Hadronic Production of Colored SUSY Particles with Electroweak NLO Contributions

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Outline

- **Introduction**

  supersymmetry

  production of colored SUSY particles

- **Production of Squarks and Gluinos**

  classification of processes

  QCD and EW contributions

- $\tilde{t}_1\tilde{t}_1^*$ **and** $\tilde{g}\tilde{q}$ **production at EW NLO**

  handling singularities

  numerical results

- **Summary**

Supersymmetry (SUSY)

SUSY relates fermions and bosons!

$Q|boson >= |fermion >$
$Q|fermion >= |boson >$

- **new partner particles** for all SM particles:
  same quantum numbers but **differ in spin by 1/2**

- SUSY as an exact symmetry predicts **m(SM)** $\stackrel{!}{=}$ **m(SUSY)**
  $\longrightarrow$ **SUSY** must be **broken** if realized

- **M**inimal **S**upersymmetric **S**tandard **M**odel (MSSM):

SM particle		**SUSY particle**		
quarks	q_L, q_R	squarks	$\tilde{q}_L, \tilde{q}_R$	
leptons	ℓ_L, ℓ_R	sleptons	$\tilde{\ell}_L, \tilde{\ell}_R$	
gluon	g	gluino	$\tilde{g}$	
W/Z boson	W, Z	wino/zino	$\tilde{W}, \tilde{Z}$	neutralinos $\tilde{\chi}^0$
photon	γ	photino	$\tilde{\gamma}$	charginos $\tilde{\chi}^\pm$
Higgs	H_1, H_2	higgsino	$\tilde{H}_1, \tilde{H}_2$	

(wino/zino, photino and higgsino together form the neutralinos $\tilde{\chi}^0$ and charginos $\tilde{\chi}^\pm$)

SUSY Breaking

SUSY as exact symmetry predicts **m(SM)** $\stackrel{!}{=}$ **m(SUSY)**

but **no SUSY particle observed** so far

$\longrightarrow$ **SUSY** must be **broken** if realized in nature

- General **SUSY breaking mechanism** introduces a large amount of new, **free parameters** (**105**)
- Constrained SUSY models:

  assumption of **specific SUSY breaking** mechanisms and universal boundary conditions at the GUT scale

  $\longrightarrow$ e.g. **m**inimal**SU**per**GRA**vity models,
  only 5 **parameters** [m_0, $m_{1/2}$, A_0, $\tan\beta$, $sgn(\mu)$]

  $\longrightarrow$ low-energy spectrum from renormalization group equations

Motivation for Supersymmetry

Supersymmetry is a possible and very attractive extension of the Standard Model:

- **unique extension** of the Poincaré group
  gravitation can be **included** at the Planck scale
- SUSY particles alter the running of gauge couplings
  $\longrightarrow$ **unification of the coupling constants**
- **protective symmetry**: Higgs mass below 1 TeV is possible
  $\longrightarrow$ solution to the hierarchy problem
- provides **Dark Matter candidate**
  (if additionnal symmetry R-parity is assumed:
  stable massive, neutral, weakly interacting particle)

Top-Squarks (Stops)

- SUSY **partners of top-quarks**
- same quantum numbers as $t_{L/R}$, but are **scalar particles**
- not yet observed, heavy particles

- **large top-Yukawa coupling**

  $\longrightarrow$ **RGE's**: stops lighter than squarks of first generations

  $\longrightarrow$ **mixing**: gauge eigenstates $\tilde{t}_{L/R} \longrightarrow$ mass eigenstates $\tilde{t}_{1/2}$:

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  $\rightarrow$ **mixing**: gauge eigenstates $\tilde{t}_{L/R} \rightarrow$ mass eigenstates $\tilde{t}_{1/2}$:

$$\mathcal{L}_{\tilde{t}\tilde{t}} = -(\tilde{t}_L^*, \tilde{t}_R^*) \begin{pmatrix} m_t^2 + A_{LL} & m_t B_{LR} \\ m_t B_{LR} & m_t^2 + C_{RR} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix} = -(\tilde{t}_1^*, \tilde{t}_2^*) \begin{pmatrix} m_{\tilde{t}_1}^2 & 0 \\ 0 & m_{\tilde{t}_2}^2 \end{pmatrix} \begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix}$$

$$\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_t & \sin\theta_t \\ -\sin\theta_t & \cos\theta_t \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix},$$

$$\begin{aligned} A_{LL} &= \left(\tfrac{1}{2} - \tfrac{2}{3}\sin^2\theta_W\right) m_Z^2 \cos 2\beta + m_{\tilde{Q}_3}^2 \\ B_{LR} &= A_t - \mu \cot\beta \\ C_{RR} &= \tfrac{2}{3}\sin^2\theta_W \, m_Z^2 \cos 2\beta + m_{\tilde{U}_3}^2 \end{aligned}$$

$$m_{\tilde{t}_{1,2}}^2 = m_t^2 + \frac{1}{2}\left(A_{LL} + C_{RR} \mp \sqrt{(A_{LL} - C_{RR})^2 + 4 m_t^2 B_{LR}^2}\right)$$

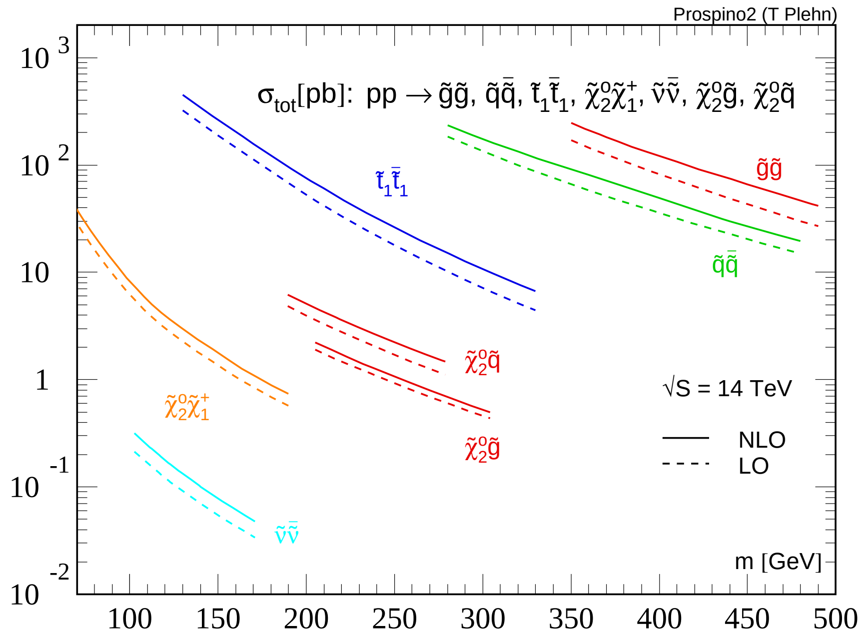
$\rightarrow$ **$\tilde{t}_1$ lightest squark in many SUSY models!**

Colored SUSY Particles at LHC

Why studying production of colored SUSY particles at the LHC?

- pair production of gluinos and squarks proceeds via **strong interaction**
  $\longrightarrow$ **large cross sections**

- large top-Yukawa coupling: **top-squark** $\tilde{t}_1$ candidate for **lightest squark**
  $\longrightarrow$ **high production rate**



- **cross section depend** essentially **on final state masses**
  $\longrightarrow$ bounds on cross section allow for lower mass bounds without specifying all other SUSY parameters

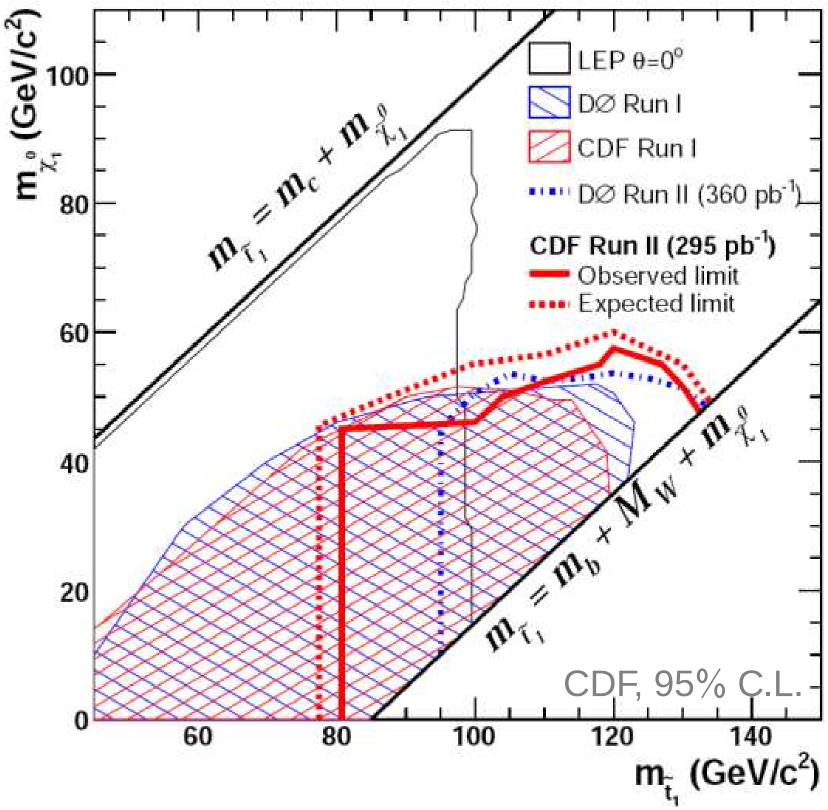
Experimental Searches

stop mass limits @ CDF Run II:

[0707.2567 hep-ex]

$m_{\tilde{t}_1} > 132$ GeV for $m(\tilde{\chi}_1^0) = 48$ GeV

QCD corr's included; $BR(\tilde{t}_1 \to c\tilde{\chi}_1^0) = 100\%$



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Gluino & squark mass limits:

at D0 Run II [0712.3805 hep-ex]

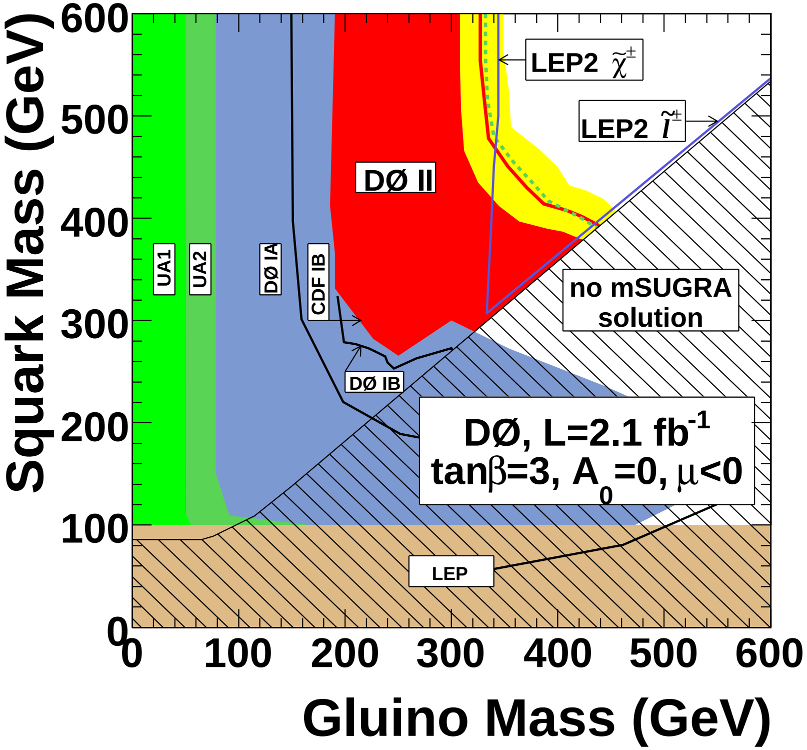
$m_{\tilde{q}\approx\tilde{g}} > 390$ GeV; $m_{\tilde{g}} > 308$ GeV

$A_0 = 0$ GeV, $\tan\beta = 3$, $\mu < 0$; L=2.1 fb^{-1}

at ALEPH & OPAL [0804.2477 hep-ph]

$m_{\tilde{g}} > 51$ GeV

model-independent; from thrust data



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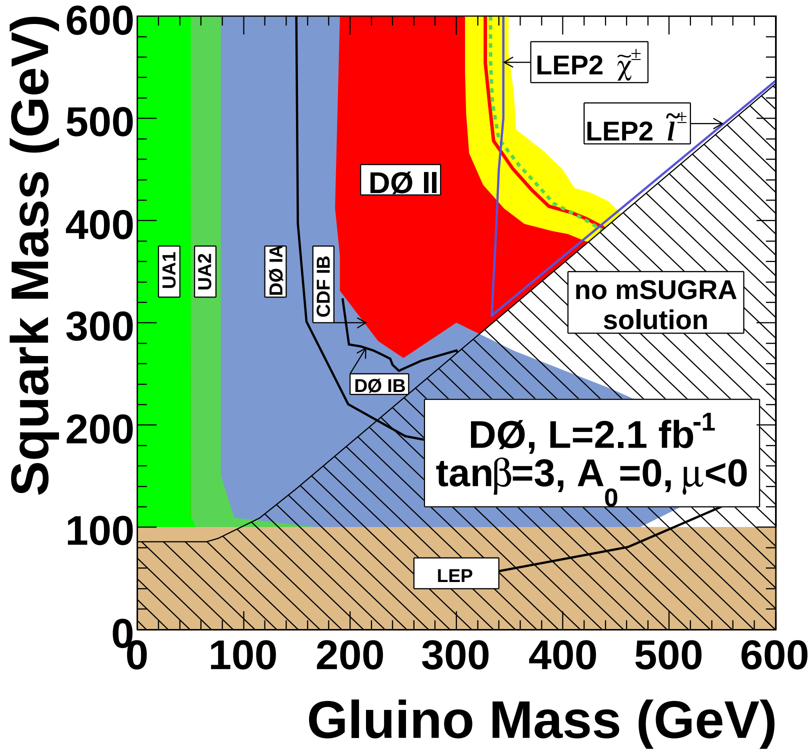

→ **until now: agreement** between data and SM expectations

→ comparison of exp. limits & theor. cross sections: restrictions on SUSY parameter space

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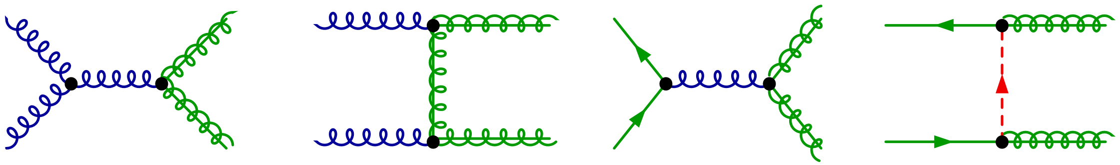
Overview: Squark & Gluino Production @ LO

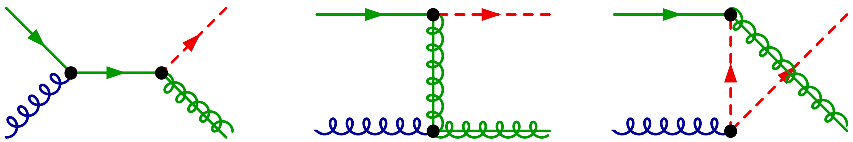
Squark and gluino production at LO is well known since many years

[Kane & Leveille '82, Harrison & Llewellyn Smith '83,
Reya & Roy '85, Dawson, Eichten, Quigg '85, Baer & Tata '85]

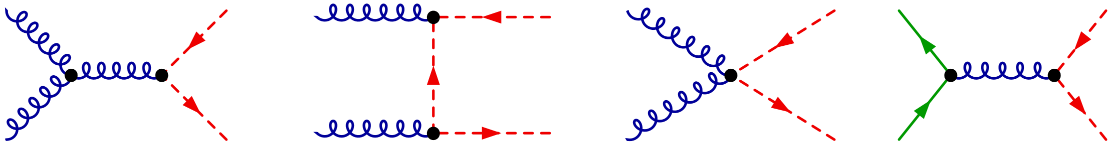
- $\mathcal{O}(\alpha_s^2)$: – $\tilde{g}\tilde{g}$ production



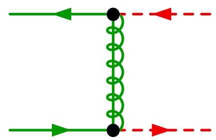
  – $\tilde{g}\tilde{q}$ production



  – $\tilde{q}\tilde{q}^*$, $\tilde{b}_i\tilde{b}_i^*$, $\tilde{t}_i\tilde{t}_i^*$ production



$(\tilde{q} \neq \tilde{t})$
$+$



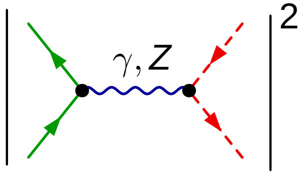
- stops & sbottoms: L–R mixing cannot be neglected; exp. distinguishable
- top-squark pair production is diagonal at LO

Tree-level Electroweak Contributions

$\tilde{t}\tilde{t}^*$ and $\tilde{q}\tilde{q}^*$ production is also possible by tree-level EW contributions!

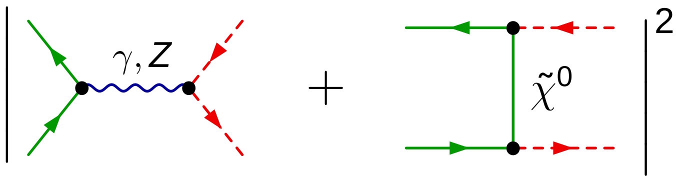
- $\mathcal{O}(\alpha^2)$: EW tree-level contributions to $\tilde{t}\tilde{t}^*$ production

[Bozzi, Fuks, Herrmann, Klasen '07]

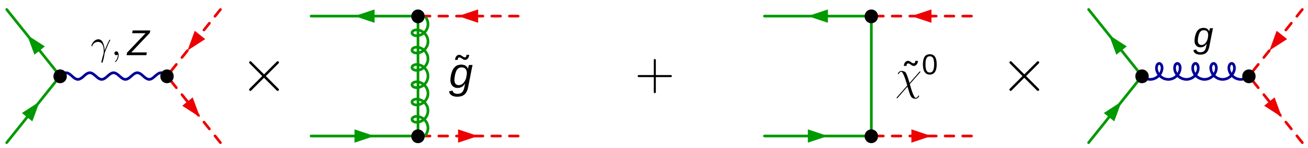


- $\mathcal{O}(\alpha^2 + \alpha_s\alpha)$: EW tree-level contributions to $\tilde{q}\tilde{q}^*$ production

[Bornhauser, Drees, Dreiner, Kim '07]



