CALCULATING PRECISE EW PREDICTIONS

- IN THESE NOTES I WANT TO PRESENT SOME HIGHLIGHTS REGARDING <u>ELECTROWEAK (EW) CORRECTIONS</u> IN PRESENT-DAY PARTICLE PHYSICS PHENOMENOLOGY

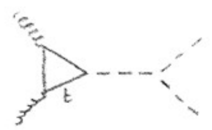
- I WILL ADDRESS THE FOLLOWING QUESTIONS:
  1. WHY ARE EW CORRECTIONS GENERALLY <u>SMALL</u> COMPARED TO QED/QCD ONES?
  2. IN WHAT DO EW CORRECTIONS <u>DIFFER</u> W.R.T. QED/QCD?
  3. HOW CAN WE <u>EASE</u> THE CALCULATION OF EW CORRECTIONS?

- I WILL BASE THESE NOTES ON TWO FIELDS OF PARTICLE PHYSICS PHENOMENOLOGY:
  - SINGLE AND DOUBLE HIGGS BOSON PRODUCTION FROM HADRONS
  - DETERMINATION OF THE ANOMALOUS MAGNETIC MOMENT OF THE MUON

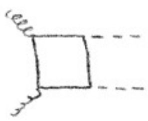
HIGGS PRODUCTION

- THE HIGGS BOSON IS STUDIED NOWADAYS IN CONNECTION TO THE CERN LHC AND ASSOCIATED EXPERIMENTS (ATLAS AND CMS PRIMARLY AND FOR PRECISION MEASUREMENTS).

- THE MAIN CHANNEL OF PRODUCTION (AND THE ONE I WILL FOCUS ON) IS <u>GLUON-GLUON FUSION</u>, WHICH IS A LOOP-MEDIATED PROCESS OF THE FORM



$\alpha_s \quad \alpha$



$\alpha_s \quad \alpha$

- WHEN CONSIDERING HIGHER-ORDER CORRECTIONS qg, $\bar{q}g$, AND $q\bar{q}$ CHANNELS ARE ALSO INCLUDED.
- THE PROPERTIES OF THE HIGGS BOSON AS A PARTICLE (ITS MASS, WIDTH AND SPIN) HAVE BEEN MEASURED SO FAR AT THE LHC, WHILE SELF-INTERACTIONS (IN PARTICULAR THE TRILINEAR COUPLING) WILL BE ONE OF THE GOALS OF THE UPCOMING LH-LHC.

MUON $g-2$

- THE ANOMALOUS MAGNETIC MOMENT OF THE MUON

$$a_\mu = \frac{g_\mu - 2}{2} = 1.16592033\,(62) \cdot 10^{-3}$$

IS ONE OF THE MOST PRECISE MEASUREMENTS IN PARTICLE PHYSICS.

- a_μ IS DETERMINED BY PROBING THE INTERACTION OF A MUON WITH AN EXTERNAL MAGNETIC FIELD (I.E. AN OFF-SHELL PHOTON) IN THE LIMIT FOR TRANSFERRED MOMENTUM APPROACHING ZERO.

- LET US CONSIDER THE MOST GENERAL FORM OF AN INTERACTION BETWEEN THE EM FIELD AND A MUON

$$= \bar{u} \left\{ F_1 \gamma^\mu + \frac{i \sigma^{\mu\nu}}{2m} q_\nu F_2 \right\} u$$

- γ^μ COMES FROM $\bar{\psi} \not{A} \psi$, SO F_1 EMBEDS THE ELECTRIC CHARGE

- $\sigma^{\mu\nu} q_\nu$ COMES FROM $\bar{\psi} \sigma^{\mu\nu} \partial_\nu A_\mu \psi$, WHICH CAN BE RELATED TO

$$\vec{B} \cdot \vec{\sigma} \longrightarrow \underbrace{\vec{B} \cdot \vec{S}}_{\text{MAGNETIC DIPOLE ENERGY}}$$

SO MEASURING a_μ IS DIRECTLY RELATED TO MEASURING $F_2(q^2 \to 0)$. NOTICE THAT THIS TENSOR STRUCTURE IS NOT PRESENT AT LO, RESULTING FROM LOOP EFFECTS.

- THE PROCEDURE SHOWED ABOVE TO WRITE THE μ-γ VERTEX IN FULL GENERALITY IS CALLED TENSOR DECOMPOSITION:

$$\underbrace{\mathcal{M}_{\{\lambda\}}}_{\text{AMPLITUDE}} = \sum_i \underbrace{\mathbb{T}_{i,\{\lambda\}}}_{\text{TENSORS}} \underbrace{F_i}_{\text{SCALAR FORM FACTORS}}$$

THIS IS A COMMON PROCEDURE IN PRECISION CALCULATIONS. ONCE THE DECOMPOSITION IS OBTAINED, THE FORM FACTORS CAN BE EXTRACTED BY CONTRACTING THE AMPLITUDE WITH PROJECTORS $\mathbb{P}_{\{\lambda\},i}$, OBTAINED AS LINEAR COMBINATIONS OF THE $\mathbb{T}^*_{\{\lambda\},i}$.

1. THE SIZE OF EW CORRECTIONS

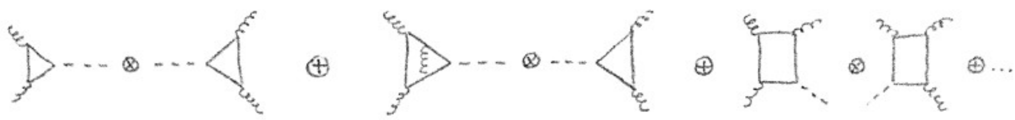
- WE CONSIDER THE THEORETICAL PREDICTIONS FOR (DOUBLE) HIGGS GLUON FUSION AND MUON $g-2$, INSPECTING THE ROLE OF EW CONTRIBUTIONS THEREIN AND DISCUSSING THE PHYSICAL GROUNDS BEHIND THEIR SIZE

gg H

- THE THEORETICAL PREDICTION FOR THE TOTAL CROSS SECTION READS [1]

$\sigma = 47.93$ pb $=$	16.00 pb +	LO	
	18.79 pb +	NLO QCD	$+117\%\ \sigma_{LO}$
	9.90 pb +	NNLO QCD	$+62\%\ \sigma_{LO}$
	1.49 pb +	N^3LO QCD	$+9\%\ \sigma_{LO}$
	1.10 pb +	NLO EW	$+5\%\ \sigma_{LO}$
	0.65 pb	NLO QCD × NLO EW	$+4\%$ LO

- THERE IS A HUGE DIFFERENCE BETWEEN NLO QCD AND NLO EW. WE COMPARE NOW THE TWO CONTRIBUTIONS.
- NEXT-TO-LEADING ORDER (NLO) CORRECTIONS FEATURE AN EXTRA POWER OF α_s (IF QCD) OR α (IF QED/EW) W.R.T. LEADING ORDER (LO) AT THE LEVEL OF THE AMPLITUDE MODULUS SQUARED $|\mathcal{M}|^2$. WE HAVE THEN (E.G. IN QCD)

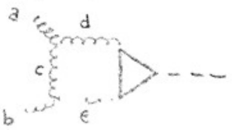
$|\mathcal{M}|^2 \Rightarrow$ 

LO: $\alpha_s^2 \alpha$ VIRTUAL NLO $\alpha_s^3 \alpha$ REAL NLO $\alpha_s^3 \alpha$

(INTEGRATED OVER THE PHASE SPACE OF THE EXTRA GLUON)

- LET US CONSIDER VIRTUAL CORRECTIONS FIRST.
  - CORRECTIONS TO GLUONS

QCD



$\sim C_A = N_c = 3$

EW

ABSENT

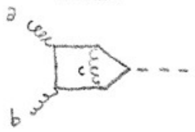
$f^a{}_{cd}\, f^{bc}{}_e\, \mathrm{Tr}(T^d T^e)$ $\underbrace{\quad}_{\delta^{de}}$

$f^a{}_{cd}\, f^{bcd}$

$N_c\, \delta^{ab}$

· CORRECTIONS TO TOP QUARKS

QCD

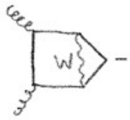


$\sim C_F = \dfrac{N_c^2 - 1}{2N_c} = \dfrac{4}{3}$

$Tr(T^a T^b \underbrace{T^c T_c}_{\frac{N_c^2-1}{2N_c}\mathbb{1}})$

$\dfrac{N_c^2-1}{2N_c}\,\delta^{ab}$

EW



$\sim C_L = \dfrac{3}{4}$

$\oplus$

$\sim Q_t^2 = \dfrac{4}{9}$ (QED)

THE TWO CONTRIBUTIONS ARE DIAGRAMMATICALLY VERY SIMILAR: A GOOD APPROXIMATION IS TO TAKE THE TOP QUARK MASS TO BE MUCH LARGER THAN ANY OTHER SCALE IN THE PROBLEM, RESULTING IN W AND Z PROPAGATORS TO ACT AS MASSLESS $SU(2)_L \times U(1)_Y$ GAUGE BOSONS. THE MAIN DAMPING EFFECT IS NOW GIVEN BY THE COUPLING CONSTANTS, SINCE

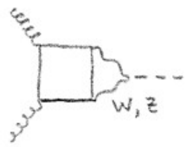
$$\left.\begin{array}{l} \text{NLO EW} \propto \alpha = 1/137 \sim 0.0078 \\ \text{NLO QCD} \propto \alpha_s \quad \sim \quad 0.11 \end{array}\right\} \quad \frac{\text{NLO EW}}{\text{NLO QCD}} = \frac{\alpha}{\alpha_s} \sim 0.07$$

· YUKAWA - INDEPENDENT CORRECTIONS [2,3]

