Matching and merging
Matrix Elements with Parton Showers
III

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Outline of Lectures

- Lecture I: Introduction, Tree-level ME, NLO, PS, ordering, basic strategies, ...
- Lecture II: Tree-level ME merging with PS, CKKW(-L), Pseudo Shower, MLM, e^+e^- comparison, ...
- Lecture III: ME+PS merging in *pp*, NLO matching with PS, MC@NLO, POWHEG, NL^3, ...

Outline

ME+PS merging in hadronic collisions

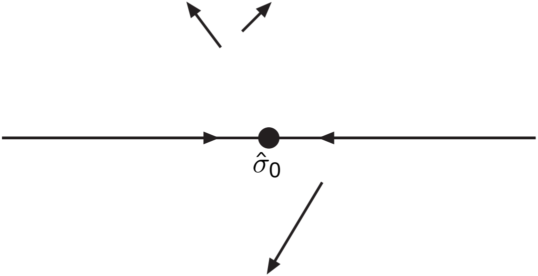
The main difference here is that we have incoming partons

We have to worry about

- parton densities
- initial-state showers
- algorithm to construct scales (and intermediate states).

(Will assume W+jets production throughout,
but it can easily be generalized.)

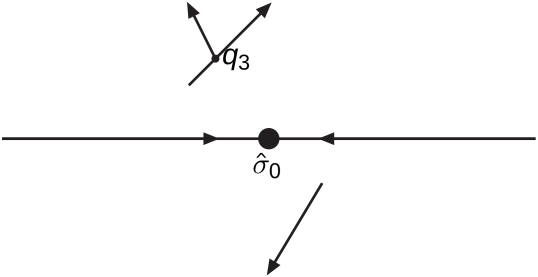
Longitudinally invariant $k_\perp$-algorithm



$$d_{ij} = \min(k_{\perp i}^2, k_{\perp j}^2)\left[\Delta\eta_{ij}^2 + \Delta\phi_{ij}^2\right]$$

$$d_i = k_{\perp i}$$

Longitudinally invariant $k_\perp$-algorithm



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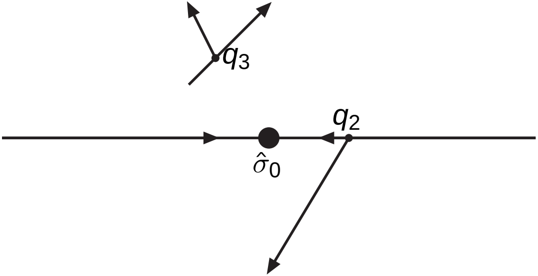
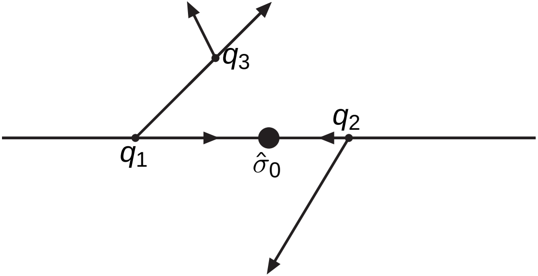
Longitudinally invariant $k_\perp$-algorithm



$$d_{ij} = \min(k_{\perp i}^2, k_{\perp j}^2) \left[\Delta\eta_{ij}^2 + \Delta\phi_{ij}^2 \right]$$

$$d_i = k_{\perp i}$$

Longitudinally invariant $k_\perp$-algorithm



$$d_{ij} = \min(k_{\perp i}^2, k_{\perp j}^2)\left[\Delta\eta_{ij}^2 + \Delta\phi_{ij}^2\right]$$

$$d_i = k_{\perp i}$$

The leading order W cross section:

$$d\sigma_{pp\to\mathrm{W}} = \sum_{\mathrm{q},\bar{\mathrm{q}}'} xf_{\mathrm{q}}(x_+, m_W) xf_{\bar{\mathrm{q}}'}(x_-, m_W)\hat{\sigma}_{\mathrm{q}\bar{\mathrm{q}}'\to\mathrm{W}}(sx_+x_-)\frac{dx_+}{x_+}\frac{dx_-}{x_-}$$

Making one step ($\mathrm{g}\to\mathrm{q}$) backward in the initial-state shower

$$dP(\rho, z) = \frac{\alpha_{\mathrm{s}}}{2\pi} P_{\mathrm{g}\to\mathrm{q}}(z)\frac{xf_{\mathrm{g}}(\frac{x_+}{z}, \rho)}{xf_{\mathrm{q}}(x_+, \rho)}\Delta_{S_0}(\rho_{\max}, \rho)\frac{d\rho}{\rho}dz,$$

Comparing with the cross section for getting the same thing from the matrix element:

$$d\sigma_{pp\to \mathrm{W\bar{q}}} = \sum_{\mathrm{\bar{q}},\mathrm{\bar{q}}'} x f_{\mathrm{g}}(\frac{x_+}{z},\mu) x f_{\mathrm{\bar{q}}'}(x_-,\mu)\hat{\sigma}_{\mathrm{g\bar{q}}'\to \mathrm{W\bar{q}}}(s\frac{x_+}{z}x_-,z,\rho)\frac{dx_+}{x_+}\frac{dx_-}{x_-}d\rho dz$$

And we identify

$$\frac{\hat{\sigma}_{\mathrm{g\bar{q}}'\to \mathrm{W\bar{q}}}(\frac{x_+}{z}x_- S,z,\rho)}{\hat{\sigma}_{\mathrm{q\bar{q}}'\to \mathrm{W}}(sx_+x_-)}d\rho dz$$

with

$$\frac{\alpha_{\mathrm{s}}}{2\pi}P_{\mathrm{g}\to\mathrm{q}}(z)\frac{d\rho}{\rho}dz,$$

CKKW-L

In CKKW-L, the way to handle this is to construction scales and intermediate states, using the "inverse-PS" algorithm, taking care to separate initial- from final-state emission.

