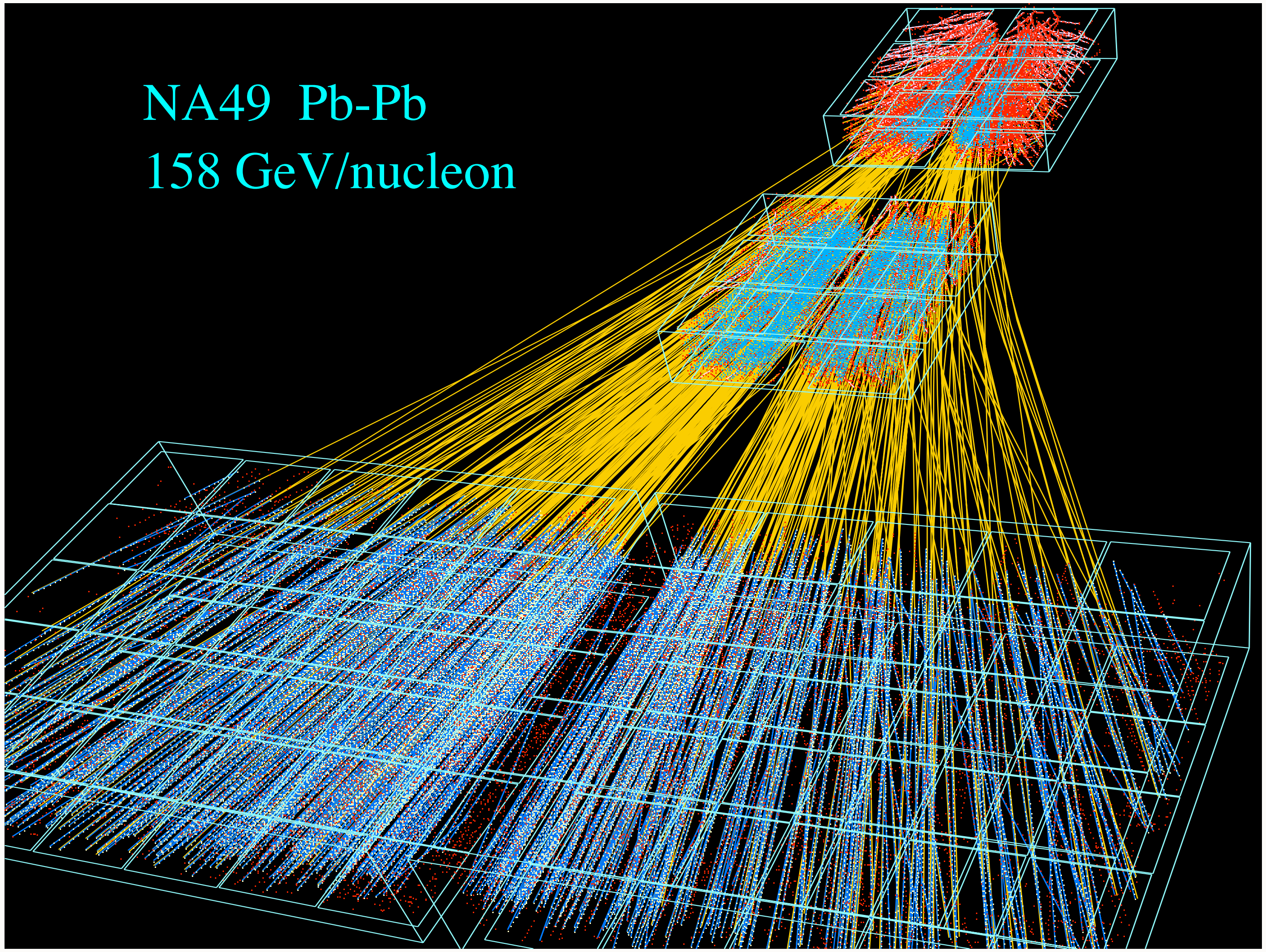
Light-Front Holography and Hadronization at the Amplitude Level



with
Robert Shrock
and
Guy de Teramond

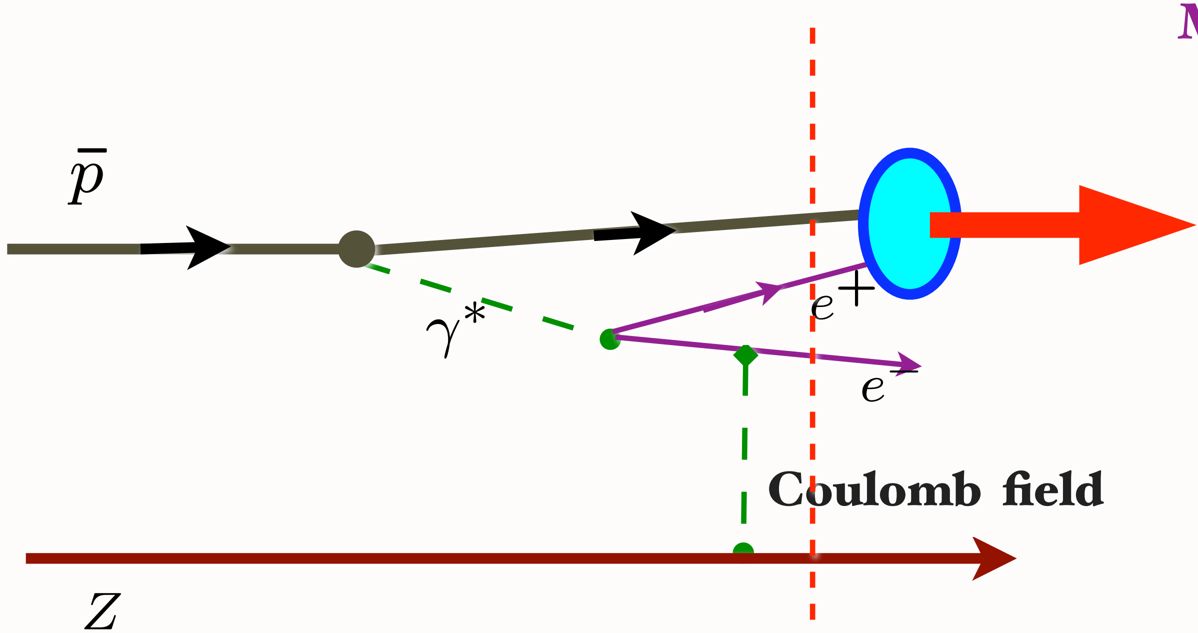
Stan Brodsky , SLAC/IPPP

Rutherford Workshop: New Ideas on Hadronization May 30, 2008

Formation of Relativistic Anti-Hydrogen

Measured at CERN-LEAR and FermiLab

Munger, Schmidt, sjb



$\bar{H}(\bar{p}e^+)$

$$b_\perp \leq \frac{1}{m_{red}\alpha}$$

$$y_{\bar{p}} \simeq y_{e^+}$$

Coalescence of off-shell co-moving positron and antiproton

Wavefunction maximal at small impact separation and equal rapidity

"Hadronization" at the Amplitude Level

Huet, sjb

$$\lim N_C \to 0 \text{ at fixed } \alpha = C_F\alpha_s, n_\ell = n_F/C_F$$

$$\text{QCD} \to \text{Abelian Gauge Theory}$$

Analytic Feature of SU(Nc) Gauge Theory

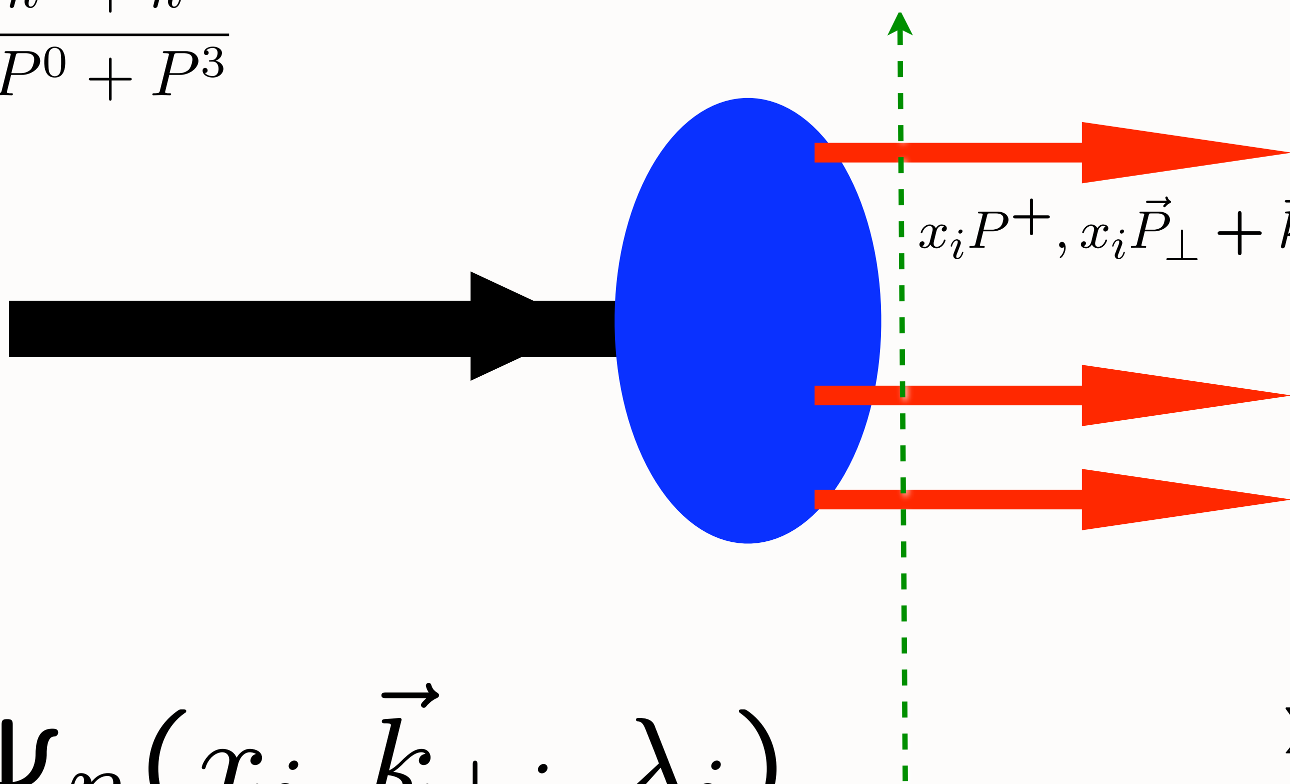
Procedures for QCD
should be valid for QED

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Fixed $\tau = t + z/c$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

$P^+, \vec{P}_\perp$



$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

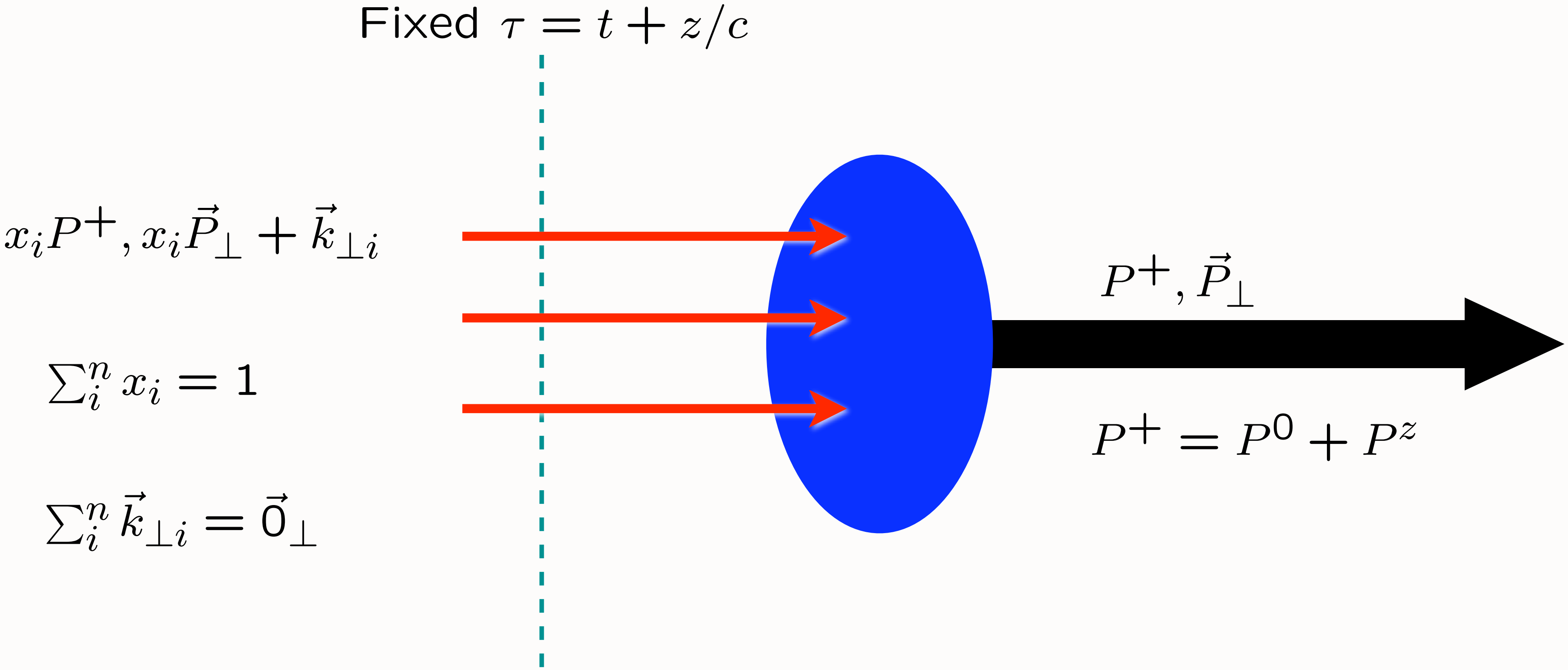
Conserved
LF Fock state by Fock State

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Nonzero Anomalous Moment -->Nonzero orbital angular momentum

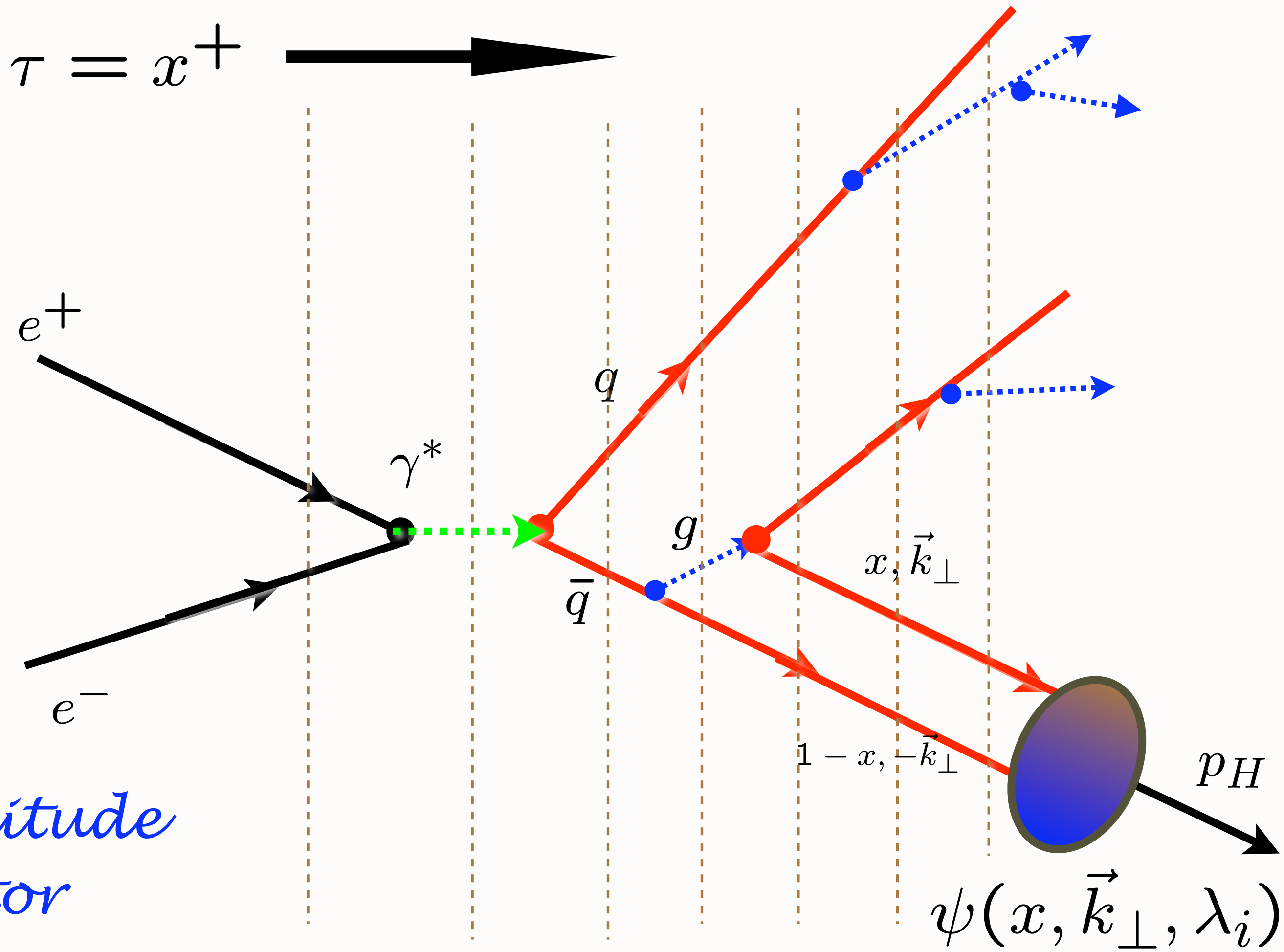
Light-Front Wavefunctions



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Invariant under boosts! Independent of P^μ

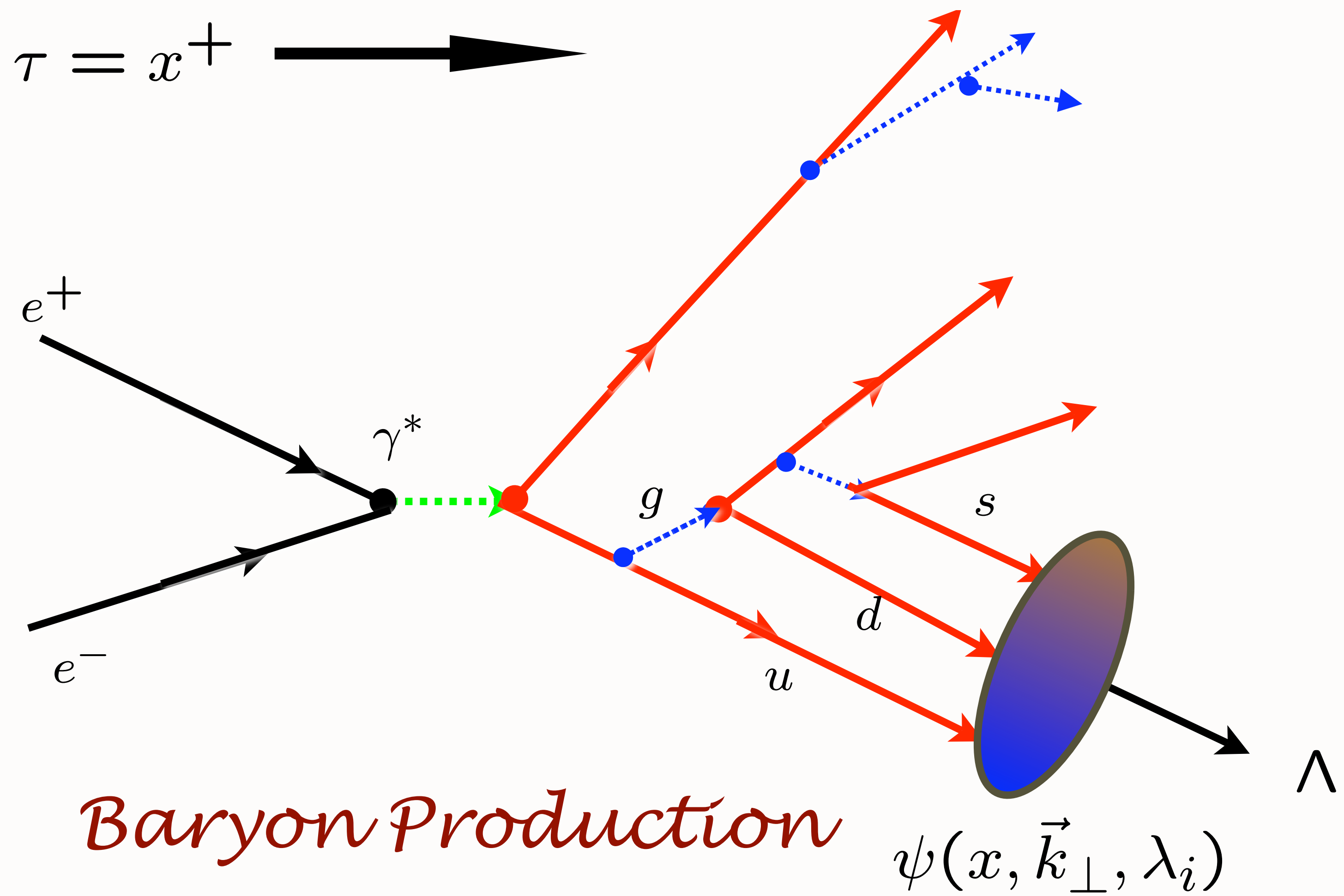
Hadronization at the Amplitude Level



Event amplitude generator

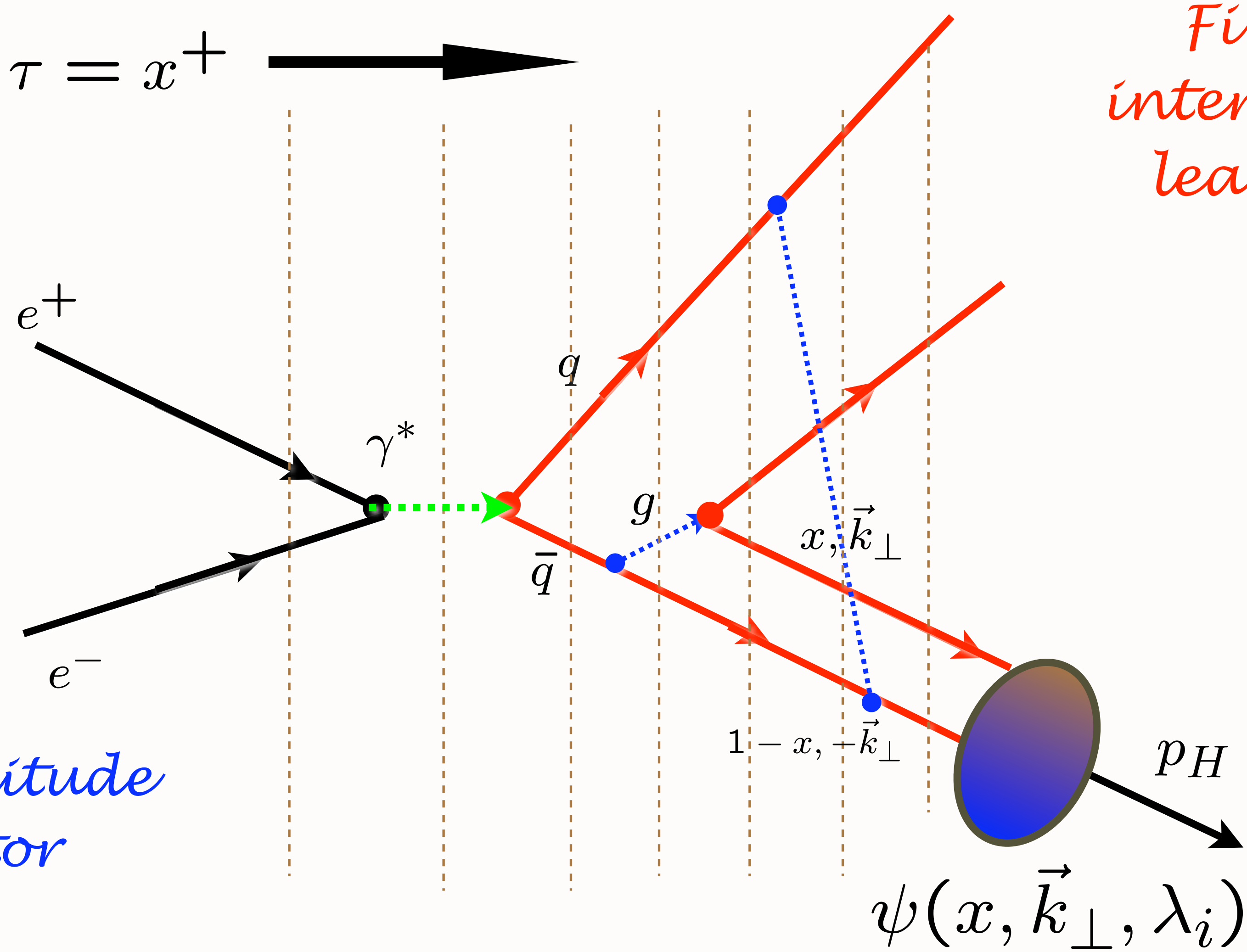
Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Hadronization at the Amplitude Level



Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Hadronization at the Amplitude Level

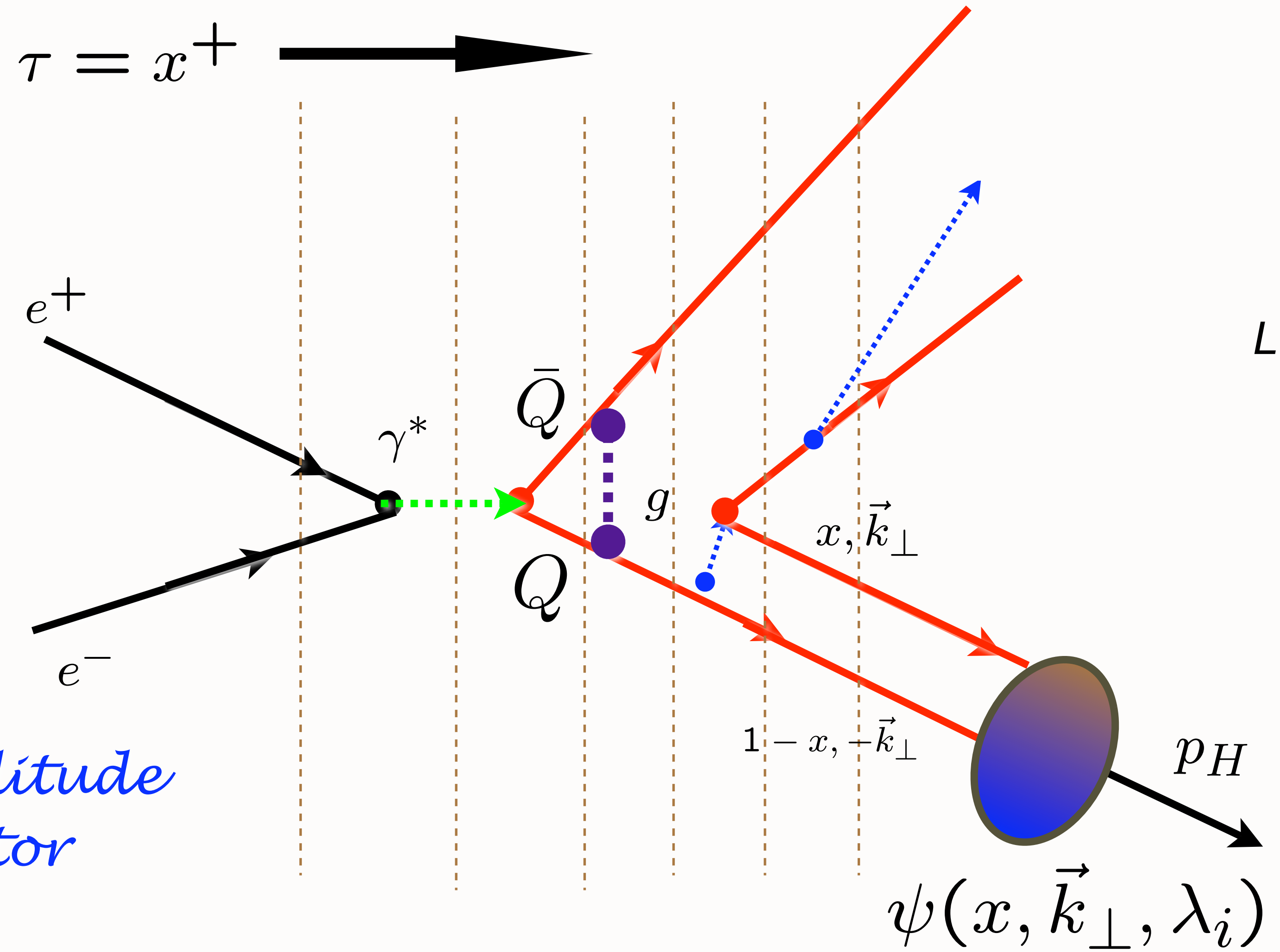


Final-state interactions are leading twist

Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Hadronization at the Amplitude Level



$$\frac{\pi \alpha_s}{\beta}$$

Large Threshold
Corrections
Sommerfeld-
Schwinger
Factor

Event amplitude
generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

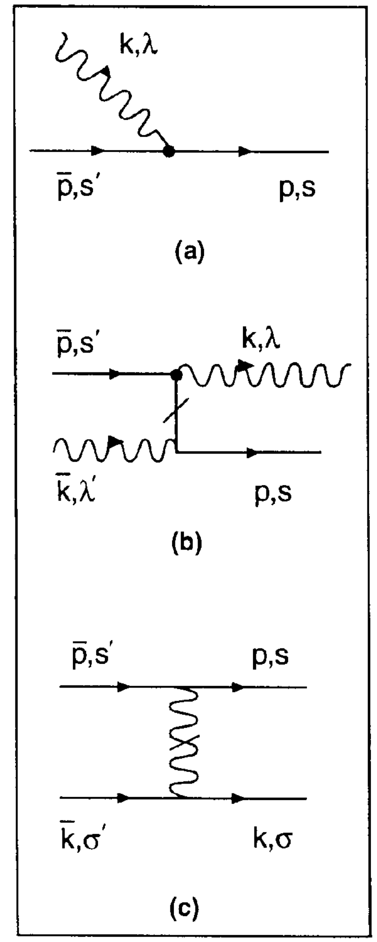
Light-Front QCD

Heisenberg Matrix Formulation

Physical gauge: $A^+ = 0$

$$L^{QCD} \rightarrow H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$



H^{int}_{LF}: Matrix in Fock Space

$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$

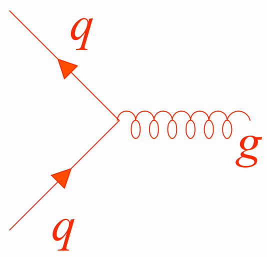
Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