QCD

ABSENT

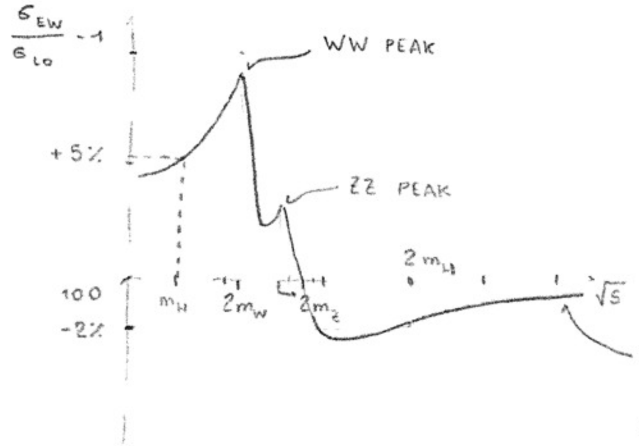
EW



$\propto g\, m_W$

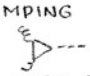
· NO YUKAWA COUPLING: LIGHT QUARKS RELEVANT

· DIFFERENTLY W.R.T. DIAGRAMS CONTAINING THE TOP, LIGHT - QUARK CONTRIBUTIONS ARE ENHANCED AROUND $s = 4m_W^2$ OR $s = 4m_Z^2$ (THE CORRESPONDING PROPAGATOR APPROACHES ZERO):



$\sigma_{EW} \propto Re\left(\mathcal{M}_{LO}\, \mathcal{M}^*_{NLO\ EW}\right)$

DAMPING FROM



AT HIGH ENERGIES

· DIAGRAMMATICALLY THESE CONTRIBUTIONS ARE ~ 4 TIMES THE VIRTUAL

QCD ONES. REINSTATING THE α/α_s SUPPRESSION WE GO DOWN TO

$$\text{vNLO EW} = \frac{\text{vNLO QCD}}{4}$$

SO WHY ARE EW CORRECTIONS SO SMALL AT NLO?

· LET US NOW CONSIDER REAL EMISSIONS. IN THE EW CASE THEY WOULD AMOUNT TO

$$gg \to H\gamma$$

BUT THIS AMPLITUDE IS ZERO FOR FURRY THEOREM, SINCE

$$gg H\gamma \longrightarrow \delta^{ab}\, \gamma\gamma H\gamma$$

↳ ZERO FOR FURRY THEOREM

· IN QCD WE HAVE INSTEAD

$$gg \to Hg$$

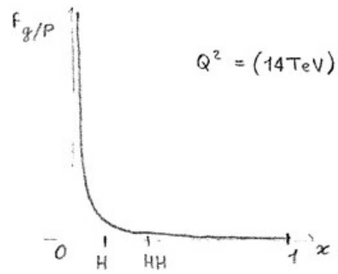
WHICH IS NON-ZERO THANKS TO ITS RICHER COLOUR STRUCTURE W.R.T. $ggH\gamma$.

· THE EMISSION OF AN EXTRA PARTICLE REQUIRES ADDITIONAL ENERGY IN THE PARTONIC PROCESS. THE ENERGY OF THE INCOMING GLUONS IS DETERMINED BY THE PARTON DISTRIBUTION FUNCTIONS (PDFS), WHICH DESCRIBE HOW TO EXTRACT A PARTON WITH THE FRACTION x OF THE ENERGY OF THE PROTON AT A GIVEN C.O.M. ENERGY.

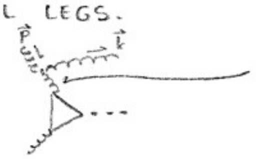
· FOR LHC ENERGIES THE GLUON PDFS ARE STEEPLY FALLING AROUND

$$x^2 \sim \frac{m_H^2}{14\ \text{TeV}} \sim (10^{-2})^2$$

ASSUMING SYMMETRIC COLLISION (MOST PROBABLE)



SO SOFT GLUONS (I.E. $E_g \to 0$) WILL BE PRODUCED THROUGH REAL EMISSIONS. LET US CONSIDER THE EMISSION OF SUCH SOFT GLUON FROM ONE OF THE EXTERNAL LEGS.



$$\frac{i}{(p-k)^2} \Rightarrow \frac{i}{2p\cdot k} \sim \frac{1}{E_k}$$

· THE PROPAGATOR IS SINGULAR IN $E_k \to 0$. INTEGRATING OVER THE UN-RESOLVED GLUON GIVES

$$\int \frac{d^3\vec{k}}{2E_k} |\mathcal{M}|^2 \longrightarrow \int \frac{E_k^2 \, dE_k \, d\Omega}{2E_k} \frac{1}{E_k^2} R \longrightarrow \log E_k$$

· AFTER COMBINING REAL, VIRTUAL, AND PDF CONVOLUTION THE RESULT IS FINITE (KLN THEOREM), BUT THE LOGARITHMIC ENHACEMENT FOR $E_k \to 0$ REMAINS.

· THE COMBINATION OF PDFS FAVOURING SOFT EMISSIONS AND OF LARGE LOGARITHMIC EFFECTS IN THE SOFT REGION MAKES REAL EMISSIONS HUGE, ABOUT FOUR OR FIVE TIMES THE NLO QCD VIRTUAL CONTRIBUTIONS.

· COMBINING THE RESULTS, WE SEE NOW

EW		QCD	
· ONLY VIRTUAL:	$+5\% \, \sigma_{LO}$	· VIRTUAL:	$\sim +20\% \, \sigma_{LO}$
		· REAL:	$\sim +80\% \, \sigma_{LO}$
NLO EW:	$+5\% \, \sigma_{LO}$	NLO QCD:	$+100\% \, \sigma_{LO}$

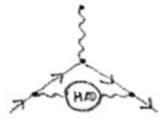
· WE SEE FOR SINGLE HIGGS PRODUCTION THAT EW CORRECTIONS ARE DISFAVOURED FOR
- WEAK COUPLING: $\alpha/\alpha_S \sim 0.05$
- ABSENCE OF REAL RADIATION

· $(g-2)_\mu$

· THE THEORETICAL PREDICTIONS FOR $(g-2)/2 = a$ READ

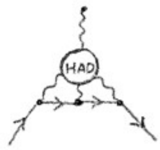
$a_\mu^{SM} = 116\,584\,718.9 \cdot 10^{-11} +$	QED (UP TO FIVE LOOPS)	
$6\,845 \cdot 10^{-11} +$	HVP	$5 \cdot 10^{-5}$
$154 \cdot 10^{-11} +$	EW (UP TO TWO LOOPS)	10^{-6}
$92 \cdot 10^{-11}$	HLbL	$8 \cdot 10^{-7}$

- THE DIFFERENT CONTRIBUTIONS CONSIST OF:
  - QED : QED LOOP CORRECTIONS UP TO FIVE LOOPS.
  - HVP : HADRONIC VACUUM POLARIZATIONS. QCD CORRECTIONS TO THE PHOTON PROPAGATOR. SINCE THE ENERGY SCALE OF THE PROCESS IS $m_\mu \sim 0.1$ GeV QCD IS NON-PERTURBATIVE AND HADRONS MUST BE CONSIDERED



  - EW : COMPLETE EW CORRECTIONS (W, Z, H)

  - HLbL : HADRONIC LIGHT-BY-LIGHT. FOUR-PHOTON VERTICES MEDIATED BY HADRONS



- LET US ESTIMATE THE SIZE OF THE CORRECTIONS. SINCE WE ARE CONSIDERING a_μ, WE NEED TO CONSIDER THE TERM

$$\frac{i\sigma^{\mu\nu}}{2m_\mu} q_\nu F_2$$

- FIRST OF ALL, THE MASS DIMENSION OF THIS TERM IS ZERO, SINCE IT MUST BE THE SAME OF γ^μ (LO $\bar{\mu}\mu\gamma^*$ DIAGRAM). AS A RESULT, F_2 MUST BE A FUNCTION OF THE RATIO OF m_μ AND ANOTHER MASS SCALE. SINCE WE ARE IN THE LIMIT $q^2 \to 0$, THE OTHER SCALE CAN ONLY BE THE MASS OF THE INTERMEDIATE PARTICLE(S) $\bar{m}$. SO F_2 CAN BE EXPANDED AS

$$F_2 = \left(\frac{m_\mu}{\bar{m}}\right)^n \left[C_0 + C_1 \frac{m_\mu}{\bar{m}} + \dots \right]$$

- $\sigma^{\mu\nu}$ CONTAINS TWO GAMMA MATRICES, INDICATING THAT A SPIN FLIP OF THE FERMION IS NECESSARY TO CONTRIBUTE TO F_2. IN THE SM (STANDARD MODEL) A SPIN FLIP CAN COME EITHER FROM A MASS TERM OR A YUKAWA COUPLING, BOTH HERE PROPORTIONAL TO m_μ. SO WE NEED

$$\frac{1}{m_\mu}\left(\frac{m_\mu}{\bar{m}}\right)^n \propto m_\mu$$

$$\downarrow$$

$$\underline{\underline{n = 2}}$$

- WE FIND THAT F_2 IS SUPPRESSED BY A FACTOR $m_\mu^2/\bar{m}^2$.

- LET US SEE WHAT THIS IMPLIES FOR QED, EW, AND HADRONIC CONTRIBUTIONS:
  - QED: SINCE ONLY m_μ IS PRESENT, WE TAKE $\overline{m} = m_\mu$, GETTING

$$1$$

THIS IS OUR BASELINE.

