(Old) Hadronization Models

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`New Ideas on Hadronization'

RAL, 30 May 2008

What is Hadronization?

In practice, two rather distinct meanings:

- General features/models of soft QCD
  - Local parton-hadron duality (LPHD)
  - Universal low-scale effective α_S (ULSEA)

- Formation of individual hadrons
  - Monte Carlo models
  - Thermal models
  - AdS/QCD

General Ideas

- Local parton-hadron duality
  - Momentum & flavour follows parton flow
  - Predicts asymptotic spectra
  - Predicts two-particle correlations
- Universal low-scale effective α_S
  - Related to “tube” model
  - Regulates IR renormalons in PT
  - Predicts power corrections to event shapes
  - Predicts jet shapes and energy corrections

Local parton-hadron duality

- Evolution equation for fragmentation function has extra z^2 due to soft gluon coherence

$$t\frac{\partial}{\partial t}F(x,t) = \int_x^1 \frac{dz}{z}\frac{\alpha_S}{2\pi}P(z)F(x/z,z^2t)$$

- Solution by moments

$$\tilde{F}(N,t) \sim \exp\left[\int_{t_0}^{t}\gamma(N,\alpha_S)\frac{dt'}{t'}\right]\tilde{F}(N,t_0)$$

$$\gamma(N,\alpha_S) = \frac{\alpha_S}{2\pi}\int_0^1 z^{N-1+2\gamma(N,\alpha_S)}P(z)$$

- Anomalous dimension dominates asymptotically

$$\gamma(N,\alpha_S) = \frac{\alpha_S}{2\pi}\int_0^1 z^{N-1+2\gamma(N,\alpha_S)} P(z)$$

$$\sim \frac{C_A\alpha_S}{\pi}\frac{1}{N-1+2\gamma(N,\alpha_S)}$$

- This is regular at N=1

$$\gamma(N,\alpha_S) = \frac{1}{4}\left[\sqrt{(N-1)^2+\frac{8C_A\alpha_S}{\pi}}-(N-1)\right]$$

$$= \sqrt{\frac{C_A\alpha_S}{2\pi}} - \frac{1}{4}(N-1) + \frac{1}{32}\sqrt{\frac{2\pi}{C_A\alpha_S}}(N-1)^2 + \cdots$$

$$\int^{t} \gamma(N, \alpha_S(t'))\frac{dt'}{t'} = \int^{\alpha_S(t)} \frac{\gamma(N, \alpha_S)}{\beta(\alpha_S)}\, d\alpha_S$$

where $\beta(\alpha_S) = -b\alpha_S^2 + \cdots$. Hence

$$\tilde{F}(N,t) \sim \exp\left[\frac{1}{b}\sqrt{\frac{2C_A}{\pi\alpha_S}} - \frac{1}{4b\alpha_S}(N-1) + \frac{1}{48b}\sqrt{\frac{2\pi}{C_A\alpha_S^3}}(N-1)^2 + \cdots\right]_{\alpha_S=\alpha_S(t)}$$

Mean multiplicity | Position of peak | Width of peak

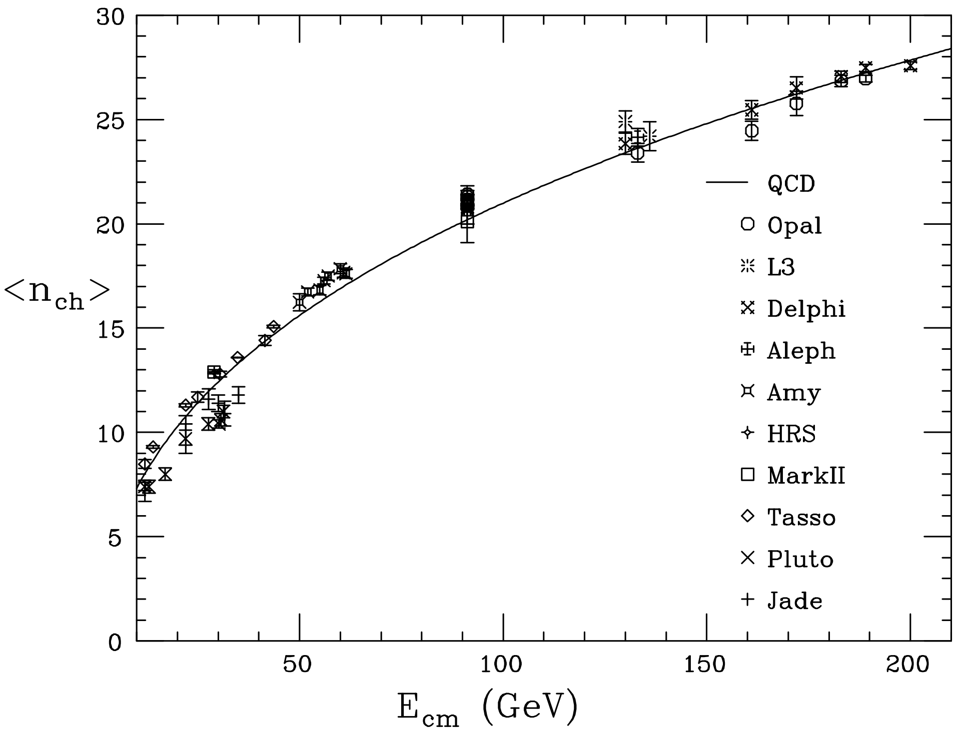
- Gaussian in $N \leftrightarrow$ Gaussian in $\xi \equiv \ln(1/x)$
- Mean multiplicity

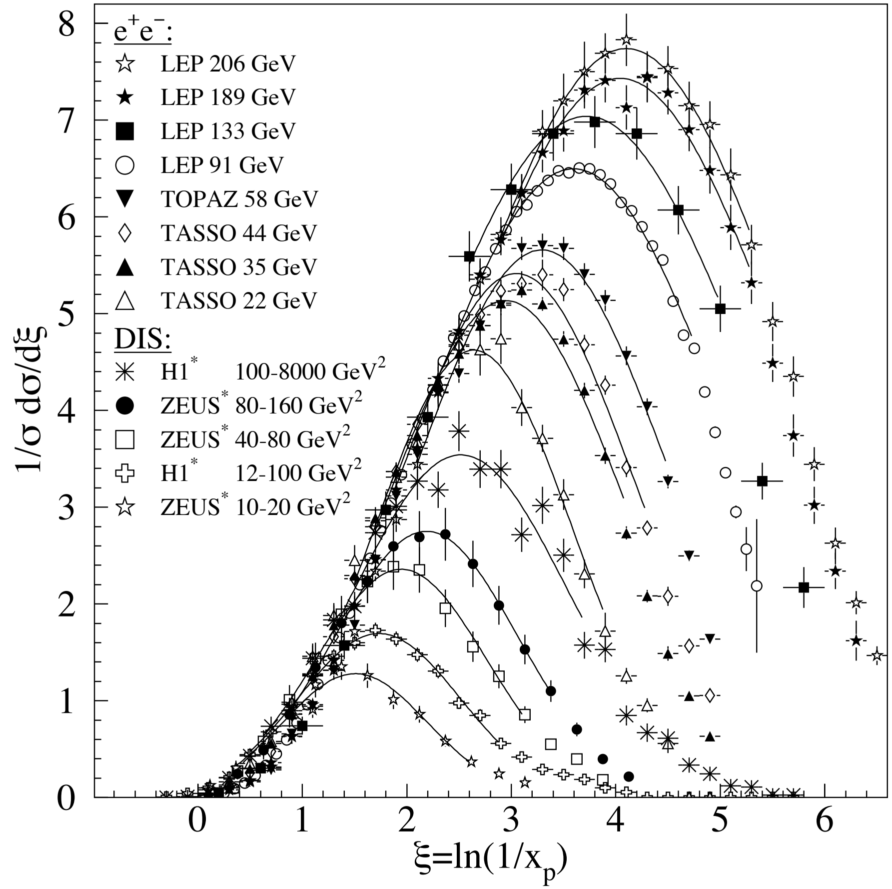
$$\langle n(s)\rangle = \int_0^1 dx\, F(x,s) = \tilde{F}(1,s)$$

$$\sim \exp\frac{1}{b}\sqrt{\frac{2C_A}{\pi\alpha_S(s)}} \sim \exp\sqrt{\frac{2C_A}{\pi b}\ln\left(\frac{s}{\Lambda^2}\right)}$$

