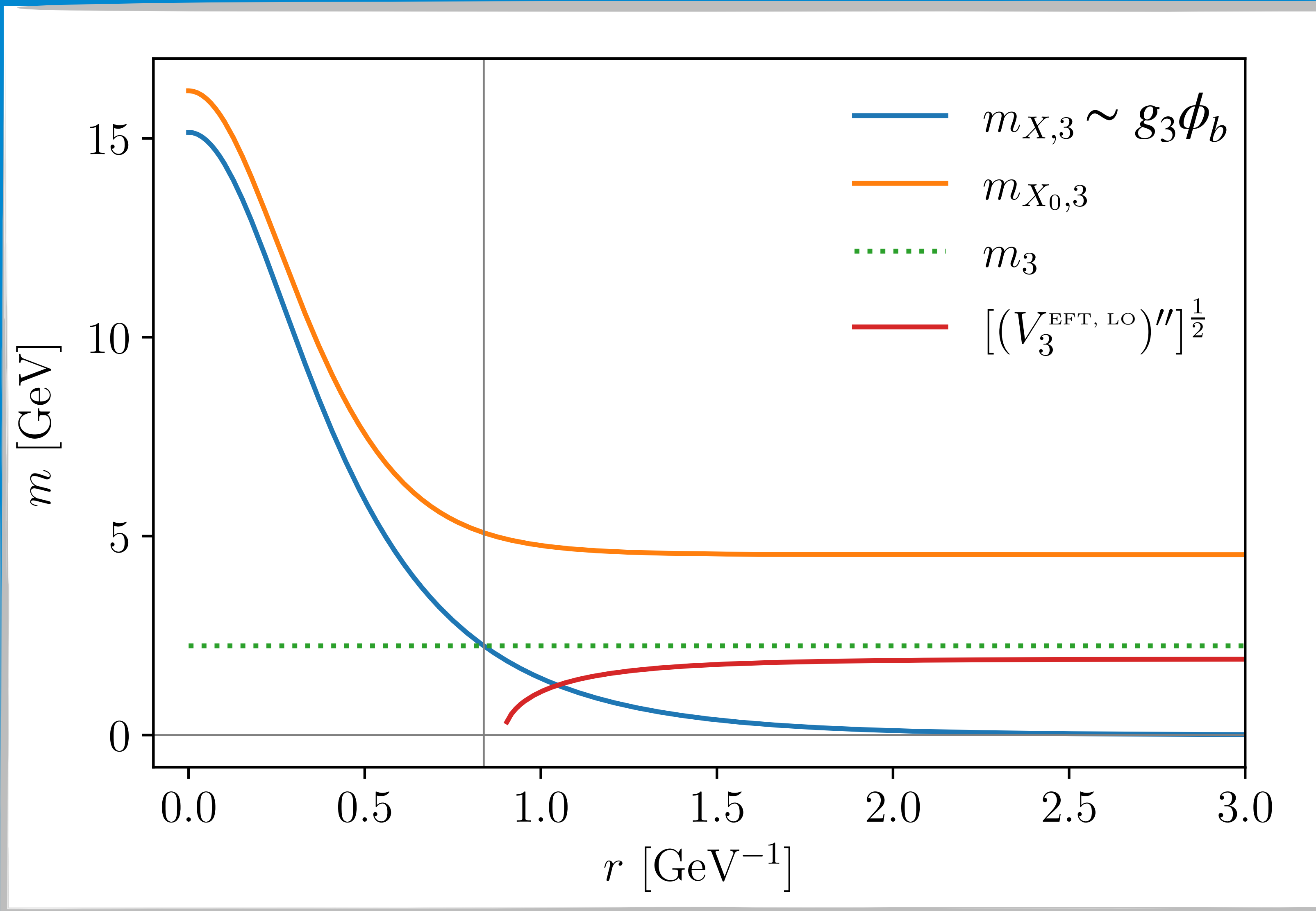


$$V_{\text{tree}} = \frac{1}{4} (\lambda_1 h^4 + \lambda_2 h^2 \phi^2 + \lambda_3 \phi^4)$$

LO bounce solution

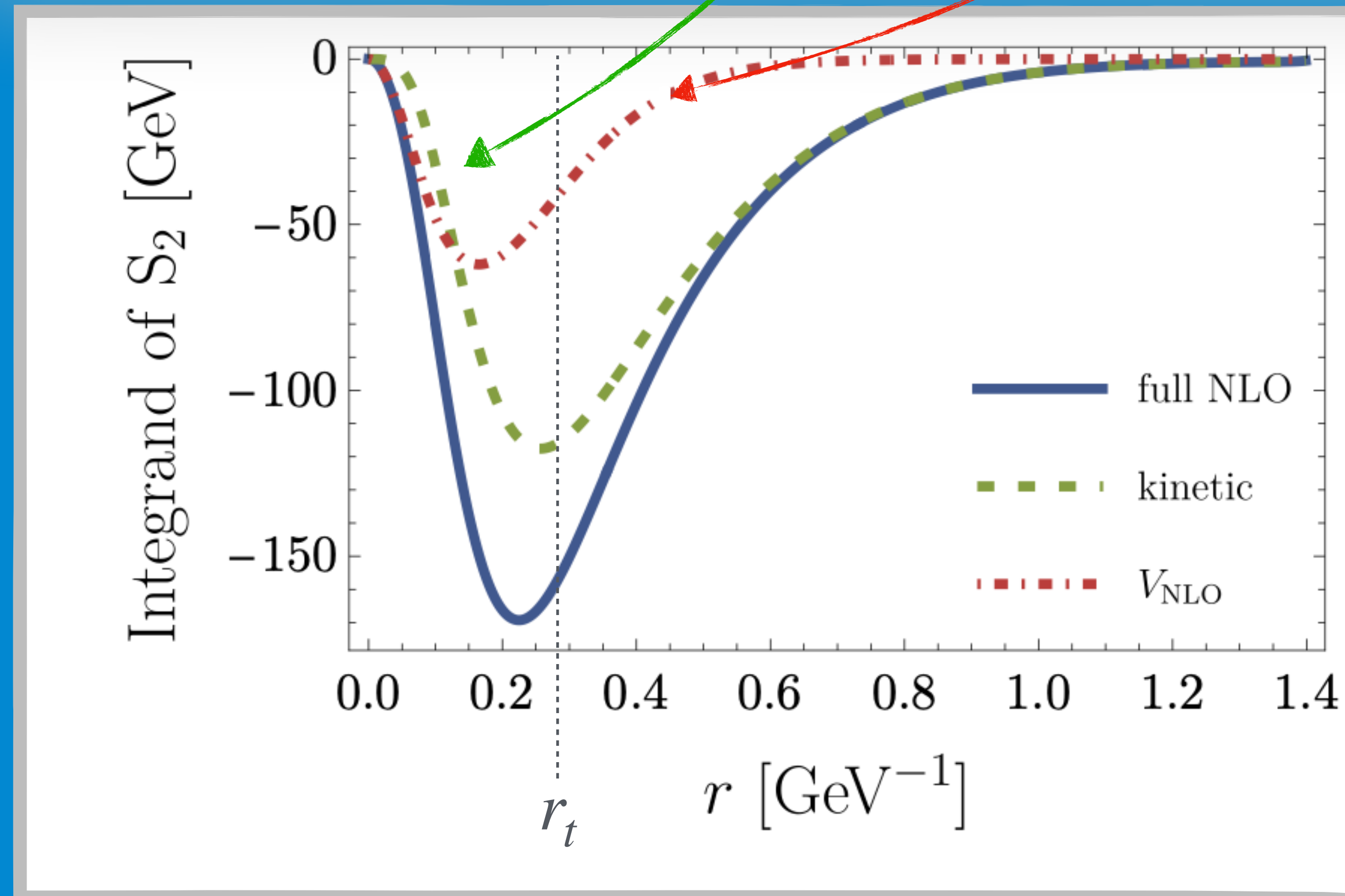


Scale-shifters in SU(2)cSM



Z-factor contribution dominates the NLO terms

$$S_2 = S^{\text{NLO}}[\phi_b^{\text{LO}}] = 4\pi \int dr \, r^2 \left[\frac{1}{2} \delta Z[\phi_b] (\partial \phi_b)^2 + V^{\text{NLO}}[\phi_b^{\text{LO}}] \right]$$