Then reweight a produced $n-$parton event with

$$\frac{xf_{p_0}(x_{+0}, m_W) \cdot xf_{q_0}(x_{-0}, m_W)}{xf_{p_n}(x_{+n}, \mu) \cdot xf_{q_n}(x_{-n}, \mu)}$$

where the partons p_0 and q_0 and their energy fractions x_{+0} and x_0- are obtained from the constructed S_0 state.

Then for each constructed state, S_i, the event is weighted by the running α_s, the no-emission probability, $\Delta_{S_i}(q_i, q_{i+1})$, using the Sudakov veto algorithm, and a PDF ratio

$$\frac{xf_{p_{i+1}}(x_{+(i+1)}, q_{i+1}) \cdot xf_{q_{i+1}}(x_{-(i+1)}, q_{i+1})}{xf_{p_i}(x_{+i}, q_{i+1}) \cdot xf_{q_i}(x_{-i}, q_{i+1})}$$

In this way we get exactly the same Sudakov form factors and PDF ratios as in the shower.

Note however that the total cross section is forced to be the LO one (as in the original shower).

CKKW

Here we use the boost-invariant $k_\perp$-algorithm to construct the emission scales.

In each step we either cluster two final-state partons together, or cluster one parton to the beam, corresponding to an final- and initial-state splitting respectively.

The reweighting of α_s and analytical Sudakov form factors works in the same way as for e^+e^-.

No PDF weighting is necessary!

This is because the Sudakov form factors are not proper no-emission probabilities:

$$P_{\mathrm{no-emission}} = \frac{xf_p(x,q_2)}{xf_p(x,q_1)} \frac{\Delta_p(q_1,\mu)}{\Delta_p(q_2,\mu)}$$

and all PDF ratios cancel!

Life becomes simpler and the total cross section becomes beyond LO (although not NLO).

The reason CKKL-L anyway insists on including the ratios is that the correspondence between the no-emission probability and the Sudakov form factors is only true for DGLAP evolution.

And CKKW-L was originally implemented for ARIADNE, which is not strictly DGLAP
(it includes a resummation of large logarithms of x).

In CKKW-L, the maximum scale on the shower applied to S_0 is the same as for the stand-alone shower. For a strict DGLAP shower this is typically m_W.

In ARIADNE and PYTHIA (in case the first emission is ME-reweighted), the maximum scale is given by the kinematical limit, e.g. $p_\perp < \sqrt{s}/2$.

In CKKW, the maximum scale for the vetoed shower from the S_n state, and for the Sudakov form factors, is typically taken to be $m_{\perp W}$.
I.e. no no-emission probability above $m_{\perp W}$, which is fine as long as there are no large logarithms of $\sqrt{s}/m_W \sim x$.

MLM

Again we use the boost-invariant $k_\perp$-algorithm to define the ME generation cutoff, μ_o, and the constructed emission scales.

The maximum scale in the shower is $m_{\perp W}$.

In the original (ALPGEN) implementation a cone algorithm is used to cluster the showered even. Here the minimum $E_\perp$ of the jets is set to $\mu > \mu_0$, and the R_{cone} is larger than the minimum ΔR in the ME generation.

A jet is matched to a parton if the $\Delta R_{\mathrm{jet,parton}} < 1.5 \times R_{\mathrm{cone}}$

Other MLM implementations

MADEVENT has another implementation which uses the $k_\perp$-algorithm throughout in much the same way as was described for $\mathrm{e}^+\mathrm{e}^-$.

HELAC is very close to ALPGEN.

Pseudo-Shower

Follows the modifications used in CKKW-L when going from e^+e^- to *pp*.

The W+jets benchmark

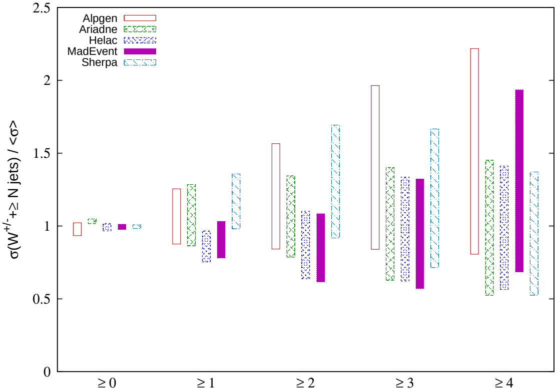
W+jets is a good benchmark to compare the algorithms in *pp*. What follows is a comparison between:

- Sherpa, implementing CKKW (with PYTHIA-like virtuality ordered shower).
- CKKW-L using ARIADNE.
- ALPGEN using MLM merging with HERWIG.
- MADEVENT using MLM merging with PYTHIA, virtuality-ordered shower.
- HELAC using MLM merging with PYTHIA, virtuality-ordered shower.

