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Debrecen 8/8/08

Heavy Quarks

8/8/08

The word for "eight" (八, 捌) in Chinese (Pinyin: bā) sounds similar to the word which means "prosper" or "wealth" (发 - short for "发财", Pinyin: fā)

from Wikipedia

Outline

- History: discoveries and interpretation
- Masses and their consequences
- Perturbative calculations and resummations
- Charm and bottom hadronisation
- Top mass and experimental studies
- Won't cover charm and bottom decays and oscillations, and many aspects of top physics

Previous lectures and reviews

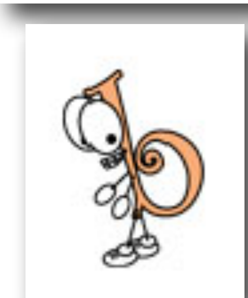
CTEQ 2007: Zack Sullivan	Top role in SM and beyond
CTEQ 2006: Carlo Oleari	Details of pQCD calculations
arXiv:0805.1333 W. Bernreuther	Top quark physics at the LHC
arXiv:0712.2733 R. Kehoe et al	CDF and D0 results
hep-ph/0003033 M. Beneke et al	LHC Workshop: top
hep-ph/0003142 P. Nason et al	LHC Workshop: bottom

... and many more of course

Definition

By definition, "heavy quarks" are the ones whose mass is larger than the QCD scale Λ:

Charm, m ~ 1.5 GeV



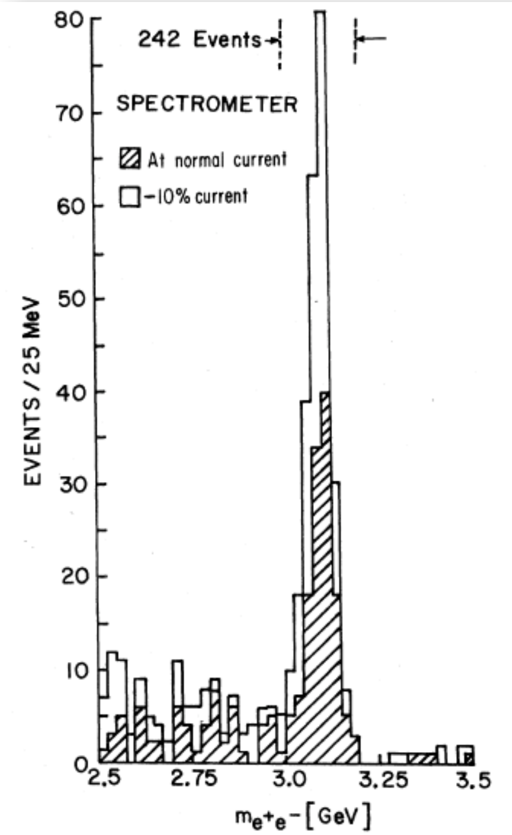
Bottom, m ~ 5 GeV

Top, m ~ 170 GeV

$m \gg \Lambda \sim 300$ MeV

Discovery: charm

The first heavy quark, charm, was simultaneously discovered in 1974 (the "November revolution") in ppbar collisions at BNL and e+e- at SLAC



Observation of HUGE PEAK with extremely NARROW WIDTH

[Phys. Rev. Lett. 33 , 1406 (1974), Phys. Rev. Lett. 33 , 1404 (1974)]

This was very soon interpreted as due to a **charm-anticharm bound state**

If we assume that the recently announced resonance[5] with mass ≈ 3 GeV is orthocharmonium, Eq. (5) fixes α_s. Preliminary estimates give[6] $\Gamma_l \approx 3$ keV and $\Gamma_h \approx 75$ keV. Their ratio gives $\alpha_s \approx 0.26$. This, along with $m_{\mathcal{P}'} \approx 1.5$ GeV, implies $\Gamma_l \approx 0.8$ keV and $\Gamma_h \approx 20$ keV, surely low

[Appelquist, Politzer, PRL 34]

[De Rujula, Glashow PRL 34]

We say that it is a hadron with $J^P = 1^-$, $I^G = 0^-$: a $^3S_1$ bound state of a charmed[7,8] quark $\mathcal{P}'$ and its antiquark with mass below twice the mass of the lightest charmed hadron: "orthocharmonium."[9] Thus,[10]

The existence of a FOURTH quark had been predicted a few years earlier:

PHYSICAL REVIEW D VOLUME 2, NUMBER 7 1 OCTOBER 1970

Weak Interactions with Lepton-Hadron Symmetry*

S. L. Glashow, J. Iliopoulos, and L. Maiani†

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02139

(Received 5 March 1970)

The November revolution

The charm discovery was a big deal because:

- It completed the second family, superseding Gell-Mann's 'Eightfold way', $SU(3)_{\text{flavour}}$
- Made SU(2)xU(1) consistent ➨ **Standard** Model
- It cemented our belief in QCD (asymptotic freedom)

It is indeed worth recalling that in those early years the extremely important role of charm was well recognized:

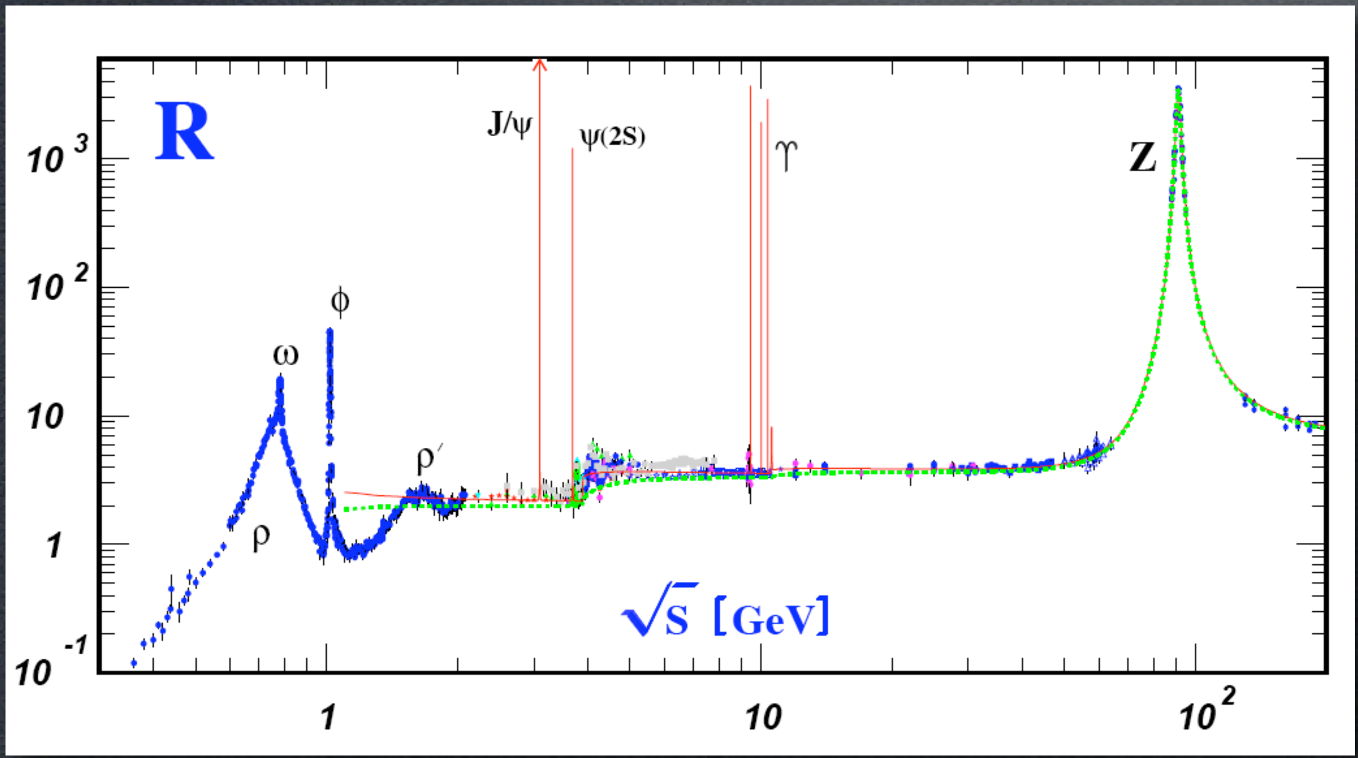
to experiment. For the sake of simplicity we specialize to the standard sequential Weinberg–Salam–Glashow-Iliopoulos-Maiani[5] model of weak interactions and the color gauge theory of strong interactions[6]; it will be obvious that our remarks

[Collins, Wilczek, Zee, PRD 18 (1978) 242]

Why a 'Revolution'

Hadronic resonances are normally LARGE, since they decay by strong interaction and have therefore very short lifetime:

$$10^{-22} - 10^{-23}\ \mathrm{s} \simeq \tau = \frac{1}{\Gamma} \quad \Rightarrow \quad \Gamma \simeq 10 - 200\ \mathrm{MeV}$$



$$R \equiv \frac{\sigma(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$\Gamma_\rho \simeq 150\ \mathrm{MeV}$ $\Gamma_\omega \simeq 8.5\ \mathrm{MeV}$ $\Gamma_\phi \simeq 4.3\ \mathrm{MeV}$ $\Gamma_{J/\psi} \simeq 0.1\ \mathrm{MeV}$

How could the new resonance have a width a factor of 100/1000 smaller, and yet be a strongly interacting particle?

The J/ψ width

Long answer:
see extra material at the end of lectures

Short answer:
it's due to the existence of a **small** strong coupling at the 'large' scale set by the charm quark mass

Is the J/ψ really a charm-anticharm bound state?

How can a hadronic system not be (much) sensitive to the strong force?

Obvious answer: it's a **small** system!

NB. Proton radius ~ 1 fm ~ 1/(200 MeV) ~ 1/Λ

Two masses m orbiting each other +
Heisenberg uncertainty principle:

$$r \sim \frac{1}{2p} \sim \frac{1}{2}\frac{1}{(m/2)v} = \frac{1}{mv}$$

To estimate v, consider the Virial Theorem

$$\langle T \rangle = -\frac{1}{2}\langle V \rangle$$

and first energy level: $E_1 = -\frac{1}{2}\frac{m}{2}\left(\frac{4}{3}\alpha_S\right)^2$

==> $\left(\frac{m}{2}\right)v^2 = 2\langle T\rangle = -2E_1 = \frac{m}{2}\left(\frac{4}{3}\alpha_S\right)^2$

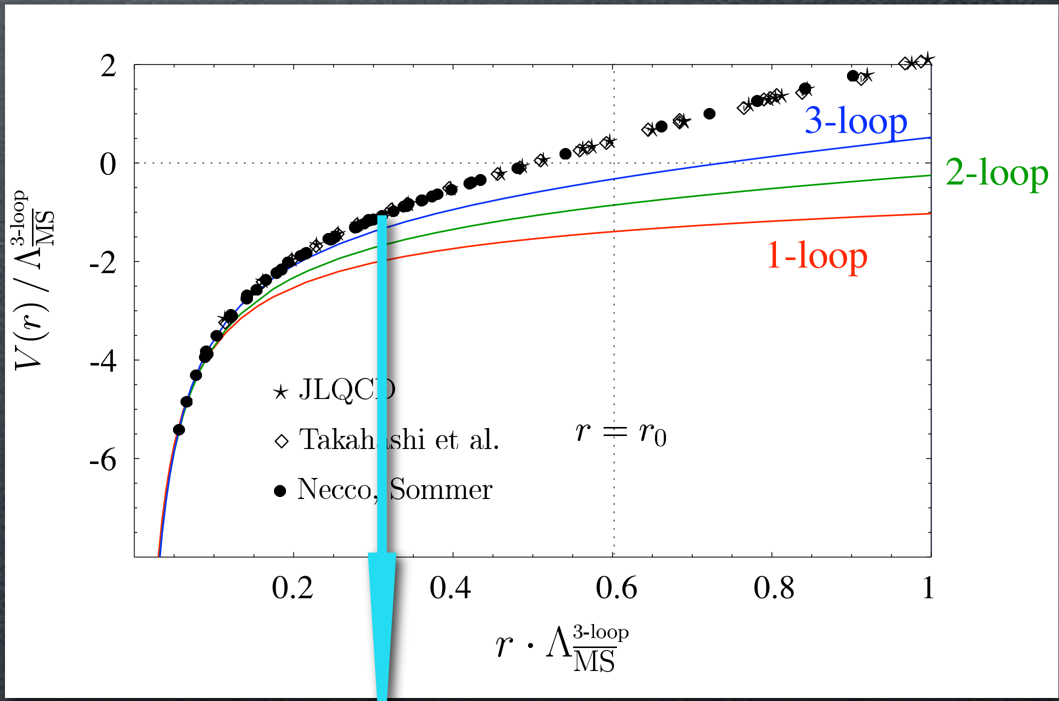
==> $v \simeq \frac{4}{3}\alpha_S((1/mv)^2)$ ==> $v \simeq 0.5c$

and consequently

$$\boxed{r \simeq \frac{1}{750\ \mathrm{MeV}} \simeq \frac{1}{3\Lambda} \simeq 0.3\,fm}$$

QCD potential: Coulomb + linear

$$V(r) \sim -\frac{4}{3}\frac{\alpha_S(1/r^2)}{r} + Kr$$



J/ψ

NB. Tight bound system below threshold for DDbar decay
(on the contrary, phi -> KKbar)
=> further explanation for small width

"*For every complex problem, there is a solution that is simple, neat and wrong*" -- H.L. Mencken

$$\Gamma_{J/\psi} \simeq 100 \text{ KeV} \qquad \Gamma_{\eta_c} \simeq 25 \text{ MeV}$$

Further evidence of asymptotic freedom (i.e. small coupling):

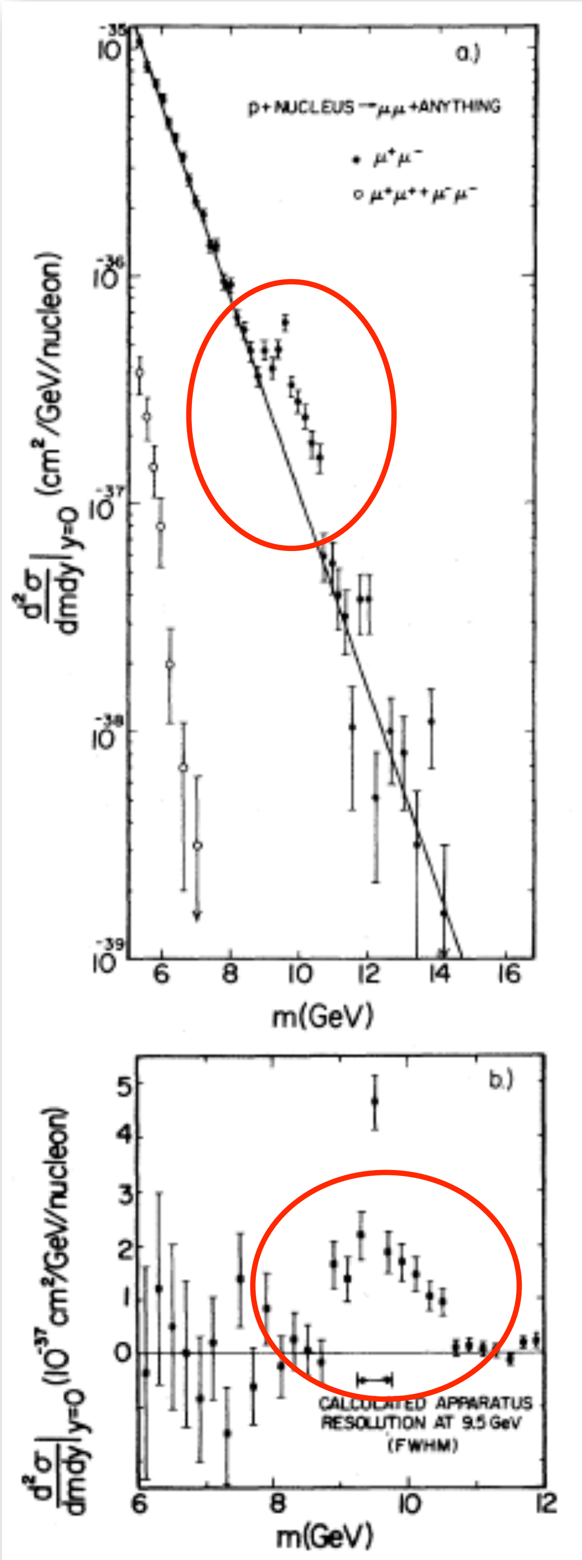
$J/\psi(^3S_1)$ is 1^{--} → decays to **three** gluons

$\eta_c(^1S_0)$ is 0^{-+} → decays to **two** gluons

Suppressed by a factor of α_S, helped by a small coefficient:

$$\frac{\Gamma_h(\text{ortho})}{\Gamma_h(\text{para})} = \frac{5}{6}\,\frac{2}{9\pi}(\pi^2 - 9)\alpha_s$$

Discovery: bottom



In 1977 the Upsilon (bbar bound state) was observed for the first time at Fermilab

[Phys. Rev. Lett. 39 (1977) 252]

The discovery of the bottom quark, the **FIFTH**, points to a new family, the third.
Hence, we'll then need to find a **SIXTH** quark

Less than three years had passed between the discoveries of charm and bottom.
But then, the waiting got longer.....

[NB. Upsilon also very narrow.
Large width here due to experimental resolution]

.....or did it?

UA1 "almost" discovers the top quark with m=40 GeV in 1984

ABSTRACT

A clear signal is observed for the production of an isolated large-transverse-momentum lepton in association with two or three centrally produced jets. The two-jet events cluster around the $W^{\pm}$ mass, indicating a novel decay of the Intermediate Vector Boson. The rate and features of these events are not consistent with expectations of known quark decays (charm, bottom). They are, however, in agreement with the process $W \to t\bar{b}$ followed by $t \to b\ell\nu$, where t is the sixth quark (top) of the weak Cabibbo current. If this is indeed so, the bounds on the mass of the top quark are $30\ \mathrm{GeV}/c^2 < m_t < 50\ \mathrm{GeV}/c^2$.

