Hard Multi-Jet Predictions using High Energy Factorisation

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Durham
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What, Why, How?

What?

Develop a framework for reliably calculating many-parton rates inclusively (ensemble of $2, 3, 4, \ldots$ parton rates) and in a flexible way (jets, W+jets, Higgs+jets,. . .)

Why?

$(n+1)$-jet rate not necessarily small compared to n-jet rate
Need inclusive perturbative corrections for realistic jet studies

How?

Factorisation of QCD Amplitudes in the High Energy Limit.
New Technique. Validation.

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Why Study Multi-jet Observables?

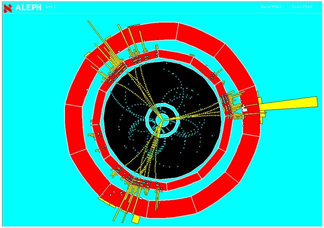
What is a jet (-algorithm)?

Organisational principle for events, which allows for a relation between the perturbative calculations with a few, hard partons (theory) and the many-hadron events observed in experiments.

Why Study Multi-jet Observables?

What is a jet?

- Experimentally: Collimated spray of (colour s.) particles
- Theoretically:
  - LO: A single coloured particle (parton–hadron duality)
  - NLO: Possibly two particles
  - Parton Shower and Hadronisation MC (e.g. Herwig): Collimated spray of (colour singlet) particles



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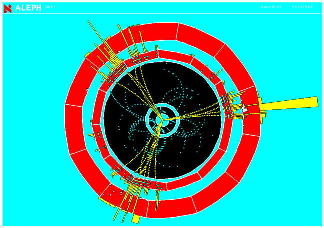

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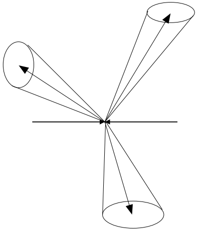

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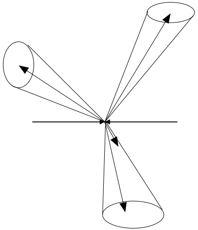
     But tends to describe only few hard jets



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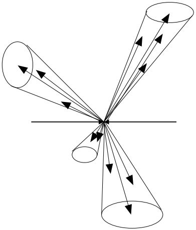

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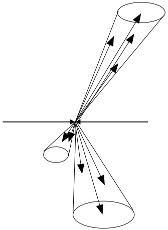

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The current discussion is independent on the exact jet-definition (*kt*, *SIS*cone,. . .), although some reasonable (i.e. IR-safe) algorithm obviously is necessary to guarantee the relation between theoretical calculation and experimental observation

Why Study Multi-jet Observables?

We don't have a choice!

1. Many BSM (e.g. SUSY) particles will have *decay chains* involving the production of jets (e.g. lepton, 4 jets + $\not{p}_T$). Calculation of signal is easy (one process), SM contribution is very hard (several processes).
2. All LHC processes involves QCD-charged particles; sometimes the (n+1)-jet cross section is as large as the n-jet cross section!
3. It is a challenge we cannot ignore !

Why Study Multi-jet Observables?

Just a few important examples

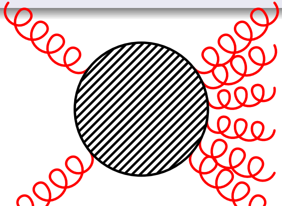
1. Pure Multi-jets
2. $W + (n >= 2)$ jets
3. Higgs + 2 jets

Will discuss how all these observables can be described in a framework tailored to the description of multiple, also (but not limited to) hard gluon emission

Why Study Multi-jet Observables?

Pure Multi-jets : High Rate

1. High rate: Possibility to look for interesting QCD effects in new corners of phase space and to further our understanding of the behaviour of field theories. (Not just looking for 2 high $p_\perp$ jets in search of quark compositeness, but now have energy for several hard jets)
2. Need to push number of both loops and legs in order to answer the necessary questions posed by experimental searches. Tree-level alone is often insufficient. Known: dijet@NLO



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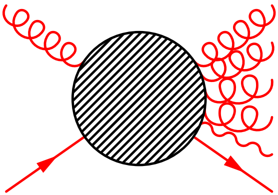
$W + (n \geq 2)$ jets

1. Important for various new physics signatures involving leptons, jets, and missing transverse energy
2. Enters on the “wish-list” for higher order calculations in preparation for LHC physics
3. Dominated by diagrams with an incoming quark at lowest order $\rightarrow$ multi-jet rates have larger relative contribution
4. Known: Wjj@NLO



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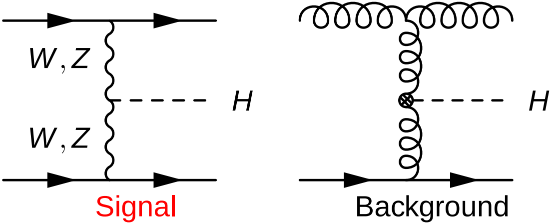
Why Study Multi-jet Observables?

$H + (n \geq 2)$ jets

1. When(!) a fundamental scalar has been found at the LHC we need to determine whether this one is responsible for the observed EWSB
2. Determine the couplings to Z or W by studying the angular distribution of the jets

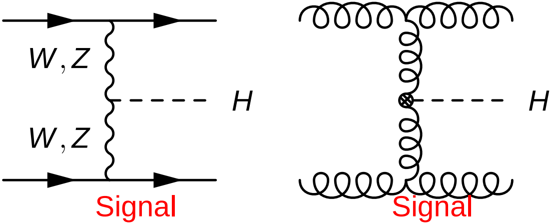


Important to understand the behaviour of the **QCD process** in order to separate **the two channels**

Why Study Multi-jet Observables?

$H + (n \geq 2)$ jets

Search for the **Higgs Boson**! May **relax** traditional Weak Boson Fusion cuts; then the QCD process can dominate. The two jets may help **give significance over background** compared to fully inclusive Higgs boson production.



Important to understand the behaviour of the QCD process in order to separate **from non-Higgs boson related background**

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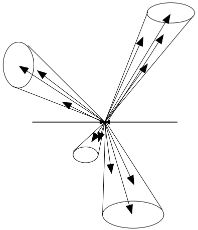


Do we need a new approach?

Already know how to calculate...