DLCQ: Periodic BC in x^-. Discrete k^+; frame-independent truncation

QCD

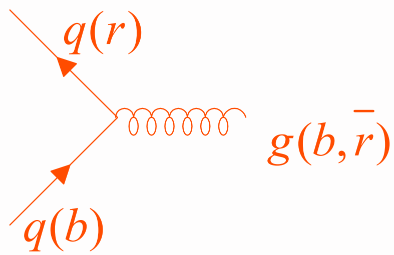
Fundamental Couplings

Only quarks and gluons involve basic vertices: Quark-gluon vertex

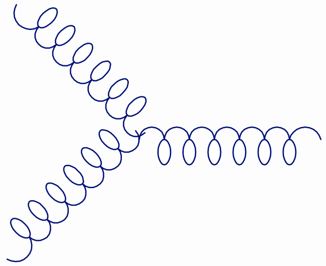


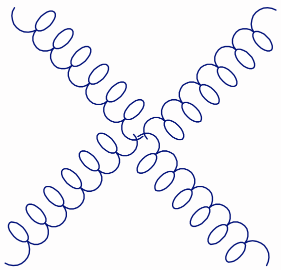
Similar to QED

More exactly



Gluon vertices

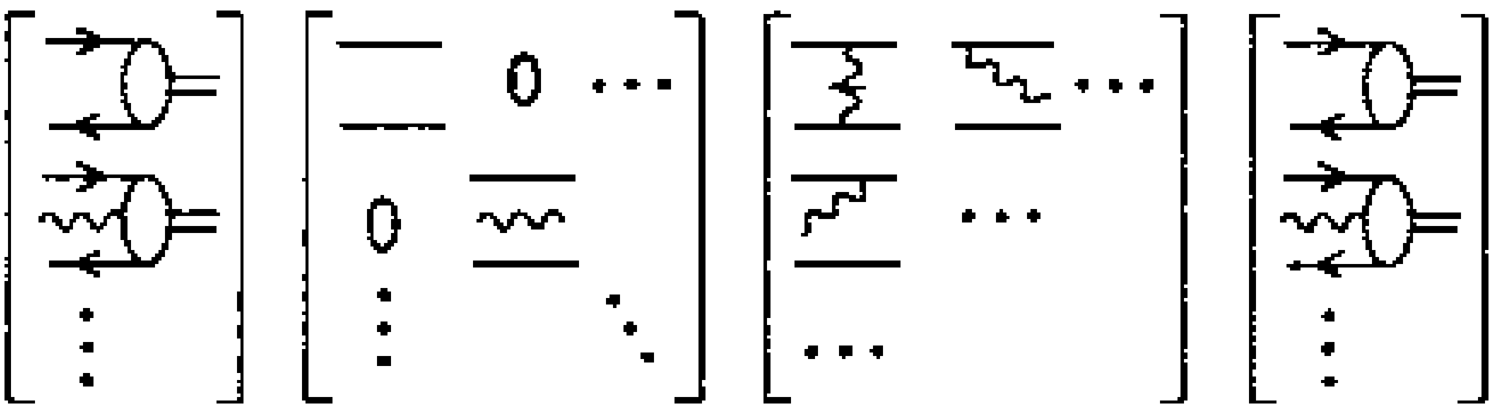




colored particles couple to gluons

LIGHT-FRONT SCHRODINGER EQUATION

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i}\right) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q}|\, V \,|q\bar{q}\rangle & \langle q\bar{q}|\, V \,|q\bar{q}g\rangle & \cdots \\ \langle q\bar{q}g|\, V \,|q\bar{q}\rangle & \langle q\bar{q}g|\, V \,|q\bar{q}g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$



$A^+ = 0$

G.P. Lepage, sjb

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) | n; \vec{k}_{\perp_i}, \lambda_i >$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

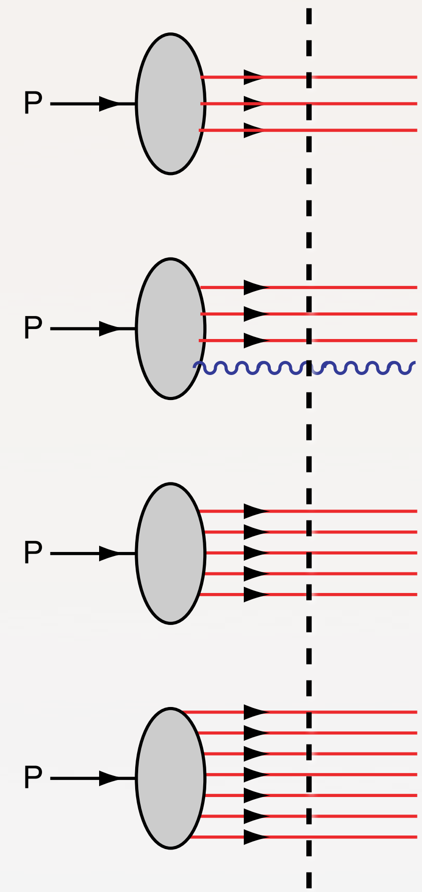
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time

Intrinsic heavy quarks $\bar{u}(x) \neq \bar{d}(x)$

Mueller: BFKL DYNAMICS $\bar{s}(x) \neq s(x)$

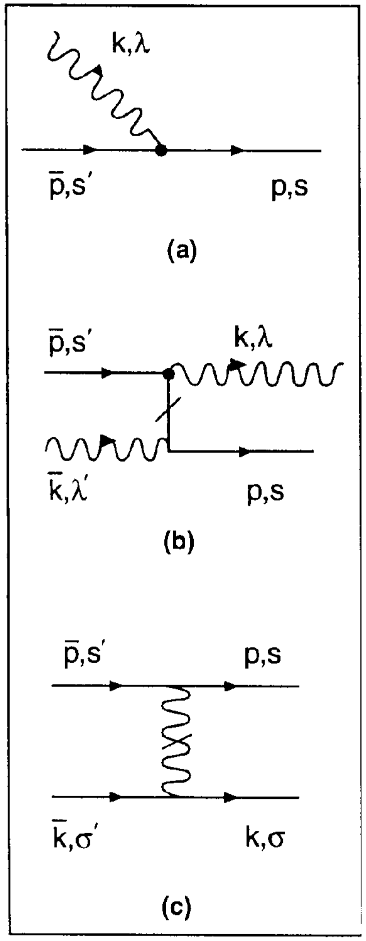
Light-Front QCD

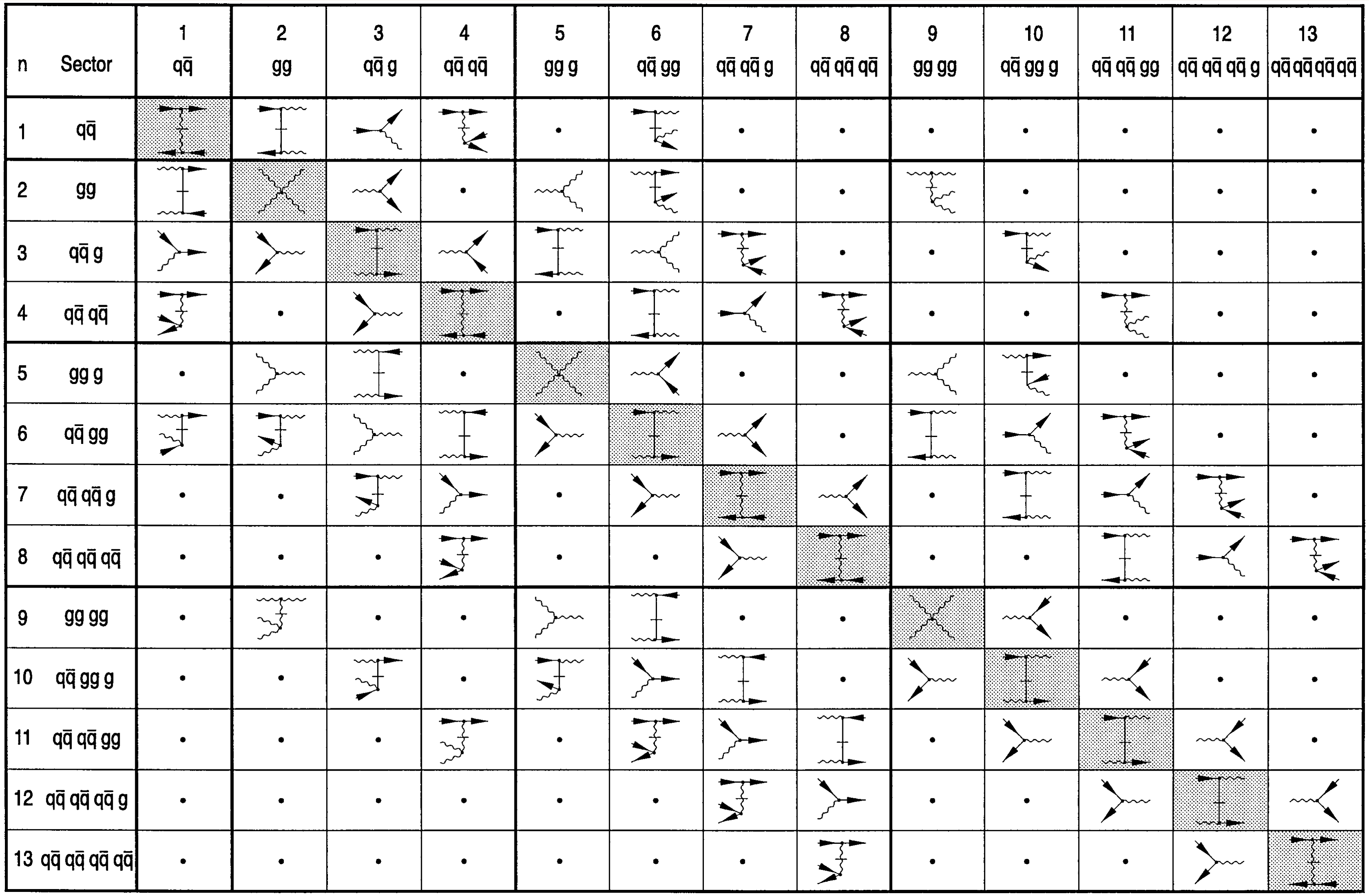
Heisenberg Matrix Formulation

$$H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>$$

DLCQ

Discretized Light-Cone Quantization





Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

H.C. Pauli & sjb

DLCQ: Frame-independent, No fermion doubling; Minkowski Space

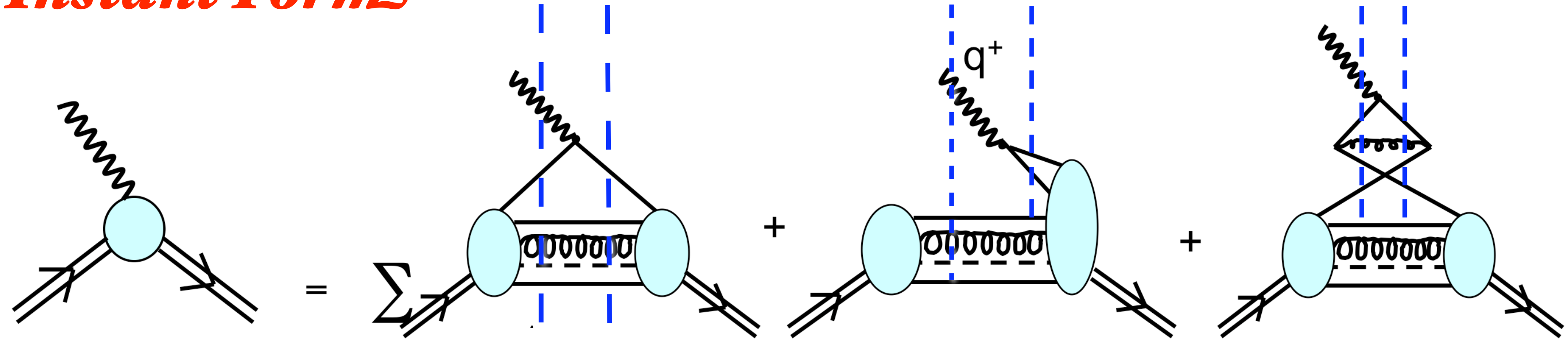
Each element of
flash photograph
illuminated
at same LF time

$$\tau = t + z/c$$

HELEN BRADLEY - PHOTOGRAPHY

Calculation of Form Factors in Equal-Time Theory

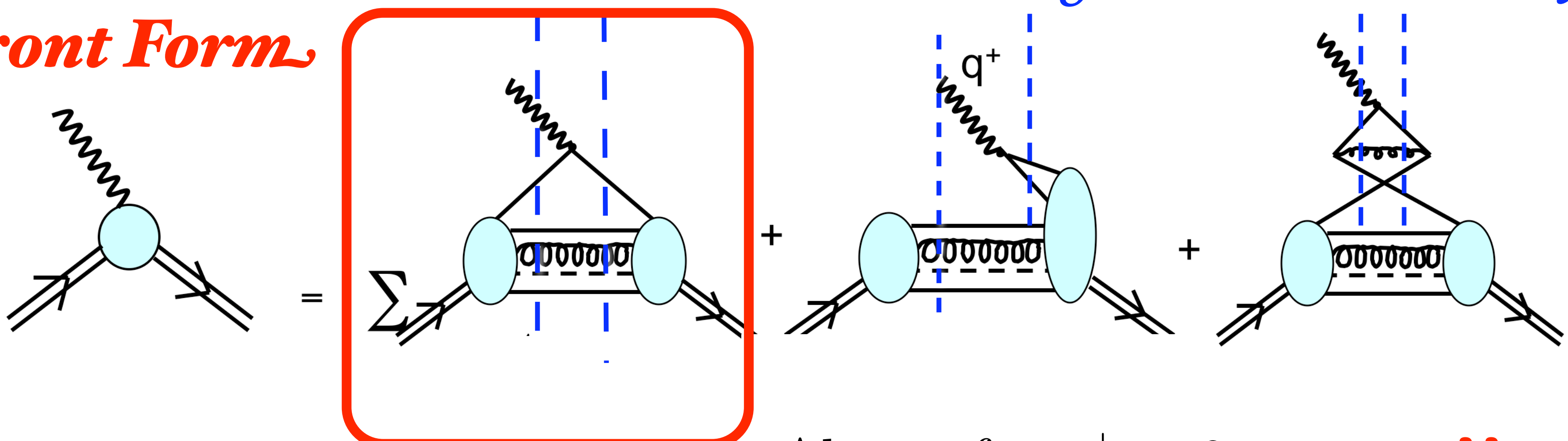
Instant Form



Need vacuum-induced currents

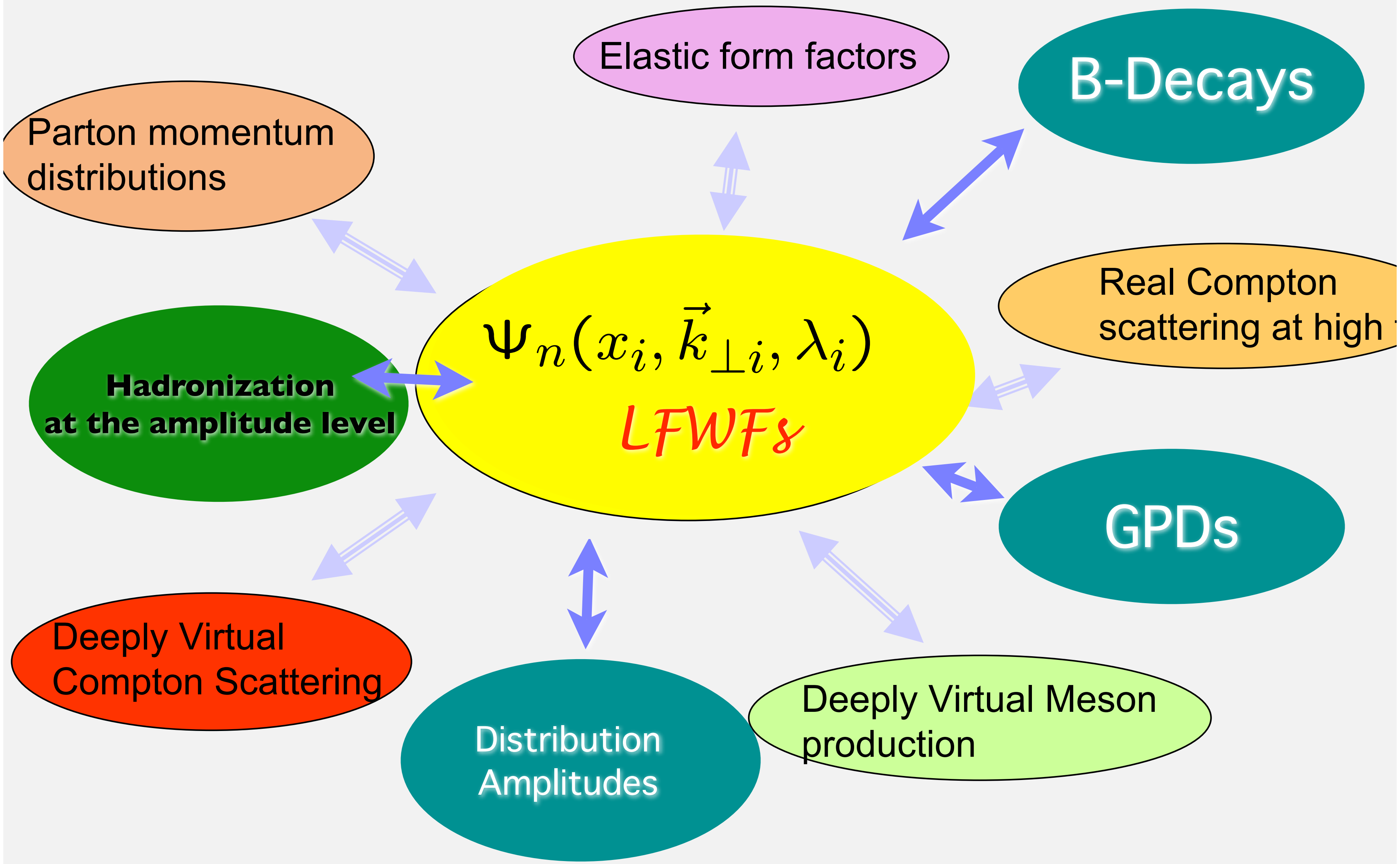
Calculation of Form Factors in Light-Front Theory

Front Form

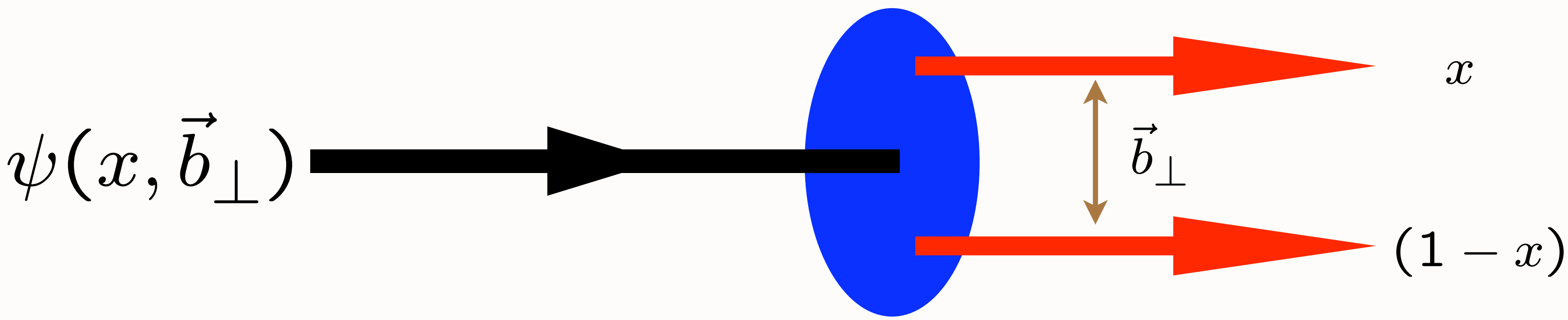


Absent for $q^+ = 0$ **zero !!**

A Unified Description of Hadron Structure



LF(3+1)		AdS_5
$\psi(x,\vec{b}_\perp)$	$\leftrightarrow$	$\phi(z)$
$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$	$\leftrightarrow$	z



$$\psi(x,\zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements: **em and gravitational!**

Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

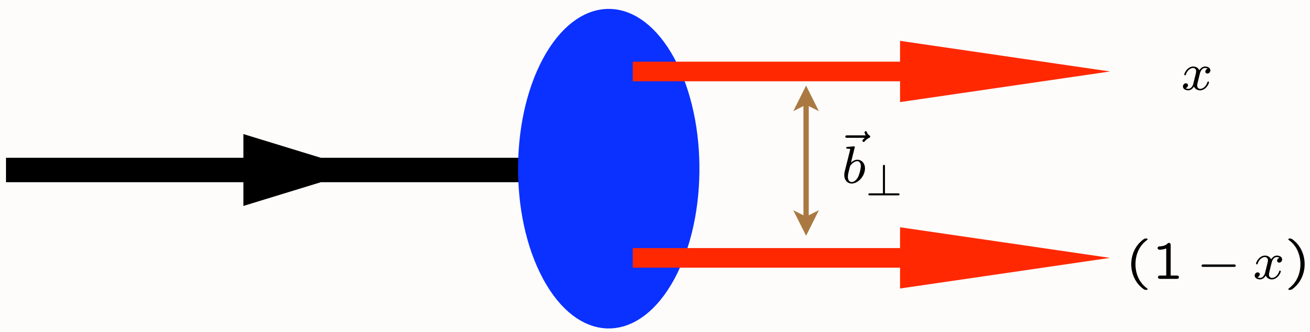
Relativistic LF radial equation

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

G. de Teramond, sjb

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



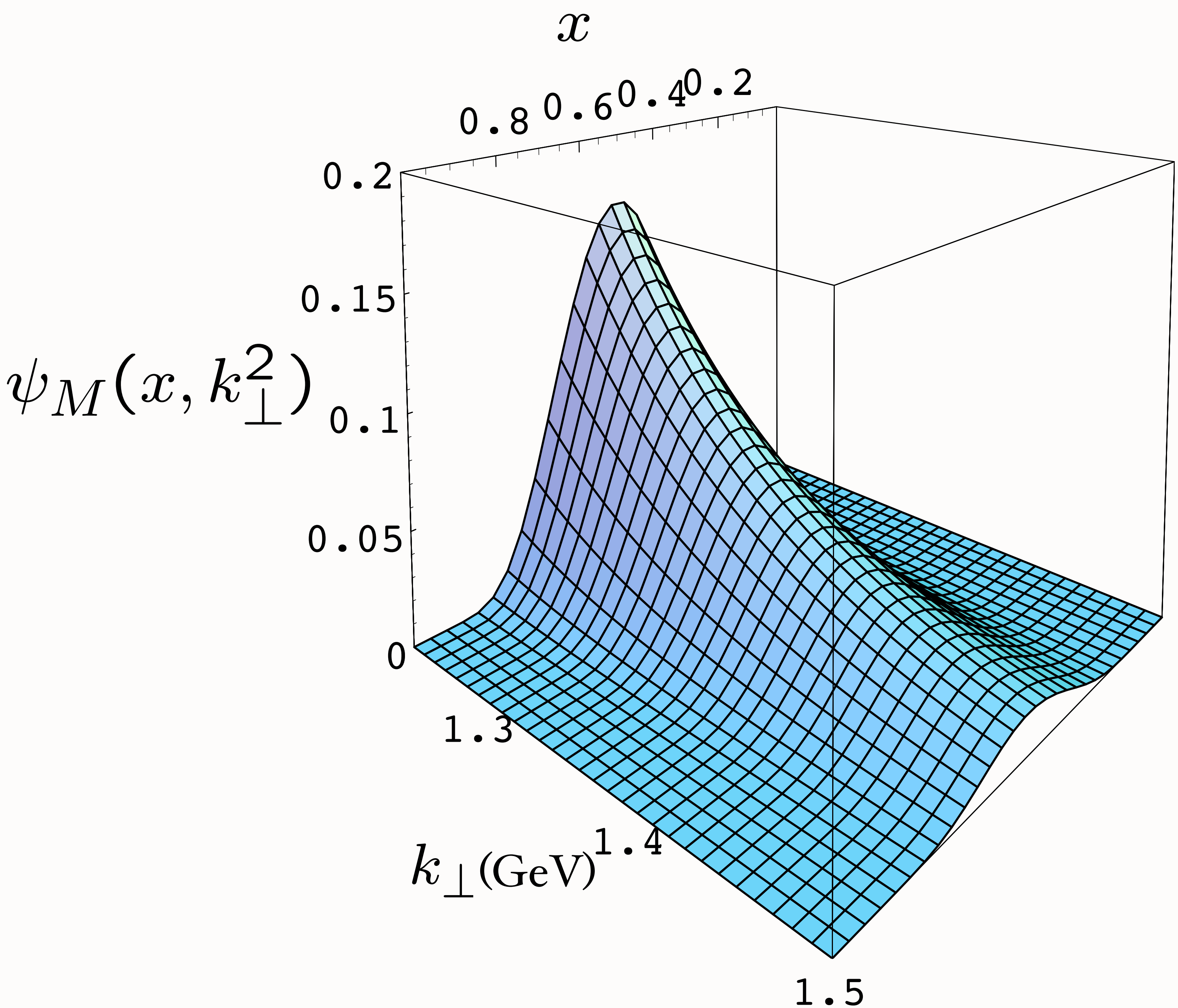
Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

$$+\kappa^4\zeta^2$$

confining potential:

Prediction from AdS/CFT: Meson LFWF



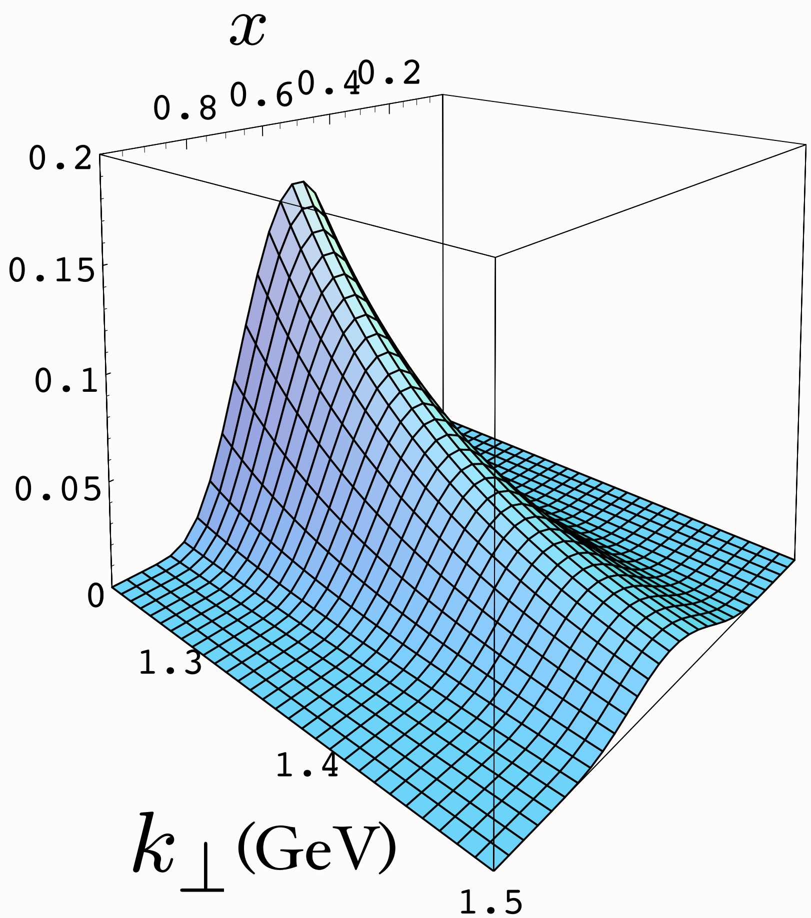
"Soft Wall" model

de Teramond, sjb

Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb

$$\psi_M(x, k_\perp^2)$$



"Soft Wall" model

$\kappa = 0.375$ GeV

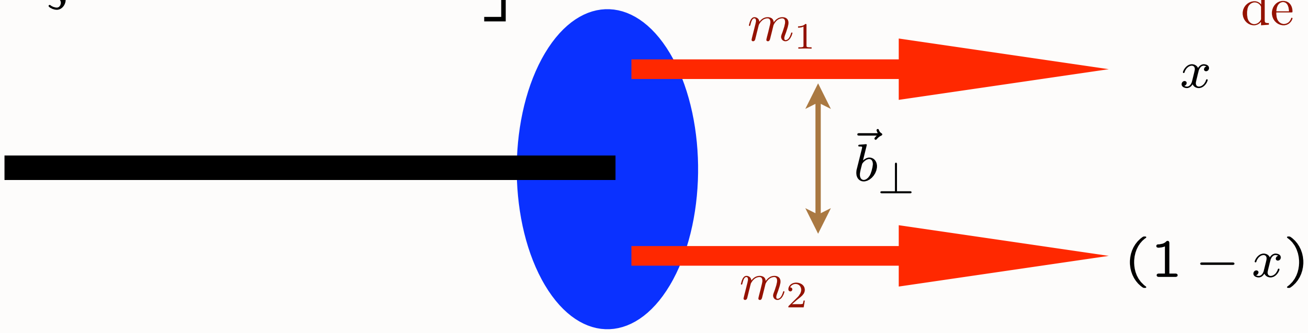
massless quarks

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\boxed{\phi_M(x, Q_0) \propto \sqrt{x(1-x)}}$$