+ EW-QCD tree level interferences

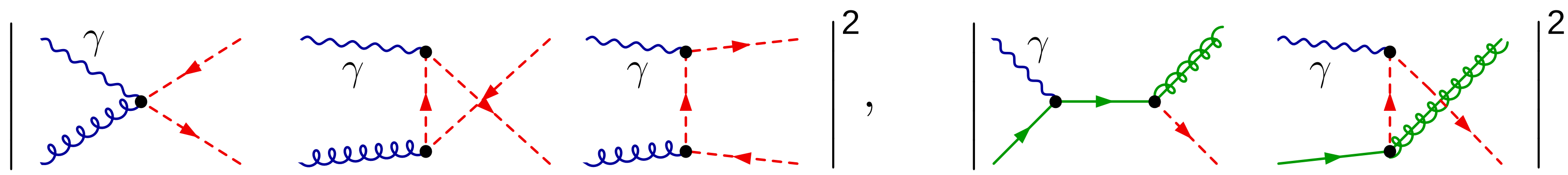


Tree-level Electroweak Contributions II

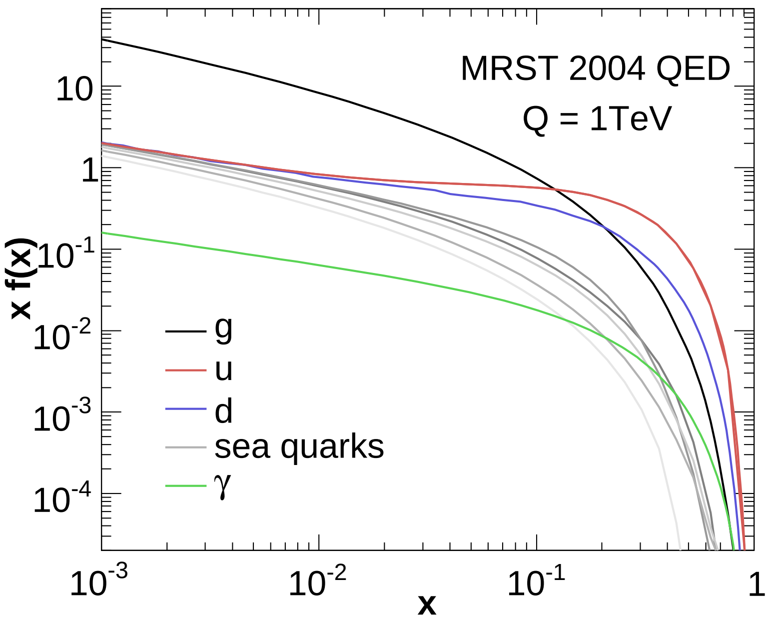
New production channel for $\tilde{q}\tilde{q}^*$, $\tilde{t}\tilde{t}^*$, and $\tilde{q}\tilde{g}$ production:

- $\mathcal{O}(\alpha_s \alpha)$: photon induced processes

[Hollik, Kollar, MT '07], [Hollik, Mirabella '08]
[Hollik, Mirabella, MT '08]



- not present at LO at the hadronic level
- **MRST 2004 QED**: inclusion of **NLO QED effects** in the evolution of PDFs
  - $\longrightarrow$ non-zero photon distribution
  - $\longrightarrow$ non-zero hadronic contributions

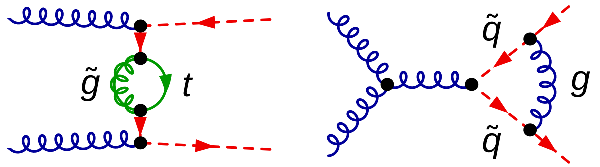


Higher Order Corrections – Squark Production

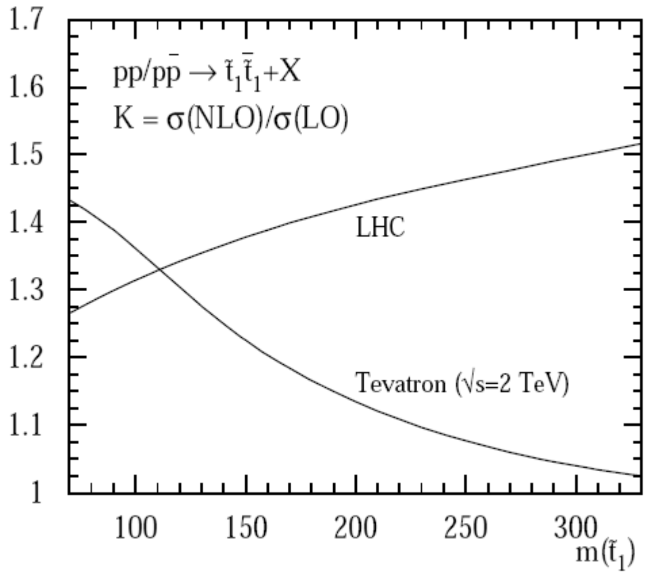
Important **higher order effects** due to **QCD corrections**:

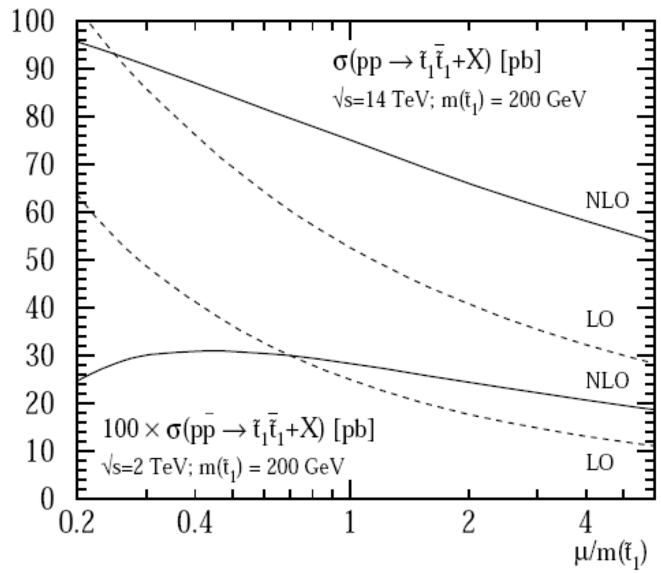
- $\mathcal{O}(\alpha_s^3)$: QCD NLO corrections

[Beenakker, Höpker, Spira, Zerwas '95 & '97] &
[Beenakker, Krämer, Plehn, Spira, Zerwas '98]
$\longrightarrow$ PROSPINO, also for $\tilde{g}\tilde{q}$, $\tilde{g}\tilde{g}$

QCD Born $\times$  $+ \ldots$

$\tilde{t}_1\tilde{t}_1^*$ production:





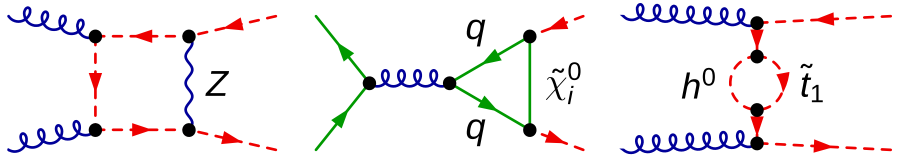
- large positive corrections
- reduced scale dependence

Higher Order Corrections – Squark Production II

Known from SM processes: also **EW corrections** can be important!

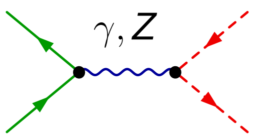
- $\mathcal{O}(\alpha_s^2\alpha)$: EW NLO corrections

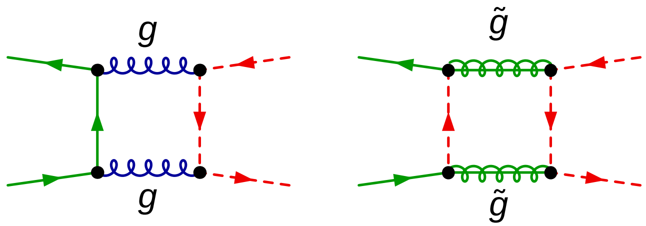
[Hollik, Kollar, MT '07], [Beccaria et. al. '08]
[Hollik, Mirabella '08]

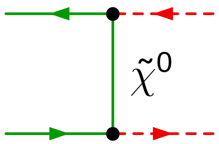
QCD Born $\times$ 

$+ \ldots$

+ EW-QCD one-loop interferences

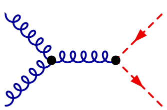
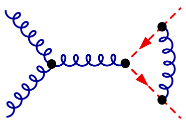
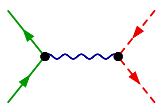
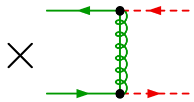
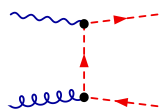
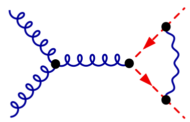


$\times$ 

$(\tilde{q} \neq \tilde{t})$
$+$ 

$\times$ full QCD 1-loop amplitude

Overview: Squark and Gluino Production @ LHC

	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha^2)$	$\mathcal{O}(\alpha_s\alpha)$	$\mathcal{O}(\alpha_s\alpha)$	$\mathcal{O}(\alpha_s^2\alpha)$
$\tilde{g}\tilde{g}$	+	+	–	–	–	+
$\tilde{g}\tilde{q}$	+	+	–	–	+	+
$\tilde{t}\tilde{t}^*$	+	+	+	–	+	+
$\tilde{q}\tilde{q}^*$	+	+	+	+	+	+
						

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EW NLO Corrections: **Singularities**

- **UV singularities** (self energies, vertices) from **loop integrals**

  $\longrightarrow$ **renormalization** of quarks & squarks

  (see talk by Edoardo)

  [no renorm. of gluon, gluino, and α_s at this order;

  stop prod. is diagonal $\longrightarrow$ no renorm. of mixing angle required]