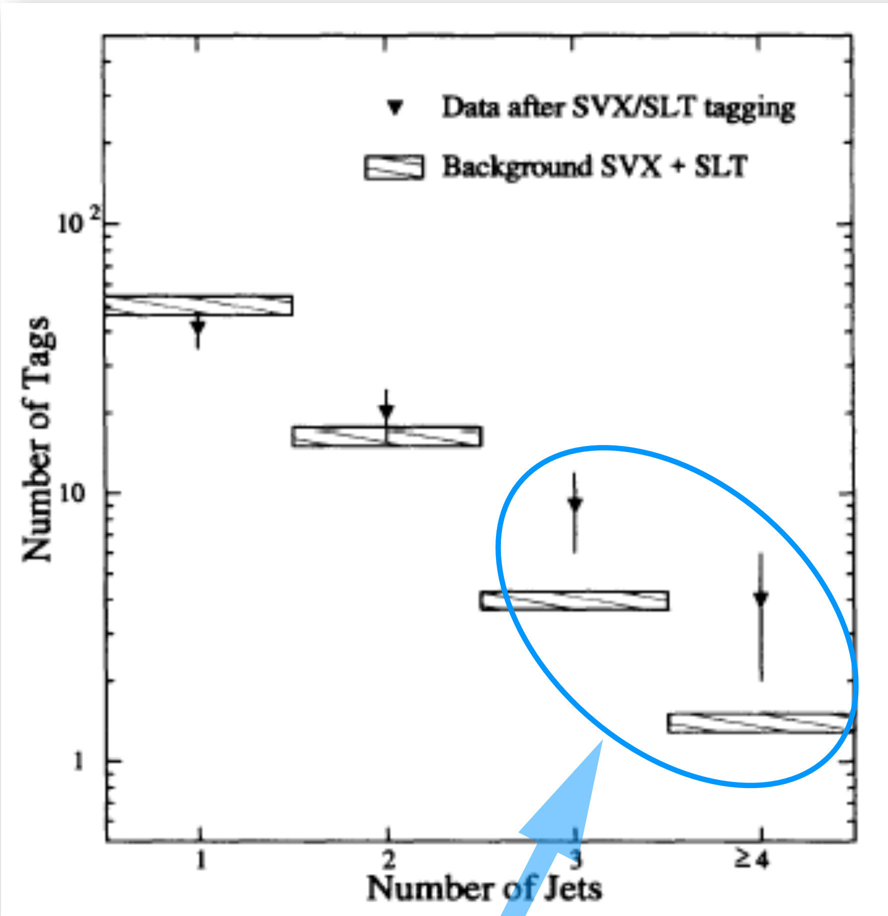
[Phys. Lett. B147 (1984) 493]

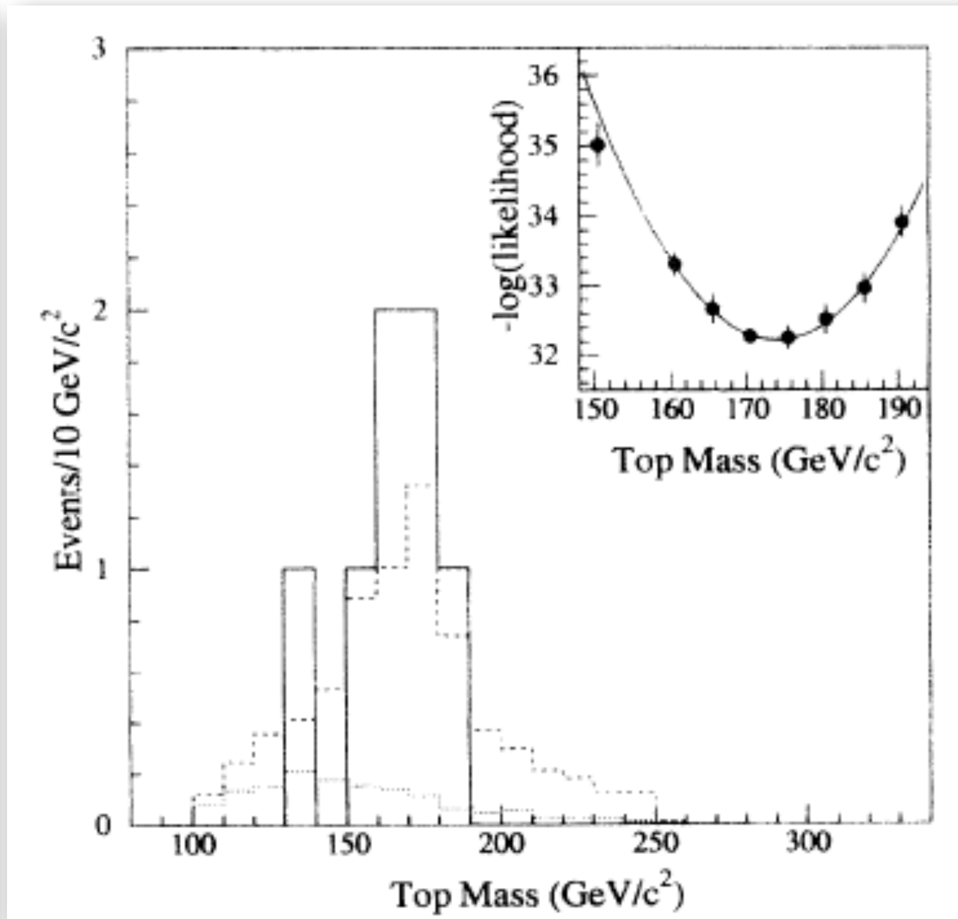
.....oooops!

Discovery: top

The top quark is finally really found at Fermilab by the CDF collaboration in 1994, with a much larger mass, ~ 175 GeV

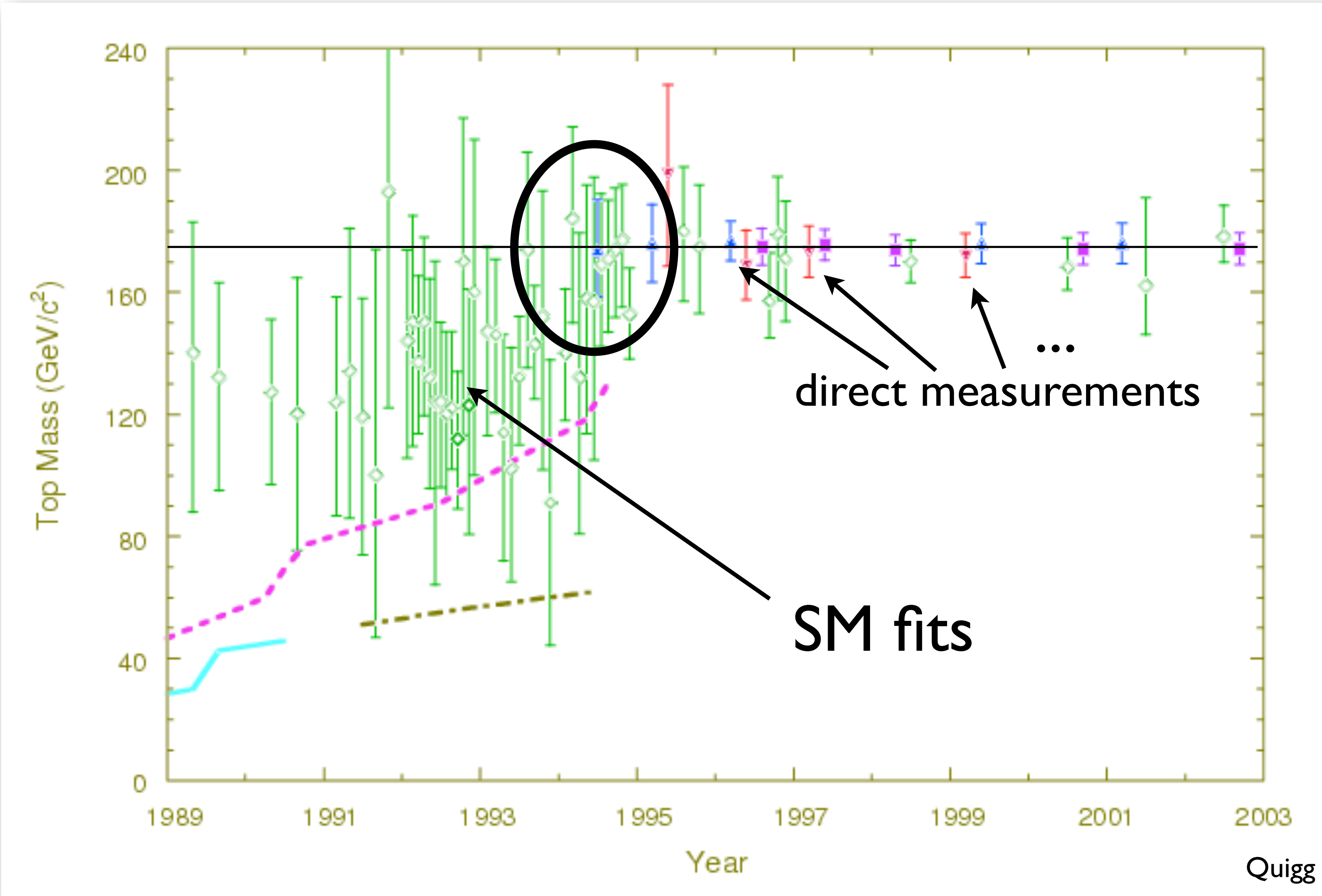
Phys. Rev. Lett. 73 (1994) 225





Excess of events with many jets.
Needs very good control of background

Such a heavy top was a surprise. However, the lower limit had been increasing and there had been hints from analysis of electroweak data, where the top mass enters via loop corrections



You might notice, however, how knowing the top mass helps a lot in predicting it......

Heavy Quark Masses

The PDG tells you:

c $\quad I(J^P) = 0(\frac{1}{2}^+)$

Mass $m = 1.27^{+0.07}_{-0.11}$ GeV $\quad$ Charge $= \frac{2}{3}\,e$ $\quad$ Charm $= +1$

b $\quad I(J^P) = 0(\frac{1}{2}^+)$

Charge $= -\frac{1}{3}\,e$ $\quad$ Bottom $= -1$

Mass $m = 4.20^{+0.17}_{-0.07}$ GeV $\quad$ ($\overline{\mathrm{MS}}$ mass)

t $\quad I(J^P) = 0(\frac{1}{2}^+)$

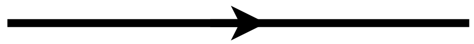
Charge $= \frac{2}{3}\,e$ $\quad$ Top $= +1$

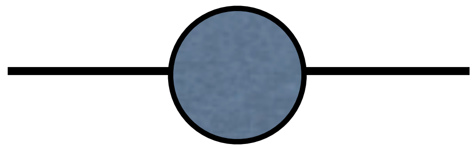
Mass $m = 171.2 \pm 2.1$ GeV [b] (direct observation of top events)

Same symbol '*m*' but different objets: not their best choice of notation

Only for bottom it's at least (partially) clear which mass they are quoting
Charm and top, anybody's guess (or knowledge)

Heavy Quark Masses

Leading order:  $\sim \dfrac{1}{\not{p} - m}$ (pole) mass = m

Higher orders:  $\sim \dfrac{1}{\not{p} - m_0 - \Sigma}$ m_0 = **bare** mass

Need for renormalisation

(At least) two possible renormalisation schemes: **MSbar** and **on-shell,** leading to to different mass definitions:

The pole mass *m* (or *M*)

(real part of the pole of the propagator)

The MSbar mass $\bar{m}$(μ)

(A short-distance mass, evalutated at the renormalisation scale μ)

Heavy Quark Masses: pros and cons

The **pole mass** is more physical (pole = propagation of particle, though a quark doesn't usually really propagate -- hadronisation!) but is affected by long-distance effects: it can never be determined with accuracy better than Λ_{QCD}

The **MSbar mass** is a fully perturbative object, not sensitive to long-distance dynamics. It can be determined as precisely as the perturbative calculation allows. Of course, it is also fully artificial.

The two masses are related by the perturbative relation:

$$\boxed{\begin{aligned}\frac{M}{\overline{m}(\overline{m})} &= 1 + \frac{4}{3}\left(\frac{\bar{\alpha}_s}{\pi}\right) + \left(\frac{\bar{\alpha}_s}{\pi}\right)^2 (-1.0414\ N_L + 13.4434) \\ &\quad + \left(\frac{\bar{\alpha}_s}{\pi}\right)^3 (0.6527\ N_L^2 - 26.655\ N_L + 190.595) \\ &\quad + \ldots + O(\Lambda_{QCD})\end{aligned}}$$

Heavy Quark Masses: summary

	MSbar: $\bar{m}(\bar{m})$	Pole: M
Charm	1.27 +0.7 -0.11 GeV ?	1.3 -- 1.7 GeV
Bottom	4.20 +0.17 -0.07 GeV	4.5 -- 5 GeV
Top	~ 163 GeV	172.6 ± 1.4 GeV

Heavy quarks are different: the dead cone

The time a coloured particle takes to hadronize is that taken by the colour field to travel a distance of the order of the typical hadron size: t′ ~ R ~ 1/Λ

Boosting to the lab frame we find

$$t_q^{\mathrm{hadr}} = t'\gamma = R\frac{E}{\Lambda} = ER^2 = \frac{E}{\Lambda^2}$$ light quarks

$$t_Q^{\mathrm{hadr}} = t'\gamma = R\frac{E}{m}$$ heavy quarks

Consider now 'shaking' (i.e. accelerating) a quark.
The regeneration time of a gluon field of momentum k around it is given by

$$t_g^{\mathrm{regen}}(k) = \frac{k_\parallel}{k_\perp^2}$$

For gluons such that $k_\perp \sim \Lambda,\ k_\parallel \sim E$ we have $t_g^{\mathrm{regen}}(k) \simeq t_q^{\mathrm{hadr}}$

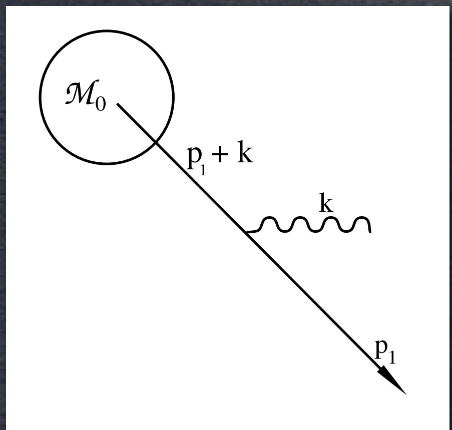
A heavy quark will therefore behave like a light one only if $t_Q^{\mathrm{hadr}} > t_g^{\mathrm{regen}}(k) \Leftrightarrow \frac{E}{m}\frac{1}{\Lambda} > \frac{k_\parallel}{k_\perp^2} \simeq \frac{1}{\Theta}\frac{1}{\Lambda} \Leftrightarrow \Theta > \frac{m}{E} \equiv \Theta_0$

Gluon transverse momenta leading to longer regeneration times will instead be **suppressed** (as the heavy quark is not there any more!!)

$\Theta < \Theta_0$ is called the 'dead cone' (no radiation from the heavy quark in a collinear region close to the quark)

The 'Dead Cone' in perturbative QCD

Consider gluon emission off a heavy quark using perturbation theory:

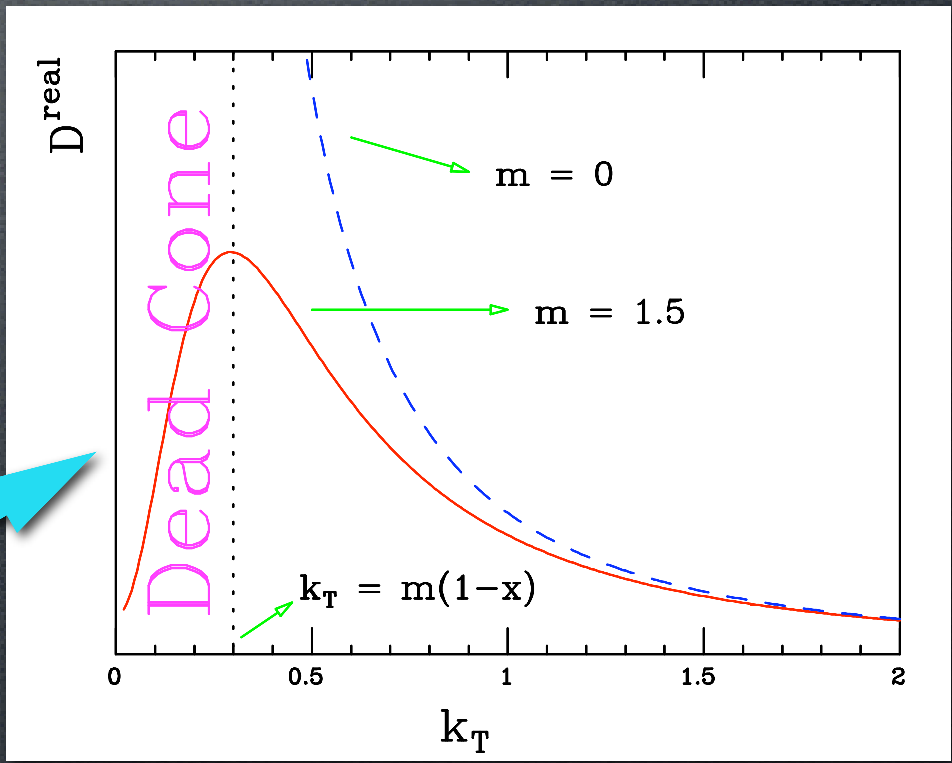


$$D^{real}(x,k_\perp^2,m^2) = \frac{C_F\alpha_S}{2\pi}\left[\frac{1+x^2}{1-x}\,\frac{1}{k_\perp^2+(1-x)^2m^2} - x(1-x)\frac{2m^2}{(k_\perp^2+(1-x)^2m^2)^2}\right]$$

In the **massless case** (m=0) we have a non-integrable collinear singularity:

$$\int_0 D(x,k_\perp^2)dk_\perp^2 = \frac{1+x^2}{1-x}\int_0 \frac{dk_\perp^2}{k_\perp^2} = \infty$$

The presence of the heavy quark mass suppresses instead the radiation at small transverse momenta and allows the integration down to zero

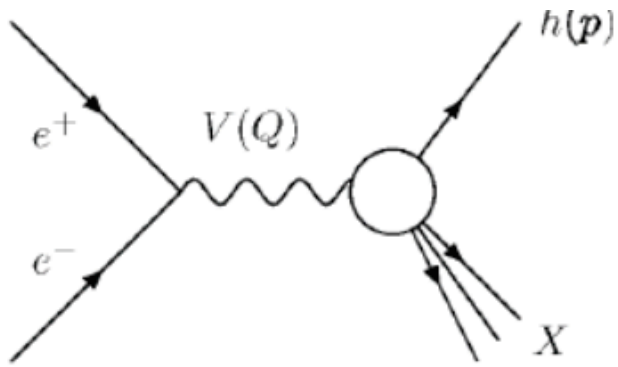


[NB. The cone is not really fully dead, just feeling unwell...]

