Matching and merging
Matrix Elements with Parton Showers III

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Outline of Lectures

- Lecture I: Introduction, Tree-level ME, NLO, PS, ordering, basic strategies, ...
- Lecture II: Tree-level ME merging with PS, CKKW(-L), Pseudo Shower, MLM, $e^+e^-$ comparison, ...
- Lecture III: ME+PS merging in $pp$, NLO matching with PS, MC@NLO, POWHEG, NL$^3$, ...
Outline

ME+PS merging in hadronic collisions
  CKKW-L
  CKKW
  MLM
  Pseudo-Shower

The W+jets benchmark
  The models
  Tevatron
  LHC
  Conclusions

NLO matching
  MC@NLO
  POWHEG
  CKKW-L@NLO

Summary
ME+PS merging in hadronic collisions

The main difference here is that we have incoming partons. We have to worry about:

- parton densities
- initial-state showers
- algorithm to construct scales (and intermediate states).

(Will assume $W+$jets production throughout, but it can easily be generalized.)
Longitudinally invariant $k_\perp$-algorithm

$d_{ij} = \min(k_{\perp i}, k_{\perp j}) \left[ \Delta \eta_{ij}^2 + \Delta \phi_{ij}^2 \right]$

$d_i = k_{\perp i}$
Longitudinally invariant $k_\perp$-algorithm

$$d_{ij} = \min(k_{\perp i}^2, k_{\perp j}^2) \left[ \Delta \eta_{ij}^2 + \Delta \phi_{ij}^2 \right]$$

$$d_i = k_{\perp i}$$
Longitudinally invariant $k_\perp$-algorithm

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Longitudinally invariant $k_{\perp}$-algorithm

$$d_{ij} = \min(k_{\perp i}^2, k_{\perp j}^2) \left[ \Delta \eta_{ij}^2 + \Delta \phi_{ij}^2 \right]$$

$$d_i = k_{\perp i}$$
The leading order $W$ cross section:

$$d\sigma_{pp\rightarrow W} = \sum_{q,\bar{q}'} xf_q(x_+, m_W) xf_{q'}(x_-, m_W) \hat{\sigma}_{q\bar{q}'\rightarrow W}(sx_+x_-) \frac{dx_+}{x_+} \frac{dx_-}{x_-}$$

Making one step ($g \rightarrow q$) backward in the initial-state shower:

$$dP(\rho, z) = \frac{\alpha_s}{2\pi} P_{g\rightarrow q}(z) \frac{xf_g(x_+/z, \rho)}{xf_q(x_+, \rho)} \Delta S_0(\rho_{\text{max}}, \rho) \frac{d\rho}{\rho} dz,$$
Comparing with the cross section for getting the same thing from the matrix element:

\[
d\sigma_{pp \to Wq} = \sum_{\bar{q},q'} x_f g \left( \frac{x_+}{z}, \mu \right) x_f \bar{q}' \left( x_-, \mu \right) \hat{\sigma}_{gq'} \to W\bar{q} \left( s \frac{x_+}{z} x_-, z, \rho \right) \frac{dx_+}{x_+} \frac{dx_-}{x_-} d\rho dz
\]

And we identify

\[
\frac{\hat{\sigma}_{gq'} \to W\bar{q} \left( \frac{x_+}{z} x_-, S, z, \rho \right)}{\hat{\sigma}_{q\bar{q}' \to W} \left( s x_+ x_- \right)} d\rho dz
\]

with

\[
\frac{\alpha_s}{2\pi} P_{g \to q} (z) \frac{d\rho}{\rho} dz,
\]
In CKKW-L, the way to handle this is to construction scales and intermediate states, using the “inverse-PS” algorithm, taking care to separate initial- from final-state emission.