- Shower MC: at most 2→2 "hard" processes with additional parton shower
- Flexible Tree level calculators:
  MadGraph, AlpGen, SHERPA,...
  Allow most $2 \to 4$, some $2 \to 6$ processes to be calculated at tree level.
  Interfaced with Shower MC makes for a powerful mix!
- MCFM: Many relevant $2 \to 3$ processes at up to NLO (i.e. including $2 \to 4$-contribution).
- ...⟨your favourite method here⟩

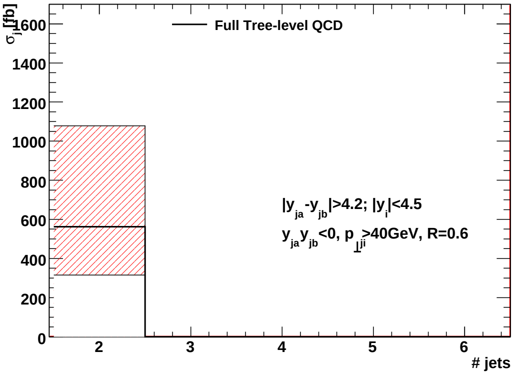
Could all be labelled “Standard Model contribution”, but give vastly different results depending on the question asked!

All Order Resummation Necessary?

Are tree-level (or generally fixed order) calculation always sufficient?

Sometimes the $(n+1)$-jet rate is as large as the n-jet rate

Higgs Boson plus n jets at the LHC at leading order



C.D. White, JRA

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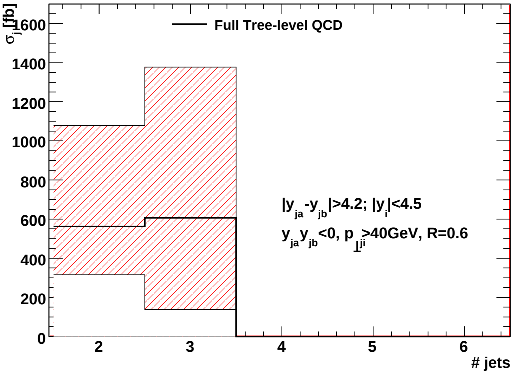

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Indication that we need to go further! However, fixed order tools **exhausted** (full $2 \rightarrow 3$ with a massive leg at two loops **untenable!**).

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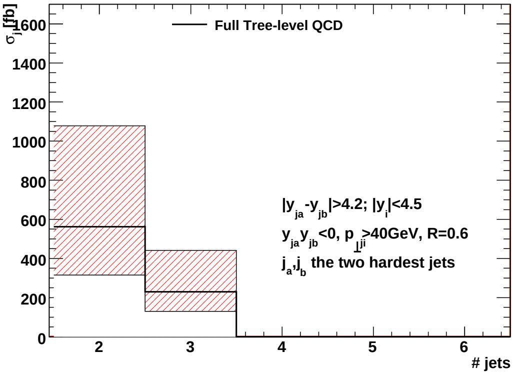

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Could require that the two jets passing the cuts are also the two hardest jets. This reduces the three-jet phase space and the higher order corrections. Sensitivity to pert. corrections?

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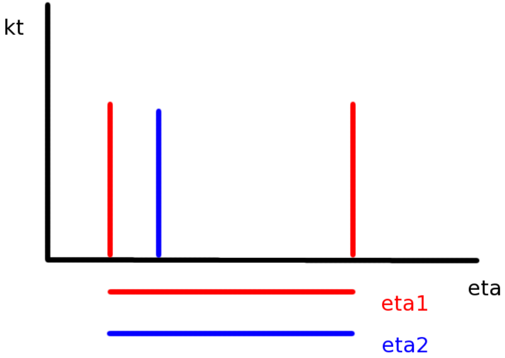

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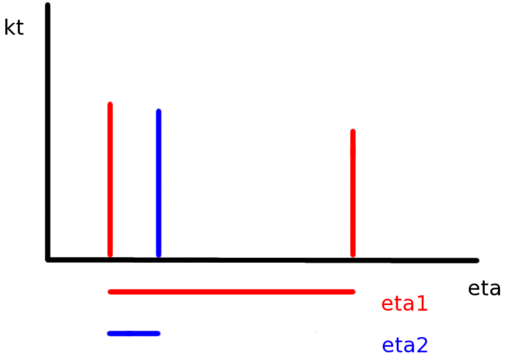

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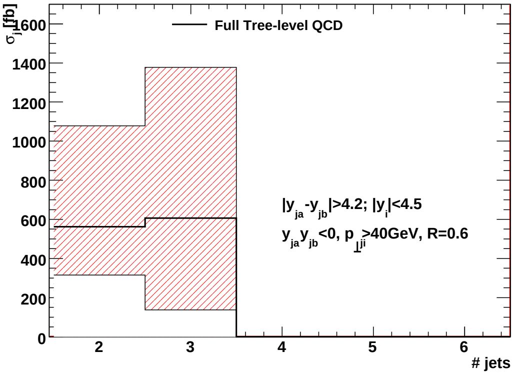

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The method we develop will be applicable to both set of cuts, but crucially will allow a **stabilisation** of the perturbative series by **resummation**

Resummation and Matching

Consider the **perturbative expansion** of an observable

$$R = r_0 + r_1\alpha_s + r_2\alpha^2 + r_3\alpha^3 + r_4\alpha^4 + \cdots$$

Fixed order pert. QCD will calculate a fixed number of terms in this expansion. r_n may contain **large logarithms** so that $\alpha_s \ln(\cdots)$ is large.

$$R = r_0 + \left(r_1^{LL}\ln(\cdots) + r_1^{NLL}\right)\alpha_s + \left(r_2^{LL}\ln^2(\cdots) + r_2^{NLL}\ln(\cdots) + r_2^{SL}\right)\alpha_s^2 + \cdots$$
$$= r_0 + \sum_n r_n^{LL}(\alpha_s\ln(\cdots))^n + \sum_n r_n^{NLL}\alpha_s(\alpha_s\ln(\cdots))^n + \text{sub-leading terms}$$

Need simplifying assumptions to get to all orders - useful **iff** the terms really do describe **the dominant part** of the **full pert. series**. **Matching** combines **best of both worlds**:

$$R = r_0 + r_1\alpha_s + r_2\alpha^2 + \left(r_3^{LL}\ln^3(\cdots) + r_3^{NLL}\ln^2(\cdots) + r_3^{SL}\right)\alpha^3 + \cdots$$

Factorisation of QCD Matrix Elements

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It is **well known** that QCD matrix elements **factorise** in certain kinematical limits:
Soft limit → **eikonal approximation** → enters all parton shower (and much else) resummation.

