Introduction to Monte Carlo Event Generators

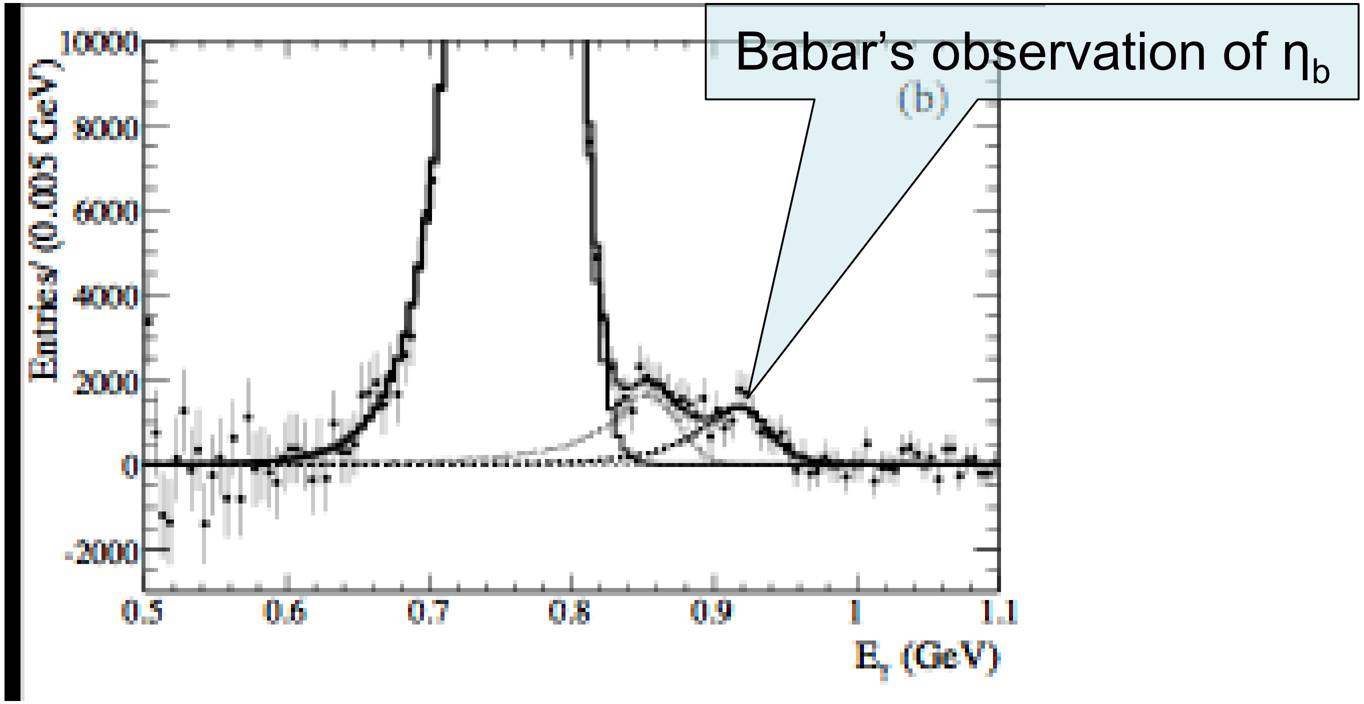
Mike Seymour
University of Manchester
& CERN

