Matching and merging
Matrix Elements with Parton Showers II

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Outline of Lectures

- Lecture I: Introduction, Tree-level ME, NLO, PS, ordering, basic strategies, ...
- Lecture II: Tree-level ME merging with PS, CKKW(-L), Pseudo Shower, MLM, e^+e^- comparison, ...
- Lecture III: ME+PS merging in *pp*, NLO matching with PS, MC@NLO, POWHEG, NL^3, ...

Outline

Merging Tree-level ME with PS

$$
\begin{aligned}
d\sigma_0 &= C_0^{\mathrm{ME}}(\boldsymbol{p}_{1..m};\mu)d\Phi_m \\
d\sigma_{+1}(\mu) &= \alpha_s C_1^{\mathrm{ME}}(\boldsymbol{p}_{1..m},\rho_1,\boldsymbol{x}_1;\mu)d\Phi_{m+1} \\
d\sigma_{+2}(\mu) &= \alpha_s^2 C_2^{\mathrm{ME}}(\boldsymbol{p}_{1..m},\rho_1,\boldsymbol{x}_1,\rho_2,\boldsymbol{x}_2;\mu)d\Phi_{m+2} \\
&\vdots
\end{aligned}
$$

- Start out with ME generated (inclusive) *n*-jet states.
- Reweight with Sudakov form factors to get exclusive states.
- Reweight with running α_s.
- Add PS below cutoff, μ.

Merging Tree-level ME with PS

$$
\begin{aligned}
d\sigma_0 &= C_0^{\mathrm{ME}}(\boldsymbol{p}_{1..m};\mu)\times\Delta_{S0}(\rho_0,\rho_c;\mu)d\Phi_m \\
d\sigma_{+1}(\mu) &= \alpha_s C_1^{\mathrm{ME}}(\boldsymbol{p}_{1..m},\rho_1,\boldsymbol{x}_1;\mu)\times \\
&\quad \Delta_{S0}(\rho_0,\rho_1;\mu)\Delta_{S1}(\rho_1,\rho_c;\mu)d\Phi_{m+1} \\
d\sigma_{+2}(\mu) &= \alpha_s^2 C_2^{\mathrm{ME}}(\boldsymbol{p}_{1..m},\rho_1,\boldsymbol{x}_1,\rho_2,\boldsymbol{x}_2;\mu)\times \\
&\quad \Delta_{S0}(\rho_0,\rho_1;\mu)\Delta_{S1}(\rho_1,\rho_2;\mu)\Delta_{S2}(\rho_2,\rho_c;\mu)d\Phi_{m+2} \\
&\vdots
\end{aligned}
$$

- Start out with ME generated (inclusive) *n*-jet states.
- Reweight with Sudakov form factors to get exclusive states.
- Reweight with running α_s.
- Add PS below cutoff, μ.

The general procedure

(We will here assume $\mathrm{e}^+\mathrm{e}^-$ and introduce *pp* collisions later)

Assuming you have a ME generator producing LO order events and up to N extra partons using some jet cutoff μ.

1. Choose a parton multiplicity $n \leq N$ according to the integrated cross sections and generate a corresponding state.
2. Construct a series of emission scales $q_1, \ldots, q_n$.
3. Reweight event with running coupling $\prod_i^n \frac{\alpha_{\mathrm{s}}(q_i)}{\alpha_{\mathrm{s}}^{\mathrm{ME}}}$.
4. Model the Sudakov form factors and reweight.
5. Add a parton shower, but veto any emission with a jet-scale above μ, except if $n = N$: veto above q_N.

Catani-Krauss-Kuhn-Webber

The first procedure to hit the market.

Use $k_\perp$-algorithm to define a jet cutoff.

Also use the $k_\perp$-algorithm to define emission scales, but only allow physical mergings:

- only merge colour-connected partons
- don't merge e.g. a u- and $\bar{\mathrm{c}}$-quark.

Note that this jet algorithm is like a parton shower, with $k_\perp$ as ordering variable, run backwards.

Now we can calculate the Sudakov form-factors this "parton shower" would have used to produce the state.