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

de Teramond, sjb



$$\zeta = \sqrt{x(1-x)\vec{b}^2_\perp}$$

Holographic Variable

$$-\frac{d}{d\zeta^2} \equiv \frac{k^2_\perp}{x(1-x)}$$

LF Kinetic Energy in momentum space

Assume LFWF is a dynamical function of the quark-antiquark invariant mass squared

$$-\frac{d}{d\zeta^2} \to -\frac{d}{d\zeta^2} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \equiv \frac{k^2_\perp + m_1^2}{x} + \frac{k^2_\perp + m_2^2}{1-x}$$

Result: Soft-Wall LFWF for massive constituents

$$\psi(x, \mathbf{k}_\perp) = \frac{4\pi c}{\kappa\sqrt{x(1-x)}} e^{-\frac{1}{2\kappa^2}\left(\frac{\mathbf{k}_\perp^2}{x(1-x)} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right)}$$

LF WF in impact space: soft-wall model with massive quarks

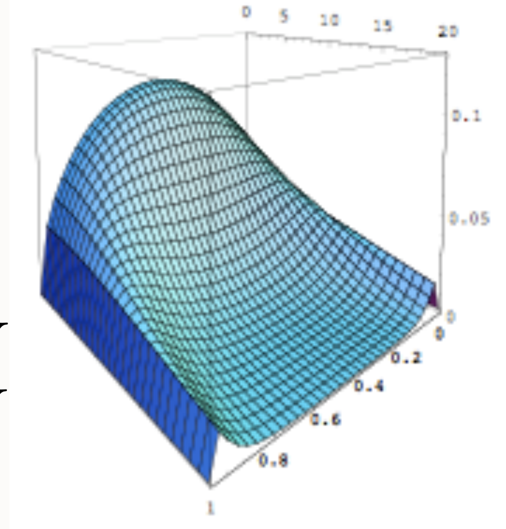
$$\psi(x, \mathbf{b}_\perp) = \frac{c\,\kappa}{\sqrt{\pi}} \sqrt{x(1-x)}\, e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2 - \frac{1}{2\kappa^2}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right]}$$

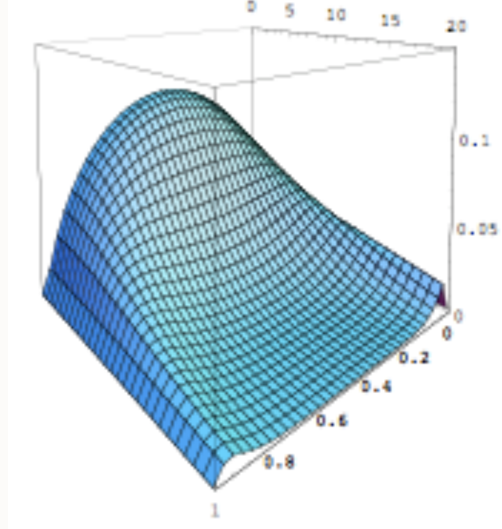
$$z \to \zeta \to \chi$$

$$\chi^2 = b^2 x(1-x) + \frac{1}{\kappa^4}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right]$$

$$|\pi^+ >= |u\bar{d}>$$

$$m_u = 2\ MeV$$
$$m_d = 5\ MeV$$



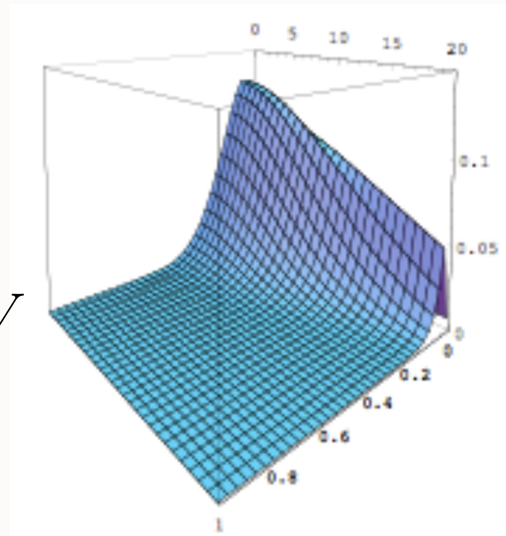


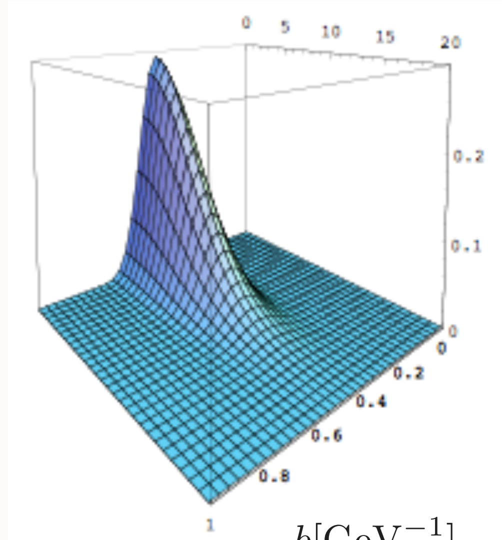
$$|K^+ >= |u\bar{s}>$$

$$m_s = 95\ MeV$$

$$|D^+ >= |c\bar{d}>$$

$$m_c = 1.25\ GeV$$

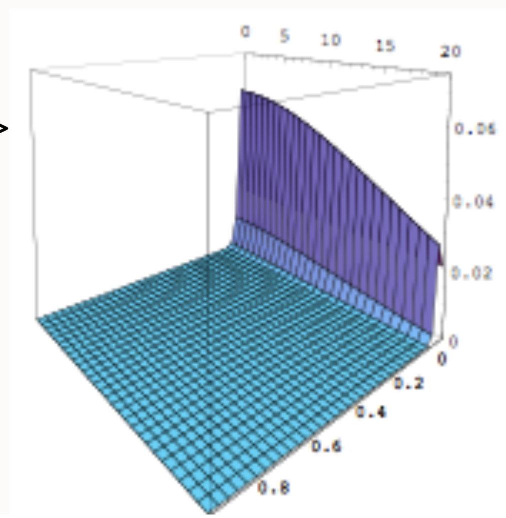


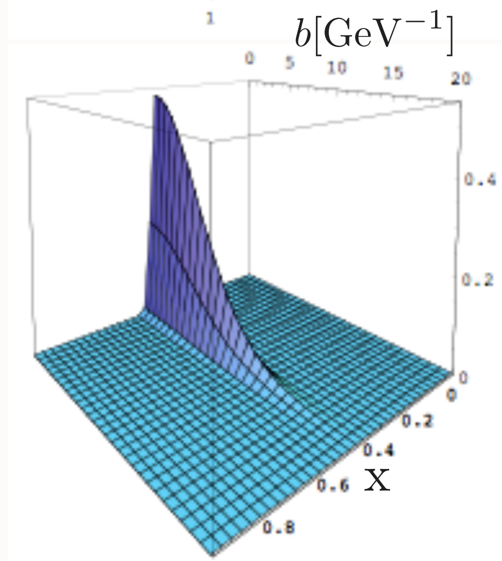


$$|\eta_c >= |c\bar{c}>$$

$$|B^+ >= |u\bar{b}>$$

$$m_b = 4.2\ GeV$$

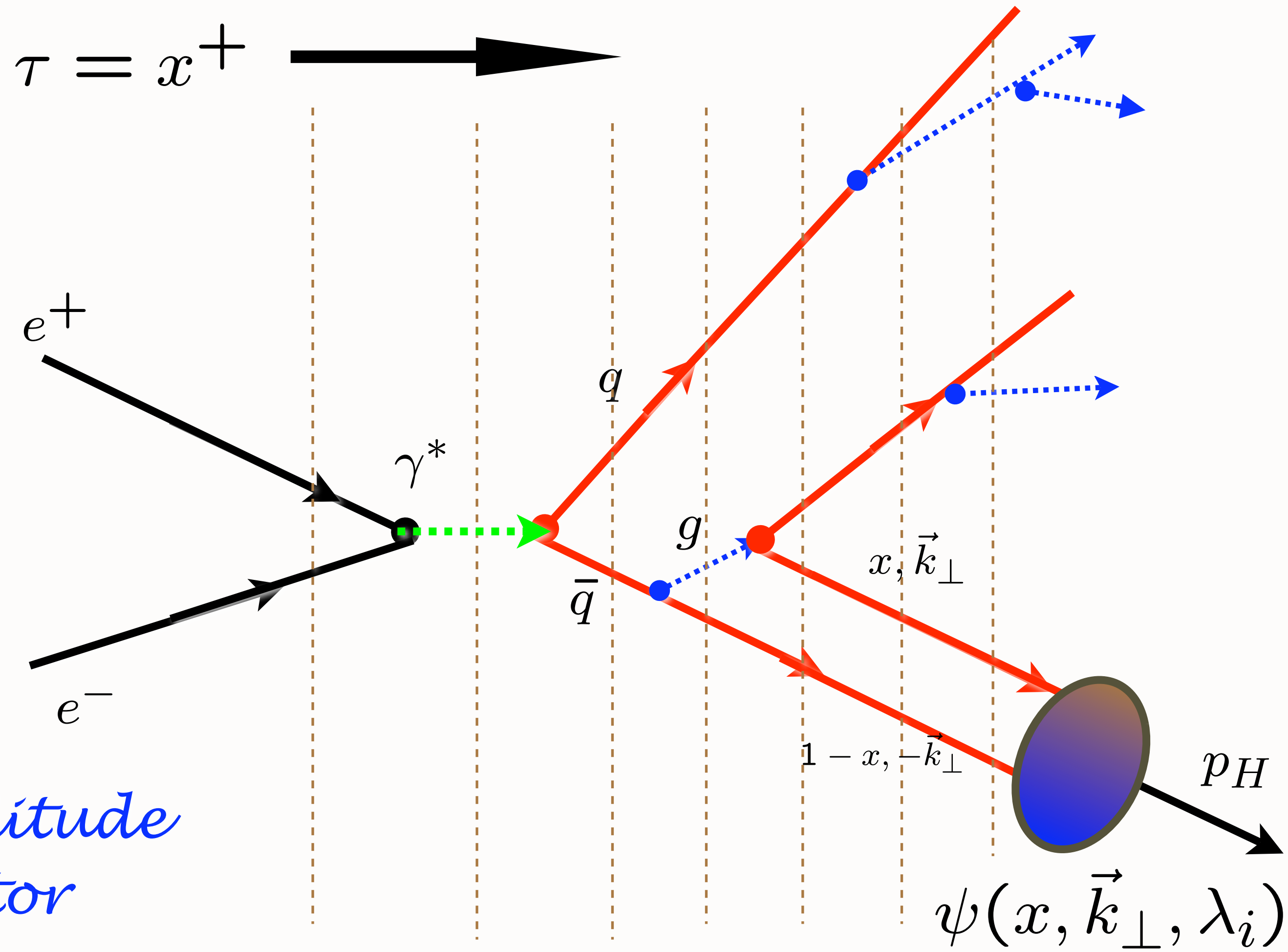




$$|\eta_b >= |b\bar{b}>$$

$$\kappa = 375\ MeV$$

Hadronization at the Amplitude Level



Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Features of LF T-Matrix Formalism

"Event Amplitude Generator"

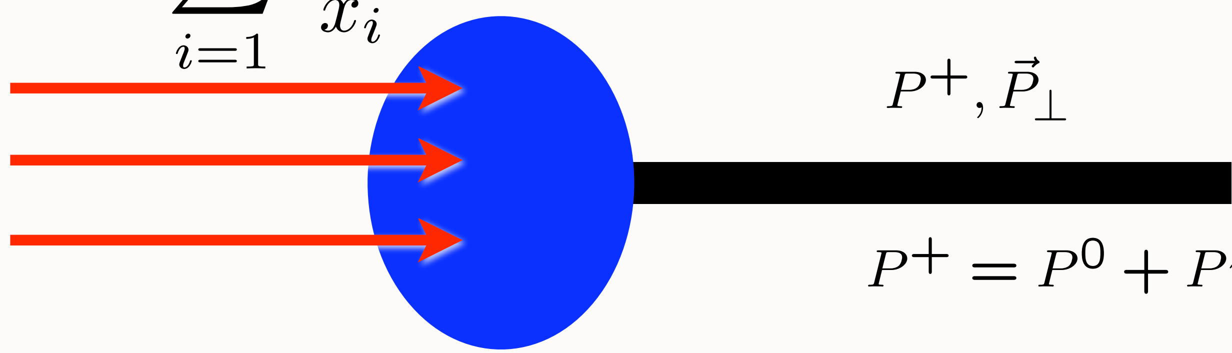
For each color-singlet cluster

If $\mathcal{M}_n^2 \le \Lambda_{QCD}^2$ coalesce to hadron

If $\mathcal{M}_n^2 \ge \Lambda_{QCD}^2$ continue to evolve

avoids gluon avalanche in jet evolution, heavy hadron decays

$$\mathcal{M}_n^2 = \sum_{i=1}^{n} \frac{k_{\perp i}^2}{x_i}$$

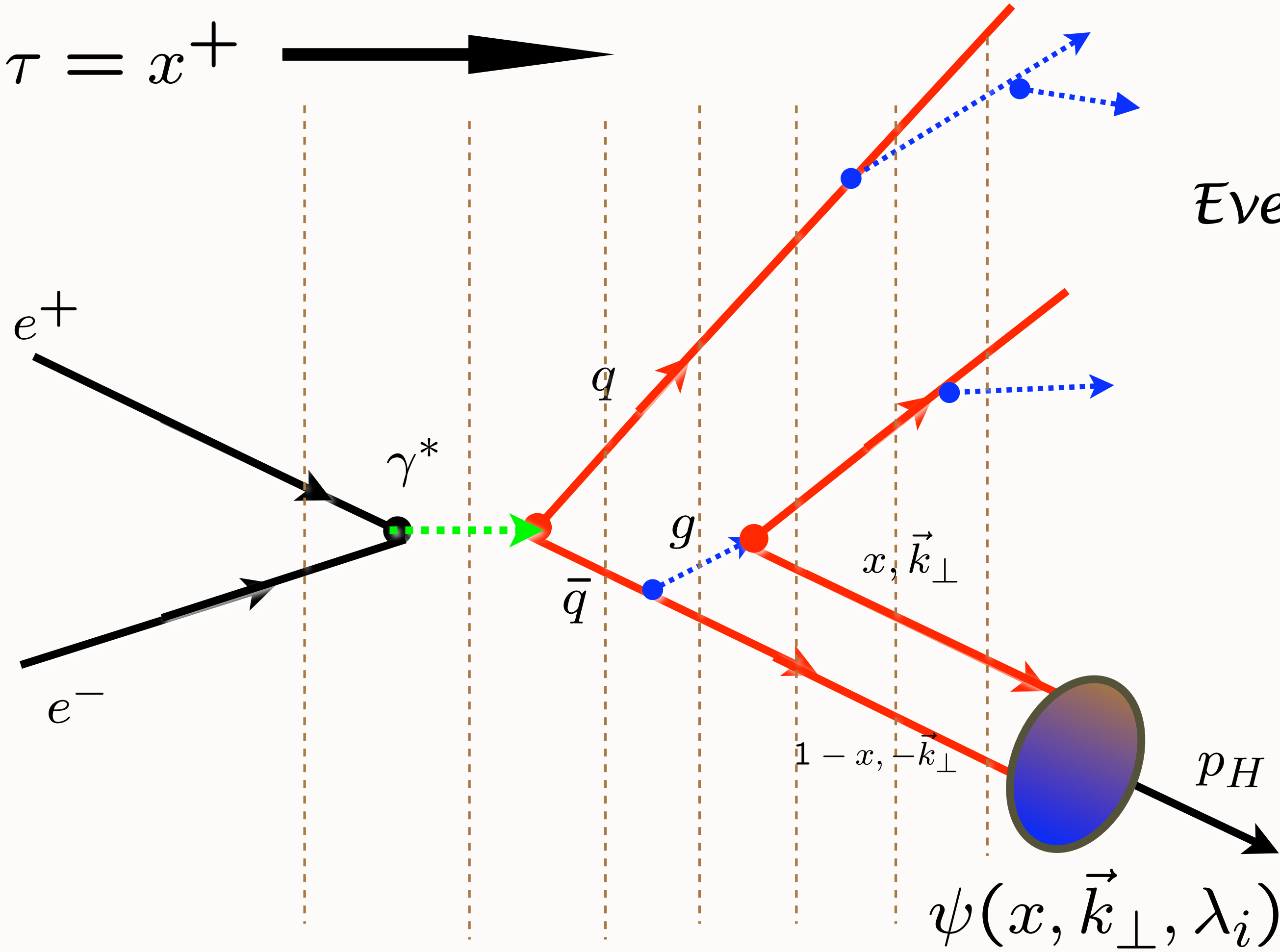


$P^+, \vec{P}_\perp$

$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$P^+ = P^0 + P^z$

Hadronization at the Amplitude Level



Event amplitude generator

Capture if $\zeta^2 = x(1-x)b_\perp^2 > \frac{1}{\Lambda^2_{QCD}}$

i.e.,

$\mathcal{M}^2 = \frac{k_\perp^2}{x(1-x)} < \Lambda^2_{QCD}$

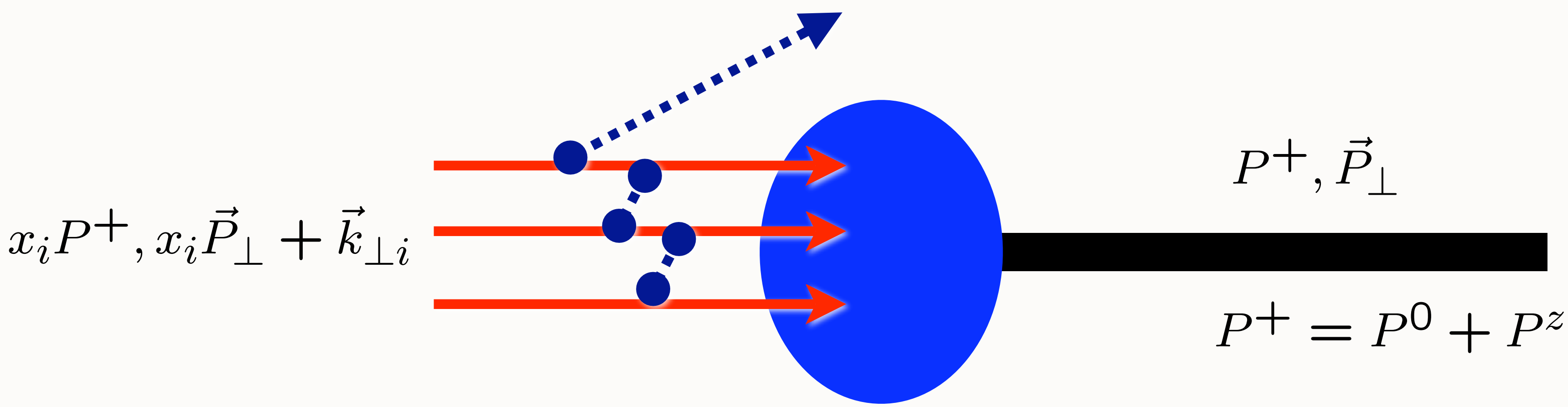
AdS/QCD Hard Wall Confinement:

Features of LF T-Matrix Formalism
"Event Amplitude Generator"

If $\mathcal{M}_n^2 \geq \Lambda_{QCD}^2$ use PQCD hard gluon exchange

- DGLAP and ERBL Evolution from gluon emission and exchange

- Factorization Scale for structure functions and fragmentation functions set: $\mu_{fact} = \Lambda_{QCD}$

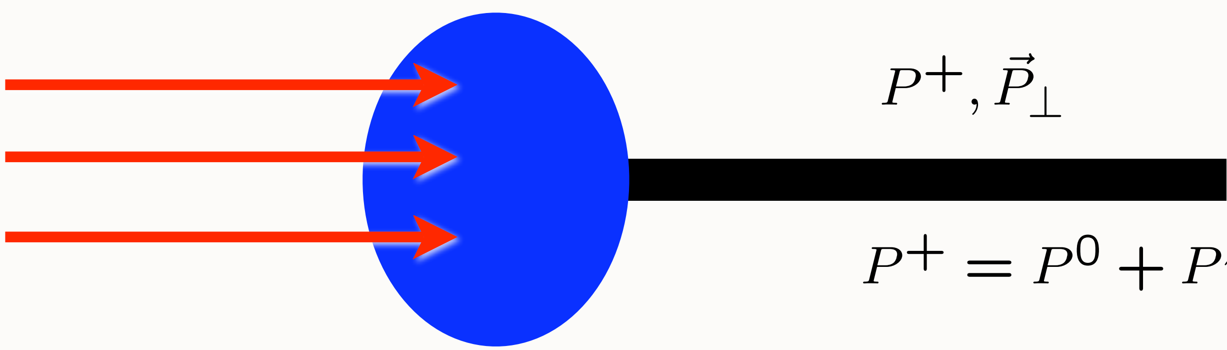


Features of LF T-Matrix Formalism

- Only positive + momenta; no backward time-ordered diagrams
- Frame-independent! Independent of P^+ and P^z
- LC gauge: No ghosts; physical helicity
- $J^z = L^z + S^z$ conservation at every vertex
- Sum all amplitudes with same initial-and final-state helicity, then square to get rate
- Renormalize each UV-divergent amplitude using "alternating denominator" method
- Multiple renormalization scales (BLM)

Features of LF T-Matrix Formalism
“Event Amplitude Generator”

- Same principle as antihydrogen production: off-shell coalescence
- coalescence to hadron favored at equal rapidity, small transverse momenta
- leading heavy hadron production: D and B mesons produced at large z
- hadron helicity conservation if hadron LFWF has $L^z = 0$
- Baryon AdS/QCD LFWF has aligned and anti-aligned quark spin



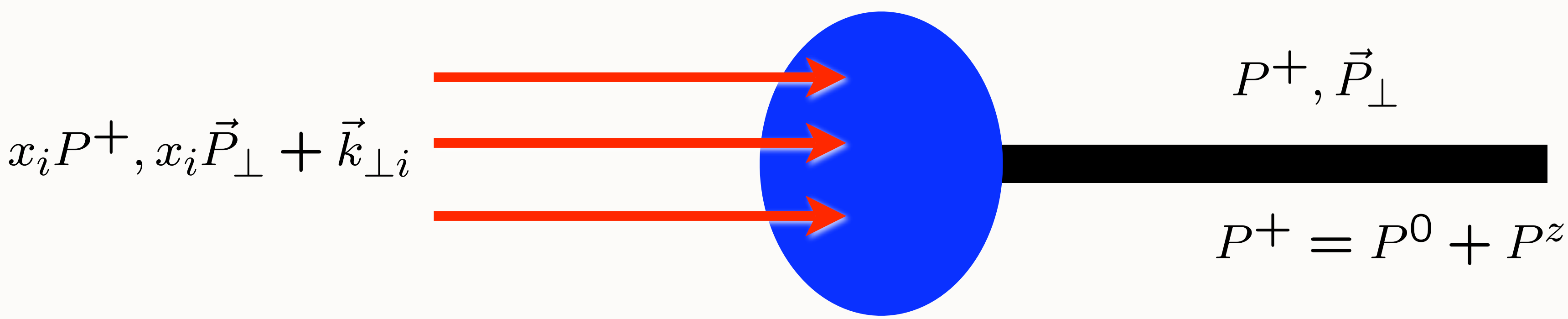
Features of LF T-Matrix Formalism
"Event Amplitude Generator"

- Coalesce color-singlet cluster to hadronic state if

$$\mathcal{M}_n^2 = \sum_{i=1}^{n} \frac{k_{\perp i}^2 + m_i^2}{x_i} < \Lambda_{QCD}^2$$

- The coalescence probability amplitude is the LF wavefunction $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

- No IR divergences: Maximal gluon and quark wavelength from confinement



Features of LF T-Matrix Formalism
"Event Amplitude Generator"

- Includes Effects of Initial and Final State Interactions from gluon exchange
- Sivers, Collins, Boer-Mulders Effects
- Diffractive Channels
- Heavy quark threshold corrections
- Intrinsic Heavy Quark Effects
- s(x) versus anti-s(x) asymmetry

Features of LF T-Matrix Formalism
"Event Amplitude Generator"

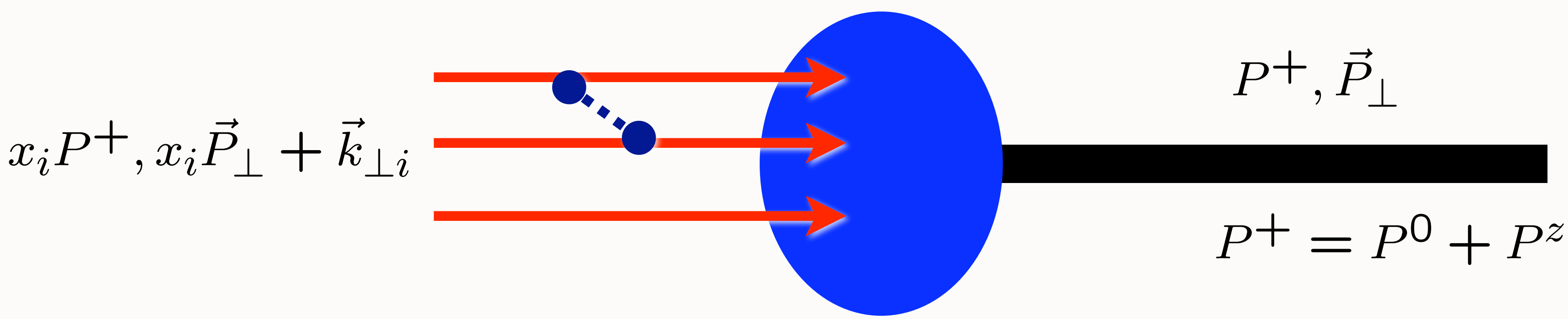
If $\mathcal{M}_n^2 \geq \Lambda^2_{QCD}$ use PQCD hard gluon exchange