CTEQ–MCnet Summer School

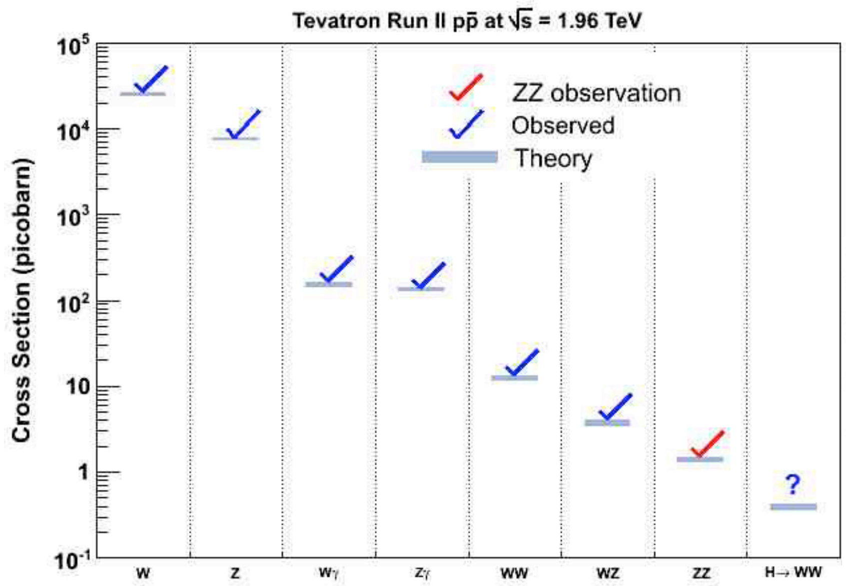
August 8th – 16th 2008

http://www.cteq-mcnet.org/

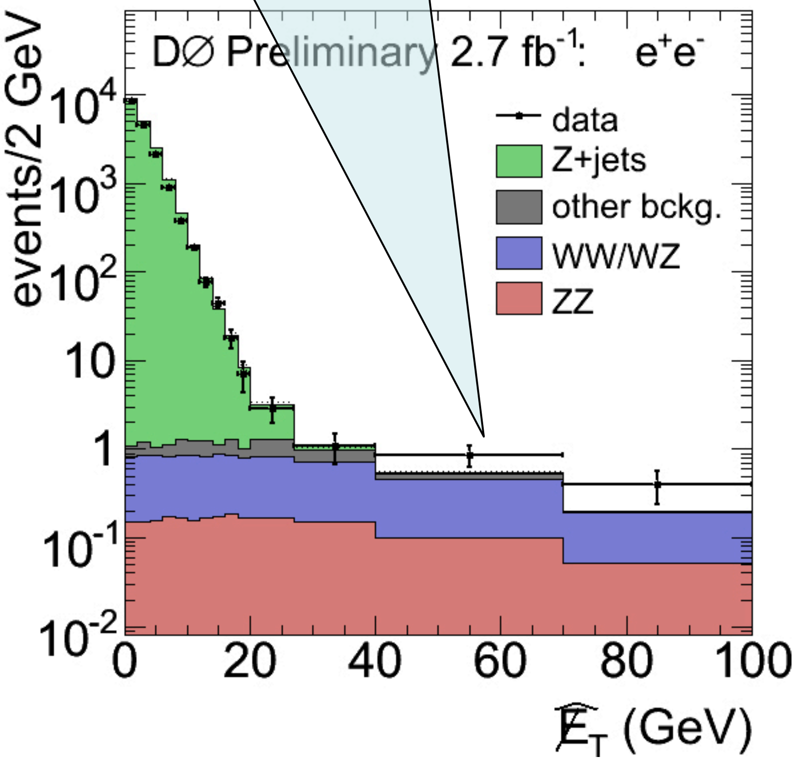
Overview and Motivation



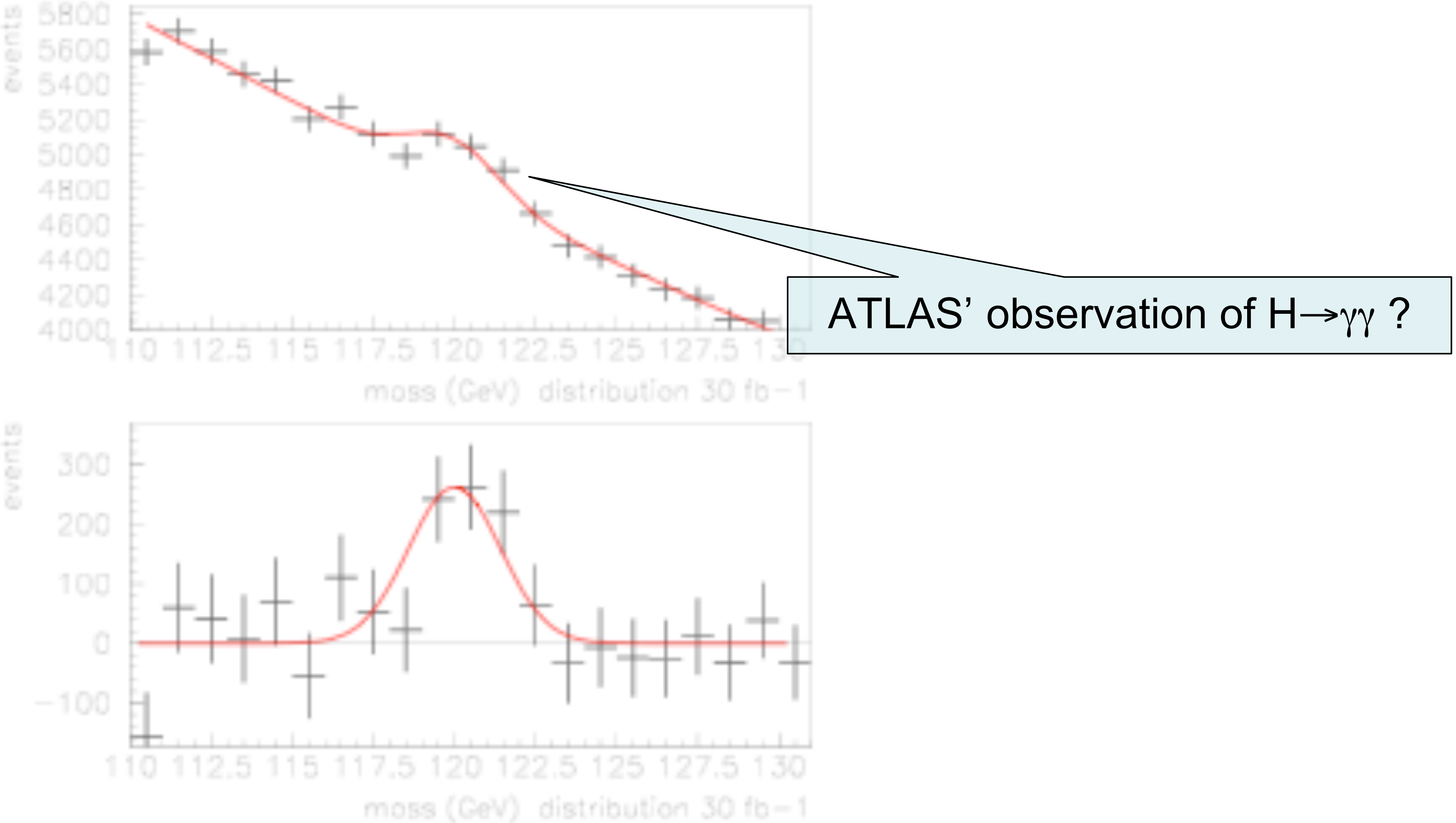
Overview and Motivation



D0's observation of ZZ production



Overview and Motivation

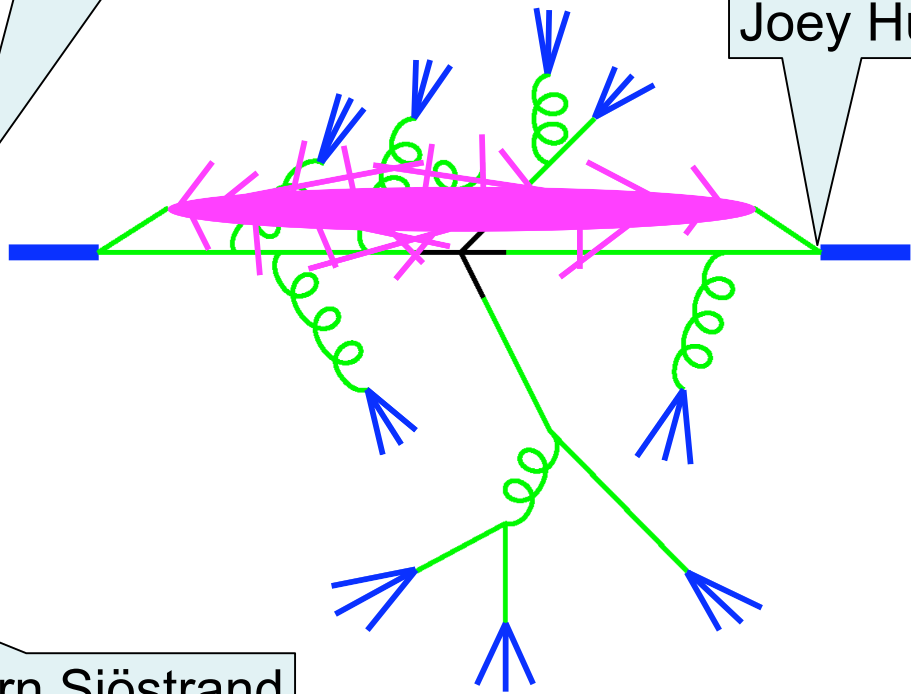


Structure of LHC Events

Leif Lönnblad

Joey Huston

1. Hard process
2. Parton shower
3. Hadronization
4. Underlying event



Torbjörn Sjöstrand

- Marie Curie Research Training Network
- for Monte Carlo event generator
  - development
  - validation and tuning
- Approved for four years from 1st Jan 2007

MCnet

CERN

Mike Seymour
Alberto Ribon
Peter Skands
Lars Sonnenschein
Andrzej Siodmok

Durham (incl. Cambridge)

Peter Richardson
Frank Krauss
Bryan Webber
Seyi Latunde-Dada
Sasha Sherstnev
Jonathan Tully
Tanju Gleisberg
Stefan Hoeche
Steffen Schumann
Jan Winter
Martyn Gigg
David Grellscheid
Keith Hamilton
Frank Siegert
Marek Schoenherr

Userlink

Jon Butterworth
Andy Buckley
Paula Eerola
Beate Heinemann
Ian Hinchliffe
James Monk
Filip Moortgat
Steve Mrenna
Paolo Nason
Emily Nurse
Patrick Robbe
Peter Skands
Ben Waugh
Mike Whalley
Matthew Wing

Lund

Leif Lönnblad
Torbjörn Sjöstrand
Hendrik Hoeth
Nils Lavesson
Richard Corke
Christopher Flensburg

Karlsruhe (incl. Krakow)

Stefan Gieseke
Simon Plaetzer
