Theoretical aspects of neutrino masses

G.G.Ross, Cosener's House, April 2008

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- Dirac or Majorana? — $m_D \overline{\nu_L} \nu_R$ or $m_M \overline{\nu_R^c} \nu_R$
- How many neutrinos? — Steriles? Modulini?
- $q \leftrightarrow l$? — See-saw?
- Mass hierarchy? — Degenerate or not?
- Origin of masses and mixings? — Symmetries?
- $\cancel{C}P$

DATA :

3 neutrinos (?)

… additional sterile components?

LSND $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ $m_{\nu_s} \simeq 1eV\,?$

MiniBoone $\nu_\mu \rightarrow \nu_e$ no evidence for sterile

LSND and MiniBoone consistent in (3+2), (3+2) schemes (new ~~CP~~ phases)

… but tension between appearance/disappearance expts

Maltoni, Schwetz
Goldman et al; Goswani et al

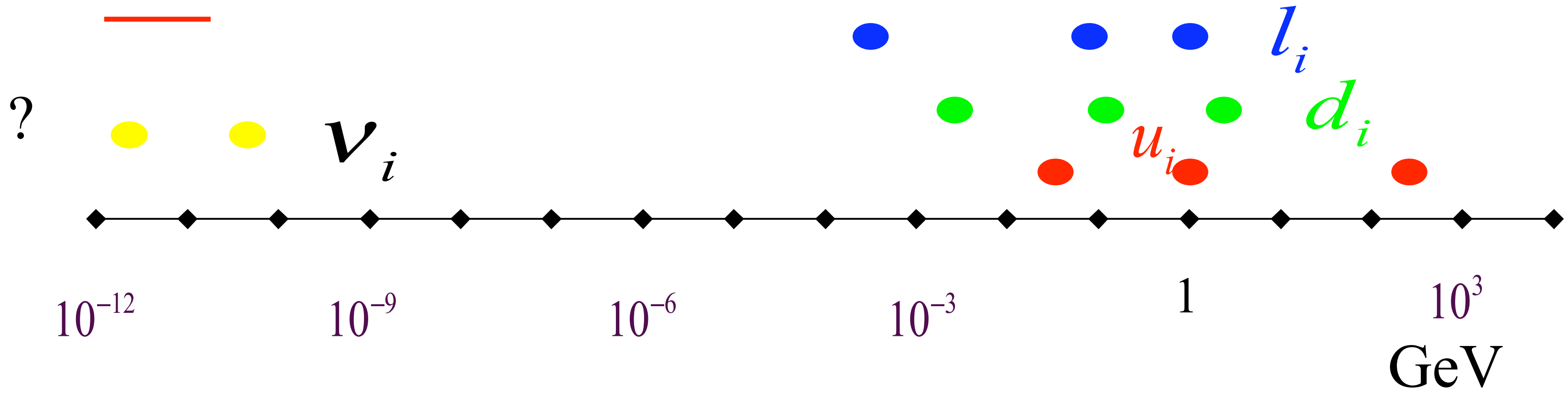
Supernovae provide more sensitive test of sterile neutrino components

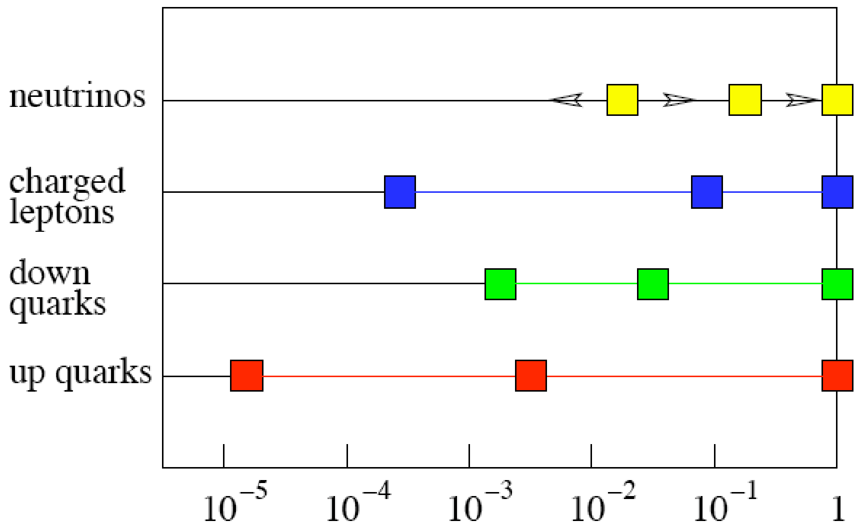
Raffelt et al, Choubey et al

DATA :