=> We can calculate in pQCD heavy quark total cross sections and momentum distributions

A Massive Calculation

$$e^+ + e^- \to V(Q) \to h(p) + X$$



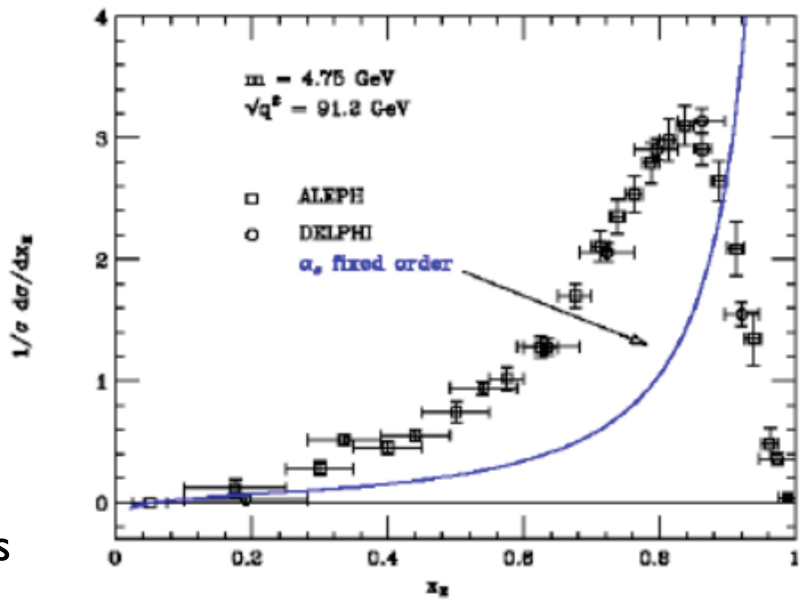
$$x = x_E = \frac{E_h}{E_{beam}}$$

Perturbative calculation:

$$\begin{aligned}\frac{1}{\sigma}\frac{d\sigma}{dx} &= \delta(1-x)+\frac{\alpha_s(Q^2)}{2\pi}\left\{C_F+C_F\left[\ln\frac{Q^2}{m^2}\left(\frac{1+x^2}{1-x}\right)_+\right.\right.\\ &+ 2\frac{1+x^2}{1-x}\log x-\left(\frac{\ln(1-x)}{1-x}\right)_+(1+x^2)+\frac{1}{2}\left(\frac{1}{1-x}\right)_+(x^2-6x-2)\\ &+ \left.\left.\left(\frac{2}{3}\pi^2-\frac{5}{2}\right)\delta(1-x)\right]\right\} + \mathcal{O}\left(\frac{m}{Q}\right)\end{aligned}$$

heavy quark mass

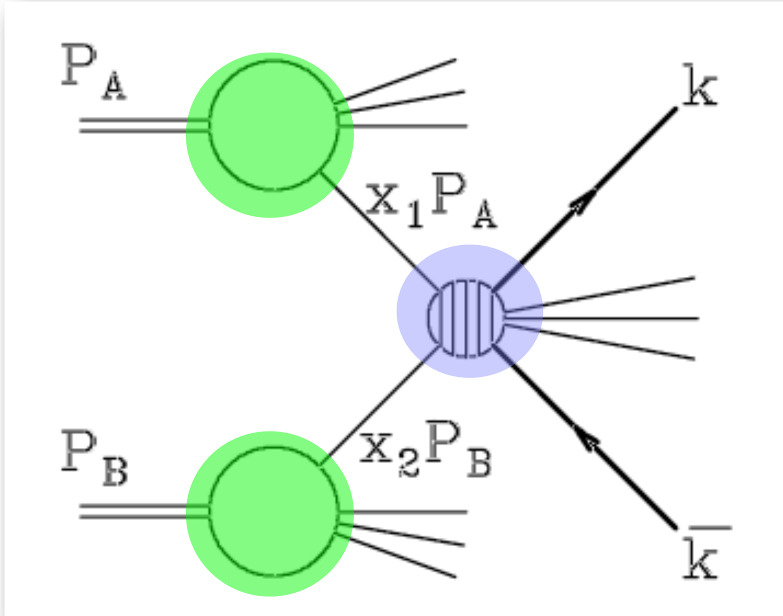
B mesons at LEP



Obviously, finite $\neq$ good description of data

Back to this problem later on

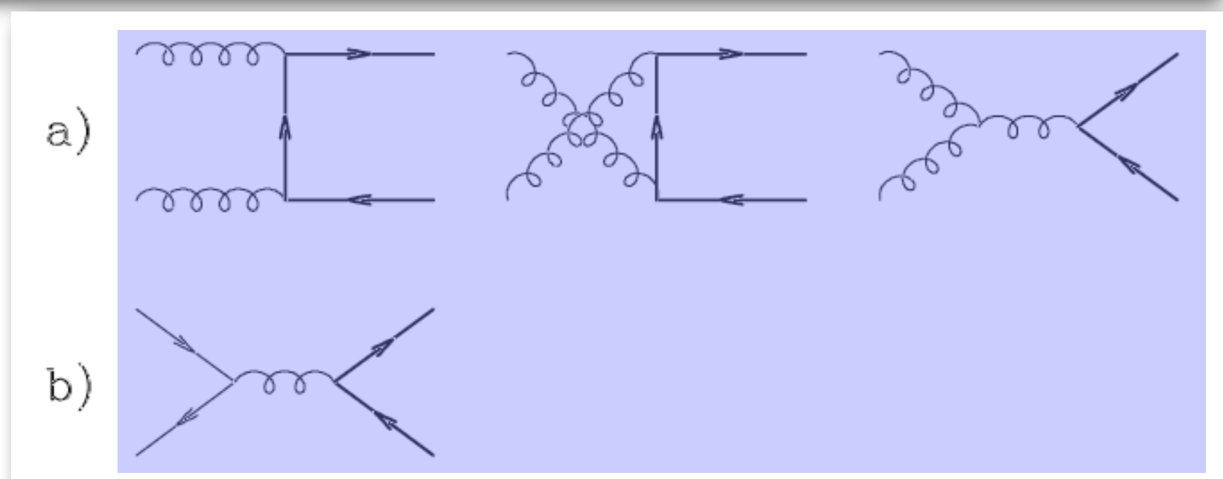
Heavy Quark hadroproduction



NB: light quarks and gluons only, in the initial state

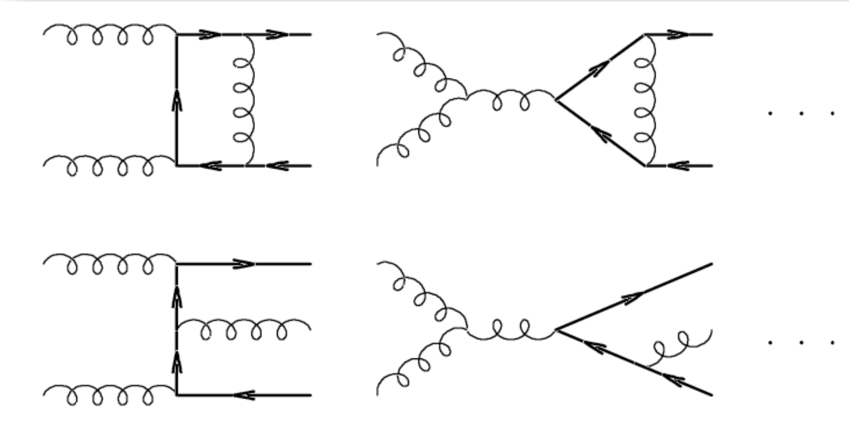
$$d^3\sigma = \sum_{i,j} \int dx_1\, dx_2\, d^3\hat{\sigma}_{ij}(x_1 P_A, x_2 P_B, k, \bar{k}, m, \mu)\, F_i^A(x_1, \mu)\, F_j^B(x_2, \mu).$$

Leading Order diagrams:



Next-to-Leading Order calculation also available:

[Nason, Dawson, Ellis; Beenakker et al, 1988-1989]



Virtual corrections

Real corrections

[At the time, quite a massive (no pun intended) calculation NNLO started only very recently and still in progress]

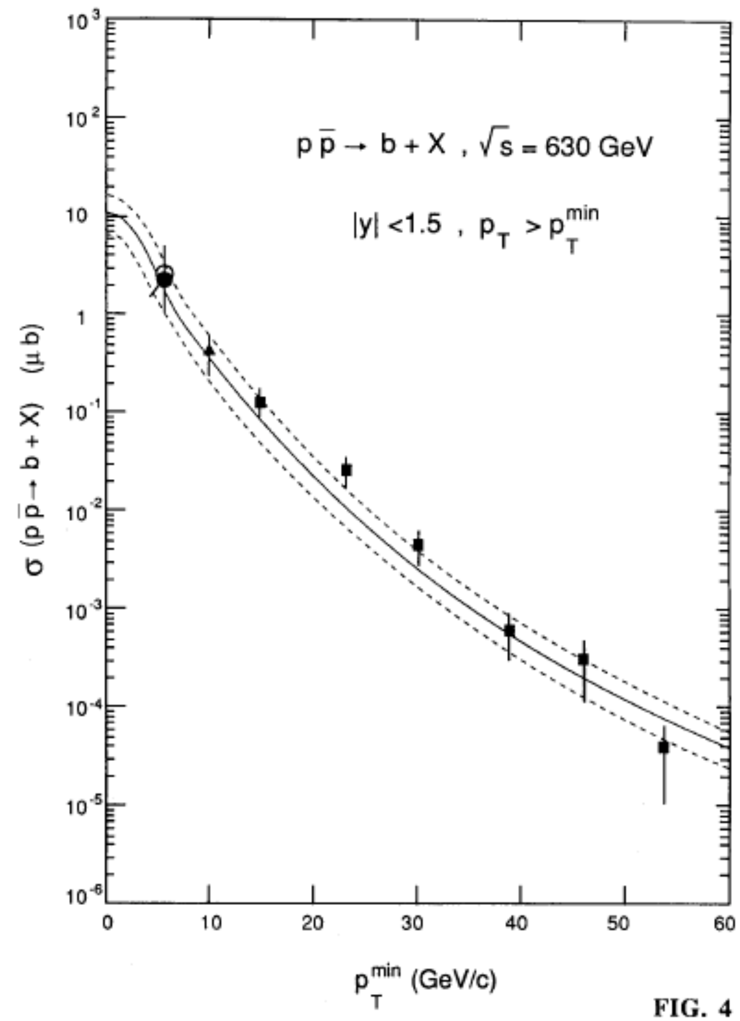


FIG. 4

First data: UA1 in 1990

Good agreement with NLO QCD predictions WITHIN UNCERTAINTIES

How does the NLO calculation look like?

$$\sigma(S) = \sum_{i,j} \int dx_1 dx_2 \, \hat{\sigma}_{ij}(x_1 x_2 S, m^2, \mu^2) \; F_i^A(x_1, \mu) F_j^B(x_2, \mu)$$

LO **NLO**

$$f_{ij}\left(\rho, \frac{\mu^2}{m^2}\right) = f_{ij}^{(0)}(\rho) + g^2(\mu^2)\left[f_{ij}^{(1)}(\rho) + \overline{f}_{ij}^{(1)}(\rho)\ln\left(\frac{\mu^2}{m^2}\right)\right] + O(g^4)$$

Must be calculated explicitly

Can be derived from LO

$$f_{q\bar{q}}^{(0)}(\rho) = \frac{\pi\beta\rho}{27}\left[2+\rho\right]$$

$$f_{gg}^{(0)}(\rho) = \frac{\pi\beta\rho}{192}\left[\frac{1}{\beta}(\rho^2+16\rho+16)\ln\left(\frac{1+\beta}{1-\beta}\right) - 28 - 31\rho\right]$$

$$f_{gq}^{(0)}(\rho) = f_{g\bar{q}}^{(0)}(\rho) = 0$$

velocity of the heavy quark

$$\rho = \frac{4m^2}{s}, \quad \beta = \sqrt{1-\rho}$$

NLO short distance cross sections

qqbar

$$f^{(1)}_{q\bar{q}} = \frac{\rho}{72\pi}\left[\frac{16}{3}\beta\,\underline{\ln^2(8\beta^2)} - \frac{82}{3}\beta\,\underline{\ln(8\beta^2)} - \frac{\pi^2}{6}\right]$$
$$+\beta\rho\left[a_0 + \beta^2\left(a_1\ln(8\beta^2) + a_2\right) + \beta^4\left(a_3\ln(8\beta^2) + a_4\right) + a_5\beta^6\ln(8\beta^2) + a_6\ln\rho + a_7\ln^2\rho\right]$$
$$+\frac{1}{8\pi^2}(n_{lf} - 4)f^{(0)}_{q\bar{q}}(\rho)\left[\frac{2}{3}\ln\left(\frac{4}{\rho}\right) - \frac{10}{9}\right] \qquad (22)$$

gg

$$f^{(1)}_{gg} = \frac{7}{1536\pi}\left[12\beta\,\underline{\ln^2(8\beta^2)} - \frac{366}{7}\beta\,\underline{\ln(8\beta^2)} + \frac{11}{42}\pi^2\right]$$
$$+\beta\left[a_0 + \beta^2\left(a_1\ln(8\beta^2) + a_2\right) + a_3\beta^4\ln(8\beta^2) + \rho^2\left(a_4\ln\rho + a_5\ln^2\rho\right)\right.$$
$$\left.+\rho\left(a_6\ln\rho + a_7\ln^2\rho\right)\right] + (n_{lf} - 4)\frac{\rho^2}{1024\pi}\left[\ln\left(\frac{1+\beta}{1-\beta}\right) - 2\beta\right]$$

gq

$$f^{(1)}_{gq} = \beta\left[\beta^2\left(a_0\ln\beta + a_1\right) + \beta^4\left(a_2\ln\beta + a_3\right) + \rho^2\left(a_4\ln\rho + a_5\ln^2\rho\right)\right.$$
$$\left.+\rho\left(a_6\ln\rho + a_7\ln^2\rho\right)\right]$$

Threshold resummation

Large logs in the threshold region ($\beta \to 0$) prompt for all-order resummation

Various calculation, to different levels of accuracy.

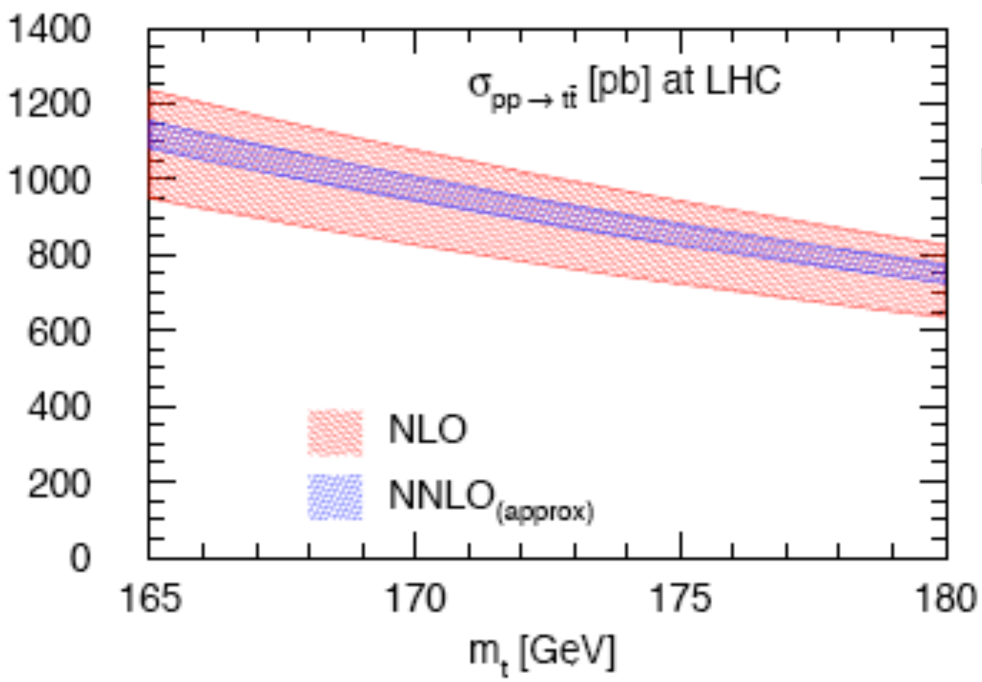
[See Sullivan's lectures for details, references]

Good nomenclature: NLO+NLL, $\text{NNLO}_{\text{approx}}$, ...

New: Moch, Uwer, arXiv:0804.1476

Bad nomenclature: NNLO+NNNLL (and variants thereof)

[Nothing "NN" in here, just a series expansion of the NLL Sudakov exponent to higher orders]



$\text{NNLO}_{\text{approx}}$ would seem to be a big improvement.

Not entirely clear <u>why</u> (production at LHC not necessarily threshold-dominated).