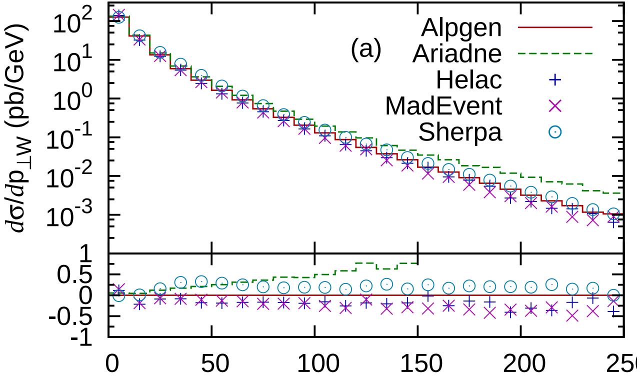
Notes:

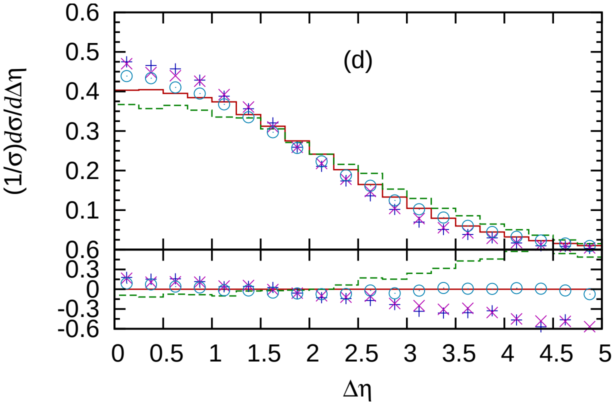
- Pseudo-Shower not included here (see refs.)
- ARIADNE is not DGLAP-based.
- PYTHIA with CKKW and two flavours of MLM.
- Same flavour of MLM with both PYTHIA and HERWIG.
- All use similar cuts on ME and up to 4 jets.

Tevatron energies



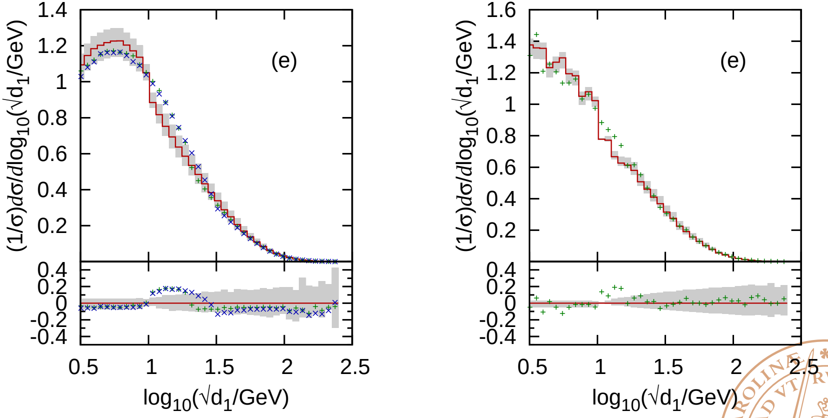
All procedures allowed different variations of parameters.
Merging scale and scale choice in α_s-reweighting in common.


(a)
Alpgen
Ariadne
Helac
MadEvent
Sherpa
$d\sigma/dp_{\perp W}$ (pb/GeV)
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
1
0.5
0
-0.5
-1
0
50
100
150
200
250



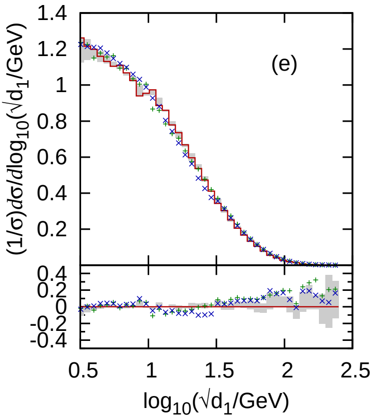
$$\Delta\eta = \left|\eta_W - \eta_{\rm jet}\right|$$

ALPGEN HELAC

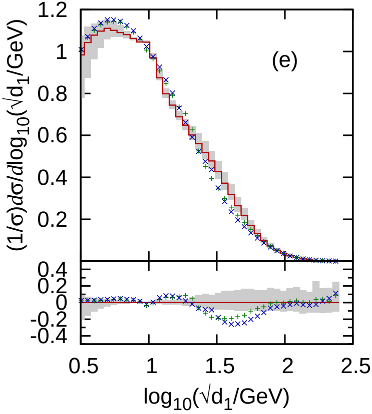


$\mu = 10, 20, 30$ GeV, ME: $\sqrt{d_1} > \mu$, PS: $\sqrt{d_1} < \mu$.

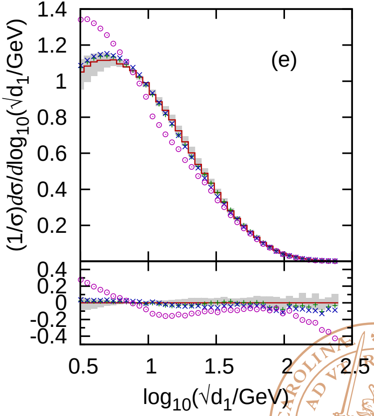
MadEvent


(e)
$(1/\sigma)d\sigma/d\log_{10}(\sqrt{d_1}/\text{GeV})$
$\log_{10}(\sqrt{d_1}/\text{GeV})$
1.4
1.2
1
0.8
0.6
0.4
0.2
0.4
0.2
0
-0.2
-0.4
0.5
1
1.5
2
2.5