  - EW: WE GET AT BEST, FOR $\overline{m} = m_W$

$$\frac{m_\mu^2}{m_W^2} \sim \frac{0.1^2}{80^2} \sim 10^{-6}$$

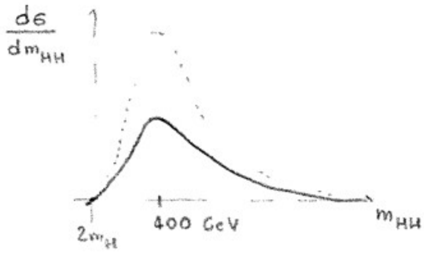
  - HADRONIC: $\overline{m} = \Lambda_{QCD} \sim 1$ GeV AND WE HAVE A FACTOR α/π MORE, SINCE THE HVP BUBBLE NEEDS TO COUPLE TWICE TO THE PHOTON

$$\frac{m_\mu^2}{\Lambda_{QCD}^2}\,\frac{\alpha}{\pi} \sim 10^{-5}$$

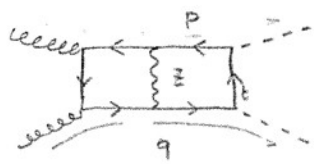
- WE SEE THAT IN THIS CASE THE MASS SCALE DISFAVOURS EW CORRECTIONS.

- gg HH

- QCD CORRECTIONS BEHAVE SIMILARLY TO THE SINGLE-HIGGS CASE: PDF SUPPRESSION FAVOURS SOFT-GLUON EMISSION, WHICH PRODUCES LARGE LOGARITHMS, THIS HAPPENS INDEPENDENTLY OF THE ENERGY OF THE HIGGS BOSON PAIR, PRODUCING A $+60\%\,\sigma_{LO}$ INCREASE [4]



- EW CORRECTIONS EXHIBIT HERE A NEW BEHAVIOUR W.R.T. SINGLE HIGGS PRODUCTION. LET US CONSIDER DIAGRAMS OF THE FORM



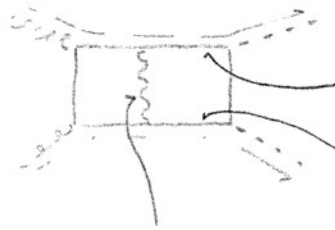
WHERE p AND q ARE EXTREMELY ENERGETIC MOMENTA ON THE TOP QUARK MASS-SHELL, IN COMBINATION WITH A LOOP MOMENTUM MUCH SMALLER THAN p, q BUT STILL LARGER THAN $m_{W/Z}$:

$$(p+q)^2 = s \gg 4 m_H^2$$

$$p^2 \sim q^2 \sim m_t^2$$

$$\sqrt{s} \sim p, q \gg k \gg m_{W/Z}$$

IN THIS CONFIGURATION, WE GET



$$\frac{1}{(p+k)^2 - m_t^2} \sim \frac{1}{p \cdot k}$$

$$\frac{1}{(q+k)^2 - m_t^2} \sim \frac{1}{q \cdot k}$$

$$\frac{1}{k^2 - m_V^2} \sim \frac{1}{k^2}$$

- WE INVESTIGATE THE DIVERGENCIES GENERATED BY THESE TERMS UPON INTEGRATION OVER k_1. THE OTHER PROPAGATORS ARE NOT SINGULAR IN THIS REGIME, SO THEY ARE NOT RELEVANT FOR THE DISCUSSION.

$$\int \frac{d^D k}{(p \cdot k)(q \cdot k)\, k^2}$$

- $d^D k = E_k^{D-1}\, dE_k\, d\cos\vartheta\, d^{D-2}\varphi$
- $p \cdot k = E_p E_k \left(1 - \cos\vartheta\right)$
- $q \cdot k = E_q E_k \left(1 - \cos\bar{\vartheta}\right)$
- $k^2 = E_k^2$

WE GET THEN

$$\underbrace{\int \frac{E_k^{D-1}\, dE_k}{E_k^4}}_{\text{"SOFT"}} \quad \underbrace{\int \frac{d\cos\vartheta}{(1-\cos\vartheta)}}_{\text{"COLLINEAR"}} \quad \ldots$$

- WE SEE THAT (FOR D=4) THE SOFT INTEGRAL RETURNS A LOGARITHM. THE INTEGRATION DOMAIN GOES FROM $E_k \sim \sqrt{s}$ DOWN TO $E_k \sim m_V$, SINCE WE HAVE SET $\sqrt{s} \gg k \gg m_V$. WE GET THEN

$$\int_{m_V}^{\sqrt{s}} \frac{dE_k}{E_k} \sim \log\frac{\sqrt{s}}{m_V}$$

- THE COLLINEAR INTEGRAL IS LOGARITHMIC AS WELL. IT WILL EXTEND

FROM $\cos\theta = -1$ TO AN UPPER LIMIT GIVEN BY THE $\sqrt{s} \gg k \gg m_W$ LIMIT. IN PARTICULAR,

$$k \cdot p = E_k E_p - \underbrace{\|\vec{k}\|}_{\sqrt{E_k^2 - m_V^2}} \underbrace{\|\vec{p}\|}_{E_p \text{ SINCE } p \gg m_t \text{ AND FIXED}} \cos\theta = E_p \left(E_k - \sqrt{E_k^2 - m_V^2} \cos\theta \right)$$

· A LIMIT WHERE OUR CONSTRAINT ON k FAILS IS $\cos\theta \to 1$, SINCE

$$E_k - \sqrt{E_k^2 - m_V^2} \sim m_V^2$$

(A LARGE CANCELLATION OCCURS). IF WE KEEP $\cos\theta > \frac{m_V}{E_k}$ INSTEAD, WE GET AT WORST

$$E_k - m_V \sim E_k$$

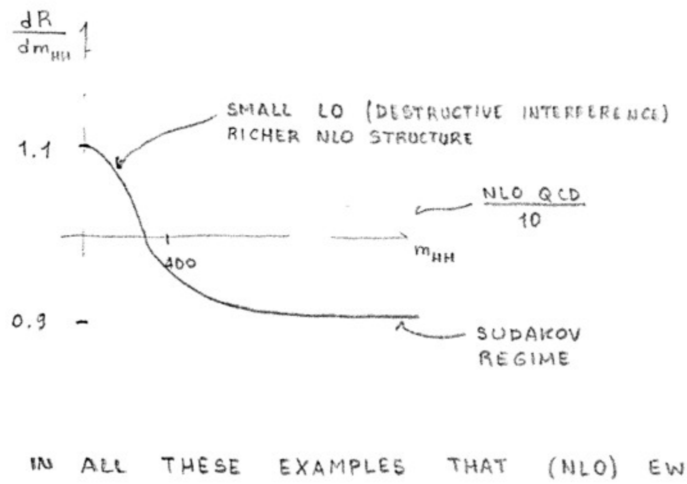
AND THE REGIME FOR k HOLDS. THE ONLY SCALE LEFT FOR E_k IS $\sqrt{s}$, SO

$$\int_{m_V/\sqrt{s}}^{-1} \frac{d\cos\theta}{1-\cos\theta} \sim \log \frac{\sqrt{s}}{m_V}$$

· THESE UNIVERSAL LOGARITHMS ARE CALLED <u>SUDAKOV LOGARITHMS</u>, AND ARE TYPICAL OF EW CORRECTIONS. TO BE RELEVANT, THE DISPROPORTION BETWEEN $\sqrt{s}$ AND m_V MUST BE HUGE. THEREFORE, THEIR EFFECT BECOMES RELEVANT AT VERY HIGH ENERGIES.

· SUDAKOV LOGS ARE ALWAYS <u>NEGATIVE</u>: COMING FROM VIRTUAL CORRECTIONS, THEY ARE RELATED TO THE PROBABILITY THAT A VERY ENERGETIC PARTICLE DOES NOT EMIT SOFT/COLLINEAR OBJECTS, BUT THE MORE A PARTICLE IS ENERGETIC, THE SMALLER THE DEVIATION FROM ON-SHELLNESS FOR EXTRA EMISSIONS.

· CONSIDER THE NLO EW / LO RATIO FOR $gg \to HH$ [5]:



· WE HAVE OBSERVED IN ALL THESE EXAMPLES THAT (NLO) EW CORRECTIONS ARE SYSTEMATICALLY SMALLER THAN MASSLESS GAUGE CORRECTIONS (I.E. QCD AND QED), BUT DEPENDING ON THE PROCESS DIFFERENT MECHANISMS ARE INVOLVED:

- <u>$\alpha / \alpha_s \sim 0.07$</u> IS A LARGE OBSTACLE IN HIGGS PHYSICS: EW CORRECTIONS CAN BE DIAGRAMMATICALLY LARGE, THANKS TO THE MASS OF THE EW BOSONS (THRESHOLD EFFECTS IN n_ℓ ggH, SUDAKOV LOGS

AT HIGH ENERGIES IN $ggHH$), BUT THE COUPLING WEAKENS THEIR CONTRIBUTION.

- ABSENCE OF REAL CORRECTIONS: NO SOFT-ENHANCEMENT OR PDF ENHANCEMENT
- DISFAVORABLE KINEMATICS: $m_\mu^2 / m_V^2 \sim 10^{-6}$.

- TO SUMMARIZE: ON A DIRECT CONFRONTATION EW CORRECTIONS ARE DISFAVOURED.

2. THE FORM OF EW CORRECTIONS

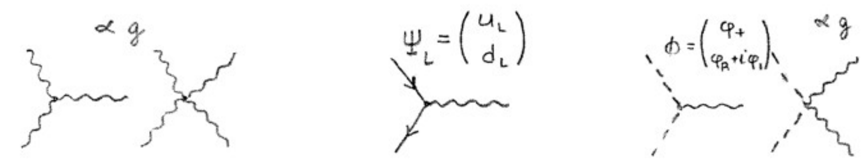
· WE WILL PRESENT SOME ASPECTS PECULIAR TO THE EW THEORY AND ITS PHENOMENOLOGICAL EFFECTS.