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CKKW use analytical Sudakov form factors, based on analytically integrated splitting functions according to an angular ordered scenario to ensure NLL accuracy.

$$
\begin{aligned}
\Delta_q(q_1, q_2) &= \exp(-\int_{q_2}^{q_1} dq' \Gamma_q(q_1, q')) \\
\Delta_g(q_1, q_2) &= \exp(-\int_{q_2}^{q_1} dq' (\Gamma_g(q_1, q') + \Gamma_f(q_1, q'))).
\end{aligned}
$$

Where Γ_i is the integrated splitting function:

$$
\Gamma_i(Q, q) = \frac{\alpha_s(q)}{2\pi q} \int_{q/Q}^{1-q/Q} P_i(z) dz
$$

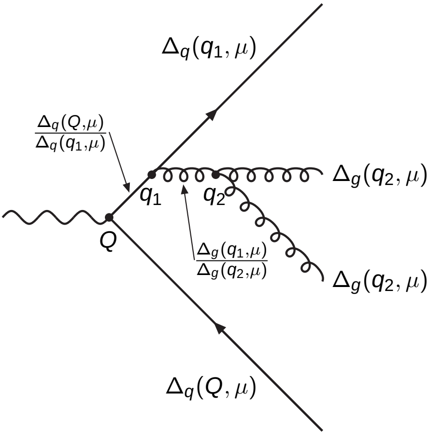
To NLL accuracy we have.

$$\begin{aligned}
\Gamma_q(Q,q) &= \frac{2C_F}{\pi}\frac{\alpha_s(q)}{q}\left(\ln\frac{Q}{q}-\frac{3}{4}\right), \\
\Gamma_g(Q,q) &= \frac{2C_A}{\pi}\frac{\alpha_s(q)}{q}\left(\ln\frac{Q}{q}-\frac{11}{12}\right), \\
\Gamma_f(q) &= \frac{N_f}{3\pi}\frac{\alpha_s(q)}{q}.
\end{aligned}$$

Note that these can become negative which may result in Sudakov form factors larger than unity.

Optionally we can cutoff the Sudakovs to recover the interpretations as no-emission probabilities.

$\Delta(Q,\mu)/\Delta(q,\mu)$ is the probability to have no emissions above μ during the evolution from Q down to q.



Then we can add a parton shower. Any parton shower. Even one ordered in $\rho \neq k_{\perp}$.

We start at the maximum scale $\rho = Q$, and generate successive emissions.

However, we veto any emission with $k_{\perp} > \mu$ (only veto the emission, not the entire event).

In this way we avoid double-counting:

- Any splitting with $k_{\perp} > \mu$ is given by the ME.
- Any splitting with $k_{\perp} < \mu$ is given by the PS.

We need to handle the maximum multiplicity events, $n = N$, differently, otherwise we would have under-counting with no events with $N + 1$ partons above μ.

- Use q_N instead of μ in the Sudakov form factors
- Veto shower emissions with $k_\perp > q_N$.

I.e. we allow PS to give splittings above μ as long as they are softer than q_N.

In all merging procedures we will have a dependence on μ.

- $C_n^{\mathrm{ME}} \neq C_n^{\mathrm{PS}}$.
- There may be differences in the implementation of the Sudakovs.
- There are differences in ordering if $\rho \neq k_\perp$.
  (e.g. a gluon at a given phase-space point may be emitted from a 3-parton state when ordered in $k_\perp$, but from a 4-parton state when ordered in ρ)

But if the parton shower is correct to NLL, the dependence on the cutoff cancels to NLL accuracy.

The whole CKKW procedure

1. Choose a parton multiplicity $n \leq N$ according to the integrated cross sections and generate a state.
2. Construct a series of emission scales $q_1, \ldots, q_n$ using the $k_\perp$-algorithm.
3. Reweight event with running coupling $\prod_i^n \frac{\alpha_s(q_i)}{\alpha_s^{\mathrm{ME}}}$.
4. Calculate analytical Sudakov form factors and reweight.
   - Each internal line gives a factor $\frac{\Delta_i(q_p,\mu)}{\Delta_i(q_d,\mu)}$
   - Each external line gives a factor $\Delta_i(q_p, \mu)$.