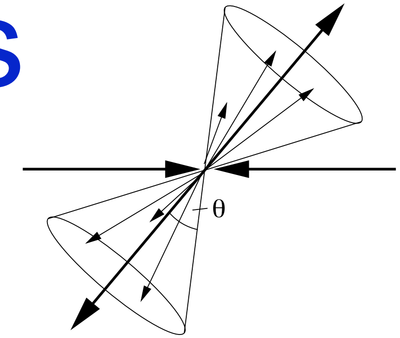
LPHD Predictions

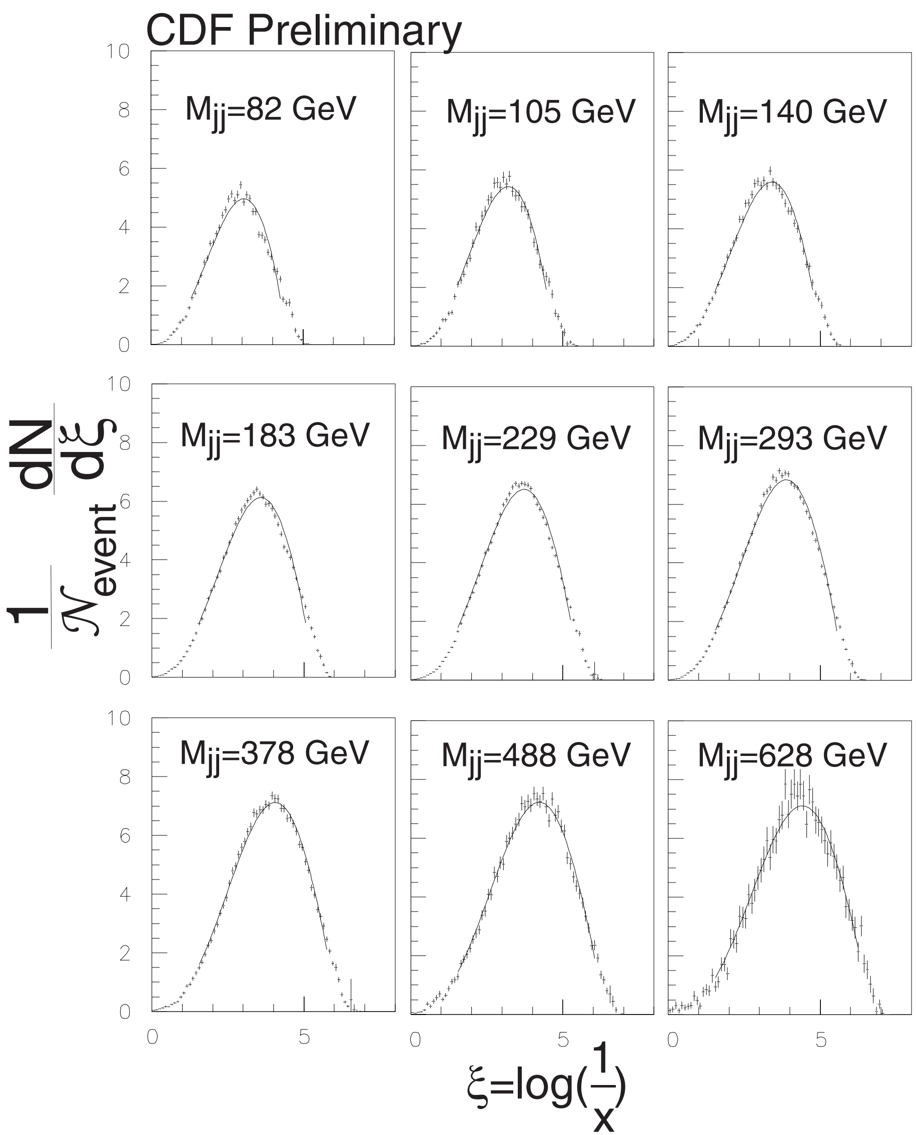
- Good agreement with data

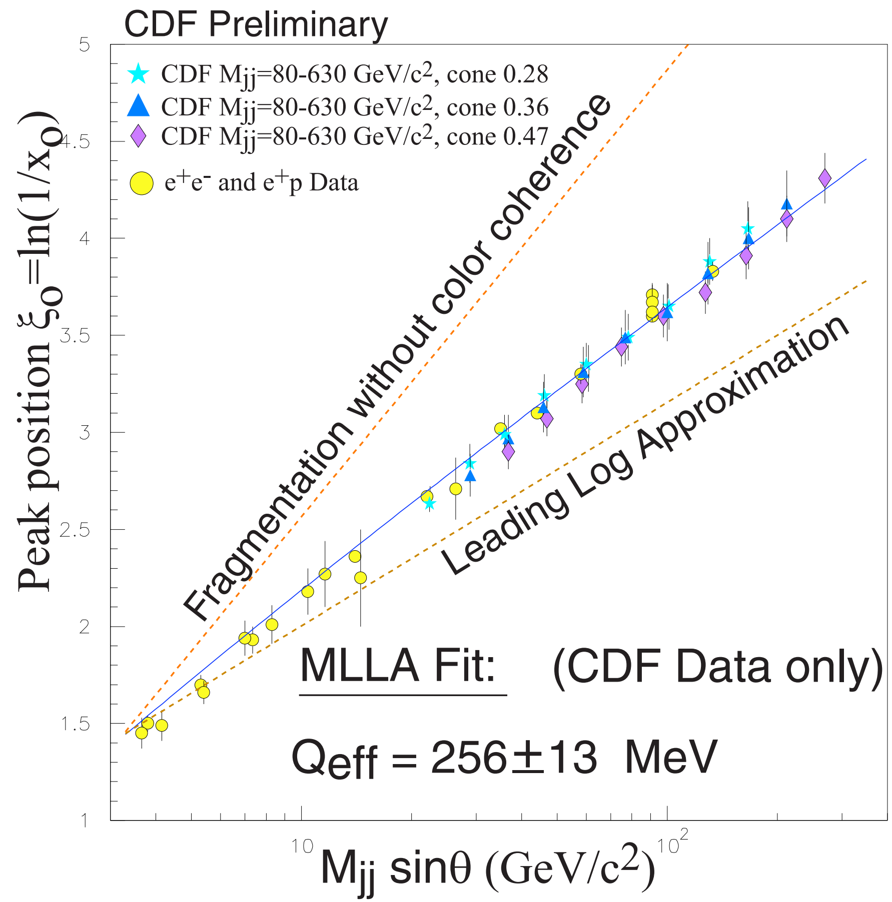




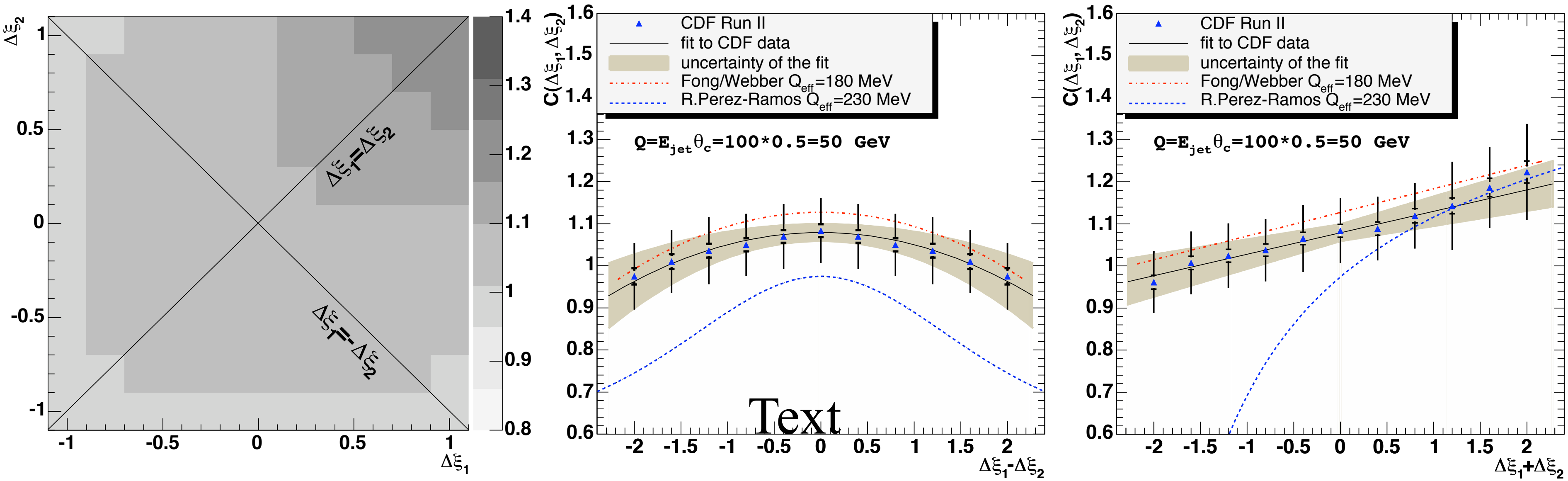
LPHD in $p\bar{p} \to$ dijets







Two-particle energy correlations



CDF, arXiv:0802.3182

$$R(\Delta\xi_1, \Delta\xi_2) = r_0 + r_1(\Delta\xi_1 + \Delta\xi_2) + r_2(\Delta\xi_1 - \Delta\xi_2)^2$$

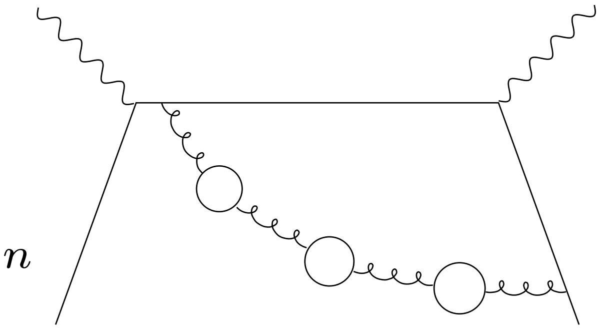
$$r_0^q = 1.75 - \frac{0.64}{\sqrt{\tau}}, \qquad r_1^q = \frac{1.6}{\tau^{3/2}}, \qquad r_2^q = -\frac{2.25}{\tau^2} \qquad \tau = \ln(Q/Q_{eff})$$

$$r_0^g = 1.33 - \frac{0.28}{\sqrt{\tau}}, \qquad r_1^g = \frac{0.7}{\tau^{3/2}}, \qquad r_2^g = -\frac{1.0}{\tau^2}.$$

CP Fong & BW,
NP B355(1991)54

Universal low-scale effective α_S

- Infrared renormalon



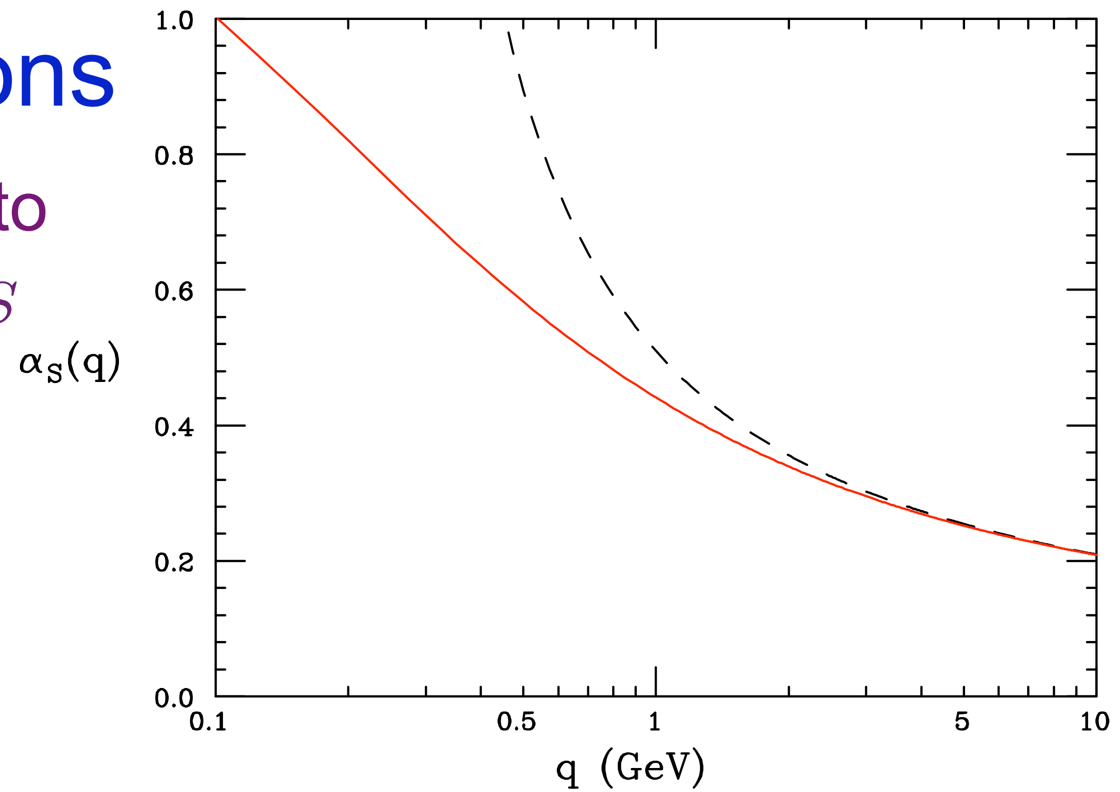
$$F \sim \int_0^Q \frac{dp_t}{Q}\alpha_S(p_t)$$
$$= \alpha_S(Q)\sum_n \int_0^Q \frac{dp_t}{Q}\left[b\alpha_S(Q)\ln\frac{Q^2}{p_t^2}\right]^n$$
$$= \alpha_S(Q)\sum_n n![2b\alpha_S(Q)]^n$$

- Divergent series: truncate at smallest term ($n_m = [2b\alpha_S(Q)]^{-1}$) $\Rightarrow$ uncertainty

$$\delta F \sim n_m![2b\alpha_S(Q)]^{n_m} \sim e^{-n_m} = \frac{\Lambda}{Q}$$

Power Corrections

- Renormalon is due to IR divergence of α_S
- Postulate universal IR-regular α_S
- Power corrections depend on