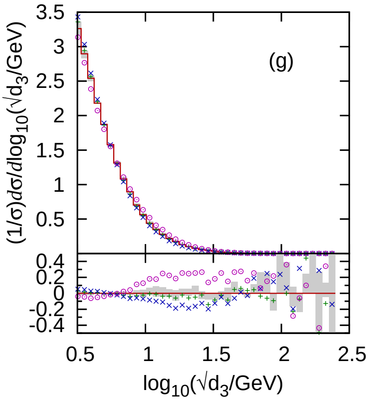
SHERPA



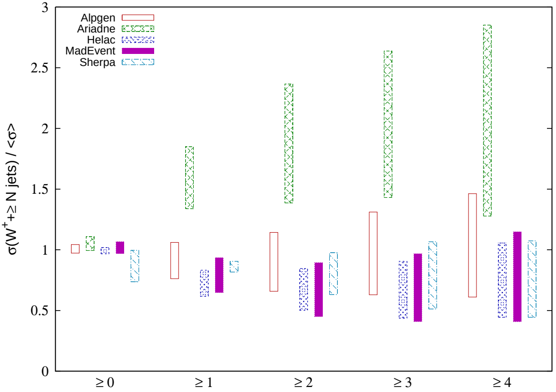
ARIADNE

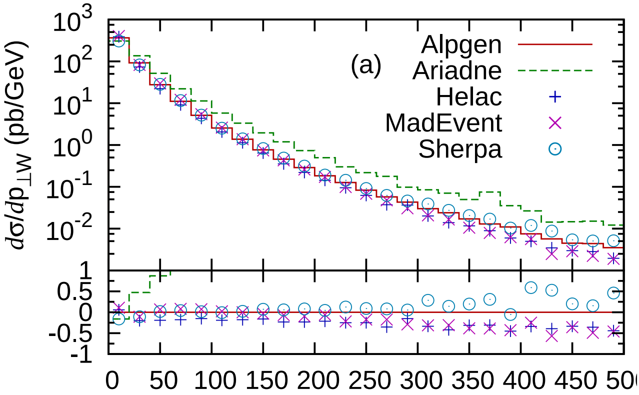


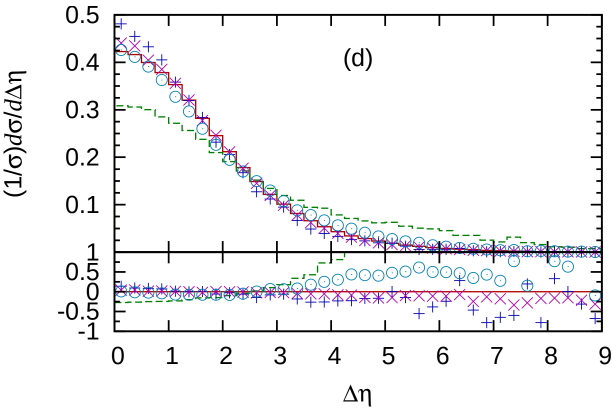
ARIADNE


(g)
(1/σ)dσ/dlog10(√d3/GeV)
3.5
3
2.5
2
1.5
1
0.5
0.4
0.2
0
-0.2
-0.4
0.5
1
1.5
2
2.5
log10(√d3/GeV)

LHC energies




10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
$d\sigma/dp_{\perp W}$ (pb/GeV)
(a)
Alpgen
Ariadne
Helac
MadEvent
Sherpa
1
0.5
0
-0.5
-1
0
50
100
150
200
250
300
350
400
450
500



$$\Delta\eta = \left|\eta_W - \eta_{\text{jet}}\right|$$

- Clear wiggles near cutoff μ.
- Far above μ, results are independent of μ.
- Different models differ, but within systematic uncertainties of each model.
- Largest difference found from ARIADNE (not DGLAP).

NLO matching

$$
\begin{aligned}
d\sigma_0 &= \left[C_0^{\mathrm{ME}}(\boldsymbol{p}_{1..m};\mu) + \alpha_s C_{0,1}^{\mathrm{PS}}(\boldsymbol{p}_{1..m};\mu) + \alpha_s^2 C_{0,2}^{\mathrm{PS}}(\boldsymbol{p}_{1..m};\mu) + \ldots\right] d\Phi_m \\
d\sigma_{+1}(\mu) &= \left[\alpha_s C_1^{\mathrm{ME}}(\boldsymbol{p}_{1..m},\boldsymbol{q}_1;\mu) + \alpha_s^2 C_{1,1}^{\mathrm{PS}}(\boldsymbol{p}_{1..m},\boldsymbol{q}_1;\mu) + \ldots\right] d\Phi_{m+1} \\
d\sigma_{+2}(\mu) &= \left[\alpha_s^2 C_2^{\mathrm{ME}}(\boldsymbol{p}_{1..m},\boldsymbol{q}_1,\boldsymbol{q}_2;\mu) + \alpha_s^3 C_{2,1}^{\mathrm{PS}}(\boldsymbol{p}_{1..m},\boldsymbol{q}_1,\boldsymbol{q}_2;\mu) + \ldots\right] d\Phi_{m+2} \\
&\vdots
\end{aligned}
$$

NLO matching

$$d\sigma_0 = \left[C_0^{\mathrm{ME}}(\boldsymbol{p}_{1..m};\mu) + \alpha_{\mathrm{s}} C_{0,1}^{\mathrm{ME}}(\boldsymbol{p}_{1..m};\mu) + \alpha_{\mathrm{s}}^2 C_{0,2}^{\mathrm{PS}}(\boldsymbol{p}_{1..m};\mu) + \dots\right] d\Phi_m$$

$$d\sigma_{+1}(\mu) = \left[\alpha_{\mathrm{s}} C_1^{\mathrm{ME}}(\boldsymbol{p}_{1..m},\boldsymbol{q}_1;\mu) + \alpha_{\mathrm{s}}^2 C_{1,1}^{\mathrm{ME}}(\boldsymbol{p}_{1..m},\boldsymbol{q}_1;\mu) + \dots\right] d\Phi_{m+1}$$