- Generates PQCD Hard Tail of LFWF at high x and high transverse momentum

- Dimensional Counting rules and Color Transparency for Hard Exclusive Channels

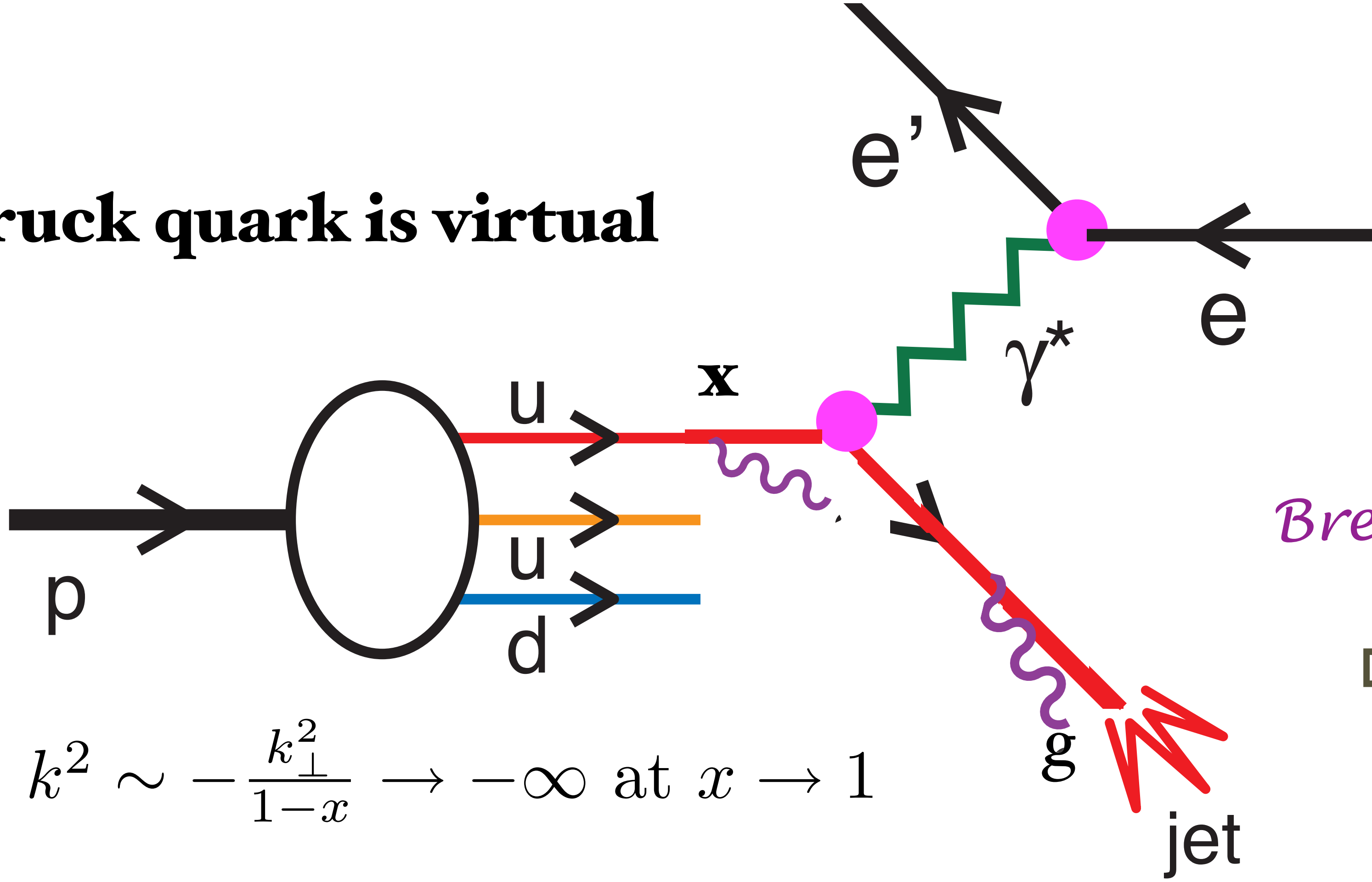
- Counting rules for structure functions and fragmentation functions at large x and z:

$$(1-x)^{2n_{spect}-1}, (1-z)^{2n_{spect}-1}$$



Deep Inelastic Electron-Proton Scattering

Struck quark is virtual



Gluonic Bremmstrahlung

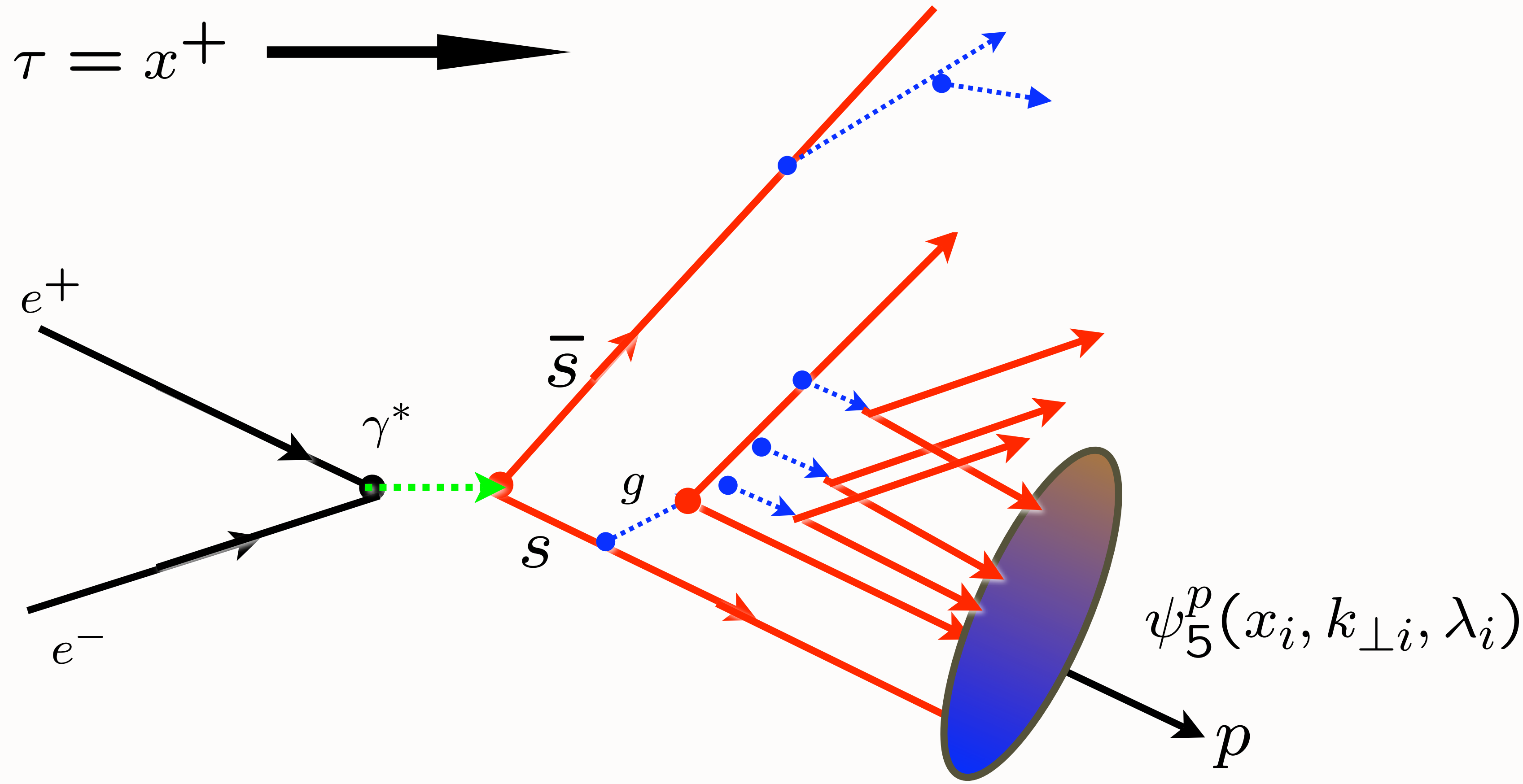
DGLAP Evolution

$$k^2 \sim -\frac{k_\perp^2}{1-x} \rightarrow -\infty \text{ at } x \rightarrow 1$$

Off-shell Effect: Breakdown of DGLAP at x ~1 !

Off-shell Effect: Breakdown of DGLAP at z ~1 !

Hadronization at the Amplitude Level



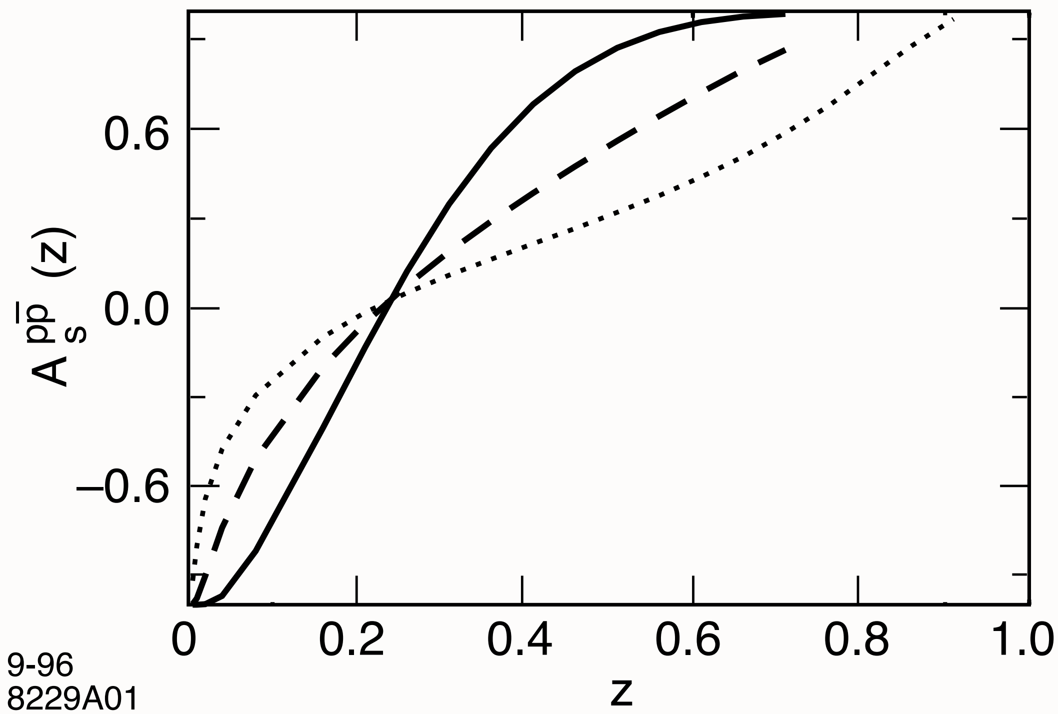
Higher Fock State Coalescence $|uuds\bar{s}>$

Asymmetric Hadronization ! $D_{s\to p}(z) \neq D_{s\to\bar{p}}(z)$

B-Q Ma, sjb

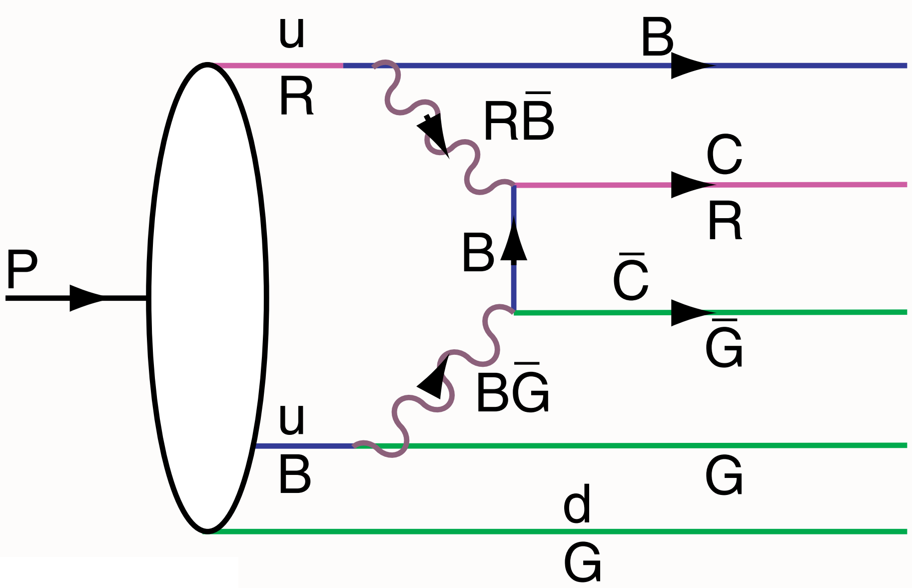
B-Q Ma, sjb

$$D_{s\to p}(z) \neq D_{s\to\bar{p}}(z)$$



$$A_s^{p\bar{p}}(z) = \frac{D_{s\to p}(z) - D_{s\to\bar{p}}(z)}{D_{s\to p}(z) + D_{s\to\bar{p}}(z)}$$

Consequence of $s_p(x) \neq \bar{s}_p(x)$ $\quad |uuds\bar{s}> \simeq |K^+\Lambda>$



$|uud c\bar{c}>$ Fluctuation in Proton

QCD: Probability $\frac{\sim\Lambda^2_{QCD}}{M^2_Q}$

$|e^+e^-\ell^+\ell^->$ Fluctuation in Positronium

QED: Probability $\frac{\sim(m_e\alpha)^4}{M^4_\ell}$

OPE derivation - M.Polyakov et al.

$< p|\frac{G^3_{\mu\nu}}{m^2_Q}|p >$ vs. $< p|\frac{F^4_{\mu\nu}}{m^4_\ell}|p >$

$c\bar{c}$ in Color Octet

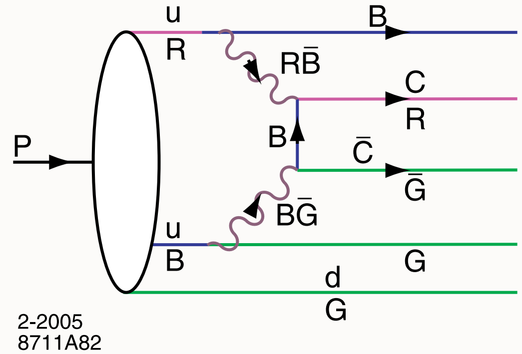
Distribution peaks at equal rapidity (velocity)
Therefore heavy particles carry the largest momentum fractions

$$\hat{x}_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

High x charm!

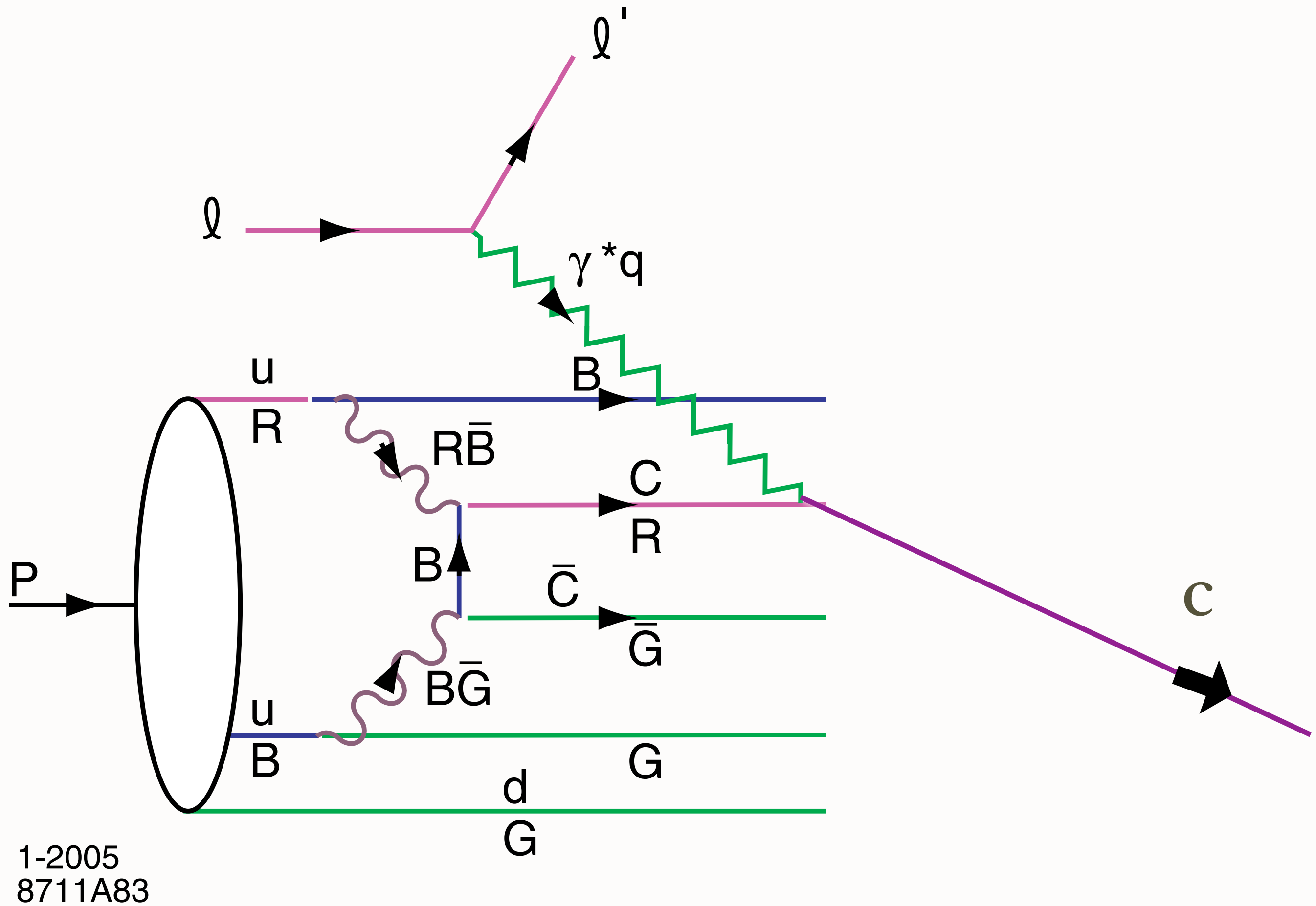
Hoyer, Peterson, Sakai, sjb

Intrinsic Heavy-Quark Fock States

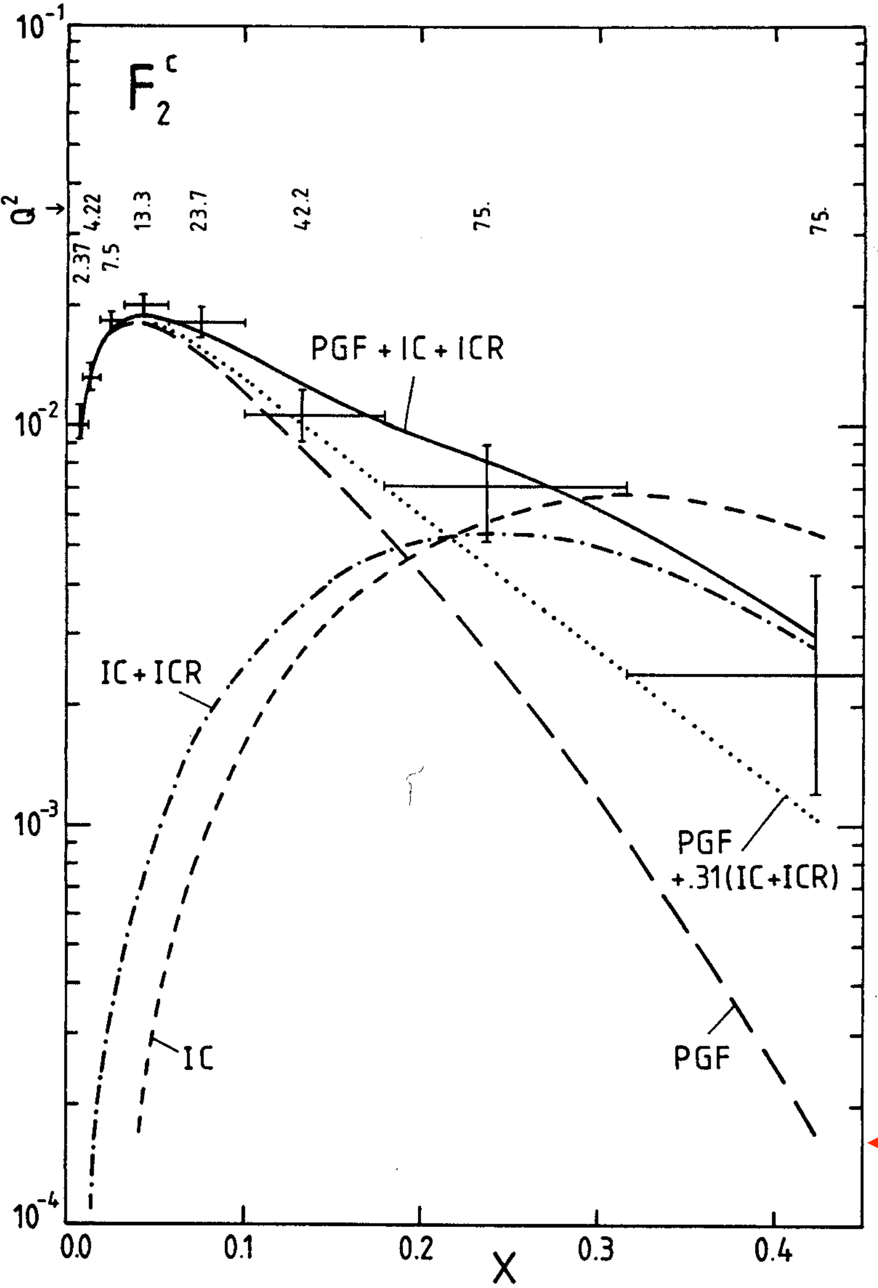


- Rigorous prediction of QCD, OPE
- Color - Octet + Color - Octet Fock State!
- Probability $P_{Q\bar{Q}} \propto \frac{1}{M_Q^2}$ $P_{Q\bar{Q}Q\bar{Q}} \sim \alpha_s^2 P_{Q\bar{Q}}$ $P_{c\bar{c}/p} \simeq 1\%$
- Large Effect at high x
- Greatly increases kinematics of colliders such as Higgs production (Kopeliovich, Schmidt, Soffer, sjb)
- Severely underestimated in conventional parameterizations of heavy quark distributions (Pumplin, Tung)
- Many empirical tests

Measure c(x) in Deep Inelastic Lepton-Proton Scattering



Hoyer, Peterson, SJB

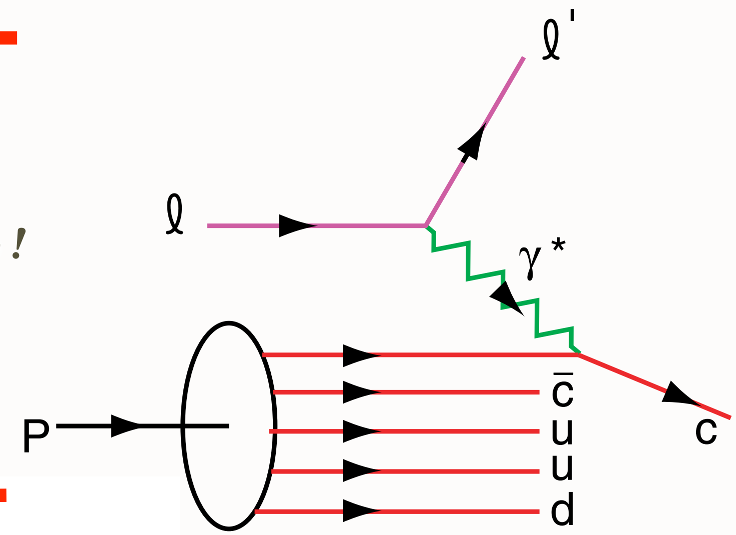


Measurement of Charm Structure Function

J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-Gev Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).

First Evidence for Intrinsic Charm

factor of 30 !



DGLAP / Photon-Gluon Fusion: factor of 30 too small

- EMC data: $c(x, Q^2) > 30 \times \text{DGLAP}$
  $Q^2 = 75\ \text{GeV}^2$, $x = 0.42$

- High x_F $pp \to J/\psi X$

- High x_F $pp \to J/\psi J/\psi X$

- High x_F $pp \to \Lambda_c X$

- High x_F $pp \to \Lambda_b X$

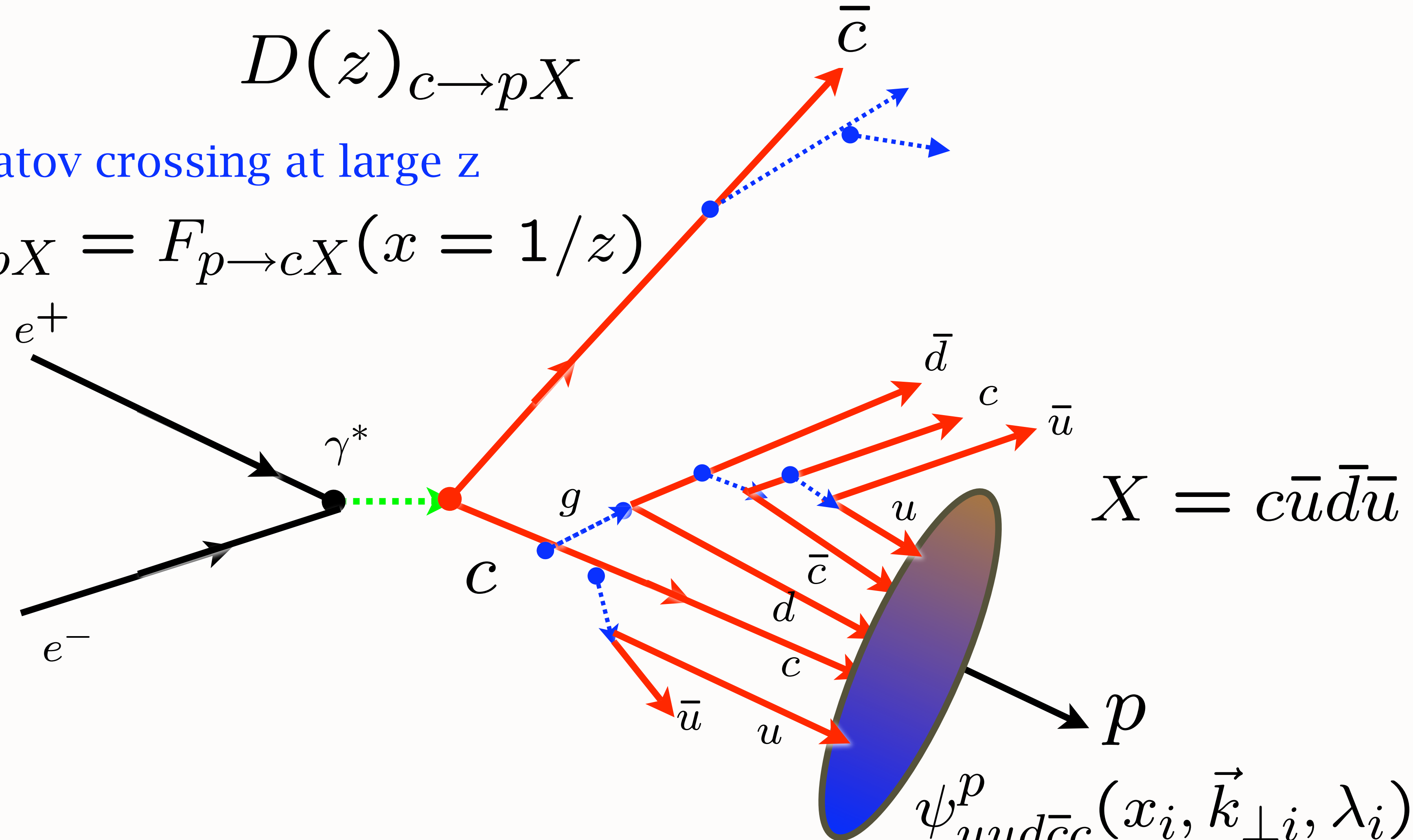
- High x_F $pp \to \Xi(ccd) X$ (SELEX)

Timelike Test of Charm Distribution in Proton

$$D(z)_{c \to pX}$$

Gribov-Lipatov crossing at large z

$$zD(z)_{c \to pX} = F_{p \to cX}(x = 1/z)$$

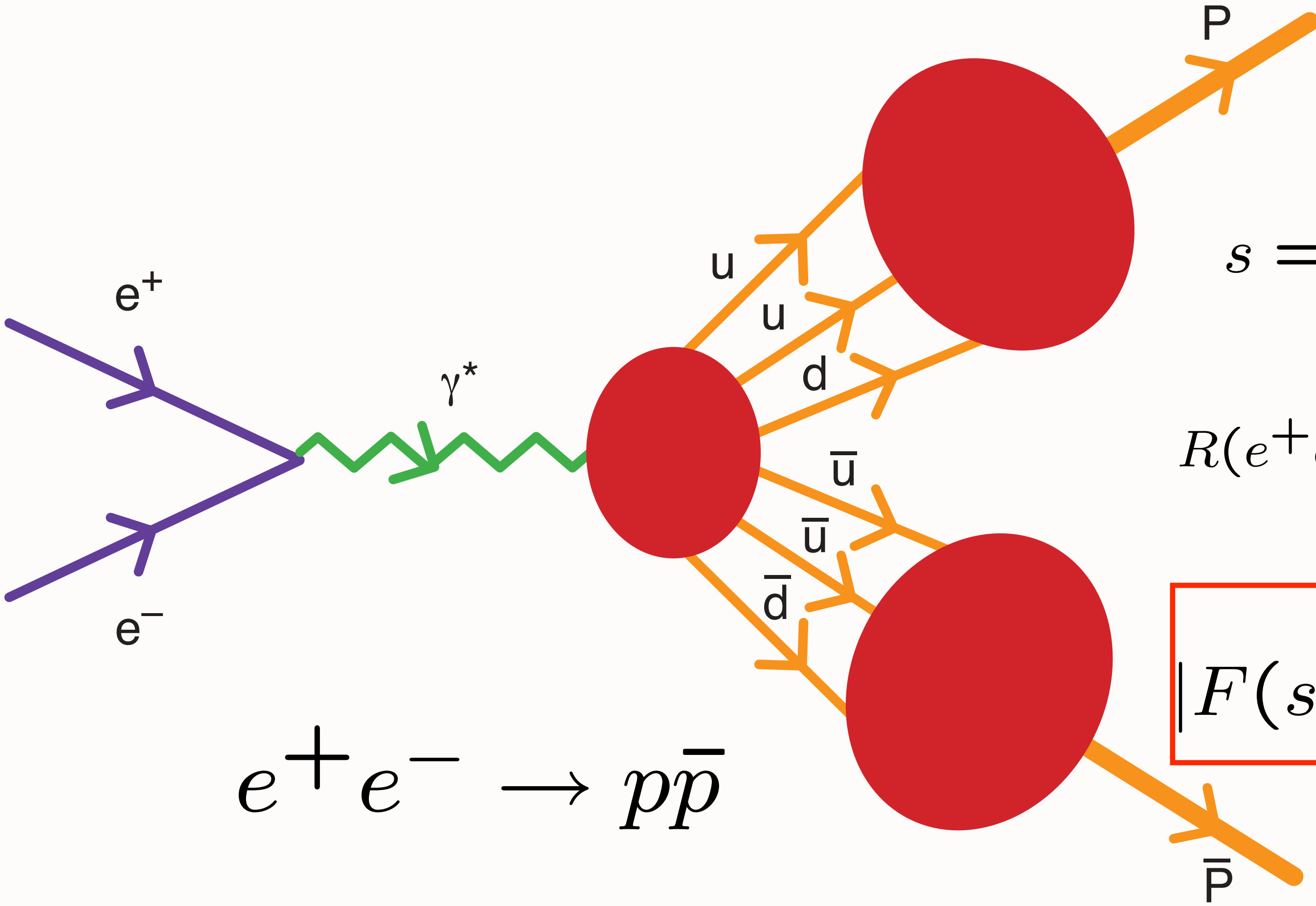


Intrinsic charm model:
predict proton at same rapidity as charm quark: high z

$$z_i \propto m_{\perp i} = \sqrt{m_i^2 + k_\perp^2}$$

Exclusive Processes

What if we ask for a specific final state?