- **IR (soft) singularities** from $m_\gamma = m_g = 0$

  $\longrightarrow$ real **photon** and **gluon bremsstrahlung**

  [technical: mass regularization + phase space slicing]

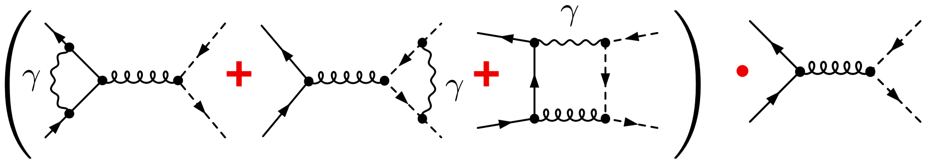
- **collinear singularities** from $m_q = 0$

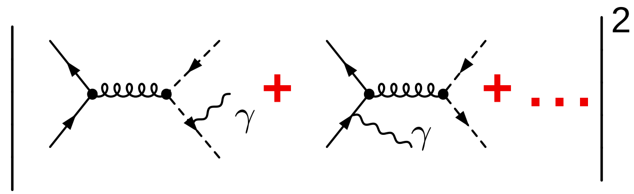
  $\longrightarrow$ real photon bremsstrahlung

  $\longrightarrow$ factorization and **redefinition of PDFs**

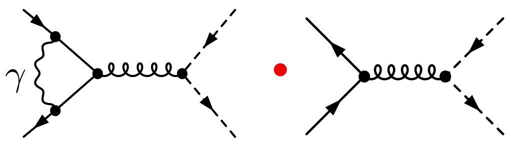
How to obtain a IR-finite cross section for $q\bar{q} \to \tilde{t}\tilde{t}^*$

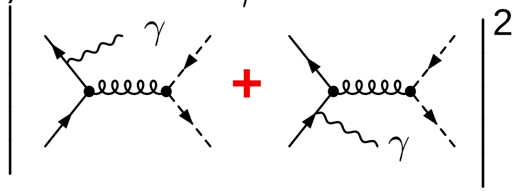
- soft divergent diagrams and soft photon bremsstrahlung





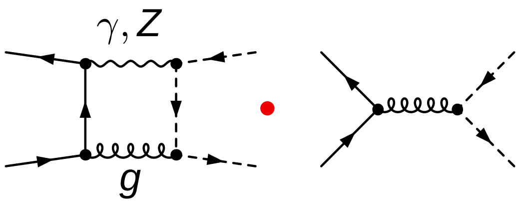
- collinear divergent diagram and hard, collinear γ bremsstrahlung

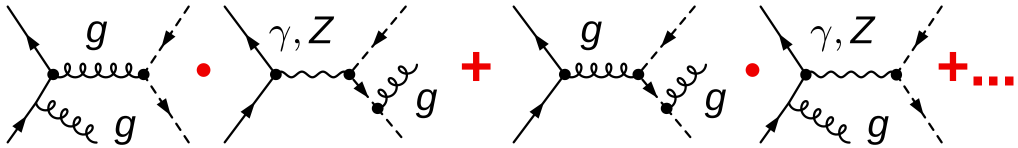




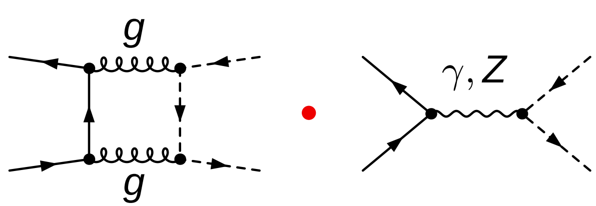
+ redefinition of PDFs: subtract $\ln(m_q^2)$-terms from $\sigma_{q\bar{q}}$

- soft gluon divergent diagrams and soft gluon bremsstrahlung





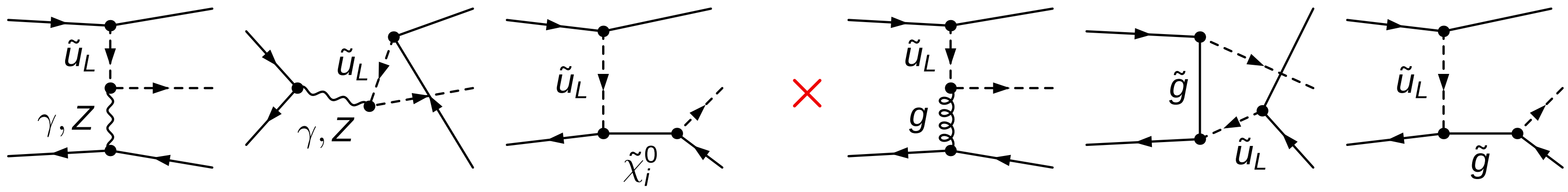
- interference of QCD boxes and EW born



Real (Anti-)Quark Radiation at $\mathcal{O}(\alpha_s^2\alpha)$

- **non-zero interference** of **EW** and **QCD** diagrams!

  $\longrightarrow$ many channels & diagrams, some examples for $u\bar{u} \to \tilde{g}\tilde{u}_L\bar{u}$:



$\longrightarrow$ IR soft and collinear finite
[$\tilde{t}\tilde{t}^*$: contribution finite & negligible; $\tilde{q}\tilde{q}^*$ collinear singular]

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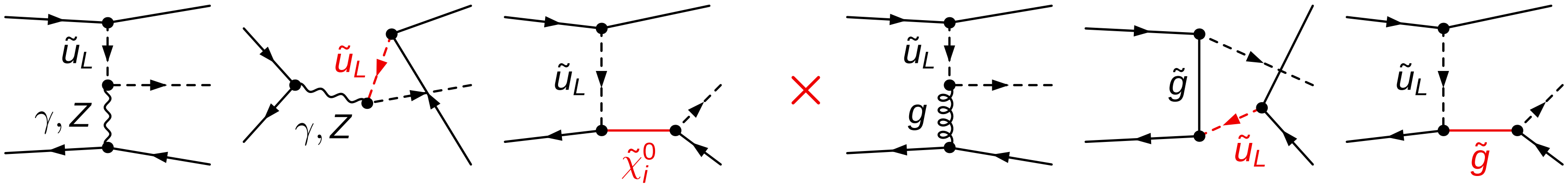

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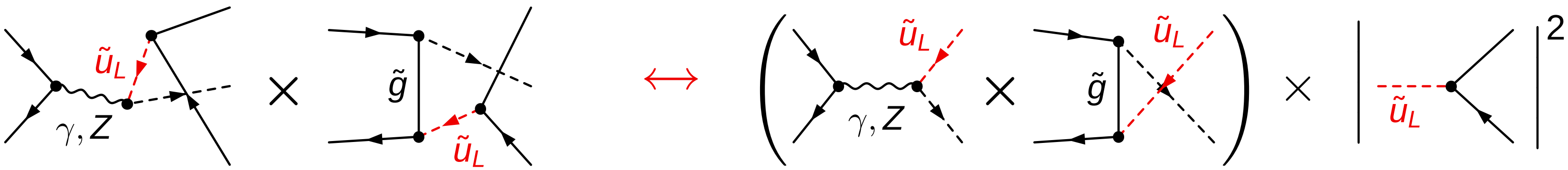
$\longrightarrow$ **on-shell internal particles**: insert widths to regularize propagators

$\longrightarrow$ in order to **avoid double counting**: subtract possible **resonances**

