technicolor on the lattice

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Dynamical Electroweak Symmetry Breaking

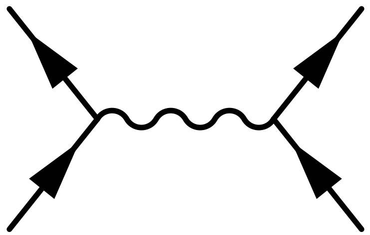
- mechanism for EWSB $\longrightarrow$ hierarchy problem: $\Lambda_{\mathrm{EW}} \ll \Lambda_{\mathrm{UV}}$
- elegant solution: dynamical EW symmetry breaking [weinberg, susskind]
  $\hookrightarrow$ new strong interaction: $\Lambda_{\mathrm{EW}} \sim \Lambda_{\mathrm{UV}} e^{-g_c^2/g_{\mathrm{UV}}^2}$
- mechanism already realized in QCD
- EW sector strongly coupled at TeV scale $\simeq 4\pi v$
- new resonances $\mathcal{O}(1\ \mathrm{TeV})$
- TC Goldstone bosons $\longrightarrow$ longitudinal EW gauge dof

Problems with technicolor

- too much freedom: flavors, colors, representations?
- difficult to compute due to strong dynamics
- precision EW data: do **not** rule out DEWSB
- **do** rule out QCD–like behaviour
- but **no** large number N of degrees of freedom
  $\hookrightarrow$ large technicolor, Randall–Sundrum models
- flavor physics from TC: extended TC

Flavor physics

- ETC: mass term for the SM fermions, FCNC



$$\longrightarrow \qquad \Delta\mathcal{L} \propto \frac{1}{M_{\mathrm{ETC}}^2}\mathcal{O}_{\mathrm{ETC}}\,\bar{\psi}\psi,\; \frac{1}{M_{\mathrm{ETC}}^2}\bar{\psi}\psi\,\bar{\psi}\psi$$

- running of $\mathcal{O}$: **(near) conformal behaviour**

$$\boxed{\mathcal{O}_{\mathrm{ETC}} = \mathcal{O}_{\mathrm{TC}}\, \exp\left(\int_{\Lambda_{\mathrm{TC}}}^{M_{\mathrm{ETC}}} \frac{d\mu}{\mu}\,\gamma(\mu)\right)}$$

Flavor physics

- QCD–like technicolor: $m \sim \Lambda_{\rm TC}^3/M_{\rm ETC}^2$

$$\Lambda_t \sim \Lambda_{\rm TC}\left(\frac{\Lambda_{\rm TC}}{m_t}\right)^{1/2} \sim 5\ {\rm TeV}$$

- non QCD–like technicolor: $m \sim \Lambda_{\rm TC}^d/M_{\rm ETC}^{d-1}$

$$\Lambda_t \sim \Lambda_{\rm TC}\left(\frac{\Lambda_{\rm TC}}{m_t}\right)^{1/(d-1)}$$

- technicolor + ladder: $d = 2, \gamma = 1$; smaller values of d

- conformal technicolor [luty 04]