Estimation of theoretical uncertainties often a delicate issue

Theoretical uncertainties

Ideally $$\frac{d}{d\ln\mu^2}\ln\sigma^{phys} = 0$$ i.e. independence of cross sections on artificial scales

This only holds for all-order calculations.
In real life: residual dependence at one order higher than the calculation

↓

Vary scales (around a physical one) to ESTIMATE the uncalculated higher order

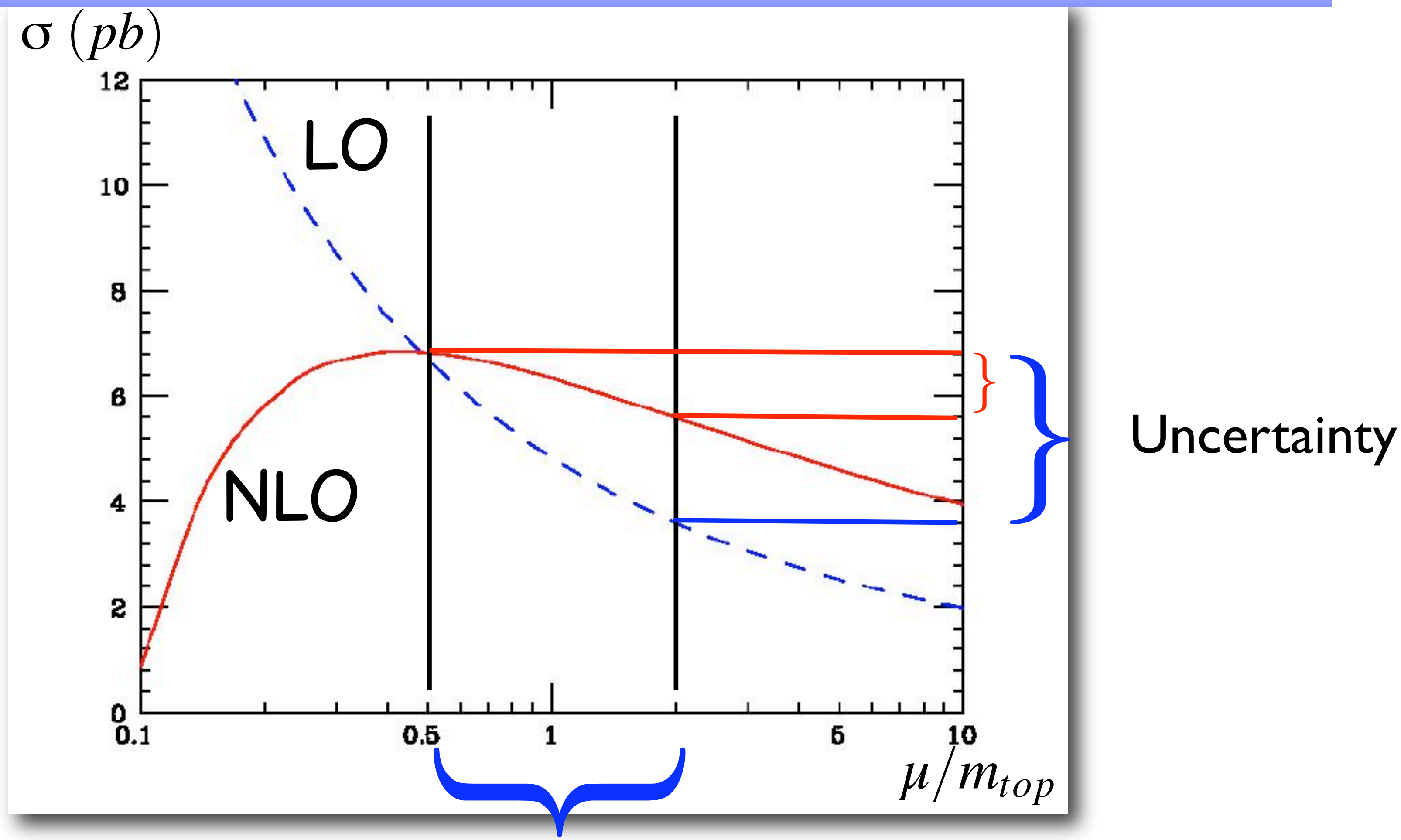
NB. Such uncertainty is a....known unknown, but still an unknown

The 'best value' of a scale <u>cannot</u> be fixed using data, as if it were a physical parameter.

[High energy physics version of 'There's no free lunch']

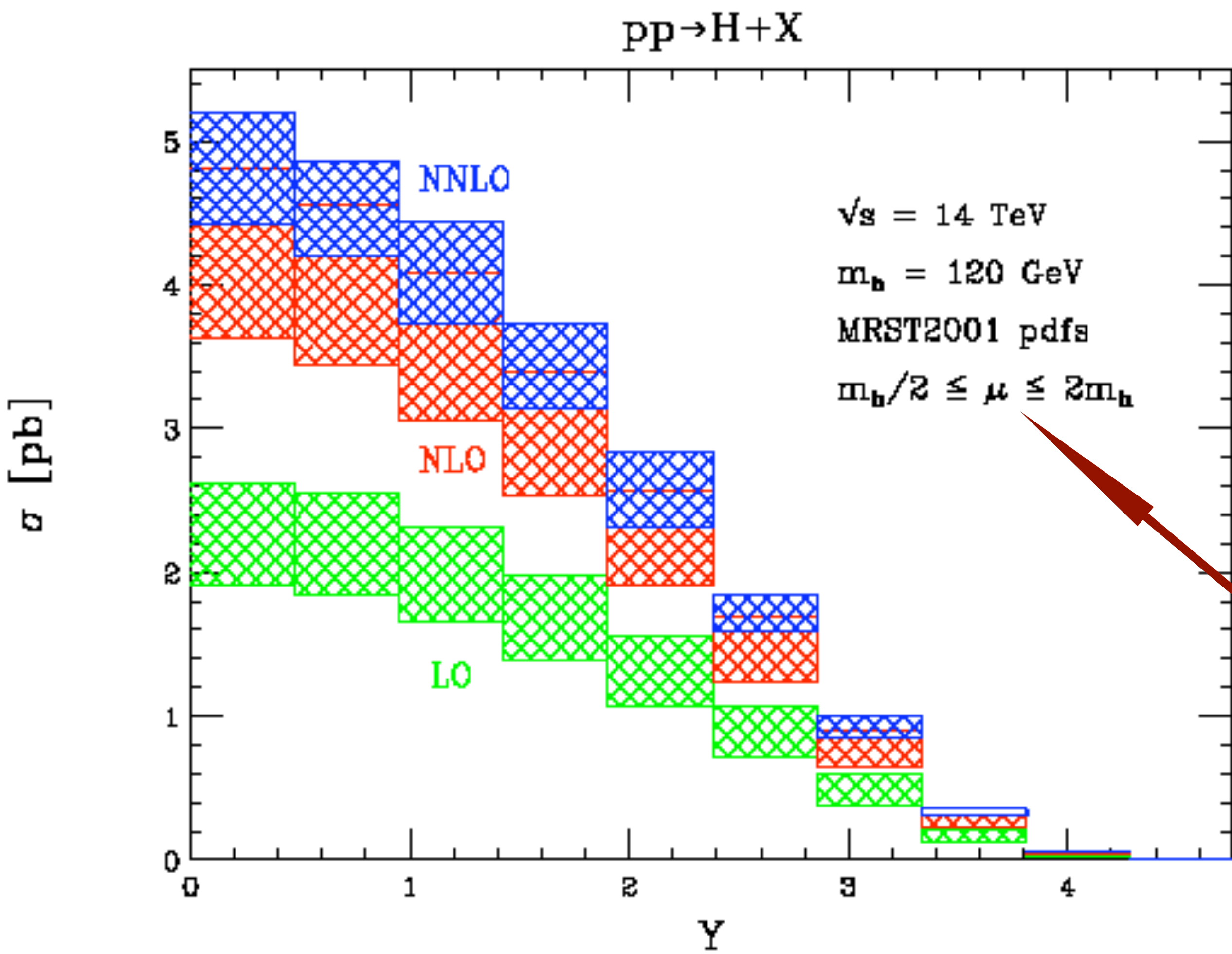
The rule of thumb on uncertainties

"Typical" behaviour of a cross-section w.r.t. scale variations



- A **LO** calculation gives you a **rough estimate** of the cross section
- A **NLO** calculation gives you a **good estimate** of the cross section and a **rough estimate** of the uncertainty
- A **NNLO** calculation gives you a **good estimate** of the uncertainty

Theoretical uncertainty: an example



Example:
Higgs boson production
at the LHC

Anastasiou, Melnikov. Petriello, hep-ph/0501130

Scale variations

NB. This example shows that the center of the NLO band has nothing to do with the most accurate theoretical prediction.

Theoretical uncertainty bands are not gaussian errors!

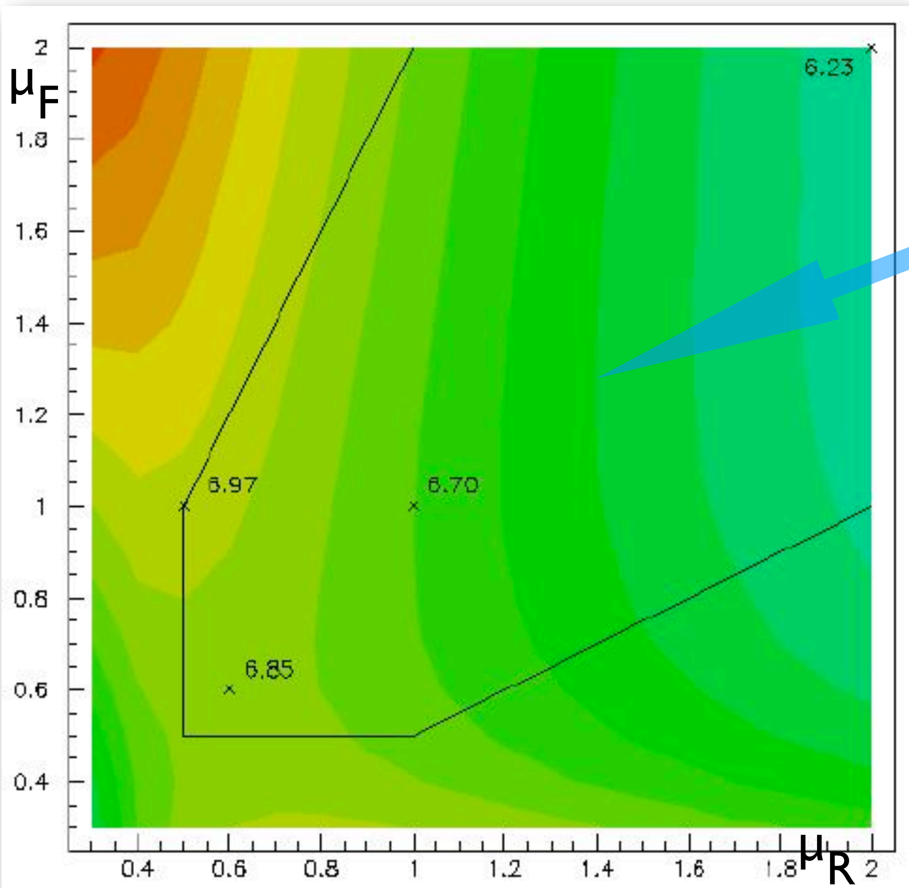
Top @ Tevatron

Standard procedure: vary renormalisation and factorisation scales.
But, better do so independently

(NLO+NLL, m=175 GeV)

σ: 6.82 > 6.70 > 6.23 pb $\qquad 0.5 < \mu_{R,F}/m < 2$

σ: 6.97 > 6.70 > 6.23 pb $\qquad 0.5 < \mu_{R,F}/m < 2 \;\&\&\; 0.5 < \mu_R/\mu_F < 2$



"Fiducial" region

Order **±5%** uncertainty along the diagonal, a little more considering independent scale variations

BTW, the PDF uncertainty (±10-15%) is probably the dominant one here

Independent scale variations

Sometimes, varying scales together can be misleading!

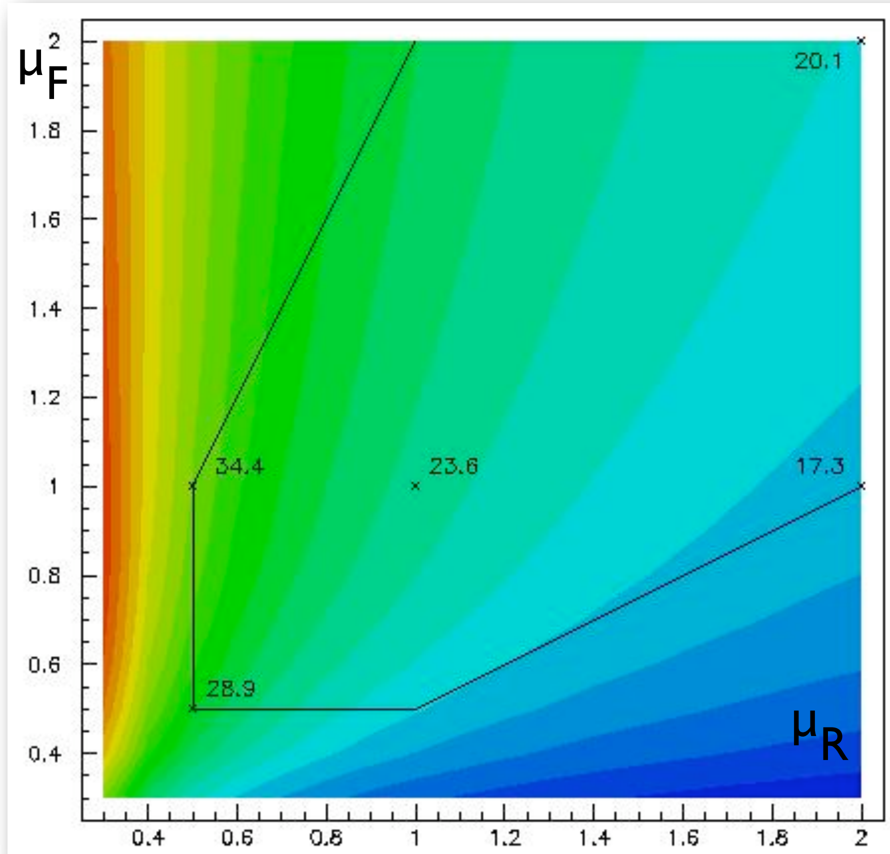
bottom at the Tevatron

$\sigma(|y|<1)$: 28.9 > 23.6 > 20.1 μb

$0.5 < \mu_{R,F}/\mu_0 < 2$

$\sigma(|y|<1)$: 34.4 > 23.6 > 17.3 μb

$0.5 < \mu_{R,F}/\mu_0 < 2$ && $0.5 < \mu_R/\mu_F < 2$



The scales uncertainty increases from **±18%** to **±35%** when going off-diagonal

Independent scale variations

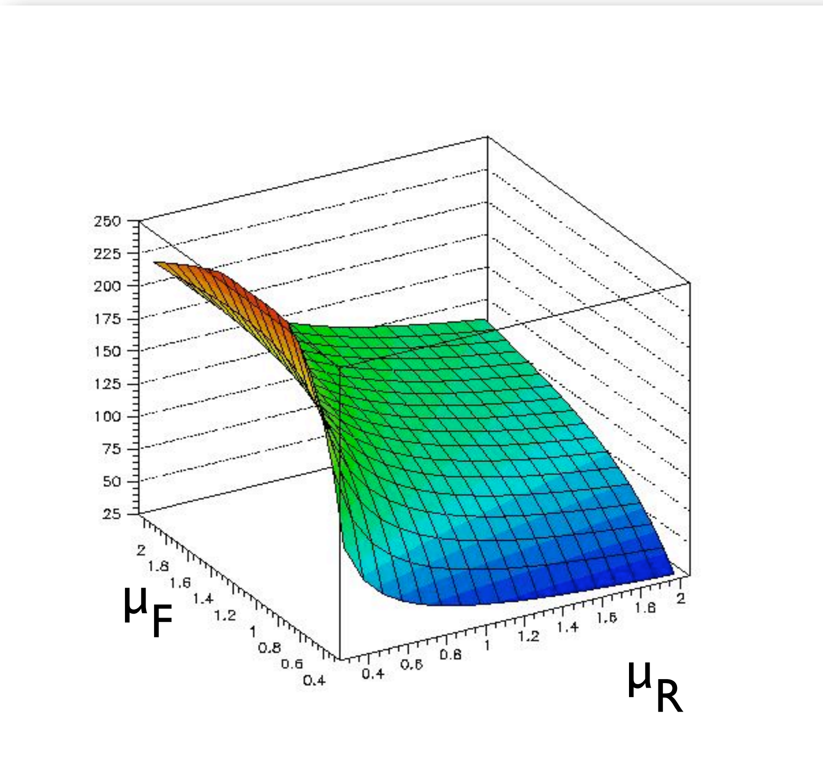
Sometimes, varying scales together can be <u>very</u> misleading!

Case in point: bottom cross section at the LHC:

σ(|y|<1): 122 > 120 > 115 μb

$0.5 < \mu_{R,F}/\mu_0 < 2$

Only a **±4%** uncertainty when varying the scales together.......



σ(|y|<1): 178 > 120 > 75 μb

$0.5 < \mu_{R,F}/\mu_0 < 2$ && $0.5 < \mu_R/\mu_F < 2$

....which becomes a **±40%** one when going off-diagonal!

Top @ LHC

Going to independent scale variations matters more at the LHC
(NLO+NLL, m = 171 GeV, scale uncertainties only)

σ: 970 > 908 > 860 pb $\quad 0.5 < \mu_{R,F}/m < 2$

σ: 990 > 908 > 823 pb $\quad 0.5 < \mu_{R,F}/m < 2$ && $0.5 < \mu_R/\mu_F < 2$

This would not have been obvious looking only at NLO

σ: 977 > 875 > 774 pb $\quad 0.5 < \mu_{R,F}/m < 2$

$0.5 < \mu_{R,F}/m < 2$ && $0.5 < \mu_R/\mu_F < 2$

**Lesson to take home here: every process/energy can be different.
Uncertainty estimates should always be carried out in detail,
and not 'carried over' from a supposedly (or hopefully) similar case**

Top @ LHC: one more lesson?

Compare

NLO+NLL, same as before:

Choice '1'

σ: 970 > 908 > 860 pb $\quad 0.5 < \mu_{R,F}/m < 2$

σ: 990 > 908 > 823 pb $\quad 0.5 < \mu_{R,F}/m < 2$ && $0.5 < \mu_R/\mu_F < 2$

with

NLO+NLL, **different choice for subleading terms**:

Choice '2'

σ: 964 > 945 > 939 pb $\quad 0.5 < \mu_{R,F}/m < 2$

σ: 1041 > 945 > 861 pb $\quad 0.5 < \mu_{R,F}/m < 2$ && $0.5 < \mu_R/\mu_F < 2$

Lessons:

* different central values, but within uncertainties of 'choice 1'
* 'choice 2' has very small uncertainty for equal scales, O(2%).
* when the scales are kept different, both choices are compatible (as they should)

In NNLO$_{\text{approx}}$ the 'choice 2' was made and uncertainties were studied equal scales. Could this explain its very small uncertainty?

Differential cross sections

Do we actually observe charm and bottom quarks?

Of course not!

Real measurements are done with (decay products of) charmed and bottom hadrons, i.e. mainly D and B

The 'old school' called for 'reconstructing' from such measurements the bare quark cross section, present the data in this way (see plot) and compare the latter to pQCD predictions.

Is this a good idea?