Higher-order momentum terms of gauge-goldstone det

$$S_3^{\text{EFT}}[v_{3,b}] \supset \int_{\mathbf{x}} \left[V^{\text{EFT}}(v_3) + \frac{1}{2} Z_{2,3} (\partial_i v_3)^2 + \frac{1}{2} Z_{4,3} (\partial^2 v_3)^2 \right. \\ \left. + \frac{1}{2} Y_{3,3} (\partial_i v_3)^2 \partial^2 v_3 + \frac{1}{8} Y_{4,3} (\partial_i v_3)^2 (\partial_j v_3)^2 + \mathcal{O}(\partial^6) \right],$$

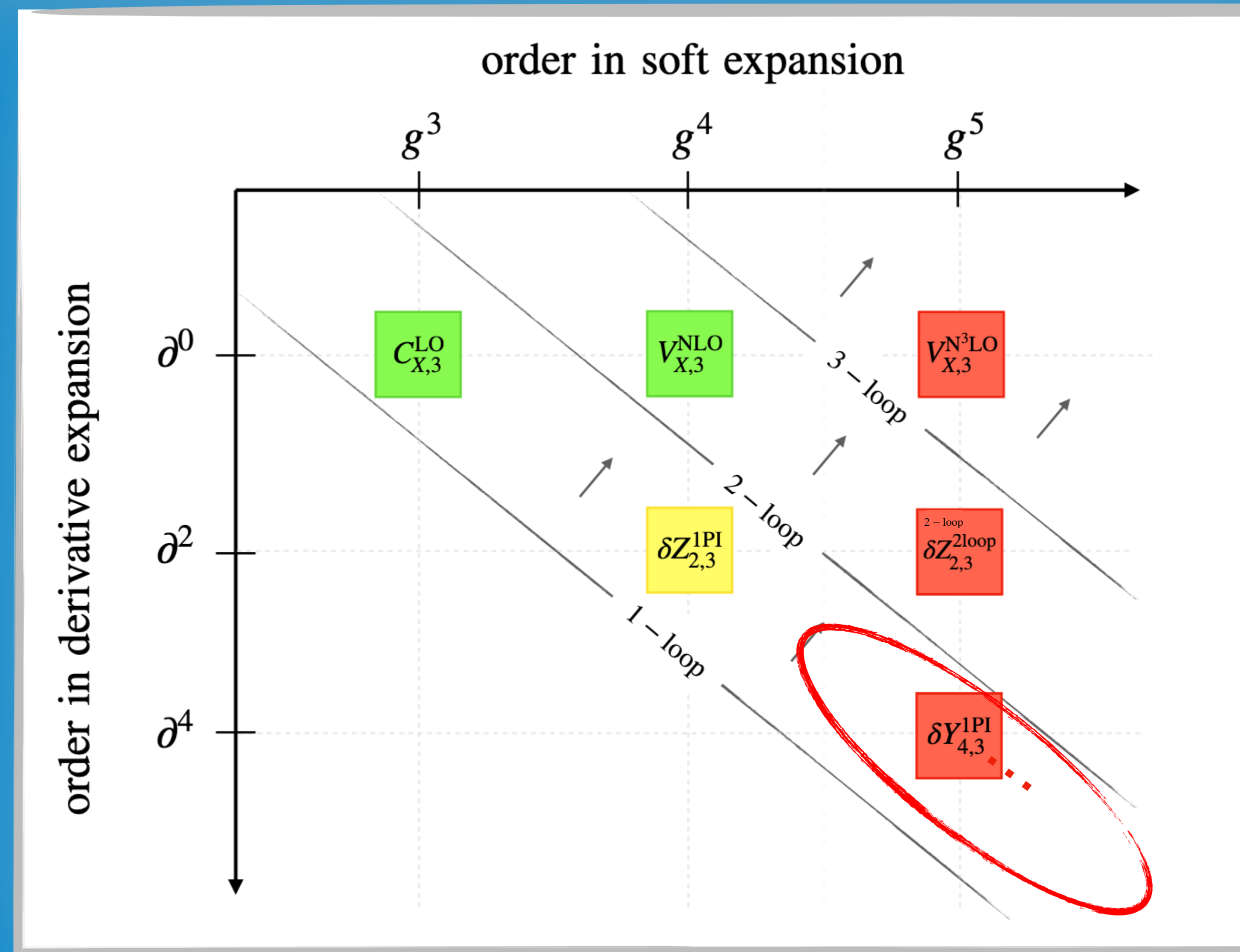
Higher-order momentum terms

(Also blows-up at 2-loop)

$$S_3^{\text{EFT}}[v_{3,b}] \supset \int_{\mathbf{x}} \left[V^{\text{EFT}}(v_3) + \frac{1}{2} Z_{2,3} (\partial_i v_3)^2 + \frac{1}{2} Z_{4,3} (\partial^2 v_3)^2 \right. \\ \left. + \frac{1}{2} Y_{3,3} (\partial_i v_3)^2 \partial^2 v_3 + \frac{1}{8} Y_{4,3} (\partial_i v_3)^2 (\partial_j v_3)^2 + \mathcal{O}(\partial^6) \right],$$

All of the red terms are divergent at large radius

$$S_3^{\text{EFT}}[v_{3,b}] \supset \int_{\mathbf{x}} \left[V^{\text{EFT}}(v_3) + \frac{1}{2} Z_{2,3} (\partial_i v_3)^2 - \frac{1}{2} Z_{4,3} (\partial^2 v_3)^2 + \frac{1}{2} Y_{3,3} (\partial_i v_3)^2 \partial^2 v_3 + \frac{1}{8} Y_{4,3} (\partial_i v_3)^2 (\partial_j v_3)^2 + \mathcal{O}(\partial^6) \right],$$



Q: Is derivative expansion always valid?