Like all good limits, the eikonal approximation is applied **outside its strict region of validity**.

Will discuss the **less well-studied factorisation** of scattering amplitudes in a different kinematic limit, better suited for describing perturbative corrections from **hard parton emission**

Factorisation only **becomes exact** in a region **outside** the reach of any collider. . .

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To boldly go. . .

In a previous episode of a CERN seminar series:
A wise man said. . .

"Use known results to gain deeper insights. . . "

young* postdoc

"Use insight to gain yet unknown results. . . "

It has become very fashionable to **claim** ⟨my favourite method⟩ can predict observables important for the LHC programme. Will actually **validate**** the claims

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Regge and High Energy Factorisation

In the High Energy Limit, $2 \to 2$ **scattering amplitudes** are **dominated** by the ***t*-channel exchange** of the particle of the **highest spin** allowed by the scattering theory

$$\mathcal{M}^{p_a p_b \to p_1 p_2} \xrightarrow{\text{Regge limit}} \hat{s}^{\hat{\alpha}(\hat{t})} \gamma(\hat{t})$$

Regge (1959)

$\hat{s} = (p_a + p_b)^2, \hat{t} = (p_a - p_1)^2$, Regge limit: $\hat{s} \to \infty, \hat{t}$ fixed.

Multi-particle generalisation?

$$\mathcal{M}^{p_a p_c \to p_{a'} p_b p_{c'}} \xrightarrow{\text{Multi Regge limit}} \hat{s}_1^{\hat{\alpha}(\hat{t}_1)} \hat{s}_2^{\hat{\alpha}(\hat{t}_2)} \gamma(\hat{t}_1, \hat{t}_2, \frac{s_{12}}{s_1 s_2})$$

MRK: $\hat{s}_{12}, \hat{s}_1, \hat{s}_2 \to \infty, t_1, t_2$ fixed

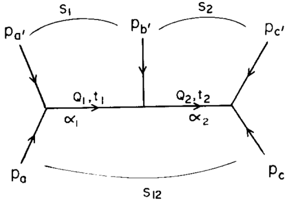
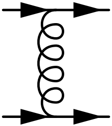
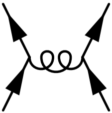
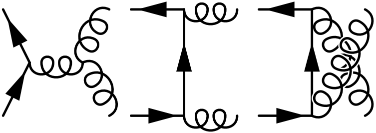


Fig. 2.1. Five-particle diagram showing notation.

Brower, DeTAR, Weis (1974)

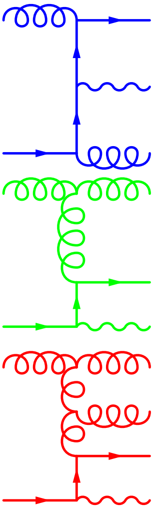
High Energy Factorisation - t–channel dominance

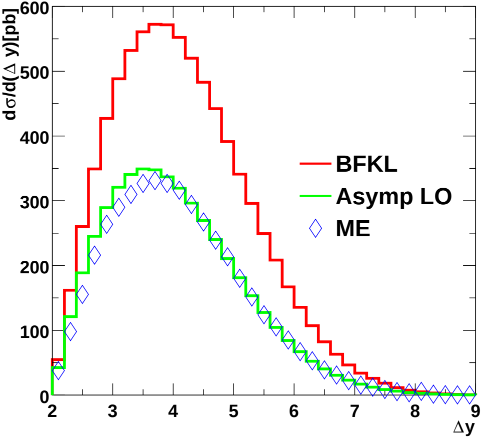
Process	Diagrams	$\overline{\sum} \lvert \mathcal{M} \rvert^2 / g^4$
$qq' \to qq'$		$\frac{4}{9} \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$
$q\bar{q} \to q'\bar{q}'$		$\frac{4}{9} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$
$q\bar{q} \to gg$		$\frac{32}{27} \frac{\hat{t}^2 + \hat{u}^2}{\hat{t}\hat{u}} - \frac{8}{3} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$

High Energy Limit: $\lvert \hat{t} \rvert$ fixed, $\hat{s} \to \infty$

t–channel dominance

Example: W+n-jet production at the LHC



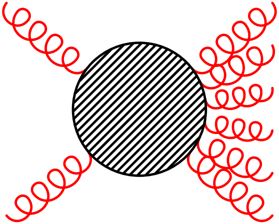


$$\Delta y = y_{j_2} - y_{j_1}, y_W, y_{j_2} \geq 1, y_{j_1} \leq -1$$

V. Del Duca, F. Maltoni, W.J. Stirling, JRA

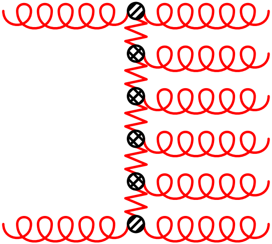
The Possibility for Predictions of *n*-jet Rates

The Power of Reggeisation



High Energy Limit $\longrightarrow$

$|\hat{t}|$ fixed, $\hat{s} \to \infty$



$$\mathcal{A}^R_{2\to 2+n} = \frac{\Gamma_{A'A}}{q_0^2}\left(\prod_{i=1}^{n} e^{\omega(q_i)(y_{i-1}-y_i)}\frac{V^{J_i}(q_i,q_{i+1})}{q_i^2 q_{i+1}^2}\right) e^{\omega(q_{n+1})(y_n-y_{n+1})}\frac{\Gamma_{B'B}}{q_{n+1}^2}$$

$q_i = \mathbf{k}_a + \sum_{l=1}^{i-1} \mathbf{k}_l$

LL: Fadin, Kuraev, Lipatov; NLL: Fadin, Fiore, Kozlov, Reznichenko

Maintain (at LL) terms of the form

$$\left(\alpha_s \ln \frac{\hat{s}_{ij}}{|\hat{t}_i|}\right)$$

to all orders in α_s.

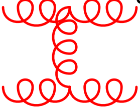
At LL only gluon production; at NLL also quark–anti-quark pairs produced.

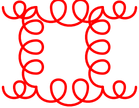
Approximation of any-jet rate possible.