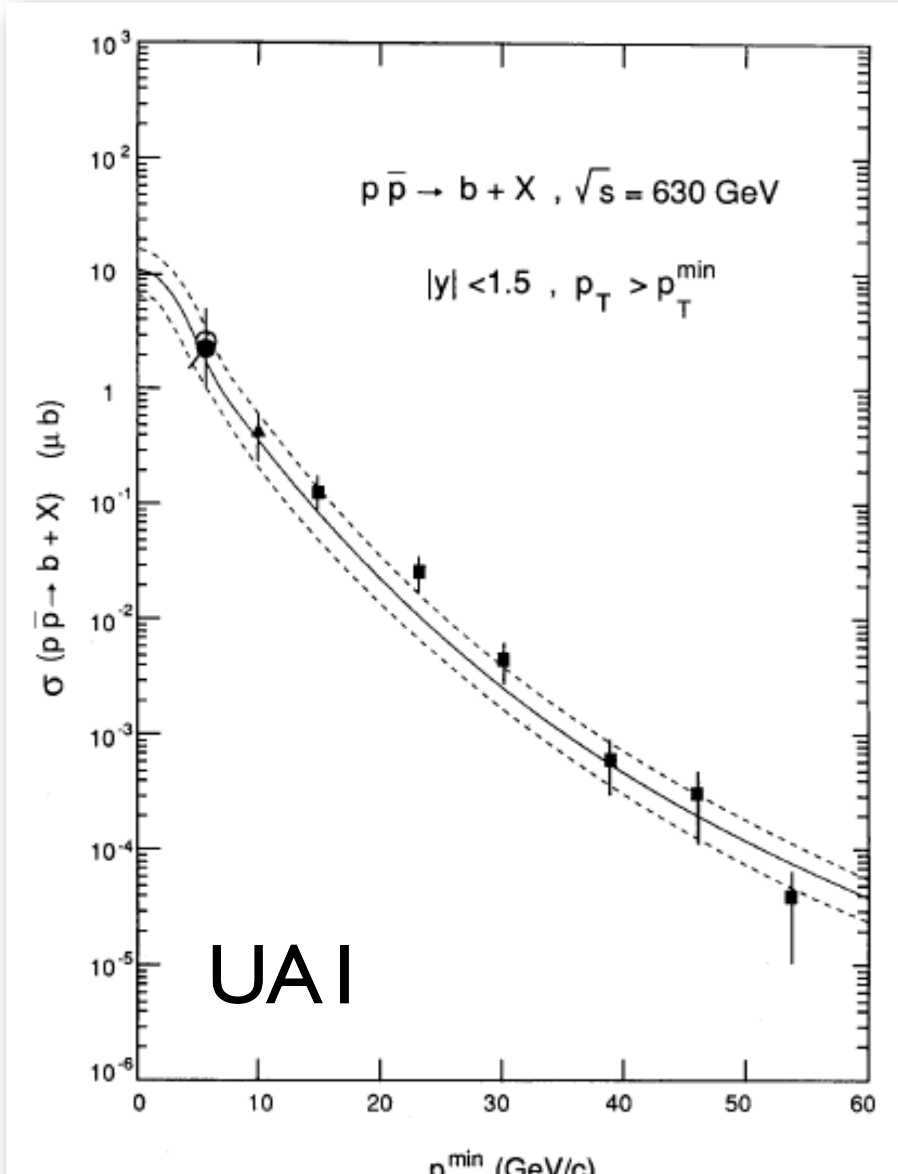
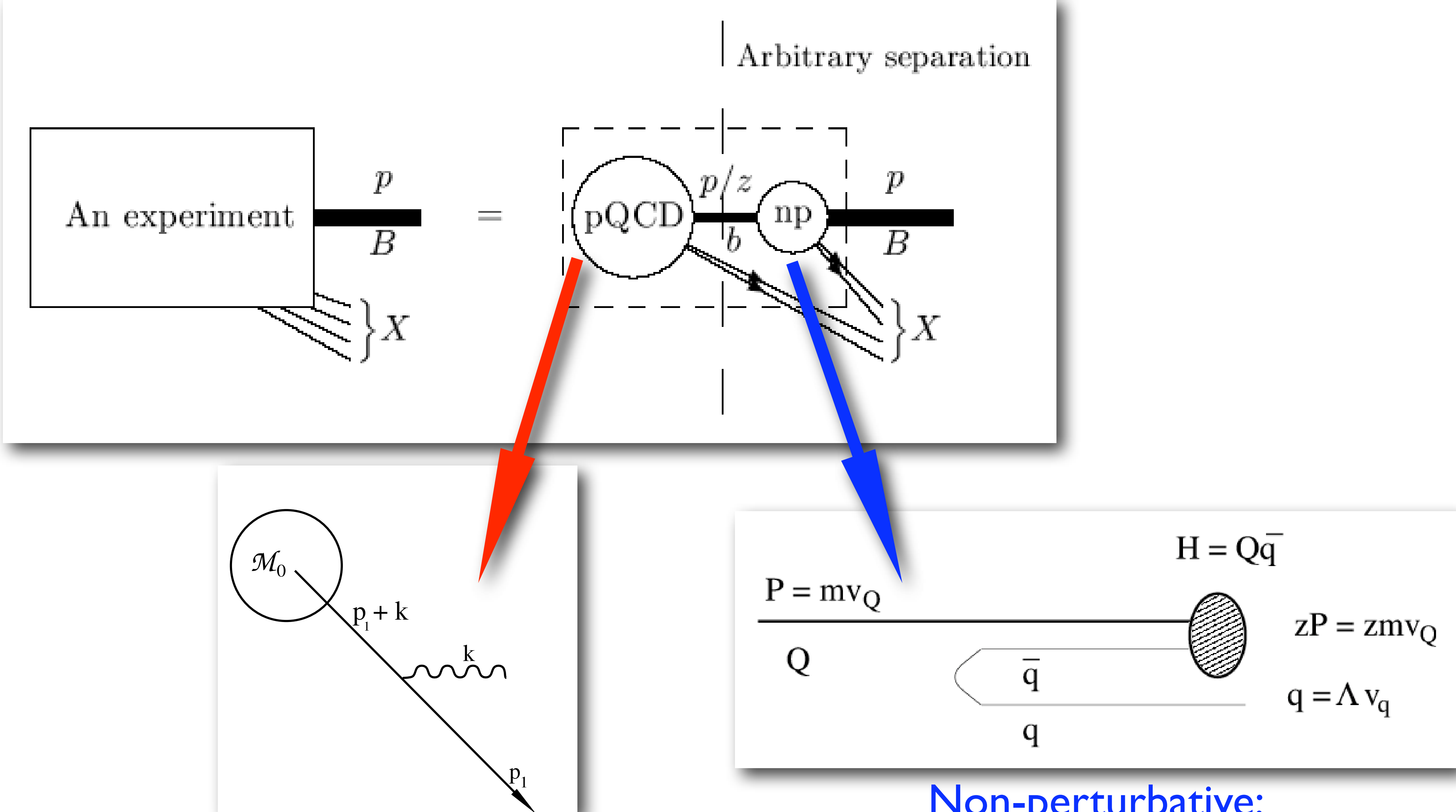


FIG. 4



Perturbative:
gluon radiation

Non-perturbative:
hadronization

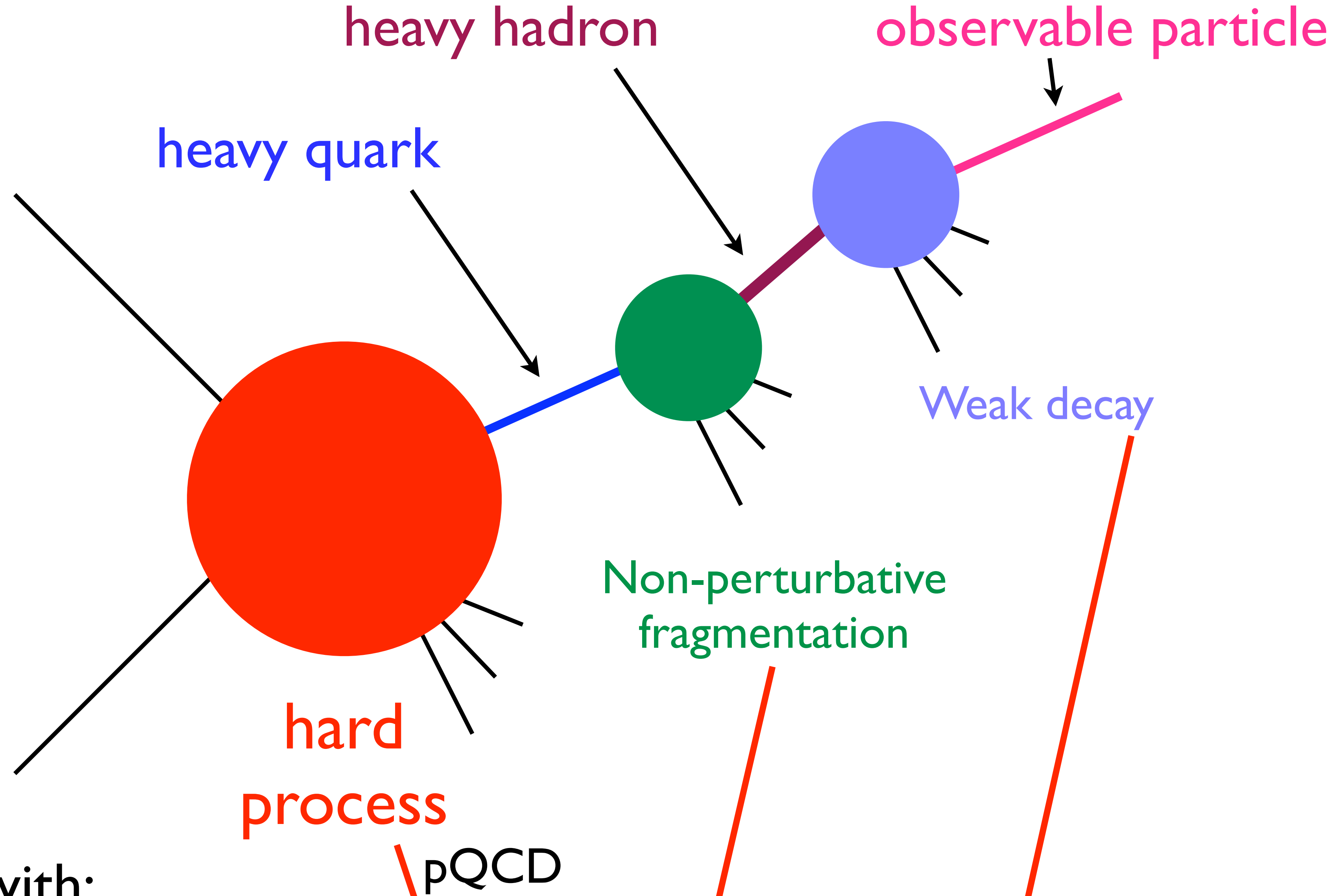
Not being the b quark a physical particle,
the quark-to-meson transition cannot be a physical observable:
its details depend on the perturbative calculation it is interfaced with.
Deconvoluting to the quark level is therefore AMBIGUOUS

Observables

More modern attitude
(also made more easily feasible by computer power):
compare at (or as close as possible to) the observable level

Full process $$pp \stackrel{pQCD}{\longrightarrow} Q \stackrel{NP fragm.}{\longrightarrow} H_Q \stackrel{decay}{\longrightarrow} e$$

Sketch of inclusive production



Describe it with:

$$\frac{d\sigma(b \to B \to J/\psi)}{dp_T} = \frac{d\sigma(b)}{d\hat{p}_T} \otimes f(b \to B) \otimes g(B \to J/\psi)$$

Non-perturbative fragmentation

What do we know about it?

If the quark is light, not much. It's a process-independent artificial object (factorisation theorem) which we must extract from data

(e.g. pion fragmentation functions)

If the quark is heavy, its fragmentation function is still ambiguous, but we can tell something more about it:

* we know it's a (parametrically) small effect

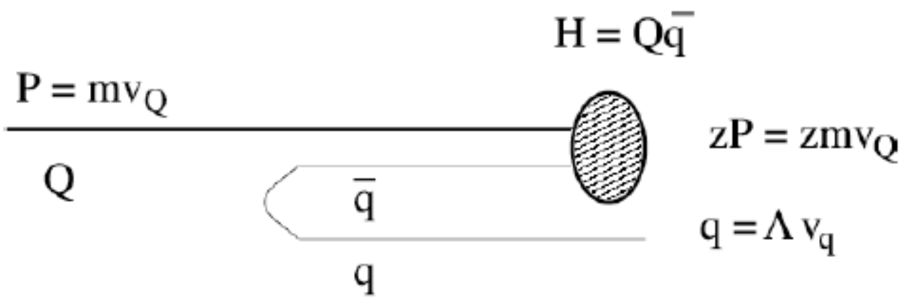
* we can relate it to the hadronisation scale and to the heavy quark mass

* we can test this on D and B data

Bjorken and Suzuki (circa 1977)

It boils down to: a heavy object is hard to slow down

We can see it in the following way (likely another Mencken's simple and wrong solution....)



The heavy quark has momentum $P = mv_Q$, the light quarks have momentum $q = \Lambda v_q$, with Λ a hadronic mass scale

For the binding we need $v_Q \simeq v_q = v$. We have then

$$P = zP + q \qquad\qquad mv = zmv + \Lambda v$$

and therefore

$$\langle z \rangle \simeq 1 - \frac{\Lambda}{m}$$

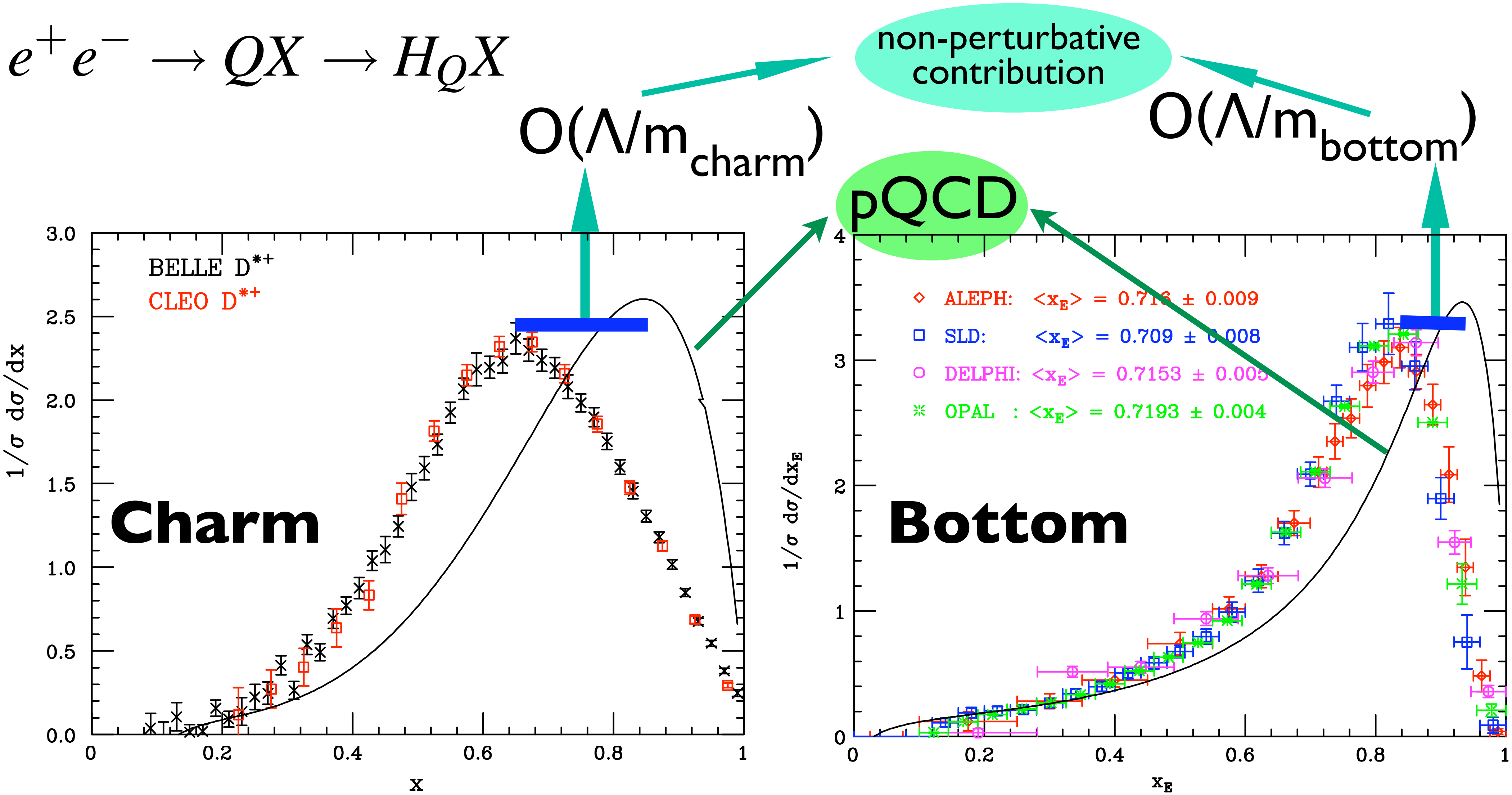
[NB. Bjorken and Suzuky used different, better, derivations]

Final result: a heavy quark only loses on average a fraction Λ/m of its momentum when hadronising.
A heavy quark fragmentation function will be peaked near z=1

pQCD substantiates this by indicating: $\langle z^{N-1} \rangle = 1 - \frac{(N-1)\overline{\Lambda}}{m} + \mathcal{O}\left(\frac{\overline{\Lambda}^2}{m^2}\right)$