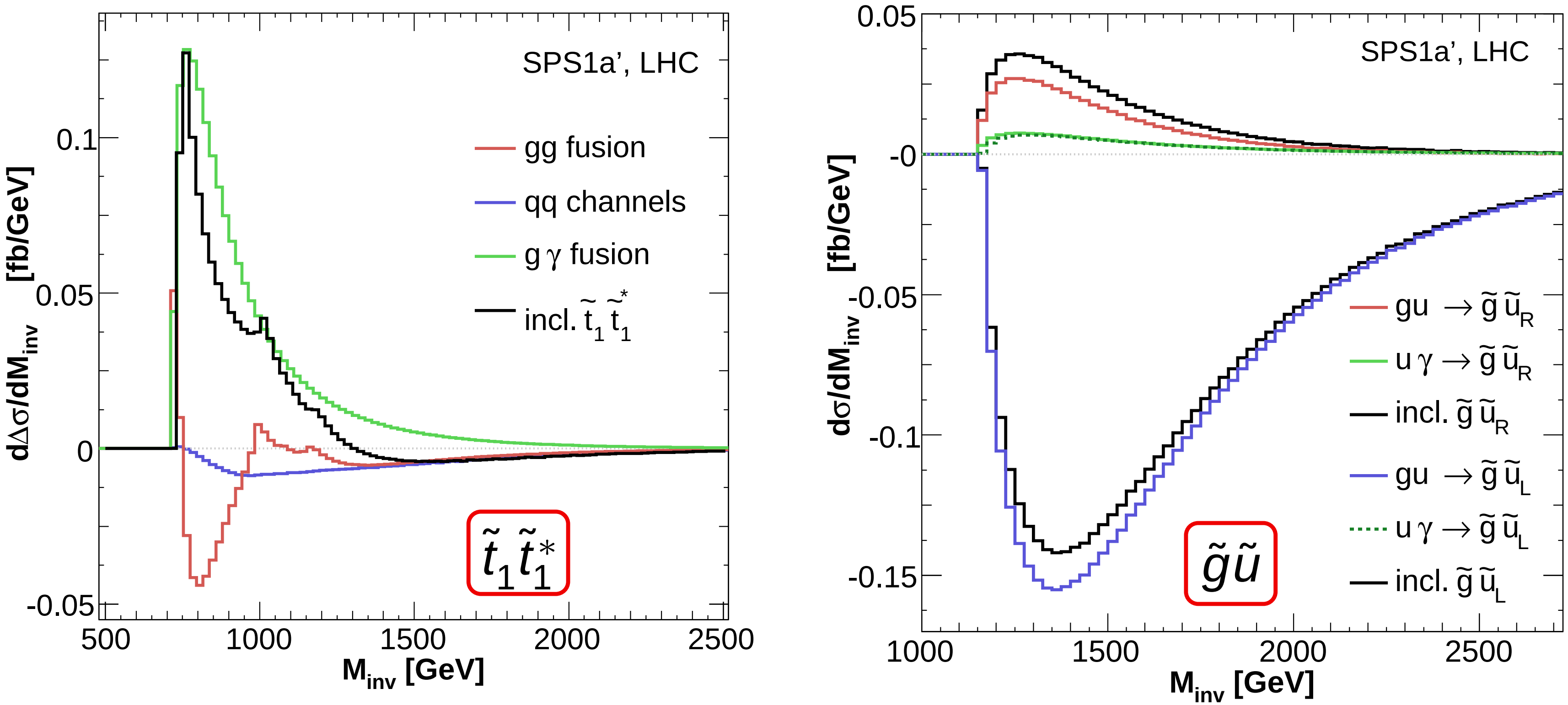


$\sigma(u\bar{u} \to \tilde{u}_L\,\tilde{g}\bar{u})$ $\qquad$ $\sigma(u\bar{u} \to \tilde{u}_L\tilde{u}_L^*) \quad \times \; BR(\tilde{u}_L^* \to \tilde{g}\bar{u})$

Numerical Results: Absolute EW Contributions

- Interplay of the production channels? Invariant mass distribtuions:

(real quark radiation not shown expl.)



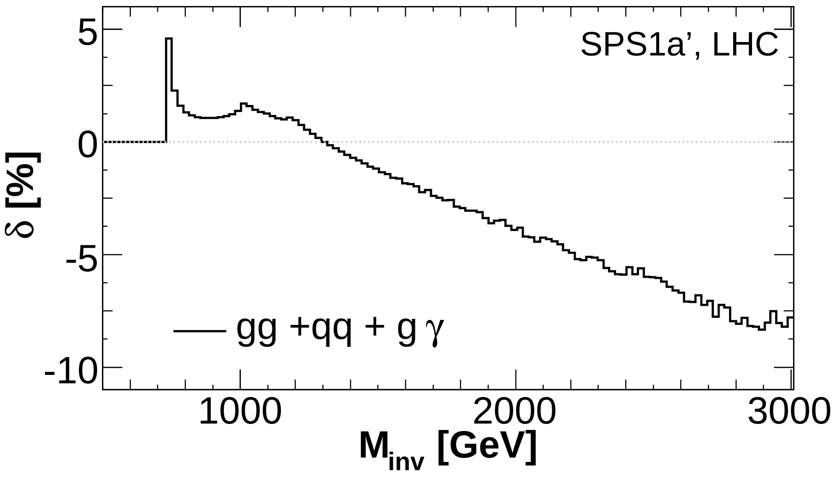
→ $\tilde{t}_1\tilde{t}_1^*$: **γg corrections** are **of the same size** and change the total EW corrections substantially!

→ $\tilde{g}\tilde{q}$: **γq corrections** do not depend on helicity state; contribute only moderate.

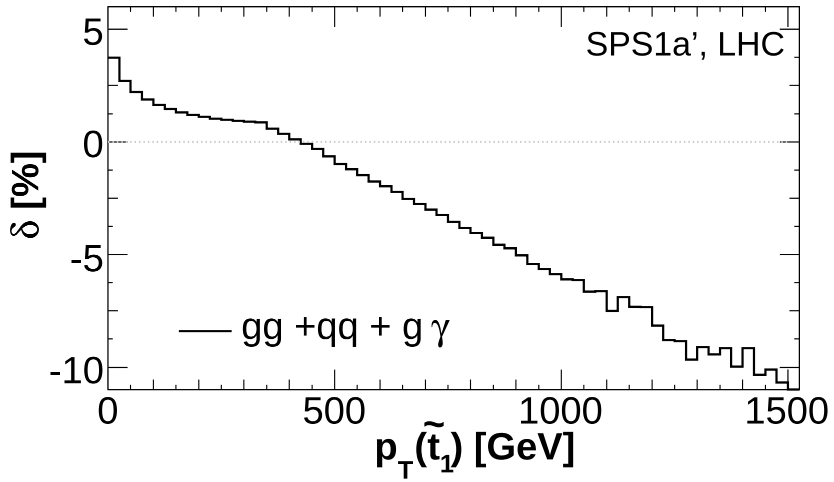
Relative Corrections $\delta = \Delta\sigma^{NLO}/\sigma^{LO}$

$\tilde{t}_1\tilde{t}_1^*$ prod.:

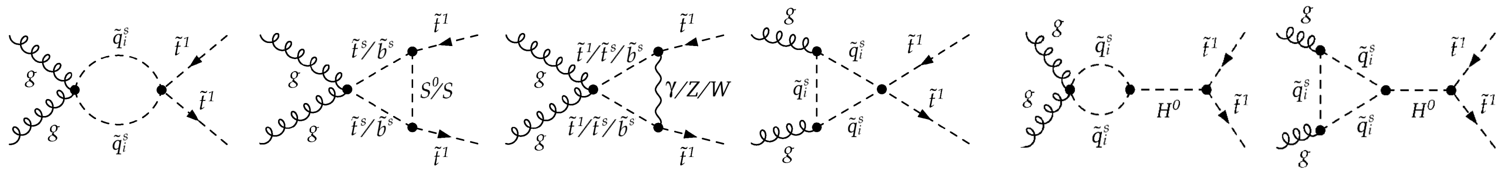
$M_{inv}(\tilde{t}_1\tilde{t}_1^*)$



$p_T(\tilde{t}_1)$



$\longrightarrow$ threshold effects from stop & sbottom pairs in loops

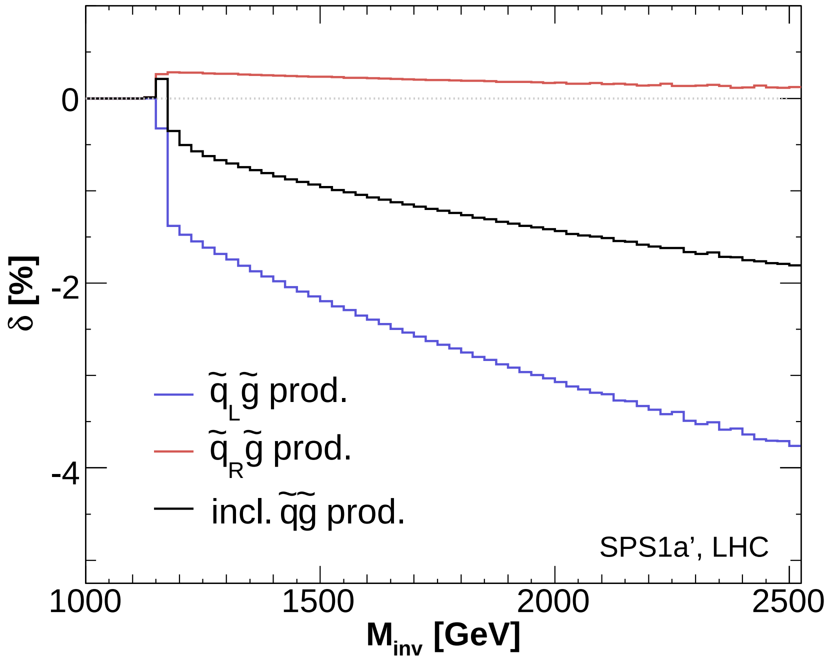


$\longrightarrow$ **EW corrections** grow **up to $\sim$ 10%** for large values of p_T & $M_{\tilde{t}_1\tilde{t}_1^*}$
(large double log's from W,Z exchange, not cancelled by real radiation)

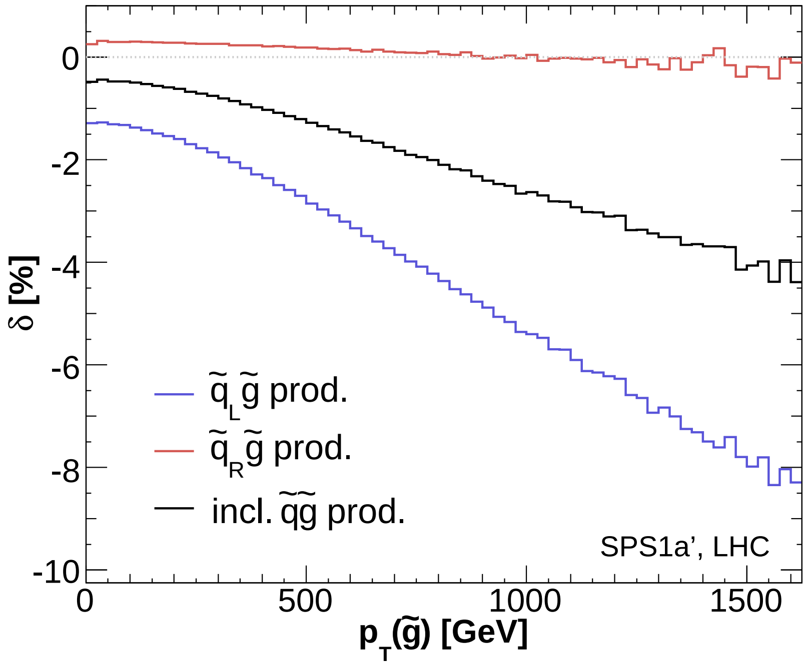
Relative Corrections $\delta = \Delta\sigma^{NLO}/\sigma^{LO}$