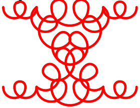
FKL at Leading Logarithmic Accuracy

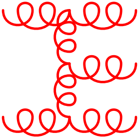
Fadin, Kuraev, Lipatov

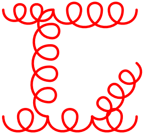
Which diagrams contribute beyond lowest order?





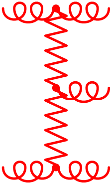






etc.

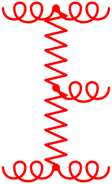
All these contributions can be calculated using **effective vertices** and propagators for the **reggeized gluon**.



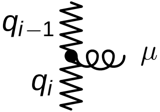
General form proved using s-channel unitarity and a set of bootstrap relations NLL: Fadin, Fiore, Kozlov, Reznichenko

FKL formalism (Fadin, Kuraev, Lipatov)

FKL: Identification of the **dominant contributions** to the **perturbative series** for processes with two large (perturbative) and disparate energy scales $\hat{s} \gg |\hat{t}|$ ($\hat{s}$: $E^2_{\rm cm}$, $\hat{t}$: $p^2_\perp$)



$$\frac{1}{q^2}\exp\left(\hat{\alpha}(q)\Delta y\right)$$



$$C^\mu_L(q_{i-1}, q_i)$$

$$C^\mu_L(q_{i-1}, q_i) = \left[-(q_i + q_{i+1})^{\mu_i} + p^\mu_a\left(\frac{q_i^2}{k_i.p_1} + 2\frac{k_i.p_b}{p_a.p_b}\right) - p_b\left(\frac{q_{i+1}^2}{k_i.p_b} + 2\frac{k_i.p_a}{p_a.p_b}\right)\right]$$

Framework exact in the limit of Multi Regge Kinematic (MRK)

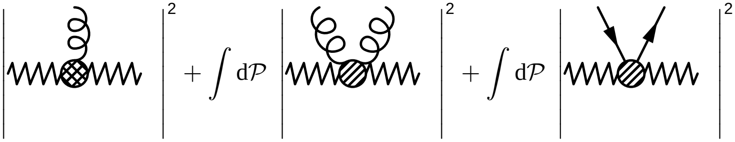
$$y_0 \gg y_1 \gg \ldots \gg y_n, \quad |k_{i\perp}| \approx |k_{j\perp}|, \quad q_i^2 \approx q_j^2$$

Reproduces the **MHV** Parke-Taylor **amplitudes** in the **High Energy Limit**

Checking effective vertices

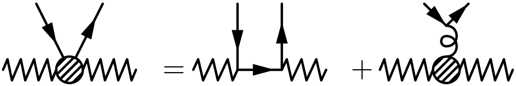
The Ingredients of the NLL Vertex

$V(\mathbf{q}_1, \mathbf{q}_2) =$

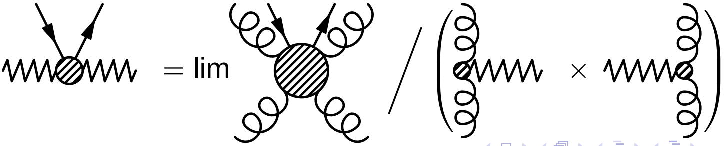


Two methods for obtaining the vertices at NLL:

- Fadin & Lipatov:

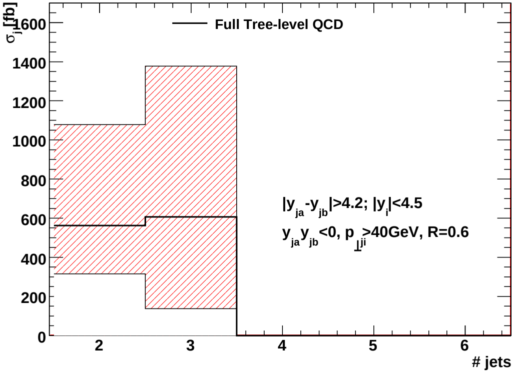


- V. Del Duca:



Case study and Validation

Tree level results for $pp \to Higgs + \text{jets}$



Necessary to understand multi-emission topologies in order to

- cleanly extract WBF signal (c. jet veto, angular dist. of jets,. . .)
- use H+jets as a discovery channel

Previous studies of Higgs Boson plus jets

- hjj@full NLO: Increase in cross section over LO estimate of factors 1.2-1.3 or 1.7-1.8 depending on cuts (note: discussion of K-factors not really useful for a multi-scale problem).

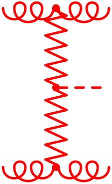
  J. Campbell, K. Ellis, G. Zanderighi

- hjj@LO+parton showers: Focus on effects of soft and collinear radiation to all orders. Find significant effects beyond NLO.

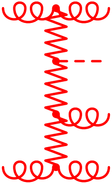
  V. Del Duca, G. Klämke, D. Zeppenfeld, M.L. Mangano, M. Moretti, F. Piccinini, R. Pittau, A.D. Polosa

Will focus on **developing a framework** which captures an alternative part of the **perturbative series to all orders** (**not relying** on soft and collinear factorisation) - and **compare it order by order** to the full result where known.

Higgs Boson plus $n \geq 2$ jets in the HE limit



Extract the effective Higgs Boson vertex using the method of VDD



Only two diagrams contribute to the process Higgs Boson plus 3 jets in the High Energy Limit!

Some contributions have suppressed HE limit. . .

$pp \to h + \text{jets}$ with vanishing HE limit

sub-processes **not contributing at all** in the HE limit:

$u\bar{u} \to ghg(g)$, $gg \to uh\bar{u}(g)$

or not in **special rapidity configurations** (at LL):

$gu \to uhg, ud \to dhu$, $gu \to ghug, \ldots$

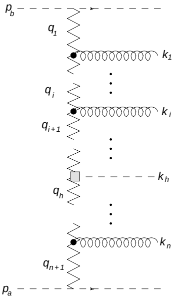
Total contribution from full QCD ME of these contributions:

$\sigma_{hjj}^{\mathrm{non-FKL-conf.}} < 0.3\%$

$\sigma_{hjjj}^{\mathrm{non-FKL-conf.}} < 10\%$ (most will be captured by allowing one t-channel quark propagator)

Contributes less than 10% of the cross section. The HE limit will approximate the remaining configurations (will later add back the missing pieces by matching to the fixed order results)