Test of scaling in D and B fragmentation

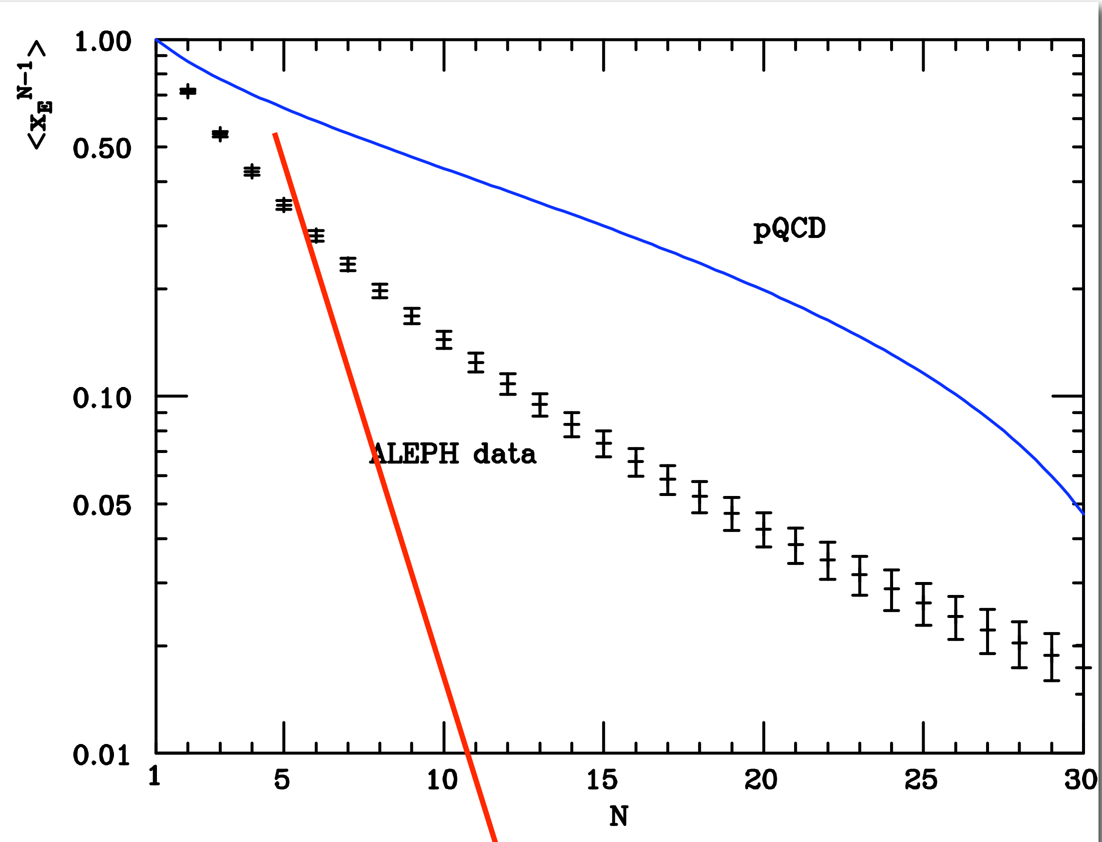
$$e^+e^- \to QX \to H_QX$$



- non-perturbative contribution limited in size and compatible with expectations
- high-accuracy expt. data allow it to be precisely determined

Test of scaling in D and B fragmentation

LEP B meson data translated to Mellin space:



$$f_N \equiv \int_0^1 x^{N-1} f(x)\, dx = \langle x^{N-1} \rangle$$

In this space
convolutions become products

$$\langle x \rangle_{expt} = \langle x \rangle_{pQCD} \langle x \rangle_{np}$$

This gap:
non-perturbative QCD

Test of scaling in D and B fragmentation

$\langle x^{N-1} \rangle$ moments can give a more quantitative picture:

N	2
c @ 10.58 GeV	0.7359
c @ 91.2 GeV (NS)	0.5858
c @ 91.2 GeV (full)	0.5954
b @ 91.2 GeV	0.7634
BELLE $D^{*+} \to D^0$ (ISR corr.)	0.6418 ± 0.0042
ALEPH D^{*+} (ISR corr.)	0.4920 ± 0.0152
ALEPH B	0.7163 ± 0.0085
CLEO D^{*+}	$0.877^{+0.009}_{-0.010}$
BELLE $D^{*+} \to D^0$	$0.872^{+0.005}_{-0.006}$
ALEPH D^{*+}	$0.840^{+0.022}_{-0.031}$
Tab. 2 and eq. (4.2)	0.868
ALEPH B	$0.938^{+0.009}_{-0.014}$
SLD B	$0.931^{+0.016}_{-0.030}$

N=2 moments (i.e. $\langle x \rangle$)

pQCD (NLL)

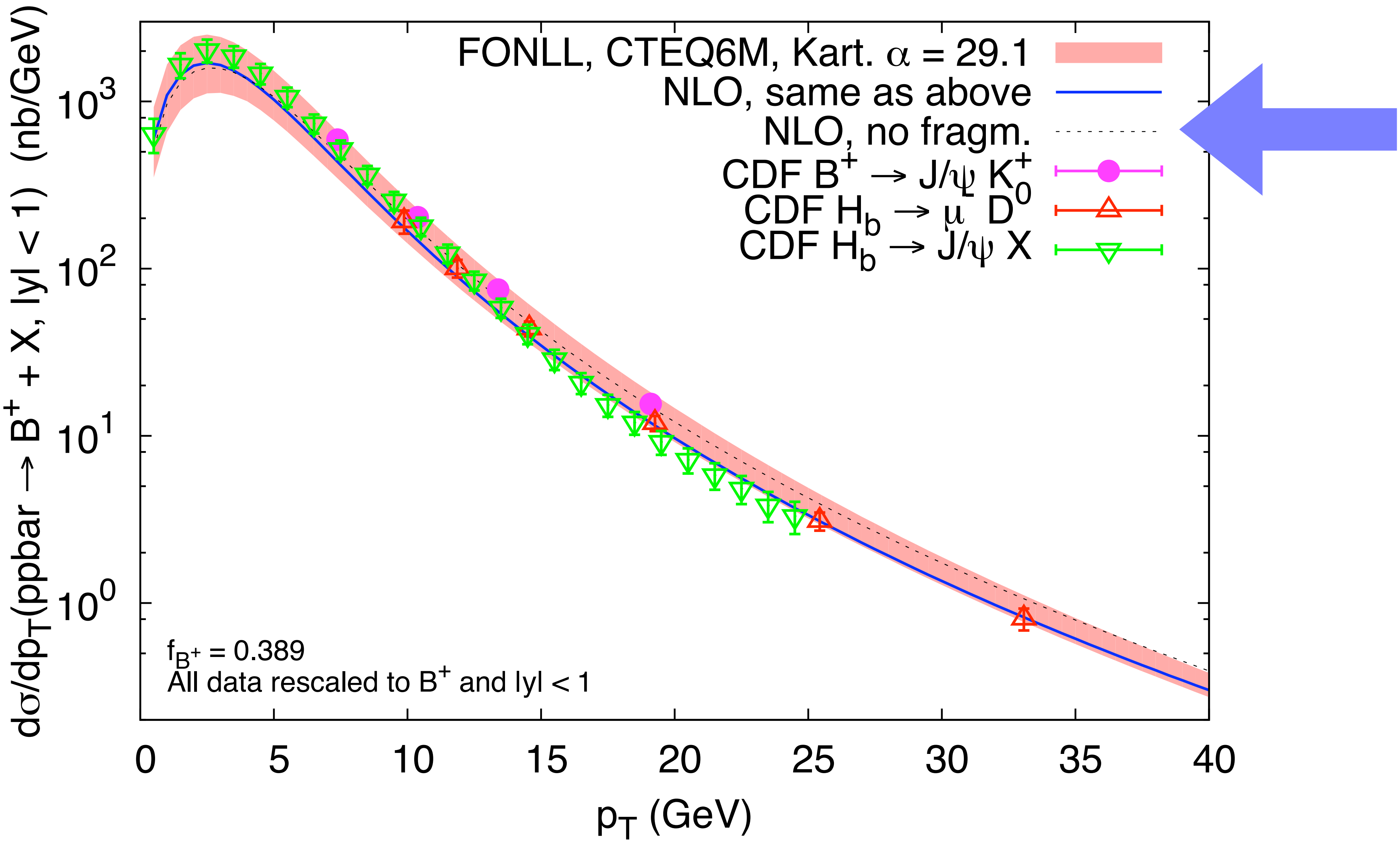
data
(very precise!)

$$D^{np} = \frac{\text{data}}{\text{pQCD}}$$

charm ~ 1 - 0.16
bottom ~ 1 - 0.06

Compatible with $D_N^{np} = 1 - \frac{(N-1)\Lambda}{m} + \cdots$ and $\Lambda \simeq 0.25$ GeV

B-mesons differential cross sections @ Tevatron



Good agreement, with **minimal** non-perturbative correction

NLO is sufficient for correct total rate prediction

Lessons in heavy quark fragmentation

- Charm and bottom are heavy and have limited non-perturbative contributions, but still hadronise
- We can predict to some extent their non-perturbative fragmentation functions
- After pQCD has done its job (gluon radiation, possibly resummed) the remaining contribution is small and scales as predicted
- the non-perturbative fragmentation function is **ambiguous** and non-observable, and must be matched properly with the pQCD part
- Even a small contribution can be enhanced by steeply falling spectra (i.e. transverse momentum distributions) and lead to large effects. Hence, importance of proper treatment of fragmentation in hadronic collisions

The top exception

You'll have noticed that the top was not discovered first as a bound state. Why ?

The revolution time of a ttbar bound state goes like $t_R \sim \dfrac{1}{m_t \alpha_S^2}$

For the top quark this yields $t_R \sim 10^{-25}$ s

On the other hand, as member of a weak isospin doublet, a heavy top can <u>decay weakly</u>:

$$t \longrightarrow bW^+ \qquad t_{\text{decay}} = \frac{1}{\Gamma_{bW}} \simeq 1/\left(\frac{G_F m_t^3}{8\pi\sqrt{2}}\right) \sim \frac{1}{G_F m_t^3} \sim \frac{m_W^2}{m_t^3} \sim 10^{-28} \text{ s}$$

[NB. The 'right' number with all the numerical factors it's actually a lot closer to 10^{-25} s]

So a heavy (>> M_W) top vanishes before a toponium can be formed

[Bigi, Dokshitzer, Khoze, Kuehn, Zerwas, PLB 181 (1986) 157]

The top exception

A similar, even more stringent, argument applies to standard hadronisation, i.e. the formation of t-(light quark) states

Hadronisation takes a certain time, namely the time for gluons to propagate the distance of a typical hadron radius R ~ 1 fm:

$$t_{\mathrm{hadr}} \sim R/c \sim 1/\Lambda \sim 10^{-24}\ s$$

Recalling the top weak decay:

$$t_{\mathrm{decay}} = \frac{1}{\Gamma_{bW}} \simeq 1/\left(\frac{G_F m_t^3}{8\pi\sqrt{2}}\right) \sim 1/(G_F m_t^3) \sim \frac{M_W^2}{m_t^3} = \frac{1}{\Lambda}\frac{M_W^2 \Lambda}{m_t^3}$$

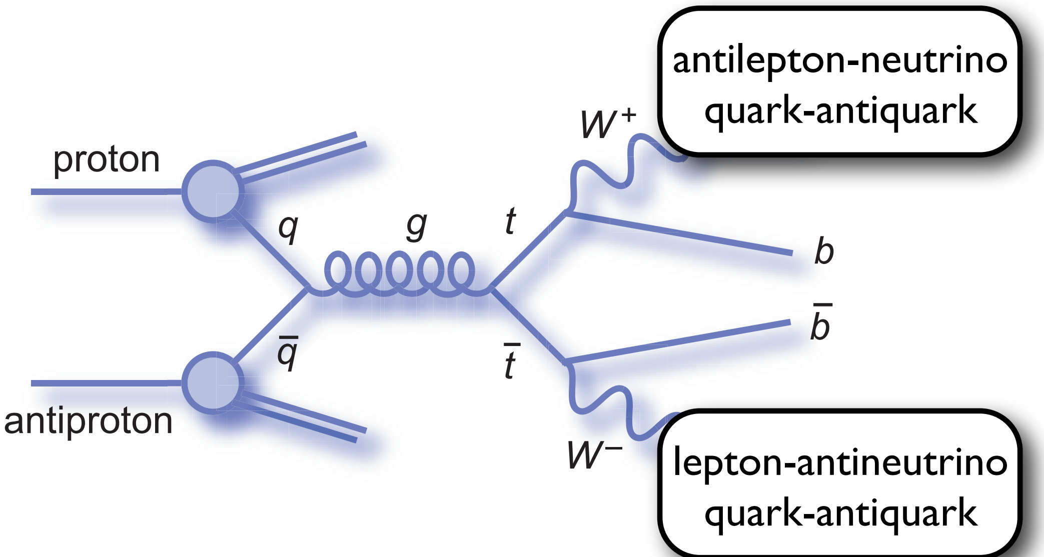
so that $t_{\mathrm{decay}} < t_{\mathrm{hadr}}$ if $\boxed{m_t > (M_W^2\Lambda)^{1/3} \simeq 10\ \mathrm{GeV}}$

NB. Neglected pretty big numerical factors. Real limit larger.

One more, a heavy top quark with mass larger than the W boson will therefore decay before hadronising

Top decays

Classified according to the W decay

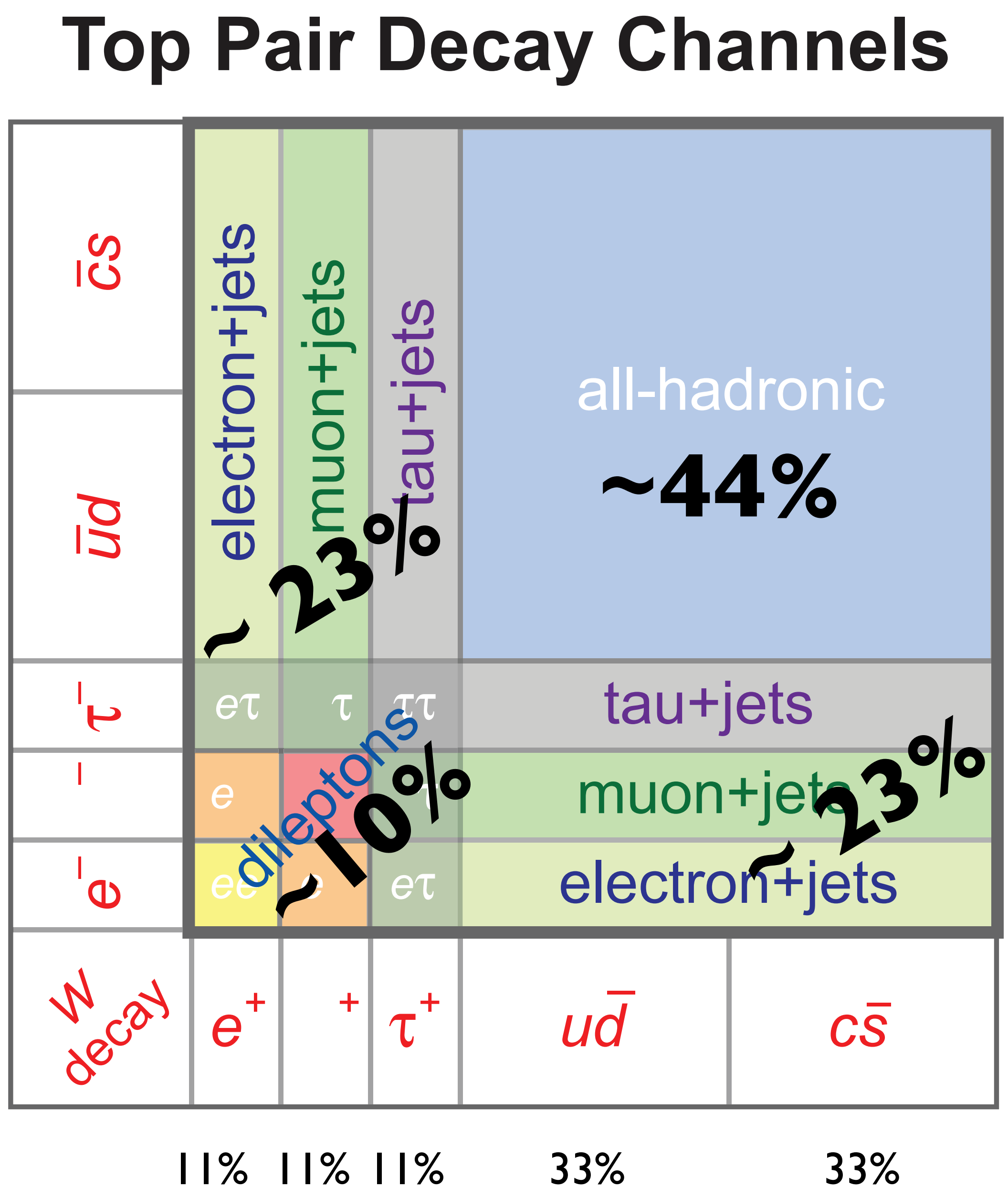


dilepton: low yield, low bckgd

lepton+jet: higher yield, moderate bckgd

all hadronic: highest yield, huge bckgd

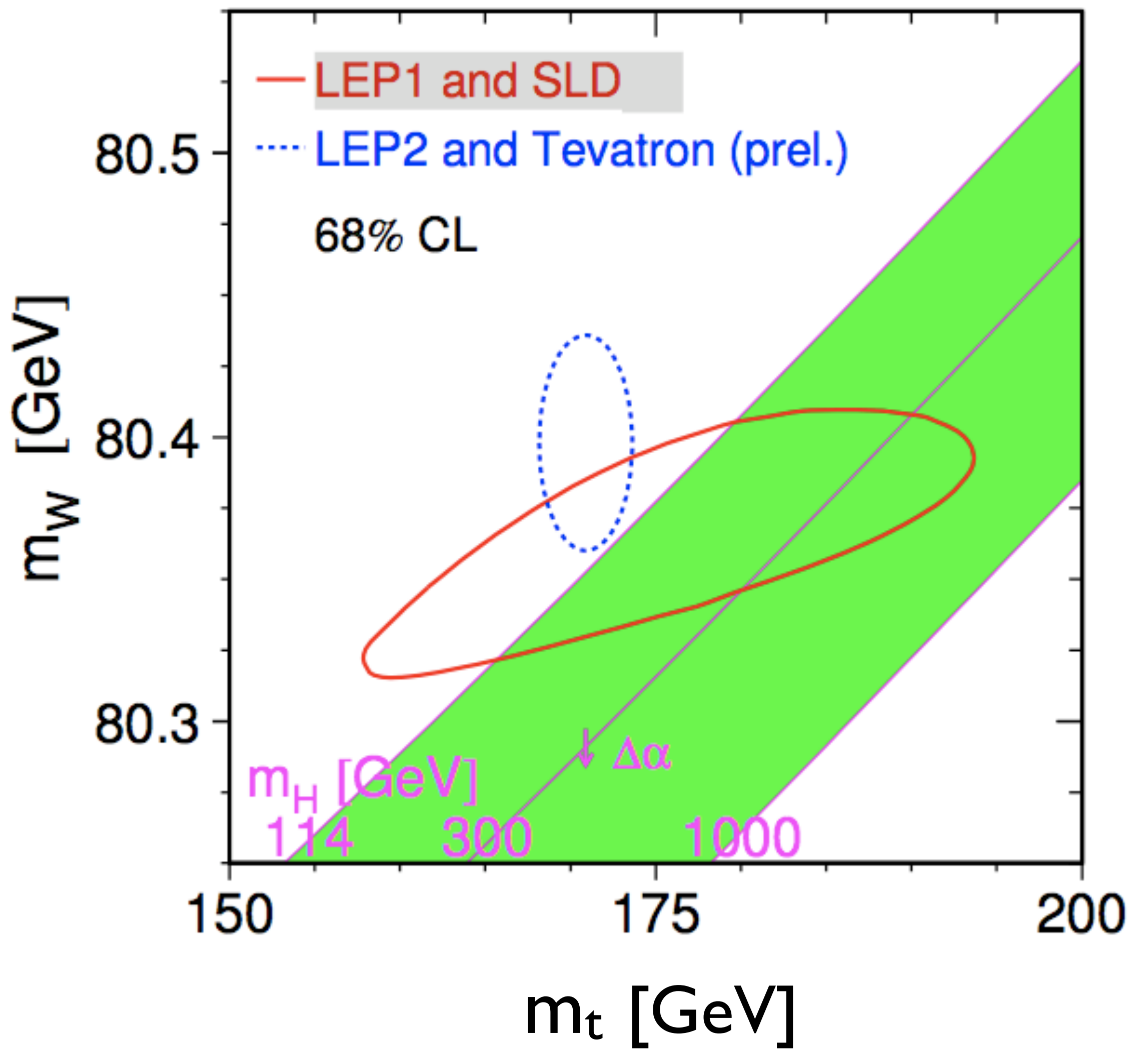
[NB. the tau is usually considered a 'hadron']