Manuel Baehr
Luca D'Errico

External Advisors

Mark Gibbs
Paolo Nason
Filip Moortgat
Beate Heinemann

MCnet

CEDAR

Jon Butterworth
Lars Sonnenschein
Andy Buckley
James Monk
Emily Nurse
Ben Waugh
Mike Whalley
Paula Eerola
Beate Heinemann
Ian Hinchliffe
Filip Moortgat
Paolo Nason
Patrick Robbe
Matthew Wing
Hendrik Hoeth

Herwig

Peter Richardson
Bryan Webber
Mike Seymour
Alberto Ribon
Stefan Gieseke
Andrzej Siodmok
Martyn Gigg
David Grellscheid
Keith Hamilton
Seyi Latunde-Dada
Sasha Sherstnev
Jonathan Tully
Simon Plaetzer
Manuel Baehr
Luca D’Errico

PYTHIA

Torbjörn Sjöstrand
Steve Mrenna
Peter Skands
Richard Corke

ThePEG Ariadne

Leif Lönnblad
Nils Lavesson
Christopher Flensburg

SHERPA

Frank Krauss
Tanju Gleisberg
Stefan Hoeche
Steffen Schumann
Jan Winter
Frank Siegert
Marek Schoenherr

MCnet objectives

Training:

- To train a large section of the user base in the physics and techniques of event generators
- To train the next generation of event generator developers

Through Research:

- To develop the next generation of event generators intended for use throughout the lifetimes of the LHC and ILC experiments
- To play a central role in the analysis of early LHC data and the discovery of new particles and interactions there
- To extract the maximum potential from existing data to constrain the modeling of the data from the LHC and other future experiments.