Then reweight a produced $n$–parton event with

$$\frac{xf_{p_0}(x_+^0, m_W) \cdot xf_{q_0}(x_-^0, m_W)}{xf_{p_n}(x_+^n, \mu) \cdot xf_{q_n}(x_-^n, \mu)}$$

where the partons $p_0$ and $q_0$ and their energy fractions $x_+^0$ and $x_-^0$ are obtained from the constructed $S_0$ state.
Then for each constructed state, $S_i$, the event is weighted by the running $\alpha_s$, the no-emission probability, $\Delta_{S_i}(q_i, q_{i+1})$, using the Sudakov veto algorithm, and a PDF ratio

$$\frac{xf_{p_{i+1}}(x_{+(i+1)}, q_{i+1}) \cdot xf_{q_{i+1}}(x_{-(i+1)}, q_{i+1})}{xf_{p_i}(x_{+i}, q_{i+1}) \cdot xf_{q_i}(x_{-i}, q_{i+1})}$$

In this way we get exactly the same Sudakov form factors and PDF ratios as in the shower.

Note however that the total cross section is forced to be the LO one (as in the original shower).
Here we use the boost-invariant $k_\perp$-algorithm to construct the emission scales.

In each step we either cluster two final-state partons together, or cluster one parton to the beam, corresponding to an final- and initial-state splitting respectively.

The reweighting of $\alpha_s$ and analytical Sudakov form factors works in the same way as for $e^+e^-$. 
No PDF weighting is necessary!

This is because the Sudakov form factors are not proper no-emission probabilities:

\[ P_{\text{no-emission}} = \frac{xf_p(x, q_2)}{xf_p(x, q_1)} \frac{\Delta_p(q_1, \mu)}{\Delta_p(q_2, \mu)} \]

and all PDF ratios cancel!

Life becomes simpler and the total cross section becomes beyond LO (although not NLO).
The reason CKKL-L anyway insists on including the ratios is that the correspondence between the no-emission probability and the Sudakov form factors is only true for DGLAP evolution. And CKKW-L was originally implemented for ARIADNE, which is not strictly DGLAP (it includes a resummation of large logarithms of $x$).
In CKKW-L, the maximum scale on the shower applied to \( S_0 \) is the same as for the stand-alone shower. For a strict DGLAP shower this is typically \( m_W \).

In ARIADNE and PYTHIA (in case the first emission is ME-reweighted), the maximum scale is given by the kinematical limit, e.g. \( p_\perp < \sqrt{s}/2 \).

In CKKW, the maximum scale for the vetoed shower from the \( S_n \) state, and for the Sudakov form factors, is typically taken to be \( m_\perp W \).

I.e. no no-emission probability above \( m_\perp W \), which is fine as long as there are no large logarithms of \( \sqrt{s}/m_W \sim x \).
Again we use the boost-invariant $k_\perp$-algorithm to define the ME generation cutoff, $\mu_0$, and the constructed emission scales.

The maximum scale in the shower is $m_\perp W$.

In the original (A\textsc{L}P\textsc{G}E\textsc{N}) implementation a cone algorithm is used to cluster the showered even. Here the minimum $E_\perp$ of the jets is set to $\mu > \mu_0$, and the $R_{\text{cone}}$ is larger than the minimum $\Delta R$ in the ME generation.

A jet is matched to a parton if the $\Delta R_{\text{jet,parton}} < 1.5 \times R_{\text{cone}}$. 
Other MLM implementations

\textbf{MADEvent} has another implementation which uses the $k_\perp$-algorithm throughout in much the same way as was described for $e^+e^-$.  

\textbf{HELAC} is very close to \textbf{ALPGEN}.  