   If $n = N$ use q_N instead of μ.
5. Add a parton shower with the maximum scale set to $\rho_0 = Q$, but veto any emission with a jet-scale above μ. (for $n = N$, veto above q_N).

Catani-Krauss-Kuhn-Webber

CKKW-L is very close to CKKW in spirit but differs in the way Sudakovs are determined an how the shower is applied.

The procedure requires a parton shower with complete on-shell intermediate states which can be stopped and restarted at any point.

This is true for e.g. ARIADNE and $p_\perp$-ordered PYTHIA, but not for e.g. HERWIG.

(HERWIG first generated splitting variables for all emissions and only reconstructs the exact kinematics in the end.)

Catani-Krauss-Kuhn-Webber-and-me

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(HERWIG first generated splitting variables for all emissions and only reconstructs the exact kinematics in the end.)

In CKKW-L we use the parton shower to define a clustering algorithm. For each state generated by the ME we try to answer the question
how would the parton shower have generated this state?

We reconstruct the emission scales $q_i = \rho_i$
and the complete kinematics of the intermediate states S_i

We can still use e.g. the $k_\perp$-algorithm to define the ME cutoff, μ.

Sometimes there are several different shower histories possible. Then we choose one according to the product of the PS splitting functions for the different histories.

Sometimes no proper history can be found. Then we only cluster as far as possible.

Sometimes no *ordered* history can be found. Then if $q_i < q_{i+1}$ we set eg. $q_i = q_{i+1}$. (No Sudakov for the state S_i.)

The Sudakov-veto algorithm.

We interpret the Sudakov form factor strictly as a no-emission probability.

We have the states $S_0, \ldots, S_n$ and emission scales $\rho_1, \ldots, \rho_n$.

We want $\Delta_{S_i}(\rho_i, \rho_{i+1})$, which is the probability that there is no emission from the state S_i between emission scales ρ_i and ρ_{i+1}.

Start the parton shower from the state S_i with ρ_i as maximum scale. (for S_0, use $\rho_0 = \rho_{\mathrm{max}}$)

Generate one emission giving a emission scale ρ.

The probability that $\rho < \rho_{i+1}$ is exactly $\Delta_{S_i}(\rho_i, \rho_{i+1})$.

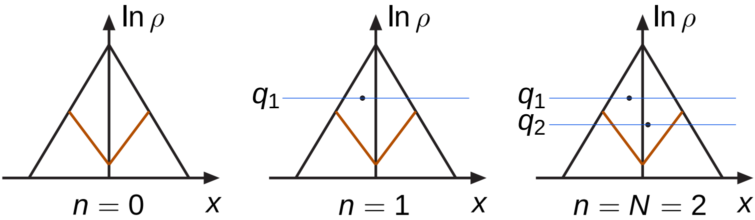
Throwing away the event if $\rho > \rho_{i+1}$ corresponds exactly to weighting with $\Delta_{S_i}(\rho_i, \rho_{i+1})$.

For $S_{n\neq N}$ we need to calculate $\Delta_{S_n}(\rho_n, \rho_c; \mu)$, the probability that there is no emission from S_n above the jet cutoff, μ.

So we generate one emission from S_n with ρ_n as maximum scale, and if the resulting state S_{n+1} passes the jet cutoff we veto the event. Otherwise we simply continue the shower.

For $n = N$, we simply add the cascade from S_N with ρ_N as maximum scale.

CKKW-L $[\mu = \mu(\rho, \boldsymbol{x})]$



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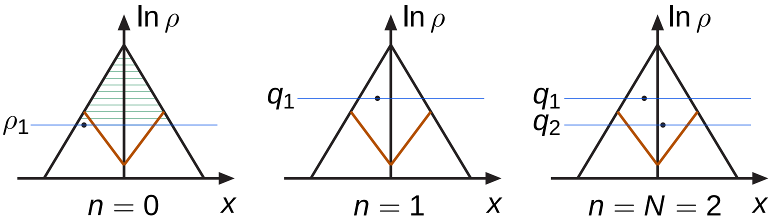