$\tilde{g}\tilde{q}$ prod.:

$M_{inv}(\tilde{q}\tilde{g})$



$p_T(\tilde{g})$



$\longrightarrow$ for $\tilde{g}\tilde{d}_L$ + $\tilde{g}\tilde{u}_L$ production EW NLO corrections grow **up to 10%**

$\longrightarrow$ corrections negligible for right-handed squarks

Total Cross Sections for SPS1a'

final state	σ^{LO} $\mathcal{O}(\alpha_s^2)$	$\Delta\sigma^{NLO}$ $\mathcal{O}(\alpha_s^2\alpha)$	$\sigma^{\gamma g/\gamma q}$ $\mathcal{O}(\alpha_s\alpha)$	$\delta = \frac{\sigma^{NLO}-\sigma^{LO}}{\sigma^{LO}}$
$\tilde{t}_1\tilde{t}_1^*$	**1830 fb**	**$-$15.0 fb**	**34.1 fb**	**1.0 %**
$\tilde{g}\,\tilde{u}_R$	5690 fb	12.6 fb	4.32 fb	0.3%
$\tilde{g}\,\tilde{d}_R$	3210 fb	1.97 fb	0.73 fb	0.1%
$\tilde{g}\,\tilde{u}_L$	5340 fb	$-$119 fb	3.98 fb	$-$2.2%
$\tilde{g}\,\tilde{d}_L$	2880 fb	$-$78.3 fb	0.64 fb	$-$2.7%
$\tilde{g}\tilde{q}$	**17120 fb**	**$-$183 fb**	**9.67 fb**	**$-$1.0%**

[$\mu_F = \mu_R =$ 1 TeV, MRST 2004 QED, $m_t = 170.9$ GeV;
$m(\tilde{t}_1) = 360$ GeV, $m(\tilde{u}_R) = 543$ GeV, $m(\tilde{d}_R) = 539$ GeV,
$m(\tilde{u}_L) = 561$ GeV, $m(\tilde{d}_L) = 566$ GeV, $m(\tilde{g}) = 609$ GeV]

- $\tilde{t}_1\tilde{t}_1^*$ production at $\mathcal{O}(\alpha^2)$: $\sigma^{EW,LO} = 1.11$ fb
- $\tilde{g}\tilde{q}$ processes: production of anti-squarks and of squarks of 2nd generation included (differing only in required PDF)

Summary

- Exciting times ahead: SUSY will be probed at the LHC
  Squarks and gluinos will be produced at a **very high rate**

- QCD corrections already well known,
  missing **EW NLO corrections:** for $\tilde{t}\tilde{t}^*$, $\tilde{q}\tilde{q}^*$ & $\tilde{g}\tilde{q}$ **completed**,
  for $\tilde{g}\tilde{g}$ publication in preparation

- **EW corrections** to the total cross section are small,
  but **important in the high-p_T** & high-M_{inv} range

- **PDF's include QED and QCD** contributions at NLO
  $\longrightarrow$ non-zero photon PDF opens **new production channel**
  $\longrightarrow$ need to include **QCD corrections for consistent picture** and for reduced scale dependence

Backup

Numerical Results: Input Parameters

- **SPA convention**: SUSY parameters defined in $\overline{DR}$ scheme
  here: (s)particles renormalized on-shell

  $\longrightarrow$ need consistent set of on-shell input parameters

  $\longrightarrow$ **translation $\overline{DR} \longrightarrow$ OS** required:

$$m^2_{\overline{\text{DR}}} + \delta m^2_{\overline{\text{DR}}} = m^2_{\text{OS}} + \delta m^2_{\text{OS}}$$

- **SU(2) invariance**: soft-breaking parameter $\boldsymbol{m_{\tilde{Q}}}$ **identical** for up- and down-type squarks

  $\longrightarrow$ fourth squark is dependent, receives mass corrections

$$\left(m^2_{\tilde{d}_L}\right)^{\text{1loop}} = \left(m^2_{\tilde{d}_L}\right)^{\text{dep.}} + \delta m^2_{\tilde{d}_L} - \Re\Sigma_{\tilde{d}_{LL}}(m^2_{\tilde{d}_L})$$

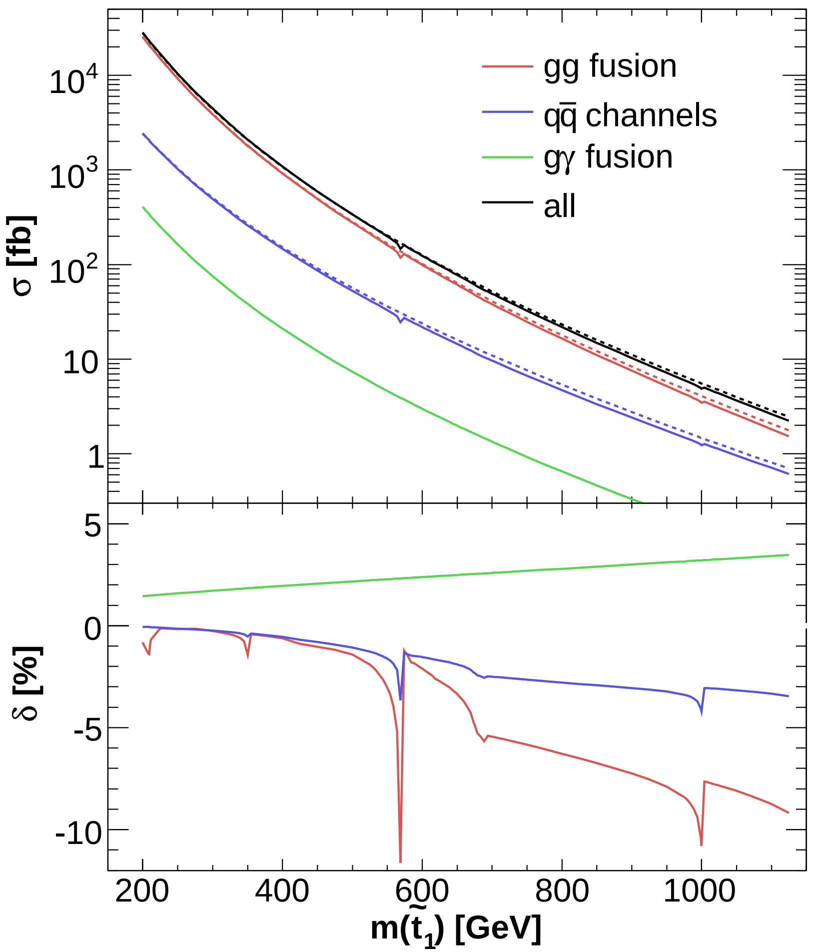
- Within the **SPS1a$'$ scenario**, the physical masses are

$$\begin{aligned} m_{\tilde{u}_R} &= 543\ \text{GeV}, & m_{\tilde{u}_L} &= 561\ \text{GeV}, & m_{\tilde{d}_R} &= 539\ \text{GeV}, \\ m_{\tilde{d}_L} &= 566\ \text{GeV}, & m_{\tilde{g}} &= 609\ \text{GeV}, & m_{\tilde{t}_1} &= 360\ \text{GeV}. \end{aligned}$$

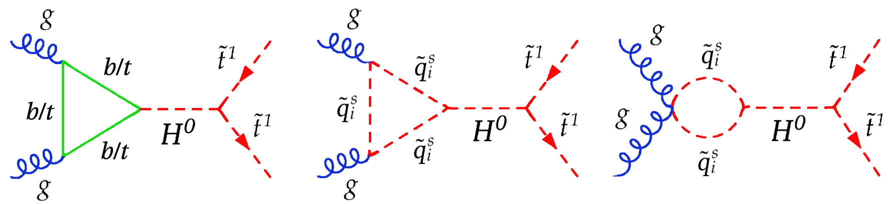
SUSY Parameter Dependence

$\tilde{t}_1\tilde{t}_1^*$ prod.:

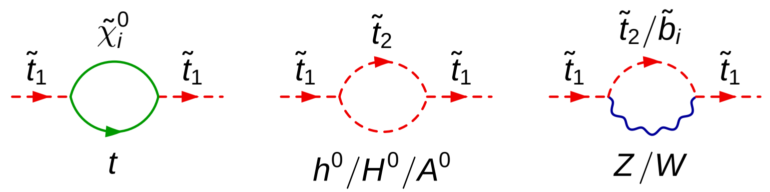
- Relative corrections δ with respect to total born cross section ($gg + q\bar{q}$), stop mass $m(\tilde{t}_1)$ **varied** around SPS 1a' value, all other parameters fixed



- moderate contributions, at percent level
- thresholds & resonances in H^0 diagrams