Pseudo-Shower

Follows the modifications used in CKKW-L when going from $e^+e^-$ to $pp$. 
The W+jets benchmark

W+jets is a good benchmark to compare the algorithms in $pp$. What follows is a comparison between:

- Sherpa, implementing CKKW (with PYTHIA-like virtuality ordered shower).
- CKKW-L using ARIADNE.
- ALPGEN using MLM merging with HERWIG.
- MADEVENT using MLM merging with PYTHIA, virtuality-ordered shower.
- HELAC using MLM merging with PYTHIA, virtuality-ordered shower.
Notes:

- Pseudo-Shower not included here (see refs.)
- **ARIADNE** is not DGLAP-based.
- **PYTHIA** with CKKW and two flavours of MLM.
- Same flavour of MLM with both **PYTHIA** and **HERWIG**.
- All use similar cuts on ME and up to 4 jets.
Tevatron energies

All procedures allowed different variations of parameters. Merging scale and scale choice in $\alpha_s$-reweighting in common.
ME+PS merging in hadronic collisions

The W+jets benchmark

NLO matching

The models

Tevatron

LHC

\[
\frac{d\sigma}{dp_T} (\text{pb/GeV})
\]

(a) Alpgen

Ariadne

Helac

MadEvent

Sherpa

Matching and Merging III

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\[ \Delta \eta = |\eta_W - \eta_{\text{jet}}| \]
ALPGEN

\( \mu = 10, 20, 30 \text{ GeV}, \text{ ME: } \sqrt{d_1} > \mu, \text{ PS: } \sqrt{d_1} < \mu. \)
ME+PS merging in hadronic collisions
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SHERPA

ARIADNE

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LHC energies

![Graph showing LHC energies comparison between different generators: Alpgen, Ariadne, Helac, MadEvent, Sherpa. The x-axis represents the number of jets (≥ 0, ≥ 1, ≥ 2, ≥ 3, ≥ 4), and the y-axis represents the ratio of cross-sections σ(W+ ≥ N jets) / <σ> for different generators.](image)
ME+PS merging in hadronic collisions

The W+jets benchmark

NLO matching

\[ \frac{d\sigma}{dp_T} (\text{pb}/\text{GeV}) \]

(a) Alpgen

Ariadne

Helac

MadEvent

Sherpa

Matching and Merging III
\[ \Delta \eta = |\eta_W - \eta_{\text{jet}}| \]
- Clear wiggles near cutoff $\mu$.
- Far above $\mu$, results are independent of $\mu$.
- Different models differ, but within systematic uncertainties of each model.
- Largest difference found from ARIADNE (not DGLAP).
NLO matching

\[ d\sigma_0 = \left[ C_0^{\text{ME}}(p_{1..m}, \mu) + \alpha_s C_{0,1}^{\text{PS}}(p_{1..m}; \mu) + \alpha_s^2 C_{0,2}^{\text{PS}}(p_{1..m}; \mu) + \ldots \right] d\Phi_m \]

\[ d\sigma_{+1}(\mu) = \left[ \alpha_s C_1^{\text{ME}}(p_{1..m}, q_1; \mu) + \alpha_s^2 C_{1,1}^{\text{PS}}(p_{1..m}, q_1; \mu) + \ldots \right] d\Phi_{m+1} \]

\[ d\sigma_{+2}(\mu) = \left[ \alpha_s^2 C_2^{\text{ME}}(p_{1..m}, q_1, q_2; \mu) + \alpha_s^3 C_{2,1}^{\text{PS}}(p_{1..m}, q_1, q_2; \mu) + \ldots \right] d\Phi_{m+2} \]

\vdots
NLO matching

\[ d\sigma_0 = \left[ C^\text{ME}_0(p_{1..m}; \mu) + \alpha_s C^\text{ME}_{0,1}(p_{1..m}; \mu) + \alpha_s^2 C^\text{PS}_{0,2}(p_{1..m}; \mu) + \ldots \right] d\Phi_m \]

\[ d\sigma_{+1}(\mu) = \left[ \alpha_s C^\text{ME}_1(p_{1..m}, q_1; \mu) + \alpha_s^2 C^\text{ME}_{1,1}(p_{1..m}, q_1; \mu) + \ldots \right] d\Phi_{m+1} \]