3 neutrinos

Masses



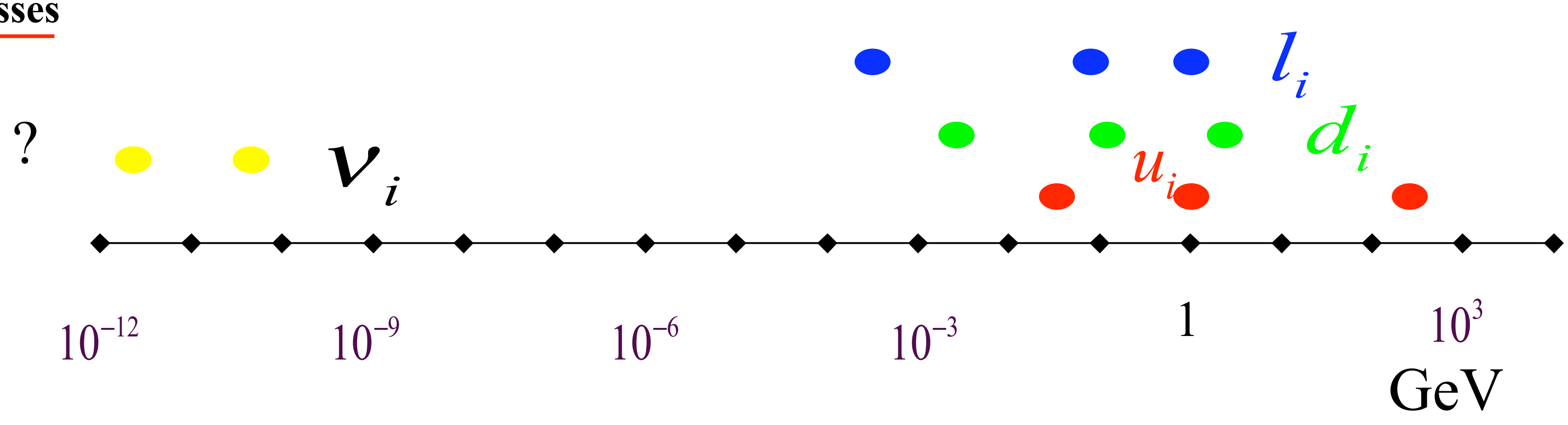


Mass ratios

DATA :

3 neutrinos

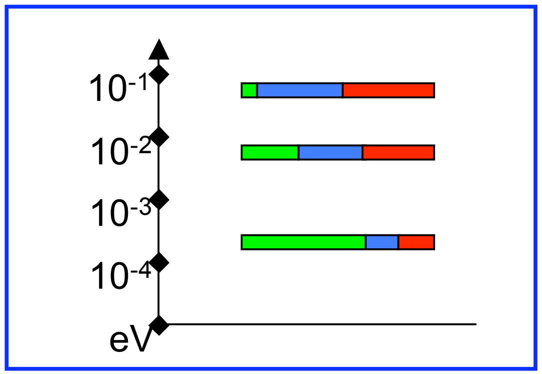
Masses



Mixing

$$V_{CKM} \approx \begin{pmatrix} 1 & 0.218-0.224 & 0.002-0.005 \\ 0.218-0.224 & 1 & 0.032-0.048 \\ 0.004-0.015 & 0.03-0.048 & 1 \end{pmatrix}$$

$$V_{MNS} = \begin{pmatrix} 0.72-0.89 & 0.45-0.69 & <0.2 \\ 0.24-0.58 & 0.39-0.76 & 0.52-0.84 \\ 0.24-0.58 & 0.39-0.76 & 0.53-0.84 \end{pmatrix}$$



$$\approx \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & \sim 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Bi-Tri Maximal Mixing …

Non Abelian Structure?

Harrison, Perkins, Scott

- Dirac or Majorana?