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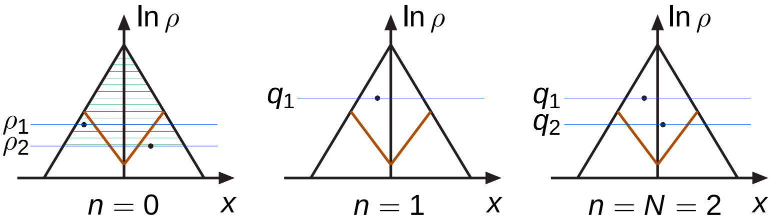

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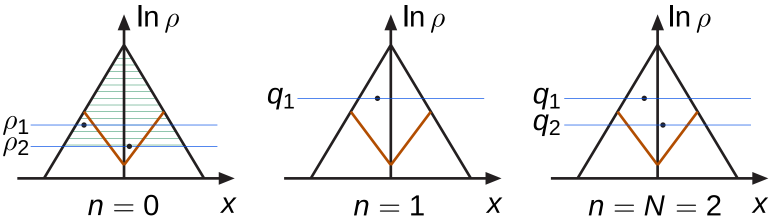

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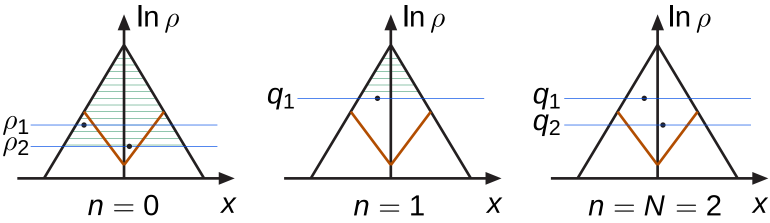

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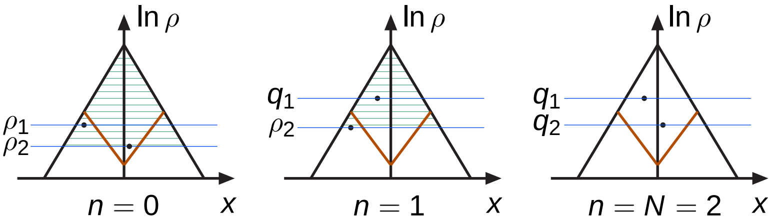

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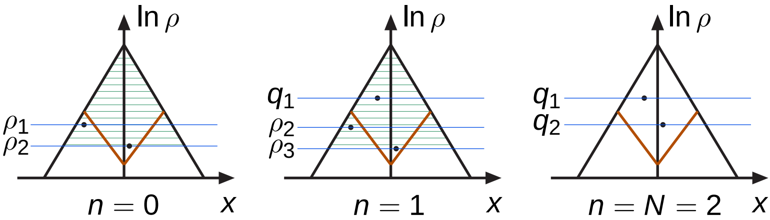

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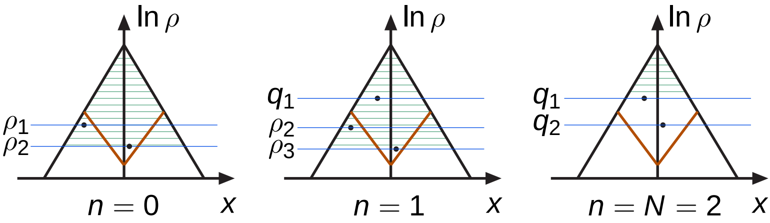

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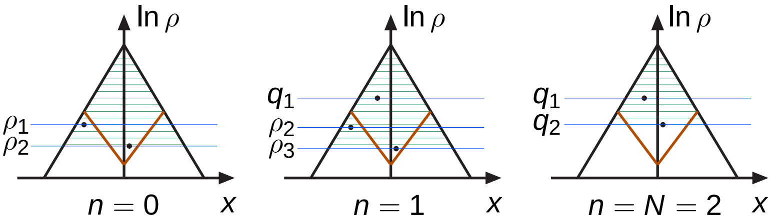
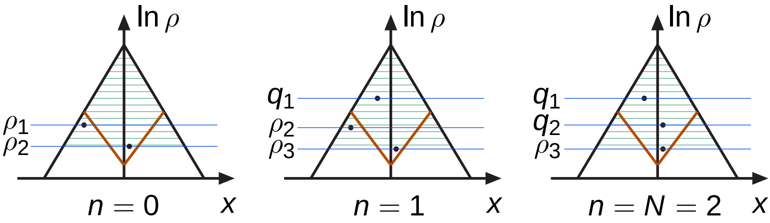