\[ d\sigma_{+2}(\mu) = \left[ \alpha_s^2 C^\text{ME}_2(p_{1..m}, q_1, q_2; \mu) + \alpha_s^3 C^\text{ME}_{2,1}(p_{1..m}, q_1, q_2; \mu) + \ldots \right] d\Phi_{m+2} \]

\[ \vdots \]
NLO matching

\[ d\sigma_0 = \left[ C_0^{\text{ME}}(p_1..m; \mu) + \alpha_s C_{0,1}^{\text{ME}}(p_1..m; \mu) + \alpha_s^2 C_{0,2}^{\text{PS}}(p_1..m; \mu) + \ldots \right] d\Phi_m \]

\[ d\sigma_{+1}(\mu) = \left[ \alpha_s C_{1,1}^{\text{ME}}(p_1..m, q_1; \mu) + \alpha_s^2 C_{1,1}^{\text{PS}}(p_1..m, q_1; \mu) + \ldots \right] d\Phi_{m+1} \]

\[ d\sigma_{+2}(\mu) = \left[ \alpha_s^2 C_{2}^{\text{PS}}(p_1..m, q_1, q_2; \mu) + \alpha_s^3 C_{2,1}^{\text{PS}}(p_1..m, q_1, q_2; \mu) + \ldots \right] d\Phi_{m+2} \]

\vdots
**MC@NLO**

Based on NLO-subtraction method

\[
d\sigma_0 = \left[ C_0^{\text{ME}}(p_{1..m}) + \alpha_s C_0^{\text{loop}}(p_{1..m}) \right. \\
+ \alpha_s \int d^3 q' C_1^{\text{CS}}(p_{1..m}, q') \left. \right] d\Phi_m \\
d\sigma_1 = \alpha_s \left[ C_1^{\text{ME}}(p_{1..m}, q') - C_1^{\text{CS}}(p_{1..m}, q') \right] d\Phi_{m+1}
\]

\[d\sigma^{\text{NLO}} = d\sigma_0 + d\sigma_1.\]

\(C_1^{\text{CS}}\) must reproduce the soft and collinear poles in \(C_1^{\text{ME}}\).
**MC@NLO**

Based on NLO-subtraction method

\[
 d\sigma_0 = \left[ C_{0}^{\text{ME}}(p_1..m) + \alpha_s C_{0}^{\text{loop}}(p_1..m) \right. \\
 + \left. \int d^3 q' C_{1}^{\text{PS}}(p_1..m, q') \right] d\Phi_m \\
 d\sigma_{+1} = \alpha_s \left[ C_{1}^{\text{ME}}(p_1..m, q') - C_{1}^{\text{PS}}(p_1..m, q') \right] d\Phi_{m+1} \\
 d\sigma^{\text{NLO}} = d\sigma_0 + d\sigma_1.
\]

\( C_{1}^{\text{CS}} \) must reproduce the soft and collinear poles in \( C_{1}^{\text{ME}} \).

We can use the splitting function of any reasonable PS.
Note that calculating \( C_0^{\text{loop}} + \int d^3q C_1^{\text{PS}} \) is highly non-trivial.
Note that $\sigma_1$ is not a proper $+1$ parton cross section, most of this is integrated over and put into $\sigma_0$.

Now we can add a parton shower to get the no-parton cross section

$$d\sigma_0(\rho_c) = \left[ C_0^{\text{ME}}(p_1..m) + \alpha_s C_0^{\text{loop}}(p_1..m) \right. + \alpha_s \int_0^{\rho_0} d\rho \int d^2 x C_1^{\text{PS}}(p_1..m, \rho, x) \left. \right] d\Phi_m \times \Delta S_0(\rho_0, \rho_c)$$

Note difference in integration region.
Note that $\sigma_1$ is not a proper +1 parton cross section, most of this is integrated over and put into $\sigma_0$.

