pySecDec TUTORIAL

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$$\sigma = \sigma_{\mathrm{LO}} + \sigma_{\mathrm{NLO}} = \int_n \mathrm{d}\sigma^{\mathrm{B}} + \int_n \mathrm{d}\sigma^{\mathrm{V}} + \int_{n+1} \mathrm{d}\sigma^{\mathrm{R}}$$

Finite

Infinite

Infinite

$\mathrm{d}\sigma^{\mathrm{B}}$ = born cross section

$\mathrm{d}\sigma^{\mathrm{V}}$ = **virtual** correction

$\mathrm{d}\sigma^{\mathrm{R}}$ = real correction

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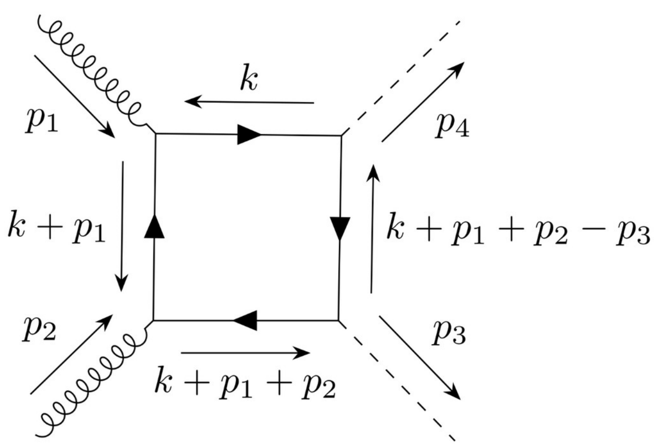
<u>Virtual correction involves **loop integrals**</u>



$$\sim \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{D_1 D_2 D_3 D_4}$$

$$D_1 = k^2 - m^2$$
$$D_2 = (k + p_1)^2 - m^2$$
$$D_3 = (k + p_1 + p_2)^2 - m^2$$
$$D_4 = (k + p_1 + p_2 - p_3)^2 - m^2$$

- These integrals can be *divergent; r*equire ***regularisation*** procedure..

Divergences & Regularisation

- **Dimensional regularisation**
  - Shift dimension such that integrals are well defined: $D = 4 - 2\epsilon$
  - Restore four-dimensional theory by: $\epsilon \longrightarrow 0$

Integration measure: $\int \frac{\mathrm{d}^D k}{(2\pi)^D}$

- Infinities "captured" as **poles in** ϵ
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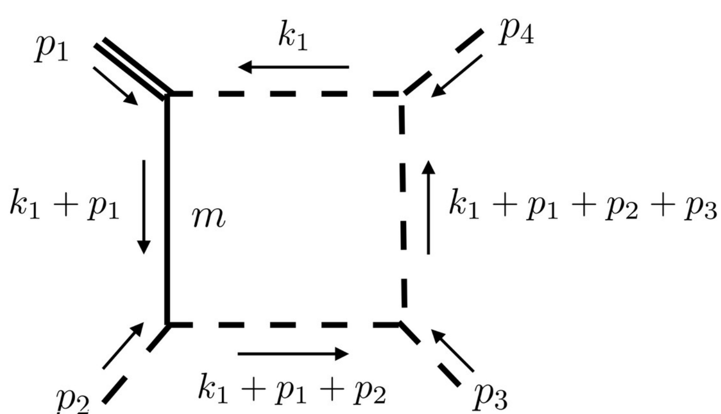
<u>A typical result can look like:</u>



<u>Box with *massless* propagators</u>

<u>Series expansion</u> in ϵ

$$\sim \frac{A(s,t)}{\epsilon^2} + \frac{B(s,t)}{\epsilon} + C(s,t) + \mathcal{O}(\epsilon)$$

- **<u>Infinities extracted as 1/eps</u>**
- A,B,C are *finite* → numerical integration

How to find this series expansion?

Feynman Parametrisation

General loop integral:

L loops

$$G = \int \prod_{l=1}^{L} [\mathrm{d}^D k_l] \, \frac{1}{\prod_{j=1}^{N} P_j^{\nu_j}(\{k\}, \{p\}, m_j^2)}$$

N propagators

$$[\mathrm{d}^D k_l] = \frac{\mu^{4-D}}{i\pi^{\frac{D}{2}}} \mathrm{d}^D k_l$$

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It is often useful to work in *parameter space.* Feyman's trick: $\frac{1}{AB} = \int_0^1 \frac{du}{[uA + (1-u)B]^2}$

Or more generally:

$$\underbrace{\frac{1}{\prod_{j=1}^{N} P_j^{\nu_j}}}_{\text{Product}} = \frac{\Gamma(N_\nu)}{\prod_{j=1}^{N} \Gamma(\nu_j)} \int_0^\infty \prod_{j=1}^{N} \underbrace{\mathrm{d}x_j x_j^{\nu_j - 1}}_{\text{Feynman Parameters}} \delta(1 - \sum_{i=1}^{N} x_i) \frac{1}{\left[\underbrace{\sum_{j=1}^{N} x_j P_j}_{\text{Sum}}\right]^{N_\nu}}$$