$$s = (E_{e^+} + E_{e^-})^2$$

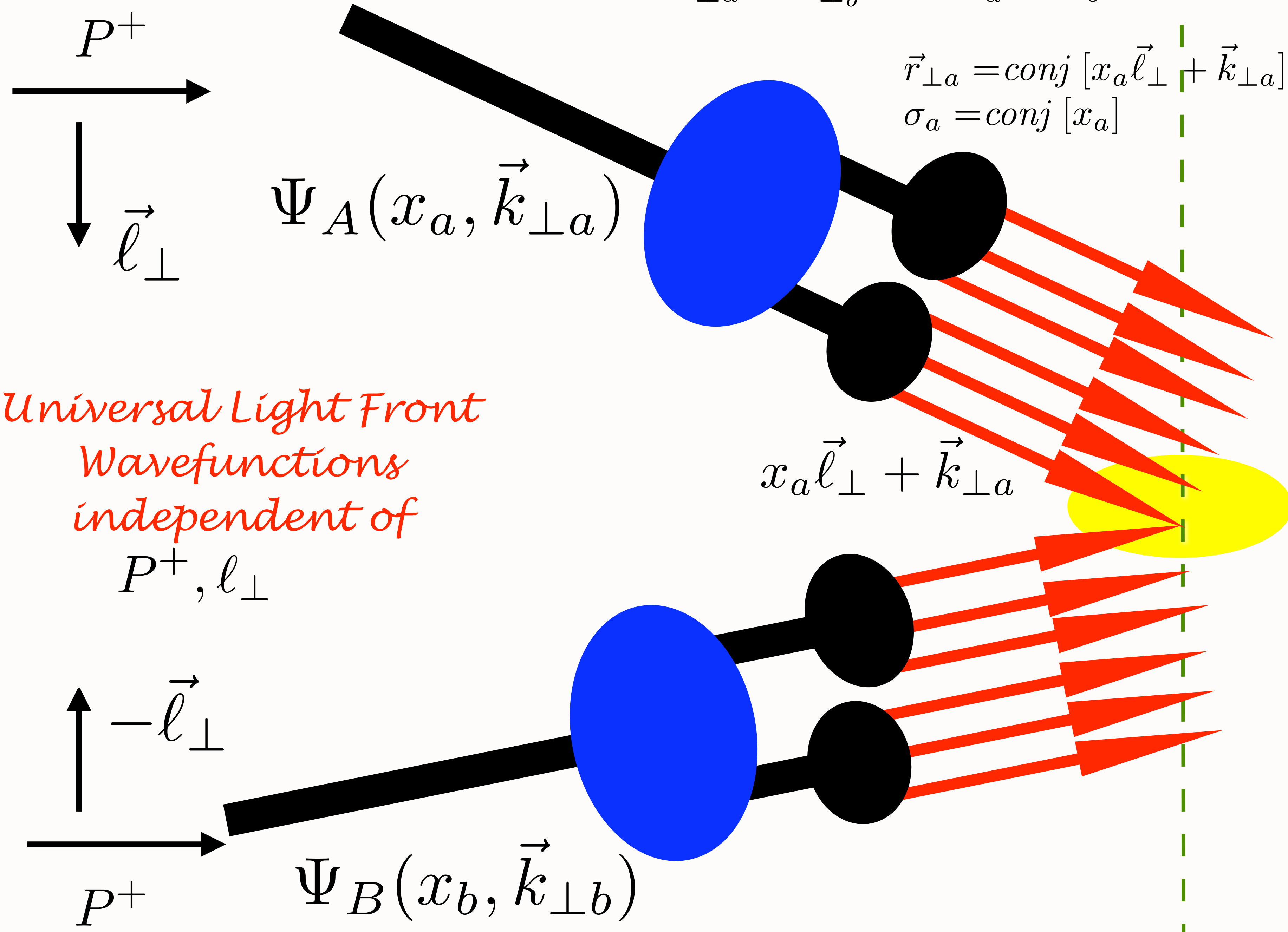
$$R(e^+e^- \to H\bar{H}) \propto |F(s)|^2$$

$$\boxed{|F(s)| \propto [\tfrac{1}{s}]^{n_q - 1}}$$

$$e^+e^- \to p\bar{p}$$

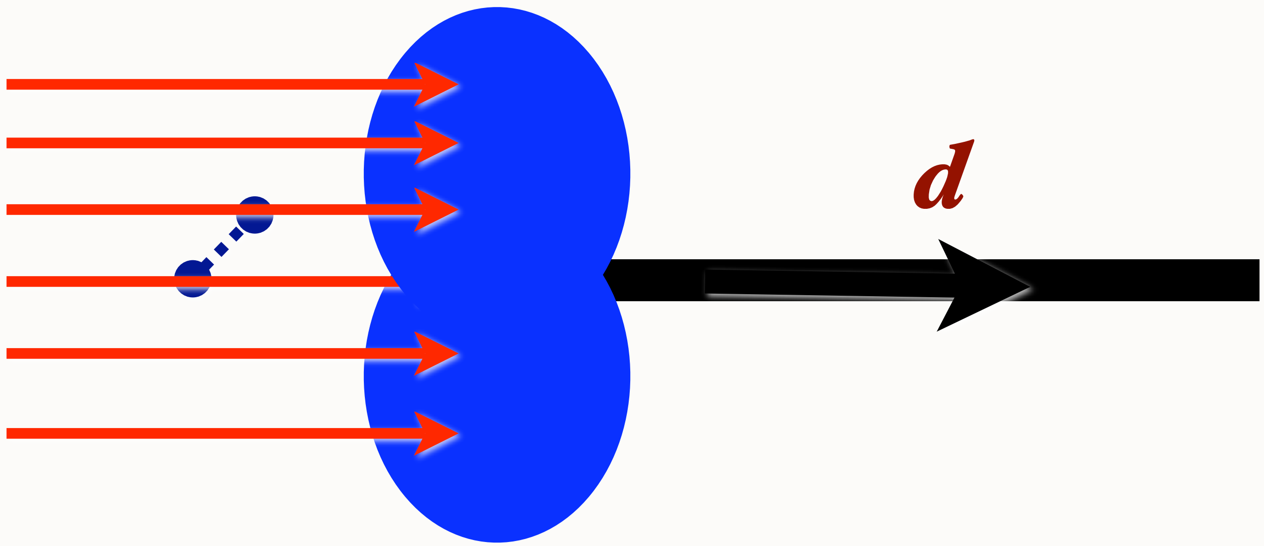
Probability decreases with number of constituents!

Interaction of a and b when $\vec{r}_{\perp a} \simeq \vec{r}_{\perp b}$ and $\sigma_a \simeq \sigma_b$



Features of LF T-Matrix Formalism
"Event Amplitude Generator"

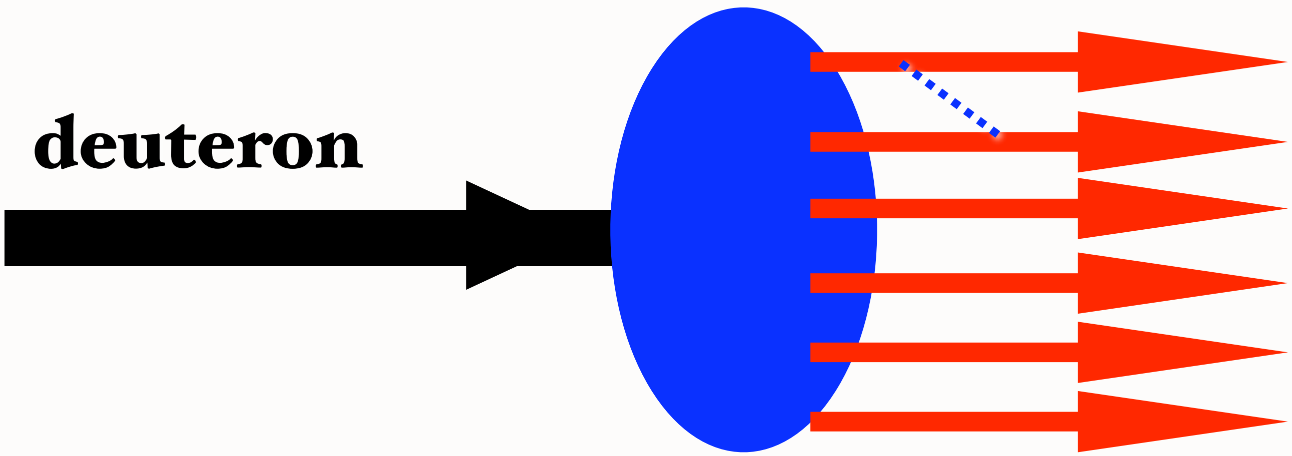
- Hidden Color: Six-quark color-singlet Fock states of deuteron from hard gluon exchange:
- Deuteron LFWF not always product of nucleon clusters



Hidden Color of Deuteron

Evolution of 5 color-singlet Fock states

$$\Psi^{\mathbf{d}}_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

$\sum_i^n x_i = 1$

$$\Phi_n(x_i, Q) = \int^{k^2_{\perp i} < Q^2} \Pi' d^2 k_{\perp j} \psi_n(x_i, \vec{k}_{\perp j})$$

Ji, Lepage, sjb

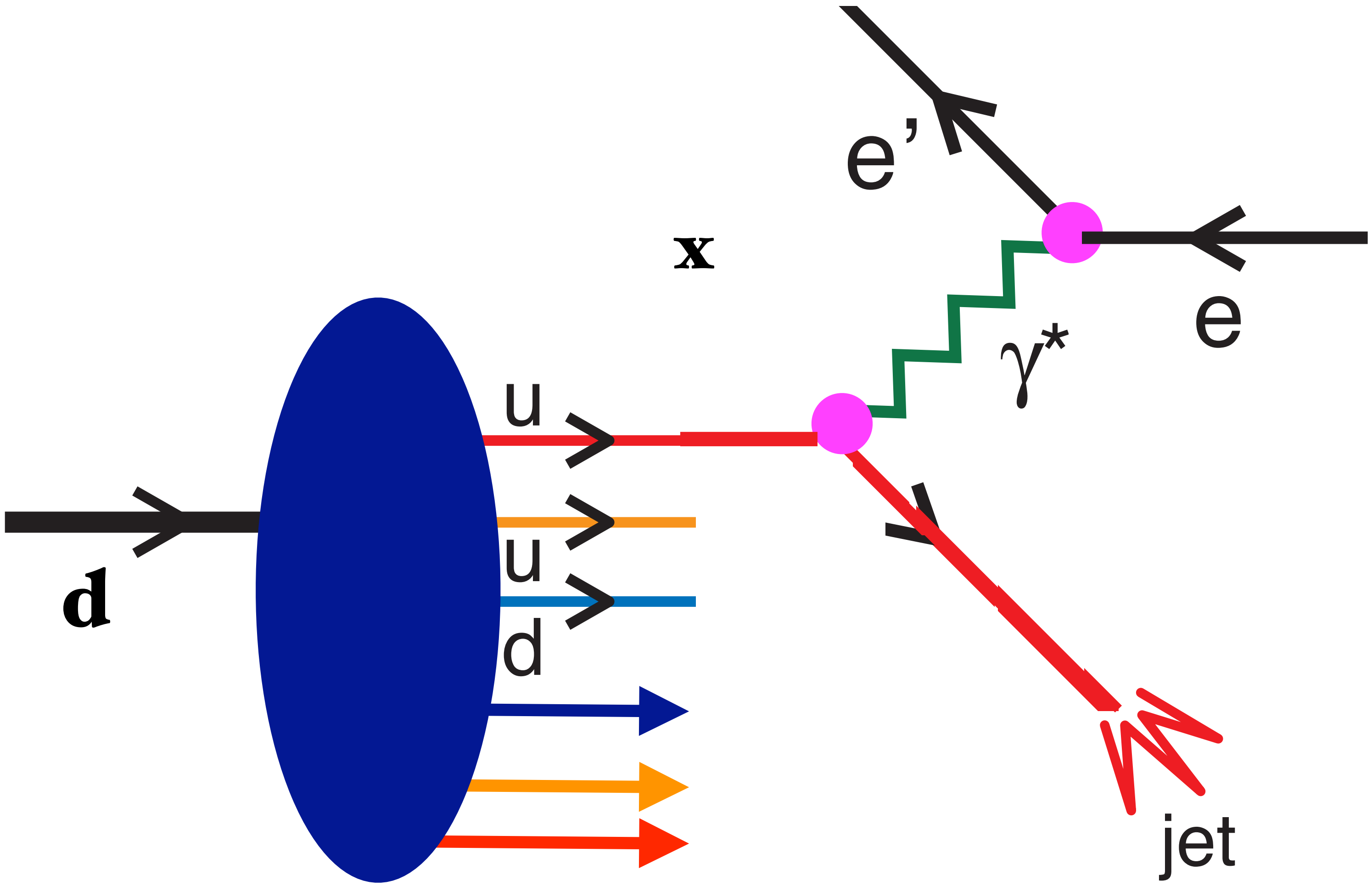
5 X 5 Matrix Evolution Equation for deuteron distribution amplitude

Hidden Color in QCD

Lepage, Ji, sjb

- Deuteron six quark wavefunction:
- 5 color-singlet combinations of 6 color-triplets -- one state is |n p>
- Components evolve towards equality at short distances
- Hidden color states dominate deuteron form factor and photodisintegration at high momentum transfer
- Predict $\frac{d\sigma}{dt}(\gamma d \to \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \to pn)$ at high Q^2

Deep Inelastic Electron-Deuteron Scattering

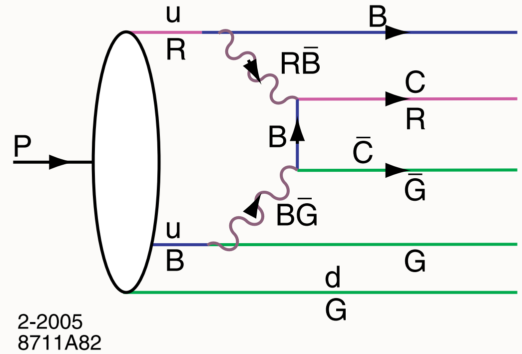


Hidden color: excited target spectator system

No nucleon spectator

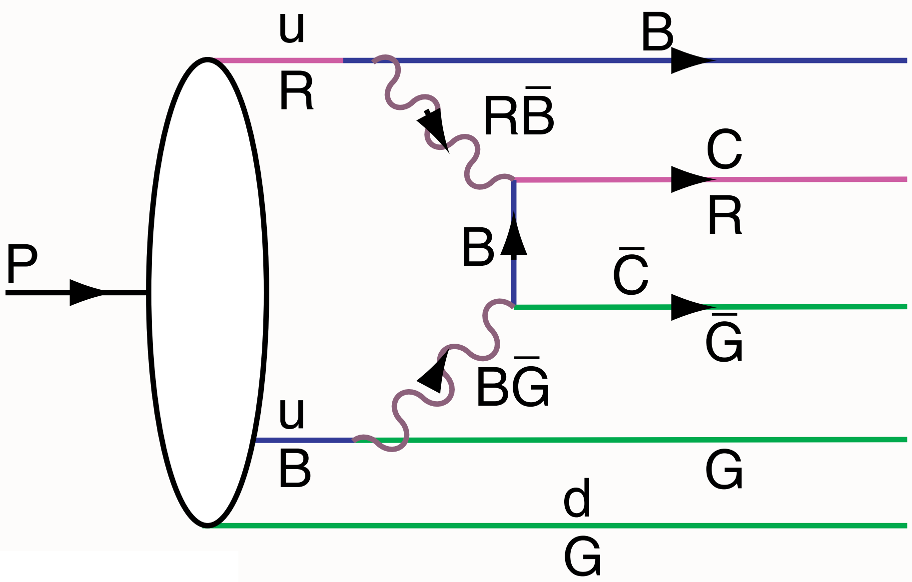
Hoyer, Peterson, Sakai, sjb

Intrinsic Heavy-Quark Fock States



- Rigorous prediction of QCD, OPE
- Color-Octet Color-Octet Fock State!
- Probability $P_{Q\bar{Q}} \propto \frac{1}{M_Q^2}$ $P_{Q\bar{Q}Q\bar{Q}} \sim \alpha_s^2 P_{Q\bar{Q}}$ $P_{c\bar{c}/p} \simeq 1\%$
- Large Effect at high x
- Greatly increases kinematics of colliders such as Higgs production (Kopeliovich, Schmidt, Soffer, sjb)
- Severely underestimated in conventional parameterizations of heavy quark distributions (Pumplin, Tung)
- Many empirical tests

Hoyer, Peterson, Sakai, sjb



$|uudc\bar{c}>$ Fluctuation in Proton

QCD: Probability $\frac{\sim\Lambda^2_{QCD}}{M^2_Q}$

$|e^+e^-\ell^+\ell^->$ Fluctuation in Positronium

QED: Probability $\frac{\sim(m_e\alpha)^4}{M^4_\ell}$

OPE derivation - M.Polyakov et al.

$$< p|\frac{G^3_{\mu\nu}}{m^2_Q}|p > \text{ vs. } < p|\frac{F^4_{\mu\nu}}{m^4_\ell}|p >$$ $c\bar{c}$ in Color Octet

Distribution peaks at equal rapidity (velocity) Therefore heavy particles carry the largest momentum fractions

$$\hat{x}_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

High x charm! *Charm at Threshold*

Electron-Electron Scattering in QED

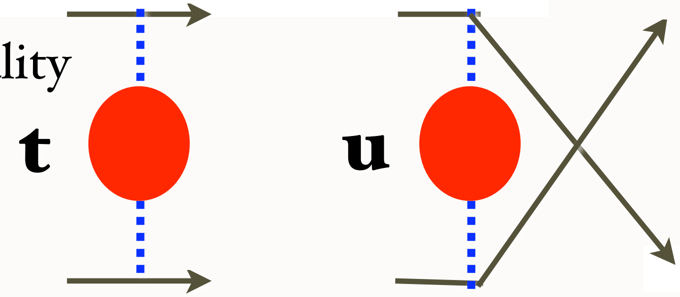
$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\ \alpha(t) + \frac{8\pi s}{u}\ \alpha(u)$$

$$\alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

Gell Mann-Low Effective Charge

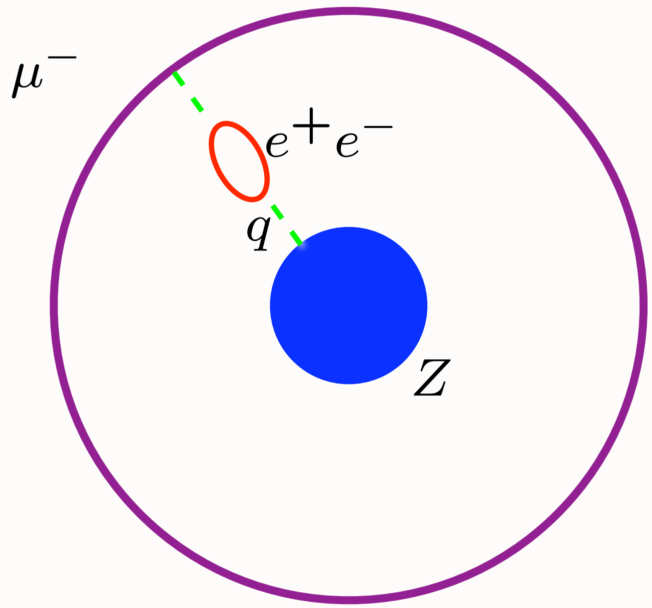
Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\ \alpha(t) + \frac{8\pi s}{u}\ \alpha(u)$$



- Two separate physical scales: t, u = photon virtuality
- Gauge Invariant. Dressed photon propagator
- Sums all vacuum polarization, non-zero beta terms into running coupling.
- If one chooses a different scale, one can sum an infinite number of graphs -- but always recover same result!
- Number of active leptons correctly set
- Analytic: reproduces correct behavior at lepton mass thresholds
- $\boxed{\text{No renormalization scale ambiguity!}}$

Another Example in QED: Muonic Atoms



$$V(q^2) = -\frac{Z\alpha_{QED}(q^2)}{q^2}$$

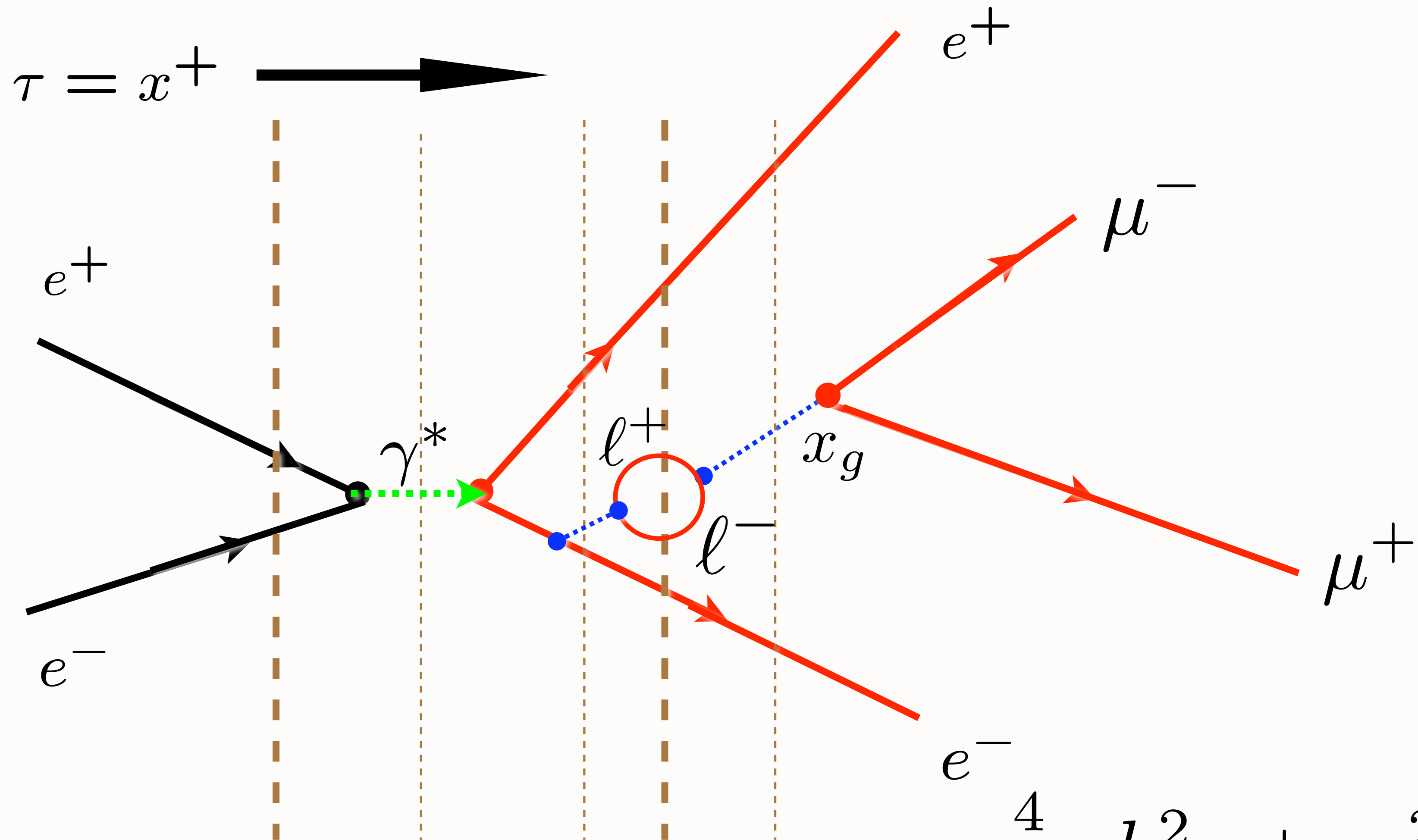
$$\mu_R^2 \equiv q^2$$

$$\alpha_{QED}(q^2) = \frac{\alpha_{QED}(0)}{1-\Pi(q^2)}$$

Scale is unique: Tested to ppm

Gyulassy: Higher Order VP verified to 0.1% precision in μ Pb

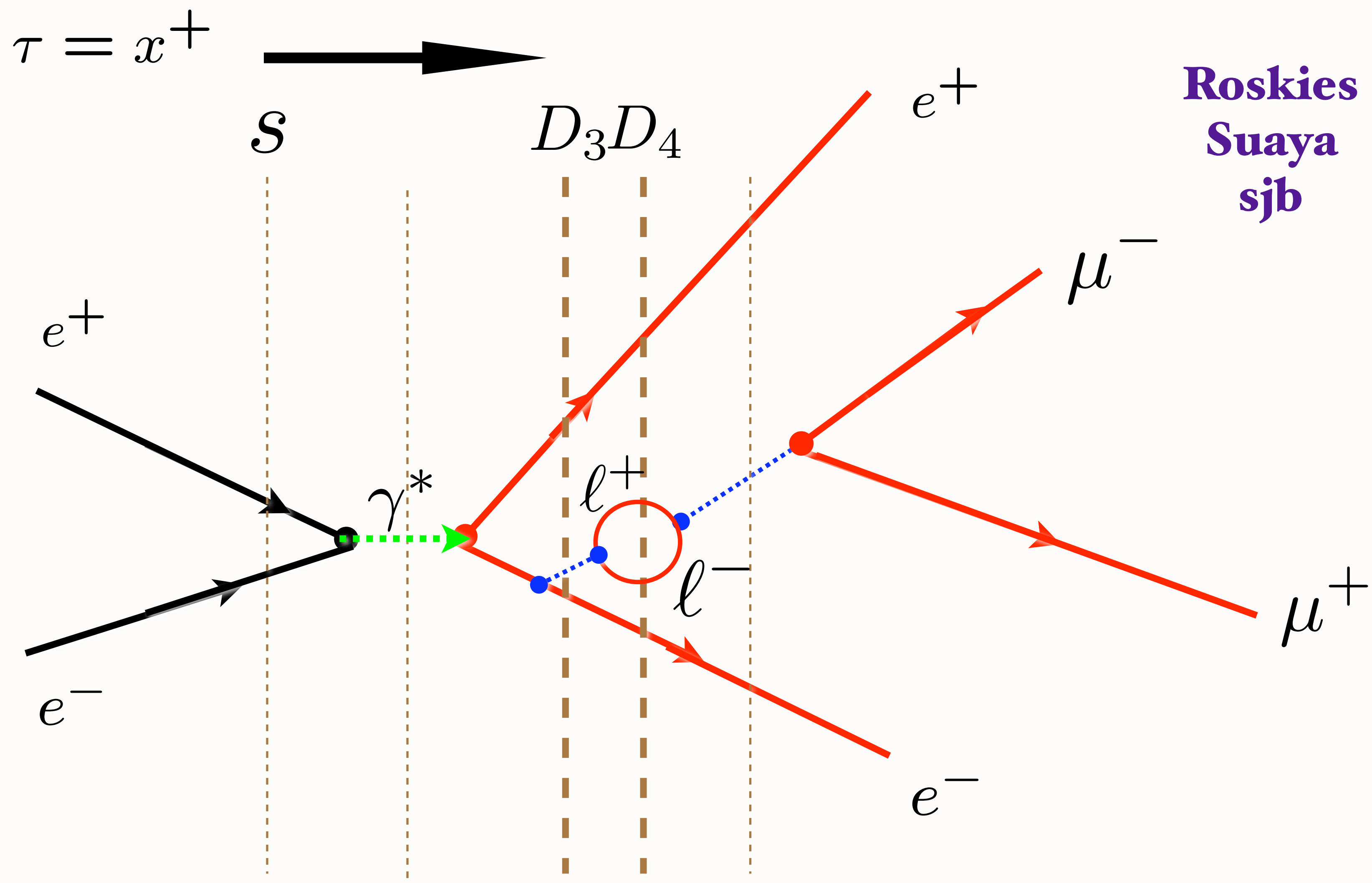
QED Renormalization Scale Setting in LFPth



$$\alpha(\mu^2 = x_g D_4) \qquad D_4 = s - \sum_{i=1}^{4} \frac{k_{\perp i}^2 + m_i^2}{x_i}$$

Scale sets effective # leptons

Alternate Denominator: UV Subtraction Method



Roskies
Suaya
sjb

$$T_{ren} : \frac{1}{s - D_4} \to \frac{1}{s - D_4} - \frac{1}{D_3 - D_4}$$

$$D_4 = s - \sum_{i=1}^{4} \frac{k_{\perp i}^2 + m_i^2}{x_i}$$

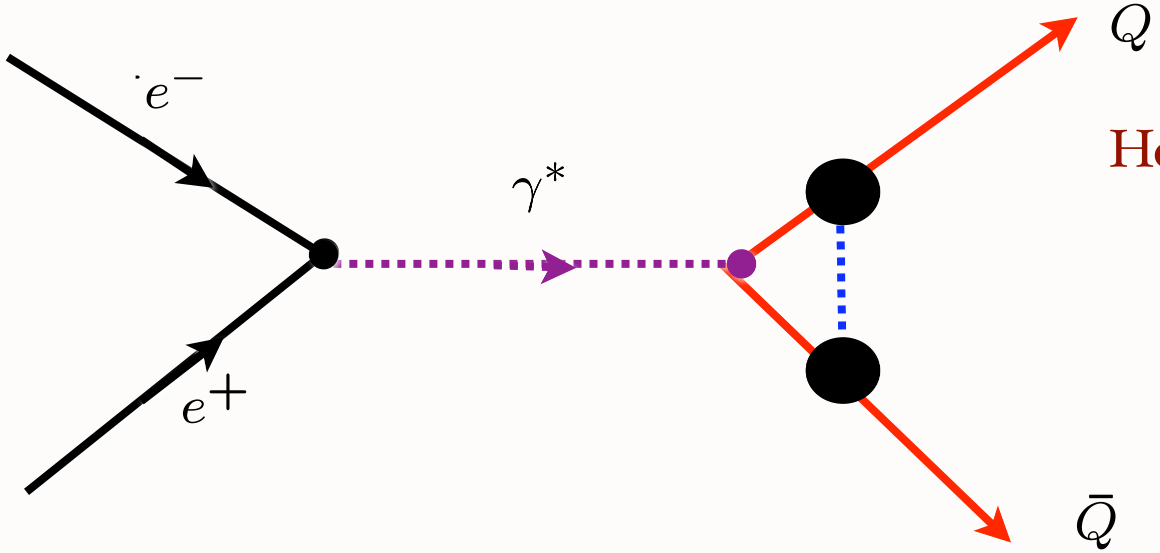
Huet, sjb

$$\lim N_C \to 0 \text{ at fixed } \alpha = C_F \alpha_s, n_\ell = n_F / C_F$$

$$\text{QCD} \to \text{Abelian Gauge Theory}$$

Analytic Feature of SU(Nc) Gauge Theory

Scale-Setting procedure for QCD must be applicable to QED



Hoang, Kuhn, Teubner, sjb

$$F_1 + F_2 = 1 + \frac{\alpha(s\,\beta^2)\,\pi}{4\,\beta} - 2\,\frac{\alpha(s\,e^{3/4}/4)}{\pi}$$

$$\cong \left(1 - 2\,\frac{\alpha(s\,e^{3/4}/4)}{\pi}\right)\left(1 + \frac{\alpha(s\,\beta^2)\,\pi}{4\,\beta}\right)$$

Example of Multiple BLM Scales

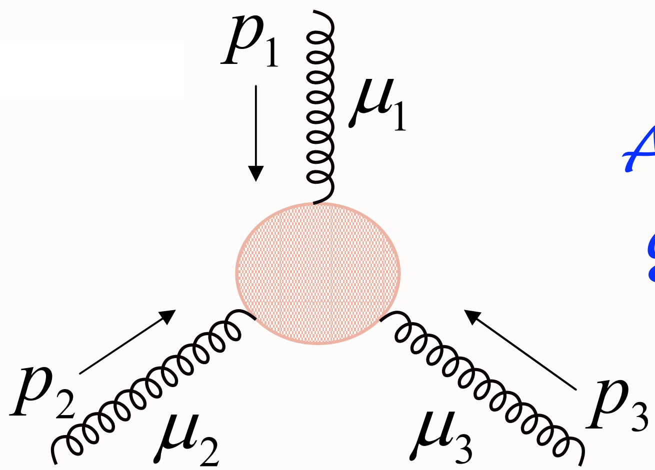
Angular distributions of massive quarks and leptons close to threshold.