· CONSTRAINTS

· THE EW SECTOR CONSISTS OF
- · AN $SU(2)_L$ GAUGE FIELD W (g)
- · A $U(1)_Y$ GAUGE FIELD B (g')
- · A COMPLEX SCALAR DOUBLET ϕ (λ)
- · LEFT-HANDED MASSLESS FERMIONS
- · RIGHT-HANDED MASSLESS FERMIONS (Y)

ALL THESE FIELDS COUPLE AMONG EACH OTHER: $\partial_\mu \to (D_{\mu,B} + D_{\mu,W})\phi$

$SU(2)_L$

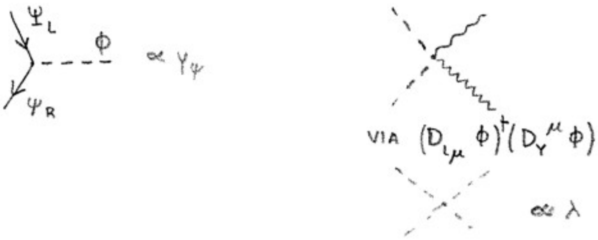


LIKE QCD — LIKE SCALAR QCD

$U(1)_Y$

LIKE QED — LIKE SCALAR QED

MIXED



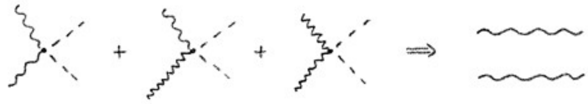
· THE SCALAR DOUBLET GETS A NON-ZERO VACUUM EXPECTATION VALUE (VEV), CAUSING A SPONTANEOUS SYMMETRY BREAKING (SSB) IN THE THEORY

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

PRODUCING

MASSIVE FERMIONS (L+R)

MASSIVE $W^\pm$

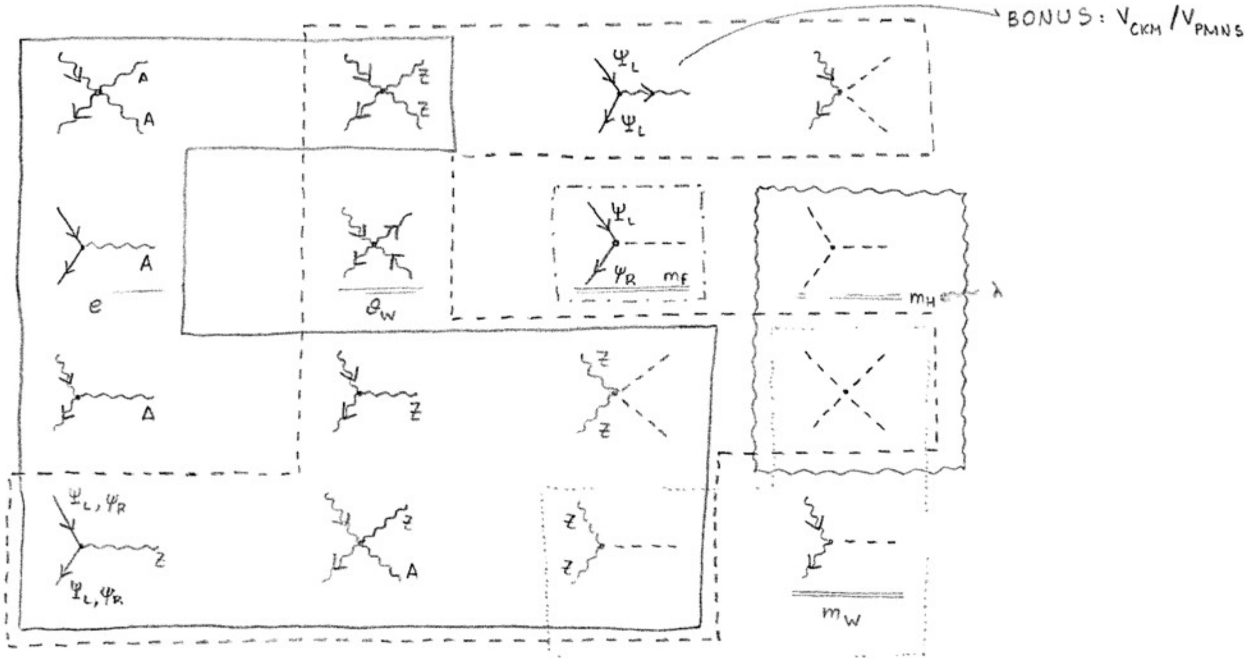


MASSIVE Z

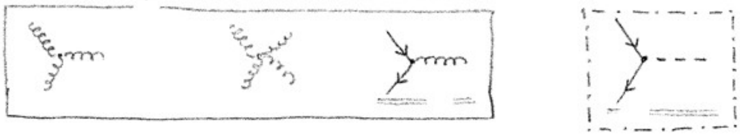
MASSLESS A

WE SEE THAT THE COUPLING STRENGTH AND CONFIGURATION PRODUCES THE MASS OF THE PARTICLES IN THE BROKEN PHASE.

· IF WE REGROUP THE EW FEYNMAN RULES ACCORDING TO THE INDEPENDENT COUPLING STRUCTURES WE GET



· IN COMPARISON, QCD + YUKAWA HAS



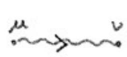
· GAUGE INVARIANCE ALSO HAS PROFUND CONSEQUENCES IN THE EW SECTOR. AS A CONSEQUENCE OF SSB, THE D.O.F. OF Φ OTHER THAN THE HIGGS FIELD (THE GOLDSTONE BOSONS) MIX WITH THE W AND Z BOSONS, PROVIDING THEM THE EXTRA POLARIZATION STATE THAT COMES WITH A NON-ZERO MASS FOR A VECTOR FIELD. THIS PROCESS IS NOT UNIQUE, AND THE "AMOUNT OF INCLUSION" OF THE GOLDSTONES INTO THE GAUGE BOSONS IS REGULATED BY GAUGE FIXING. $\sim \left(\partial^\mu Z_\mu + \xi_Z m_Z \varphi_Z\right)^2$

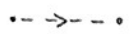
$$\mu \sim\sim\sim \nu = -i\left[\eta^{\mu\nu} - (1-\xi)\frac{k^\mu k^\nu}{k^2}\right] \xrightarrow{\text{SBB}} \begin{cases} \dfrac{-i}{k^2-m^2}\left[\eta^{\mu\nu} - (1-\xi)\dfrac{k^\mu k^\nu}{k^2-\xi m^2}\right] = \mu \sim\!\!>\!\!\sim \nu \\[2ex] \dfrac{i}{k^2-\xi m^2} = \bullet - - > - - \bullet \end{cases}$$

· BY VARYING ξ WE EMBED THE GOLDSTONE BOSON INTO THE GAUGE BOSON. AN EXTREME CASE (USED FOR THE TABLE ABOVE) IS UNITARY GAUGE ($\xi \longrightarrow \infty$) WHERE THE GOLDSTONE BOSONS BECOME INFINITELY MASSIVE AND TRANSFER ALL THEIR D.O.F. TO THE GAUGE BOSON, WHOSE PROPAGATOR BECOMES NOW THE FULL PROCA PROPAGATOR

$$\mu \sim\!\!>\!\!\sim \nu = \frac{-i}{k^2-m^2}\left(\eta^{\mu\nu} - \frac{k^\mu k^\nu}{m^2}\right) \implies 3 \text{ D.O.F.}$$

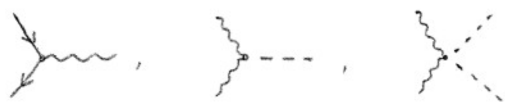
- THE OTHER EXTREMUM IS THE LANDAU GAUGE ($\xi = 0$) WHERE

 $= \dfrac{-i}{k^2 - m^2}\left(\eta^{\mu\nu} - \dfrac{k^\mu k^\nu}{k^2}\right) \Rightarrow$ 2 D.O.F.

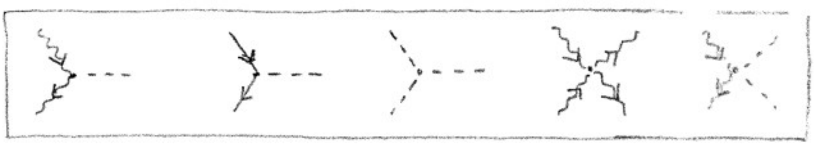
 $= \dfrac{i}{k^2} \Longrightarrow$ 1 D.O.F.

- GOLDSTONE AND GAUGE BOSONS COUPLE TO THE SAME PARTICLES AND SHARE A COMMON GAUGE PARAMETER. GAUGE INVARIANCE ENSURES THAT SUMMING OVER ALL DIAGRAMS IN AN AMPLITUDE RESULTS IN ξ DISAPPEARING FROM THE RESULT.
- GOLDSTONE AND GAUGE BOSONS DO NOT COUPLE THE SAME E.G. TO FERMIONS:
  - GOLDSTONE BOSONS ORIGINATE FROM Φ, SO THEY HAVE A YUKAWA COUPLING WITH FERMIONS AND A TRILINEAR COUPLING WITH THE HIGGS

  - GAUGE BOSONS HAVE GAUGE COUPLINGS BOTH WITH FERMIONS AND THE HIGGS



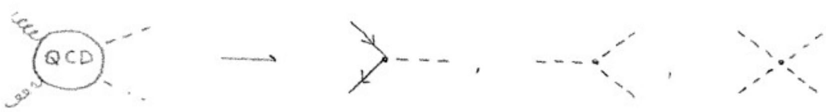
SINCE THEY SHARE THE SAME GAUGE PARAMETER AND IT MUST ULTIMATELY CANCEL, THE VERTICES ABOVE CANNOT BE CONSIDERED INDEPENDENT:



THIS COMPLETELY CONSTRAINTS THE EW SECTOR TO SM CONFIGURATION: WE CANNOT ARBITRARLY VARY THE STRENGTH OF JUST ONE COUPLING AND KEEP THE REST FIXED.