A: Nope 🥲, gauge modes always break derivative expansion

Fluctuation determinants strike again

how to install bubbledet

Tryb AI Wszystko Krótkie filmy Wideo Grafika Zakupy Wiadomości Więcej ▾

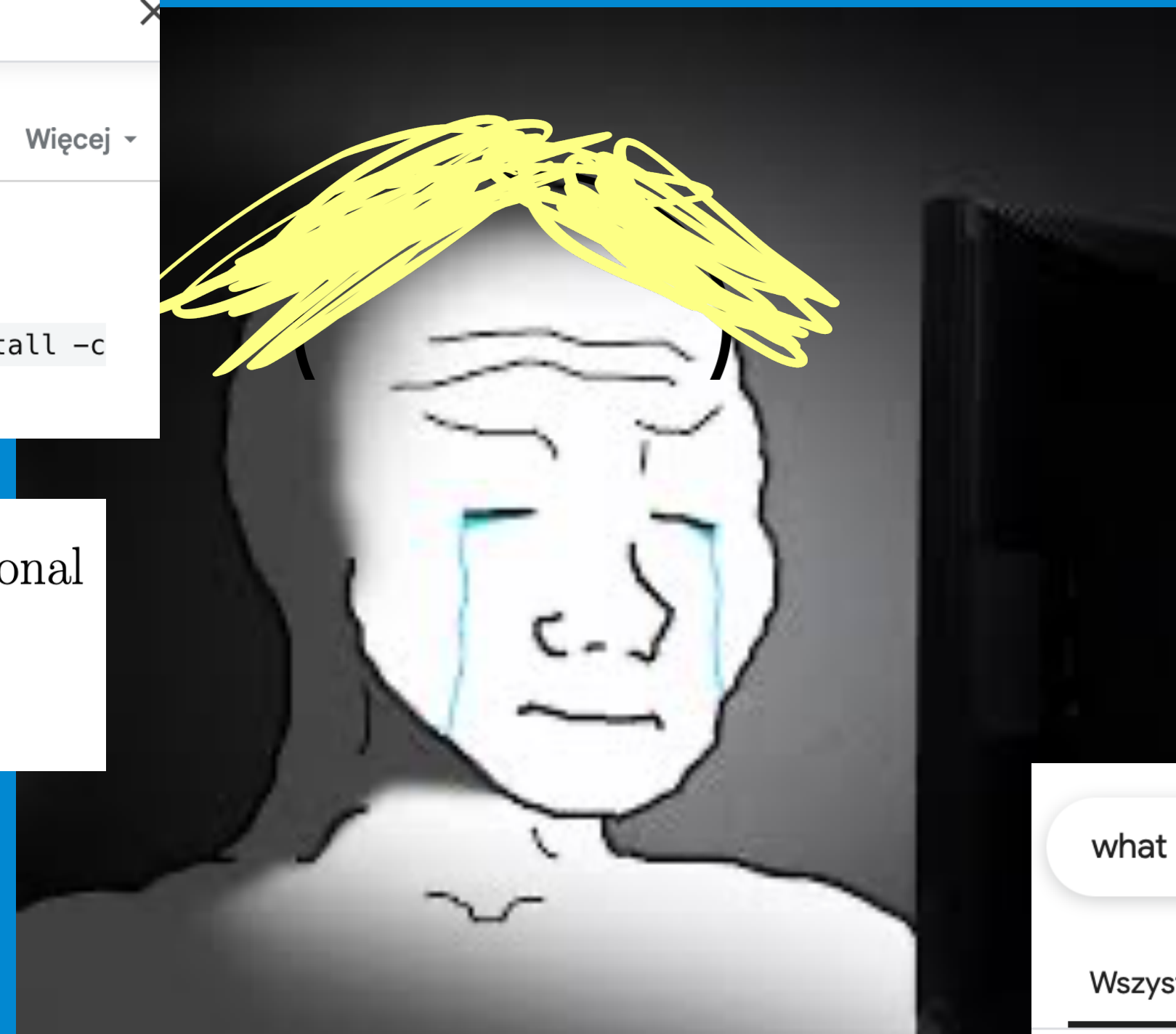
◆ Przegląd od AI

You can install BubbleDet by running `pip install BubbleDet` or `conda install --c conda-forge BubbleDet` in your terminal.  University of Helsinki

BubbleDet: A Python package to compute functional determinants for bubble nucleation

Andreas Ekstedt^{*1}, Oliver Gould^{†2}, and Joonas Hirvonen^{‡3}

(Works only for 1d operators!)



Me, spring of 2024

what is gelfand yaglom theorem

Wszystko Grafika Wideo Krótkie filmy Wiadomości Sieć Książki : Więcej

The Gelfand-Yaglom formula **relates functional determinants of the one-dimensional second order differential operators to the solutions of the corresponding initial value problem.** 8 sie 2018

Joonas & Oliver recipe for taming scale-shifters

[NLO det] recipe

Step 1. Obtain LO bounce solution ϕ_b^{LO} using S^{LO} (grad. expans.)

Step 2. Evaluate determinants numerically $\text{Det}_{VG}[\phi_b^{\text{LO}}]$

Step 2. Get the NLO bosons contribution $S^{\text{NLO}} \sim V_3^{\text{NLO}}$ on ϕ_b^{LO}

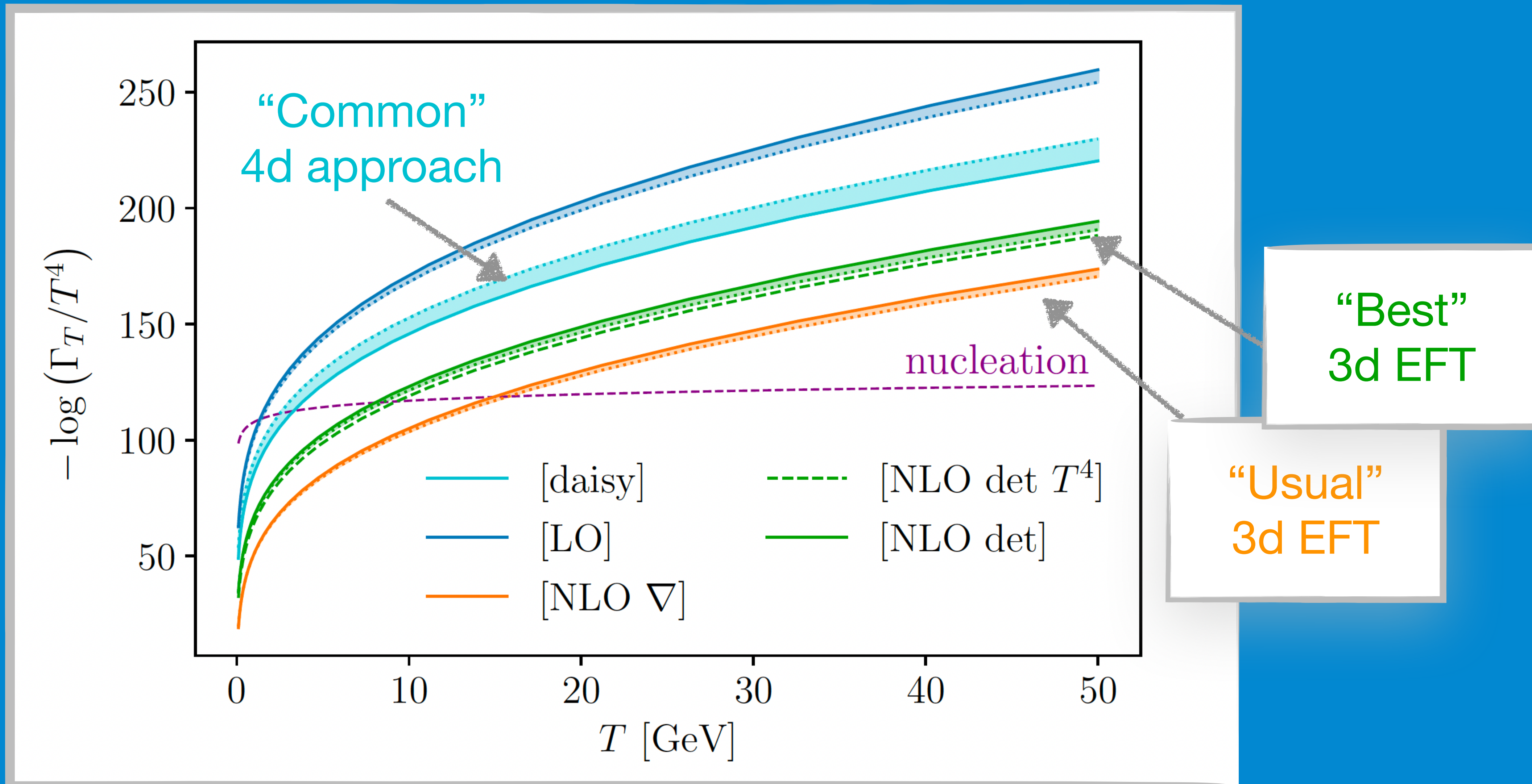
Step 3. Obtain the action

$$S_{\text{eff}} = (S^{\text{LO}} - S_{\text{bosons}}^{\text{LO}}) - \log \text{Det}_{VG} - S_{\text{NLO}}^{\text{pot}}$$

Step 4. Calculate nucleation rate

$$\Gamma_T = A_{\text{dyn}} \text{Det}_H e^{-S_{\text{eff}}[\phi_b]}$$

Results: nucleation rate from different “recipes”