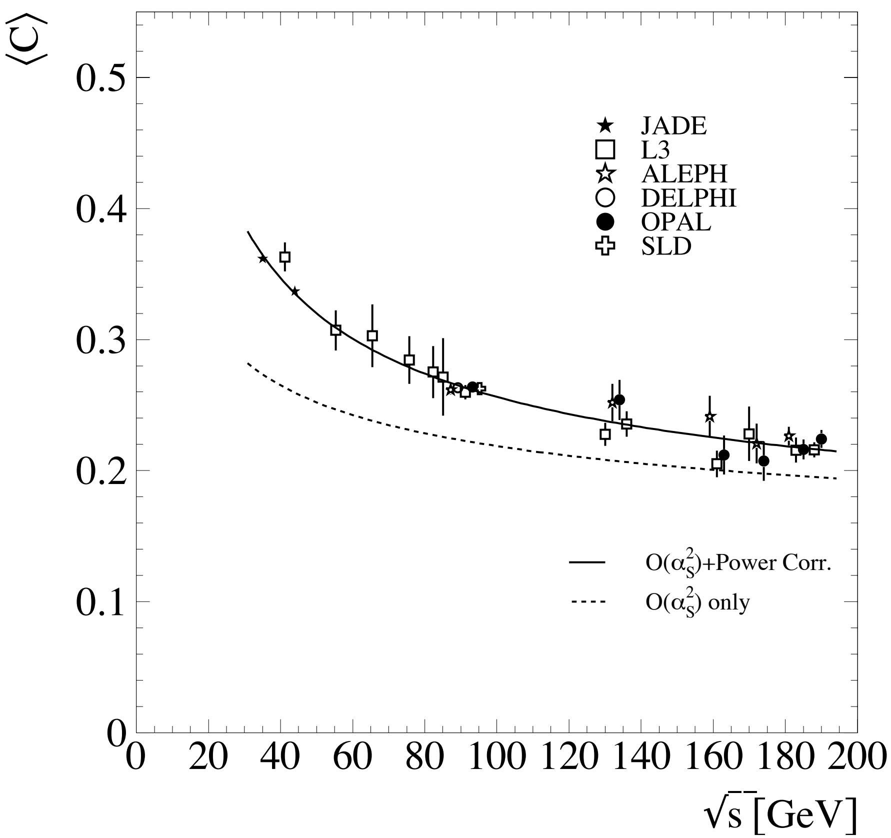


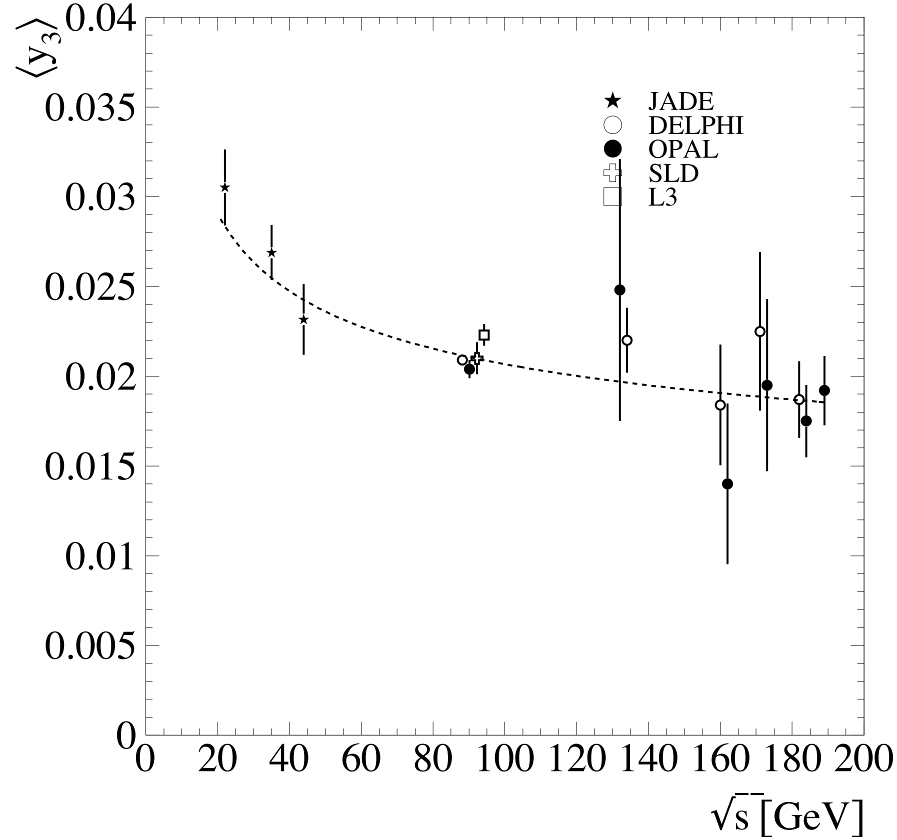
$$\alpha_0(\mu_I) = \frac{1}{\mu_I}\int_0^{\mu_I} \alpha_S(p_t)\, dp_t$$