HIGGS PRODUCTION

- A TEMPORARY WAY OUT OF THE CONSTRAINTS ABOVE IS TO CONSIDER CORRECTIONS ONLY ENCOMPASSING AN INDEPENDENT SUBSET OF THE EW RULES. THIS NATURALLY HAPPENS FOR QCD CORRECTIONS, WHERE ONLY THE YUKAWA VERTEX IS CONSIDERED AND NO EW GAUGE INVARIANCE IS TO BE CONSIDERED. HERE, A K-FRAMEWORK TO STUDY THE TRILINEAR COUPLING IS CONSISTENT.



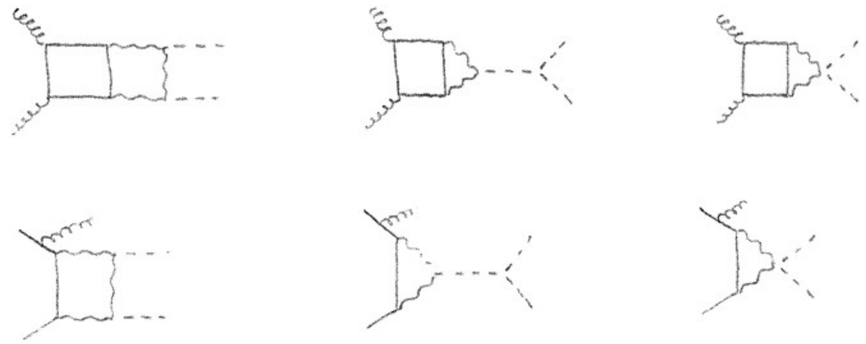
INDEPENDENT

- INCLUDING EW CORRECTIONS CHANGES THE PICTURE. IF WE CONSIDER $q\bar{q} \longrightarrow HHg$ AT ORDER $\alpha^2\sqrt{\alpha_s}$ [6, 7]

WE FIND THAT TO ENSURE GAUGE INVARIANCE WE MUST IMPOSE A SM RELATION AMONG

LINKING THE DIFFERENT BLOCKS LISTED BEFORE. NOTICE THAT THIS HAPPENS DUE TO <u>TOP QUARK CONTRIBUTIONS</u>, SINCE THESE CONTRIBUTIONS TRIGGER THE APPEARANCE OF GOLDSTONE BOSONS, WHICH IN TURN RELATE YUKAWA AND GAUGE COUPLINGS THROUGH GAUGE INVARIANCE.

- INDEED, IF WE EXCLUDE MASSIVE QUARKS WE CAN STILL MAINTAIN THE COUPLINGS TO BE INDEPENDENT. CONSIDER <u>LIGHT-QUARK CONTRIBUTIONS</u> TO $gg \to HH$ AND $q\bar{q} \to HHg$. WE CAN ISOLATE THE FOLLOWING <u>GAUGE INVARIANT</u> AND <u>FINITE BLOCKS</u> [8]



- QCD AND LIGHT QUARKS ARE ONLY PART OF THE PICTURE: ALL EW TERMS WILL ULTIMATELY CONTRIBUTE TO THE PROCESS AT SOME ORDER IN PERTURBATION THEORY, ENFORCING CONSTRAINTS. WE CAN JUSTIFY KEEPING THE COUPLINGS INDEPENDENT (AND THEREFORE JUSTIFYING THE USE OF K-FRAMEWORKS FOR STUDYING THE IMPACT OF SOME COUPLINGS) SAYING THAT THE INCONSISTENCIES WILL APPEAR AT HIGHER ORDERS IN PERTURBATION THEORY, AND WILL THEREFORE BE SUPPRESSED.
  THIS LINE OF THOUGHT IS CLEARLY FAILING ONCE WE NEED TO INCLUDE THE WHOLE EW SECTOR IN OUR PREDICTIONS, AS WE ARE DOING NOW FOR HIGGS PRODUCTION.
- A PROPER WAY OF TESTING THE SM COULD THEN BE THE USE OF <u>EFFECTIVE FIELD THEORIES</u> (EFTS), WHICH ARE BUILD TO INTRODUCE NEW LOCAL INTERACTIONS AMONG SM FIELDS RESPECTING THE SM SYMMETRIES.

· CHIRALITY

· EW CORRECTIONS DISTINGUISH BETWEEN FERMION CHIRALITIES, INTRODUCING PARITY-VIOLATING TERMS IN THE AMPLITUDES EXPRESSED THROUGH THE PRESENCE OF γ_5 AND LEVI-CIVITA SYMBOLS.

· THE FIRST CONSEQUENCE IS THAT NEW TENSOR STRUCTURES CAN APPEAR IN THE AMPLITUDE.

· $(g-2)_\mu$

· TWO NEW TENSOR STRUCTURES APPEAR FOR THE $\mu^-\mu^+\gamma^*$ VERTEX:

$$\underbrace{i\,E^{\mu\nu\alpha\beta}\,\frac{\sigma_{\alpha\beta}}{4m}\,q_\nu\,F_3}_{\overline{\psi}\,\tilde{F}_{\alpha\beta}\,\sigma^{\alpha\beta}\,\overline{\psi}} \qquad\qquad \underbrace{\frac{1}{2m}\left(q^\mu-\frac{q^2}{2m}\gamma^\mu\right)\gamma_5\,F_4}_{\overline{\psi}\,\not{j}\,\gamma_5\,\psi}$$

AS THE MAGNETIC DIPOLE MOMENT BUT FOR THE ELECTRIC FIELD

↓

ELECTRIC DIPOLE MOMENT

RELATED TO $\mathcal{CP}$ VIOLATION

DIRECT COUPLING TO THE CURRENT, NO EM MEDIATION, NO RADIATION

↓

ANAPOLE MOMENT

· gg HH

· TWO NEW TENSOR STRUCTURES APPEAR

	SINGLE HIGGS	DOUBLE HIGGS
$\mathcal{P}$-EVEN	$\eta^{\mu\nu}-\dfrac{p_2^\mu p_1^\nu}{p_{12}}$	$\eta^{\mu\nu}+\dfrac{m_H^2 p_2^\mu p_1^\nu-2p_{23}p_3^\mu p_1^\nu-2p_{13}p_2^\mu p_3^\nu+2p_{12}p_3^\mu p_3^\nu}{p_{12}\,p_T^2}$
$\mathcal{P}$-ODD	$\dfrac{E^{\mu\nu 12}}{p_{12}}$	$\dfrac{p_3^\mu E^{\nu 123}+p_3^\nu E^{\mu 123}+p_{23}E^{\mu\nu 13}+p_{13}E^{\mu\nu 23}}{p_{12}\,p_T^2}$

$$p_T^2=\frac{2p_{13}\,p_{23}}{p_{12}}-m_H^2.$$

· WE OBSERVE, THOUGH, THAT $\mathcal{P}$-ODD TENSOR STRUCTURES MULTIPLY ZERO FORM FACTORS AT NLO EW.

· THE SECOND CONSEQUENCE IS THAT THE PASSAGE FROM 4 TO D DIMENSIONS IS NO MORE SMOOTH. THIS IS DUE TO THE FACT THAT γ^5 IS RELATED TO THE LEVI-CIVITA SYMBOL, WHICH CANNOT BE EXTENDED TO D DIMENSIONS.

· IF WE WANT TO KEEP USING DIMENSIONAL REGULARIZATION, WE NEED TO

EMPLOY A <u>SCHEME FOR γ_5</u>: A SCHEME IS A SET OF PRACTICAL RULES THAT ALLOWS US TO COMPUTE A WRONG RESULT IN A CONSISTENT WAY AND THEN ADJUST IT SUCH THAT IN THE LIMIT $D \to 4$ THE CALCULATION IS CORRECT. WHAT USUALLY HAPPENS IS:

- THE SCHEME BREAKS WARD IDENTITIES IN A CONTROLLED AND UNIVERSAL WAY, BY TERMS PROPORTIONAL TO THE PREVIOUS PERTURBATIVE ORDER.
- FINITE RENORMALIZATION OF THE FORM

$$\alpha_B \to \alpha\left(1 + \delta Z_5 \alpha + \ldots\right)$$

REMOVES THE PROBLEMATIC TERM.

- FINITE RENORMALIZATION IS THEREFORE NEEDED ONLY WHEN CALCULATING CORRECTIONS TO AMPLITUDES ALREADY CONTAINING AXIAL TERMS.
- A PARTICULAR CASE IS GIVEN BY AMPLITUDES WHERE γ_5 INSERTIONS OCCUR ONLY ON OPEN FERMION LINES. IN THIS CASE γ_5 CAN BE NAIVELY ANTICOMMUTED UNTIL IT REACHES ONE OF THE EXTERNAL SPINORS AND THEN REWRITTEN AS

$$\gamma_5 = \mathbb{P}_R - \mathbb{P}_L$$

THE PRESENCE OF CHIRALITY PROJECTORS CAN BE INTERPRETED AS SELECTING SPECIFIC HELICITY CONFIGURATIONS FOR THE EXTERNAL FERMIONS. AT THIS POINT, WE CAN EXCHANGE THE AXIAL AMPLITUDES WITH VECTORIAL ONES AND CARRY OVER THE CALCULATION WITHOUT PARITY-VIOLATING TERMS, HAVING ENCAPSULATED THE LEFT-RIGHT ASYMMETRY IN DIFFERENT COEFFICIENTS FOR DIFFERENT HELICITY CONFIGURATIONS. THE KREIMER SCHEME SUPPORTS THIS PROCEDURE

- <u>$gg \to HH$</u>

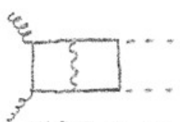
  - W AND Z BOSONS DIFFERENTIATE BETWEEN FERMION CHIRALITY.