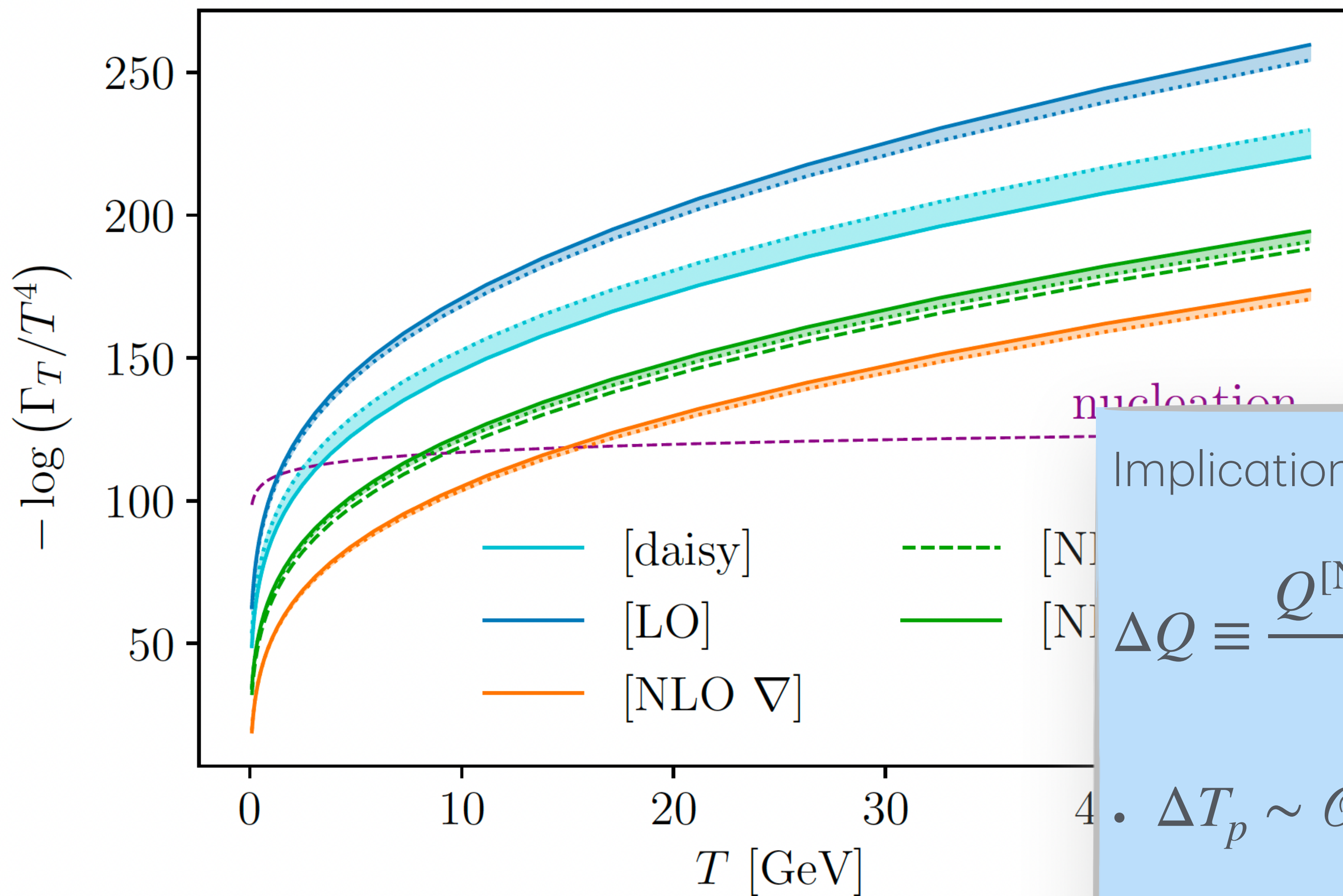


Note:

$A_{\text{dyn}} \det s \simeq T^4$
everywhere
except [NLO det]

[2503.13597, with P. Schicho, B. Świeżewska, T.V.I. Tenkanen and J. van de Vis,]

Results: nucleation rate from different “recipes”



Note:
 $A_{\text{dyn det } s} \simeq T^4$
 everywhere
 except [NLO det]

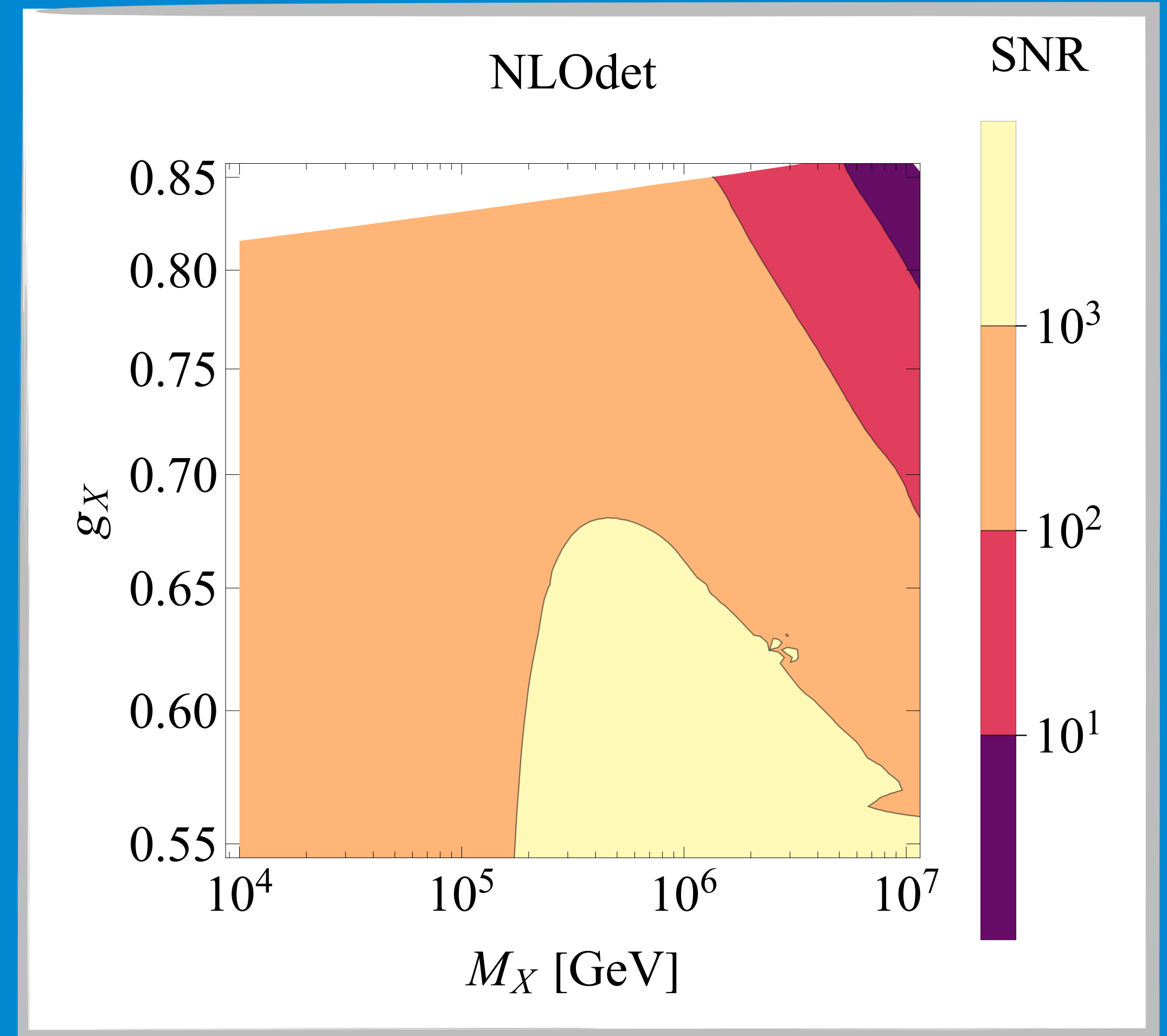
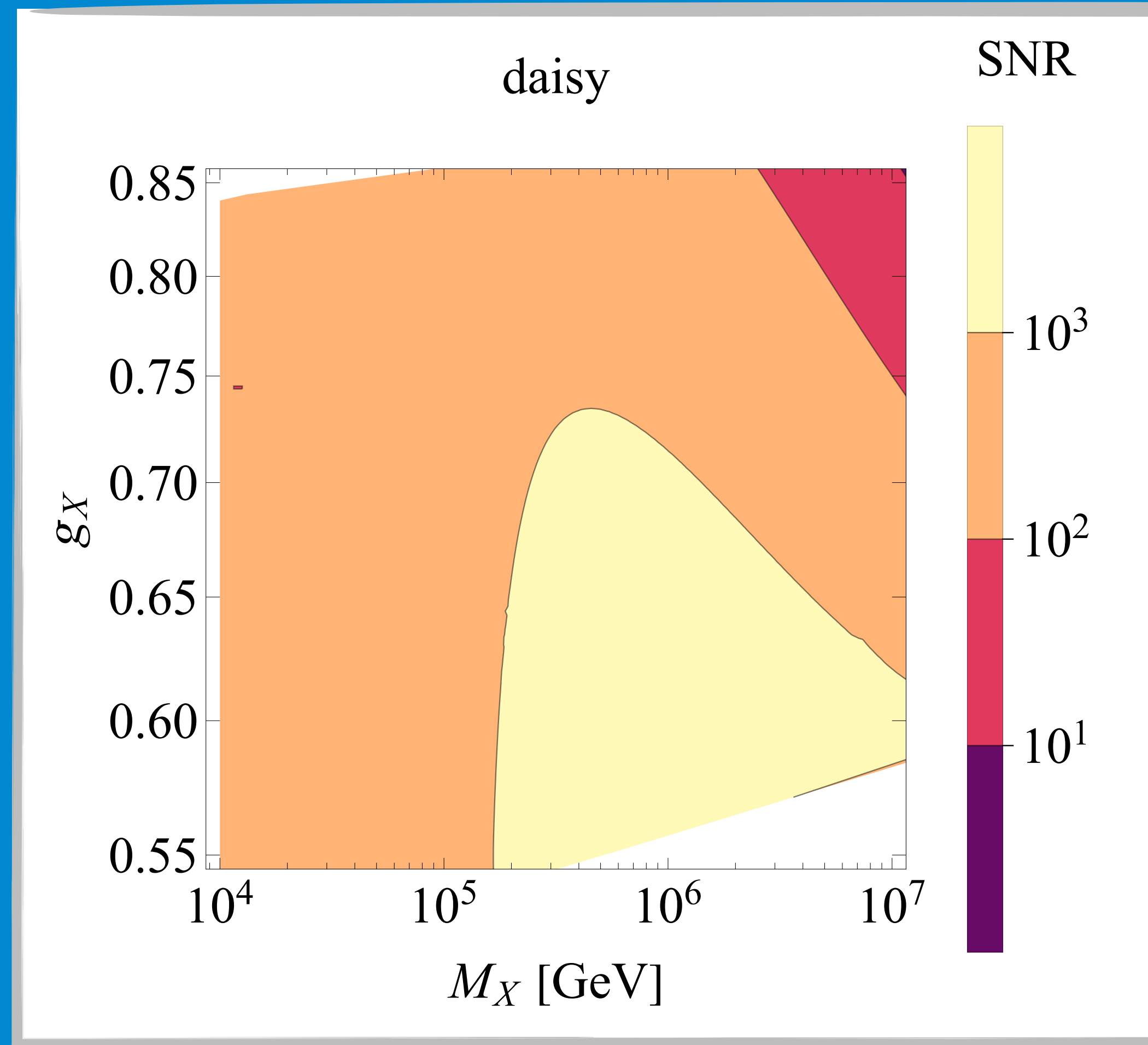
Implications for PT parameters:

$$\Delta Q \equiv \frac{Q^{[\text{NLO det}]} - Q^{[\text{daisy}]}}{Q^{[\text{NLO det}]}}$$

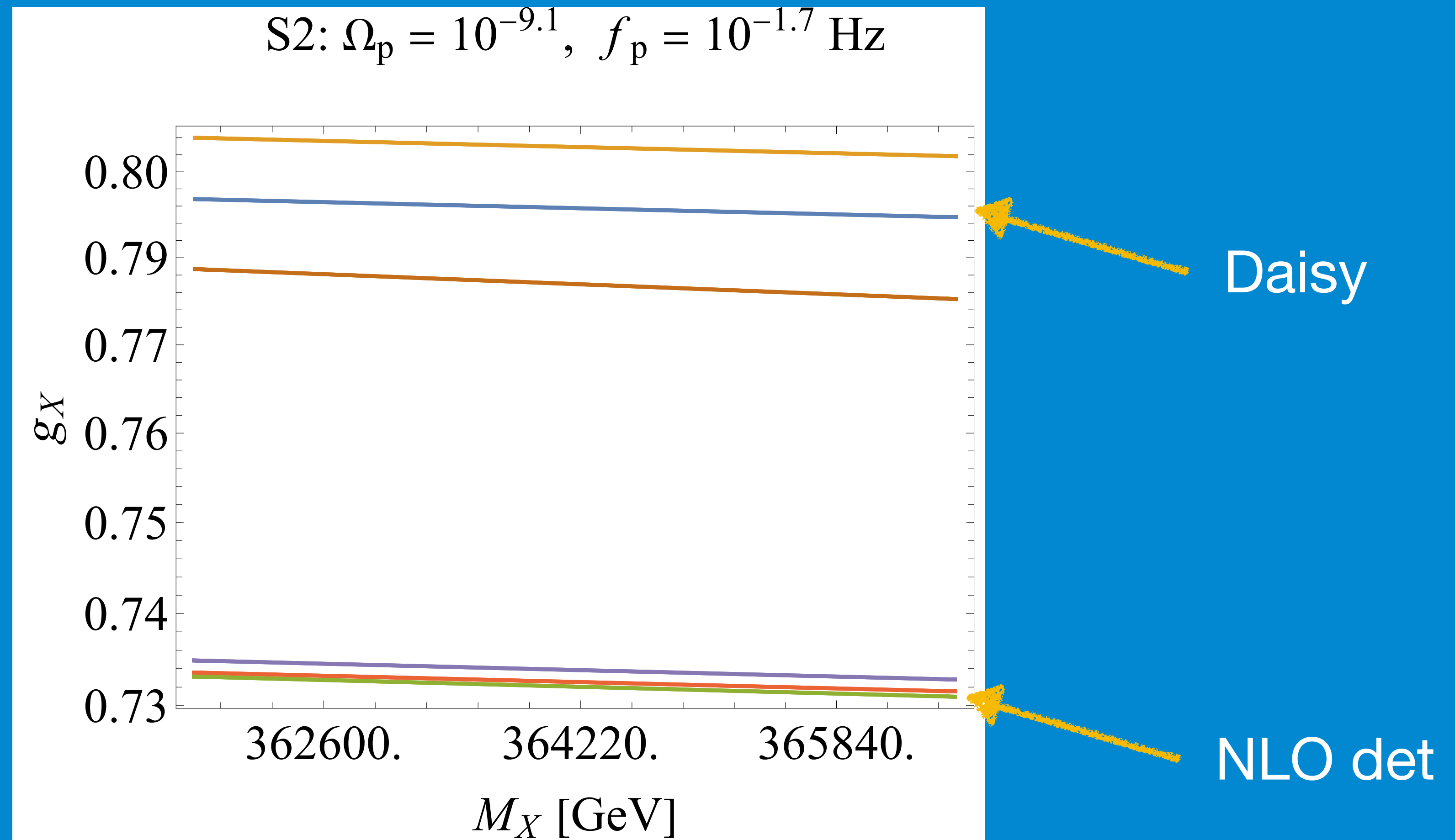
- $\Delta T_p \sim \mathcal{O}(30\%) - \mathcal{O}(80\%)$

