Electroweak Theory in the Standard Model

Selected Tutorial Problem Set

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Instructions

There is one problem corresponding to each lecture. Each problem contains two related parts and tests material retained in the revised core syllabus. State any approximations and explain the physical meaning of each result. Total: 90 marks.

Lecture 1: From Fermi Theory to Electroweak Gauge Theory

Problem 1: The range and validity of Fermi theory **[15 marks]**

At momentum transfer q, the propagator factor for exchange of a massive W boson contains

$$\frac{1}{q^2 - M_W^2}.$$

(a) Expand this expression through order q^2/M_W^2. Taking the characteristic scale in muon decay to be $q^2 \sim m_\mu^2$, estimate m_μ^2/M_W^2 and comment on the accuracy of the Fermi contact interaction for muon decay. **[8 marks]**

(b) Using dimensional analysis in four spacetime dimensions, show that a four-fermion operator has mass dimension six. Hence explain why $G_F E^2$ is the dimensionless expansion parameter and why Fermi theory must eventually fail as the energy increases. **[7 marks]**

Lecture 2: Building the Electroweak Gauge Theory

Problem 2: Electric charge and neutral gauge interactions **[15 marks]**

Use $Q = T_3 + Y$. The left-handed lepton and quark doublets have hypercharges $Y_L = -\frac{1}{2}$ and $Y_Q = \frac{1}{6}$, while the right-handed singlets have $Y_{e_R} = -1$, $Y_{u_R} = \frac{2}{3}$ and $Y_{d_R} = -\frac{1}{3}$.

(a) Verify the electric charge of every component of $L_L = (\nu_L, e_L)^T$, $Q_L = (u_L, d_L)^T$, e_R, u_R and d_R. Explain why $T_3 = 0$ for the right-handed fields. **[6 marks]**

(b) Starting from

$$\mathcal{L}_{\text{neutral}} = -\bar{\psi}\gamma^\mu (g T_3 W_\mu^3 + g' Y B_\mu)\psi,$$

use

$$A_\mu = s_W W_\mu^3 + c_W B_\mu, \qquad Z_\mu = c_W W_\mu^3 - s_W B_\mu,$$

with $e = g s_W = g' c_W$, to derive the photon coupling eQ and the Z coupling

$$\frac{g}{c_W}(T_3 - s_W^2 Q).$$

[9 marks]

Lecture 3: Electroweak Symmetry Breaking

Problem 3: Gauge-boson masses from the Higgs vacuum **[15 marks]**

Let the Higgs doublet have $Y = \frac{1}{2}$ and

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}, \qquad D_\mu = \partial_\mu - i\frac{g}{2}\tau^a W_\mu^a - ig' Y B_\mu.$$

(a) Evaluate $(D_\mu\langle\Phi\rangle)^\dagger(D^\mu\langle\Phi\rangle)$ explicitly and obtain $m_W = gv/2$. **[7 marks]**

(b) Construct and diagonalise the neutral gauge-boson mass matrix in the (W_μ^3, B_μ) basis. Find its normalised eigenvectors and eigenvalues, identify the photon and Z, and show that $m_Z = v\sqrt{g^2 + g'^2}/2$. **[8 marks]**

Lecture 4: Electroweak Phenomenology and Precision Tests

Problem 4: Neutral-current couplings and asymmetries **[15 marks]**

Use

$$g_V^f = T_3^f - 2Q_f s_W^2, \qquad g_A^f = T_3^f, \qquad A_f = \frac{2g_V^f g_A^f}{(g_V^f)^2 + (g_A^f)^2}.$$

(a) Calculate g_V^f and g_A^f for $f = \nu, e, u, d$. Why are charged-lepton asymmetries particularly sensitive to s_W^2? **[7 marks]**

(b) Near the Z pole, take the angular distribution to be proportional to

$$(1 + \cos^2\theta) + 2A_e A_f \cos\theta.$$

Integrate over the forward and backward hemispheres and derive

$$A_{\mathrm{FB}}^f = \frac{3}{4}A_e A_f.$$

[8 marks]

Lecture 5: Flavour Physics, CP Violation and Muon $g-2$

Problem 5: The origin of CKM mixing and GIM suppression **[15 marks]**

In the weak basis,

$$\mathcal{L}_{\mathrm{CC}} = -\frac{g}{\sqrt{2}}\bar{u}'_L\gamma^\mu d'_L W_\mu^+ + \text{h.c.},$$

and $u'_L = U_{uL}u_L$, $d'_L = U_{dL}d_L$.

(a) Derive $V_{\text{CKM}} = U_{uL}^\dagger U_{dL}$ in the charged current. Explain why right-handed diagonalisation matrices do not appear. **[7 marks]**

(b) A flavour-changing neutral-current amplitude contains

$$\sum_{i=u,c,t} V_{is}^* V_{id} F(m_i^2/M_W^2).$$

Use CKM unitarity to show that the mass-independent part of F cancels. What happens if all internal up-type quarks have equal masses, and what does this demonstrate physically? **[8 marks]**

Lecture 6: From the Standard Model to Grand Unification

Problem 6: Testing a grand-unified theory **[15 marks]**

At one loop,

$$\mu \frac{dg_i}{d\mu} = \frac{b_i}{16\pi^2} g_i^3, \qquad \alpha_i = \frac{g_i^2}{4\pi}.$$

A heavy GUT gauge boson X of mass M_X and coupling g_5 can mediate proton decay.

(a) Integrate the beta function to show that

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(\mu_0) - \frac{b_i}{2\pi} \ln\left(\frac{\mu}{\mu_0}\right).$$

Explain why a new particle threshold changes the slope of a unification plot. **[7 marks]**

(b) Explain why integrating out X produces an effective four-fermion coefficient of order g_5^2/M_X^2. Using dimensional analysis, show that $\tau_p \propto M_X^4$ when other parameters are held fixed. Why does this make proton decay a sensitive test of very high scales? **[8 marks]**