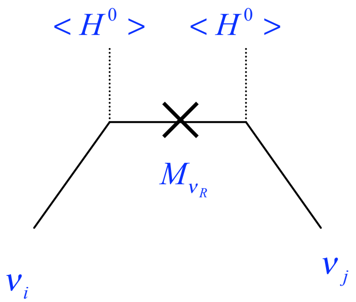
Dirac or Majorana?

$$\frac{f_{ij}}{\Lambda}\nu_i\nu_j H^0H^0 \quad\Rightarrow\quad (M_\nu)_{ij} = f_{ij}\frac{<H^0>^2}{\Lambda}$$

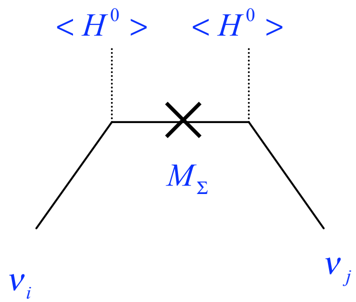
Weinberg 1979

	$SU(2)_L$ singlets		$SU(2)_L$ triplets
(A)	$\nu_iH^0 - l_iH^+$	(D)	$[\nu_iH^+, (\nu_iH^0 + l_iH^+)/\sqrt{2}, l_iH^0]$
(B)	$\nu_il_j - l_i\nu_j$	(E)	$[\nu_i\nu_j, (\nu_il_j + l_i\nu_j)/\sqrt{2}, l_il_j]$
(C)	$H_1^+H_2^0 - H_1^0H_2^+$	(F)	$[H^+H^+, \sqrt{2}H^+H^0, H^0H^0]$

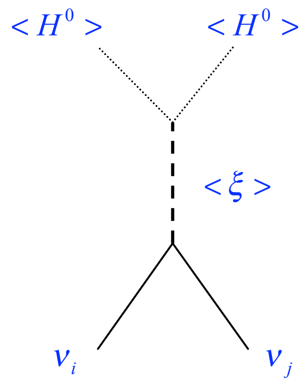
See-saw



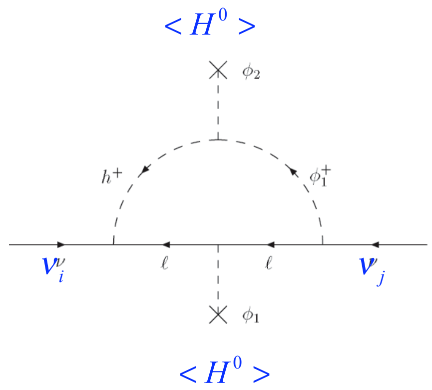
$(A)\times(A)$



$(D)\times(D)$



$(E)\times(F)$



$(B)\times(C)$

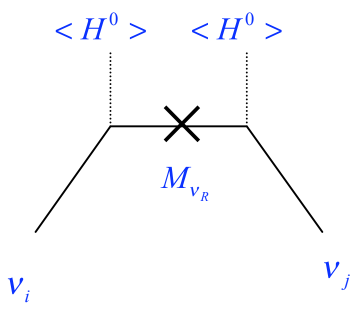
Dirac or Majorana?

$$\frac{f_{ij}}{\Lambda}\nu_i\nu_j H^0H^0 \quad \Rightarrow \quad (M_\nu)_{ij} = f_{ij}\frac{<H^0>^2}{\Lambda}$$

Weinberg 1979

	$SU(2)_L$ singlets		$SU(2)_L$ triplets
(A)	$\nu_i H^0 - l_i H^+$	(D)	$[\nu_i H^+, (\nu_i H^0 + l_i H^+)/\sqrt{2}, l_i H^0]$
(B)	$\nu_i l_j - l_i \nu_j$	(E)	$[\nu_i \nu_j, (\nu_i l_j + l_i \nu_j)/\sqrt{2}, l_i l_j]$
(C)	$H_1^+ H_2^0 - H_1^0 H_2^+$	(F)	$[H^+H^+, \sqrt{2}H^+H^0, H^0H^0]$

Minkowski mechanism (Type I see-saw)



$(A)\times(A)$

$$m_{\nu_i} \sim \frac{<H^0>^2}{M_{\nu_R}}$$

$$m_{\nu_R} \sim 10^{13}\,GeV\left(\frac{H^0}{10^2\,GeV}\right)^2$$

$$\left(M_\nu = M_D^\nu\, M_M^{-1}\, M_D^{\nu T}\right)$$

- Dirac or Majorana?

$$\frac{f_{ij}}{\Lambda}\nu_i\nu_j H^0 H^0 \quad \Rightarrow \quad (M_\nu)_{ij} = f_{ij}\frac{<H^0>^2}{\Lambda}$$