General Structure of the Three-Gluon Vertex

"THE FORM-FACTORS OF THE GAUGE-INVARIANT THREE-GLUON VERTEX"

M. Binger, sjb

$$\hat{\Gamma}_{\mu_1\mu_2\mu_3} =$$

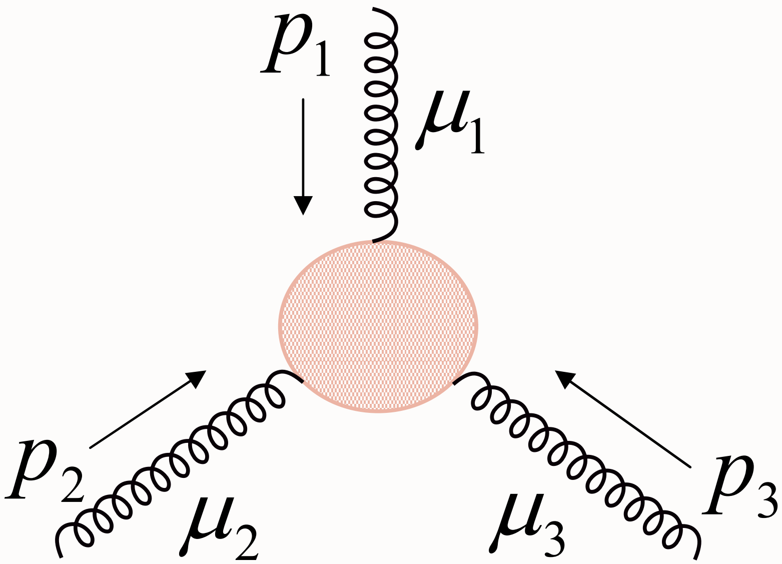


Analytic calculation: general masses, spin

3 index tensor $\hat{\Gamma}_{\mu_1\mu_2\mu_3}$ built out of $g_{\mu\nu}$ and p_1, p_2, p_3
with $p_1 + p_2 + p_3 = 0$

⇨ 14 basis tensors and form factors

$$\hat{\Gamma}_{\mu_1\mu_2\mu_3} =$$



H. J. Lu

$$\mu_R^2 \simeq \frac{p_{min}^2 p_{med}^2}{p_{max}^2}$$

Binger, sjb

Properties of the Effective Scale

$$Q^2_{eff}(a,b,c) = Q^2_{eff}(-a,-b,-c)$$

$$Q^2_{eff}(\lambda a, \lambda b, \lambda c) = |\lambda| Q^2_{eff}(a,b,c)$$

$$Q^2_{eff}(a,a,a) = |a|$$

$$Q^2_{eff}(a,-a,-a) \approx 5.54\,|a|$$

$$Q^2_{eff}(a,a,c) \approx 3.08\,|c| \quad \text{for} \quad |a| >> |c|$$

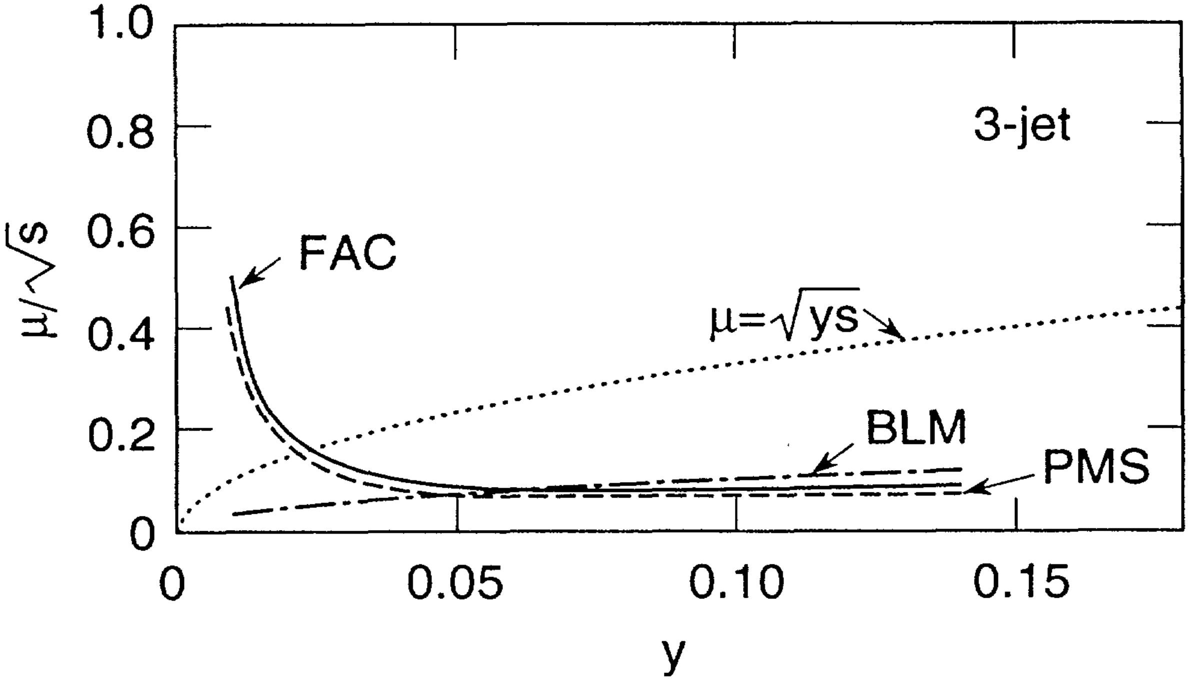
$$Q^2_{eff}(a,-a,c) \approx 22.8\,|c| \quad \text{for} \quad |a| >> |c|$$

$$Q^2_{eff}(a,b,c) \approx 22.8\frac{|bc|}{|a|} \quad \text{for} \quad |a| >> |b|, |c|$$

Surprising dependence on Invariants

BLM Method

- Satisfies Transitivity, all aspects of Renormalization Group; scheme independent
- Analytic at Flavor Thresholds
- Preserves Underlying Conformal Template
- Physical Interpretation of Scales; Multiple Scales
- Correct Abelian Limit ($N_C = 0$)
- Eliminates unnecessary source of imprecision of PQCD predictions
- Commensurate Scale Relations: Fundamental Tests of QCD free of renormalization scale and scheme ambiguities
- BLM used in many applications, QED, LGTH, BFKL, ...



Kramer & Lampe

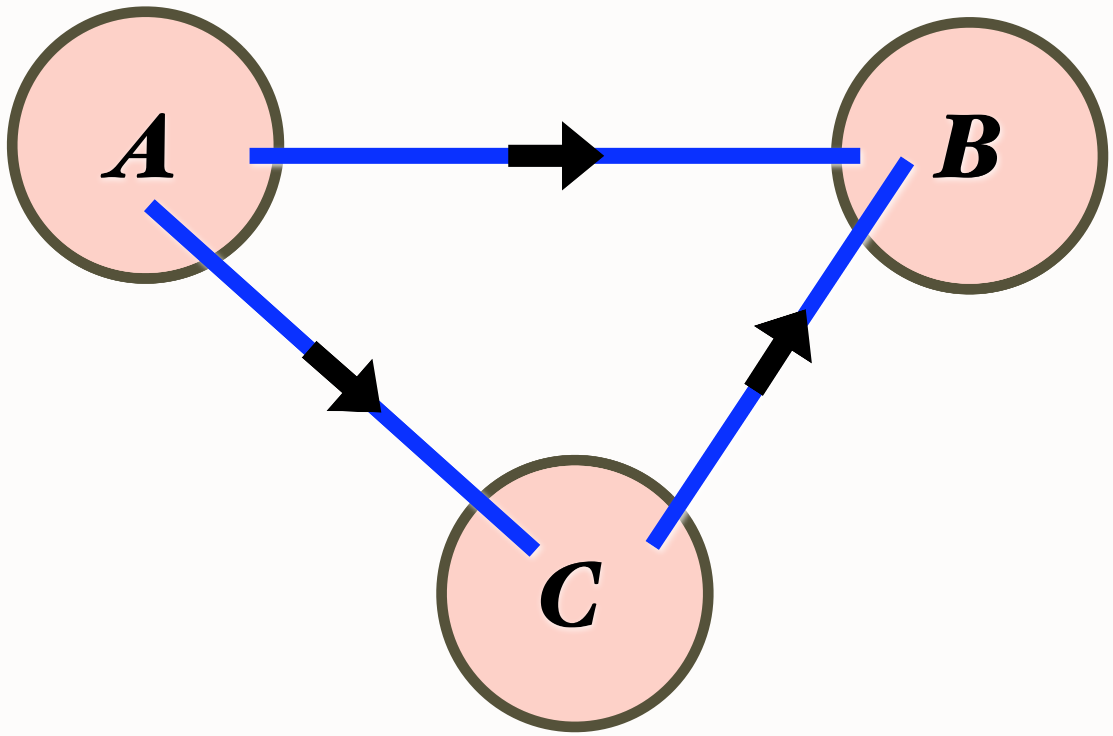
Three-Jet Rate

The scale $\mu/\sqrt{s}$ according to the BLM (dashed-dotted), PMS (dashed), FAC (full), and $\sqrt{y}$ (dotted) procedures for the three-jet rate in e^+e^- annihilation, as computed by Kramer and Lampe [10]. Notice the strikingly different behavior of the BLM scale from the PMS and FAC scales at low y. In particular, the latter two methods predict increasing values of μ as the jet invariant mass $\mathcal{M} < \sqrt{(ys)}$ decreases.

Rathsman

Other Jet Observables:

Transitivity Property of Renormalization Group



$A \rightarrow C \quad C \rightarrow B$ ***identical to*** $A \rightarrow B$

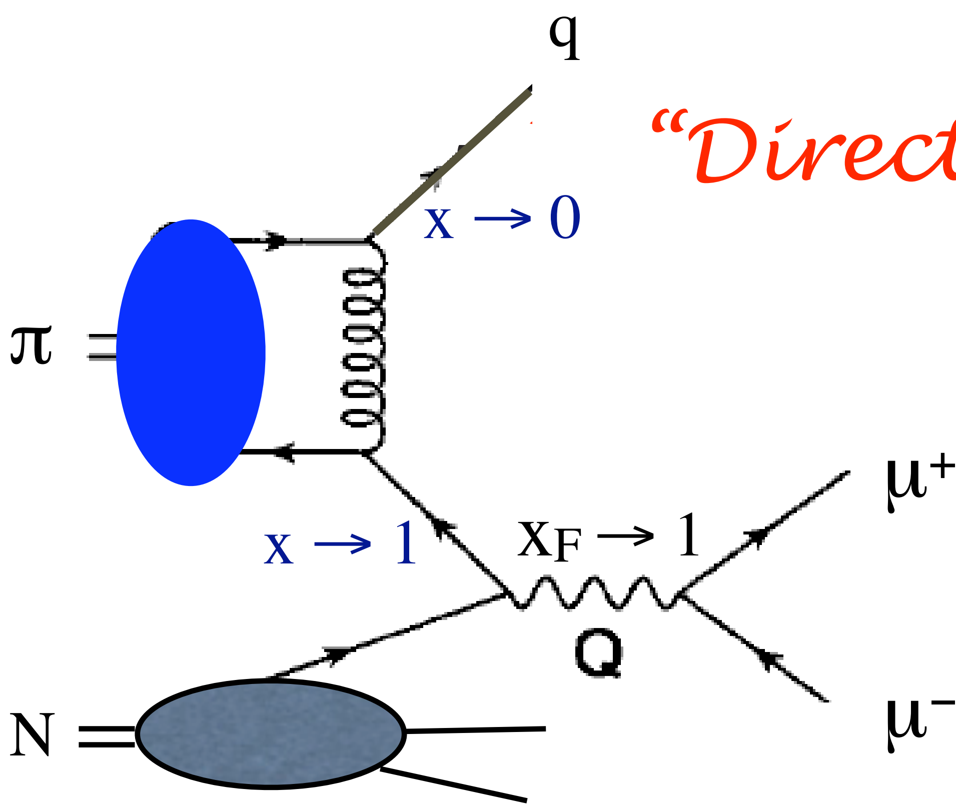
Relation of observables independent of intermediate scheme C

$\pi N \to \mu^+ \mu^- X$ at high x_F

In the limit where $(1-x_F)Q^2$ is fixed as $Q^2 \to \infty$

"Direct" Subprocess

Entire pion wf contributes to hard process



Virtual photon is longitudinally polarized

Berger and Brodsky, PRL 42 (1979) 940

$\pi^- N \to \mu^+ \mu^- X$ at 80 GeV/c

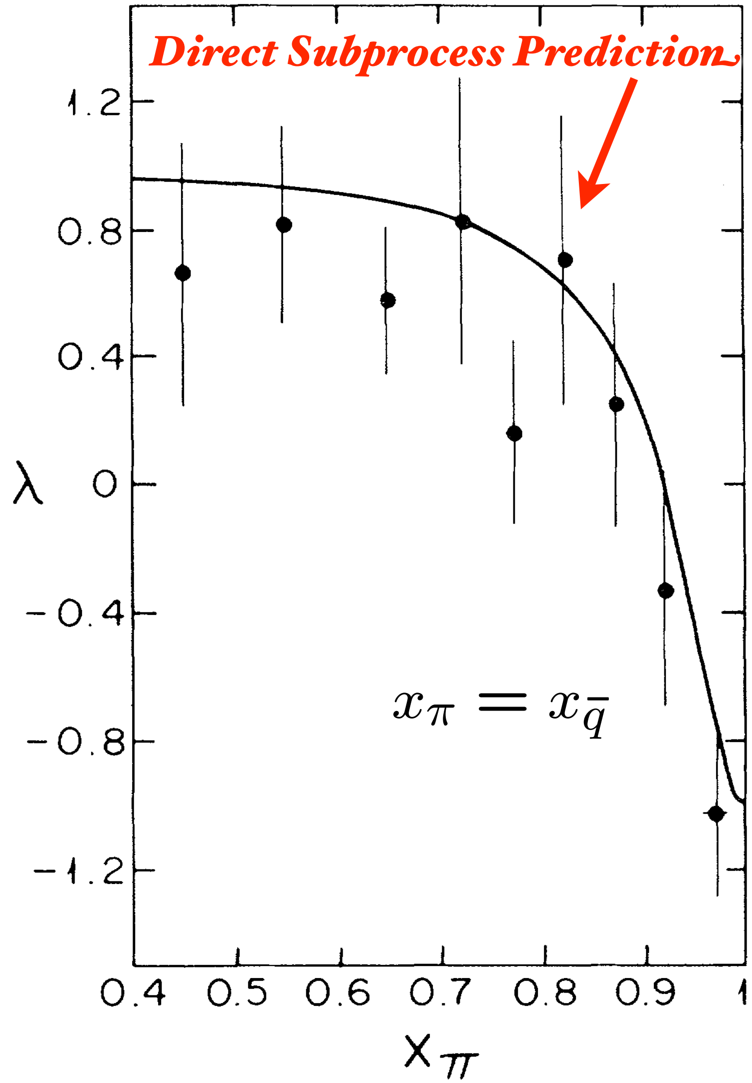
$$\frac{d\sigma}{d\Omega} \propto 1 + \lambda \cos^2\theta + \rho \sin 2\theta \cos\phi + \omega \sin^2\theta \cos 2\phi.$$

$$\frac{d^2\sigma}{dx_\pi d\cos\theta} \propto x_\pi \left[(1 - x_\pi)^2 (1 + \cos^2\theta) + \frac{4}{9} \frac{\langle k_T^2 \rangle}{M^2} \sin^2\theta \right]$$

$$\langle k_T^2 \rangle = 0.62 \pm 0.16 \ \mathrm{GeV}^2/c^2$$

Dramatic change in angular distribution at large x_F

Example of a higher-twist direct subprocess

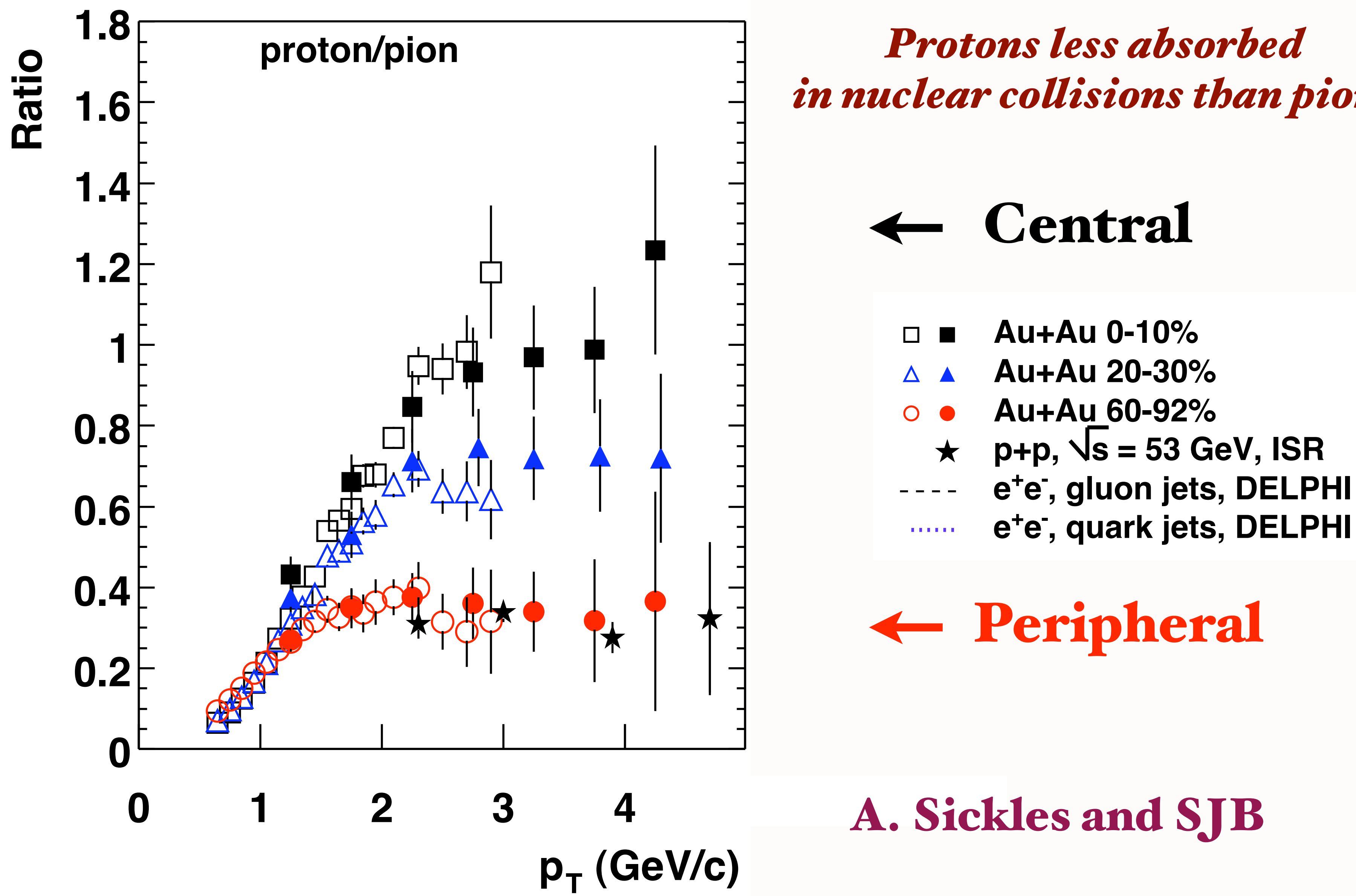


Chicago-Princeton Collaboration

Phys.Rev.Lett.55:2649,1985

S. S. Adler *et al*. PHENIX Collaboration *Phys. Rev. Lett*. **91**, 172301 (2003).

Particle ratio changes with centrality!



Protons less absorbed in nuclear collisions than pions

← **Central**

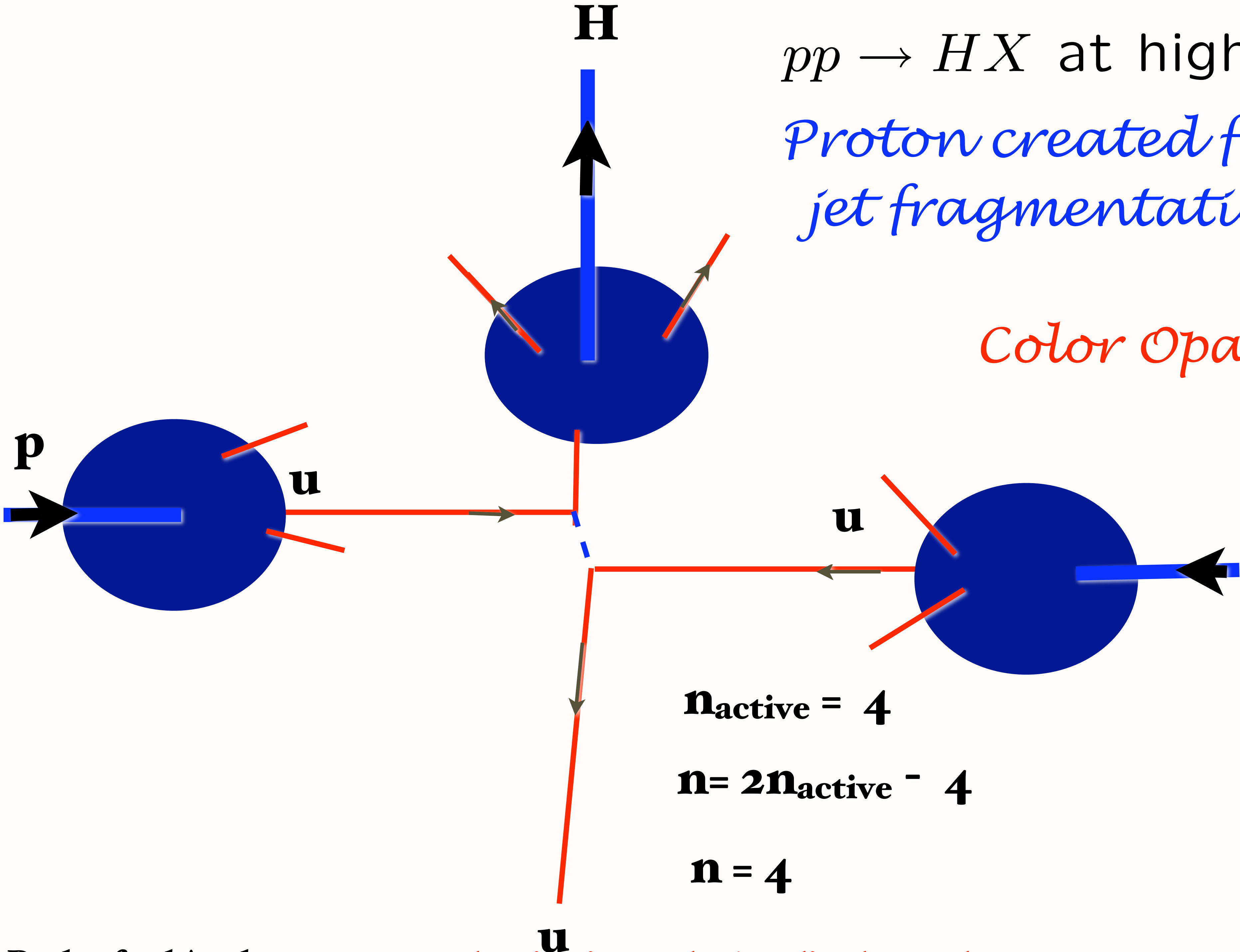
← **Peripheral**

A. Sickles and SJB

$pp \rightarrow HX$ at high p_T

Proton created from jet fragmentation

Color Opaque



$n_{active} = 4$

$n = 2n_{active} - 4$

$n = 4$

Crucial Test of Leading -Twist QCD: Scaling at fixed x_T

$$x_T = \frac{2p_T}{\sqrt{s}}$$

$$E\frac{d\sigma}{d^3p}(pN \to \pi X) = \frac{F(x_T, \theta_{CM})}{p_T^{n_{eff}}}$$

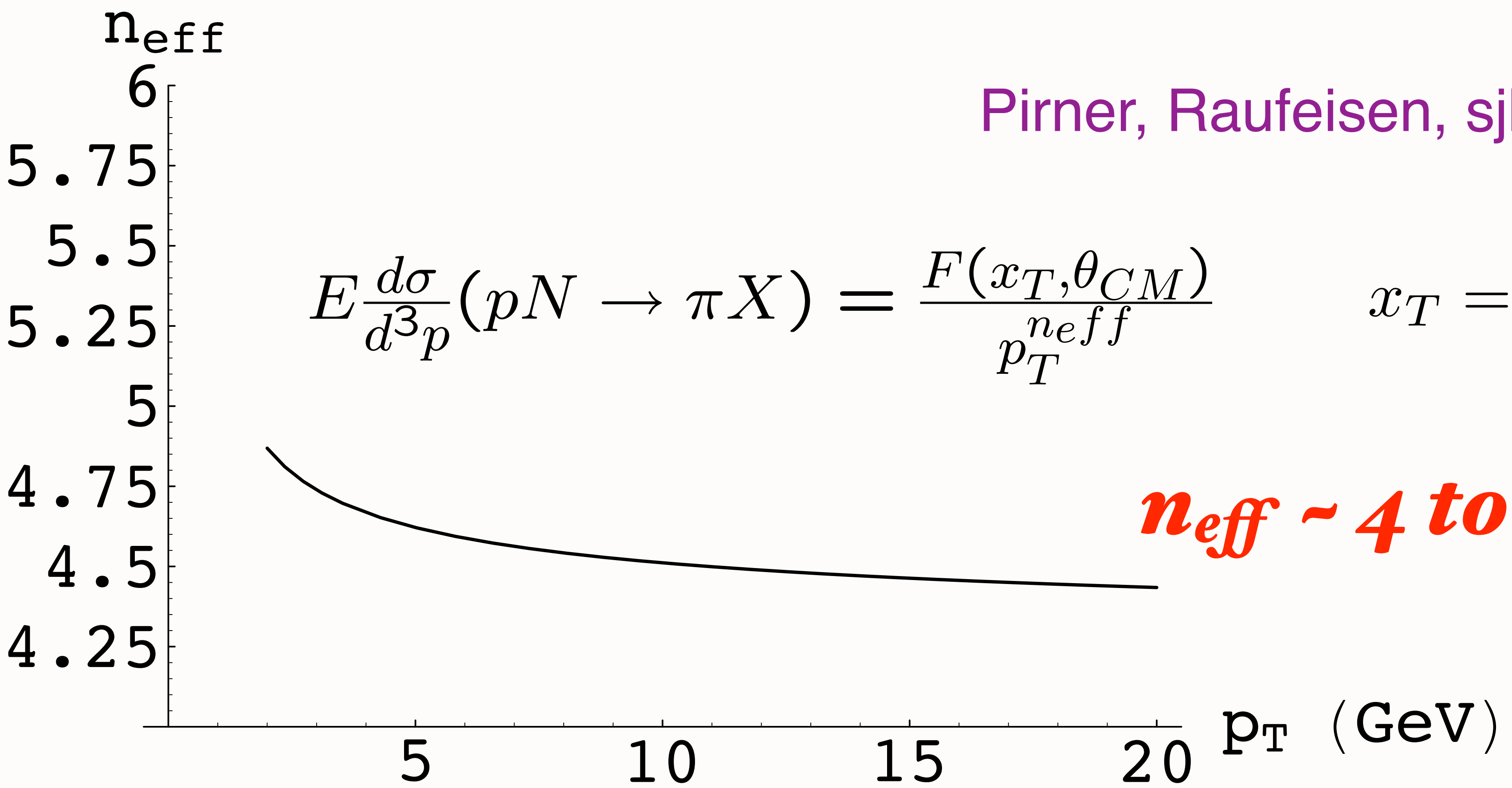
Parton model: $n_{eff} = 4$

As fundamental as Bjorken scaling in DIS

Conformal scaling: $n_{eff} = 2\, n_{active} - 4$

QCD prediction: Modification of power fall-off due to DGLAP evolution and the Running Coupling