CKKW-L $[\mu = \mu(\rho, \boldsymbol{x})]$



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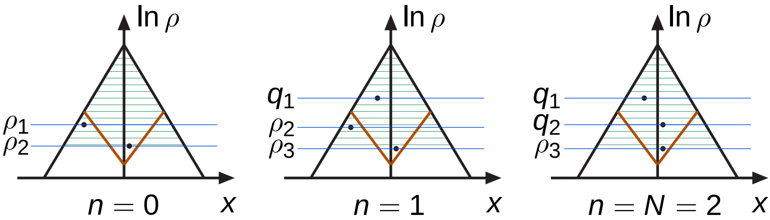
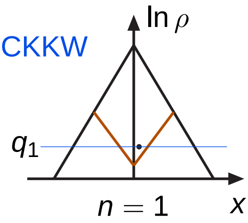
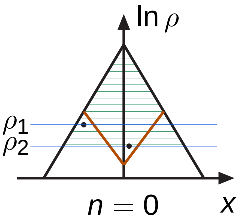
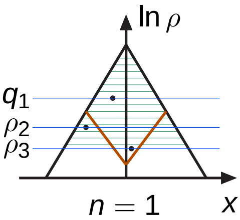


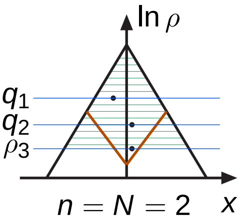
CKKW



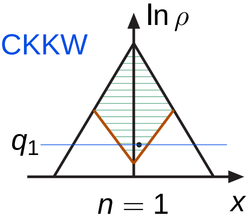
CKKW-L $[\mu = \mu(\rho, \boldsymbol{x})]$







CKKW



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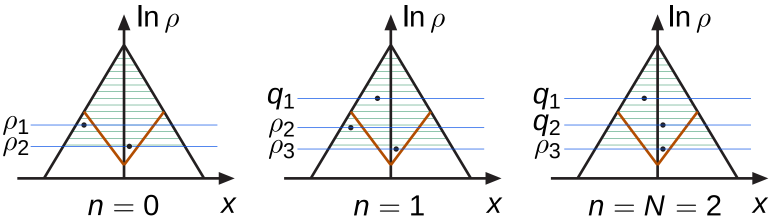
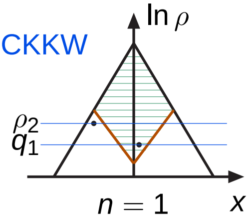


CKKW



In CKKW-L we have exactly the same Sudakov form factors for emissions above or below the jet cutoff, μ.

All shower emissions are ordered.

Only the $n \leq N$ first emissions (as ordered in ρ) will be corrected if all are above μ.

The whole CKKW-L procedure

1. Choose a parton multiplicity $n \leq N$ according to the integrated cross sections and generate a state.
2. Construct emission scales $q_1, \ldots, q_n$ and intermediate states $S_0, \ldots, S_{n-1}$ using the "inverse shower".
3. Reweight event with running coupling $\prod_i^n \frac{\alpha_s(q_i)}{\alpha_s^{\mathrm{ME}}}$.
4. For each state $S_{i<n}$ make a trial emission below q_i. If emission is larger than q_{i+1} veto event (goto 1).
5.

$n < N$ Make a trial emission from S_n below q_n. If resulting S_{n+1} passes jet cutoff, veto event (goto 1) otherwise accept emission and continue shower.

$n = N$ Add shower from S_N below q_N.

Mrennas Pseudo Shower Prescription

What if you want to use CKKW-L but don't have a shower with on-shell intermediate states?