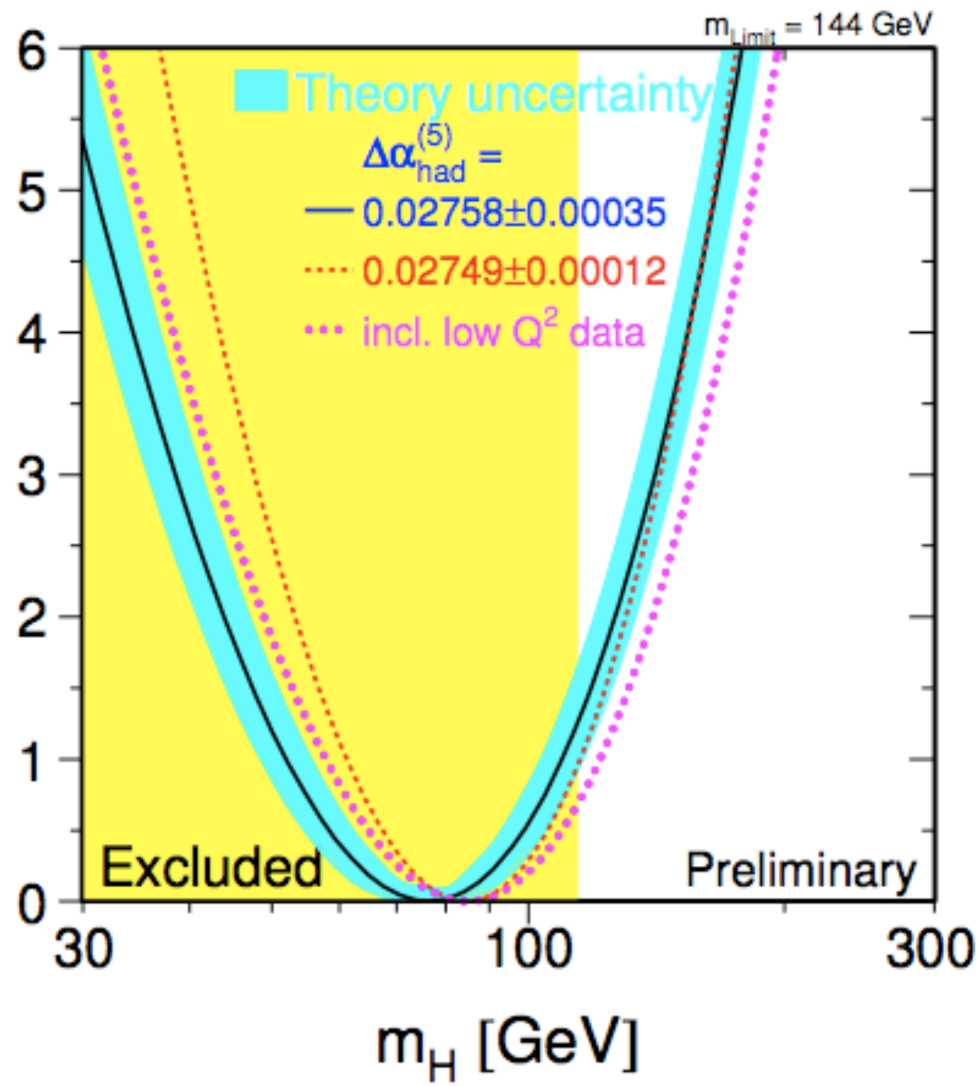


The top mass

Why are we interested in a precise measurement of the top mass?

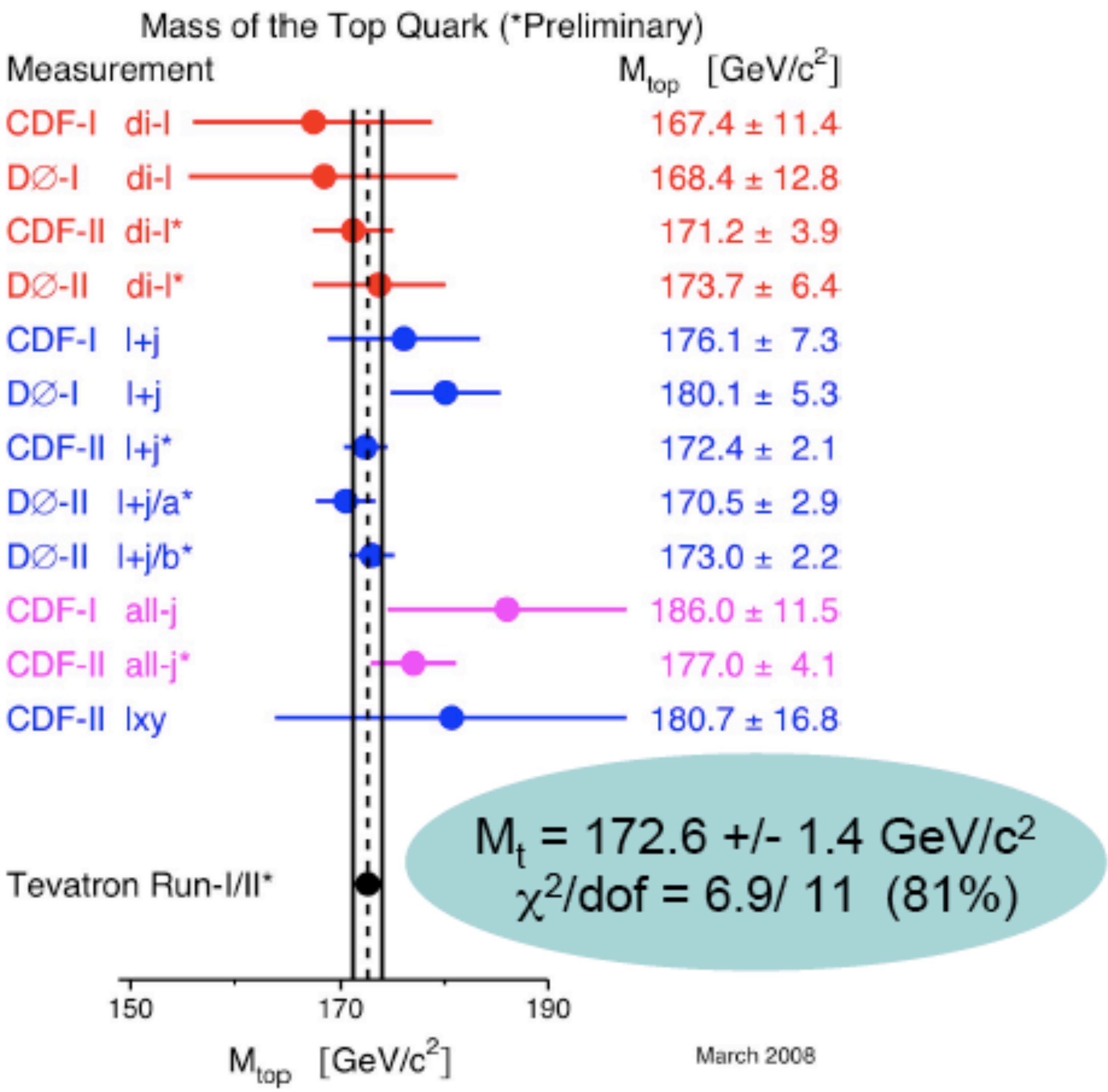
Indirect handle on the Higgs mass





A 2 GeV change in m_t changes the limit on m_H by ~20 GeV

The top mass



What is this mass?
What are we actually measuring?

- Relative uncertainty: 0.8%

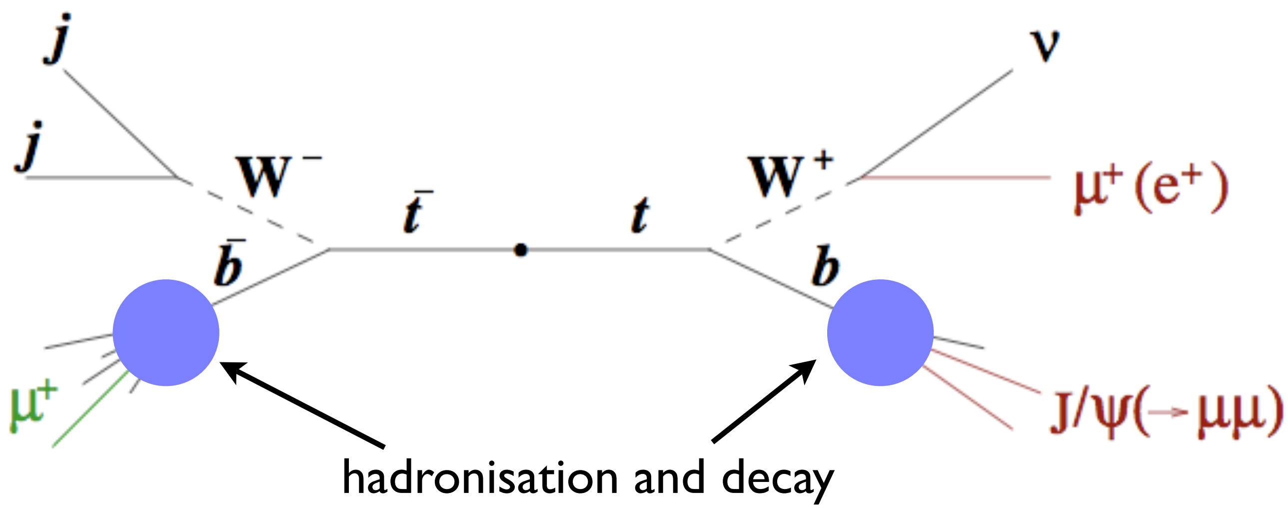
D. Glenzinski's talk at Top 2008

The top mass

Since the top does not hadronise,
can we measure its pole mass to any given accuracy?

Not really

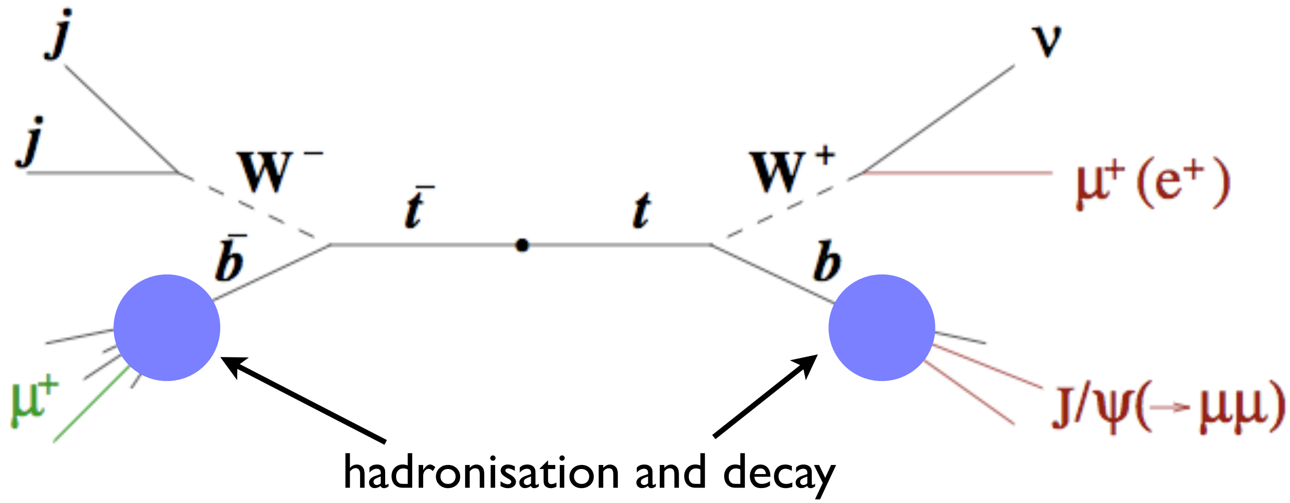
The top mass is usually measured through kinematic distributions of the top decay products, the bottom quark among them



The hadronisation (= long-distance) uncertainties enter the top mass determination through its decay products

The top mass

A second source of uncertainty



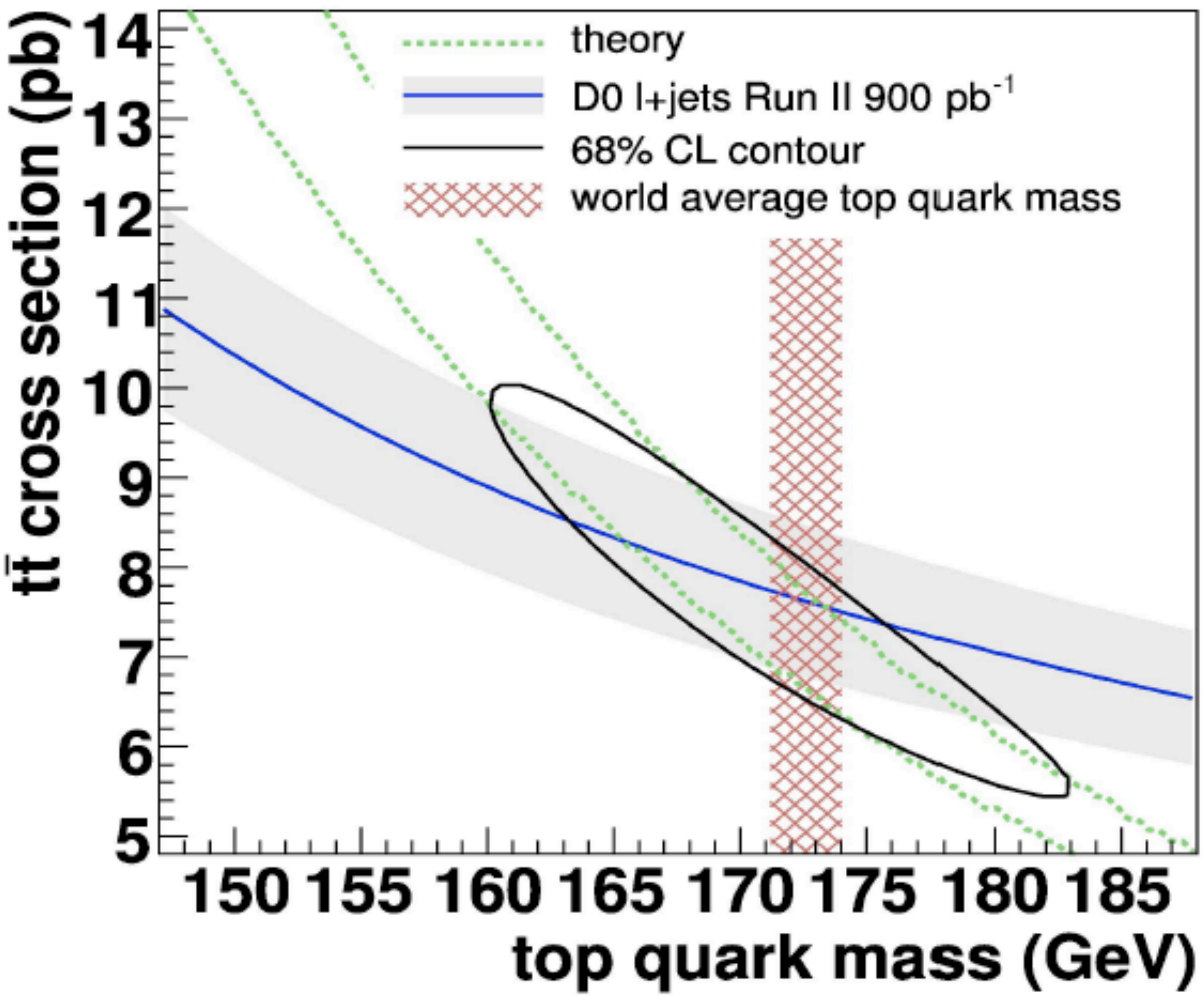
Processes of this kind are only calculated/simulated at tree level in pQCD

Presently no higher-order calculation relates a kinematical distribution used for top mass extraction to the mass parameter in the QCD lagrangian

We are therefore measuring a <u>leading order</u> pole mass

The top mass: a NLO extraction

Actually, there is an observable, dependent on the top mass, calculated at higher order in pQCD: the total ttbar production cross section