$$d\sigma_{+2}(\mu) = \left[\alpha_{\mathrm{s}}^2 C_2^{\mathrm{ME}}(\boldsymbol{p}_{1..m},\boldsymbol{q}_1,\boldsymbol{q}_2;\mu) + \alpha_{\mathrm{s}}^3 C_{2,1}^{\mathrm{ME}}(\boldsymbol{p}_{1..m},\boldsymbol{q}_1,\boldsymbol{q}_2;\mu) + \dots\right] d\Phi_{m+2}$$

$$\vdots$$

NLO matching

$$
\begin{aligned}
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d\sigma_{+1}(\mu) &= \left[\alpha_s C_1^{\mathrm{ME}}(\boldsymbol{p}_{1..m},\boldsymbol{q}_1;\mu) + \alpha_s^2 C_{1,1}^{\mathrm{PS}}(\boldsymbol{p}_{1..m},\boldsymbol{q}_1;\mu) + \ldots\right] d\Phi_{m+1} \\
d\sigma_{+2}(\mu) &= \left[\alpha_s^2 C_2^{\mathrm{PS}}(\boldsymbol{p}_{1..m},\boldsymbol{q}_1,\boldsymbol{q}_2;\mu) + \alpha_s^3 C_{2,1}^{\mathrm{PS}}(\boldsymbol{p}_{1..m},\boldsymbol{q}_1,\boldsymbol{q}_2;\mu) + \ldots\right] d\Phi_{m+2} \\
&\vdots
\end{aligned}
$$

MC@NLO

Based on NLO-subtraction method

$$
\begin{aligned}
d\sigma_0 &= \Big[C_0^{\mathrm{ME}}(\boldsymbol{p}_{1..m}) + \alpha_{\mathrm{s}} C_0^{\mathrm{loop}}(\boldsymbol{p}_{1..m}) \\
&\qquad + \alpha_{\mathrm{s}} \int d^3q' C_1^{\mathrm{CS}}(\boldsymbol{p}_{1..m}, \boldsymbol{q}') \Big] d\Phi_m \\
d\sigma_{+1} &= \alpha_{\mathrm{s}} \Big[C_1^{\mathrm{ME}}(\boldsymbol{p}_{1..m}, \boldsymbol{q}') - C_1^{\mathrm{CS}}(\boldsymbol{p}_{1..m}, \boldsymbol{q}') \Big] d\Phi_{m+1}
\end{aligned}
$$

$d\sigma^{\mathrm{NLO}} = d\sigma_0 + d\sigma_1$.

C_1^{CS} must reproduce the soft and collinear poles in C_1^{ME}.

MC@NLO

Based on NLO-subtraction method

$$
\begin{aligned}
d\sigma_0 &= \left[C_0^{\mathrm{ME}}(\boldsymbol{p}_{1..m}) + \alpha_s C_0^{\mathrm{loop}}(\boldsymbol{p}_{1..m}) \right. \\
&\qquad \left. + \alpha_s \int d^3q' C_1^{\mathrm{PS}}(\boldsymbol{p}_{1..m}, \boldsymbol{q}')\right] d\Phi_m \\
d\sigma_{+1} &= \alpha_s \left[C_1^{\mathrm{ME}}(\boldsymbol{p}_{1..m}, \boldsymbol{q}') - C_1^{\mathrm{PS}}(\boldsymbol{p}_{1..m}, \boldsymbol{q}')\right] d\Phi_{m+1}
\end{aligned}
$$

$d\sigma^{\mathrm{NLO}} = d\sigma_0 + d\sigma_1$.

C_1^{CS} must reproduce the soft and collinear poles in C_1^{ME}.

We can use the splitting function of any reasonable PS.

Note that calculating "$C_0^{\rm loop} + \int d^3q C_1^{\rm PS}$" is highly non-trivial.

Note that σ_1 is not a proper $+1$ parton cross section, most of this is integrated over and put into σ_0.

Now we can add a parton shower to get the no-parton cross section

$$
\begin{aligned}
d\sigma_0(\rho_c) &= \Big[C_0^{\mathrm{ME}}(\boldsymbol{p}_{1..m}) + \alpha_s C_0^{\mathrm{loop}}(\boldsymbol{p}_{1..m}) \\
&\qquad + \alpha_s \int_0^{\rho_0} d\rho \int \mathrm{d}^2 x C_1^{\mathrm{PS}}(\boldsymbol{p}_{1..m}, \rho, \boldsymbol{x}) \Big] d\Phi_m \\
&\times \quad \Delta_{S_0}(\rho_0, \rho_c)
\end{aligned}
$$

Note difference in integration region.

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\begin{aligned}
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& + \alpha_{\mathrm{s}} \int_0^{\rho_0} d\rho \int \mathrm{d}^2 x C_1^{\mathrm{PS}}(\boldsymbol{p}_{1..m}, \rho, \boldsymbol{x}) \Big] d\Phi_m \\
\times \quad & \left(1 - \alpha_{\mathrm{s}} \int_{\rho_c}^{\rho_0} d\rho \int \mathrm{d}^2 x \frac{C_1^{\mathrm{PS}}(\boldsymbol{p}_{1..m}, \rho, \boldsymbol{x})}{C_0^{\mathrm{ME}}(\boldsymbol{p}_{1..m})} + \mathcal{O}(\alpha_{\mathrm{s}}^2) \right)
\end{aligned}
$$

Note difference in integration region.