- Match NP & PT at $\mu_I \sim 2$ GeV

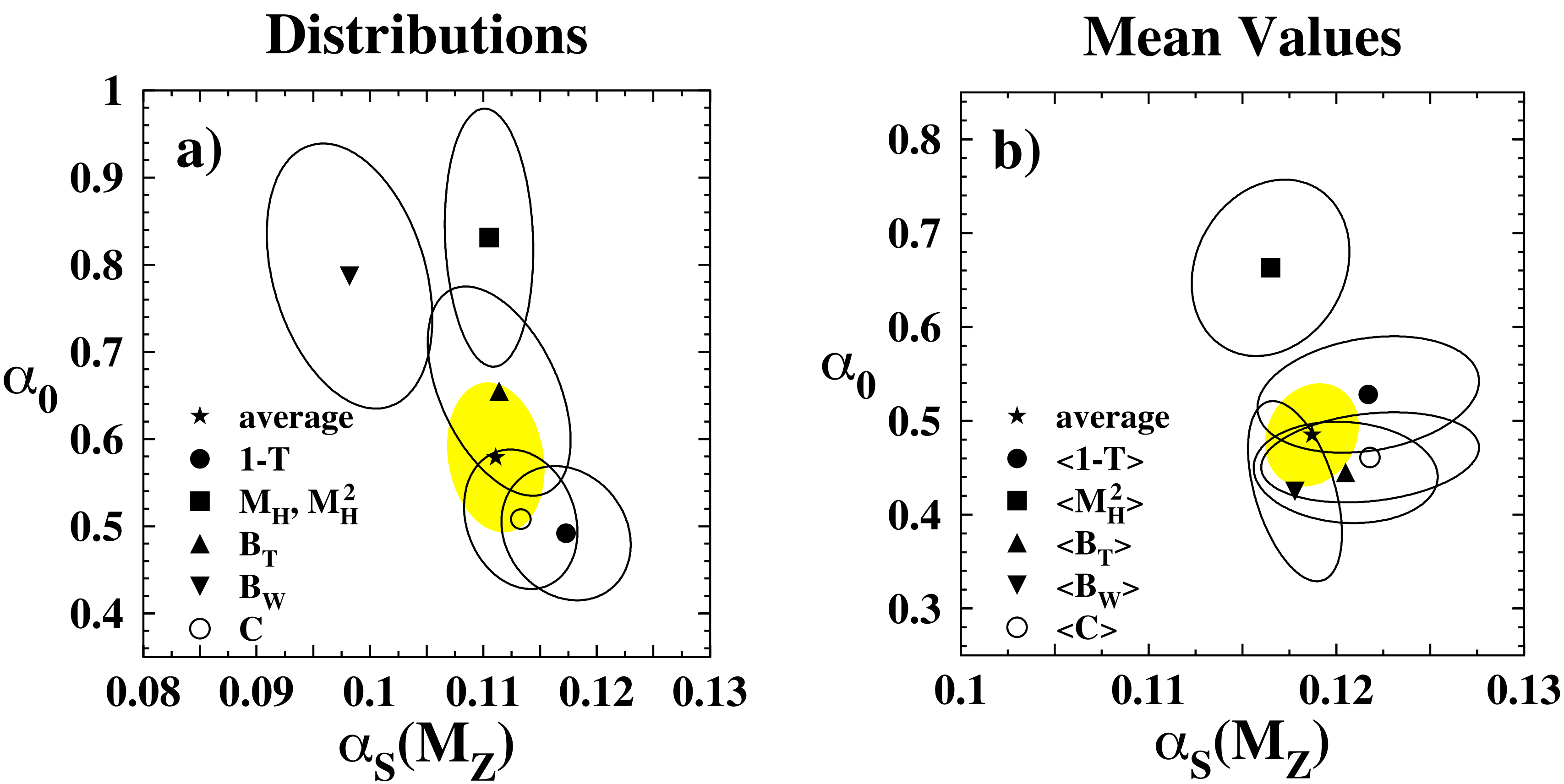
Power corrections to event shapes

- 1/Q renormalon present in C, absent in y_3



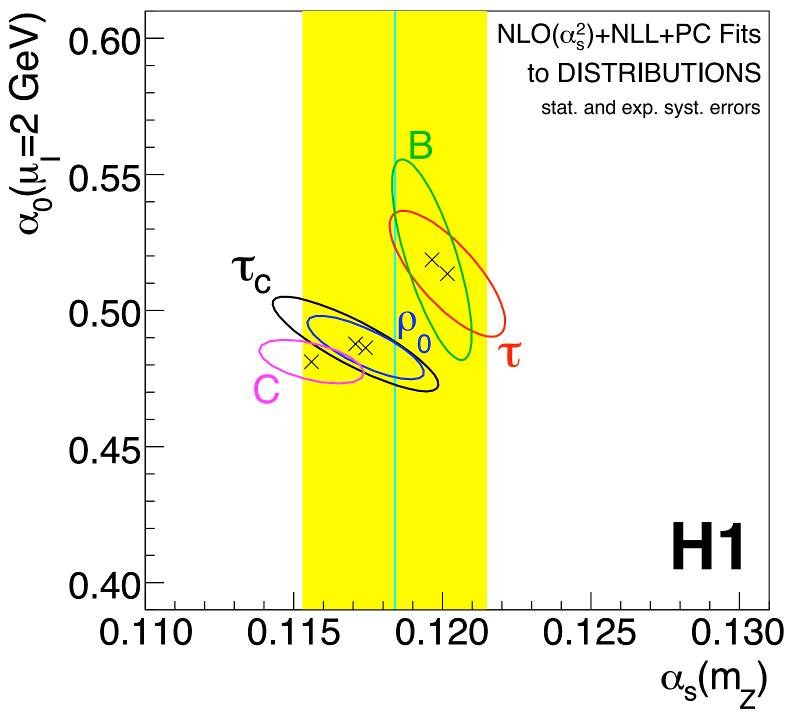


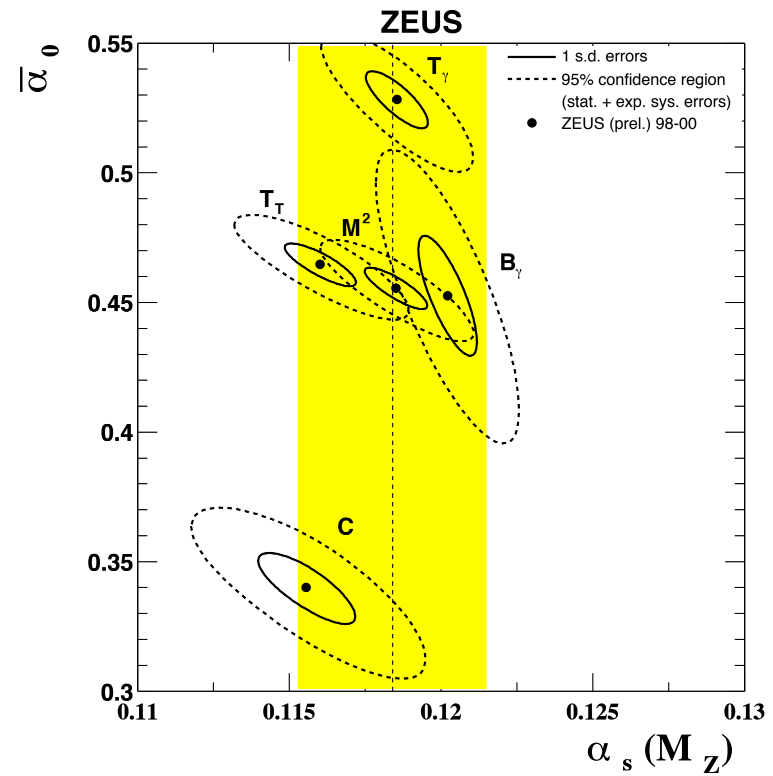
ULSEA results from e^+e^-



Movilla Fernandez, Bethke, Biebel & Kluth, EPJ C22(2001)1

ULSEA results from DIS





ULSEA hadronic jet energy correction

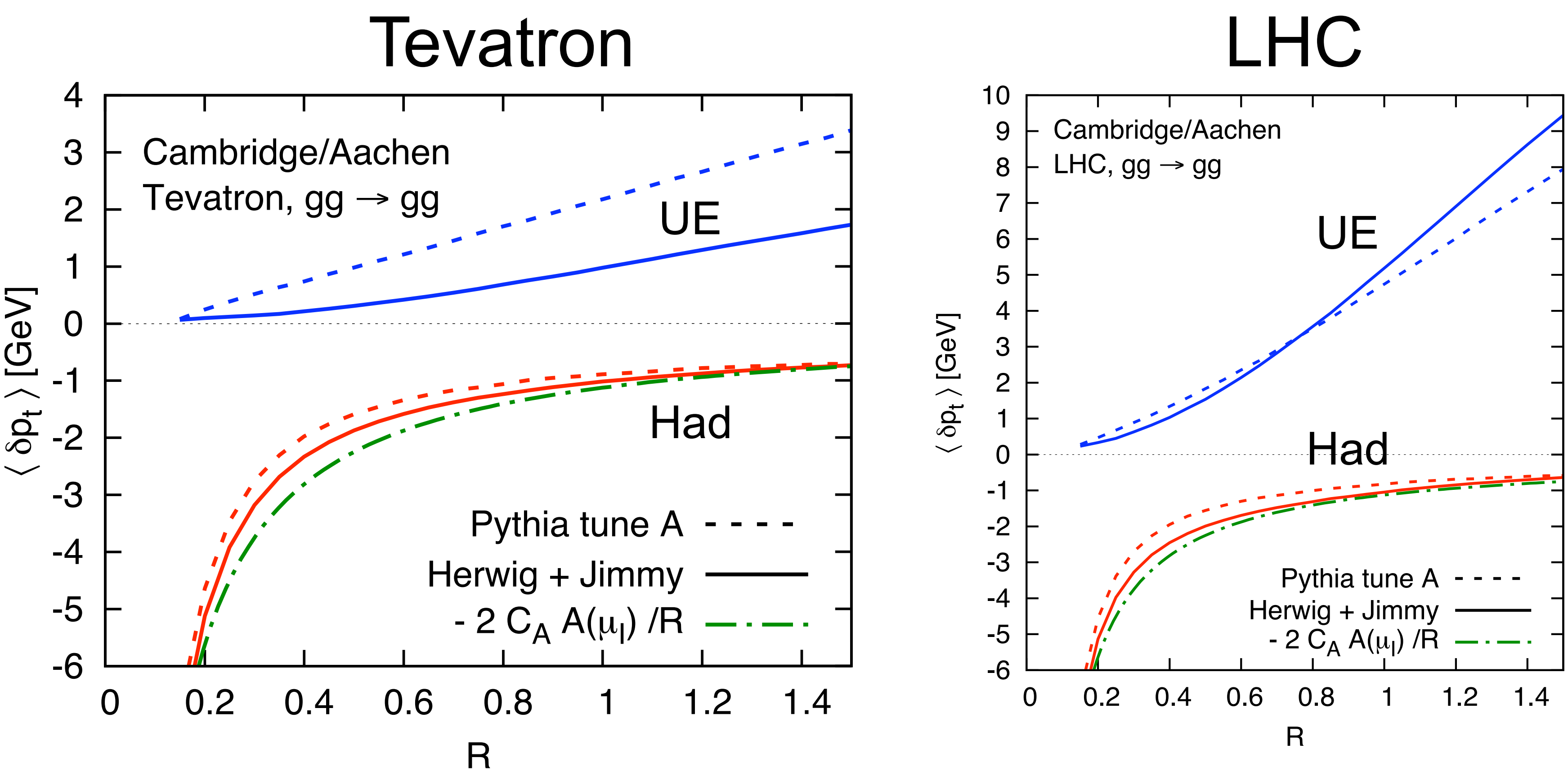
$$\langle \delta p_t \rangle_{\mathrm{h}}^{(jr)} = C_{jr}\,\mathcal{A}(\mu_I)\left(-\frac{1}{R} - \frac{1}{4}R + \frac{1}{192}R^3 - \frac{5}{2304}R^5 + \mathcal{O}\left(R^7\right)\right)$$

$$\mathcal{A}(\mu_I) = \frac{1}{\pi}\,\mu_I\left[\alpha_0\left(\mu_I\right) - \alpha_s(p_t) - \frac{\beta_0}{2\pi}\left(\ln\frac{p_t}{\mu_I} + \frac{K}{\beta_0} + 1\right)\alpha_s^2(p_t)\right]$$

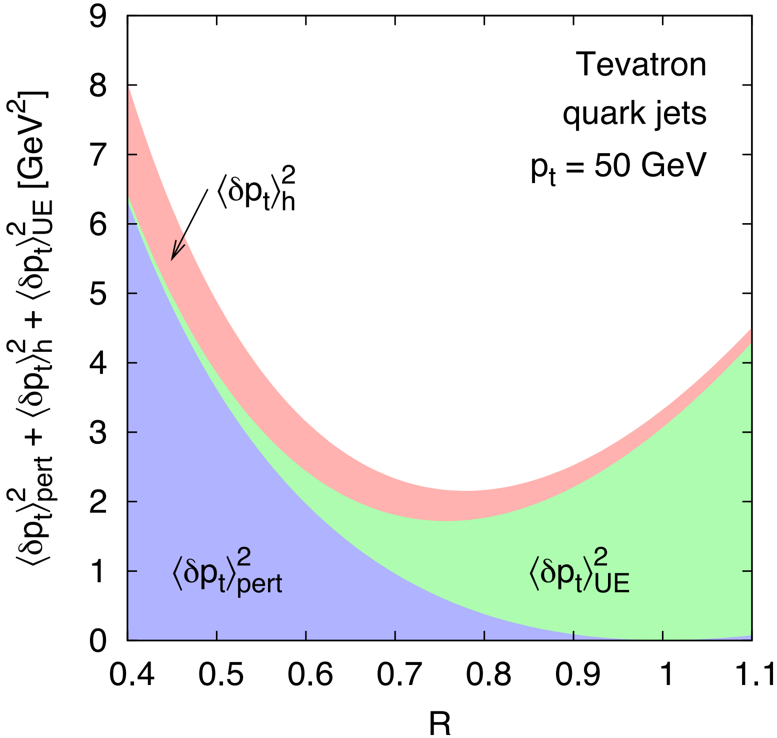
PT subtraction

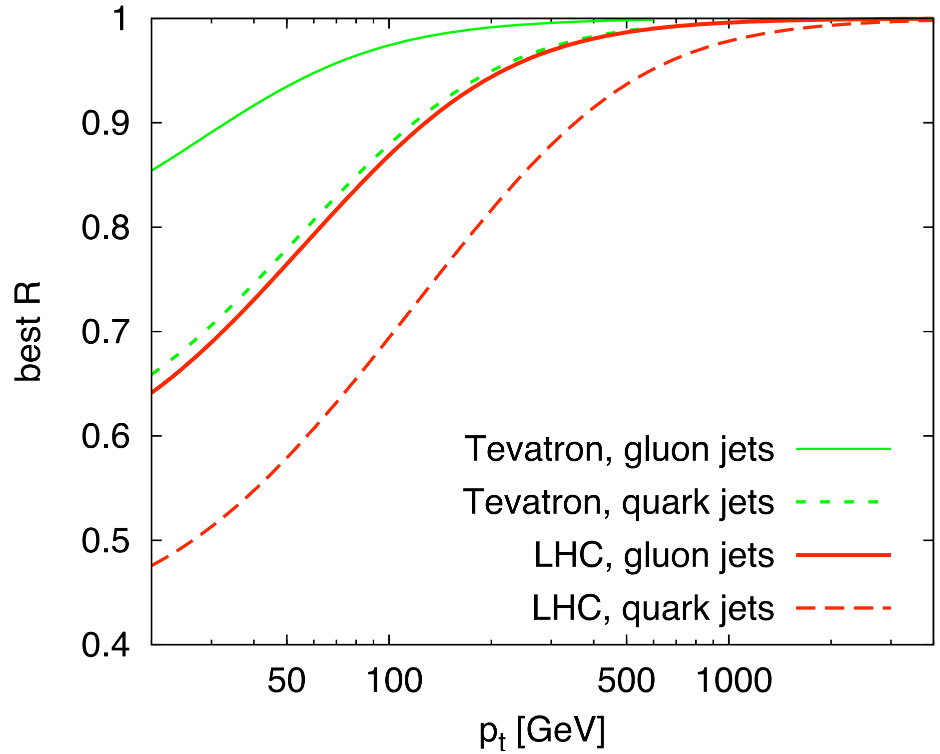
Dasgupta, Magnea & Salam, JHEP02(2008)055

ULSEA jet energy correction



Optimal Jet Cone Size



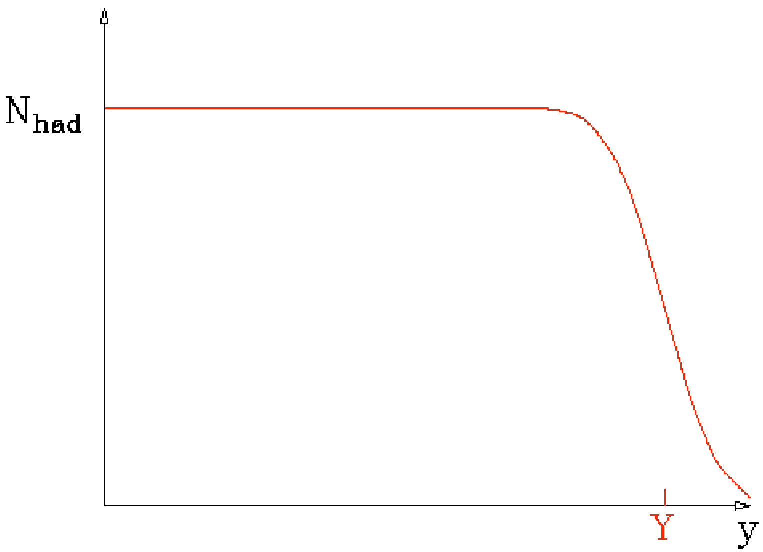


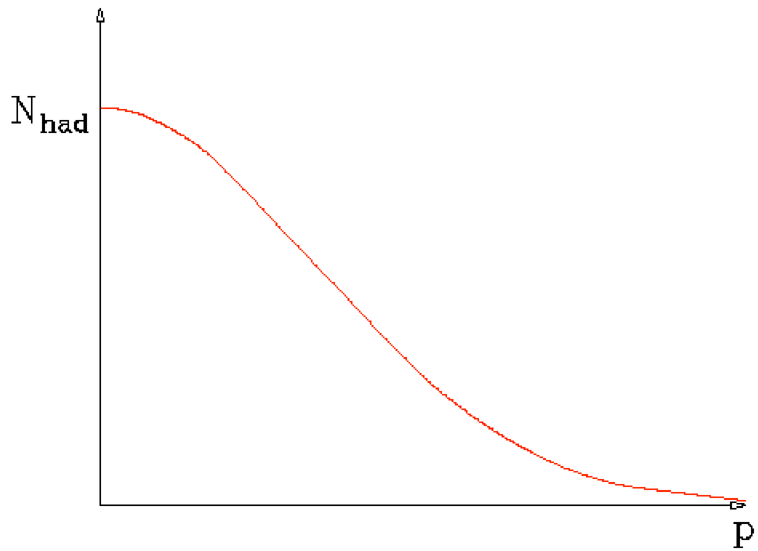
"Tube" Model for Jet Fragmentation

- Precursor of MC models
- Shows some features of ULSEA

Experimentally, $e^+e^- \to$ two jets:

Flat rapidity plateau and limited p_t, $\rho(p_t^2) \sim e^{-p_t^2/2p_0^2}$





Tube model gives simple estimates of hadronization corrections to perturbative quantities.

E.g. Jet energy and momentum:

$$E = \int_0^Y dy\, d^2p_t\, \rho(p_t^2)\, p_t\, \cosh y = \lambda \sinh Y$$

$$P = \int_0^Y dy\, d^2p_t\, \rho(p_t^2)\, p_t\, \sinh y = \lambda(\cosh Y - 1) \sim E - \lambda,$$

with $\lambda = \int d^2p_t\, \rho(p_t^2)\, p_t,$ mean transverse momentum.
Estimate from Fermi motion $\lambda \sim 1/R_{had} \sim m_{had}.$

Jet acquires non-perturbative mass: $M^2 = E^2 - P^2 \sim 2\lambda E$
Large: ~ 10 GeV for 100 GeV jets.

Independent Fragmentation Model (Field-Feynman)

MC implementation of tube model.

Longitudinal momentum distribution = arbitrary fragmentation function: parameterization of data.

Transverse momentum distribution = Gaussian.

Recursively apply $q \to q' + \text{had.}$

Hook up remaining soft q and $\bar{q}$.

Strongly frame dependent.

No obvious relation with perturbative emission.