Now we can add a parton shower to get the no-parton cross section

$$d\sigma_0(\rho_c) = \left[ C_0^{\text{ME}}(p_1\ldots m) + \alpha_s C_0^{\text{loop}}(p_1\ldots m) + \alpha_s \int_0^{\rho_0} d\rho \int d^2x C_1^{\text{PS}}(p_1\ldots m, \rho, x) \right] d\Phi_m \times \left(1 - \alpha_s \int_{\rho_c}^{\rho_0} d\rho \int d^2x \frac{C_1^{\text{PS}}(p_1\ldots m, \rho, x)}{C_0^{\text{ME}}(p_1\ldots m)} + \mathcal{O}(\alpha_s^2) \right)$$

Note difference in integration region.
Note that $\sigma_1$ is not a proper $+1$ parton cross section, most of this is integrated over and put into $\sigma_0$.

Now we can add a parton shower to get the no-parton cross section

$$d\sigma_0(\rho_c) = \left[ C_{0}^{\text{ME}}(p_1..m) + \alpha_s C_{0}^{\text{loop}}(p_1..m) \right. $$

$$\left. + \alpha_s \int_{0}^{\rho_0} d\rho \int d^2x C_{1}^{\text{PS}}(p_1..m, \rho, x) \right] d\Phi_m$$

$$\times \left( 1 - \alpha_s \int_{\rho_c}^{\rho_0} d\rho \int d^2x \frac{C_{1}^{\text{PS}}(p_1..m, \rho, x)}{C_{0}^{\text{ME}}(p_1..m)} + O(\alpha_s^2) \right)$$

Note difference in integration region.
We then get the one-parton cross section

\[ d\sigma_{+1}(\rho_c) = \alpha_s \left[ C_{1}^{\text{ME}}(p_{1..m}, \rho, x) - C_{1}^{\text{PS}}(p_{1..m}, \rho, x) \right] \Delta S_1(\rho, \rho_c) d\Phi_{m+1} \]

\[ + \left[ C_0^{\text{ME}} + \alpha_s C_0^{\text{loop}} + \alpha_s \int C_1^{\text{PS}} \right] \times \alpha_s \frac{C_{1}^{\text{PS}}(p_{1..m}, \rho, x)}{C_0^{\text{ME}}(p_{1..m})} \]

\[ \times \Delta S_0(\rho_0, \rho) \Delta S_1(\rho, \rho_c) d\Phi_{m+1} \]

To leading order we are left with \( \alpha_s C_{1}^{\text{ME}} \)
We then get the one-parton cross section

\[ d\sigma_{+1}(\rho_c) = \alpha_s \left[ C_{1}^{\text{ME}}(p_{1..m}, \rho, x) - C_{1}^{\text{PS}}(p_{1..m}, \rho, x) \right] \Delta S_{1}(\rho, \rho_{c}) d\Phi_{m+1} \]

\[ + \left[ C_{0}^{\text{ME}} + \alpha_s C_{0}^{\text{loop}} + \alpha_s \int C_{1}^{\text{PS}} \right] \times \alpha_s \frac{C_{1}^{\text{PS}}(p_{1..m}, \rho, x)}{C_{0}^{\text{ME}}(p_{1..m})} \times \Delta S_{0}(\rho_{0}, \rho) \Delta S_{1}(\rho, \rho_{c}) d\Phi_{m+1} \]

To leading order we are left with \( \alpha_s C_{1}^{\text{ME}} \)
We then get the one-parton cross section

\[
d\sigma_{+1}(\rho_c) = \alpha_s \left[ C_{1}^{\text{ME}}(p_{1..m}, \rho, x) - C_{1}^{\text{PS}}(p_{1..m}, \rho, x) \right] \Delta S_1(\rho, \rho_c) d\Phi_{m+1} \\
+ \left[ C_0^{\text{ME}} + \alpha_s C_0^{\text{loop}} + \alpha_s \int C_1^{\text{PS}} \right] \times \alpha_s \frac{C_{1}^{\text{PS}}(p_{1..m}, \rho, x)}{C_0^{\text{ME}}(p_{1..m})} \times \Delta S_0(\rho_0, \rho) \Delta S_1(\rho, \rho_c) d\Phi_{m+1}
\]

To leading order we are left with \( \alpha_s C_1^{\text{ME}} \)
If you change the shower, you have to redo the NLO calculation.