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$$\begin{aligned} d\sigma_0(\rho_c) &= \Big[C_0^{\mathrm{ME}}(\boldsymbol{p}_{1..m}) + \alpha_{\mathrm{s}} C_0^{\mathrm{loop}}(\boldsymbol{p}_{1..m}) \\ &\quad + \alpha_{\mathrm{s}} \int_0^{\rho_0} d\rho \int \mathrm{d}^2 x C_1^{\mathrm{PS}}(\boldsymbol{p}_{1..m}, \rho, \boldsymbol{x}) \Big] d\Phi_m \\ &\times \left(1 - \alpha_{\mathrm{s}} \int_{\rho_c}^{\rho_0} d\rho \int \mathrm{d}^2 x \frac{C_1^{\mathrm{PS}}(\boldsymbol{p}_{1..m}, \rho, \boldsymbol{x})}{C_0^{\mathrm{ME}}(\boldsymbol{p}_{1..m})} + \mathcal{O}(\alpha_{\mathrm{s}}^2) \right) \end{aligned}$$

Note difference in integration region.

We then get the one-parton cross section

$$
\begin{aligned}
d\sigma_{+1}(\rho_c) &= \alpha_s \left[C_1^{\mathrm{ME}}(\boldsymbol{p}_{1..m}, \rho, \boldsymbol{x}) - C_1^{\mathrm{PS}}(\boldsymbol{p}_{1..m}, \rho, \boldsymbol{x}) \right] \Delta_{S_1}(\rho, \rho_c) d\Phi_{m+1} \\
&+ \left[C_0^{\mathrm{ME}} + \alpha_s C_0^{\mathrm{loop}} + \alpha_s \int C_1^{\mathrm{PS}} \right] \times \alpha_s \frac{C_1^{\mathrm{PS}}(\boldsymbol{p}_{1..m}, \rho, \boldsymbol{x})}{C_0^{\mathrm{ME}}(\boldsymbol{p}_{1..m})} \\
&\times \Delta_{S_0}(\rho_0, \rho) \Delta_{S_1}(\rho, \rho_c) d\Phi_{m+1}
\end{aligned}
$$

To leading order we are left with $\alpha_s C_1^{\mathrm{ME}}$

We then get the one-parton cross section

$$\begin{aligned} d\sigma_{+1}(\rho_c) &= \alpha_s \left[C_1^{\mathrm{ME}}(\boldsymbol{p}_{1..m}, \rho, \boldsymbol{x}) - C_1^{\mathrm{PS}}(\boldsymbol{p}_{1..m}, \rho, \boldsymbol{x}) \right] \Delta_{S_1}(\rho, \rho_c) d\Phi_{m+1} \\ &+ \left[C_0^{\mathrm{ME}} + \alpha_s C_0^{\mathrm{loop}} + \alpha_s \int C_1^{\mathrm{PS}} \right] \times \alpha_s \frac{C_1^{\mathrm{PS}}(\boldsymbol{p}_{1..m}, \rho, \boldsymbol{x})}{C_0^{\mathrm{ME}}(\boldsymbol{p}_{1..m})} \\ &\times \Delta_{S_0}(\rho_0, \rho) \Delta_{S_1}(\rho, \rho_c) d\Phi_{m+1} \end{aligned}$$

To leading order we are left with $\alpha_s C_1^{\mathrm{ME}}$

We then get the one-parton cross section

$$
\begin{aligned}
d\sigma_{+1}(\rho_c) &= \alpha_s\left[C_1^{\mathrm{ME}}(\boldsymbol{p}_{1..m},\rho,\boldsymbol{x}) - C_1^{\mathrm{PS}}(\boldsymbol{p}_{1..m},\rho,\boldsymbol{x})\right]\Delta_{S_1}(\rho,\rho_c)d\Phi_{m+1} \\
&+ \left[C_0^{\mathrm{ME}} + \alpha_s C_0^{\mathrm{loop}} + \alpha_s\int C_1^{\mathrm{PS}}\right]\times\alpha_s\frac{C_1^{\mathrm{PS}}(\boldsymbol{p}_{1..m},\rho,\boldsymbol{x})}{C_0^{\mathrm{ME}}(\boldsymbol{p}_{1..m})} \\
&\quad\times\Delta_{S_0}(\rho_0,\rho)\Delta_{S_1}(\rho,\rho_c)d\Phi_{m+1}
\end{aligned}
$$

To leading order we are left with $\alpha_s C_1^{\mathrm{ME}}$

- If you change the shower, you have to redo the NLO calculation.
- The first Parton shower emission will be correct NLO
  Not necessarily the hardest emission if you have bad ordering.
- Requires $\Delta_{S_0}(\rho_0, \rho) \sim 1$ when ρ distributed as $C_1^{\mathrm{ME}}(\rho) - C_1^{\mathrm{PS}}(\rho)$ (No soft and collinear poles.)
- Event weights may become negative
  (sum is finite and positive for reasonable observables).