Feynman Parametrisation II

Feynman parameterizing our loop integral:

$$G = \int_{-\infty}^{\infty} \prod_{l=1}^{L} [\mathrm{d}^D k_l] \frac{1}{\prod_{j=1}^{N} P_j^{\nu_j}} = \frac{\Gamma(N_\nu)}{\prod_{j=1}^{N} \Gamma(\nu_j)} \int_0^\infty \prod_{j=1}^{N} \mathrm{d}x_j x_j^{\nu_j - 1} \delta(1 - \sum_{i=1}^{N} x_i)$$

$$\times \int_{-\infty}^{\infty} \prod_{l=1}^{L} [\mathrm{d}^D k_l] \left[\sum_{i,j=1}^{L} k_i^T M_{ij} k_j - 2 \sum_{j=1}^{L} k_j^T \cdot Q_j + J + i\delta \right]^{-N_\nu}$$

From quadratic (in k) terms of propagators

Linear (in k) terms

Feynman Parametrisation II

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$$\times \int_{-\infty}^{\infty} \prod_{l=1}^{L} [\mathrm{d}^D k_l] \left[\sum_{i,j=1}^{L} k_i^T M_{ij} k_j - 2 \sum_{j=1}^{L} k_j^T \cdot Q_j + J + i\delta \right]^{-N_\nu}$$

From quadratic (in k) terms of propagators

Linear (in k) terms

Key Point: In this form we can shift k to eliminate linear terms (obtain spherical symmetry) then do the momentum integrals!

Feynman Parametrisation III

After integration over momenta we obtain:

Master Formula

$$G = (-1)^{N_\nu} \frac{\Gamma(N_\nu - LD/2)}{\prod_{j=1}^{N} \Gamma(\nu_j)} \int_0^\infty \prod_{j=1}^{N} \mathrm{d}x_j \; x_j^{\nu_j - 1} \delta(1 - \sum_{i=1}^{N} x_i) \frac{\mathcal{U}^{N_\nu - (L+1)D/2}(\vec{x})}{\mathcal{F}^{N_\nu - LD/2}(\vec{x}, s_{ij})}$$

Graph Polynomials:

1st Symanzik Polynomial: $\mathcal{U}(\vec{x}) = \det(M)$

2nd Symanzik Polynomial: $\mathcal{F}(\vec{x}, s_{ij}) = \det(M) \left[\sum_{i,j=1}^{L} Q_i M_{ij}^{-1} Q_j - J - i\delta \right]$

We have exchanged L momentum integrals for N parameter integrals

Sector Decomposition

In parameter space we know how to algorithmically obtain series expansions

One technique **Iterated Sector Decomposition** repeat:

Binoth, Heinrich 00

$$\int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \frac{1}{(x_1+x_2)^{2+\epsilon}}$$ ← **Overlapping singularity for** $x_1, x_2 \to 0$

$$= \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \frac{1}{(x_1+x_2)^{2+\epsilon}} (\theta(x_1 - x_2) + \theta(x_2 - x_1))$$

$$= \int_0^1 \mathrm{d}x_1 \int_0^{x_1} \mathrm{d}x_2 \frac{1}{(x_1+x_2)^{2+\epsilon}} + \int_0^1 \mathrm{d}x_2 \int_0^{x_2} \mathrm{d}x_1 \frac{1}{(x_1+x_2)^{2+\epsilon}}$$

$$= \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}t_2 \frac{x_1}{(x_1+x_1 t_2)^{2+\epsilon}} + \int_0^1 \mathrm{d}x_2 \int_0^1 \mathrm{d}t_1 \frac{x_2}{(x_2 t_1+x_2)^{2+\epsilon}}$$

$$= \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}t_2 \frac{x_1^{-1-\epsilon}}{(1+t_2)^{2+\epsilon}} + \int_0^1 \mathrm{d}x_2 \int_0^1 \mathrm{d}t_1 \frac{x_2^{-1-\epsilon}}{(t_1+1)^{2+\epsilon}}$$ ← **Singularities factorised**

Sector Decomposition II

Consider a **Sector Decomposed** integral (simple case $a = -1$):

$$\int_0^1 \mathrm{d}x x^{-1-b\epsilon} f(x)$$

$$= \int_0^1 \mathrm{d}x x^{-1-b\epsilon} [f(x) - f(0) + f(0)]$$

$$= \int_0^1 \mathrm{d}x x^{-1-b\epsilon} f(0) + \int_0^1 \mathrm{d}x x^{-1-b\epsilon} [f(x) - f(0)]$$

$$= \underbrace{\frac{f(0)}{-b\epsilon}}_{\text{Poles}} + \underbrace{\int_0^1 \mathrm{d}x x^{-b\epsilon} \left[\frac{f(x) - f(0)}{x}\right]}_{\text{Finite}}$$

By Definition:

$f(0) \neq 0$

$f(0)$ finite

Key Point: Sector Decomposed integrals can be easily expanded in ϵ and **numerically** integrated!

pySecDec

pySecDec evaluates loop integrals

Python

- Divergences subtracted through **sector decomposition**
- Finite (subtracted) integrands are expanded in the dimensional regulator $I = \sum_{m=-r}^{2L} \frac{C_m}{\epsilon^m} + \mathcal{O}(\epsilon^{r+1})$
- The expansion coefficients (finite integrals) are **integrated numerically**

C++ *(with python interface)*

Installation

```
$ python3 -m pip install --user --upgrade pySecDec
```

Exercises

```
python3 -m pip install notebook
```

If you want to run the examples locally (strongly advised)

- Three Jupyter notebooks
  1. Scalar 1-loop box integral **(1_box.ipynb)**
  2. LO amplitude to gg → H **(2_ggh.ipynb)**
  3. LO amplitude to light-by-light scattering **(3_lbl.ipynb)**

Find the notebooks at: https://yeti-2526.notebooks.danielmaitre.phyip3.dur.ac.uk/