The first Parton shower emission will be correct NLO
Not necessarily the hardest emission if you have bad ordering.

Requires $\Delta_{S_0}(\rho_0, \rho) \sim 1$ when $\rho$ distributed as
$C^\text{ME}_1(\rho) - C^\text{PS}_1(\rho)$ (No soft and collinear poles.)

Event weights may become negative
(sum is finite and positive for reasonable observables).
What if we use a parton shower for which the first/hardest emission is correct to $O(\alpha_s)$, $C_1^{\text{ME}} = C_1^{\text{PS}}$

$$d\sigma_0(\rho_c) = \left[ C_0^{\text{ME}}(p_{1..m}) + \alpha_s C_0^{\text{loop}}(p_{1..m}) \right. + \alpha_s \int_0^{\rho_0} d\rho \int d^2x C_1^{\text{ME}}(p_{1..m}, \rho, x) \right] d\Phi_m$$

$$\times \Delta S_0(\rho_0, \rho_c)$$
What if we use a parton shower for which the first/hardest emission is correct to $O(\alpha_s)$, $C_{1}^{\text{ME}} = C_{1}^{\text{PS}}$.

$$d\sigma_0(\rho_c) = \left[ C_{0}^{\text{ME}}(p_{1..m}) + \alpha_s C_{0}^{\text{loop}}(p_{1..m}) \right. \\
+ \alpha_s \int_0^{\rho_0} d\rho \int d^2x C_{1}^{\text{ME}}(p_{1..m}, \rho, x) \left. \right] d\Phi_m \times \left( 1 - \alpha_s \int_{\rho_c}^{\rho_0} d\rho \int d^2x \frac{C_{1}^{\text{ME}}(p_{1..m}, \rho, x)}{C_{0}^{\text{ME}}(p_{1..m})} + O(\alpha_s^2) \right)$$
\[
\begin{align*}
d\sigma_{+1}(\rho_c) &= \left[ C_{0}^{\text{ME}} + \alpha_s C_{0}^{\text{loop}} + \int C_{1}^{\text{ME}} \right] \times \alpha_s \frac{C_{1}^{\text{ME}}(p_{1\ldots m}, \rho, x)}{C_{0}^{\text{ME}}(p_{1\ldots m})} \\
&\times \Delta_{S_0}(\rho_0, \rho) \Delta_{S_1}(\rho, \rho_c) d\Phi_{m+1}
\end{align*}
\]

**POWHEG** assumes a generic \(p_\perp\)-ordered shower, and implements this first step itself. We should then be able to continue with any \(p_\perp\)-ordered shower.
No negative weights
No need to redo NLO calculation
If our PS is not ordered in $p_{\perp}$, we can use a vetoed shower.
But we still have problems with un-ordered emissions.
Truncated (vetoed) showers

- Take the state, with some $p_{\perp}$, generated by POWHEG
- Construct the corresponding splitting variables of your PS: $(\rho, x)$.
- Undo the corresponding emission
- Evolve from maximum scale down to $\rho$ (vetoing emissions above the $p_{\perp}$).
- Perform the $(\rho, x)$ emission
- Continue evolution below $\rho$ (vetoing emissions above the $p_{\perp}$).