Pirner, Raufeisen, sjb



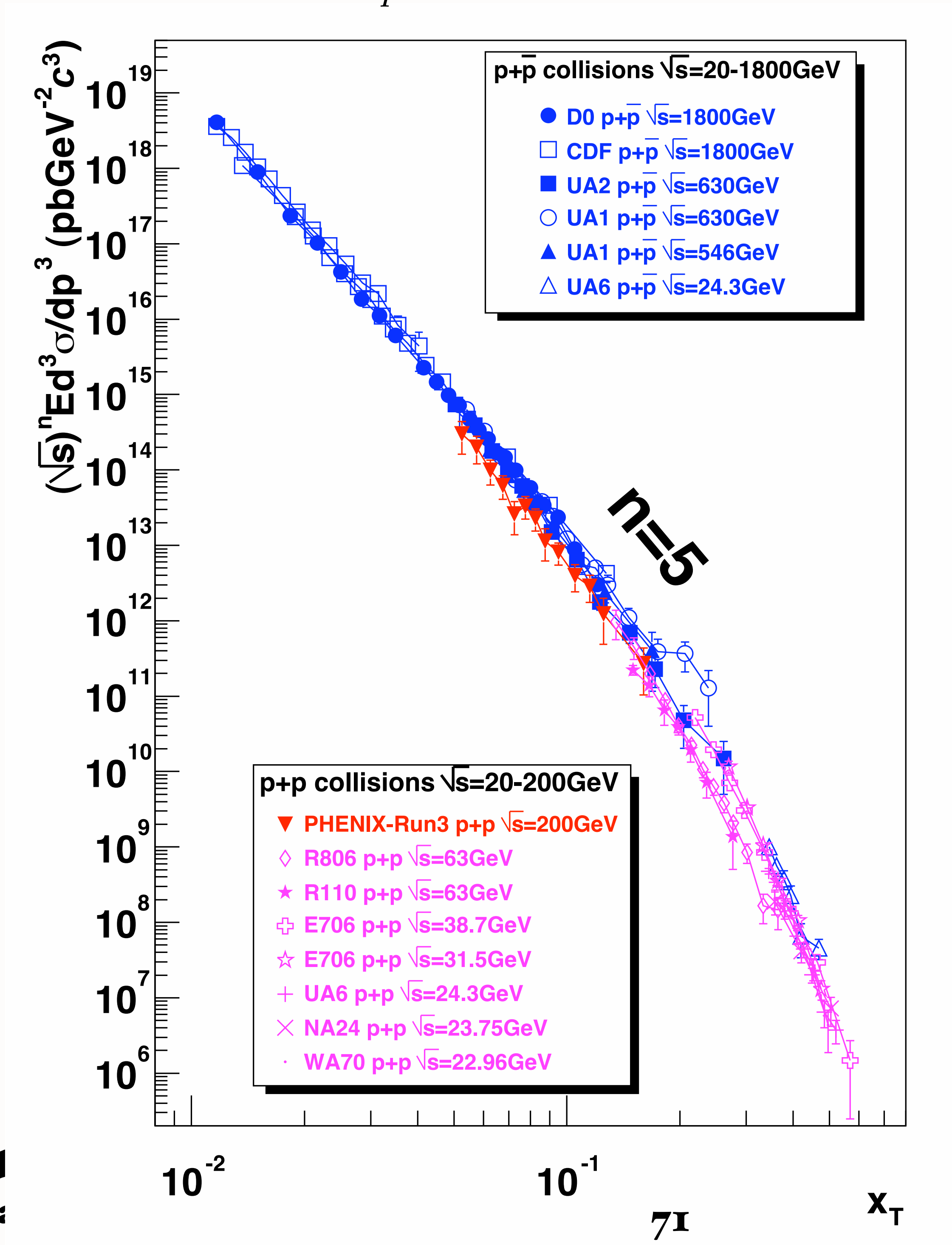
$$E\frac{d\sigma}{d^3p}(pN \to \pi X) = \frac{F(x_T, \theta_{CM})}{p_T^{n_{eff}}} \qquad x_T = \frac{2p_T}{\sqrt{s}}$$

$n_{eff} \sim 4$ to 5

Key test of PQCD: power-law fall-off at fixed x_T

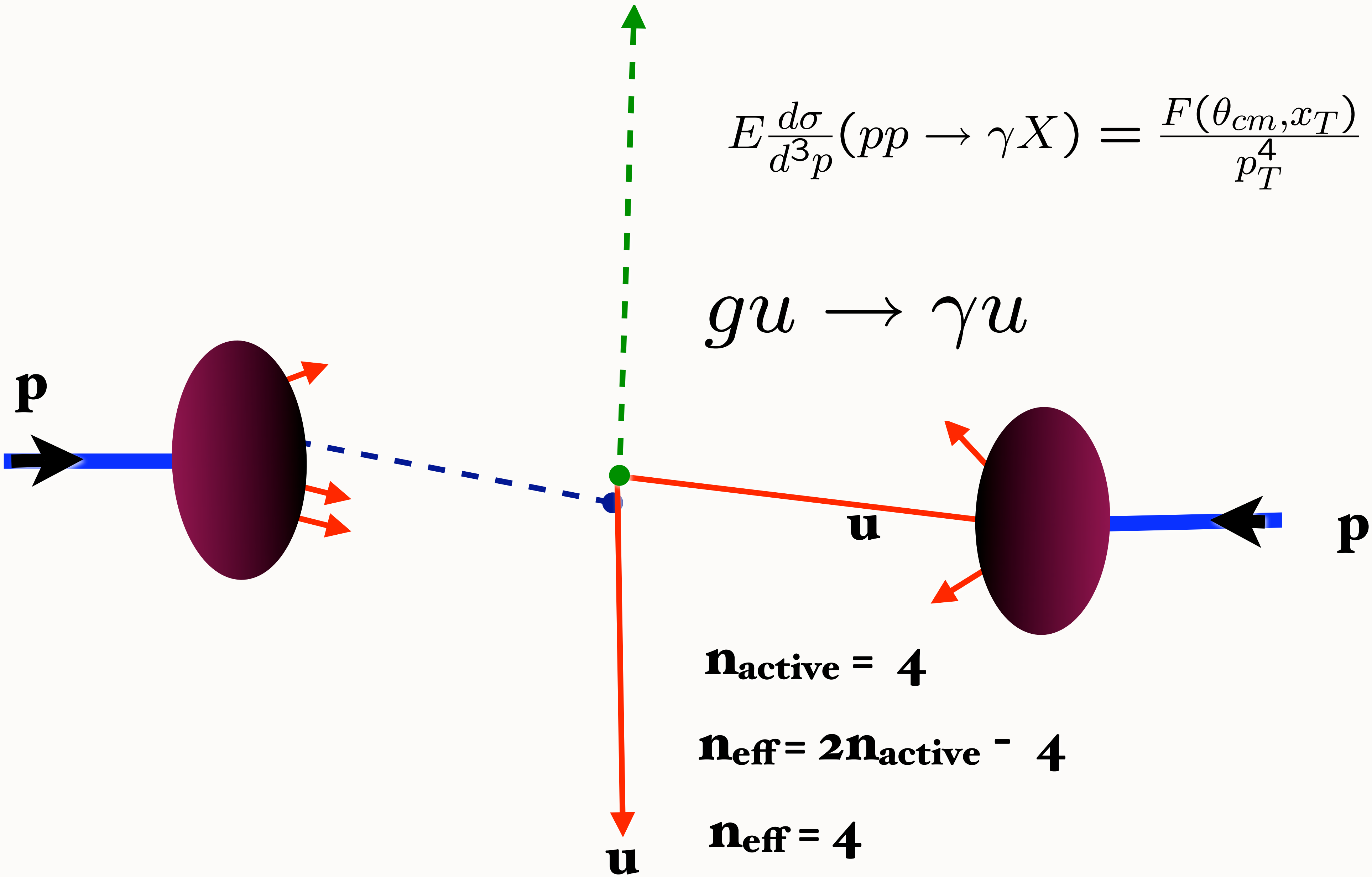
$\sqrt{s}^n E \frac{d\sigma}{d^3p}(pp \to \gamma X)$ at fixed x_T

Tannenbaum



Scaling of direct photon production consistent with PQCD

$$pp \longrightarrow \gamma X$$



$$E\frac{d\sigma}{d^3p}(pp \to \gamma X) = \frac{F(\theta_{cm}, x_T)}{p_T^4}$$

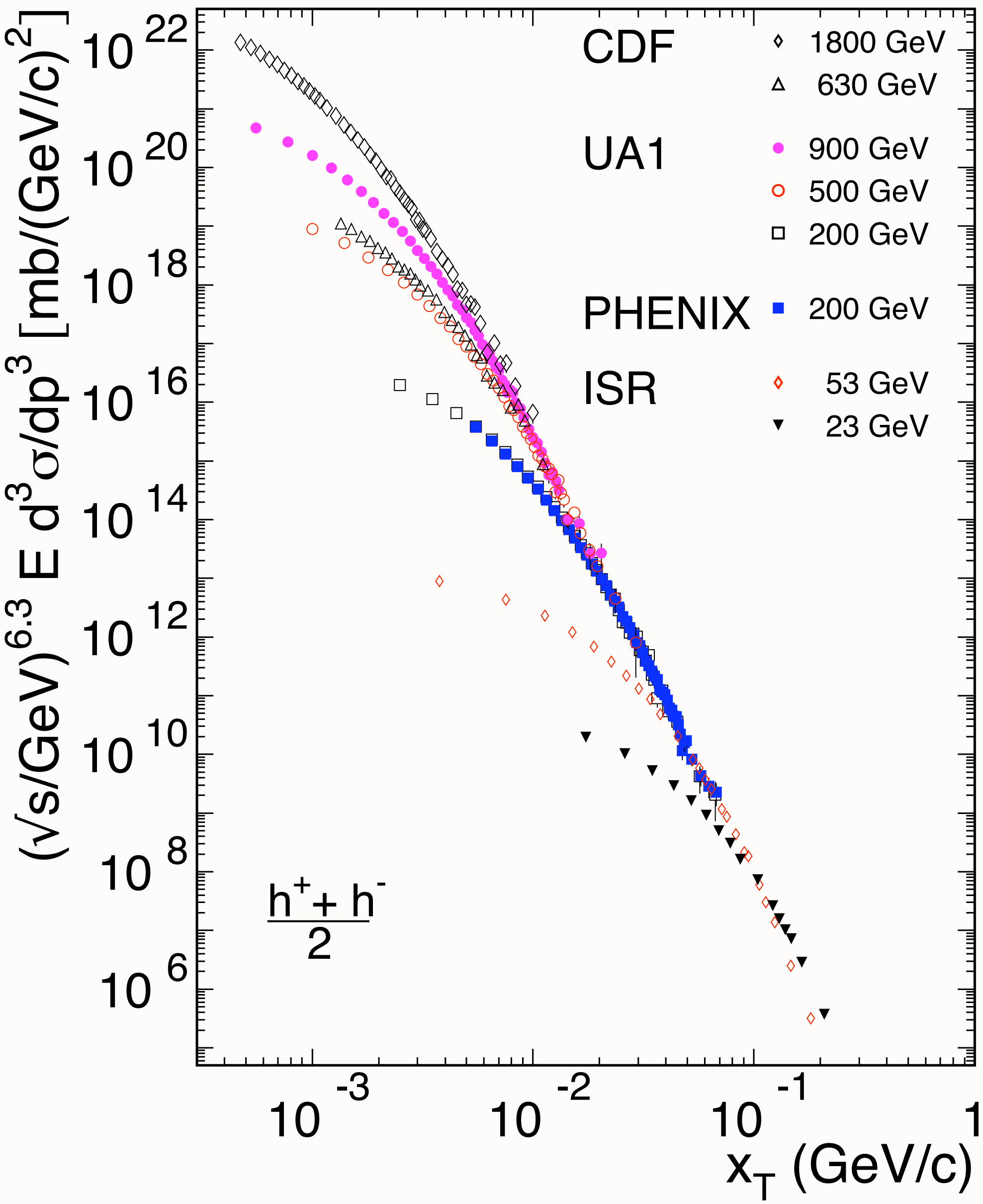
$$gu \longrightarrow \gamma u$$

$n_{active} = 4$

$n_{eff} = 2n_{active} - 4$

$n_{eff} = 4$

$$\sqrt{s}^{6.3} \times E\frac{d\sigma}{d^3p}(pp \to H^{\pm}X) \text{ at fixed } x_T$$

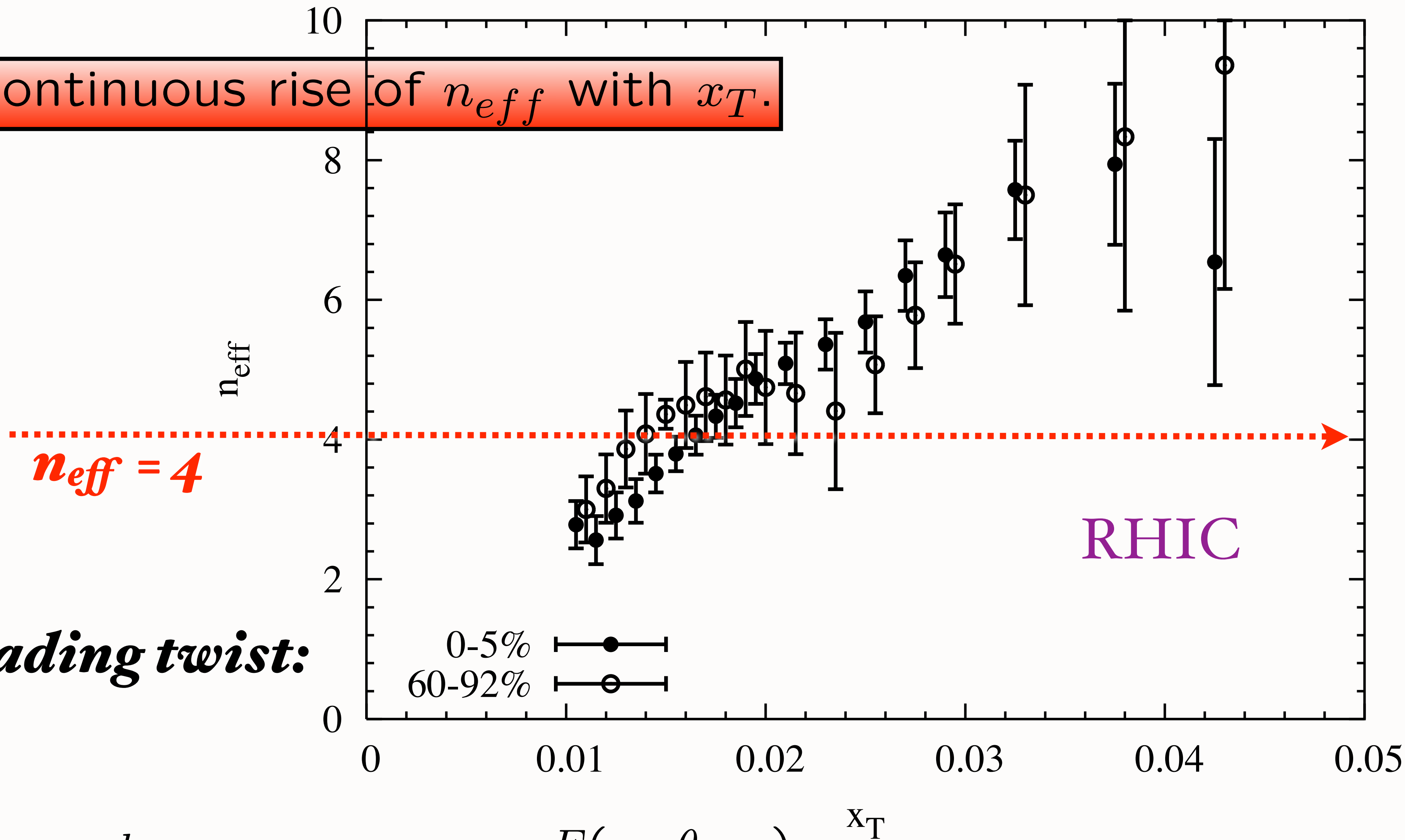


Tannenbaum

Scaling inconsistent with PQCD

Protons produced in AuAu collisions at RHIC do not exhibit clear scaling properties in the available p_T range. Shown are data for central $(0-5\%)$ and for peripheral $(60-90\%)$ collisions.

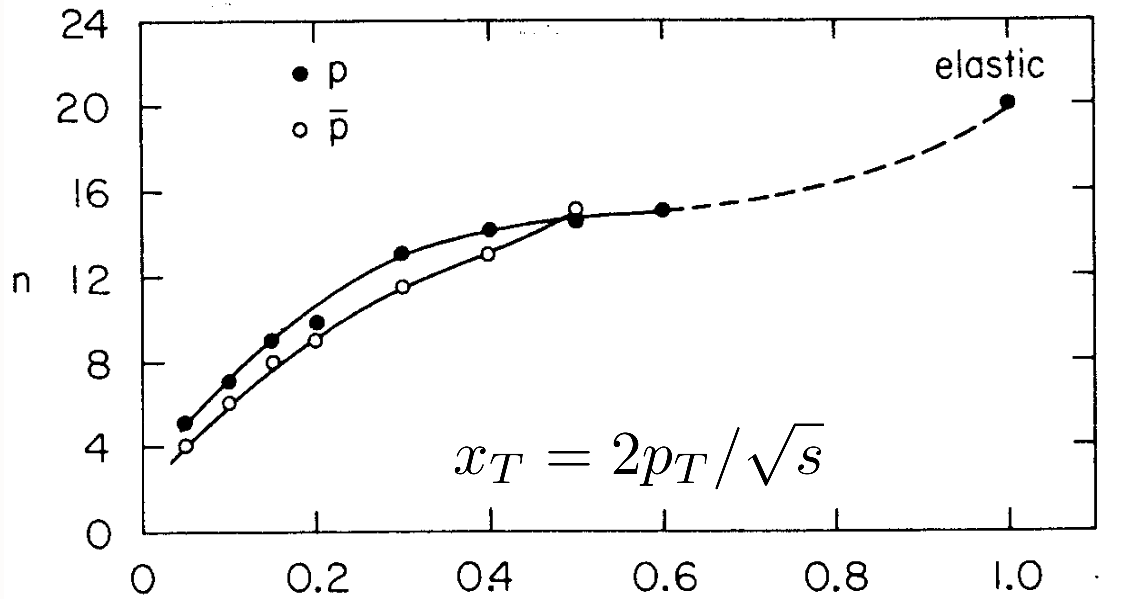
Continuous rise of n_{eff} with x_T.



Leading twist:

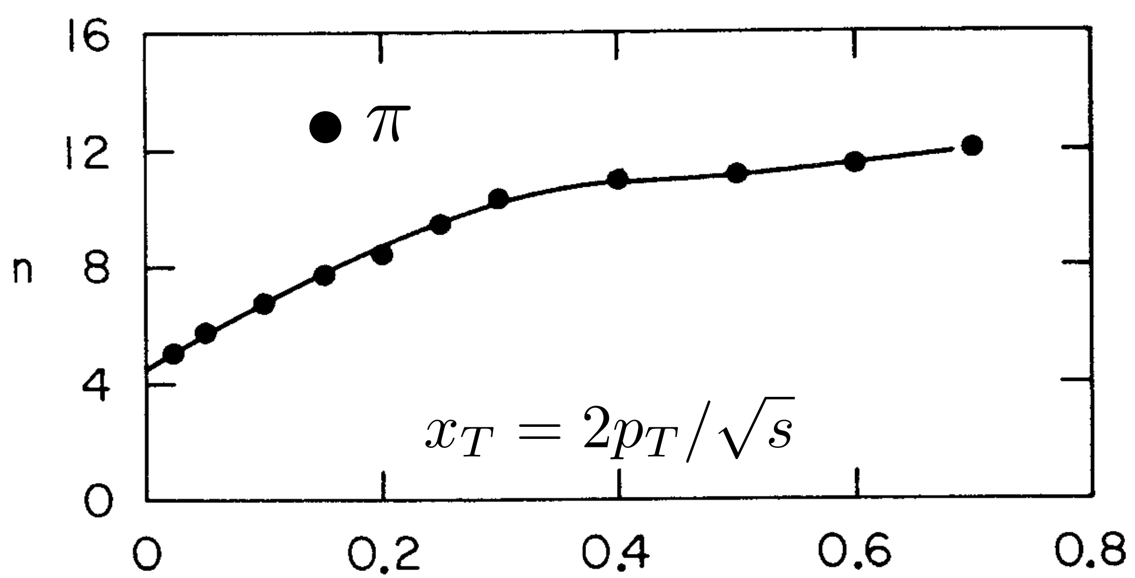
$$E\frac{d\sigma}{d^3p}(pN \to pX) = \frac{F(x_T,\theta_{CM})}{p_T^{n_{eff}}}$$

$$E\frac{d\sigma}{d^3p}(pp \to HX) = \frac{F(x_T, \theta_{cm} = \pi/2)}{p_T^n}$$

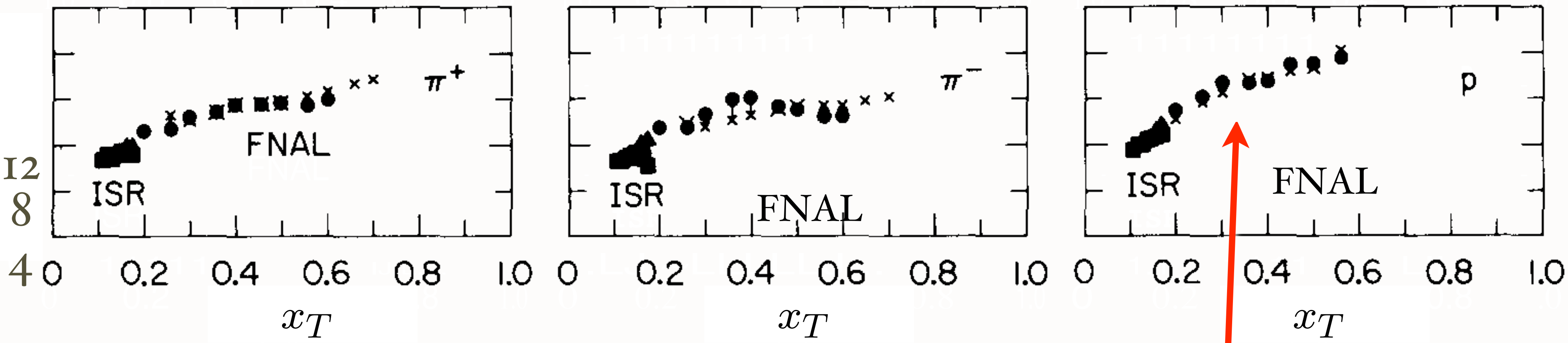


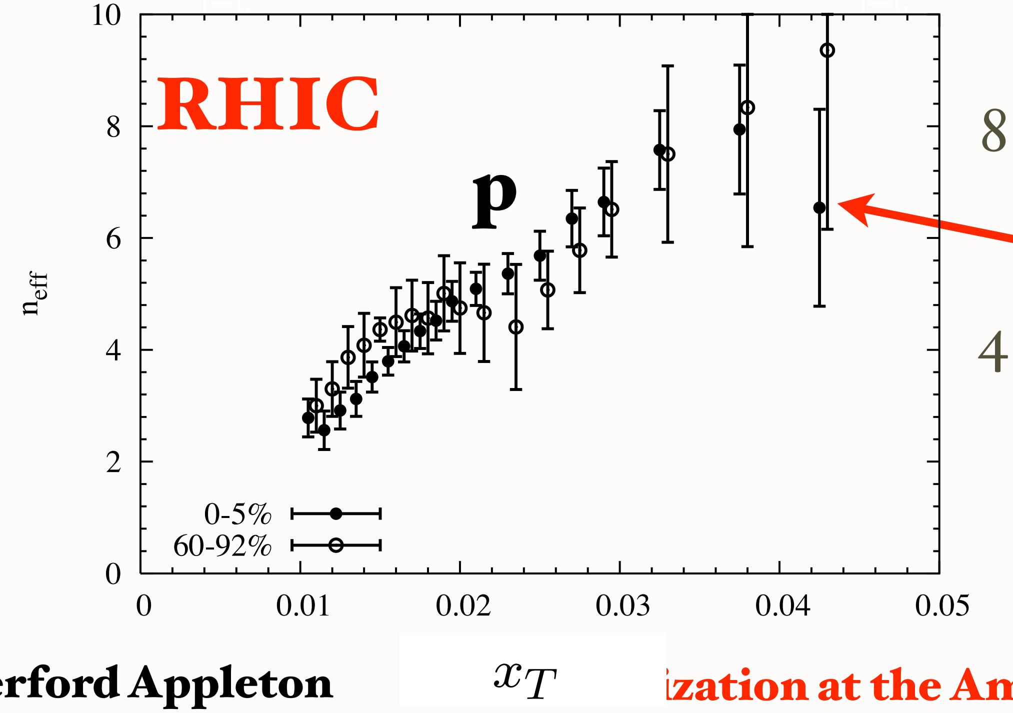
Clear evidence for higher-twist contributions

J. W. Cronin, SSI 1974



$$E\frac{d\sigma}{d^3p}(pp \to HX) = \frac{F(x_T, \theta_{CM})}{p_T^{n_{eff}}}$$



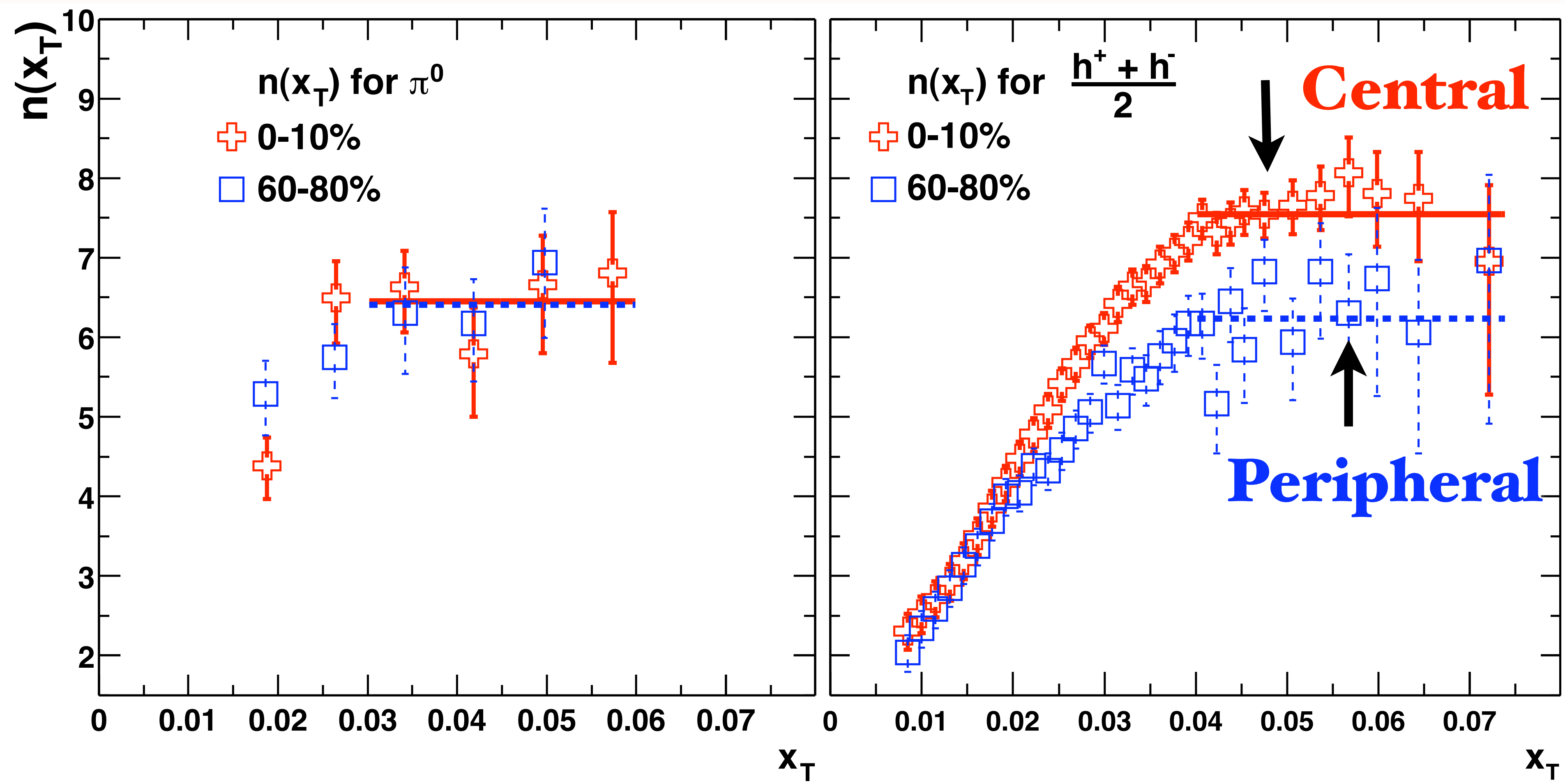


$$E\frac{d\sigma}{d^3p}(pp \to pX) = \frac{F(x_T, \theta_{CM})}{p_T^{12}}$$

$$E\frac{d\sigma}{d^3p}(pp \to pX) = \frac{F(x_T, \theta_{CM})}{p_T^{8}}$$

Trend consistent with RHIC at small x_T

$$\sqrt{s_{NN}} = 130 \text{ and } 200 \text{ GeV}$$



Proton power changes with centrality !