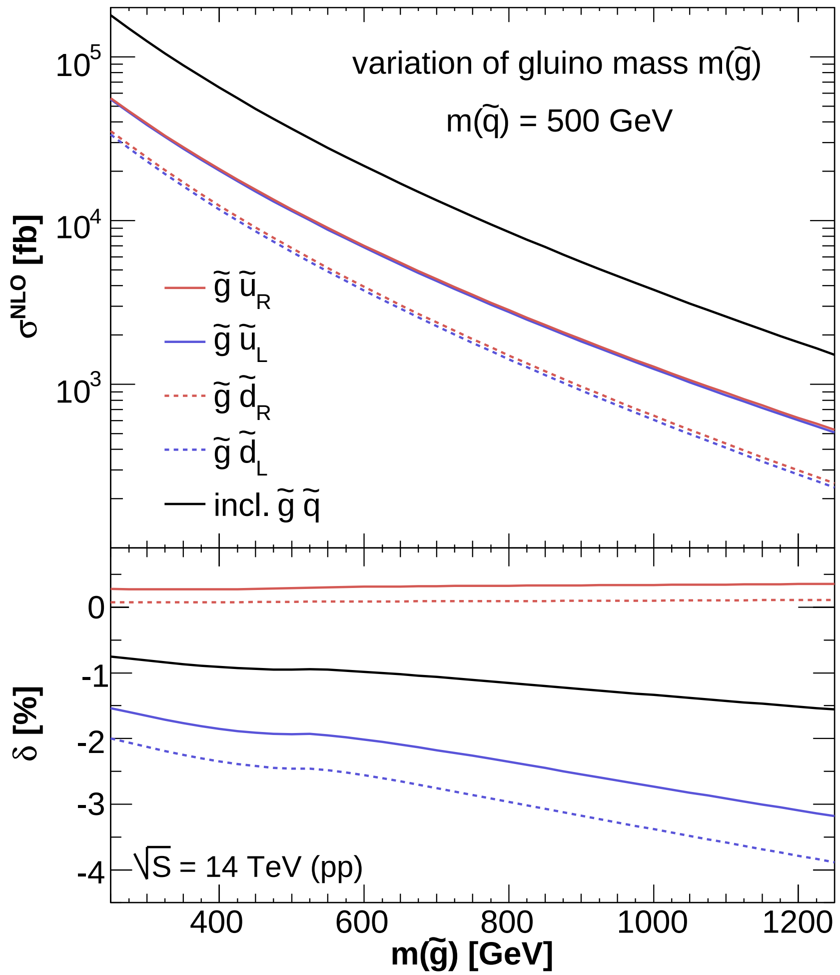


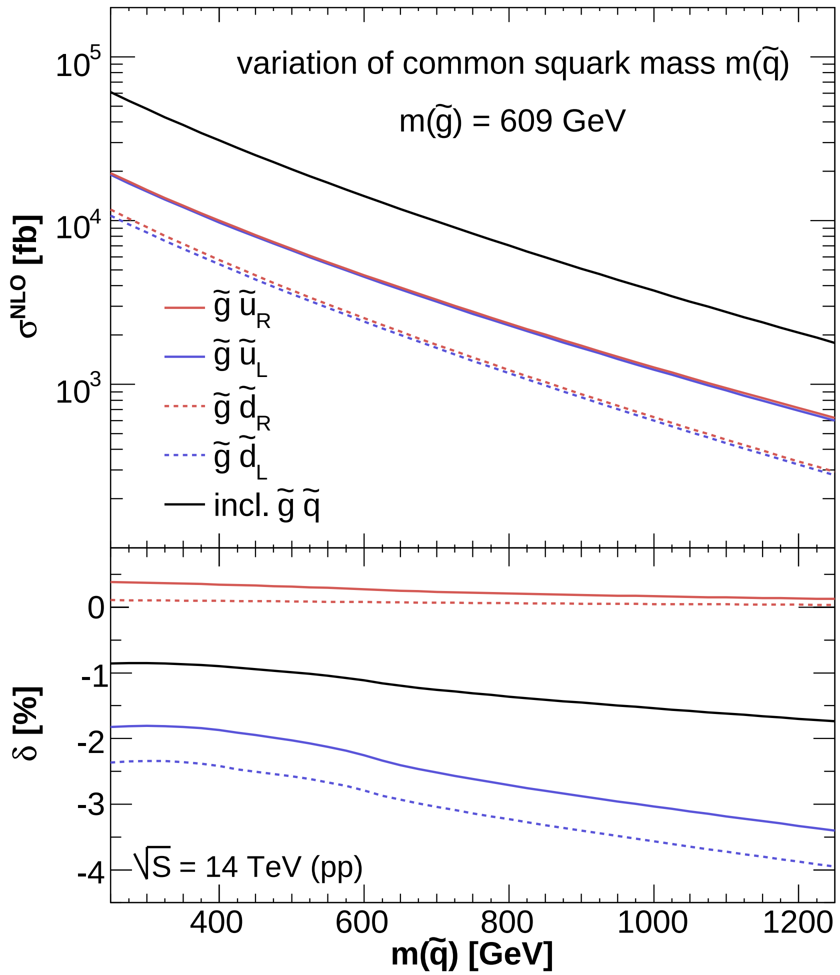
- thresholds in top-squark wave function renormalization



SUSY Parameter Dependence II

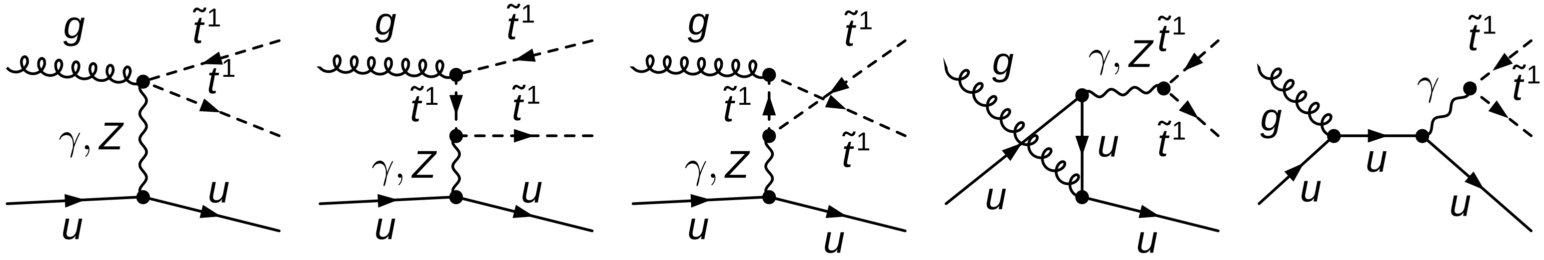
$\tilde{g}\tilde{q}$ prod.:



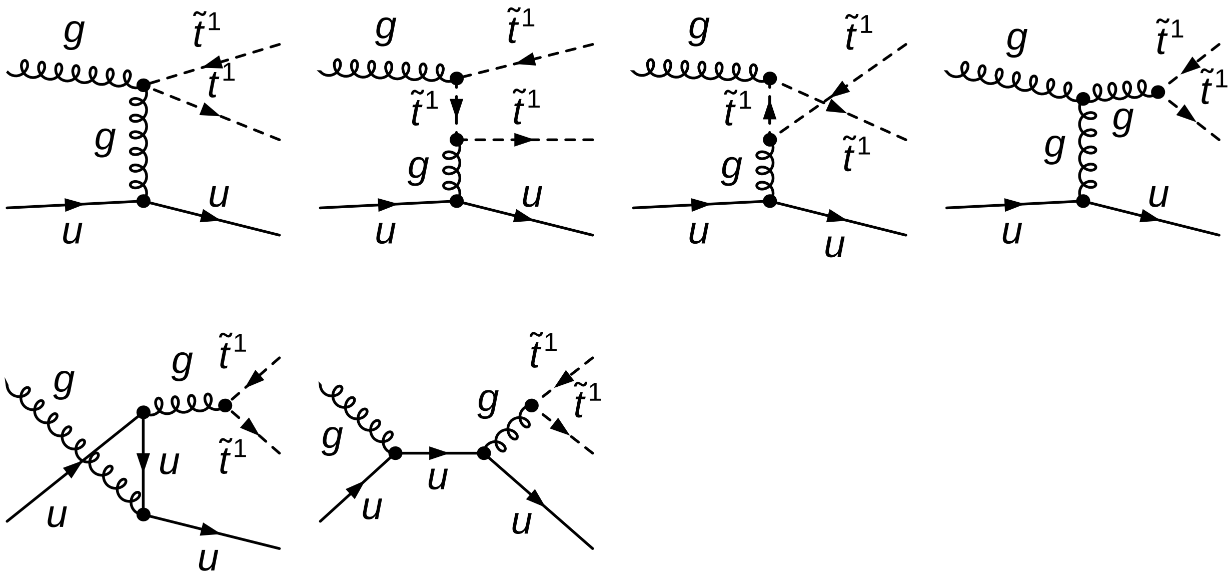


$\tilde{t}\tilde{t}^*$ prod.: Real Quark Radiation at $\mathcal{O}(\alpha_s^2\alpha)$