Weinberg 1979

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(A)	$\nu_i H^0 - l_i H^+$	(D)	$[\nu_i H^+, (\nu_i H^0 + l_i H^+)/\sqrt{2}, l_i H^0]$
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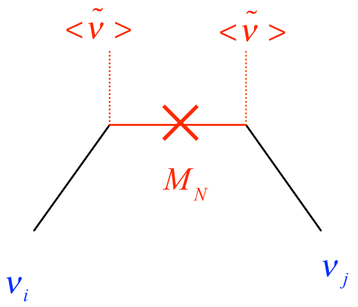
(C)	$H_1^+ H_2^0 - H_1^0 H_2^+$	(F)	$[H^+ H^+, \sqrt{2} H^+ H^0, H^0 H^0]$

$SUSY\ \not{R}_P$



$(A)\times(A)$

$$m_{\nu_i} \sim \frac{<H^0>^2}{M_{\nu_R}}$$

$$m_{\nu_R} \sim 10^{13}\,GeV\left(\frac{H^0}{10^2\,GeV}\right)^2$$

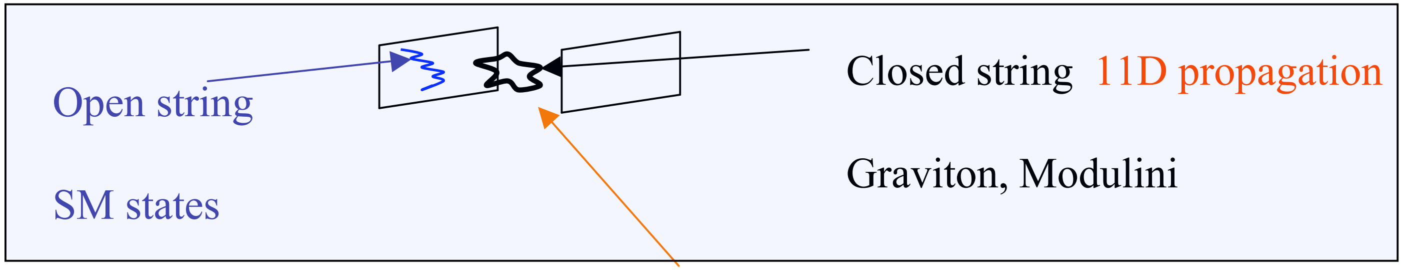
$$m_{\nu_i} \sim \frac{<\tilde{\nu}>^2}{M_N}$$

$$M_N \sim 10^3\,GeV\left(\frac{<\tilde{\nu}>}{1MeV}\right)^2$$

Dirac or Majorana? $\quad h\,\bar{\nu}_L \nu_R H, \qquad m = h < H > \quad : \quad h < 10^{-12}$?

- New space dimensions

Arkani Hamed, Dimopoulos, March Russell
Dienes,Farragi



Right-handed ν^s in the bulk

$$\lambda \int d^4x\, \nu_L(x) H(x) \nu_R(x, \underbrace{y=0}_{\sum_n \frac{1}{\sqrt{2\pi R M_*}} \nu_{R,n}(x) e^{iny/R}})$$

Flux spreading

$$m_\nu = \frac{\lambda < H > M_*}{M_{Planck}} \approx \lambda . 10^{-4} eV \left(\frac{M_*}{TeV} \right)$$

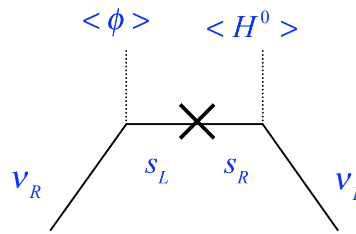
- In 4D small Dirac masses can arise through higher dimension operators

$$e.g. \quad K \supset \frac{F(\sigma,\sigma^*)}{M} L H_u \bar{N} + \frac{F'(\sigma,\sigma^*)}{M} L H_d^* \bar{N}$$

$$m_\nu \simeq \frac{\langle \phi \rangle}{M} < H >, \quad M \sim 10^{16} - 10^7\, GeV$$