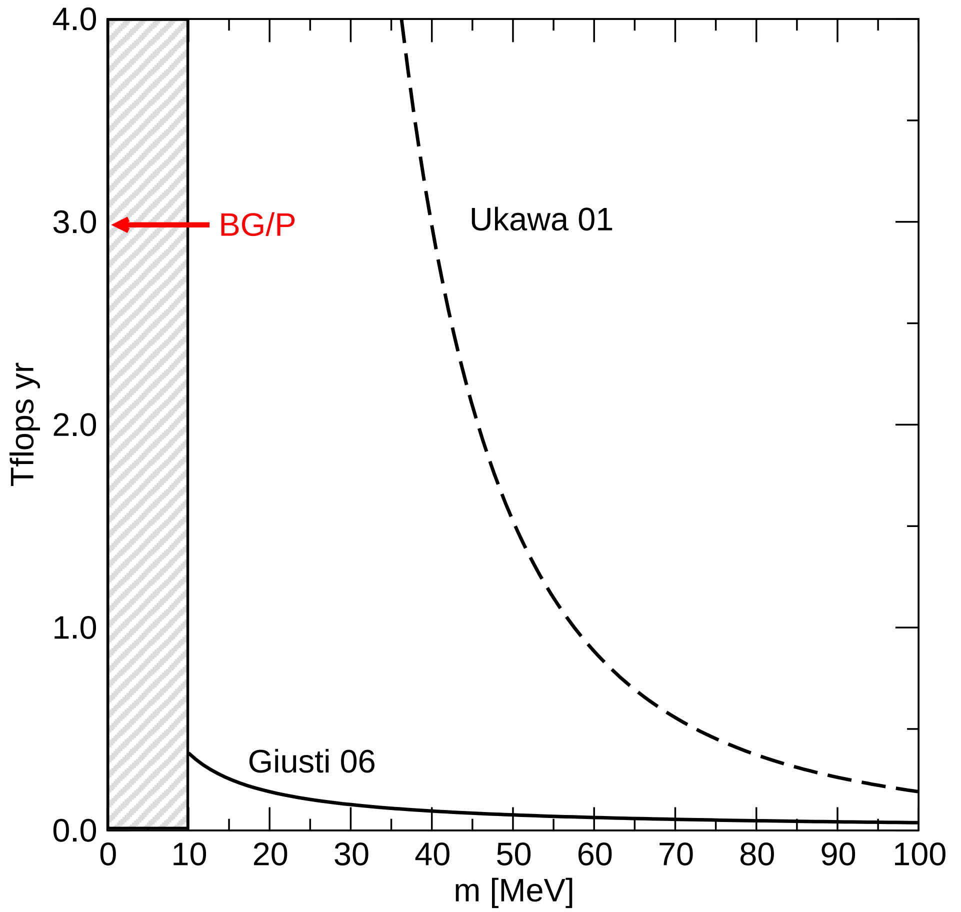
DEWSB @ LHC

- collider phenomenology via effective theories
- effective lagrangian for the low–lying states
  LHC observables in terms of the LEC [giudice et al, sannino et al]
- lattice simulations provide first principle calculations
  $\hookrightarrow$ phase structure of the theory, spectrum, LEC
- light dynamical fermions play a key role for non QCD–like behaviour
- **understand the effect of systematic errors in lattice results**

Progress with dynamical fermions

- algorithmic developments
  [hasenbusch 01, urbach et al 05, luscher 03ff, 07, kennedy et al 05]

- program efficiency & faster computers
  [PC clusters, APE, QCDOC $\longrightarrow$ BG]



Numerical studies can now access the chiral regime of QCD, at large volumes and small lattice spacing using Wilson fermions.

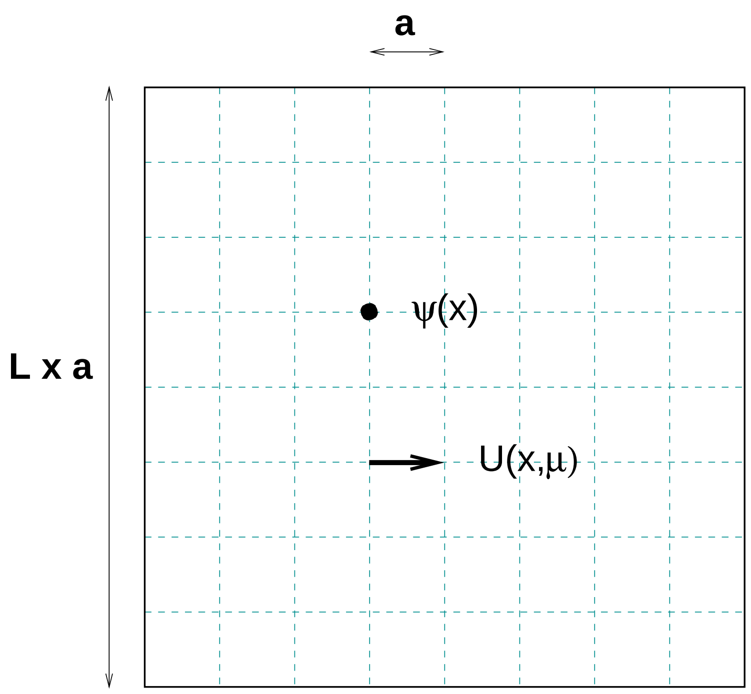
New scope for lattice QFT

- simulations with **light**, **dynamical** fermions w/out compromising the robustness of the formulation
- lattice simulations: quantitative tool for NP QFT
- dynamics should be different from QCD + we don't know the answer!!