The Scattering Amplitude



$$
\begin{aligned}
i\mathcal{M}_{\text{HE}}^{ab\to p_0\ldots p_j h p_{j+1} p_n} &= 2i\hat{s} \\
&\cdot \left(i g_s f^{a d_0 c_1} g_{\mu_a \mu_0}\right) \\
&\cdot \prod_{i=1}^{j} \left(\frac{1}{q_i^2} \exp[\hat{\alpha}(q_i^2)(y_{i-1} - y_i)] \left(i g_s f^{c_i d_i c_{i+1}}\right) C_{\mu_i}(q_i, q_{i+1}) \right) \\
&\cdot \left(\frac{1}{q_h^2} \exp[\hat{\alpha}(q_i^2)(y_j - y_h)] C_H(q_{j+1}, q_h) \right) \\
&\cdot \prod_{i=j+1}^{n} \left(\frac{1}{q_i^2} \exp[\hat{\alpha}(q_i^2)(y'_{i-1} - y'_i)] \left(i g_s f^{c_i d_i c_{i+1}}\right) C_{\mu_i}(q_i, q_{i+1}) \right) \\
&\cdot \frac{1}{q_{n+1}^2} \exp[\hat{\alpha}(q_{n+1}^2)(y'_n - y_b)] \left(i g_s f^{b d_{n+1} c_{n+1}} g_{\mu_b \mu_{n+1}}\right)
\end{aligned}
$$

Have: **exact** result in the **very exclusive limit** of **infinite separation** between **all particles**

Want: **inclusive** cross sections. . .

The Traditional Implementation Using the BFKL Eqn*

Adding one emission $\rightarrow$ emergence of extra factor in $|\mathcal{M}|^2$ of

$$\frac{-C^{\mu_i} \cdot C_{\mu_i}}{t_i \; t_{i+1}} \xrightarrow{\text{Ultimate MRK limit}} \frac{4}{p_{i\perp}^2}$$

Taking into account contraction of colour factors, the addition of an emission leads to the following factor in the colour and spin summed and averaged square of the matrix element

$$\frac{4 \; g_s^2 \; C_A}{p_{i\perp}^2}$$

Only **transverse degrees of freedom** left!

*Now is a good time to take a nap - in a few minutes I will ask you to forget all about the BFKL eqn.

The Traditional Implementation Using the BFKL Eqn*

$$\left|\mathcal{M}^{gg\to hgg}\right|^2 = \frac{4\hat{s}^2}{N_c^2-1}\frac{C_A g_s^2}{p_{0\perp}^2}\left|C^H_{HEL}\left(-p_{0\perp},p_{1,\perp}\right)\right|^2\frac{C_A g_s^2}{p_{1\perp}^2}$$

$$\left|\mathcal{M}^{gg\to hggg}\right|^2 = \frac{4\hat{s}^2}{N_c^2-1}\frac{C_A g_s^2}{p_{0\perp}^2}\left|C^H_{HEL}\left(q_{a\perp},q_{b,\perp}\right)\right|^2\frac{4\,C_A g_s^2}{p_{1\perp}^2}\frac{C_A g_s^2}{p_{2\perp}^2}$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

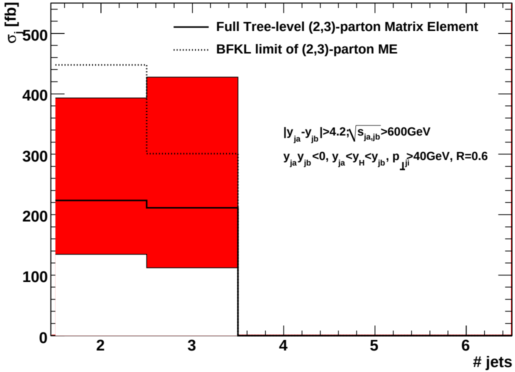
$$\frac{d\hat{\sigma}_{gg\to g\cdots h\cdots g}}{dp_{a\perp}^2\,dy_a\;dp_{b\perp}^2\,dy_b\;dp_{H\perp}^2\,dy_H} = \int d^2q_{a\perp}d^2q_{b\perp}\left(\frac{\alpha_s\,N_c}{p_{a\perp}^2}\right)f(-p_{a\perp},q_{a,\perp},\Delta y_{aH})$$

$$\cdot\left|C^H_{HEL}\left(q_{a,\perp},q_{b,\perp}\right)\right|^2 f(q_{b\perp},p_{b,\perp},\Delta y_{Hb})\left(\frac{\alpha_s\,N_c}{p_{b\perp}^2}\right)$$

$$\text{BFKL eqn}: \frac{\partial}{\partial y}f(q_a,q_b,y) = \int \mathrm{d}^{D-2}\mathbf{q}\,K(q_a,\mathbf{q})\,f(\mathbf{q},q_b,y)$$

Applies **factorisation** and **kinematic approximations** in **all of phase space**

Comparison between BFKL and Full Matrix Element



C.D. White, JRA

Not convincing*. Can obviously match to FO, but better also improve resum^{n}!

---

*And this is even the energy and momentum conserving variant of BFKL - please ask about this point if you want to see something crazy. It is actually a very important point.

Improving the Framework*

Start again from the FKL amplitudes:

$$i\mathcal{M}_{\rm HE}^{ab\to p_0\ldots p_j h p_{j+1} p_n} = 2i\hat{s}\cdots\prod_{i=1}^{j}\left(\frac{1}{q_i^2}\exp[\hat{\alpha}(q_i^2)(y_{i-1}-y_i)]\left(ig_s f^{c_i d_i c_{i+1}}\right) C_{\mu_i}(q_i,q_{i+1})\right)\cdots$$

Instead of extending the kinematic approximations valid in the MRK limit to all of phase space, impose the right analytic behaviour away from the MRK limit

1. Position of Divergences
2. Gauge invariance (also in sub-MRK region)

*Now would be a good time to wake up. Any time now. Please.

Improving the Framework*

Start again from the FKL amplitudes:

$$i\mathcal{M}_{\mathrm{HE}}^{ab\to p_0\dots p_j h p_{j+1} p_n} = 2i\hat{s}\cdots\prod_{i=1}^{j}\left(\frac{1}{q_i^2}\exp[\hat{\alpha}(q_i^2)(y_{i-1}-y_i)]\left(ig_s f^{c_i d_i c_{i+1}}\right)C_{\mu_i}(q_i,q_{i+1})\right)\cdots$$

Instead of extending the kinematic approximations valid in the MRK limit to all of phase space, impose the right analytic behaviour away from the MRK limit

1. Position of Divergences
2. Gauge invariance (also in sub-MRK region)

The full scattering amplitude is divergent for several momentum configurations, for which the BFKL approximations of is finite. These divergences obviously lie explicitly outside of MRK. However, we choose to re-instate several of these divergences by using **the full momentum dependence of all invariants**. Result differ from the BFKL equivalent only in the sub-MRK region.