POWHEG (reweighting)

What if we use a parton shower for which the first/hardest emission is correct to $\mathcal{O}(\alpha_s)$, $C_1^{\mathrm{ME}} = C_1^{\mathrm{PS}}$

$$
\begin{aligned}
d\sigma_0(\rho_c) &= \Bigg[C_0^{\mathrm{ME}}(\boldsymbol{p}_{1..m}) + \alpha_s C_0^{\mathrm{loop}}(\boldsymbol{p}_{1..m}) \\
&\quad + \alpha_s \int_0^{\rho_0} d\rho \int \mathrm{d}^2 x C_1^{\mathrm{ME}}(\boldsymbol{p}_{1..m}, \rho, \boldsymbol{x})\Bigg] d\Phi_m \\
&\times \quad \Delta_{S_0}(\rho_0, \rho_c)
\end{aligned}
$$

POWHEG (reweighting)

What if we use a parton shower for which the first/hardest emission is correct to $\mathcal{O}(\alpha_s)$, $C_1^{\mathrm{ME}} = C_1^{\mathrm{PS}}$

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\begin{aligned}
d\sigma_0(\rho_c) &= \Bigg[C_0^{\mathrm{ME}}(\boldsymbol{p}_{1..m}) + \alpha_s C_0^{\mathrm{loop}}(\boldsymbol{p}_{1..m}) \\
&\quad + \alpha_s \int_0^{\rho_0} d\rho \int \mathrm{d}^2 x C_1^{\mathrm{ME}}(\boldsymbol{p}_{1..m}, \rho, \boldsymbol{x}) \Bigg] d\Phi_m \\
&\times \left(1 - \alpha_s \int_{\rho_c}^{\rho_0} d\rho \int \mathrm{d}^2 x \frac{C_1^{\mathrm{ME}}(\boldsymbol{p}_{1..m}, \rho, \boldsymbol{x})}{C_0^{\mathrm{ME}}(\boldsymbol{p}_{1..m})} + \mathcal{O}(\alpha_s^2) \right)
\end{aligned}
$$

$$d\sigma_{+1}(\rho_c) = \left[C_0^{\mathrm{ME}} + \alpha_s C_0^{\mathrm{loop}} + \int C_1^{\mathrm{ME}}\right] \times \alpha_s \frac{C_1^{\mathrm{ME}}(\boldsymbol{p}_{1..m}, \rho, \boldsymbol{x})}{C_0^{\mathrm{ME}}(\boldsymbol{p}_{1..m})} \times \Delta_{S_0}(\rho_0, \rho)\Delta_{S_1}(\rho, \rho_c) d\Phi_{m+1}$$

POWHEG assumes a generic $p_\perp$-ordered shower, and implements this first step itself. We should then be able to continue with any $p_\perp$-ordered shower.

- No negative weights
- No need to redo NLO calculation
- If our PS is not ordered in $p_\perp$, we can use a vetoed shower.
- But we still have problems with un-ordered emissions.

Truncated (vetoed) showers

- Take the state, with some $p_\perp$, generated by POWHEG
- Construct the corresponding splitting variables of your PS: $(\rho, \boldsymbol{x})$.
- Undo the corresponding emission
- Evolve from maximum scale down to ρ (vetoing emissions above the $p_\perp$).
- Perform the $(\rho, \boldsymbol{x})$ emission
- Continue evolution below ρ (vetoing emissions above the $p_\perp$).

This can also be used for CKKW.

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CKKW-L@NLO (merging)

$$
\begin{aligned}
d\sigma_0 &= \left[C_0^{\mathrm{ME}}(\boldsymbol{p}_{1..m};\mu) + \alpha_{\mathrm{s}} C_{0,1}^{\mathrm{ME}}(\boldsymbol{p}_{1..m};\mu) + \alpha_{\mathrm{s}}^2 C_{0,2}^{\mathrm{PS}}(\boldsymbol{p}_{1..m};\mu) + \ldots \right] d\Phi_m \\
d\sigma_{+1}(\mu) &= \left[\alpha_{\mathrm{s}} C_1^{\mathrm{ME}}(\boldsymbol{p}_{1..m},\boldsymbol{q}_1;\mu) + \alpha_{\mathrm{s}}^2 C_{1,1}^{\mathrm{ME}}(\boldsymbol{p}_{1..m},\boldsymbol{q}_1;\mu) + \ldots \right] d\Phi_{m+1} \\
d\sigma_{+2}(\mu) &= \left[\alpha_{\mathrm{s}}^2 C_2^{\mathrm{ME}}(\boldsymbol{p}_{1..m},\boldsymbol{q}_1,\boldsymbol{q}_2;\mu) + \alpha_{\mathrm{s}}^3 C_{2,1}^{\mathrm{ME}}(\boldsymbol{p}_{1..m},\boldsymbol{q}_1,\boldsymbol{q}_2;\mu) + \ldots \right] d\Phi_{m+2} \\
&\vdots
\end{aligned}
$$

CKKW-L@NLO (merging) aka NL^3

$$
\begin{aligned}
d\sigma_0 &= \left[C_0^{\mathrm{ME}}(\boldsymbol{p}_{1..m};\mu) + \alpha_s C_{0,1}^{\mathrm{ME}}(\boldsymbol{p}_{1..m};\mu) + \alpha_s^2 C_{0,2}^{\mathrm{PS}}(\boldsymbol{p}_{1..m};\mu) + \ldots\right] d\Phi_m \\
d\sigma_{+1}(\mu) &= \left[\alpha_s C_1^{\mathrm{ME}}(\boldsymbol{p}_{1..m},\boldsymbol{q}_1;\mu) + \alpha_s^2 C_{1,1}^{\mathrm{ME}}(\boldsymbol{p}_{1..m},\boldsymbol{q}_1;\mu) + \ldots\right] d\Phi_{m+1} \\
d\sigma_{+2}(\mu) &= \left[\alpha_s^2 C_2^{\mathrm{ME}}(\boldsymbol{p}_{1..m},\boldsymbol{q}_1,\boldsymbol{q}_2;\mu) + \alpha_s^3 C_{2,1}^{\mathrm{ME}}(\boldsymbol{p}_{1..m},\boldsymbol{q}_1,\boldsymbol{q}_2;\mu) + \ldots\right] d\Phi_{m+2} \\
&\vdots
\end{aligned}
$$