  - <u>LARIN SCHEME</u>

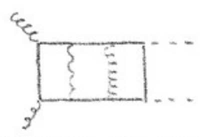
    1. AXIAL VERTICES: $\gamma^\mu \gamma^5 \to \frac{1}{2}\left(\gamma^\mu \gamma^5 - \gamma^5 \gamma^\mu\right)$
    2. IN TRACES WITH TWO γ_5: ANTICOMMUTE γ_5 UNTIL $\gamma_5^2 = 1$
    3. $\gamma_5 = \frac{i}{24} \mathcal{E}^{\mu\nu\rho\sigma} \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma$
    4. EXPRESS $\mathcal{E}^{\mu\nu\rho\sigma} \mathcal{E}_{\alpha\beta\gamma\delta}$ IN TERMS OF METRIC TENSORS

  - FINITE RENORMALIZATION NOT NECESSARY AT NLO EW: NO γ_5 AT LOWER ORDER

LO: NO γ_5



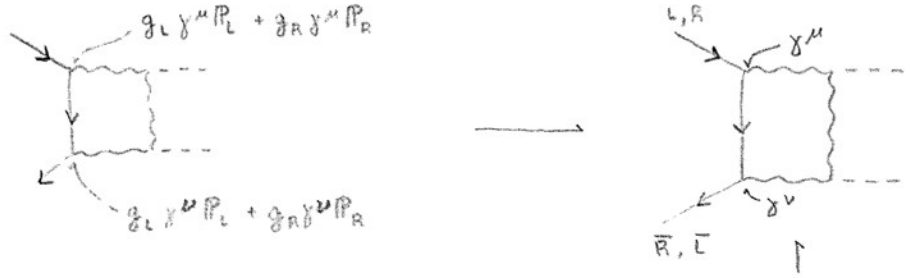
NLO EW: γ_5 NO REN.



NLO EW × NLO QCD: γ_5, REN.

- <u>$q\bar{q} \to HH(g), n_f$</u>

- WE HAVE AN OPEN FERMION LINE, SO WE MOVE $P_{L/R}$ TO TOUCH THE SPINOR u:



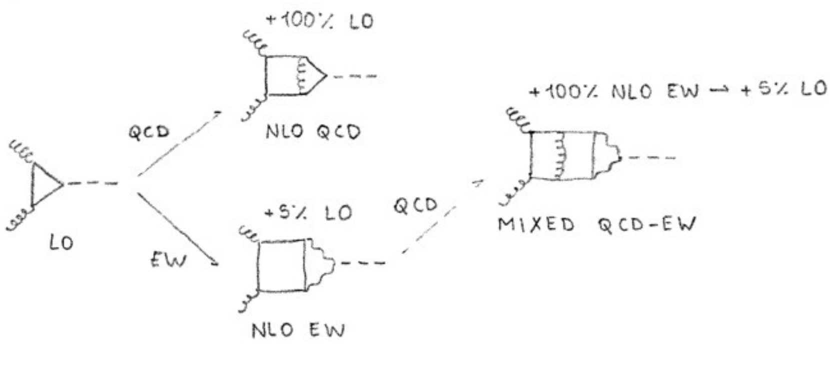
$$\bar{u}\, \gamma^\mu \left(g_L P_L + g_R P_R \right) \not{p}\, \gamma^\nu \left(g_L P_L + g_R P_R \right) u \longrightarrow g_L^2\, \bar{u}_R\, \gamma^\mu \not{p}\, \gamma^\nu u_L + g_R^2\, \bar{u}_L\, \gamma^\mu \not{p}\, \gamma^\nu u_R$$

- HAVING REMAPPED γ_5 ONTO PICKING SPECIFIC HELICITY CONFIGURATIONS WE CAN NOW EMPLOY THE TENSOR DECOMPOSITION USED FOR PURELY VECTOR INTERACTIONS, SIMPLIFYING THE STRUCTURE NEEDED FOR THE AMPLITUDE.

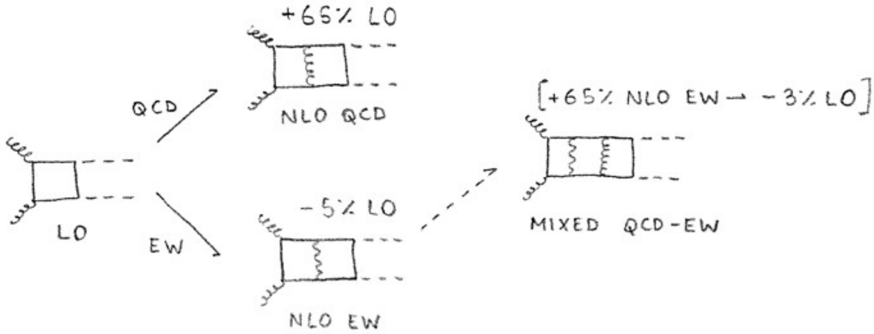
- ORTHOGONALITY

- WITHIN THE SM, QCD AND EW CORRECTIONS ARE INDEPENDENT. THIS HAS TWO IMPORTANT CONSEQUENCES.
  - RE-ITERATION OF QCD EFFECTS · APPLYING QCD CORRECTIONS ON EW ONES CAN PRODUCE A SIMILAR ENHANCEMENT AS APPLYING QCD CORRECTIONS ON THE LO, BASED ON UNIVERSALITY OF INITIAL-STATE RADIATION (WHICH IS THE DOMINANT CONTRIBUTION AT LO) AND SIMILAR COLOUR STRUCTURE.
  - NEW LO AMPLITUDES : THE NEW VERTICES AND FIELDS MAY ALLOW FOR DIAGRAMS WHICH BELONG TO THE NLO BY POWER COUNTING BUT DO NOT ADMIT DIAGRAMS AT LOWER ORDERS UPON WHICH THEY CAN BE BUILT.

- $gg \to H$



- IT HAS BEEN OBSERVED THAT NLO QCD CORRECTIONS PRODUCE THE SAME RELATIVE EFFECT WHETHER THEY ARE APPLIED TO LO OR NLO EW. THIS IS A CRITICAL POINT FOR THEORY PREDICTIONS AIMING AT PERCENT UNCERTAINTY.
- IF A SIMILAR BEHAVIOUR IS PRESENT FOR DOUBLE HIGGS PRODUCTION



· A NEW LO CONTRIBUTION IN BOTH SINGLE AND DOUBLE HIGGS IS GIVEN BY LIGHT QUARKS, IN PARTICULAR EACH BLOCK SEPARATELY

THANKS TO THE FACT THAT LIGHT QUARKS DO NOT COUPLE TO GOLDSTONE BOSONS, AVOIDING MIXING YUKAWA AND GAUGE COUPLINGS. THESE CONTRIBUTIONS ARE, INDEED ε-FINITE AND GAUGE INVARIANT, AND NO 1-LOOP DIAGRAM WITHOUT YUKAWA COUPLING IS POSSIBLE IN $gg \to H(H)$.

· NEW CHANNELS ALSO ARISE IN $q\bar{q} \to HH(g)$, WHERE IN QCD THE LOWEST ORDERS ARE

$\alpha_s^2 \alpha$ $\qquad$ $\alpha_s^{3/2} \alpha$

WHILE IN EW

α^2 $\qquad$ $\alpha_s^{3/2} \alpha$

THIS OPENS A NEW CONTRIBUTION TO ORDER $\alpha_s^2 \alpha^3$:

$\alpha_s \alpha$ $\otimes$ $\alpha_s \alpha^2$ $\oplus$ $\alpha_s^2 \alpha$ $\otimes$ α^2 $\oplus$

$\oplus$ $\alpha_s^{3/2} \alpha$ $\otimes$ $\alpha_s^{1/2} \alpha^2$

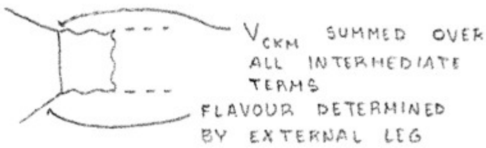
3. TIPS FOR EW CORRECTIONS

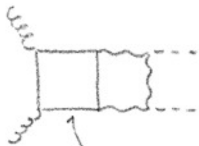
· I WILL PRESENT SOME TIPS THAT I HAVE FOUND USEFUL WHEN COMPUTING EW (LOOP) CORRECTIONS.

· USE ONLY WHAT YOU NEED

· EW CORRECTIONS RELATE MANY DIFFERENT PARTICLES. BOOKKEEPING CAN QUICKLY BECOME PROBLEMATIC, AND PROCESSING MANY CONTRIBUTIONS ONLY DIFFERING BY FLAVOUR OR CHARGE LABELS MAY WASTE COMPUTATIONAL POWER.