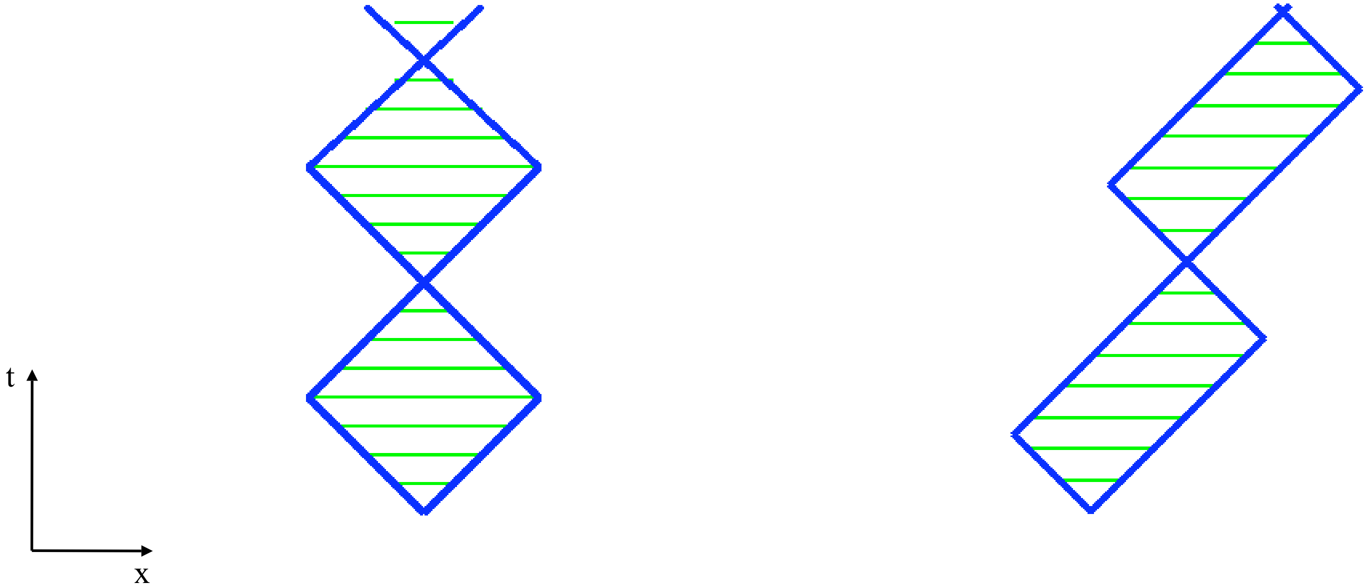
Not infrared safe.

Not a model of confinement.

2-d String Model of Mesons

Light quarks connected by string.

L=0 mesons only have 'yo-yo' modes:



Obeys area law: $m^2 = 2\kappa^2$ area

The Lund String Model

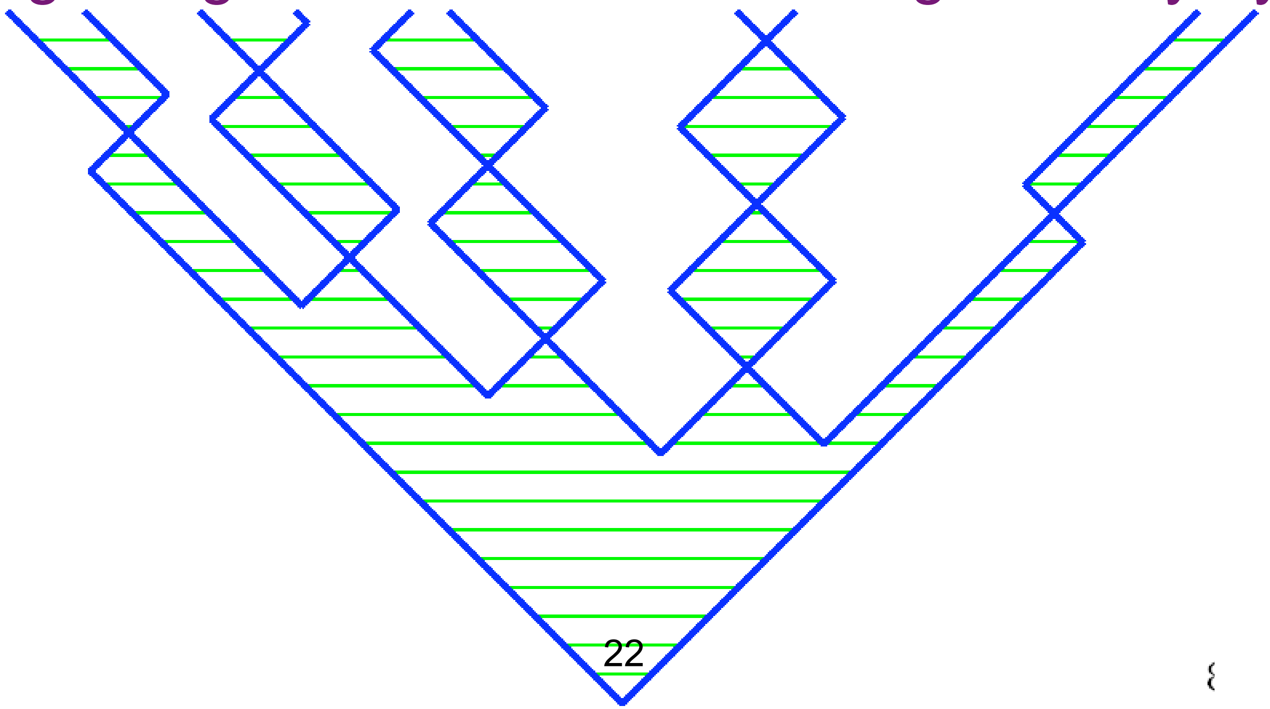
Start by ignoring gluon radiation:

e^+e^- annihilation = pointlike source of $q\bar{q}$ pairs

Intense chromomagnetic field within string → $q\bar{q}$ pairs created by tunnelling. Analogy with QED:

$$\frac{d(\text{Probability})}{dx\ dt} \propto \exp(-\pi m_q^2/\kappa)$$

Expanding string breaks into mesons long before yo-yo point.



Lund Symmetric Fragmentation Function

String picture → constraints on fragmentation function:

- Lorentz invariance
- Acausality
- Left—right symmetry

$$f(z) \propto z^{a_\alpha - a_\beta - 1}(1-z)^{a_\beta}$$

$a_{\alpha,\beta}$ adjustable parameters for quarks α and β.

Fermi motion → Gaussian transverse momentum.

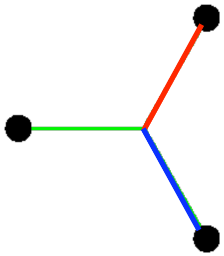
Tunnelling probability becomes

$$\exp\left[-b(m_q^2 + p_t^2)\right]$$

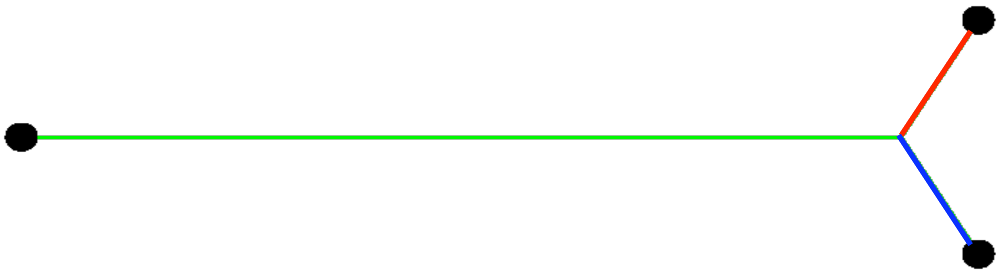
a, b and m_q^2 = main tuneable parameters of model

Baryon Production

Baryon pictured as three quarks attached to a common centre:



At large separation, can consider two quarks tightly bound: diquark



→ diquark treated like antiquark.

Two quarks can tunnel nearby in phase space: baryon—antibaryon pair

Extra adjustable parameter for each diquark!

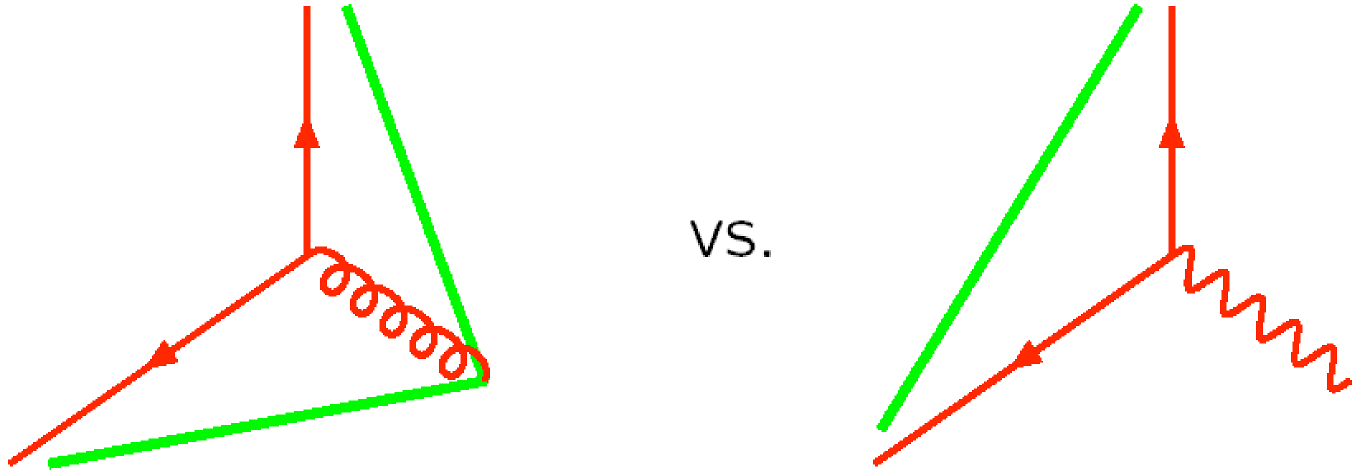
Alternative “popcorn” model:

Three-Jet Events

So far: string model = motivated, constrained independent fragmentation!

New feature: universal

Gluon = kink on string → the string effect



Infrared safe matching with parton shower: gluons with $k_\perp <$ inverse string width irrelevant.

String Model Summary

- String model strongly physically motivated.
- Very successful fit to data.
- Universal: fitted to e^+e^-, little freedom elsewhere.

- How does motivation translate to prediction?

  ~ one free parameter per hadron/effect!

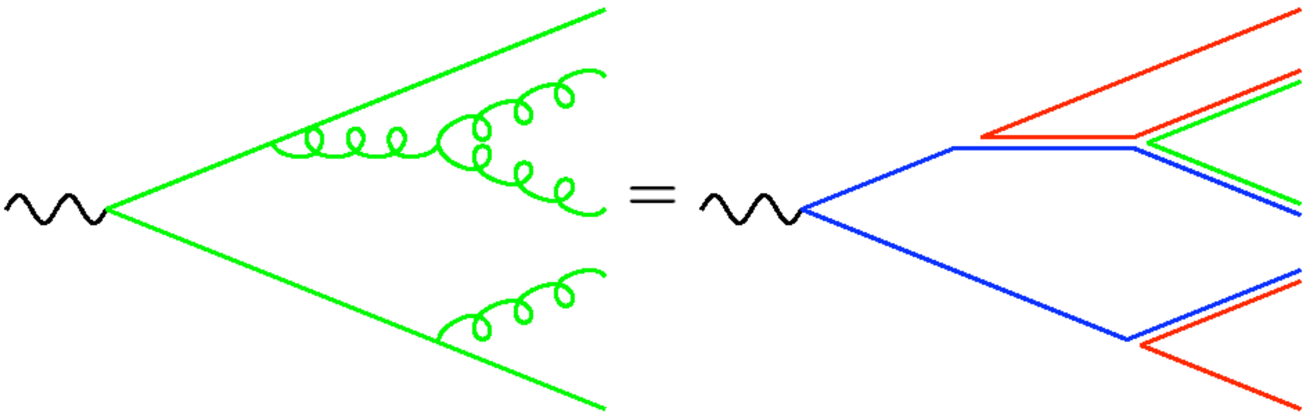
- Blankets too much perturbative information?

- Can we get by with a simpler model?

Cluster Model: Preconfinement

Planar approximation: gluon = colour—anticolour pair.