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Improving the Framework*

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Instead of extending the kinematic approximations valid in the MRK limit to all of phase space, impose the right analytic behaviour away from the MRK limit

1. Position of Divergences
2. Gauge invariance (also in sub-MRK region)

Using full expression for propagators **automatically takes into account the dominant source of NLL corrections** to *any* logarithmic accuracy. NLL corrections to Lipatov Vertex C^μ starts to address the dependence on longitudinal momenta between two neighbouring partons. We can restore the **full** propagator between all gluons.

*Now would be a good time to wake up. Any time now. Please.

Improving the Framework*

Start again from the FKL amplitudes:

$$i\mathcal{M}_{\mathrm{HE}}^{ab\to p_0\ldots p_j h p_{j+1} p_n} = 2i\hat{s}\cdots\prod_{i=1}^{j}\left(\frac{1}{q_i^2}\exp[\hat{\alpha}(q_i^2)(y_{i-1}-y_i)]\left(ig_s f^{c_i d_i c_{i+1}}\right)C_{\mu_i}(q_i,q_{i+1})\right)\cdots$$

Instead of extending the kinematic approximations valid in the MRK limit to all of phase space, impose the right analytic behaviour away from the MRK limit

1. Position of Divergences
2. Gauge invariance (also in sub-MRK region)

Choose form of Lipatov vertex satisfying $C^\mu k_\mu = 0$. The gauge terms are automatically suppressed in the HE limit, but we are seeking a form which works *everywhere*. Simultaneously ensures amplitude is positive definite.

* Now would be a good time to wake up. Any time now. Please.

Improving the Framework*

Start again from the FKL amplitudes:

$$i\mathcal{M}_{\mathrm{HE}}^{ab\to p_0\cdots p_j h p_{j+1} p_n} = 2i\hat{s}\cdots\prod_{i=1}^{j}\left(\frac{1}{q_i^2}\exp[\hat{\alpha}(q_i^2)(y_{i-1}-y_i)]\left(ig_s f^{c_i d_i c_{i+1}}\right)C_{\mu_i}(q_i,q_{i+1})\right)\cdots$$

Instead of extending the kinematic approximations valid in the MRK limit to all of phase space, impose the right analytic behaviour away from the MRK limit

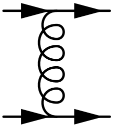
1. Position of Divergences
2. Gauge invariance (also in sub-MRK region)

These two constrains the subasymptotic form of the amplitude (and obviously does not alter the asymptotic form). Approximates the full results well where known. Sufficiently simple to allow an all-order resummation.

* Now would be a good time to wake up. Any time now. Please.

What basically does this amount to?

Consider again $qq' \to qq'$ scattering:

Process	Diagrams	$\overline{\sum}\lvert\mathcal{M}\rvert^2/g^4$
$qq' \to qq'$		$\frac{4}{9}\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$

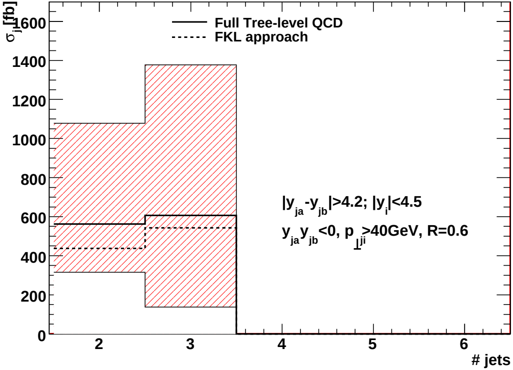
$$\hat{s}+\hat{t}+\hat{u}=0, i.e.\ \hat{u}=-(\hat{s}+\hat{t}), \hat{u}^2=\hat{s}^2+\hat{t}^2+2\hat{s}\hat{t}.$$

$$\text{BFKL result}: \frac{4}{9}\frac{2\hat{s}^2+\hat{t}^2+2\hat{s}\hat{t}}{\hat{t}^2} \to \frac{8}{9}\frac{\hat{s}^2}{k_\perp^4}$$

$$\text{FKL result}: \frac{4}{9}\frac{2\hat{s}^2+\hat{t}^2+2\hat{s}\hat{t}}{\hat{t}^2} \to \frac{8}{9}\frac{\hat{s}^2}{\hat{t}^2}$$

Formalism re-introduces the right position of divergences in the sub-MRK region to all orders.

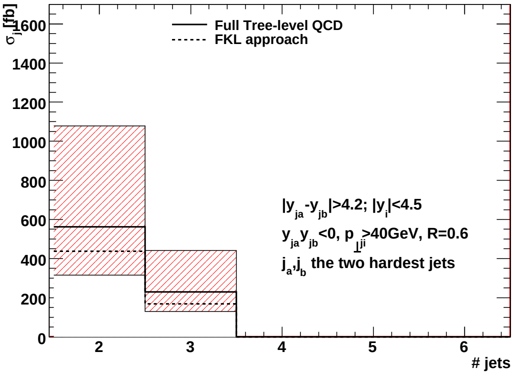
Comparison between FKL and Full Matrix Element



V. Del Duca, C. White, JRA

Difference between FKL (2 diagrams) and full result (10^3 diagrams) is much less than the renormalisation and factorisation scale uncertainty. Repair with matching corrections.