· IF MASSLESS ALL QUARK FAMILIES BEHAVE THE SAME UNDER EW INTERACTIONS, THE ONLY DIFFERENCE BEING THE ENTRIES OF V_{CKM}. A SINGLE FAMILY CAN THEN BE USED, CUTTING THE NUMBER OF DIAGRAMS BY A FACTOR OF TWO OR EVEN FOUR (W) OR FIVE (Z) WHEN USING A SINGLE QUARK FIELD. V_{CKM} AND $g_{A,Z}$, $g_{V,Z}$ COUPLINGS CAN BE UNAMBIGUOUSLY REINSTATED AT A LATER STAGE:



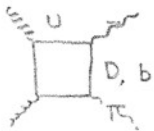


SUM OVER ALL LIGHT QUARKS

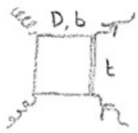
W) $2\sum_{ij} V_{ij}^* V_{ij}$

Z) $\sum_i g_{Vi}^2 + g_{Ai}^2$

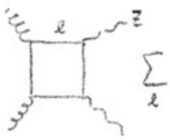
· IF YOU INCLUDE THE TOP QUARK, YOU WILL NEED AT LEAST ONE MASSLESS QUARK FIELD AND ONE MASSIVE (FOR Z) OR TWO QUARK FAMILIES $\binom{U}{D}$, $\binom{t}{b}$ (FOR W).



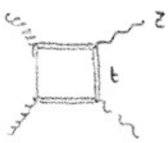
$V_{Ub}^* V_{Ub} + V_{UD}^* V_{UD}$



$V_{tD}^* V_{tD} + V_{tb}^* V_{tb}$

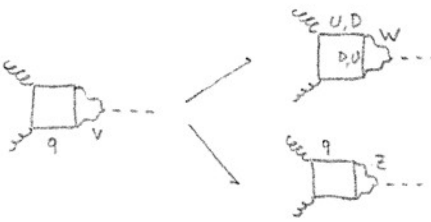


$\sum_\ell g_{V,\ell}^2 + g_{A,\ell}^2$



$g_{V,t}^2 + g_{A,t}^2 + \gamma^5 g_{V,t}\, g_{A,t}\, m_t$

· IF YOU KNOW THAT W AND Z BOSONS CANNOT APPEAR TOGETHER IN THE SAME DIAGRAM YOU CAN USE A SINGLE FIELD V WITH MASS m_V. THIS REDUCES BOTH THE NUMBER OF DIAGRAMS AND THE TYPE OF FEYNMAN INTEGRALS TO COMPUTE



$\Rightarrow m_V = m_W \quad V_{UD}^* V_{UD}$

$\Rightarrow m_V = m_Z \quad \sum g_{Vq}^2 + g_{Aq}^2$

· GOOD GAUGES

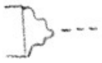
· IN QCD AND QED CALCULATIONS THE GAUGE OF THE INTERNAL LINES IS USUALLY KEPT GENERAL. ITS EXPLICIT CANCELLATION IN THE FULL AMPLITUDE IS A STRONG SANITY CHECK.

· THE EW SECTOR PRESENTS A CRITICALITY IN THIS SENSE, SINCE THE GAUGE PARAMETER DIRECTLY AFFECTS THE MASS OF GAUGE AND GOLDSTONE BOSONS AS

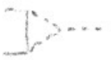
$$= \frac{i}{k^2 - m^2}\left(-\eta^{\mu\nu} + (1-\xi)\frac{k^\mu k^\nu}{k^2 - \xi m^2}\right) \longrightarrow \frac{i\left(-\eta^{\mu\nu} + \frac{k^\mu k^\nu}{m^2}\right)}{k^2 - m^2} + \frac{-\frac{k^\mu k^\nu}{m^2}\, i}{\underline{k^2 - \xi m^2}}$$

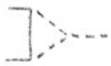
$$= \frac{i}{\underline{k^2 - \xi m^2}}$$

THIS EFFECTIVELY INTRODUCES A NEW MASS SCALE IN THE COMPUTATION, SINCE PROPAGATORS WITH DIFFERENT MASSES CAN NOW MIX


$$\Rightarrow \frac{1}{(k^2-m^2)(\ell^2-m^2)} \oplus \frac{1}{(k^2-m^2)(k^2-\xi m^2)} \oplus \frac{1}{(k^2-\xi m^2)(\ell^2-m^2)} \oplus \frac{1}{(k^2-\xi m^2)(\ell^2-\xi m^2)}$$


$$\Rightarrow \frac{1}{(k^2-m^2)(k^2-\xi m^2)}$$


$$\Rightarrow \frac{1}{(k^2-\xi m^2)(\ell^2-m^2)}$$


$$\Rightarrow \frac{1}{(k^2-\xi m^2)(\ell^2-\xi m^2)}$$

· A WAY OF STILL USING GAUGE INVARIANCE AS A SANITY CHECK AND AVOID INCREASED COMPLEXITY IS TO PICK TWO GAUGES WHERE GAUGE AND GOLDSTONE BOSONS HAVE THE SAME MASS. LUCKILY FOR US, THIS IS POSSIBLE USING:

- FEYNMAN GAUGE ($\xi \to 1$)

$$= \frac{-i\eta^{\mu\nu}}{k^2 - m^2}$$

$$= \frac{i}{k^2 - m^2}$$

- UNITARY GAUGE ($\xi \to \infty$)

$$= \frac{i}{k^2 - m^2}\left(-\eta^{\mu\nu} + \frac{k^\mu k^\nu}{m^2}\right)$$

· WE CAN THEN COMPARE THE RESULTS OBTAINED IN THE TWO GAUGES.

GOOD γ_5 SCHEMES

- SOME γ_5 SCHEMES ARE MORE SUITED FOR SPECIFIC CALCULATIONS:
  - OPEN FERMION LINE: PICK A SCHEME THAT PRESERVES ANTICOMMUTATIVITY, SO TO MAP γ_5 ONTO HELICITY AMPLITUDES.
  - LIGHT-QUARK LOOPS: IF WE HAVE TWO GAUGE-BOSON INSERTIONS WE EXPEC A RESULT PROPORTIONAL TO $g_V^2+g_A^2$, SO HERE AS WELL A SCHEME THAT PRESERVES ANTICOMMUTATIVITY IS BENEFICIAL.
  - FIRST OCCURRENCE OF γ_5: IF γ_5 APPEARS FOR THE FIRST TIME AT THE CURRENT PERTURBATIVE ORDER, WE OBSERVED THAT NO FINITE RENORMALIZATION SHOULD BE NECESSARY, SO ONE COULD USE A NAÏVE SCHEME (I.E. TREATING γ_5 EXACTLY AS IN $D=4$). THIS COULD BE DANGEROUS

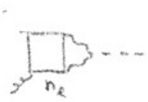
- A GOOD SANITY CHECK IS TO COMPUTE THE AMPLITUDE IN TWO DIFFERENT SCHEMES AND CHECK THAT THE RESULTS AGREE. A GOOD CHOICE COULD BE
  - FIRST OCCURRENCE + OPEN FERMION LINE : NAÏVE + KREIMER (NOT VERY ROBUST, THOUGH)
  - FIRST OCCURRENCE + LIGHT-QUARK LOOP : NAÏVE + KREIMER + LARIN
  - FIRST OCCURRENCE + FERMION LOOPS : NAÏVE + LARIN
  - GENERAL CASE: LARIN + KREIMER

PARTITIONING THE AMPLITUDE

- WE OBSERVE THAT AN AMPLITUDE IS ALMOST ALWAYS MORE REGULAR AND SIMPLER THAN THE SINGLE DIAGRAMS BUILDING IT:
  - DIAGRAMS ADDING UP TO ZERO

$+ \ldots = 0$ (FURRY THEOREM)

  - DIVERGENT DIAGRAMS IN FINITE AMPLITUDES



$\Rightarrow \frac{C_0}{\varepsilon^2} + \frac{C_1}{\varepsilon} + C_2 + \ldots$

$\Rightarrow A_0 + \ldots$

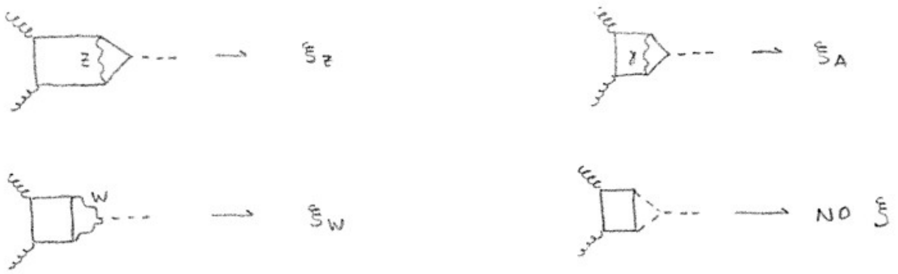
  - GAUGE INVARIANCE

- EW AMPLITUDES ARE USUALLY EXTREMELY LARGE AND BUG-HUNTING CAN BECOME UNMANAGEABLE WHEN CONSIDERING HUNDREDS OF DIAGRAM, SEVERAL MEBIBYTE EACH.

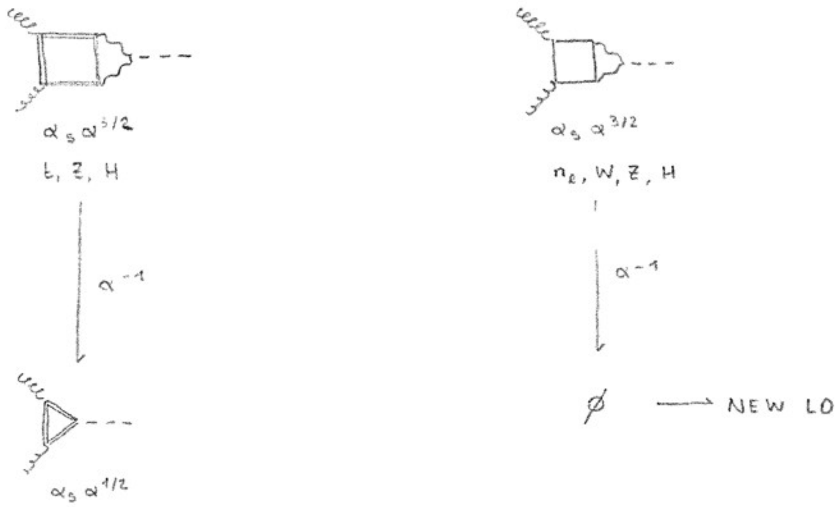
- WE NEED A TRADEOFF: WE NEED TO FIND THE FINEST PARTITION OF THE

AMPLITUDE THAT WILL RETURN US THE SIMPLEST EXPRESSIONS.