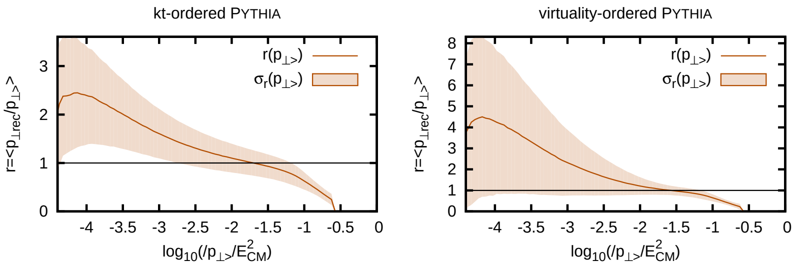
You can start your cascade from S_i from q_i and run it down to the shower cutoff. Then cluster the event back to an S_{i+1} state, using ρ as a jet measure. If $\rho_{i+1} > q_{i+1}$ you veto the event and get $\Delta_{S_i}(q_i, q_{i+1})$

In Mrennas original implementation the virtuality-ordered shower in PYTHIA was used together with a $k_\perp$-ordered jet algorithm (based on LUCLUS).

From each constructed state S_i, the shower was then started from the maximum scale $\rho = Q$ and run down to the shower cutoff, vetoing all emissions above q_i.

The resulting state was $k_\perp$-clustered back to a S'_{i+1} giving a q'_{i+1}. Basically answering the question

How would a $k_\perp$-ordered shower have produced the first emission?



Clustering back a fully showered partonic state does not give the correct emission scales, even if clustering is done in the shower evolution variable.

The whole Pseudo Shower procedure

1. Choose a parton multiplicity $n \leq N$ according to the integrated cross sections and generate a state.
2. Construct emission scales $q_1, \ldots, q_n$ and intermediate states $S_0, \ldots, S_{n-1}$ using the $k_\perp$-algorithm.
3. Reweight event with running coupling $\prod_i^n \frac{\alpha_\mathrm{s}(q_i)}{\alpha_\mathrm{s}^\mathrm{ME}}$.
4. For each state $S_{i<n}$ add a shower starting from $\rho_\mathrm{max} = Q$, vetoing emissions above q_i. Cluster to a S'_{i+1} state with the $k_\perp$-algorithm and determine q'_{i+1}.
   If $q'_{i+1} > q_{i+1} + d_\mathrm{fudge}$, veto event (goto 1).
5. Start shower from S_n vetoing emissions above q_n and cluster back to S'_{n+1}

$n < N$ If $q'_{n+1} > \mu + d_\mathrm{fudge}$, veto event (goto 1).
$n = N$ If $q'_{N+1} > q_N + d_\mathrm{fudge}$, veto event (goto 1).

Michelangelo Manganos (MLM) procedure

What if we simply add a shower to the state, S_n, produced by the ME generator with some jet cutoff, μ?

Start the shower from some maximum scale,$\rho_{\max}$ and run down to the shower cutoff. No veto on the emissions.

We then cluster back using a jet algorithm with resolution scale μ and obtain a jet state S'_m.

The probability that no extra jets were produced ($m = n$) and that the jets match the directions of the original partons should give the probability that the parton shower did not make any emissions above μ, irrespectively of the ordering in the shower.

Hence, throwing away the event if this is not the case corresponds to reweighting with a no-emission probability.

Voila! We have our Sudakov form factor!

The original implementation in ALPGEN uses a cone algorithm, but we want to test the method for e^+e^-, so we will use the $k_\perp$-algorithm instead (Similar to the implementation in MADEVENT).

Remember the scale definition of the $k_\perp$-algorithm

$$k_{\perp ij}^2 = \min(E_i^2, E_j^2)\left(1 - \cos\theta_{ij}\right)$$

We use this scale for the cutoff in the ME generator, and in the clustering of the showered state.

If the clustered state does not have the same number of jets as partons in the original state we throw away the event.