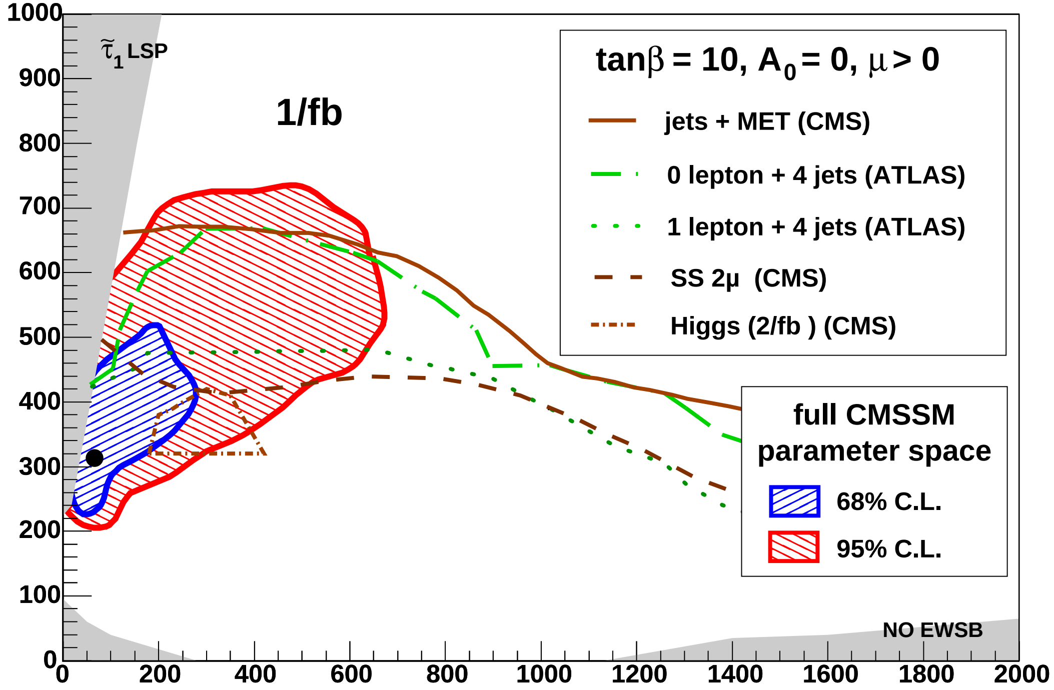
EW diagrams:



QCD diagrams:



Experimental Searches – prospects



good prospects for LHC! [Buchmüller et al. '08]

from combination of experimental, phenomenological, and cosmological information:

- 95% C. L. area in the $(m_{1/2}, m_0)$ plane of CMSSM lies largely within the region that **can be explored with 1fb^{-1} at 14 TeV**

Cross Sections at Hadron Colliders

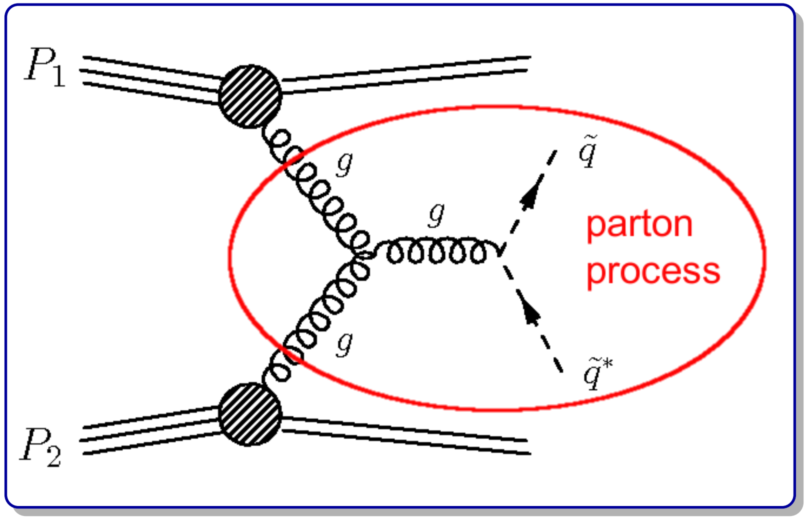
Distinguish **hadron level** and **parton level** :

$$\sigma_{P_1P_2\to\tilde{q}\tilde{q}^*}(P_1,P_2) = \sum_{i,j=g,q,\bar{q}} \int dx_1\, dx_2\; f_i(x_1)\, f_j(x_2)\; \hat{\sigma}_{ij\to\tilde{q}\tilde{q}^*}(p_1,p_2)$$

$$p_{1,2} = x_{1,2}\, P_{1,2}$$

$f_{g,q,\bar{q}}(x)$: **parton density function** (PDF)

$\longrightarrow$ probability to find a gluon g or (anti-)quark q with momentum fraction x (process independent)



Factorization

$$f_i(x) \;\longrightarrow\; f_i(x,\mu_F)$$

μ_F: factorization scale

Soft Singularities in Detail

The **soft-photon parts** factorize in **universal factors** and the **LO cross section**:

$$d\hat{\sigma}^{q\bar{q}}_{soft,\gamma}(\hat{s}) = \frac{\alpha}{\pi}\left(e_q^2\,\delta^{in}_{soft} + e_t^2\,\delta^{fin}_{soft} + 2e_q e_t\,\delta^{int}_{soft}\right) d\hat{\sigma}^{q\bar{q}}_0(\hat{s})$$

$$d\hat{\sigma}^{gg}_{soft,\gamma}(\hat{s}) = \frac{\alpha}{\pi}\,e_t^2\,\delta^{fin}_{soft}\,d\hat{\sigma}^{gg}_0(\hat{s})$$

the $q\bar{q}$ **soft-gluon part** needs special **color matrix arrangement**:

$$d\hat{\sigma}^{q\bar{q}}_{soft,g}(\hat{s}) = \frac{\alpha_s}{\pi}\,\delta^{int}_{soft}\left[T^a_{ij}T^b_{ji}T^a_{lm}T^b_{ml}\right]\cdot\overline{\sum}\left(2\widetilde{\mathcal{M}}^{q\bar{q}\,*}_{0,g}\widetilde{\mathcal{M}}^{q\bar{q}}_{0,\gamma} + 2\widetilde{\mathcal{M}}^{q\bar{q}\,*}_{0,g}\widetilde{\mathcal{M}}^{q\bar{q}}_{0,Z}\right)\frac{d\hat{t}}{16\pi\hat{s}^2}$$

with

$$\delta^{in}_{soft} = \left[\ln\frac{4(\Delta E)^2}{\hat{s}} - \ln\frac{\lambda^2}{\hat{s}}\right]\left[\ln\frac{\hat{s}}{m_q^2} - 1\right] - \frac{1}{2}\ln^2\frac{\hat{s}}{m_q^2} + \ln\frac{\hat{s}}{m_q^2} - \frac{\pi^2}{3},$$

$$\delta^{fin}_{soft} = \left[\ln\frac{4(\Delta E)^2}{\hat{s}} - \ln\frac{\lambda^2}{\hat{s}}\right]\left[\frac{\hat{s}-2m^2_{\tilde{t}_i}}{\hat{s}\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) - 1\right] + \frac{1}{\beta}\ln\left(\frac{1+\beta}{1-\beta}\right)$$
$$- \frac{\hat{s}-2m^2_{\tilde{t}_i}}{\hat{s}\beta}\left[2\mathrm{Li}_2\left(\frac{2\beta}{1+\beta}\right) + \frac{1}{2}\ln^2\left(\frac{1+\beta}{1-\beta}\right)\right], \qquad \beta = \sqrt{1 - \frac{4m_{\tilde{t}_i}}{\hat{s}}},$$

$$\delta^{int}_{soft} = \left[\ln\frac{4(\Delta E)^2}{\hat{s}} - \ln\frac{\lambda^2}{\hat{s}}\right]\ln\left(\frac{1-\beta\cos\theta}{1+\beta\cos\theta}\right) - \mathrm{Li}_2\left(1 - \frac{1-\beta}{1-\beta\cos\theta}\right)$$
$$- \mathrm{Li}_2\left(1 - \frac{1+\beta}{1-\beta\cos\theta}\right) + \mathrm{Li}_2\left(1 - \frac{1-\beta}{1+\beta\cos\theta}\right) + \mathrm{Li}_2\left(1 - \frac{1+\beta}{1+\beta\cos\theta}\right).$$

Collinear Singularities in Detail

- **Approx.** of partonic cross section in the **collinear cones** ($p_\gamma = (1-z)p_a$):

$$d\hat{\sigma}_{coll}(\hat{s}) = \frac{\alpha}{\pi} e_q^2 \int_0^{1-\delta_s} dz \; d\hat{\sigma}_0^{q\bar{q}}(\hat{s}) \; \kappa_{coll}(z), \qquad \kappa_{coll}(z) = \frac{1}{2} P_{qq}(z) \left[\ln\left(\frac{\tilde{s}}{m_q^2} \frac{\delta_\theta}{2} \right) - 1 \right] + \frac{1}{2}(1-z),$$

- **redefinition of PDFs** at NLO QED:

$$f_{a/A}(x) \rightarrow f_{a/A}(x,\mu_F) + f_{a/A}(x,\mu_F) \frac{\alpha}{\pi} e_q^2 \, \kappa_{soft}^{PDF} + \frac{\alpha}{\pi} e_q^2 \int_x^{1-\delta_s} \frac{dz}{z} f_{a/A}\left(\frac{x}{z}, \mu_F\right) \kappa_{coll}^{PDF}(z)$$

with

$$\kappa_{soft}^{PDF} = -1 + \ln\delta_s + \ln^2\delta_s - \ln\left(\frac{\mu_F^2}{m_q^2}\right) \left[\frac{3}{4} + \ln\delta_s\right] + \frac{1}{4}\lambda_{sc}\left[9 + \frac{2\pi^2}{3} + 3\ln\delta_s - 2\ln^2\delta_s\right],$$

$$\kappa_{coll}^{PDF}(z) = \frac{1}{2} P_{qq}(z) \left[\ln\left(\frac{m_q^2 (1-z)^2}{\mu_F^2} \right) + 1 \right] - \frac{1}{2}\lambda_{sc} \left[P_{qq}(z) \ln\frac{1-z}{z} - \frac{3}{2}\frac{1}{1-z} + 2z + 3 \right].$$

- at hadronic level: mass singularities in $\kappa_{coll} + \kappa_{coll}^{PDF}$ cancel
- κ_{soft}^{PDF} cancels remaining mass singularities owing to soft photons

$\sim$ [Baur, Keller, Wackeroth ’99], [Diener, Dittmaier, Hollik ’04]