This can also be used for CKKW.
Truncated (vetoed) showers

- Take the state, with some $p_\perp$, generated by POWHEG
- Construct the corresponding splitting variables of your PS: $(\rho, x)$.
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This can also be used for CKKW.
CKKW-L@NLO (merging)

\[
d\sigma_0 = \left[ C_{0}^{\text{ME}}(\mathbf{p}_{1..m}; \mu) + \alpha_s C_{0,1}^{\text{ME}}(\mathbf{p}_{1..m}; \mu) + \alpha_s^2 C_{0,2}^{\text{PS}}(\mathbf{p}_{1..m}; \mu) + \ldots \right] d\Phi_m
\]

\[
d\sigma_1(\mu) = \left[ \alpha_s C_{1}^{\text{ME}}(\mathbf{p}_{1..m}, \mathbf{q}_1; \mu) + \alpha_s^2 C_{1,1}^{\text{ME}}(\mathbf{p}_{1..m}, \mathbf{q}_1; \mu) + \ldots \right] d\Phi_{m+1}
\]

\[
d\sigma_2(\mu) = \left[ \alpha_s^2 C_{2}^{\text{ME}}(\mathbf{p}_{1..m}, \mathbf{q}_1, \mathbf{q}_2; \mu) + \alpha_s^3 C_{2,1}^{\text{ME}}(\mathbf{p}_{1..m}, \mathbf{q}_1, \mathbf{q}_2; \mu) + \ldots \right] d\Phi_{m+2}
\]

\vdots \]
CKKW-L@NLO (merging) aka NL$^3$

\[
\begin{align*}
\sigma_0 &= \left[ C_{0}^{\text{ME}}(p_{1..m}; \mu) + \alpha_s C_{0,1}^{\text{ME}}(p_{1..m}; \mu) + \alpha_s^2 C_{0,2}^{\text{PS}}(p_{1..m}; \mu) + \ldots \right] d\Phi_m \\
\sigma_{+1}(\mu) &= \left[ \alpha_s C_{1}^{\text{ME}}(p_{1..m}, q_1; \mu) + \alpha_s^2 C_{1,1}^{\text{ME}}(p_{1..m}, q_1; \mu) + \ldots \right] d\Phi_{m+1} \\
\sigma_{+2}(\mu) &= \left[ \alpha_s^2 C_{2}^{\text{ME}}(p_{1..m}, q_1, q_2; \mu) + \alpha_s^3 C_{2,1}^{\text{ME}}(p_{1..m}, q_1, q_2; \mu) + \ldots \right] d\Phi_{m+2} \\
\vdots
\end{align*}
\]
Assume we have a tree-level ME generator producing up to $N$ extra partons, using some jet cutoff $\mu$

$$d\sigma_{+n}^{\text{tree}}(\mu) = \alpha_s^n(\mu_F) C_1^{\text{ME}}(p_1..m, \rho_1..n, x_1..n; \mu) d\Phi_{m+1}$$

Also assume we have a NLO generator producing states for $n < N$

$$d\sigma_{+n}^{\text{NLO}}(\mu) = \left[ \alpha_s^n(\mu_F) C_1^{\text{ME}}(p_1..m, \rho_1..n, x_1..n; \mu) \right. \right.$$  

$$+ \alpha_s^{n+1}(\mu_F) C_{1,1}^{\text{ME}}(p_1..m, \rho_1..n, x_1..n; \mu) \left. \right] d\Phi_{m+1}$$

We can reconstruct the scales $\rho_i$ and intermediate states as in CKKW-L.
As in CKKW-L we want to

- add a shower below $\mu$;
- reweight with the running of $\alpha_s$;
- reweight with Sudakov form factors above $\mu$.
- reweight with $K$-factor to get cross sections.

But both the Sudakov form factors and the running of $\alpha_s$ contains $\alpha_{s}^{n+1}$ terms.

Hence we only want to add only the terms which are $O(\alpha_{s}^{n+2})$ and higher.
For the $\sigma_{n}^{NLO}$ term, we simply add a cascade below $\mu$. No $\alpha_s$ reweighting, or anything else.