Follow colour structure of parton shower: colour-singlet pairs end up close in phase space

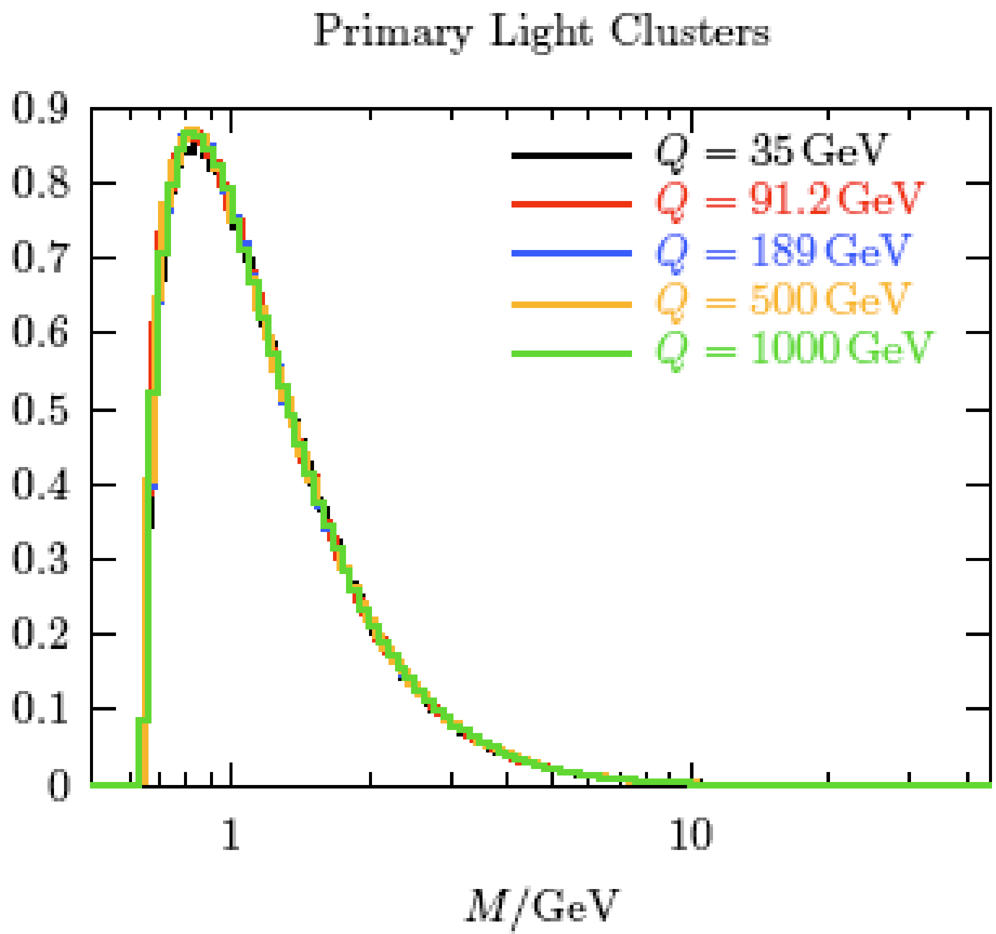


Mass spectrum of colour-singlet pairs asymptotically independent of energy, production mechanism, …

Peaked at low mass $\sim Q_0$.

Cluster mass distribution

- Independent of shower scale Q
  - depends on Q_0 and Λ



The Naïve Cluster Model

Project colour singlets onto continuum of high-mass mesonic resonances (=clusters). Decay to lighter well-known resonances and stable hadrons.

Assume spin information washed out:

decay = pure phase space.

→ heavier hadrons suppressed

→ baryon & strangeness suppression 'for free' (i.e. untuneable).

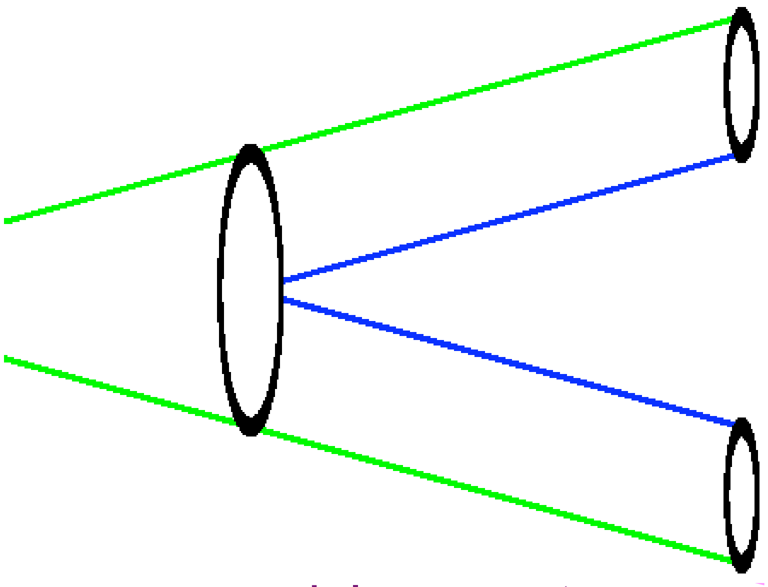
Hadron-level properties fully determined by cluster mass spectrum, i.e. by perturbative parameters.

Shower cutoff Q_0 becomes parameter of model.

The Cluster Model

Although cluster mass spectrum peaked at small m, broad tail at high m.

"Small fraction of clusters too heavy for isotropic two-body decay to be a good approximation" → Longitudinal cluster fission:



Rather string-like.

Fission threshold becomes crucial parameter.

~15% of primary clusters get split but ~50% of hadrons come from them.

The Cluster Model

"Leading hadrons are too soft"

→ 'perturbative' quarks remember their direction somewhat

$$P(\theta^2) \sim \exp(-\theta^2/2\theta_0^2)$$

Rather string-like.

Extra adjustable parameter.

Strings

"Hadrons are produced by hadronization: you must get the non-perturbative dynamics right"

Improving data has meant successively refining perturbative phase of evolution…

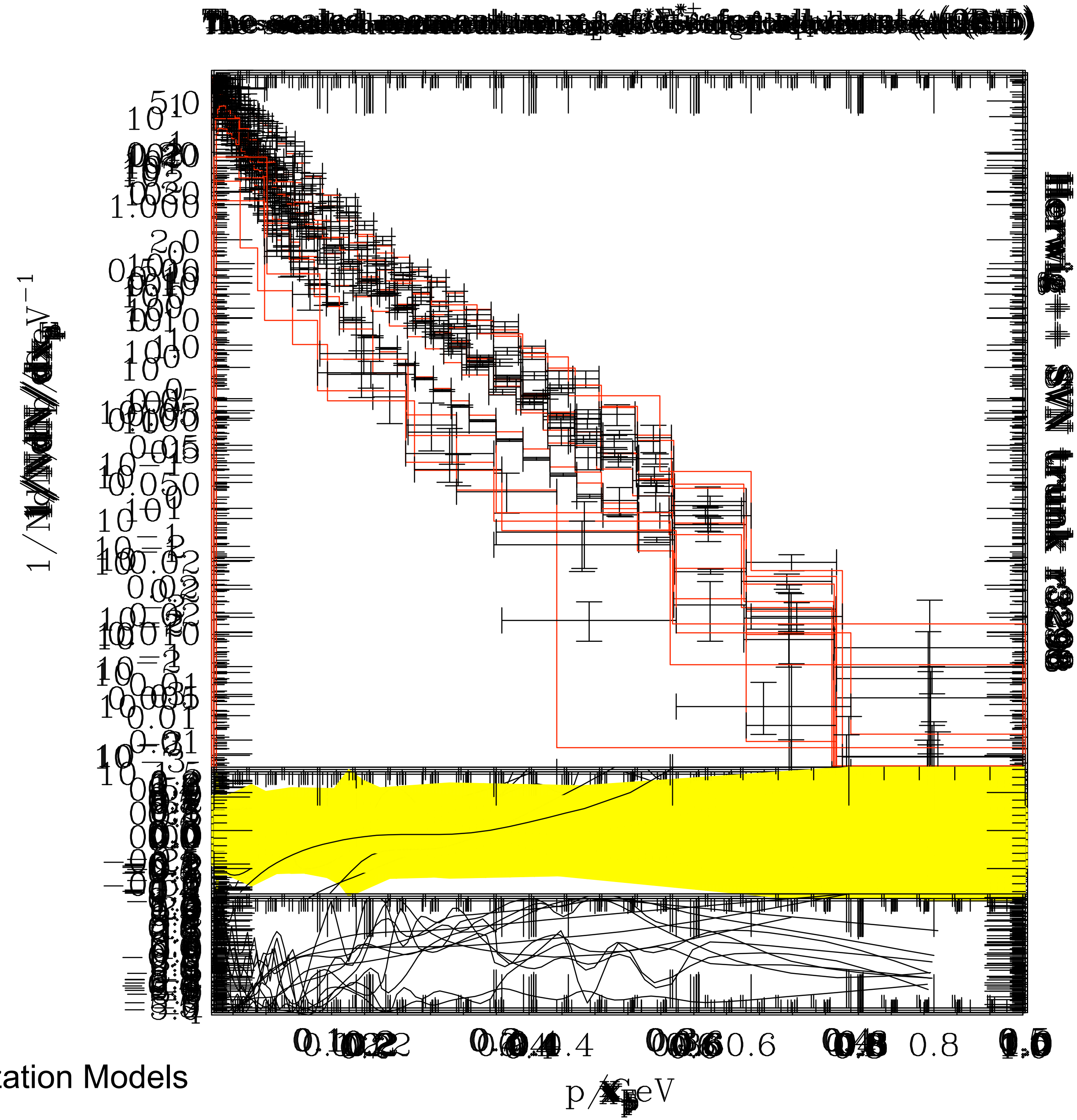
Clusters

"Get the perturbative phase right and any old hadronization model will be good enough"

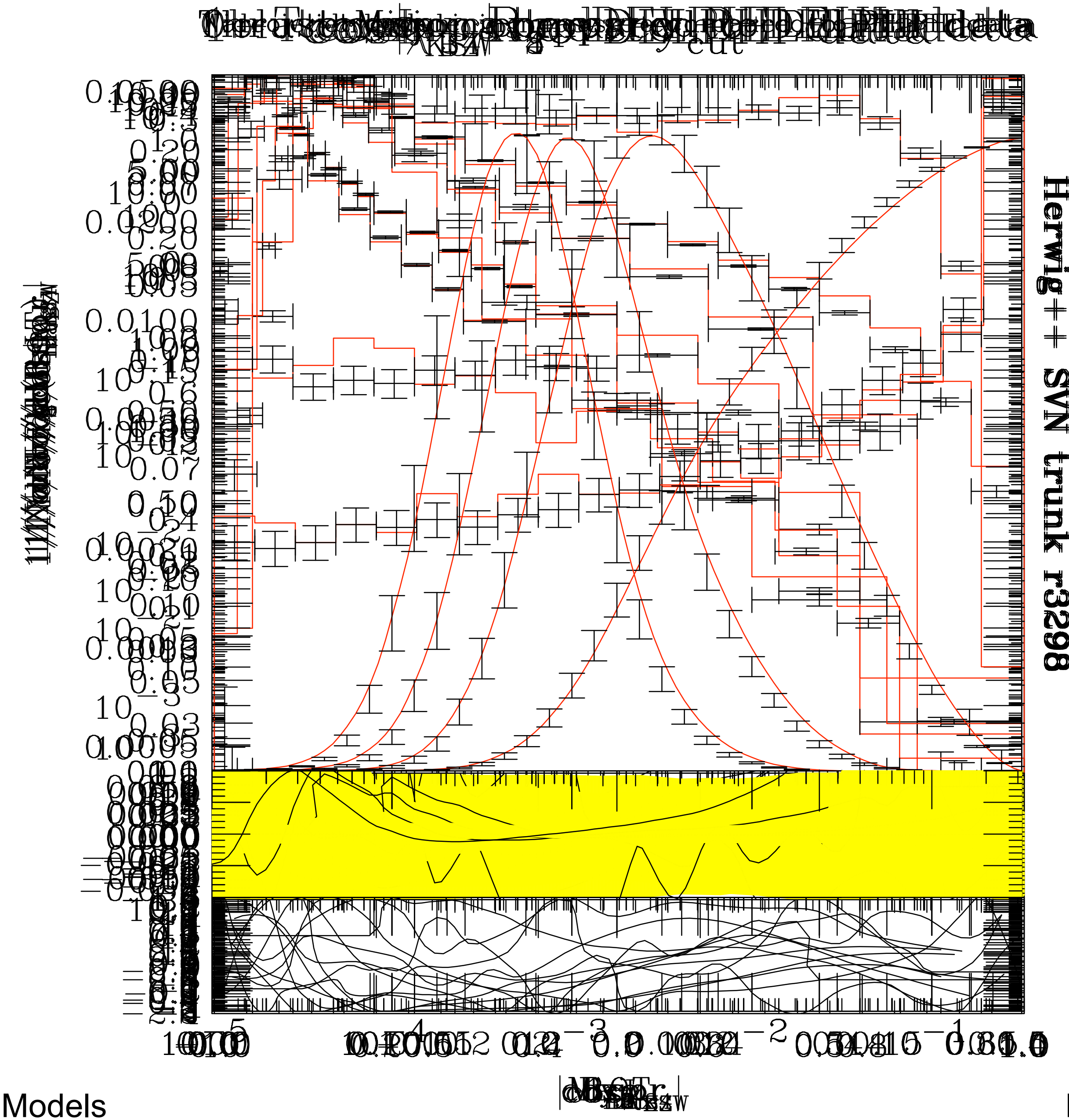
Improving data has meant successively making non-perturbative phase more string-like…

???