Abel, Dedes, Tamvakis

or

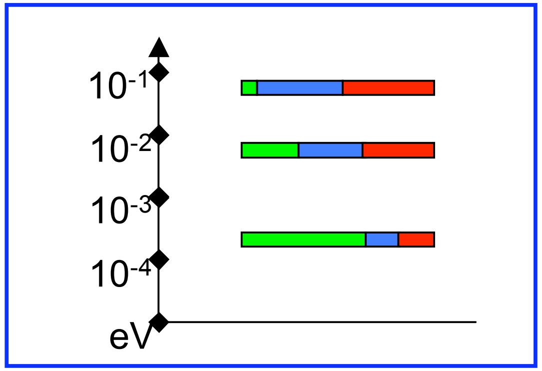


$$m_\nu \simeq \frac{\langle \phi \rangle}{M} < H >$$

Cerdeno, Dedes, Underwood

Masses and mixings

$$V_{CKM} \approx \begin{pmatrix} 1 & 0.218-0.224 & 0.002-0.005 \\ 0.218-0.224 & 1 & 0.032-0.048 \\ 0.004-0.015 & 0.03-0.048 & 1 \end{pmatrix} \qquad V_{MNS} = \begin{pmatrix} 0.72-0.89 & 0.45-0.69 & <0.2 \\ 0.24-0.58 & 0.39-0.76 & 0.52-0.84 \\ 0.24-0.58 & 0.39-0.76 & 0.53-0.84 \end{pmatrix}$$



$$\approx \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & \sim 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Bi-Tri Maximal Mixing …

$$L^{\nu}_{eff} = a\, \psi_i \theta^i_{23} \psi_j \theta^j_{23} H + b\, \psi_i \theta^i_{123} \psi_j \theta^j_{123} H \qquad L^{l,q} = c\, \psi_i \theta^i_3 \psi^c_j \theta^j_3 H + \ldots$$

$$\theta_{23} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \qquad \theta_{123} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \theta_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Can one have a unified description of quark, charged lepton and neutrinos?

$$L^{\nu}_{eff} = \beta\, \psi_i \theta^i_{23} \psi_j \theta^j_{23} H + \gamma\, \psi_i \theta^i_{123} \psi_j \theta^j_{123} H \qquad L^{l,q} = \alpha\, \psi_i \theta^i_3 \psi^c_j \theta^j_3 H + \dots$$

$$\Rightarrow L^{q,l,\nu}_{Dirac} = \alpha\, \psi_i \theta^i_3 \psi^c_j \theta^j_3 + \beta\, \psi_i \theta^i_{123} \psi^c{}_j \theta^j_{23} + \gamma\, \psi_i \theta^i_{23} \psi^c{}_j \theta^j_{123} \qquad \alpha > \beta > \gamma$$

…possible with "see-saw" plus non Abelian (discrete) family symmetry!

$e.g.\, SU(3)$ family symmetry $\qquad \psi_i, \psi^c_i \sim 3 \quad \theta_3, \theta_{23}, \theta_{123} \sim \bar{3}$

$$\begin{array}{lccc} & \theta_3 & & \\ q_i, l_i & SU(3) & \rightarrow SU(2) & \rightarrow .. \\ & 45^0 & & 35^0 \\ \nu_i & SU(3) & \rightarrow SU(2)' & \rightarrow .. \\ & \theta_{23} & & \theta_{123} \end{array}$$

??? Decoupling + vacuum alignment

$$\theta_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \theta_{23} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \theta_{123} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

- ## Decoupling of θ_3 contribution to neutrino masses

"See-saw" with sequential domination $$M_\nu = M_D^\nu\, M_M^{-1}\, M_D^{\nu T}$$ SRHD King

$$M_M \approx \begin{pmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{pmatrix} \qquad M_1 << M_2 << M_3 \qquad M_3 \text{ suppresses } \theta_3 \text{ contribution}$$

$$L_{Dirac}^{l,\nu} = \alpha\, \psi_i \theta_3^i \psi_j^c \theta_3^j + \beta\, \psi_i \theta_{123}^i \psi^c{}_j \theta_{23}^j + \gamma\, \psi_i \theta_{23}^i \psi^c{}_j \theta_{123}^j \qquad \alpha > \beta > \gamma$$

small

$$\boxed{L_{eff}^\nu = \frac{\beta^2}{M_1} \psi_i \theta_{123}^i \psi_j \theta_{123}^j + \frac{\gamma^2}{M_2} \psi_i \theta_{23}^i \psi_j \theta_{23}^j + \frac{(\alpha+\beta+\gamma)^2}{M_3} \psi_i \theta_3^i \psi_j \theta_3^j}$$

$(\phi_{23} \to \phi_{23}, \quad \phi_{123} \to -\phi_{123}$ Symmetry)