Assume we have a tree-level ME generator producing up to N extra partons, using some jet cutoff μ

$$d\sigma^{\text{tree}}_{+n}(\mu) = \alpha_s^n(\mu_F)C_1^{\text{ME}}(\boldsymbol{p}_{1..m}, \rho_{1..n}, \boldsymbol{x}_{1..n}; \mu)d\Phi_{m+1}$$

Also assume we have a NLO generator producing states for $n < N$

$$d\sigma^{\text{NLO}}_{+n}(\mu) = \Big[\alpha_s^n(\mu_F)C_1^{\text{ME}}(\boldsymbol{p}_{1..m}, \rho_{1..n}, \boldsymbol{x}_{1..n}; \mu) + \alpha_s^{n+1}(\mu_F)C_{1,1}^{\text{ME}}(\boldsymbol{p}_{1..m}, \rho_{1..n}, \boldsymbol{x}_{1..n}; \mu)\Big]d\Phi_{m+1}$$

We can reconstruct the scales ρ_i and intermediate states as in CKKW-L.

As in CKKW-L we want to

- add a shower below μ;
- reweight with the running of α_s;
- reweight with Sudakov form factors above μ.
- reweight with K-factor to get cross sections.

But both the Sudakov form factors and the running of α_s contains α_s^{n+1} terms.

Hence we only want to add only the terms which are $\mathcal{O}(\alpha_s^{n+2})$ and higher.

For the σ_n^{NLO} term, we simply add a cascade below μ. No α_{s} reweighting, or anything else.

Then we add the σ_n^{tree} term, but multiply it with

$$
\begin{aligned}
d\sigma_n^{\mathrm{PScorr}}(\mu) \quad = \quad & \alpha_{\mathrm{s}}^n(\mu_F) C_n^{\mathrm{ME}}(\rho_{1..n};\mu) \times \\
& \left[K \prod_{i=1}^{n} \frac{\alpha_{\mathrm{s}}(\rho_i)}{\alpha_{\mathrm{s}}(\mu_F)} \Delta_{S_{i-1}}(\rho_{i-1},\rho_i;\mu) \Delta_{S_n}(\rho_n,\rho_c;\mu) \right. \\
& \quad - \left\{ 1 + k_1 \alpha_{\mathrm{s}}(\mu_F) + \alpha_{\mathrm{s}}(\mu_F) \sum_{i=1}^{n} \frac{\log(\mu_F/\rho_i)}{\alpha_0} \right. \\
& \quad \left. \left. - \alpha_{\mathrm{s}}(\mu_F) \sum_{i=1}^{n+1} \int_{\rho_i}^{\rho_{i-1}} d\rho' \Gamma_{S_{i-1}}^{\mathrm{PS}}(\rho';\mu) \right\} \right] d\Phi_{m+n}
\end{aligned}
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$$d\sigma_n^{\mathrm{PScorr}}(\mu) = \alpha_{\mathrm{s}}^n(\mu_F)C_n^{\mathrm{ME}}(\rho_{1..n};\mu) \times \Bigg[K\prod_{i=1}^{n}\frac{\alpha_{\mathrm{s}}(\rho_i)}{\alpha_{\mathrm{s}}(\mu_F)}\Delta_{S_{i-1}}(\rho_{i-1},\rho_i;\mu)\Delta_{S_n}(\rho_n,\rho_c;\mu) - \left\{1 + k_1\alpha_{\mathrm{s}}(\mu_F) + \alpha_{\mathrm{s}}(\mu_F)\sum_{i=1}^{n}\frac{\log(\mu_F/\rho_i)}{\alpha_0} - \alpha_{\mathrm{s}}(\mu_F)\sum_{i=1}^{n+1}\log\Delta'_{S_{i-1}}(\rho_i,\rho_{i-1};\mu)\right\}\Bigg] d\Phi_{m+n}$$

Δ' evaluated with constant $\alpha_{\mathrm{s}}(\mu_F)$, and can be generated in a way inspired by the Sudakov veto algorithm in CKKW-L.

For $n = N$, we do the same as in CKKW-L.

- We do not change the higher order α_{s}-expansion of the PS.
- We match the α_{s} used in the NLO calculation.
  The NLO scale-dependence is reduced.
- We need a jet cutoff μ.
- If this jet-scale is very different from the PS evolution scale, ρ, we will have a mismatch.
- We will have negative weights.
- So far only derived for $\mathrm{e}^+\mathrm{e}^-$.

Same thing can be done for CKKW.

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Summary

- Why?
  - We need many-parton tree-level ME's to describe rare events.
  - We need NLO ME's to get precision.
  - We need Parton Showers to evolve partons into partonic jets.
  - We need hadronization models to get "real" jets.
  - We need to combine ME and PS.
- Tree-level ME + PS reweighting
  - Only works on first/hardest emission.
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- Tree-level ME + PS merging
  - CKKW(-L), Pseudo-Shower and MLM.
  - Split the phase space between ME and PS region using jet cutoff, μ.
  - Small μ is preferred, but makes things slow.
  - Avoid double-counting and under counting.
  - Introduce Sudakov form factors to make ME states exclusive.
  - Introduce running α_s to get the same non-leading behavior as PS.
  - Hadronic collisions straight-forward.

- NLO + PS matching
  - MC@NLO
  - Use PS splitting function to define NLO subtraction terms.
  - Adding PS is easy.
  - Only first emissions corrected (not necessarily hardest).
  - PS-dependent, negative weights.
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  - PS-independent (as long as PS is $p_\perp$-ordered).
  - Only first emission – hardest can be obtained by truncated showers.

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  - NL^3, but also implementation in Sherpa soon(?)
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  - Is very important.
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