Comparisons with LEP1/SLC: Spectra



Comparisons with LEP1/SLC: Shapes



Thermal Model

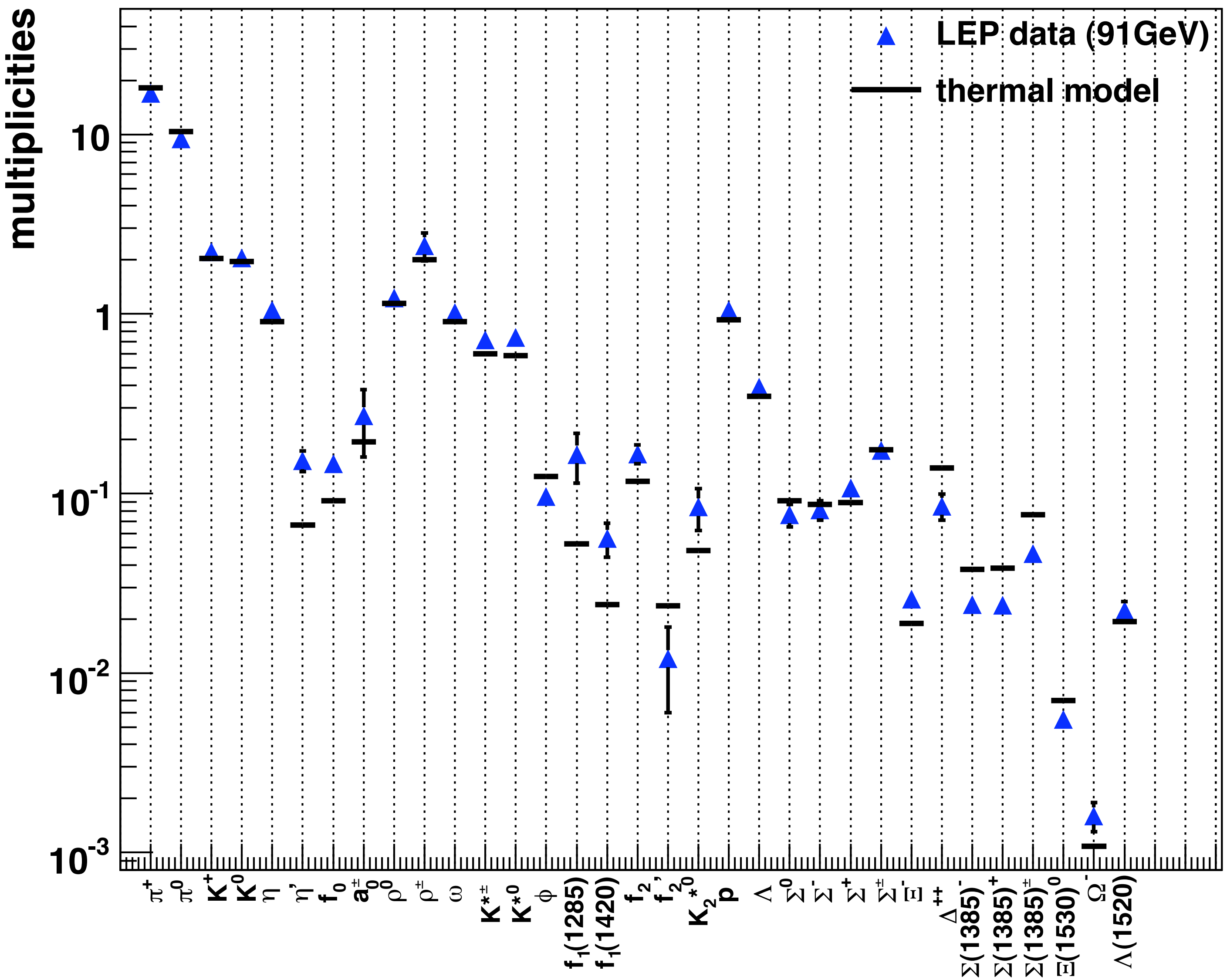
- Assume $e^+e^- \to 2$ jets in thermal equilibrium
  - 3 parameters T, V, γ_s

Values of fit parameters in e^+e^- collisions at different energies.

$\sqrt{s}$[GeV]	T[MeV]	V[fm^3]	γ_s	χ^2/dof
10	152±1.7	20±1.5	0.82±0.02	333/21
29-35	156±1.7	24±1.4	0.92±0.03	95/18
91	154±0.50	40±1.0	0.76±0.007	631/30
130-200	154±2.8	46±4.3	0.72±0.03	12/2

A Andronic et al., arXiv:0804.4132

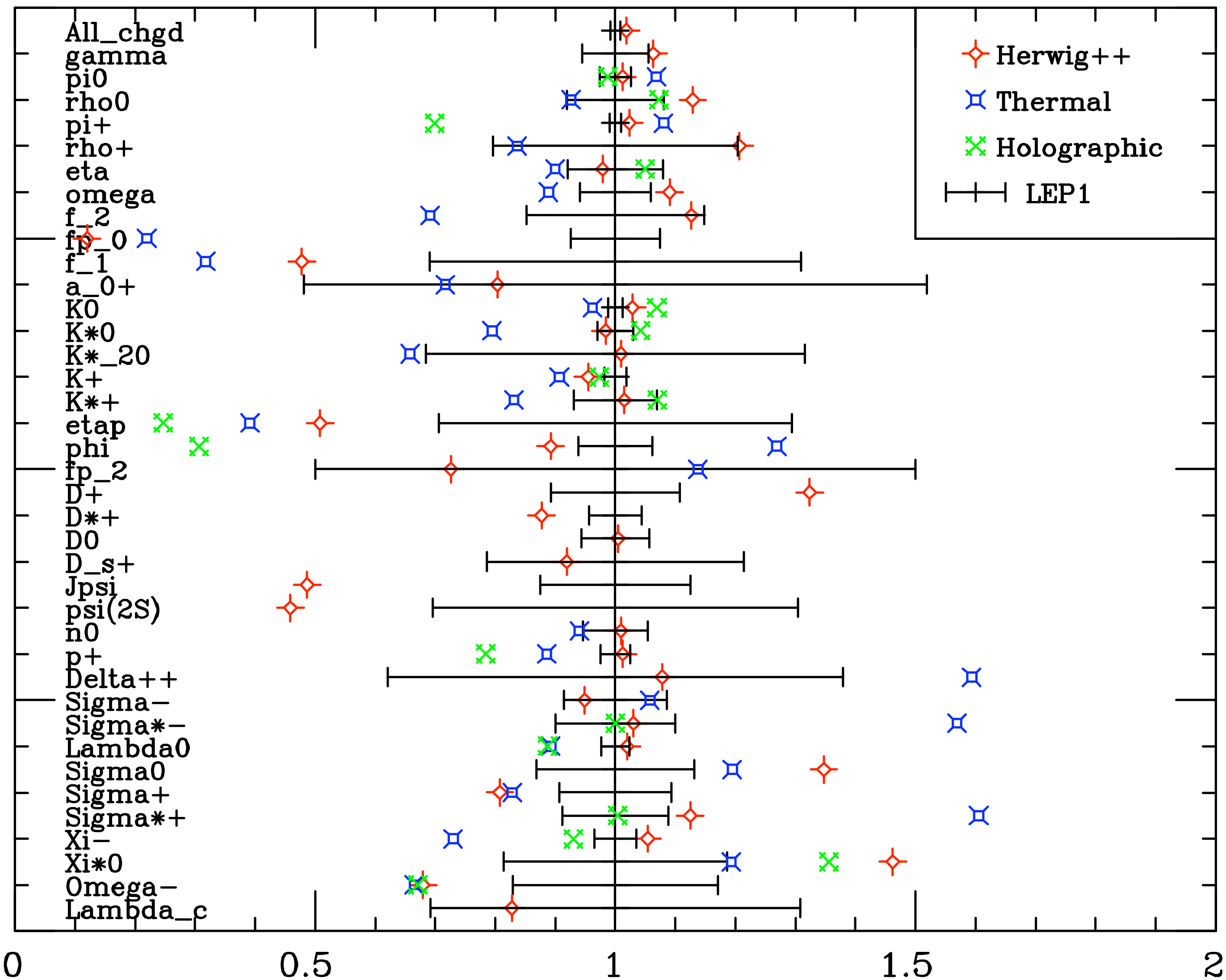
Thermal model results



Holographic Model

- Strongly-coupled gauge theory dual to weakly-coupled 5D gravity
  - Promising approach to IR behaviour of QCD
  - Relative hadron multiplicities given by 5D radial wave function overlap with common Gaussian
  - 4 parameters (1 energy dependent)
  - Nick Evans will explain more!

Hadron Yields at LEP1

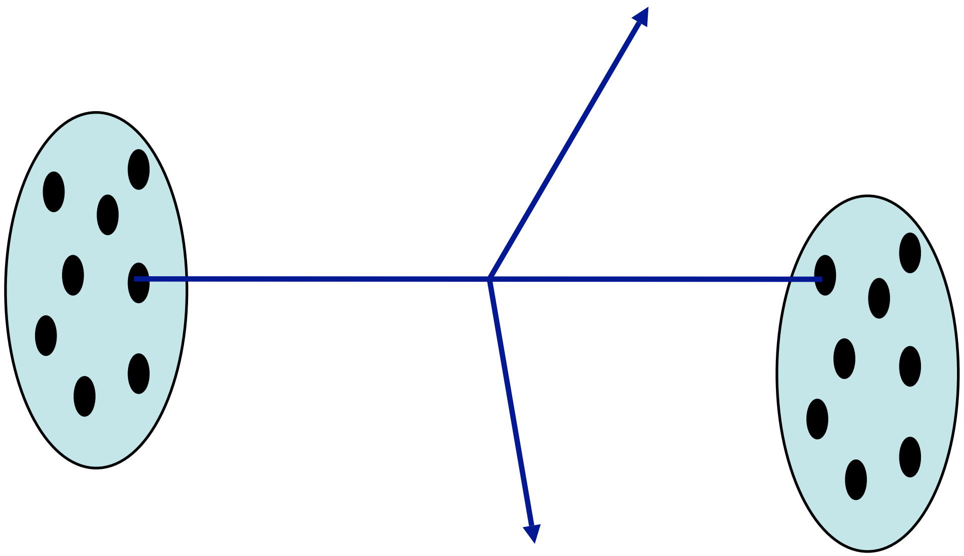


Conclusions?

- General ideas (LPHD, ULSEA) successful at 20% level, but
  - No systematic scheme for improvement
  - Don’t say anything about hadrons
- Monte Carlo models more successful
  - Complete final states
  - Matched to perturbation theory
  - But ad hoc parameters
- Other models (thermal, holographic)
  - Fewer parameters but limited predictions
  - How to match to perturbation theory?

The Underlying Event

- Protons are extended objects
- After a parton has been scattered out of each, what happens to the remnants?