Comparison between FKL and Full Matrix Element



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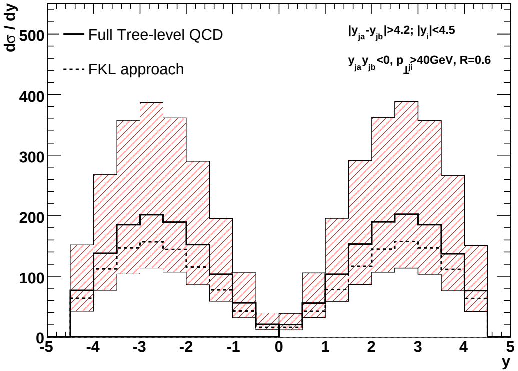

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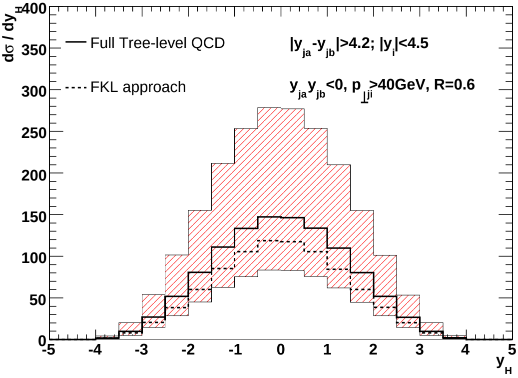

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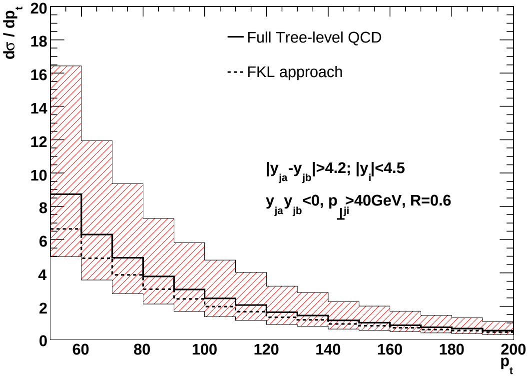

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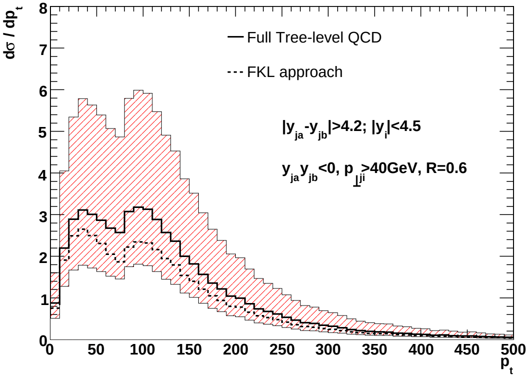

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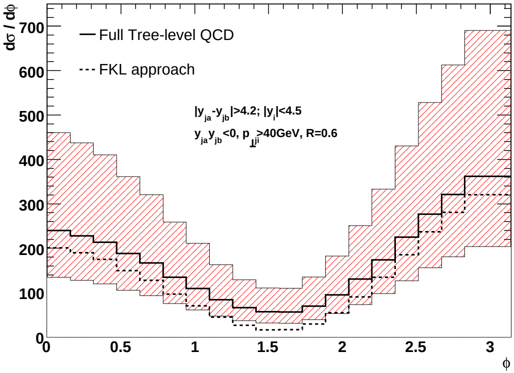

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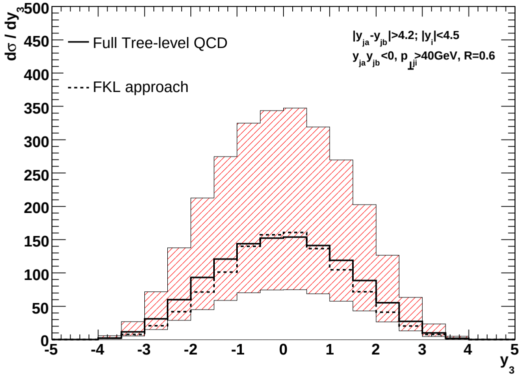

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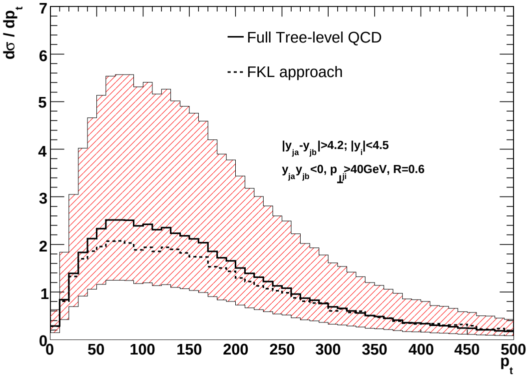

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Beyond validation. . .

Have so far demonstrated that the terms we can take into account reproduce the full tree level results to within $10 - 25\%$ where ever these are known - and reproduce distributions.

Can calculate this approximation for the tree-level Higgs Boson plus *n*-parton amplitude, and include also the corresponding virtual corrections. Can thereby form the inclusive *any*-parton sample (i.e. LO: only H+2 partons, NLO: H+2 and H+3 partons, . . .)

Fully exclusive in all particles - Can perform any analysis using your favourite jet algorithm*

* . . . or all of them. . .

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Performing the Explicit Resummation

Soft divergence from real radiation:

$$\left|\mathcal{M}_{\mathrm{HE}}^{p_a p_b \to p_0 p_1 p_h p_2}\right|^2 \xrightarrow{\mathbf{p}_1^2 \to 0} \left(\frac{4 g_s^2 C_A}{\mathbf{p}_1^2}\right) \left|\mathcal{M}_{\mathrm{HE}}^{p_a p_b \to p_0 p_h p_2}\right|^2$$

Integrate over the soft part $\mathbf{p}_i^2 < \lambda^2$ of phase space in $D = 4 + 2\varepsilon$ dimensions

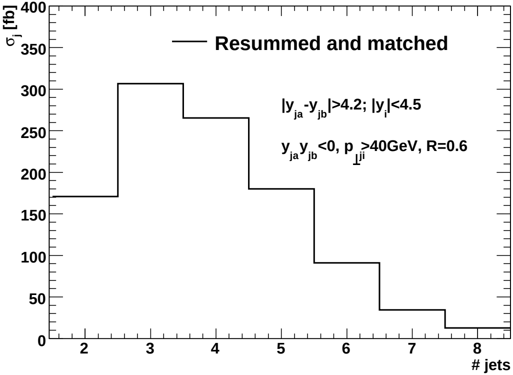
$$\int_0^\lambda \frac{\mathrm{d}^{2+2\varepsilon}\mathbf{p}\ \mathrm{d}y_1}{(2\pi)^{2+2\varepsilon}\ 4\pi} \left(\frac{4 g_s^2 C_A}{\mathbf{p}^2}\right) \mu^{-2\varepsilon}$$
$$= \frac{4 g_s^2 C_A}{(2\pi)^{2+2\varepsilon} 4\pi} \Delta y_{0h} \frac{\pi^{1+\varepsilon}}{\Gamma(1+\varepsilon)} \frac{1}{\varepsilon} (\lambda^2/\mu^2)^\varepsilon$$