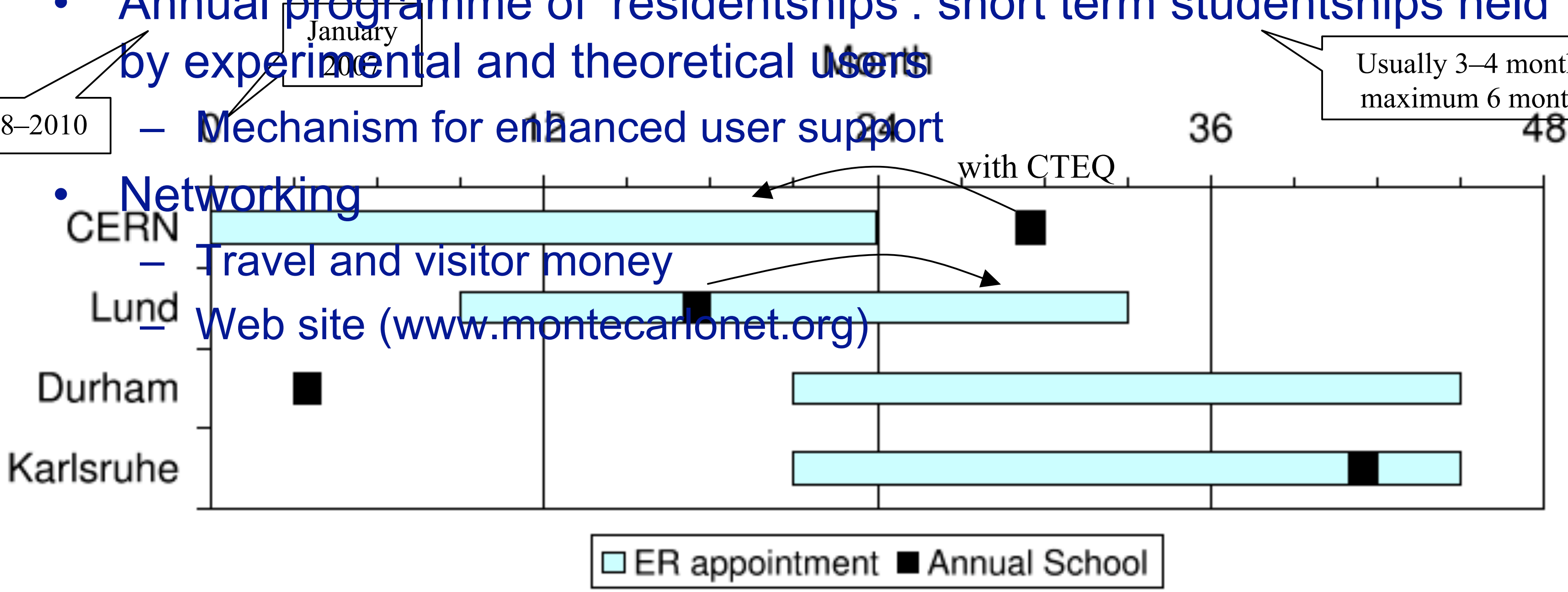
MCnet main activities

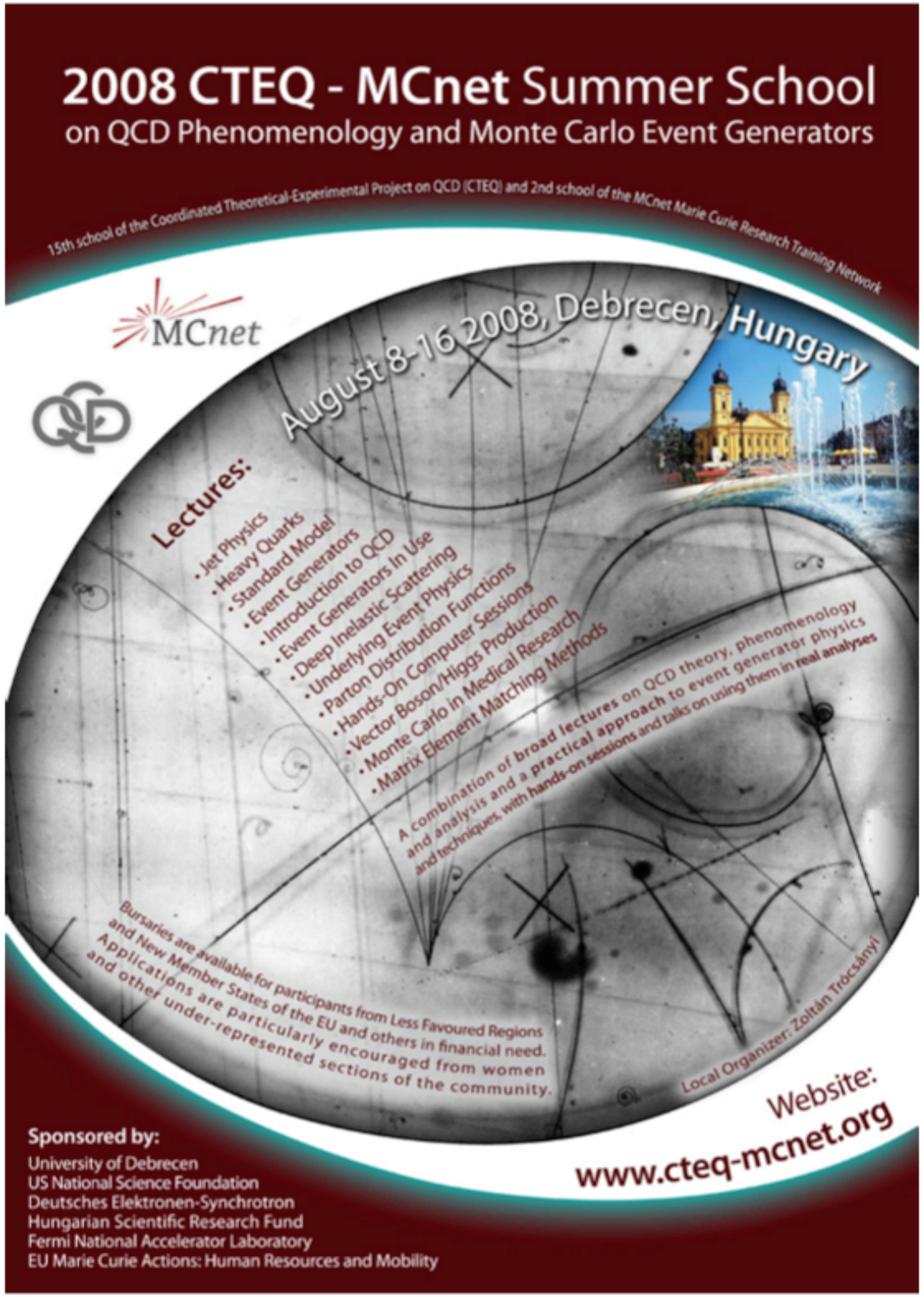
- Four postdoc positions
- Two joint studentships (Karlsruhe–Durham, Durham–UCL)
- Annual School
- Two annual meetings (Januarys @ CERN, summer with school)
- Annual programme of ‘residentships’: short term studentships held by experimental and theoretical users
  - Mechanism for enhanced user support

in 2008–2010

Usually 3–4 months, maximum 6 months

- Networking
  - Travel and visitor money
  - Web site (www.montecarlonet.org)





MCnet opportunities 2008:

- CTEQ-MCnet school
- Short-term studentships:

for th. and exptl. students to spend 3-6 months with MC authors in:

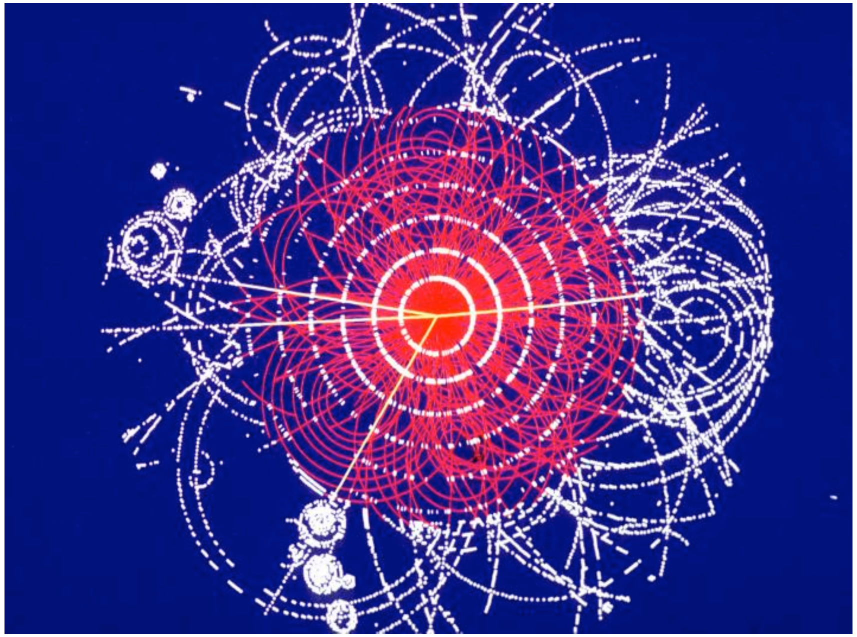
- CERN
- Durham/Cambridge
- Karlsruhe
- Lund
- UCL

on a project of their choice

- Next closing date:
    - October 6th

Introduction to Monte Carlo Event Generator Physics & Techniques

- Basic principles
- LHC event generation
- Parton showers
- Hadronization
- Underlying Events
- Practicalities
- Practical sessions



Mike Seymour

Intro to Monte Carlo Generators

1. Basic principles
2. Parton showers
3. Hadronization
4. Introduction to the MCnet Monte Carlo Event Generator projects and practical exercises

   (NB: 5 lectures in 4 chapters!)

Lecture1: Basics

- The Monte Carlo concept
- Event generation
- Examples: particle production and decay
- Structure of an LHC event

Integrals as Averages

- Basis of all Monte Carlo methods: $$I = \int_{x_1}^{x_2} f(x)\,dx = (x_2 - x_1)\,\langle f(x)\rangle$$
- Draw N values from a uniform distribution: $$I \approx I_N \equiv (x_2 - x_1)\frac{1}{N}\sum_{i=1}^{N} f(x_i)$$
- Sum invariant under reordering: randomize
- Central limit theorem: $$I \approx I_N \pm \sqrt{V_N/N}$$

$$V = (x_2 - x_1)\int_{x_1}^{x_2}\big[f(x)\big]^2 dx - \Big[\int_{x_1}^{x_2} f(x)\,dx\Big]^2$$

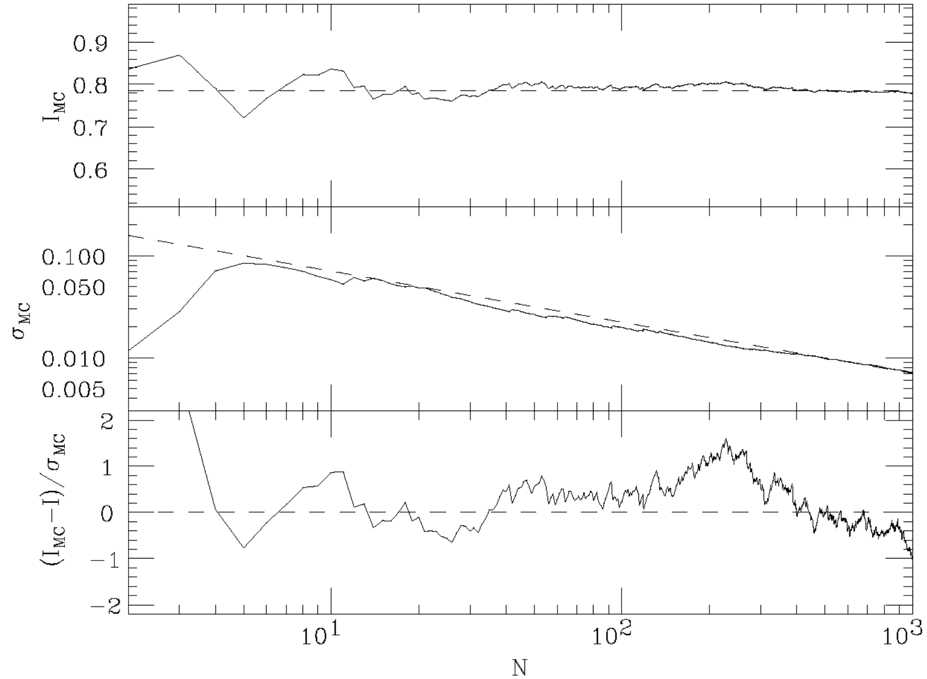
Convergence

- Monte Carlo integrals governed by Central Limit Theorem: error $\propto 1/\sqrt{N}$

cf trapezium rule $\propto 1/N^2$

Simpson's rule $\propto 1/N^4$

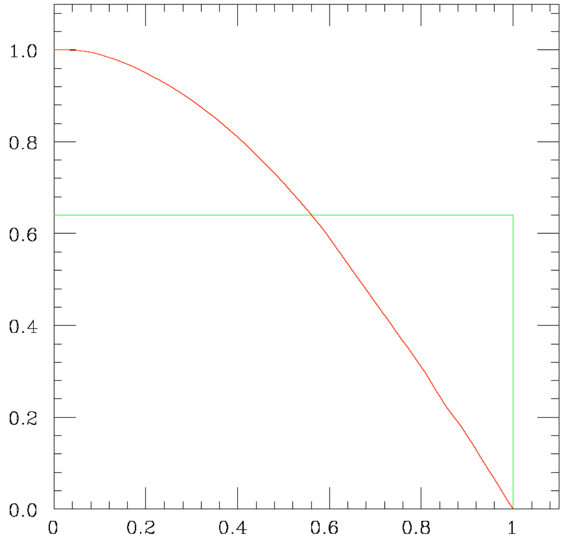
but only if derivatives exist and are finite, cf $\sqrt{1-x^2} \sim 1/N^{3/2}$

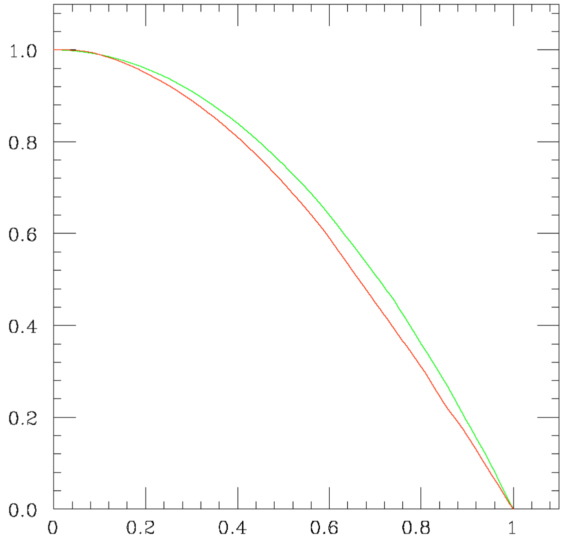


Importance Sampling

Convergence improved by putting more samples in region where function is largest.