Consider the original partons in order of decreasing energy. Find the closest jet, but use the measure

$$k'^2_{\perp ij} = E^2_{\mathrm{jet}}\left(1 - \cos\theta_{\mathrm{parton,jet}}\right)$$

(we cannot use $\min(E^2_{\mathrm{parton}}, E^2_{\mathrm{jet}})$ because then soft partons could match jets at very wide angle).

If $k'_{\perp ij} > \mu$, throw away the event, otherwise remove the matched jet and continue with the next parton.

For the maximum parton multiplicity, $n = N$, we allow extra jets and relax our matching criteria to $k'_{\perp ij} < q_N$, but require that the N hardest jets are matched.

Alternatively use q_N as resolution scale in clustering and matching.

The MLM procedure suffers from the same problems as the Pseudo Shower: The clustering will not exactly reconstruct the shower emissions.

In MLM this is alleviated by introducing a lower cut on the ME generator $\mu_c < \mu$. In this way both the ME and PS cut is applied in the same way.

The result should be independent on μ_c as long as it is small enough.

The modeling of the Sudakov form factor will never be exact.

The shower will be forced to do un-ordered emissions.

If we have two partons separated by a scale q_i, we require the shower to emit from these partons, above this scale, in the same way as from a single parton.

But different parton showers will have different ways of limiting the emissions depending on the multi-partonic state given.

In PYTHIA, the limiting factor is the maximum scale given, and the energies of the partons.

In HERWIG, also the angles between the partons will limit the shower.

But even if we had a perfect shower which could handle unordered emissions and get the Sudakov Δ_{S_0} correct, we would still have problems:

For $\mu_c \to 0$, there would always be a finite probability to get a jet which would match an infinitely soft parton, and the event could be accepted. This would give a very strong dependence on μ_c.

The whole MLM procedure (modified for e^+e^-)

1. Choose a parton multiplicity $n \leq N$ according to the integrated cross sections using a cutoff, μ_c, and generate a state.
2. Construct emission scales $q_1, \ldots, q_n$ using a jet algorithm.
3. Reweight event with running coupling $\prod_i^n \frac{\alpha_s(q_i)}{\alpha_s^{\mathrm{ME}}}$.
4. Add a parton shower starting from the maximum possible scale.
5. Cluster the partonic state using the jet algorithm using μ as resolution scale and obtain a jet state S'_m. If $m \neq n$ or if not all partons match a jet below the scale μ, throw away the event (goto 1).
   If $n = N$, replace μ with $\max(\mu, q_N)$.

The e^+e^- test bench

To understand the features of the different procedures we will use the simplest possible test case: $e^+e^- \to q\bar{q}$.

To make it even simpler we will only consider one extra jet from the ME generator: $e^+e^- \to qg\bar{q}$.

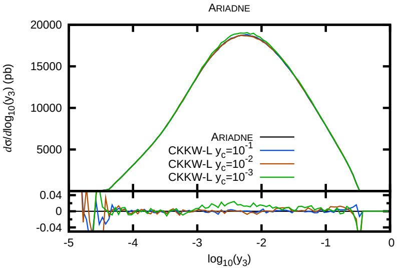
In addition we here know the “correct” answer, in that we can use the reweighting method, where there is no dependence on a jet cutoff.

In all cases we will use the $k_\perp$-algorithm to define the cutoff μ.

We will then look at the $y_3 = k_\perp^2/E_{\mathrm{cm}}^2$ distribution for the produced partonic state. I.e. the value of the resolution parameter for which the $k_\perp$-algorithm clusters a three-jet state into a two-jet state.

This distribution should be the most sensitive to the cutoff μ.

CKKW-L


ARIADNE
20000
15000
10000
5000
0.04
0
-0.04
-5
-4
-3
-2
-1
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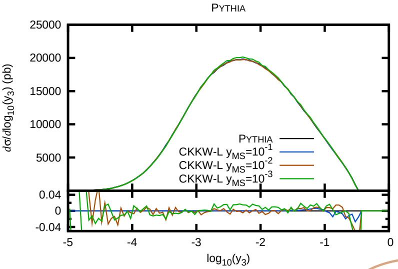
$d\sigma/d\log_{10}(y_3)$ (pb)
$\log_{10}(y_3)$
ARIADNE
CKKW-L $y_c=10^{-1}$
CKKW-L $y_c=10^{-2}$
CKKW-L $y_c=10^{-3}$