SU(N) gauge theories with fermions in higher–dimensional reps

- definition and implementation
- large-N limit with fundamental fermions [ldd et al 07]
- technicolor theories [catterall et al, svetitsky et al]
- planar orientifold equivalence [armoni et al 03ff]
- lattice SUSY

Lattice formulation



L x L sites

lattice spacing: a [fm]

Physical size: L x a [fm]

$$S(U,\psi,\bar{\psi}) = S_g(U) + a^4 \sum_{x,y} \bar{\psi}(x) D(x,y) \psi(y)$$

$$\boxed{Z = \int dU \, \exp\left[-S_g(U)\right] \, (\det D)^{n_f}}$$

Higher representations

Wilson–Dirac operator:

$$D = \frac{1}{2}\sum_{\mu}\left[\gamma_\mu\left(\nabla_\mu + \nabla^*_\mu\right) - a\nabla^*_\mu\nabla_\mu\right] + m\,,$$

i.e.

$$\begin{aligned}\sum_y D(x,y)\psi(y) \quad = \quad & -\frac{1}{2a}\Big\{\sum_\mu\Big[(1-\gamma_\mu)\,U^R(x,\mu)\psi(x+\mu)+ \\ & (1+\gamma_\mu)\,U^R(x-\mu,\mu)^\dagger\psi(x-\mu)\Big] - (8+2am)\psi(x)\Big\},\end{aligned}$$

$$\kappa = 1/(8+2am)$$

Link variables:

$$\begin{aligned}U(x,\mu) \quad &= \quad \exp\left[i\,A^a_\mu(x)\,T^a_f\right], \quad N\times N \text{ matrix} \\ U^R(x,\mu) \quad &= \quad \exp\left[i\,A^a_\mu(x,\mu)\,T^a_R\right], \quad d_R\times d_R \text{ matrix}\end{aligned}$$

HMC for higher representations

[ldd, patella, pica 08]

Including the fermionic determinant:

$$Z = \int \mathcal{D}U\mathcal{D}\phi\mathcal{D}\phi^* \exp[-S_G - \|Q^{-1}\phi\|^2]$$
$$= \int \mathcal{D}\Pi\mathcal{D}U\mathcal{D}\phi\mathcal{D}\phi^* \exp[-\frac{1}{2}(\Pi,\Pi) - S_G - \|Q^{-1}\phi\|^2], \quad Q = \gamma_5 D$$

Molecular dynamics evolution:

$$\frac{d}{dt}\Pi(x,\mu) = -\frac{\delta\mathcal{H}}{\delta U(x,\mu)} = \sum_k F_k(x,\mu)$$
$$\frac{d}{dt}U(x,\mu) = \Pi(x,\mu)U(x,\mu)$$

performed numerically through discrete leapfrog integration w. time-step $d\tau$

Total trajectory length $\tau = Nd\tau$ – $\mathrm{cost} \propto N\langle N_{\mathrm{iter}}\rangle$

Metropolis acceptance test at the end of the evolution – acceptance rate

Forces in the molecular dynamics (1)

$$\begin{aligned}
\mathcal{H}_G &= S_G = \beta \sum_{\mu<\nu} \left(1 - \frac{1}{N} \mathrm{Re}\,\mathrm{tr}\,\mathcal{P}_{\mu\nu} \right) \\
\mathcal{H}_F &= S_F = \sum_x \phi^\dagger(x) \left[Q^2 \right]^{-1} \phi(x)
\end{aligned}$$

The forces are defined as:

$$\begin{aligned}
\delta U(x,\mu) &= \delta\omega(x,\mu) U(x,\mu) \\
\delta S_G &= (\delta\omega, F_G) \\
\delta S_F &= (\delta\omega, F_F)
\end{aligned}$$