Corresponds to a Jacobian transformation.





$$I = \int_0^1 dx \cos\tfrac{\pi}{2}x$$
$$= 0.637 \pm 0.308/\sqrt{N}$$

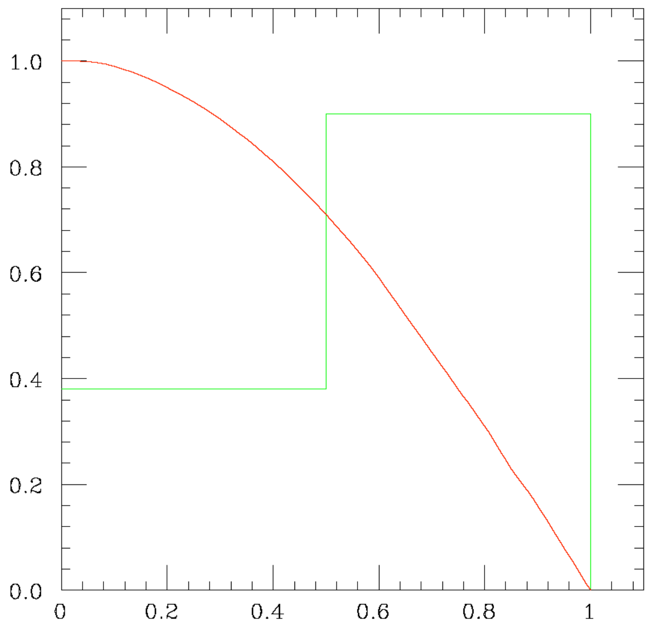
$$I = \int_0^1 dx(1-x^2)\,\frac{\cos\frac{\pi}{2}x}{1-x^2}$$
$$= \int d\rho\,\frac{\cos\frac{\pi}{2}x}{1-x^2}[x(\rho)]$$
$$= 0.637 \pm 0.032/\sqrt{N}$$

Stratified Sampling

Divide up integration region piecemeal and optimize to minimize total error.

Can be done automatically (eg VEGAS).

Never as good as Jacobian transformations.



N.B. Puts more points where rapidly varying, not necessarily where larger!

$$I = 0.637 \pm 0.147/\sqrt{N}$$

Multichannel Sampling

If $f(x) \leq g(x) = \sum_i g_i(x)$

where all g_i "nice" (but $g(x)$ not)

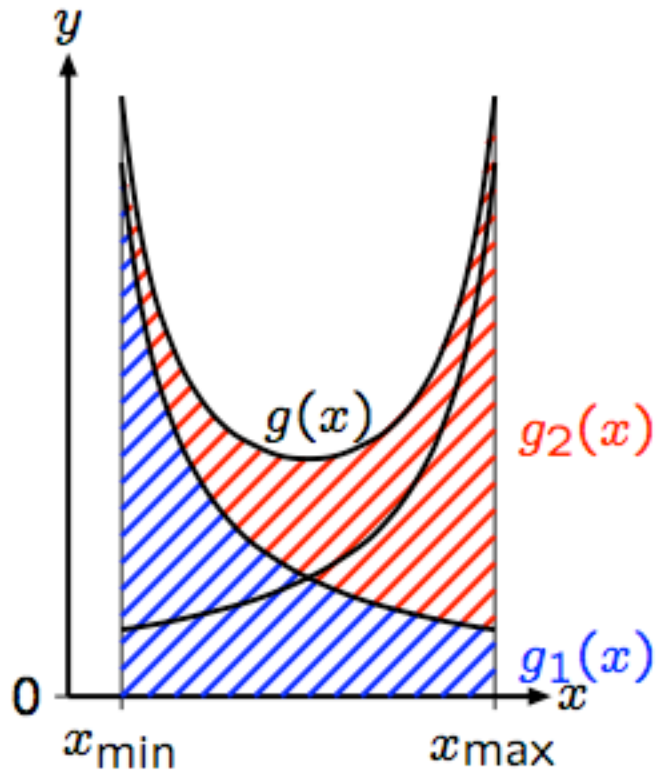
1) select i with relative probability

$$A_i = \int_{x_{min}}^{x_{max}} g_i(x') \mathrm{d}x'$$

2) select x according to $g_i(x)$

3) select $y = Rg(x) = R \sum g_i(x)$

4) while $y > f(x)$ cycle to 1)



Multi-dimensional Integration

- Formalism extends trivially to many dimensions
- Particle physics: very many dimensions,
  eg phase space = 3 dimensions per particles,
  LHC event ~ 250 hadrons.
- Monte Carlo error remains $\propto 1/\sqrt{N}$
- Trapezium rule $\propto 1/N^{2/d}$
- Simpson's rule $\propto 1/N^{4/d}$

Summary

Disadvantages of Monte Carlo:

- Slow convergence in few dimensions.

Advantages of Monte Carlo:

- Fast convergence in many dimensions.
- Arbitrarily complex integration regions (finite discontinuities not a problem).
- Few points needed to get first estimate (“feasibility limit”).
- Every additional point improves accuracy (“growth rate”).
- Easy error estimate.