- $\Delta R_* H_* \sim \mathcal{O}(15\%) - \mathcal{O}(20\%)$

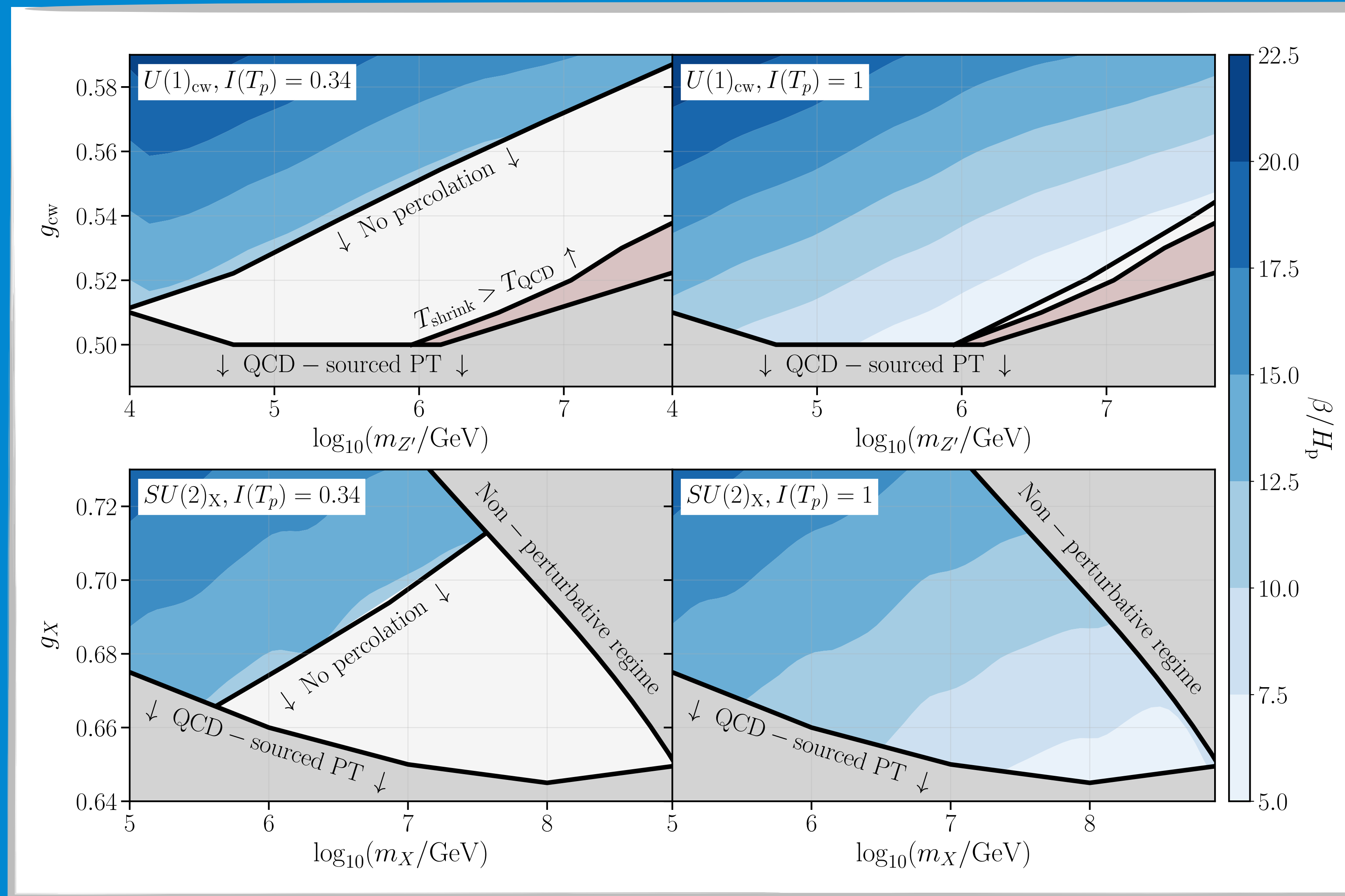
For supercooled PTs SNR looks similar for “daisy”
and “NLOdet”, but...



...NLO and daisy cannot be reconciled in signal reconstruction!



It's unlikely to go below $\beta/H \sim 5$



How do dim-6 operators affect the nucleation rate?

$$\mathcal{L}_1 = \alpha_{\phi^2 F^2} F_{ij} F_{ij} \phi^\dagger \phi + \alpha_{D^2 \phi^4} (D_i \phi^\dagger D_i \phi) (\phi^\dagger \phi) \\ + \alpha_{D^2 \phi^2 B_0^2} (D_i \phi^\dagger D_i \phi) B_0^2 + \alpha_{B_0^2 \phi^4} B_0^2 (\phi^\dagger \phi)^2 + \alpha_{\phi^6} (\phi^\dagger \phi)^3$$