The fermionic force is obtained from:

$$\begin{aligned}
\delta S_F &= -2\mathrm{Re} \left[\xi^\dagger \delta(Q) \eta \right] \\
\eta &= (Q^2)^{-1} \phi \\
\xi &= Q\eta
\end{aligned}$$

Forces in the molecular dynamics (2)

$$\delta S_F = \sum_{x,\mu,a} \delta\omega^a(x,\mu)\,\mathrm{Re}\,\mathrm{tr}_{\mathrm{s,c}}\left[iT_R^a U^R(x,\mu)\gamma_5(1-\gamma_\mu)\times \right.$$
$$\left. \times\left\{\eta(x+\mu)\otimes\xi(x)^\dagger + \xi(x+\mu)\otimes\eta(x)^\dagger\right\}\right]$$

Introducing the projectors:

$$P_R^a(F) = -\frac{1}{T_f}\mathrm{Re}\,\mathrm{tr}_{\mathrm{c}}\,\left[iT_R^a F\right]\,,$$

one obtains a compact expression:

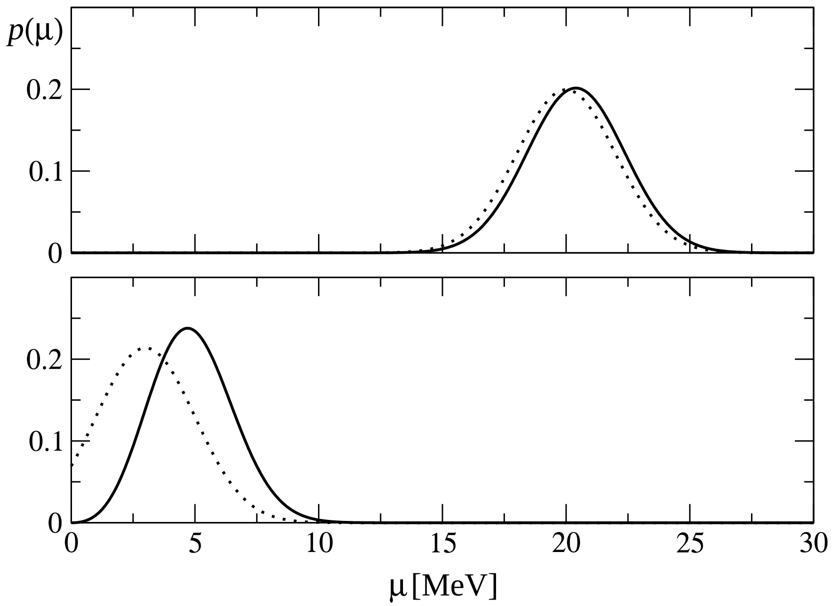
$$F_F^a(x,\mu) = P_R^a\left(U^R(x,\mu)\,\mathrm{tr}_{\mathrm{s}}\left[\gamma_5(1-\gamma_\mu)\times \right.\right.$$
$$\left.\left. \times\left\{\color{red}{\eta(x+\mu)}\otimes\xi(x)^\dagger + \xi(x+\mu)\otimes\color{red}{\eta(x)}^\dagger\right\}\right]\right)$$

Eigenvalues of the Dirac operator

[Idd et al 05]

- continuum theory: $\{\gamma_5, D\} = 0$
  $\Longrightarrow$ the massive Dirac operator is protected from small e.v.
- Wilson fermions break chiral symmetry
  $\Longrightarrow$ no protection for $D_m = D_w + m_0$
- for particular gauge fields: **exceptionally small e.v.**
  $\Longrightarrow$ instabilities / more expensive computations
- observed very smooth runs: can NOT be a property of the algorithm
  distribution of the spectral gap is a property of D_m
- scaling as $a \to 0$
  $m \to 0$
  $V \to \infty$
- analytical control: PQChPt/RMT, $O(a^2)$ effects, finite volume [sharpe 06]