Varzielas, GGR

- ## Vacuum alignment

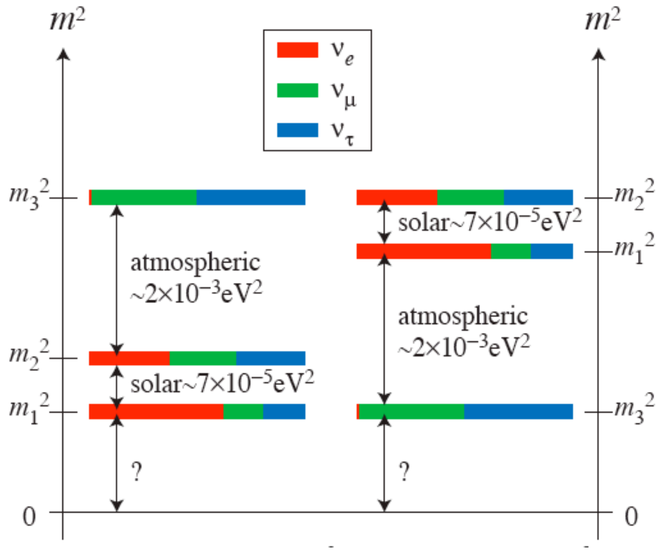
(Discrete) Non-Abelian family symmetry

$$\theta_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \theta_{23} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \quad \theta_{123} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \left(\begin{array}{c} \theta_3 \\ Z_3 \times Z'_3 \to Z_3 \text{ or } Z'_3 \\ \theta_{123} \end{array} \right)$$

Masses and mixings

- Degenerate or not?

  Normal or hierarchical?



$SO(3)$ family symmetry

$$l \equiv \left(\begin{pmatrix} e \\ \nu_e \end{pmatrix}, \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}, \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix} \right)$$

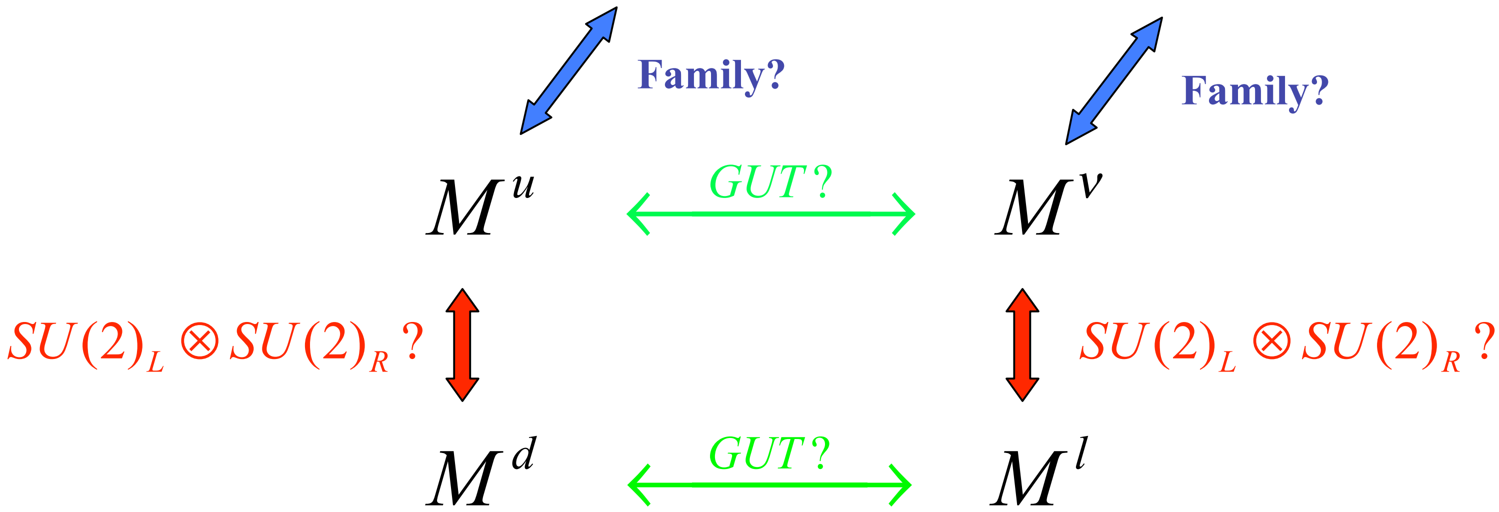
$$\underline{W_1 = \frac{f}{\Lambda}\left(\sum_i l_i l_i\right) hh}$$

degenerate

$$\boxed{W = W_1 + (l.\chi_3)^2\, hh + (l.\chi_{23})^2\, hh + (l.\phi_{23})\,\tau h + (l.\phi_{123})\,\mu h \qquad \chi_3 \propto (0,0,1),\ \chi_{23} \propto (0,i,1),\ \phi_{23} \propto (0,i,1),\ \phi_{123} \propto (1,-i,1)}$$