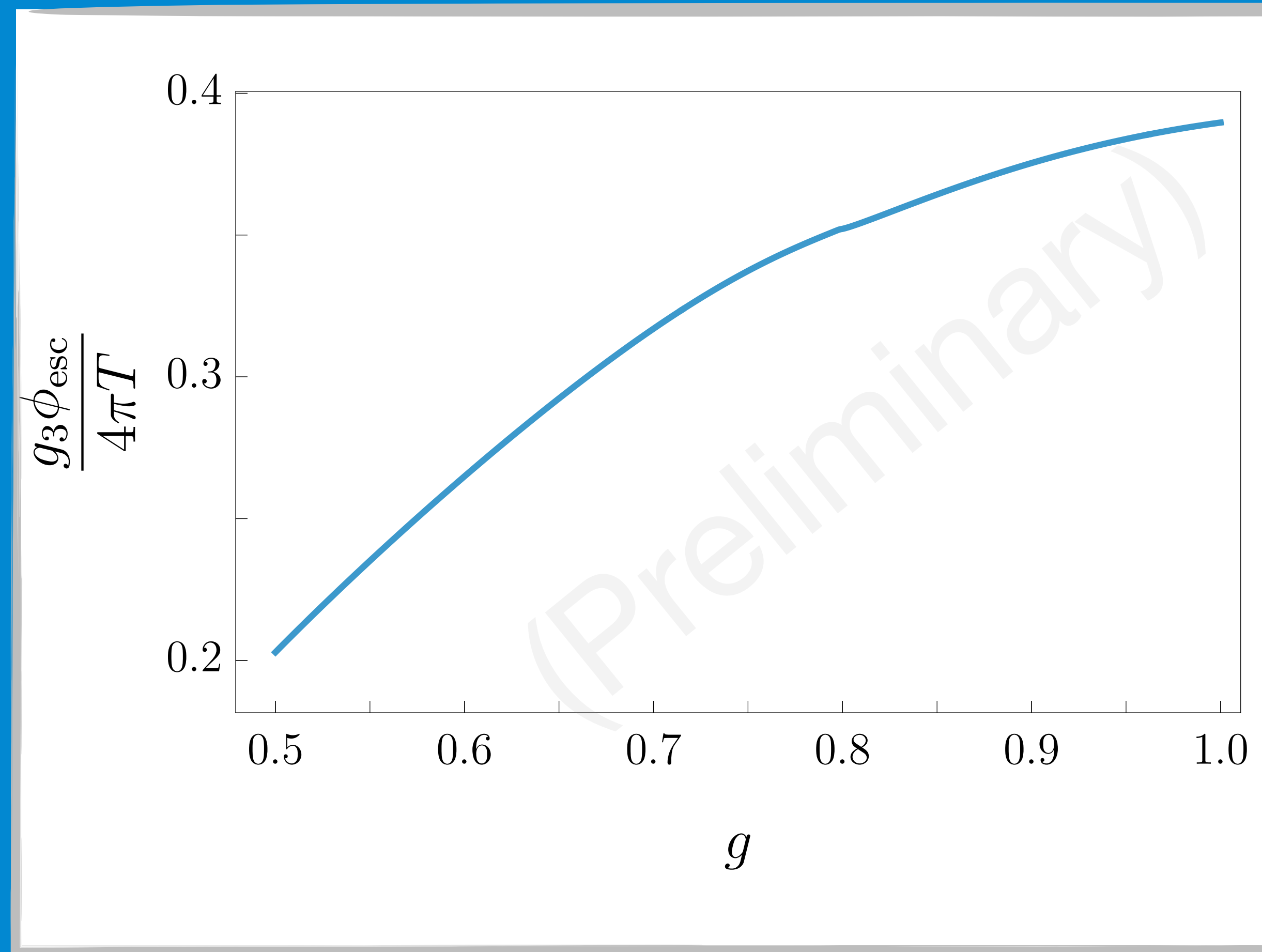
Bounce sol
gets modified

Quadratic terms
get modified

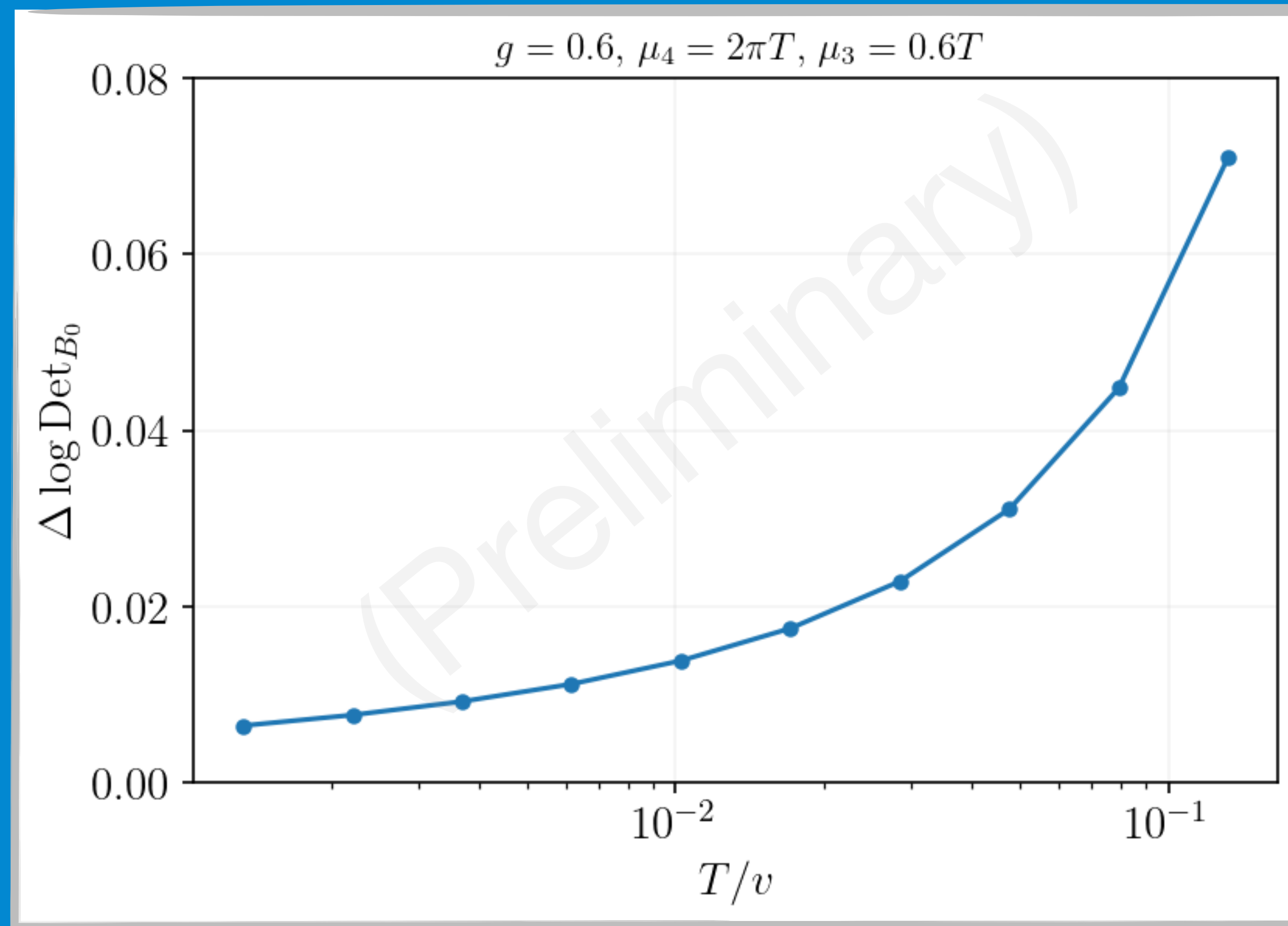
Example:

$$\text{Det}_{X_0} = \text{Det} \left(\partial^2 + m_{X_0}^2 + \frac{1}{2} \alpha_{\phi^4 B_0^2} \phi_b^4 + \alpha_{D^2 \phi^2 B_0^2} \phi_b'^2 \right)$$

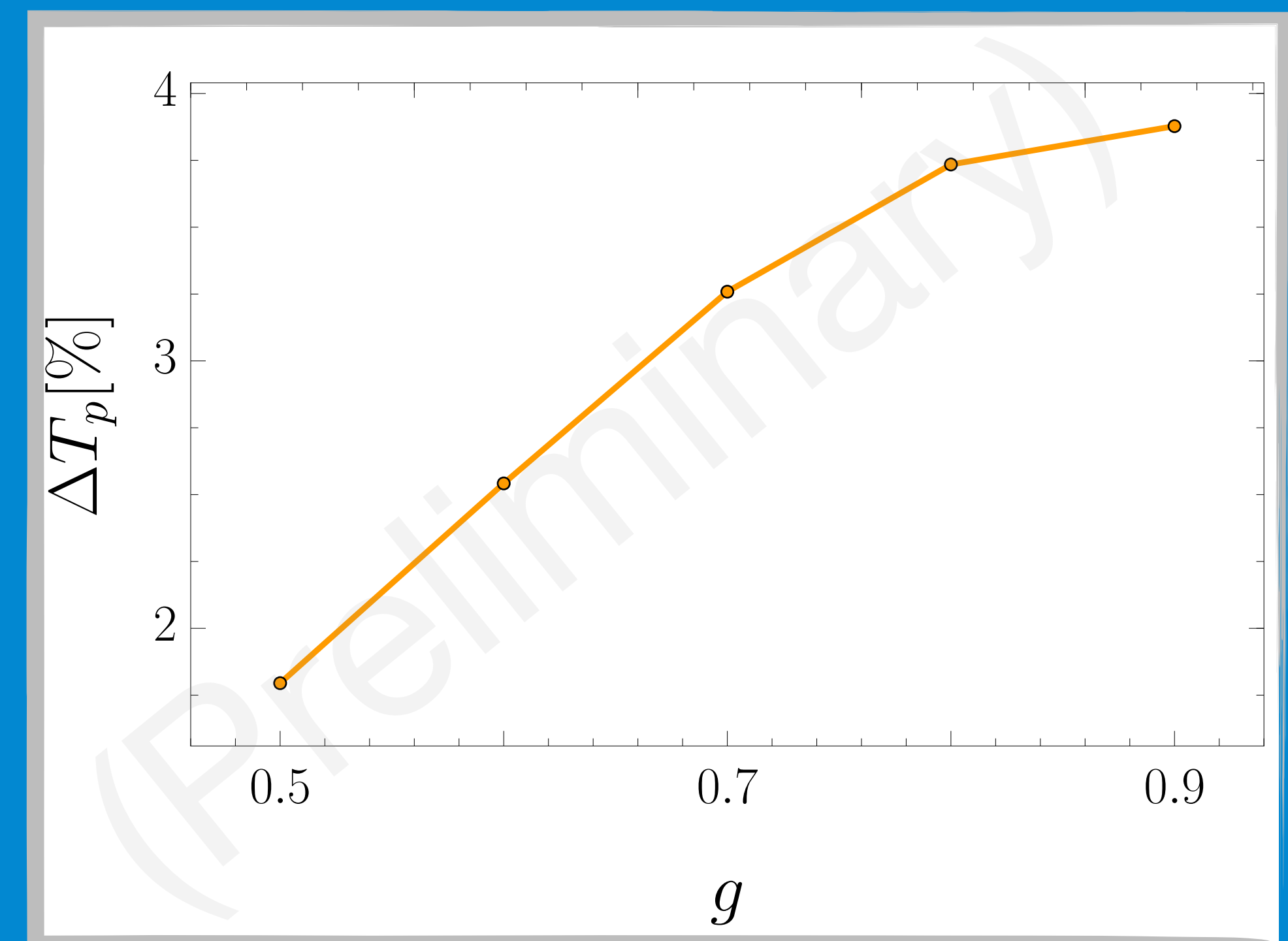
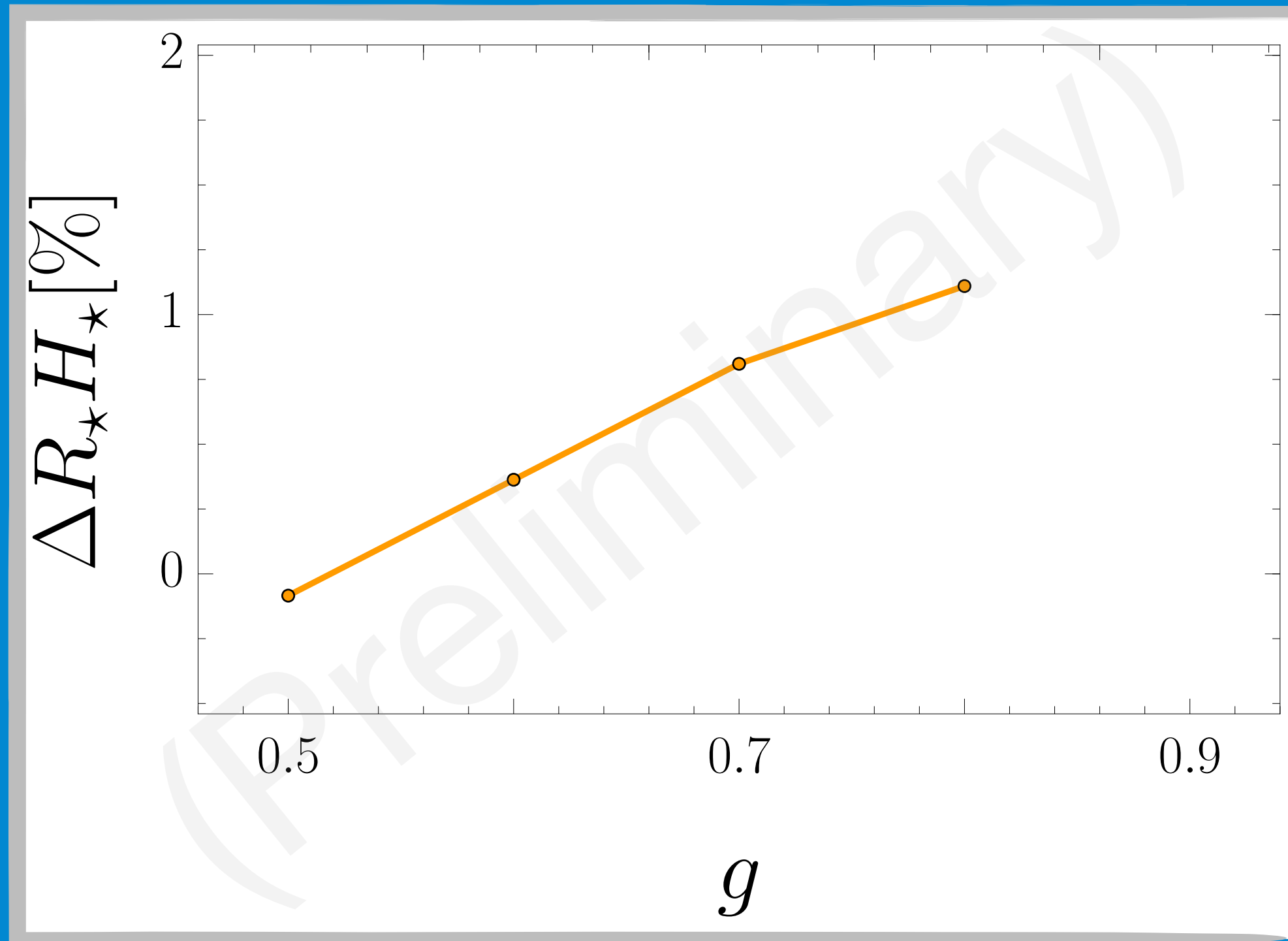
HT validity => small effect of dim-6 operators