Phase Space

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 \, d\Pi_n(\sqrt{s})$$

$$\Gamma = \frac{1}{2M} \int |\mathcal{M}|^2 \, d\Pi_n(M)$$

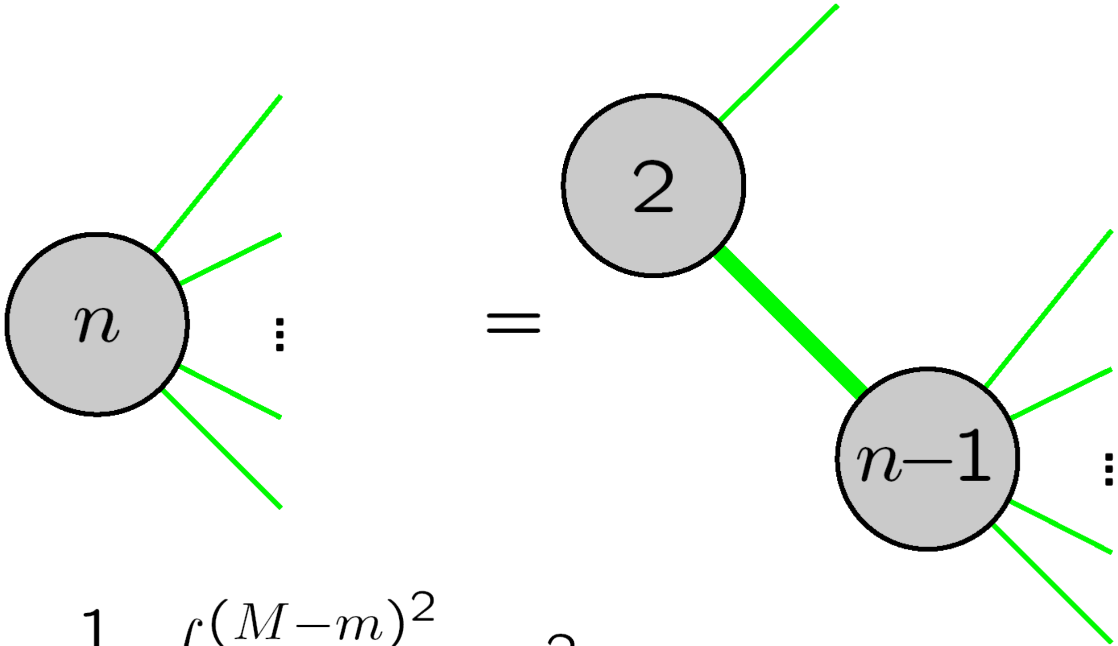
Phase space:

$$d\Pi_n(M) = \left[\prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 (2E_i)} \right] (2\pi)^4 \delta^{(4)} \left(p_0 - \sum_{i=1}^{n} p_i \right)$$

Two-body easy:

$$d\Pi_2(M) = \frac{1}{8\pi} \frac{2p}{M} \frac{d\Omega}{4\pi}$$

Other cases by recursive subdivision:



$$d\Pi_n(M) = \frac{1}{2\pi}\int_0^{(M-m)^2} dm_x^2 \, d\Pi_2(M) \, d\Pi_{n-1}(m_x)$$

Or by 'democratic' algorithms: RAMBO, MAMBO

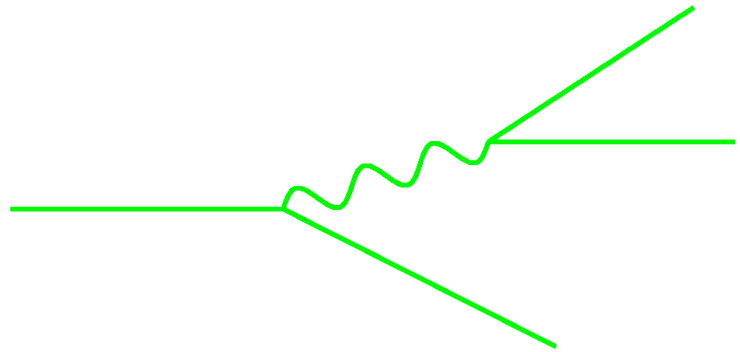
Can be better, but matrix elements rarely flat

$\rightarrow$ use knowledge of matrix elements/multichannel

Particle Decays

Simplest example

eg top quark decay:



$$|\mathcal{M}|^2 = \frac{1}{2}\left(\frac{8\pi\alpha}{\sin^2\theta_w}\right)^2 \frac{p_t\cdot p_\nu \; p_b\cdot p_\ell}{(m_W^2 - M_W^2)^2 + \Gamma_W^2 M_W^2}$$

$$\Gamma = \frac{1}{2M}\frac{1}{128\pi^3}\int |\mathcal{M}|^2 dm_W^2 \left(1 - \frac{m_W^2}{M^2}\right)\frac{d\Omega}{4\pi}\frac{d\Omega_W}{4\pi}$$

Breit-Wigner peak of W very strong: must be removed by Jacobian factor

Associated Distributions

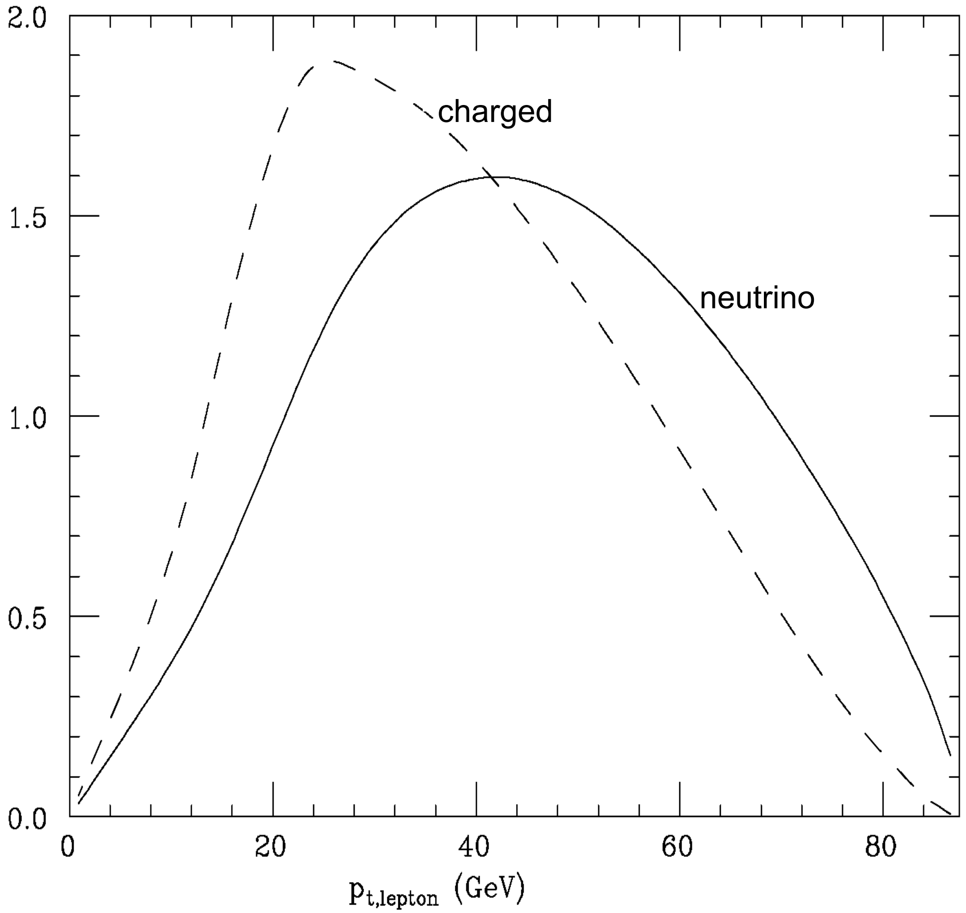
Big advantage of Monte Carlo integration:

simply histogram any associated quantities.

Almost any other technique requires new integration for each observable.

Can apply arbitrary cuts/smearing.

eg lepton momentum in top decays:



Cross Sections

Additional integrations over incoming parton densities:

$$\sigma(s) = \int_0^1 dx_1 f_1(x_1) \int_0^1 dx_2 f_2(x_2)\, \hat{\sigma}(x_1 x_2 s)$$
$$= \int_0^1 \frac{d\tau}{\tau} \hat{\sigma}(\tau s) \int_\tau^1 \frac{dx}{x}\, x f_1(x)\, \tfrac{\tau}{x} f_2(\tfrac{\tau}{x})$$

$\hat{\sigma}(\hat{s})$ can have strong peaks, eg Z Breit-Wigner: need Jacobian factors.

Hard to make process-independent

Leading Order Monte Carlo Calculations

Now have everything we need to make leading order cross section calculations and distributions

Can be largely automated…

- MADGRAPH
- GRACE
- COMPHEP
- AMAGIC++
- ALPGEN

But…

- Fixed parton/jet multiplicity
- No control of large logs
- Parton level → Need hadron level event generators

Event Generators

Up to here, only considered Monte Carlo as a numerical integration method.

If function being integrated is a probability density (positive definite), trivial to convert it to a simulation of physical process = an event generator.

Simple example: $\sigma = \int_0^1 \frac{d\sigma}{dx}\, dx$

Naive approach: 'events' x with 'weights' $d\sigma/dx$

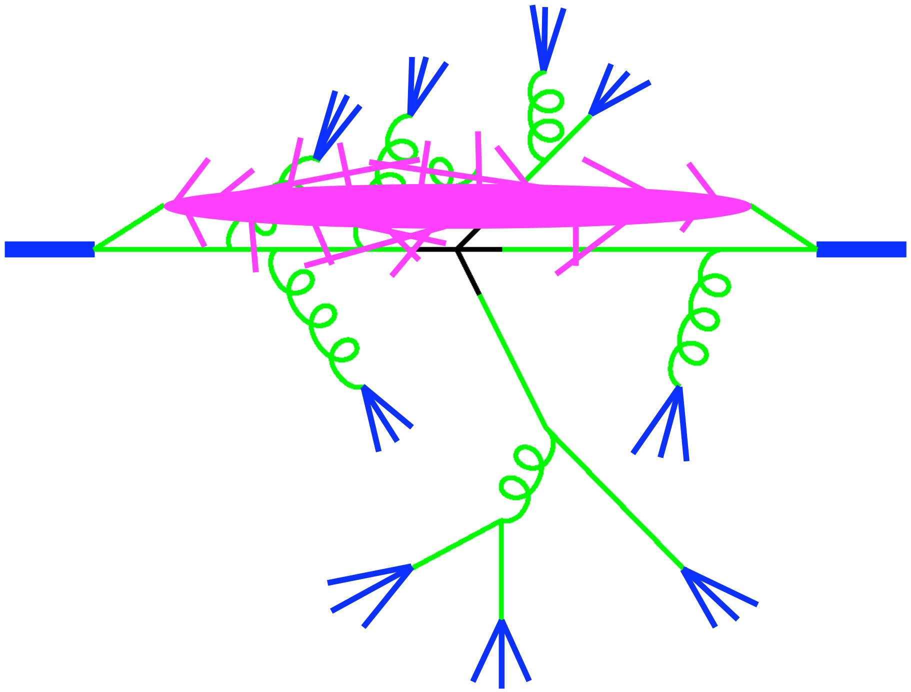
Can generate unweighted events by keeping them with probability $(d\sigma/dx)/(d\sigma/dx)_{\text{max}}$ give them all weight σ_{tot}

Happen with same frequency as in nature.

Efficiency: $\frac{(d\sigma/dx)_{\text{avge}}}{(d\sigma/dx)_{\text{max}}}$ = fraction of generated events kept.

Structure of LHC Events

1. Hard process
2. Parton shower
3. Hadronization
4. Underlying event



Monte Carlo Calculations of NLO QCD

Two separate divergent integrals:

$$\sigma_{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V$$

Must combine before numerical integration.

Jet definition could be arbitrarily complicated.

$$d\sigma^R = d\Pi_{m+1} \, |\mathcal{M}_{m+1}|^2 \, F^J_{m+1}(p_1, \ldots, p_{m+1})$$

How to combine without knowing F^J?

Two solutions:

phase space slicing and subtraction method.

Summary

- Monte Carlo is a very convenient numerical integration method.
- Well-suited to particle physics: difficult integrands, many dimensions.
- Integrand positive definite → event generator.
- Fully exclusive → treat particles exactly like in data.

→need to understand/model hadronic final state.