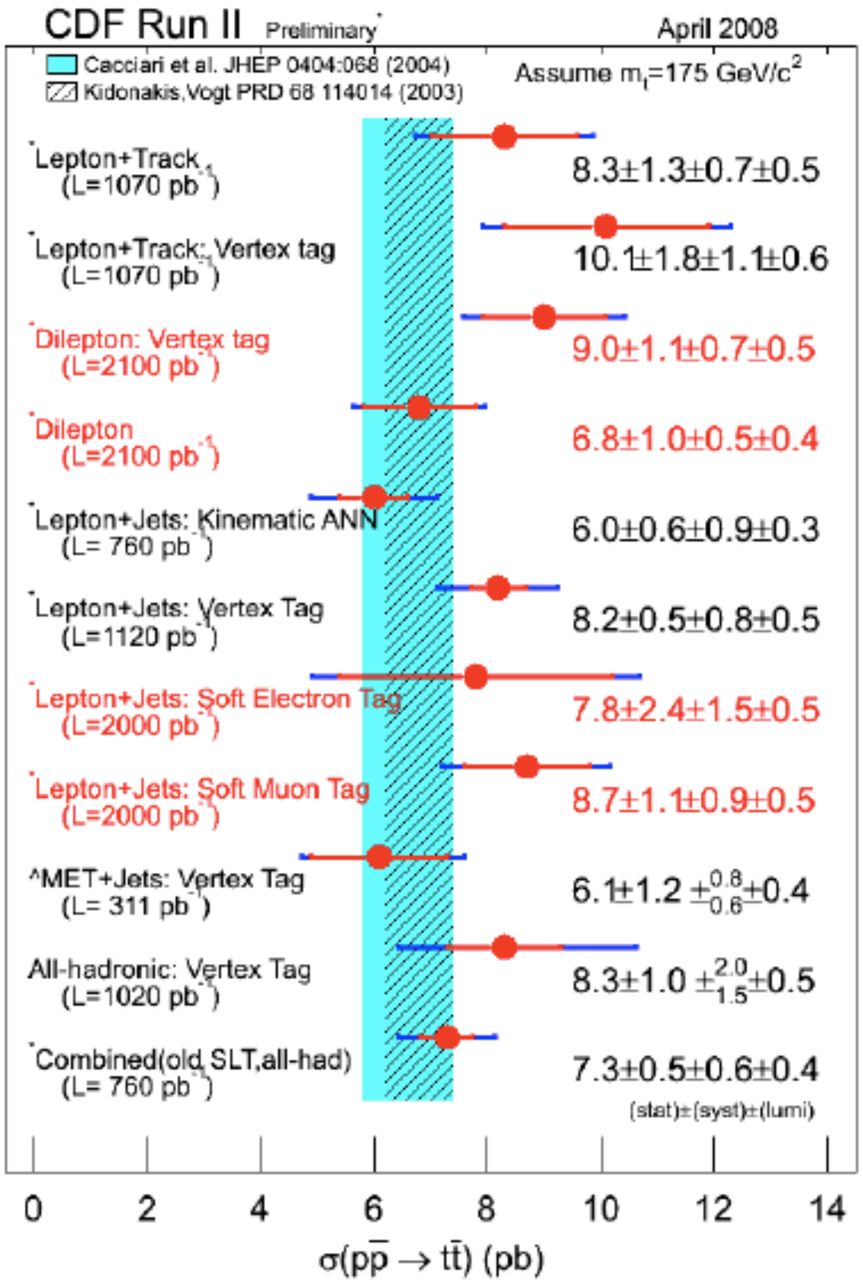
Example of extraction by D0:

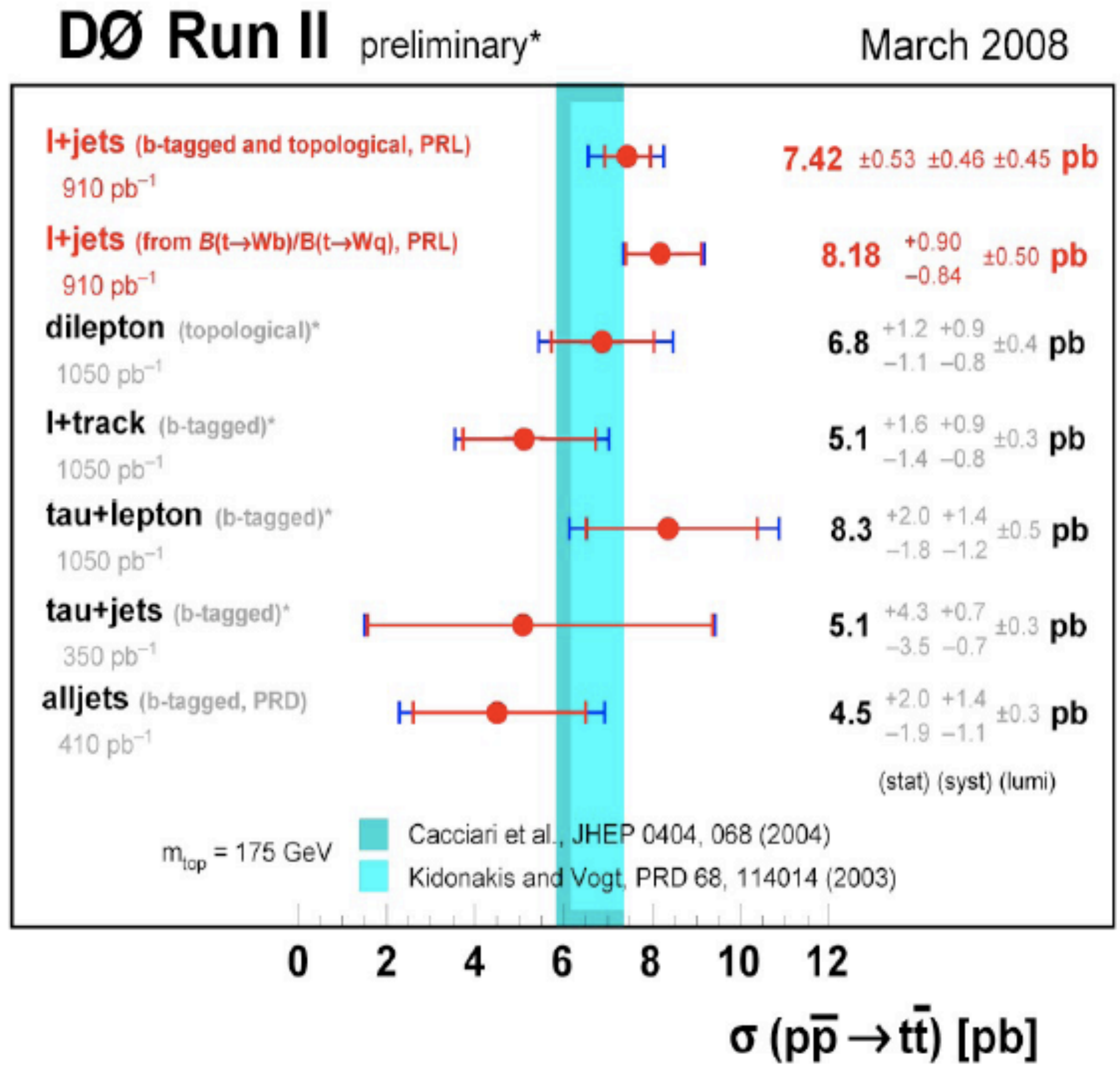
$$m_t = 170 \pm 7 \text{ GeV}$$

- Fairly large uncertainty, but
- compatible with kinematic measurements, and
- we know exactly what we are measuring.

Might become competitive with NNLO calculation and better measurement

ttbar cross section at the Tevatron





Overall uncertainty ~ 10%

A.Castro's and V. Sharyy's talks at Top 2008

Top quark perspectives at the LHC

LHC is a top factory:

8 million ttbar pairs at L = 10 fb^{-1} /year

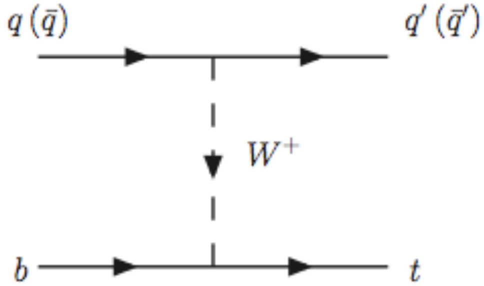
Unfortunately, it's also a background factory…. :-(

The expectations for mass and cross section measurements are therefore not significantly better than already achieved at the Tevatron:

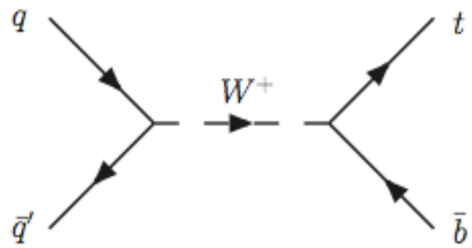
$$\Delta m_t \simeq 1 \text{ GeV} \qquad \frac{\Delta\sigma_{t\bar{t}}}{\sigma_{t\bar{t}}} \simeq 5 - 10\%$$

Single top

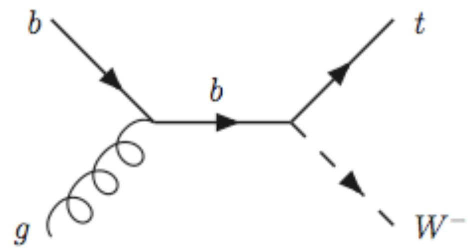
[See Z. Sullivan's lecture at CTEQ '07]

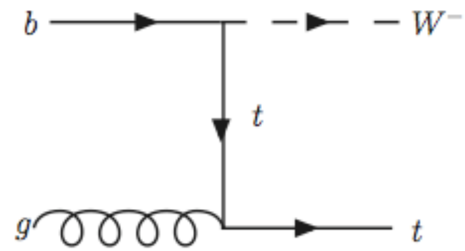


t-channel



s-channel





tW associated production

Essentially electroweak processes: proportional to (and therefore probe of) $|V_{tb}|^2$

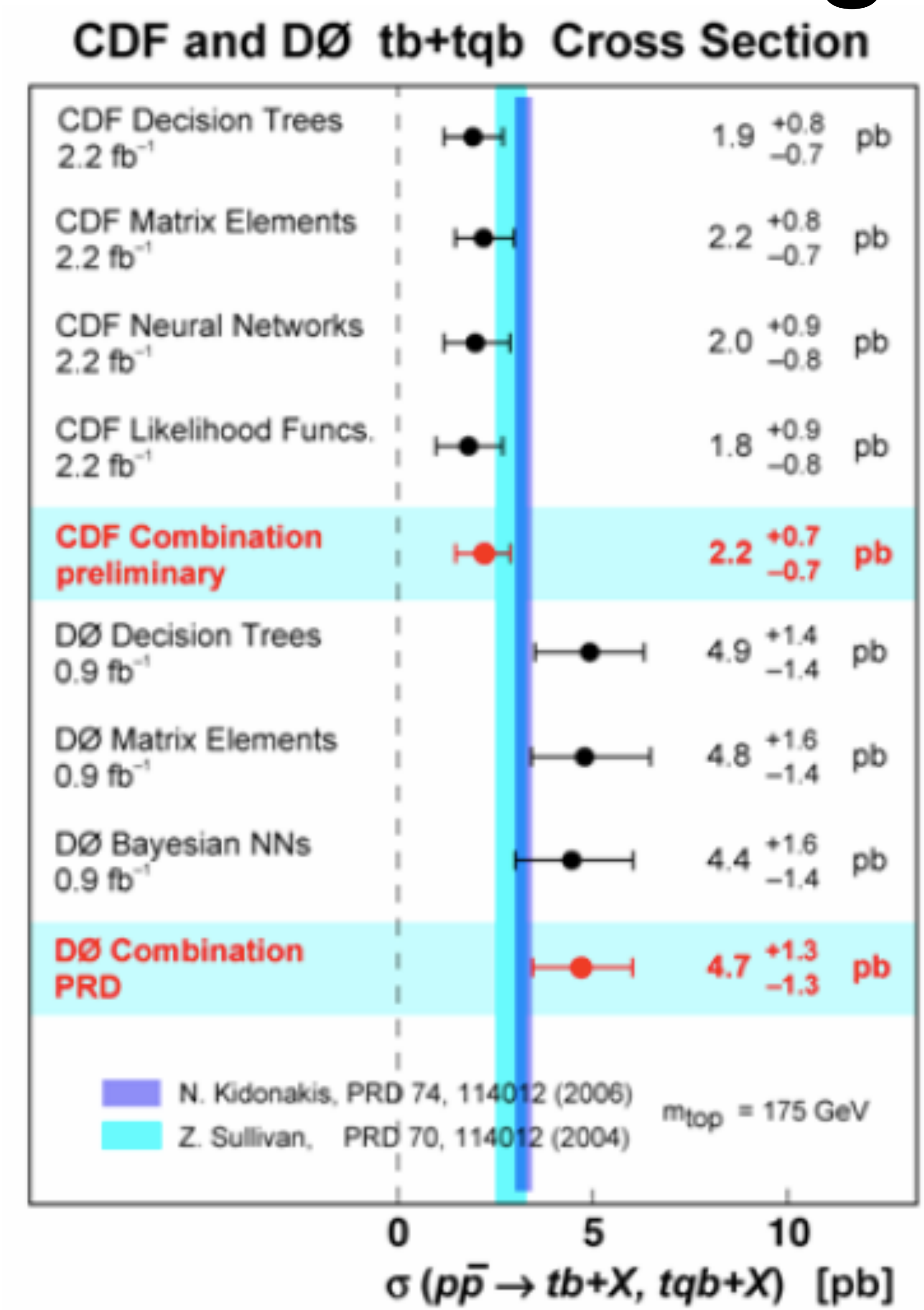
Moreover, source of highly polarized top quarks:
investigations of charged weak current interactions possible

Predicted cross sections (NLO) at the Tevatron:

t-channel	s-channel	tW associated production
2.0 ± 0.2 pb	0.9 ± 0.1 pb	~ 0.1 pb

Cross section not much smaller than ttbar, but measurement more challenging because backgrounds are larger

Single top



Measurements compatible with predictions (~ 3 pb), but still large uncertainties

J. Lueck and S.Jabeen talks at Top 2008

Conclusions

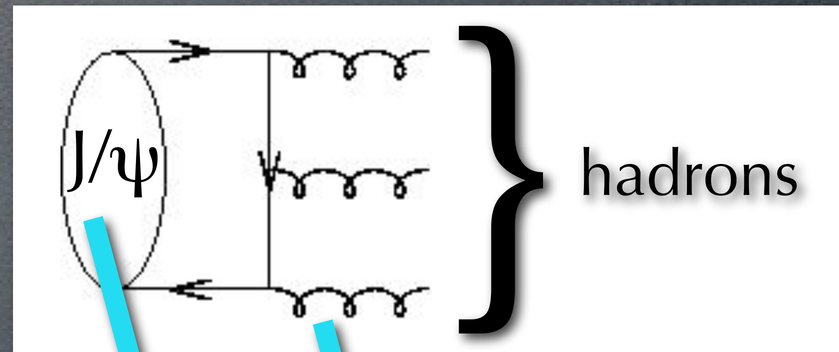
- Heavy quarks are nice to pQCD: large mass means smaller running coupling and collinear safety
- Charm and bottom hadronise, but the effect tends to be small in sufficiently inclusive observables: predictivity is maintained
- Top behaves essentially as an electroweak particle
- A number of tools which have recently appeared for studying today's (and tomorrow's) top physics: ALPGEN, MC@NLO, POWHEG, MADGRAPH, ….. , without forgetting the evergreen PYTHIA and HERWIG

Extra material

The J/ψ width

If J/ψ is produced in the interaction of an electron and a positron via a photon it must therefore have the **same quantum numbers as the photon: $J^P = 1^-$**

If we assume that its decay into hadrons goes via gluons, the Landau-Yang theorem (a vector particle cannot decay into two vector states) implies there must be at least **three of them** in the final state



We write the decay width as: $\Gamma({}^3S_1 \rightarrow 3 \text{ gluons}) = |R(0)|^2 |M(q\bar{q} \rightarrow 3 \text{ gluons})|^2$

Probability of finding the two quarks at the same point

annihilation probability at rest

We now need the **tools** to perform the calculations of the two terms. We shall use a **Coulomb approximation** for the first term and the **QCD Feynman rules** for the second

The J/ψ width

Coulomb potential: $V(r) \sim -\frac{4}{3}\frac{\alpha_S}{r}$

Solving the Schroedinger equation we find $|R(0)|^2 = \frac{4}{(\text{Bohr radius})^3} = 4\left(\frac{4}{3}\alpha_S\right)^3\left(\frac{m}{2}\right)^3$

Colour factors

The QCD probability for annihilation into 3 gluons will also be proportional to the cube of the strong coupling:

$$|M(q\bar{q} \to 3 \text{ gluons})|^2 = \frac{\alpha_S^3}{m^2}\left(\frac{5}{18}\right)\frac{4(\pi^2-9)}{9\pi}$$

Finally: $\boxed{\Gamma(^3S_1 \to 3 \text{ gluons}) \propto \alpha_S^6}$

The strong coupling runs with the scale. At what scale should I take it?

The renormalization group fixes it: $\Gamma(Q, g, \mu) = \Gamma(Q, \bar{g}(Q), Q)$

so that $\Gamma(^3S_1 \to 3 \text{ gluons}) \propto [\alpha_S(4m^2)]^6$

In 1974, however, we had no measurement for the strong coupling at a scale around 3 GeV. We did not even know if such a perturbative coupling existed!

The J/ψ width

Two options for checking the consistency of the picture

1. - Try to rescale a lower energy decay width

From $\Gamma(\phi \to 3\pi) \simeq 600\ \text{keV}$ one can extract $\alpha_S((1\ \text{GeV})^2) \simeq 0.53$

Asymptotic freedom scales this to $\alpha_S((3\ \text{GeV})^2) \simeq 0.29$

$$\Gamma(J/\psi \to \text{hadrons}) = \frac{3}{2}\frac{M_{J/\psi}}{M_\phi}\left(\frac{\alpha_S(M^2_{J/\psi})}{\alpha_S(M^2_\phi)}\right)^6 \Gamma(\phi \to 3\pi) \simeq 73\ \text{keV}$$

OK!

2. - Use leptonic width to eliminate wavefunction and extract value of strong coupling

From $\Gamma(J/\psi \to \text{leptons}) = |R(0)|^2 |M(q\bar{q} \to e^+e^-)|^2 = \frac{1}{m^2}\left(\frac{2}{3}\alpha_{em}\right)^2 |R(0)|^2 \simeq 3\ \text{keV}$

and $\frac{\Gamma(J/\psi \to \text{leptons})}{\Gamma(J/\psi \to \text{hadrons})} = \frac{18\pi\alpha_{em}^2}{5(\pi^2-9)\alpha_S^3} \simeq 0.04$

we get $\alpha_S((3\ \text{GeV})^2) \simeq 0.26$ OK!

Good consistency between strong coupling values. Good estimate of hadronic width.