The relative difference for Det_{X_0} is up to 7%



T_p changes up to 4%, while R_*H_* up to 1%



(Plots due to courtesy of N. Leister)

Dim-6 operators
seem to have **little**
effect on nucleation

Derivative expansion
for **gauge modes** introduces
significant **errors**.

Summary

NLO corrections are
mandatory for reliable
pheno predictions

Min value
of $\beta/H \simeq 5$

Go to NNLO to
assert convergence of
perturbative
expansion

Outlook

Precise predictions for
PTA data

It'd be nice to have a
comparison to nucl.
rate on a lattice!

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Summary

NLO corrections are
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Outlook

Go to NNLO to
assert convergence of
perturbative
expansion

Use functional
methods

It'd be nice to have a
comparison to lattice!

(Naive) daisy resummation

- Captures the “general” behaviour, but:
- Uncancelled RG-scale dependence
- Naive use leads to imaginary parts in the effective potential
- Gauge-dependent predictions
- Usually double-counts scalar contributions

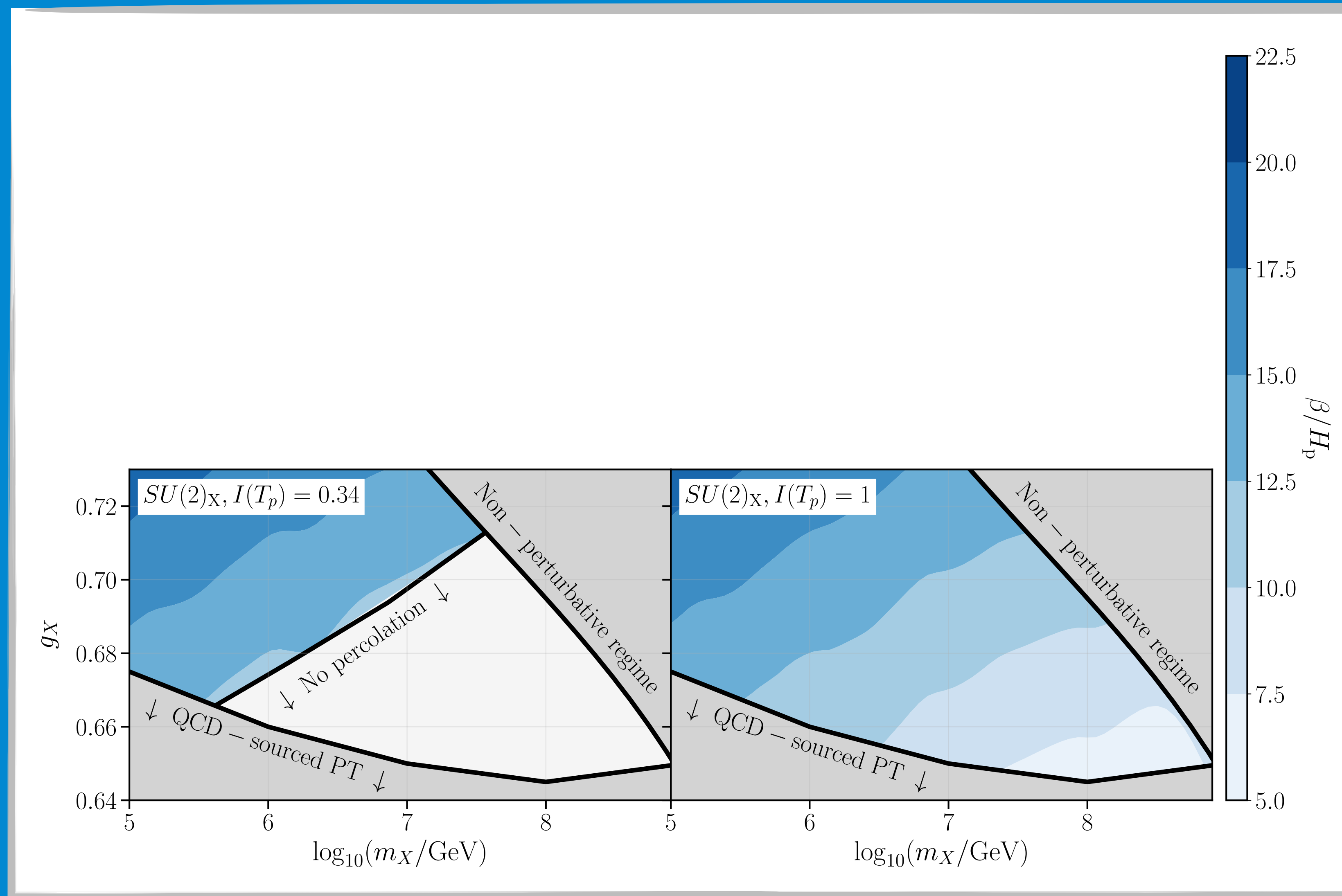
vs

Dimensional Reduction

- Gets *all* pert. contributions at a given $\mathcal{O}(g)$
- Diminished RG-scale dependence
- No imaginary parts in the effective potential
- Gauge-invariant predictions
- One can increase precision consistently, order by order
- Can reproduce lattice very well (at least for eq. thermodynamics)

Thank you!

What can happen for extreme supercooling?



What can happen for extreme supercooling?