Gap distribution



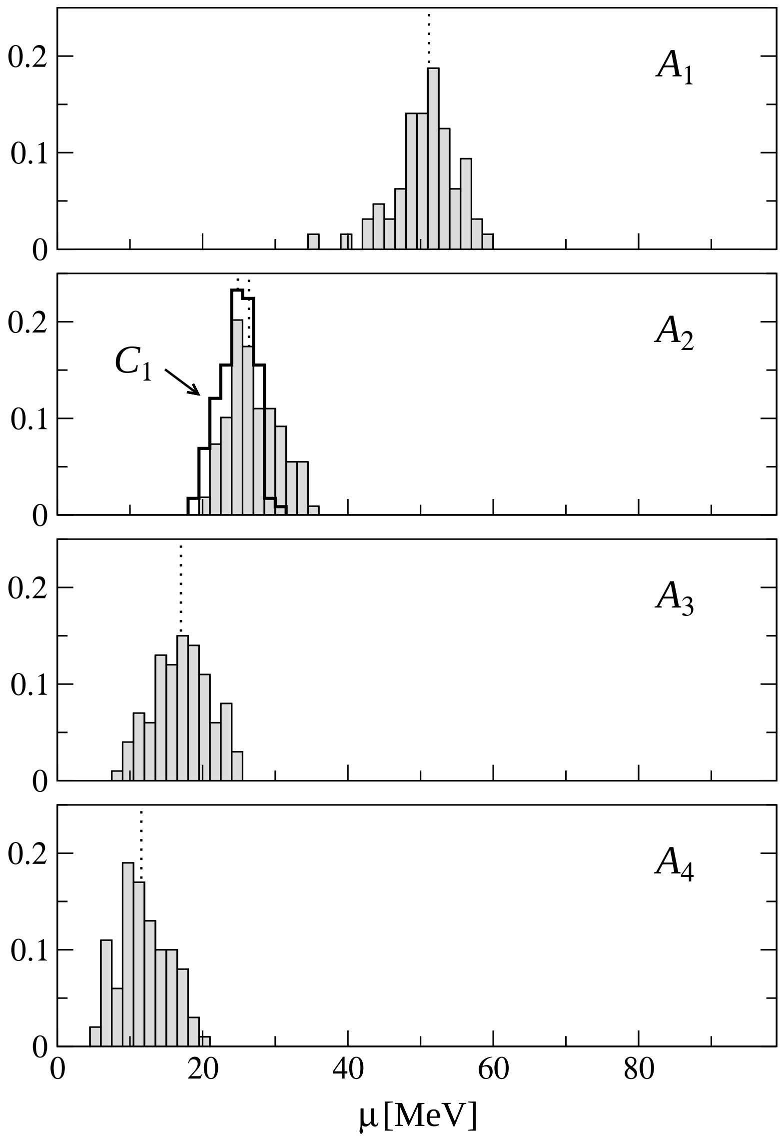
- integration instabilities / reversibility
- ergodicity problem: HMC stuck in a sector with $\eta \neq 0$
- sampling of observables: $p(\mu)/\mu^2$

difficult regime for simulations:

$$\boxed{\begin{array}{l} m \rightarrow 0 \\ a, V \ \text{ fixed} \end{array}}$$

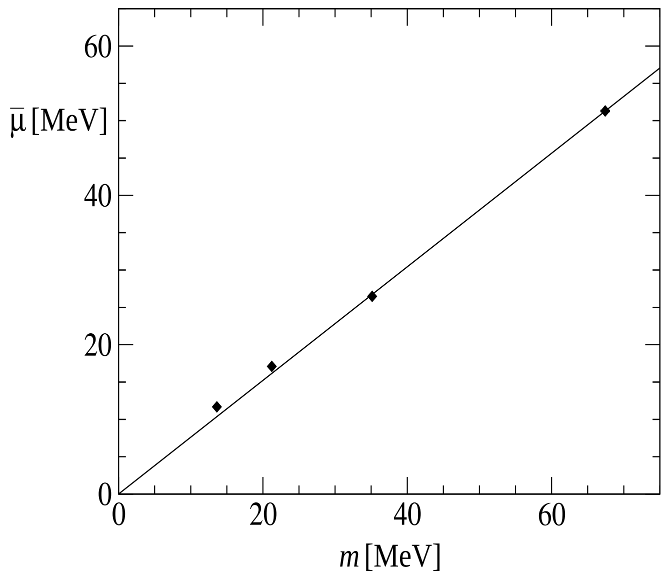
$\hookrightarrow$ is there a safe regime?