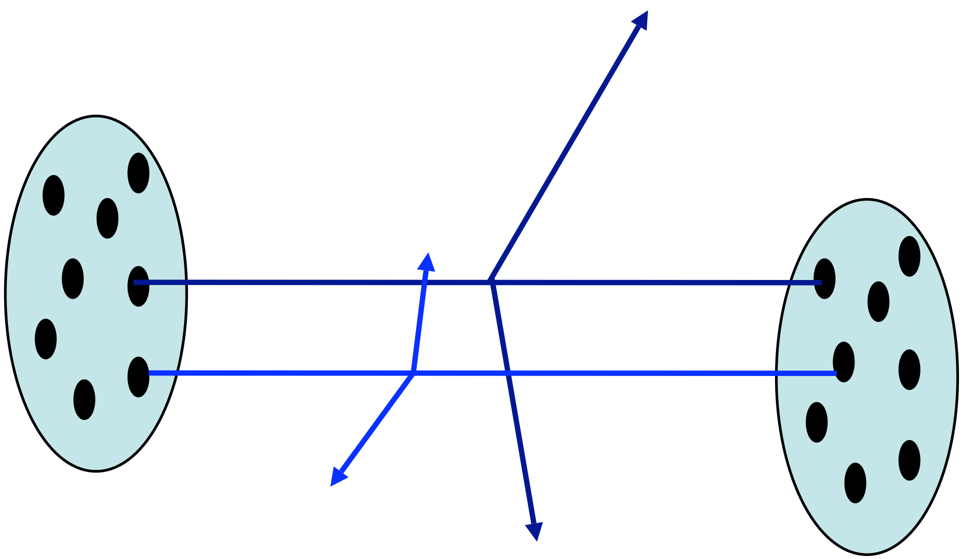


- Only viable current model: multiple parton interactions

Multiple Parton Interaction Model (PYTHIA/JIMMY)

For small $p_{t\ min}$ and high energy inclusive parton—parton cross section is larger than total proton—proton cross section.

→ More than one parton—parton scatter per proton—proton



Need a model of spatial distribution within proton

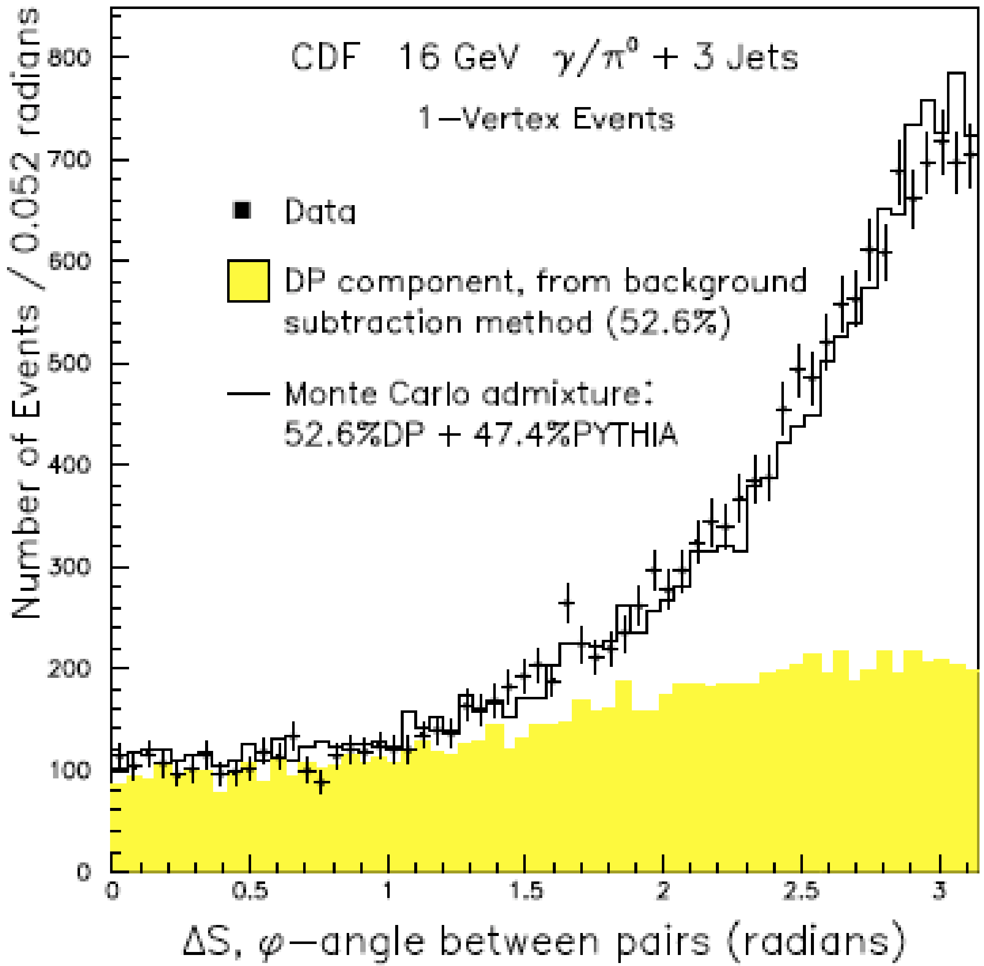
→ Perturbation theory gives n-scatter distributions

Double Parton Scattering

- CDF Collaboration, PR D56 (1997) 3811

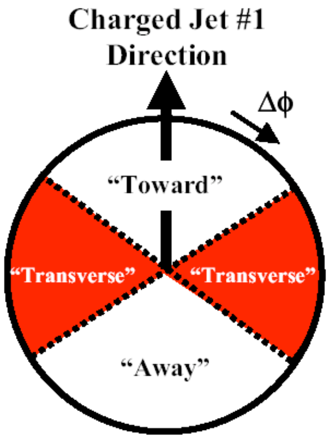
$$\sigma_{\rm DPS} = \frac{\sigma_{\gamma j}\sigma_{jj}}{\sigma_{\rm eff}}$$

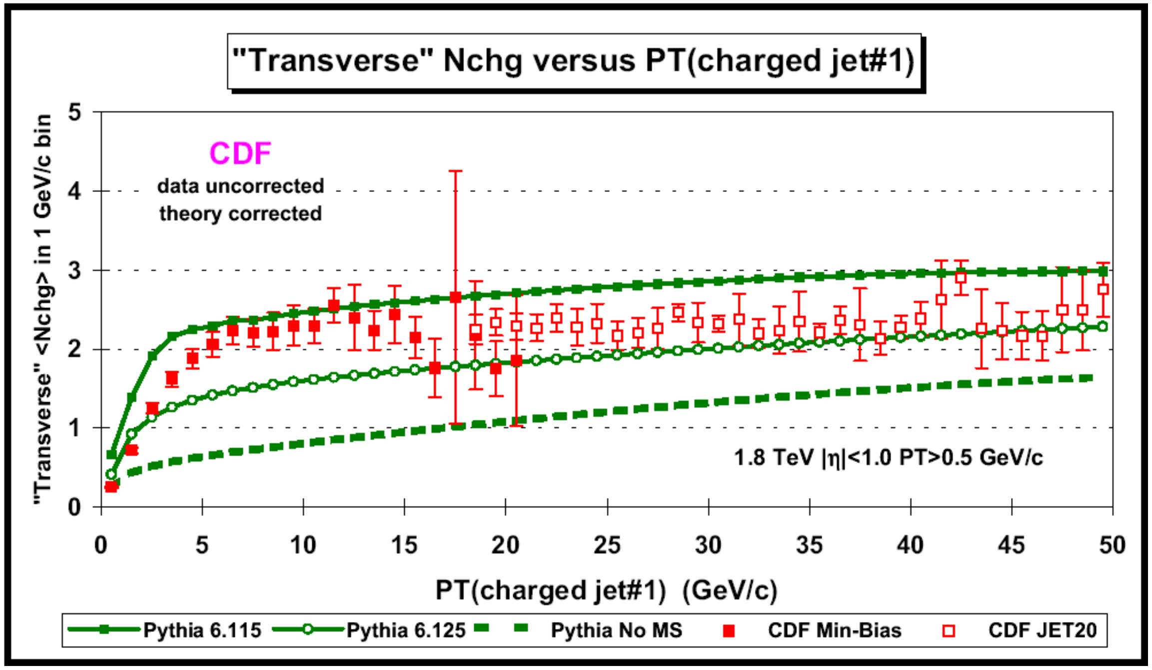
$$\sigma_{\rm eff} = 14 \pm 1.7^{+1.7}_{-2.3}\ {\rm mb}$$



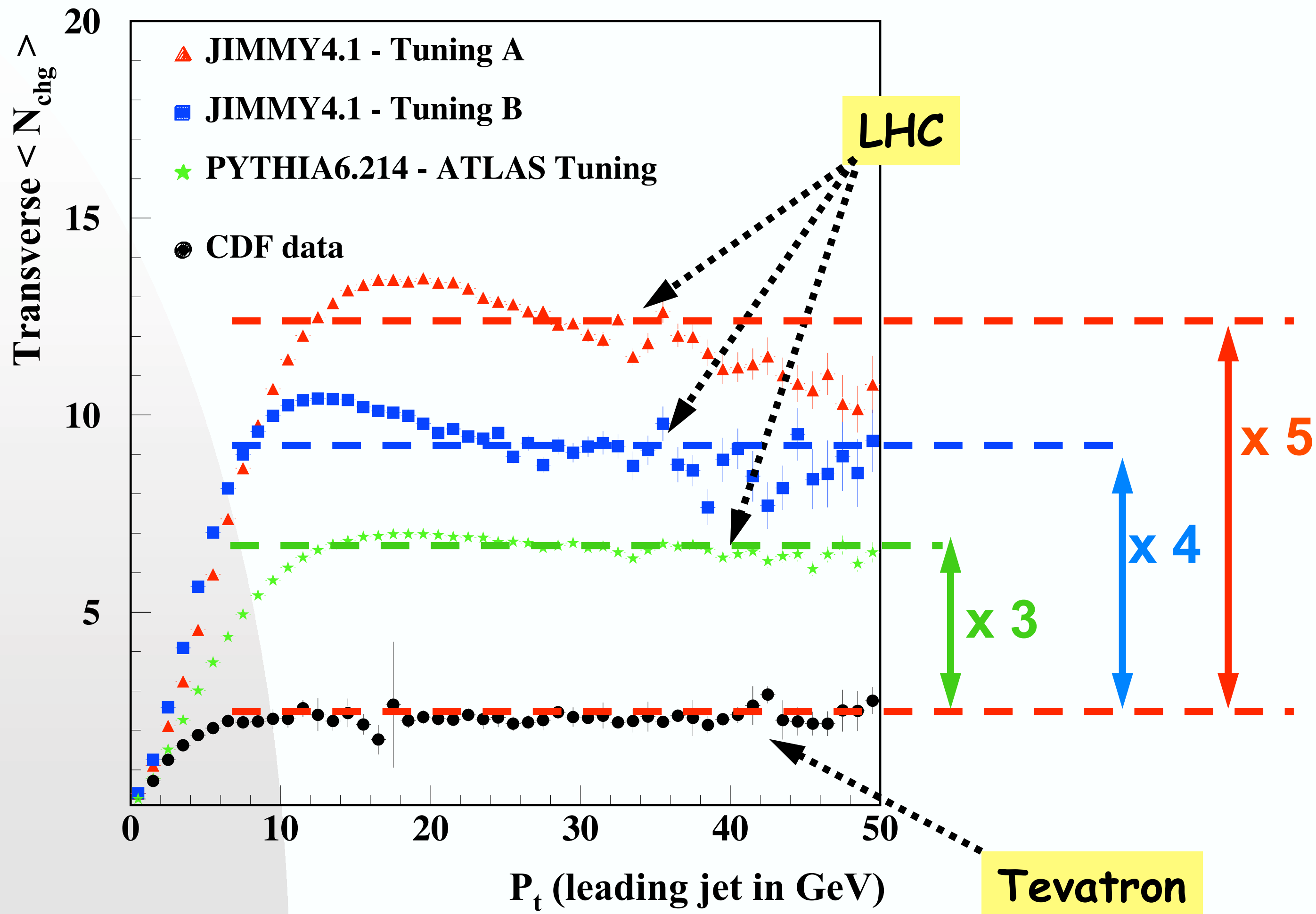
Tuning PYTHIA to the Underlying Event

- Rick Field (CDF): keep all parameters that can be fixed by LEP or HERA at their default values. What's left?
- Underlying event. Big uncertainties at LHC...





LHC predictions: JIMMY4.1 Tunings A and B vs. PYTHIA6.214 – ATLAS Tuning (DC2)



A. M. Moraes

Minimum-bias and the Underlying Event at the LHC

5th November 2004