Barbieri, Hall, Kane, GGR
Serna, Varzielas
Antusch, King

- $q \leftrightarrow l$? GUT symmetry



GUT relations

$$Det(M^l) = Det(M^d)|_{M_X}$$

$$|V_{us}| = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\delta} \sqrt{\frac{m_u}{m_c}} \right|$$

$$\frac{M^{d,l}}{m_3} = \begin{pmatrix} 0 & \varepsilon^3 & ?\varepsilon^3 \\ \varepsilon^3 & 3\varepsilon^2 & ?\varepsilon^2 \\ ?\varepsilon^2 & ?\varepsilon^2 & 1 \end{pmatrix}$$

$$\begin{array}{ccc} M^u & \xleftrightarrow{GUT?} & M^\nu \\ SU(2)_L \otimes SU(2)_R? \updownarrow & & \updownarrow SU(2)_L \otimes SU(2)_R? \\ M^d & \xleftrightarrow{GUT?} & M^l \end{array}$$

GUT factor

$$\frac{m_s}{m_\mu}(M_X) = \frac{1}{3}$$

3 colours

Georgi Jarlskog SO(10)

$$\frac{m_b}{m_\tau}(M_X) = 1$$

$$\frac{m_b}{m_\tau}(M_X) = \begin{cases} \underline{1} & \tan\beta = 2 \\ 0.6 & \tan\beta = 30 \\ \underline{1.0} & Anomaly\ Mediation \end{cases}$$

$$\frac{3m_s}{m_\mu}(M_X) = \begin{cases} \underline{0.9} \\ 0.82 \\ \underline{1.05} \end{cases}$$

$$\frac{Det(M^l)}{Det(M^d)} = \begin{cases} 0.6 \\ 0.3 \\ \underline{1.04} \end{cases}$$

Serna, GGR

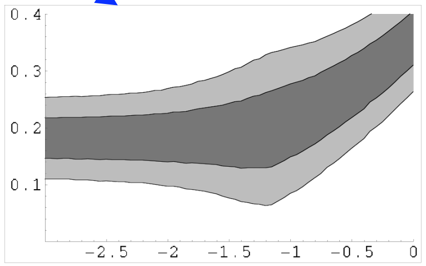
- Mixing angle sum rules.

Texture zero

$$\frac{M^{d,l}}{m_3} = \begin{pmatrix} 0 & \varepsilon^3 & ?\varepsilon^3 \\ \varepsilon^3 & 3\varepsilon^2 & ?\varepsilon^2 \\ ?\varepsilon^2 & ?\varepsilon^2 & 1 \end{pmatrix} \sim \frac{M_D^\nu}{m_3}$$

$$|V_{us}| = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\delta} \sqrt{\frac{m_u}{m_c}} \right|$$

θ_{13}



Gatto, Sartori, Tonin; Oakes; Fritzsch; Roberts et al

Ibarra, GGR

Tri-bi-maximal mixing

$$\theta_{13} \simeq \frac{1}{\sqrt{2}} \theta_{12}^l \simeq \sqrt{m_e / 2m_\mu} \simeq 3^0$$

$35^o, (45^0)?$

$$\theta_{12} - \theta_{13} \cos(\delta) \simeq \theta_{12}^\nu \qquad \left(\theta_{13}^e, \theta_{13}^\nu \quad small\right)$$

Antusch, King, Malinsky

Summary

- Majorana or Dirac - probe of underlying mechanism

- Due to the see-saw mechanism, the quark, charged lepton and neutrino masses and mixing angles can be consistent with a similar structure for their Dirac mass matrices.

  Hints at an underlying (spontaneously broken) family symmetry? SO(10)XSU(3)?

- Tri-bi maximal mixing - first indication of discrete non-Abelian symmetry?

- Neutrino spectrum

  Nearly degenerate, normal hierarchy, inverted hierarchy all possible

- Sum rule tests for various structures

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