PYTHIA
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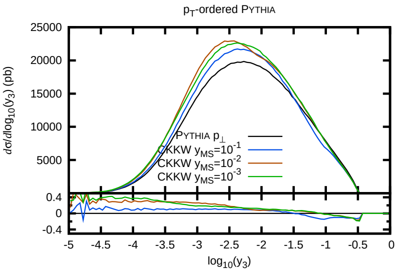
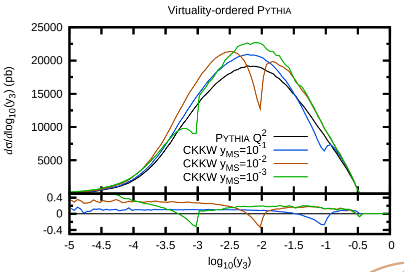
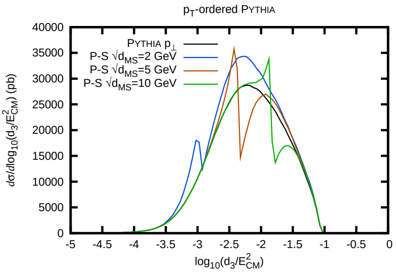
CKKW-L $y_{MS}=10^{-3}$

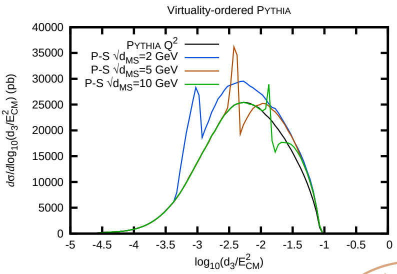
CKKW


pT-ordered PYTHIA
dσ/dlog10(y3) (pb)
log10(y3)
PYTHIA p⊥
CKKW yMS=10^-1
CKKW yMS=10^-2
CKKW yMS=10^-3

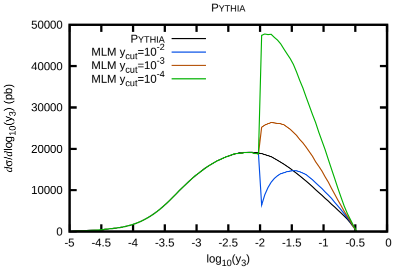

Virtuality-ordered PYTHIA
dσ/dlog10(y3) (pb)
log10(y3)
PYTHIA Q^2
CKKW yMS=10^-1
CKKW yMS=10^-2
CKKW yMS=10^-3

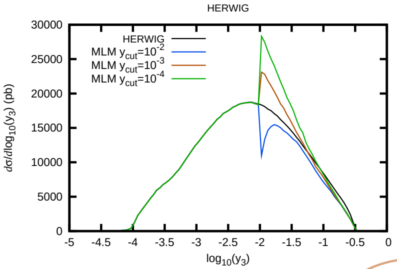
Pseudo Shower





MLM


PYTHIA
PYTHIA
MLM ycut=10-2
MLM ycut=10-3
MLM ycut=10-4
dσ/dlog10(y3) (pb)
log10(y3)
50000
40000
30000
20000
10000
0
-5
-4.5
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0


HERWIG
HERWIG
MLM ycut=10-2
MLM ycut=10-3
MLM ycut=10-4
dσ/dlog10(y3) (pb)
log10(y3)
30000
25000
20000
15000
10000
5000
0
-5
-4.5
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0

This may look bad for some procedures. But

- Hadronization effects tend to smooth things out.
- We are may only interested in observables far above μ.
- Formally they may still be NLL correct.
- In *pp* the problems seem to be smaller (especially for MLM).

Outline of Lectures

- Lecture I: Introduction, Tree-level ME, NLO, PS, ordering, basic strategies, . . .
- Lecture II: Tree-level ME merging with PS, CKKW(-L), Pseudo Shower, MLM, e^+e^- comparison, . . .
- Lecture III: ME+PS merging in *pp*, NLO matching with PS, MC@NLO, POWHEG, NL^3, . . .