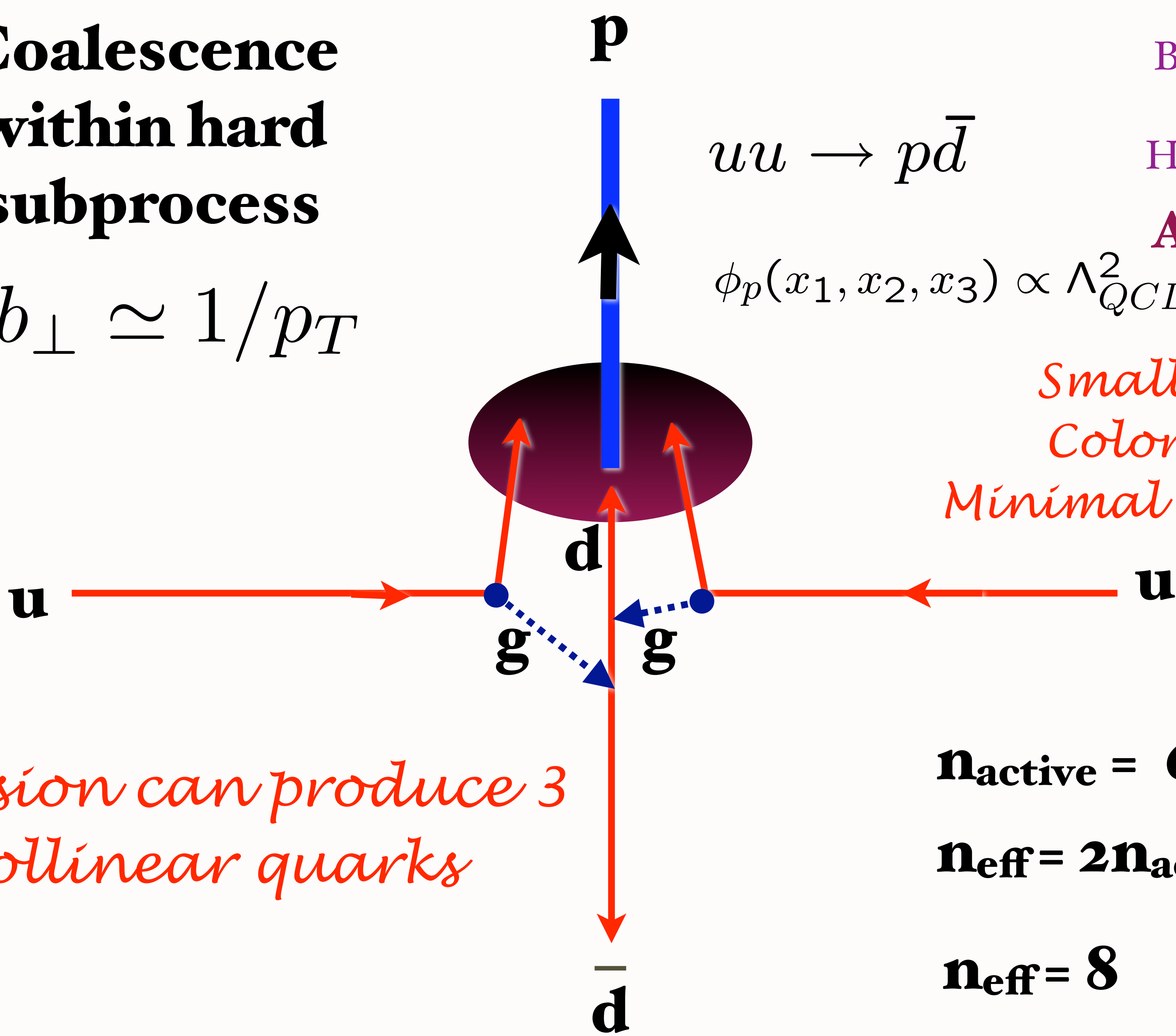
Baryon can be made directly within hard subprocess

Coalescence within hard subprocess

$$b_\perp \simeq 1/p_T$$

$$uu \to p\bar{d}$$

$$\phi_p(x_1, x_2, x_3) \propto \Lambda^2_{QCD}$$



Bjorken
Blankenbecler, Gunion, sjb
Berger, sjb
Hoyer, et al: Semi-Exclusive

A. Sickles and SJB

Small color-singlet
Color Transparent
Minimal same-side energy

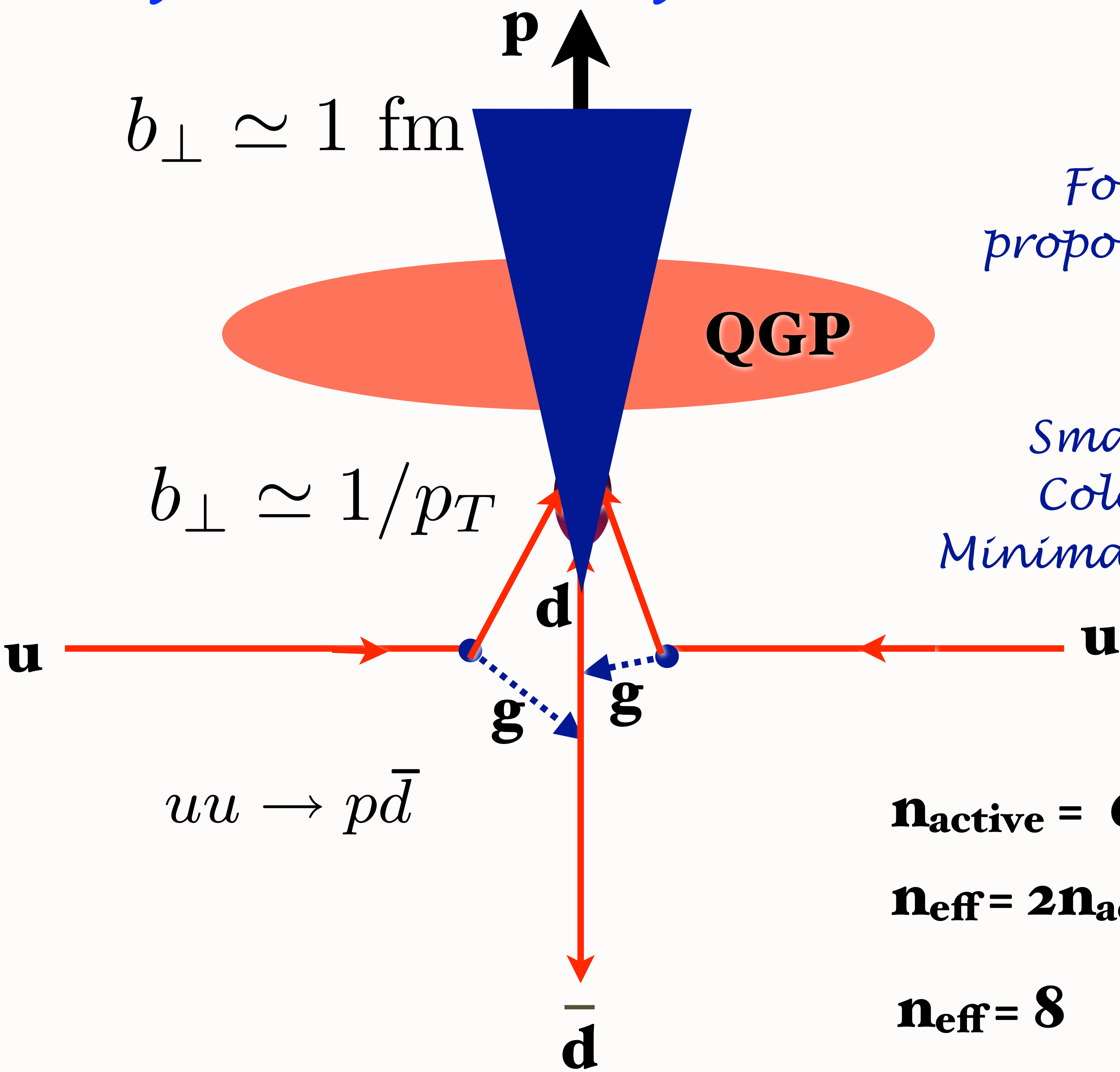
Collision can produce 3 collinear quarks

$n_{active} = 6$

$n_{eff} = 2n_{active} - 4$

$n_{eff} = 8$

Baryon made directly within hard subprocess



Formation Time
proportional to Energy

Small color-singlet
Color Transparent
Minimal same-side energy

$n_{active} = 6$

$n_{eff} = 2n_{active} - 4$

$n_{eff} = 8$

Dimensional counting rules provide a simple rule-of-thumb guide for the power-law fall-off of the inclusive cross section in both p_T and $(1 - x_T)$ due to a given subprocess:

$$E\frac{d\sigma}{d^3p}(AB \to CX) \propto \frac{(1-x_T)^{2n_{spectator}-1}}{p_T{}^{2n_{active}-4}}$$

where n_{active} is the "twist", i.e., the number of elementary fields participating in the hard subprocess, and $n_{spectator}$ is the total number of constituents in A, B and C not participating in the hard-scattering subprocess. For example, consider $pp \to pX$. The leading-twist contribution from $qq \to qq$ has $n_{active} = 4$ and $n_{spectator} = 6$. The higher-twist subprocess $qq \to p\bar{q}$ has $n_{active} = 6$ and $n_{spectator} = 4$. This simplified model provides two distinct contributions to the inclusive cross section

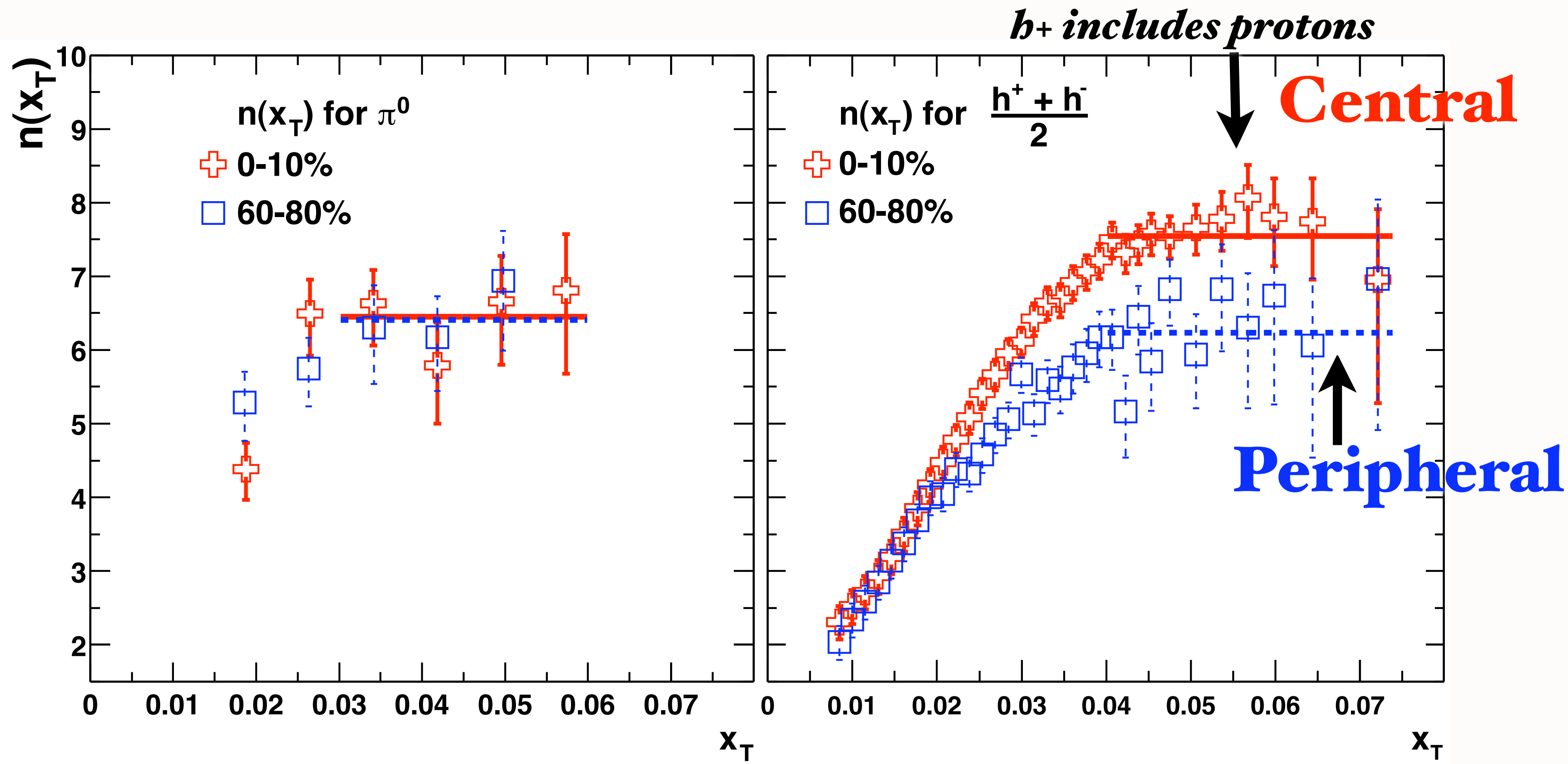
$$\frac{d\sigma}{d^3p/E}(pp \to pX) = A\frac{(1-x_T)^{11}}{p_T^4} + B\frac{(1-x_T)^7}{p_T^8}$$

and $n = n(x_T)$ increases from 4 to 8 at large x_T.

Small color-singlet
Color Transparent
Minimal same-side energ

Power-law exponent $n(x_T)$ for π^0 and h spectra in central and peripheral Au+Au collisions at $\sqrt{s_{NN}} = 130$ and 200 GeV

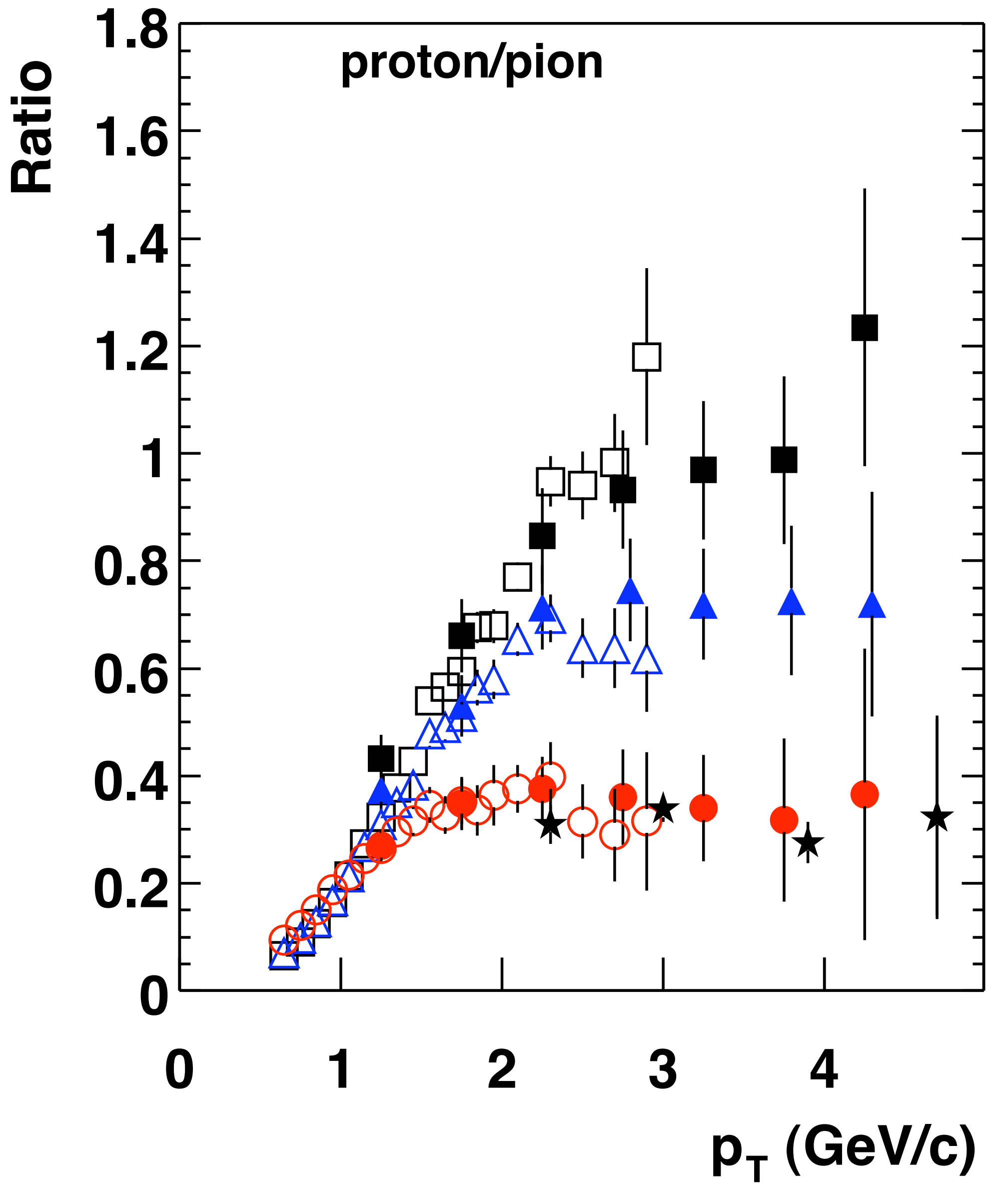
S. S. Adler, *et al.*, PHENIX Collaboration, *Phys. Rev.* C **69**, 034910 (2004) [nucl-ex/0308006].



Proton production dominated by color-transparent direct high n_{eff} subprocesses

S. S. Adler *et al*. PHENIX Collaboration *Phys. Rev. Lett*. **91**, 172301 (2003).

Particle ratio changes with centrality!

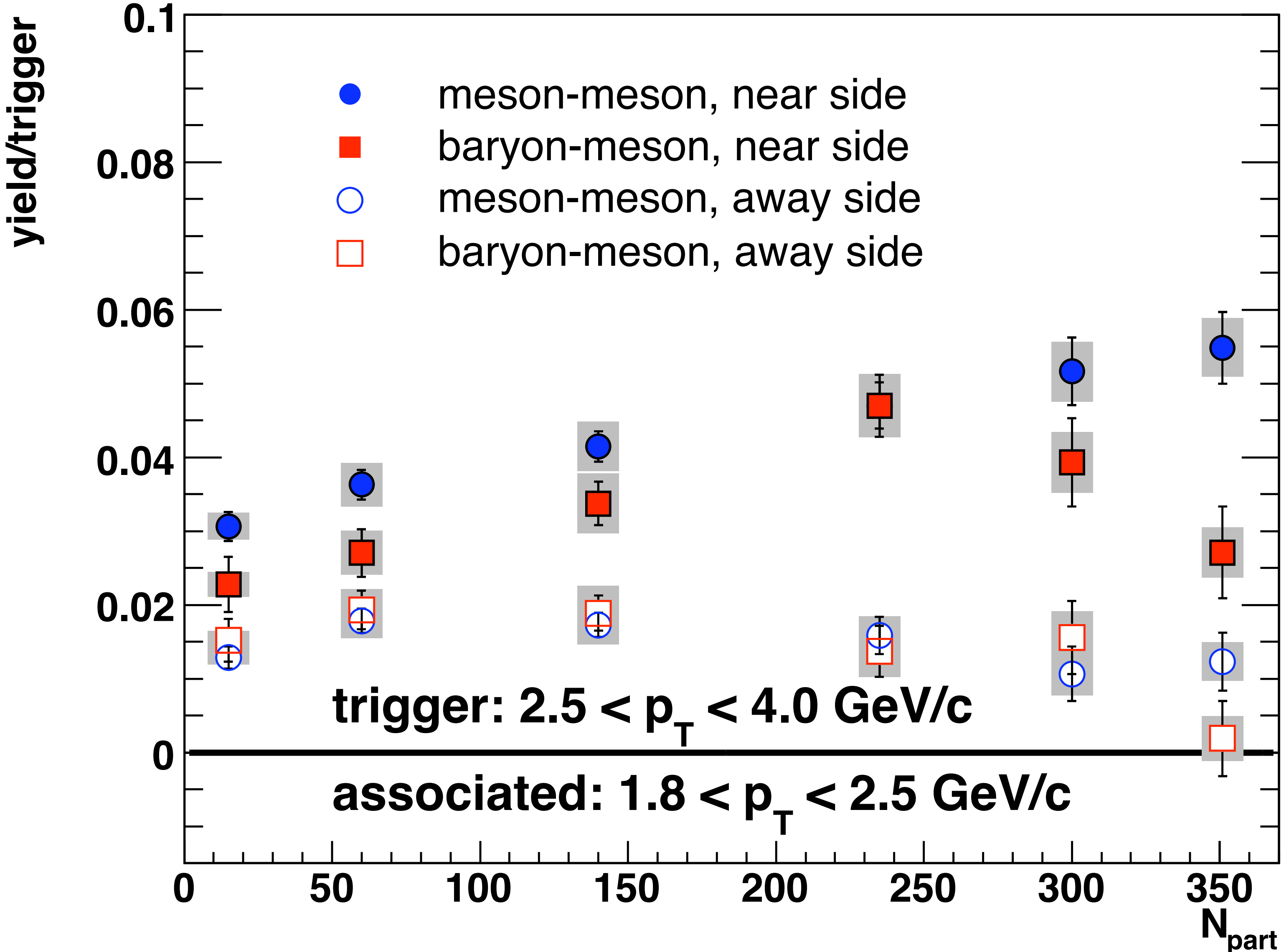


Protons less absorbed in nuclear collisions than pions because of dominant color transparent higher twist process

← **Central**

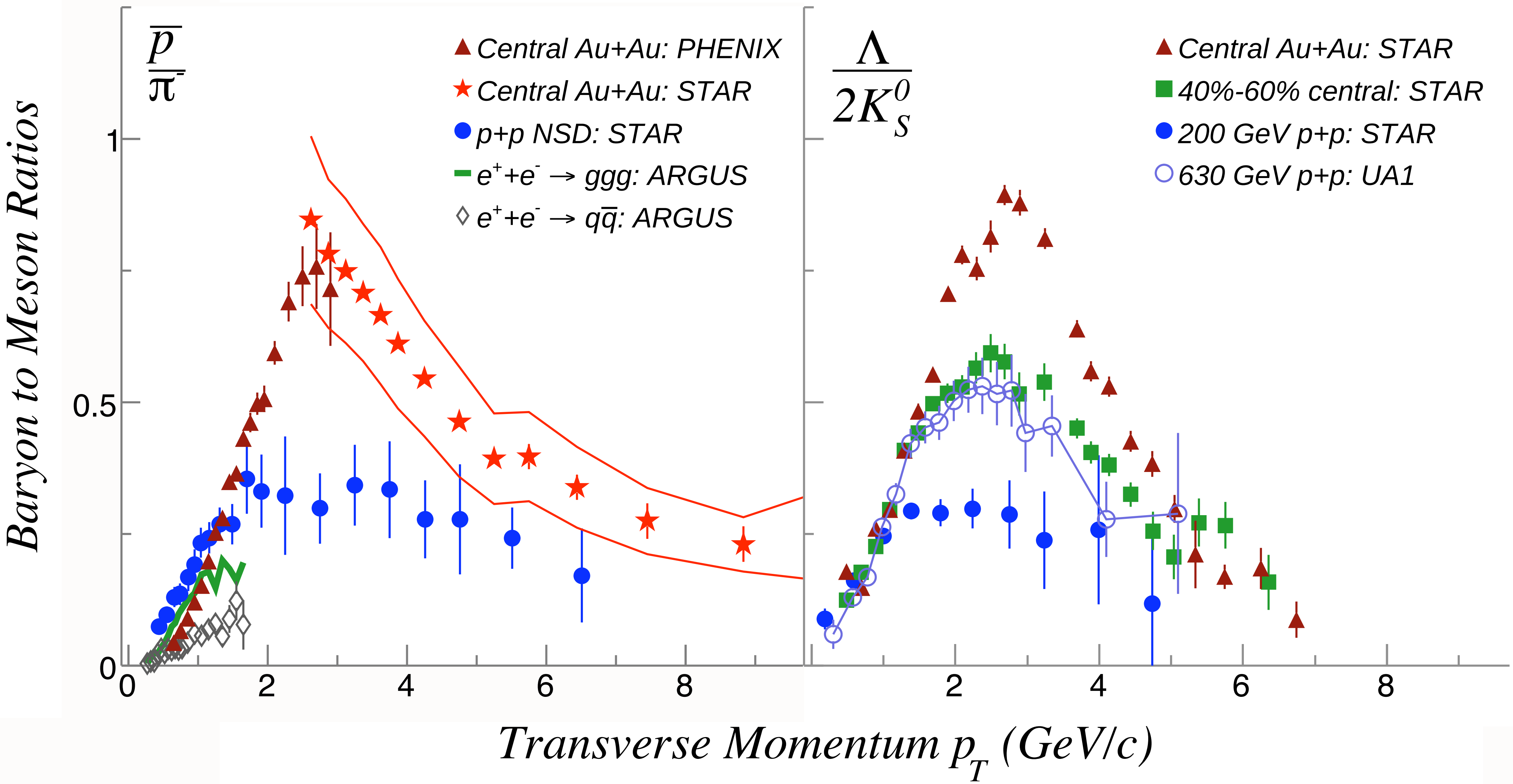
← **Peripheral**

Anne Sickles



proton trigger: # same-side particles decreases with centrality

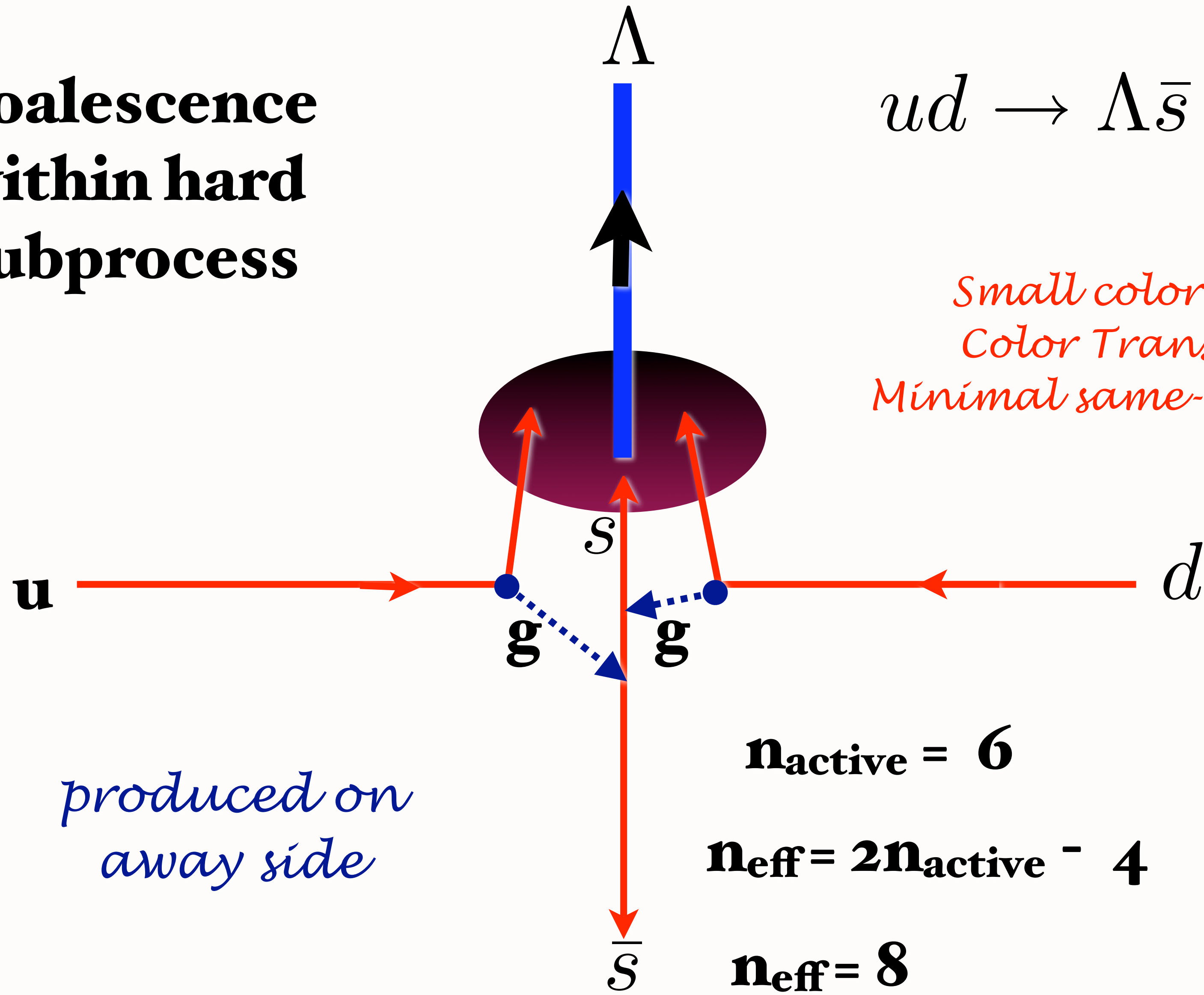
Proton production more dominated by color-transparent direct high-n_{eff} subprocesses


$\frac{\bar{p}}{\pi^-}$
Central Au+Au: PHENIX
Central Au+Au: STAR
p+p NSD: STAR
$e^+ + e^- \rightarrow ggg$: ARGUS
$e^+ + e^- \rightarrow q\bar{q}$: ARGUS
$\frac{\Lambda}{2K_S^0}$
Central Au+Au: STAR
40%-60% central: STAR
200 GeV p+p: STAR
630 GeV p+p: UA1
Baryon to Meson Ratios
Transverse Momentum p_T (GeV/c)
0
0.5
1
0
2
4
6
8

Lambda can be made directly within hard subprocess

Coalescence within hard subprocess

$$ud \to \Lambda \bar{s}$$



Small color-singlet
Color Transparent
Minimal same-side energy

$\bar{s}$ produced on away side

$n_{active} = 6$

$n_{eff} = 2n_{active} - 4$

$n_{eff} = 8$

Evidence for Direct, Higher-Twist Subprocesses

- Anomalous power behavior at fixed x_T
- Protons more likely to come from direct subprocess than pions
- Protons less absorbed than pions in central nuclear collisions because of color transparency
- Predicts increasing proton to pion ratio in central collisions
- Exclusive-inclusive connection at $x_T = 1$

- Renormalization scale is not arbitrary; multiple scales, unambiguous at given order

- Heavy quark distributions do not derive exclusively from DGLAP or gluon splitting -- component intrinsic to hadron wavefunction

- Initial and final-state interactions are not always power suppressed in a hard QCD reaction

- LFWFS are universal, but measured nuclear parton distributions are not universal -- antishadowing is flavor dependent

- Hadroproduction at large transverse momentum does not derive exclusively from 2 to 2 scattering subprocesses

- Hadronization at the Amplitude Level