Pole in ε cancels with that from the virtual corrections

$$\hat{\alpha}(t) = -\frac{g_s^2 C_A \Gamma(1-\varepsilon)}{(4\pi)^{2+\varepsilon}} \frac{2}{\varepsilon} \left(\mathbf{q}^2/\mu^2\right)^\varepsilon.$$

FKL All Order Resummation Incl. Matching

σ_{hjj}^{LO} : 562fb; $\sigma_{hjj}^{\text{resummed+matched}}$: 1050fb

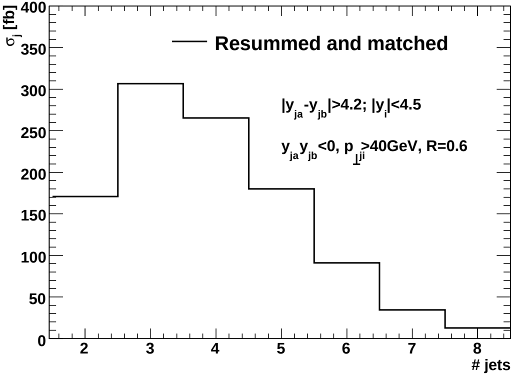


C.D. White, JRA

Can sum over n-parton inclusive samples (both real and virtual contributions included). Matching to the tree level n-parton matrix elements.

FKL All Order Resummation Incl. Matching

σ^{LO}_{hjj} : 562fb; $\sigma^{\text{resummed+matched}}_{hjj}$: 1050fb



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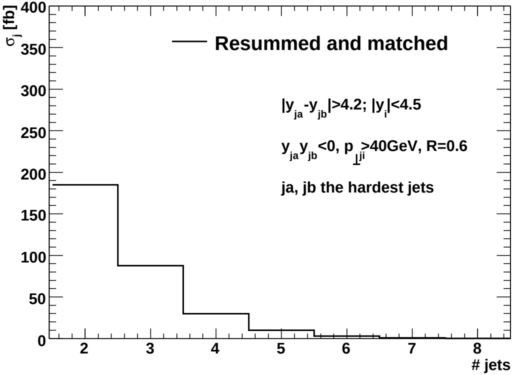
Significant jet activity

FKL All Order Resummation Incl. Matching

σ_{hjj}^{LO} : 562fb; $\sigma_{hjj}^{\text{resummed+matched}}$: 316fb



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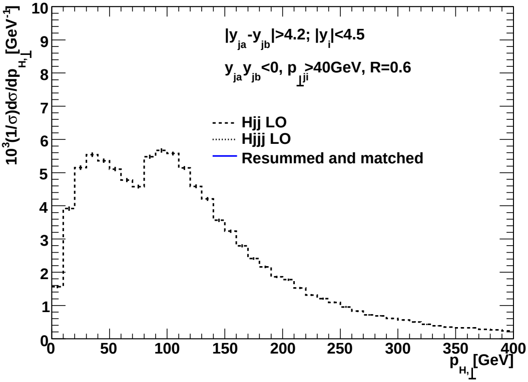
Most events are rejected because of the central jet activity

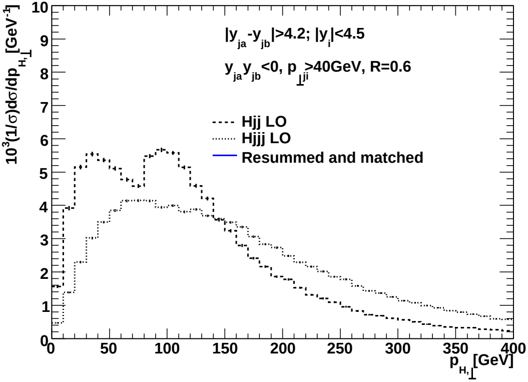
Impact on Observables



C.D. White, JRA

Strong features of Higgs boson transverse momentum spectrum (caused by strong azimuthal correlation coupled with cuts on jets) disappears at higher orders.

Impact on Observables



C.D. White, JRA

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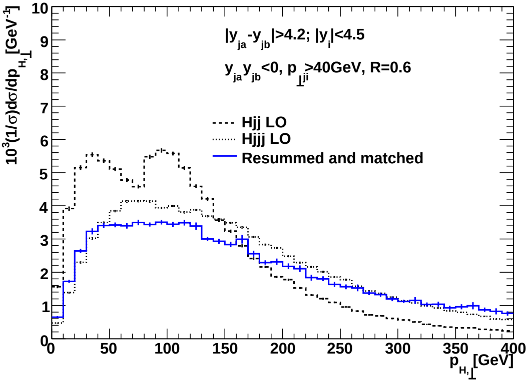


C.D. White, JRA

Strong features of Higgs boson transverse momentum spectrum (caused by strong azimuthal correlation coupled with cuts on jets) disappears at higher orders.

Outlook and Conclusions

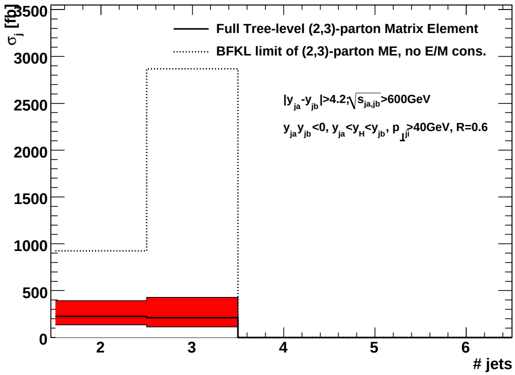
Conclusions

- Emerging framework for the study of processes with multiple hard jets
- Working implementation, including matching to the known fixed order results; public code available
- Impact many studies: jet correllations,...
- Les Houches Interface to study effects of showering

Outlook

- Implement other processes and test against Tevatron Data
- Include t-channel quark propagators (include more partonic channels)
- Matching to shower algorithms
- ...

Thank you for asking that question. . .



Formulation valid for $\hat{s} \to \infty$, $|t|$ fixed. But $\hat{s} < s$ fixed at any collider! E/M conserv. not just “subleading corrections” in partonic scattering, but stops the evolution all together (even before the strict MRK limit is reached!).