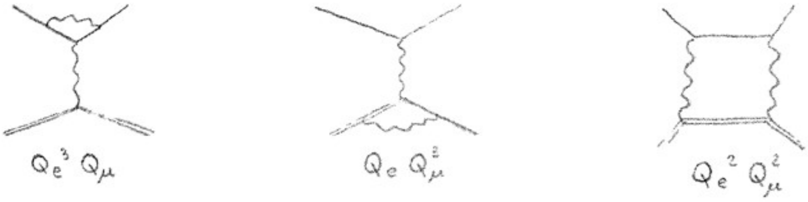
- SOME RULES OF THUMB TO ACHIEVE THIS, MOSTLY BASED ON WHAT WE HAVE SEEN SO FAR, ARE:
  - <u>KEEP DIFFERENT GAUGE SECTORS SEPARATED</u>. ALL DIAGRAMS CONTAINING THE SAME GAUGE PARAMETER (ξ_G, ξ_W, ξ_Z, OR ξ_A) ARE REGROUPED ACCORDING TO IT. SINCE CANCELLATIONS CAN ONLY HAPPEN AMONG DIAGRAMS WITH THE SAME GAUGE PARAMETER, WE OBTAIN SELF-SUFFICIENT GROUPS. IF MULTIPLE GAUGE PARAMETERS ARE PRESENT IN A DIAGRAM, THIS DIAGRAM BELONGS TO A SEPARATE SET.



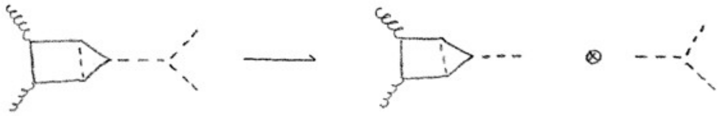
  - <u>NEW LO ARE INDEPENDENT</u>. DIAGRAMS WHOSE FIELD AND COUPLING CONTENT IS NOT ABLE TO PRODUCE VALID, NON-ZERO DIAGRAMS AT ONE PERTURBATIVE ORDER LESS ARE NEW LO CONTRIBUTIONS. THANKS TO THIS, WE EXPECT THEM TO FORM A FINITE AND GAUGE-INVARIANT SUBSET OF THE AMPLITUDE



  - <u>ELECTRIC CHARGE IS NOT QUANTIZED</u>. ABELIAN GAUGE THEORIES DO NOT HAVE MECHANISMS TO ENFORCE CHARGE QUANTIZATION. WE CAN USE THIS FORMAL PROPERTY TO PARTITION THE AMPLITUDE INTO SUBSETS ACCORDING TO THE POWERS OF THE CHARGES OF THE DIFFERENT PARTICLES PRODUCED BY EACH DIAGRAM [9]

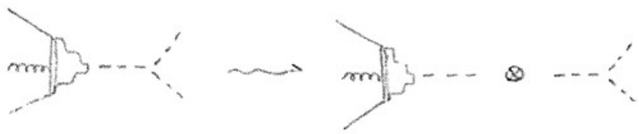


· SUB-PROCESSES. IF DIAGRAMS ADMIT A SUB-DIAGRAM WHICH IS A VALID, SELF-CONTAINED PROCESS (OR A PARTITION OF IT), YOU CAN COLLECT THEM INTO THE SAME PARTITION.



ggH, HIGGS ONLY — TRILINEAR COUPLING

IT IS WORTH MENTIONING THAT THIS CRITERION MAY ONLY WORK WHEN ENFORCING A STRICTER SET OF RELATIONS AMONG THE VERTICES (E.G., THE FULL SM) THAN THE SET USED FOR THE SUB-PROCESS. E.G. WE FIND THAT DIAGRAMS OF THE FORM



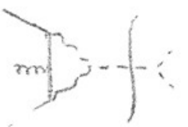
PRESENT A DECOMPOSITION LIKE

$$A \frac{i}{s_{125} - m_H^2} + B(\xi)$$

GAUGE INVARIANCE CAN THEN BE ACHIEVED EITHER

· BY ENFORCING FULL SM RELATIONS OR
· BY APPLYING CUTOWSKI RULES (WHICH ARE, INDEED, THE CORRECT WAY TO EXTRACT A SUB-PROCESS)

$$\frac{i}{s - m^2} \longrightarrow \delta(s - m_H^2)$$



$$\longrightarrow A\,\delta(s_{125} - m^2) + B(\xi)\,\underbrace{(s_{125} - m_H^2)\,\delta(s_{125} - m_H^2)}_{0} = A\,\delta(s_{125} - m_H^2)$$

WE CAN INTERPRET THIS RESULT AS EVIDENCE THAT, CONSIDERING EW CORRECTIONS, WE ARE NOT AT LIBERTY TO VARY ONLY ONE OF THE PARAMETERS EXPECTING A MEANINGFUL COMPARISON WITH EXPERIMENTS. WE SHOULD INSTEAD RELY ON EFT FRAMEWORKS, SUCH AS SMEFT OR HEFT.

FINDING BALANCE

· ALMOST ALL THE TIPS SHOWN ABOVE RELY ON REFRAINIG FROM USING A SPECIFIC PROPERTY OR FREEDOM OF THE THEORY TO SIMPLIFY OUR CALCULATION AND CHECK THAT OUR RESULT VERIFY THE PROPERTY NEVERTHELESS. THIS TRADES EASINESS WITH SAFETY. WHERE TO DRAW THE LINE DEPENDS ON THE PROCESS, THE RESULTS ALREADY AVAILABLE, AND THE CONTEXT.

WRAP-UP

- IN THESE NOTES I POINTED OUT SOME ELEMENTS OF EW CORRECTIONS THAT I FIND INTERESTING OR WORTH-MENTIONING, FOCUSING ON THE HIGGS BOSON AT THE LHC AND ON SOME PROPERTIES OF $(g-2)_\mu$.

- WE DISCUSSED THREE TOPICS:

  1. THE SIZE OF EW CORRECTIONS. EW CORRECTIONS ARE SMALL BECAUSE OF A WEAKER COUPLING CONSTANT W.R.T. QCD, THE LACK OF REAL EMISSIONS AT NLO IN HIGGS PRODUCTION, THE PRESENCE OF LARGELY MASSIVE GAUGE BOSON FOR $(g-2)_\mu$ ENERGIES.

  2. THE FORM OF EW CORRECTIONS. EW CORRECTIONS PRESENT A RICH YET CONSTRAINED DYNAMICS COMING FROM SSB. THEY ALLOW NEW CONTRIBUTIONS AND EFFECTS W.R.T. QCD AND QED AND ARE POTENTIALLY ENHANCED BY QCD CORRECTIONS, INTRODUCING NEW CRITICALITIES AT THE SAME TIME.

  3. SOME TIPS FOR EW CORRECTIONS. EW CORRECTIONS CAN BE TACKLED MAKING GOOD USE OF GAUGE AND SCHEMES FREEDOM, AND FINDING GOOD WAYS TO REGROUP DIAGRAMS INTO SIMPLER SUBSETS OF THE AMPLITUDE BY MEANS OF GAUGE, COUPLINGS, AND DYNAMICS.

- A CRUCIAL POINT THAT I AM NOT CONSIDERING IN THESE NOTES IS HOW TO DEAL WITH THE FEYNMAN INTEGRALS APPEARING IN LOOP AMPLITUDES. THIS IS A RICH FIELD OF STUDY, REGARDING HOW TO SIMPLIFY THEM EFFICIENTLY, AS WELL AS HOW TO EVALUATE THEM (NUMERICALLY OR ANALITICALLY) AND PRODUCE MATERIAL (EXPRESSIONS OR GRIDS) WHICH CAN THEN BE USED TO EXTRACT PHENOMENOLOGICAL RESULTS.

REFERENCES

[1] HANDBOOK OF LHC HIGGS CROSS SECTIONS: 4. DECIPHERING THE NATURE OF THE HIGGS SECTOR, [1610.07922].

[2] TWO-LOOP LIGHT FERMION CONTRIBUTION TO HIGGS PRODUCTION AND DECAY, [hep-ph/0404071].

[3] TWO-LOOP ELECTROWEAK CORRECTIONS TO HIGGS PRODUCTION AT HADRON COLLIDERS, [hep-ph/0407249].

[4] FULL TOP QUARK MASS DEPENDENCE IN HIGGS BOSON PAIR PRODUCTION AT NLO, [1608.04798].

[5] ELECTROWEAK CORRECTIONS TO DOUBLE HIGGS PRODUCTION AT THE LHC, [2311.16963].

[6] NLO QCD CORRECTIONS TO THE ELECTROWEAK PRODUCTION OF A HIGGS BOSON PAIR IN THE QUARK-ANTIQUARK CHANNEL, [2601.16924].

[7] [QCD x EW CONTRIBUTIONS TO DOUBLE HIGGS PRODUCTION FROM A LIGHT QUARK-ANTIQUARK PAIR], IN PREPARATION.

[8] TWO-LOOP LIGHT-QUARK ELECTROWEAK CORRECTIONS TO HIGGS BOSON PAIR PRODUCTION IN GLUON FUSION, [2503.16620]

[9] MUON-ELECTRON SCATTERING AT NNLO, [2212.06481].