Then we add the $\sigma_{n}^{\text{tree}}$ term, but multiply it with

$$d\sigma_{n}^{\text{PScorr}}(\mu) = \alpha_s^n(\mu_F) C_n^\text{ME}(\rho_{1..n}; \mu) \times$$

$$\left[ K \prod_{i=1}^{n} \frac{\alpha_s(\rho_i)}{\alpha_s(\mu_F)} \Delta S_{i-1}(\rho_{i-1}, \rho_i; \mu) \Delta S_n(\rho_n, \rho_c; \mu) \right]$$

$$- \left\{ 1 + k_1 \alpha_s(\mu_F) + \alpha_s(\mu_F) \sum_{i=1}^{n} \frac{\log(\mu_F/\rho_i)}{\alpha_0} \right\}$$

$$d\Phi_{m+n}$$
For the \( \sigma^{\text{NLO}}_n \) term, we simply add a cascade below \( \mu \). No \( \alpha_s \) reweighting, or anything else.

Then we add the \( \sigma^{\text{tree}}_n \) term, but multiply it with

\[
\frac{d\sigma^{\text{PScorr}}_n(\mu)}{d\Phi_{m+n}} = \alpha_s^n(\mu_F)C^\text{ME}_n(\rho_{1..n}; \mu) \times \\
K \prod_{i=1}^{n} \frac{\alpha_s(\rho_i)}{\alpha_s(\mu_F)} \Delta_{S_{i-1}}(\rho_{i-1}, \rho_i; \mu) \Delta_{S_n}(\rho_n, \rho_c; \mu) \\
- \left\{ 1 + k_1 \alpha_s(\mu_F) + \alpha_s(\mu_F) \sum_{i=1}^{n} \frac{\log(\mu_F/\rho_i)}{\alpha_0} \right. \\
- \alpha_s(\mu_F) \sum_{i=1}^{n+1} \log \Delta'_{S_{i-1}}(\rho_i, \rho_{i-1}; \mu) \right\} \\
\Delta' \text{ evaluated with constant } \alpha_s(\mu_F), \text{ and can be generated in a way inspired by the Sudakov veto algorithm in CKKW-L.}
For $n = N$, we do the same as in CKKW-L.

- We do not change the higher order $\alpha_s$-expansion of the PS.
- We match the $\alpha_s$ used in the NLO calculation. The NLO scale-dependence is reduced.
- We need a jet cutoff $\mu$.
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Summary

Why?

- We need many-parton tree-level ME’s to describe rare events.
- We need NLO ME’s to get precision.
- We need Parton Showers to evolve partons into partonic jets.
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Tree-level ME + PS reweighting

- Only works on first/hardest emission.
- Does not affect total cross section.
- Similar to POWHEG.
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Tree-level ME + PS merging

- CKKW(-L), Pseudo-Shower and MLM.
- Split the phase space between ME and PS region using jet cutoff, $\mu$.
- Small $\mu$ is preferred, but makes things slow.
- Avoid double-counting and under-counting.
- Introduce Sudakov form factors to make ME states exclusive.
- Introduce running $\alpha_s$ to get the same non-leading behavior as PS.
- Hadronic collisions straight-forward.
NLO + PS matching
- MC@NLO
- Use PS splitting function to define NLO subtraction terms.
- Adding PS is easy.
- Only first emissions corrected (not necessarily hardest).
- PS-dependent, negative weights.

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- Is very important.
- For any merging/matching procedure.
- Ordering in $p_\perp$ is good.
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References

- Veto algorithm for processes with Sudakov form factors.
  - T. Sjöstrand, Pythia 6.4 Physics and Manual (ch 4.2)
- Reweighting (Correcting the hardest emission)
- CKKW
- CKKW-L
The W+jets benchmark
NLO matching

Summary

- **MLM**
  - M. Mangano (Talk at tuning workshop at Fermilab)
- **Pseudo-Shower**
- **Comparisons**
MC@NLO
- S. Frixione, B. Webber, (manual)

POWHEG

NL$^3$
- N. Lavesson, L. Lönnblad, (preprint in preparation)