Numerical results in QCD



Small lattice 32×24^3, 4 bare masses [Idd et al 05]

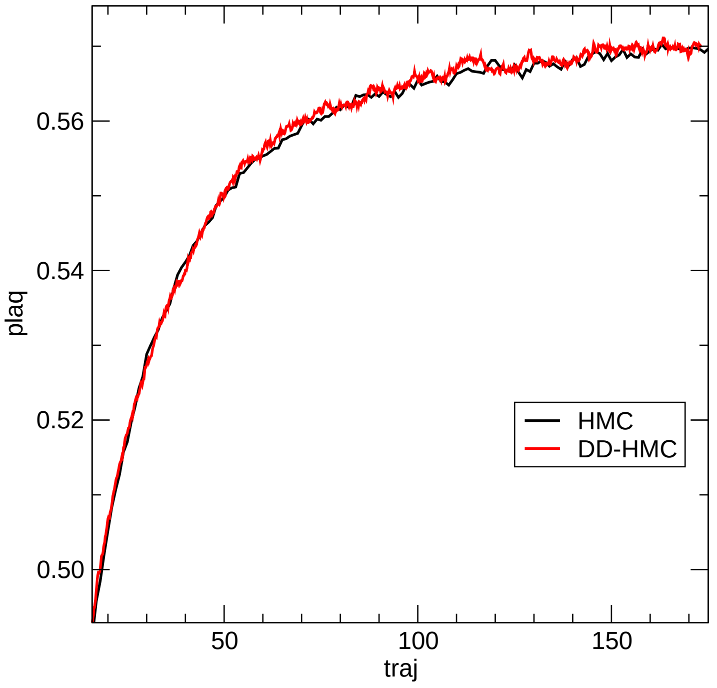
- stability is related to the width of the distribution
- $\langle \mu \rangle$ shifts linearly with m
- width independent of the mass
- scaling with a and V: $\sigma \propto aV^{-1/2}$



Tests of the algorithm (1)

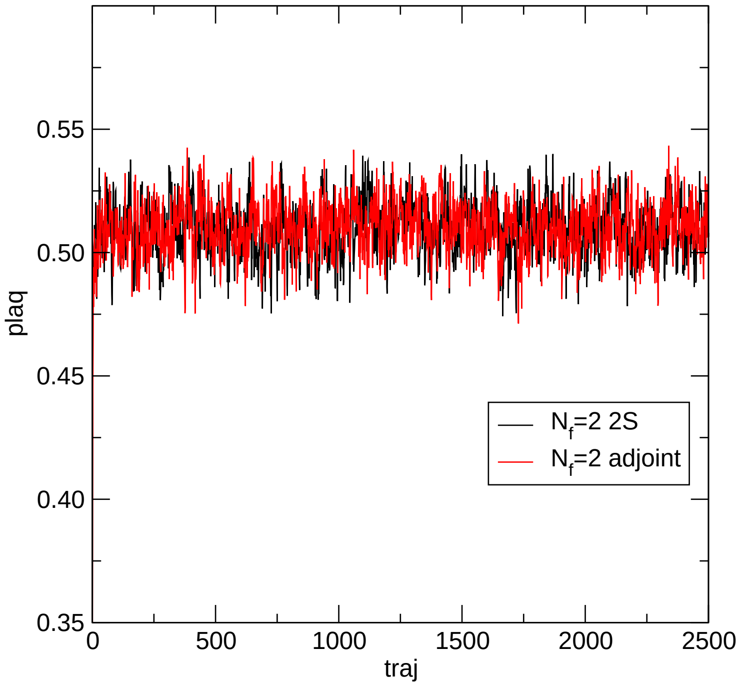
SU(3) fund repr

$16^3 \times 16, \beta = 5.6, \kappa = 0.15750$

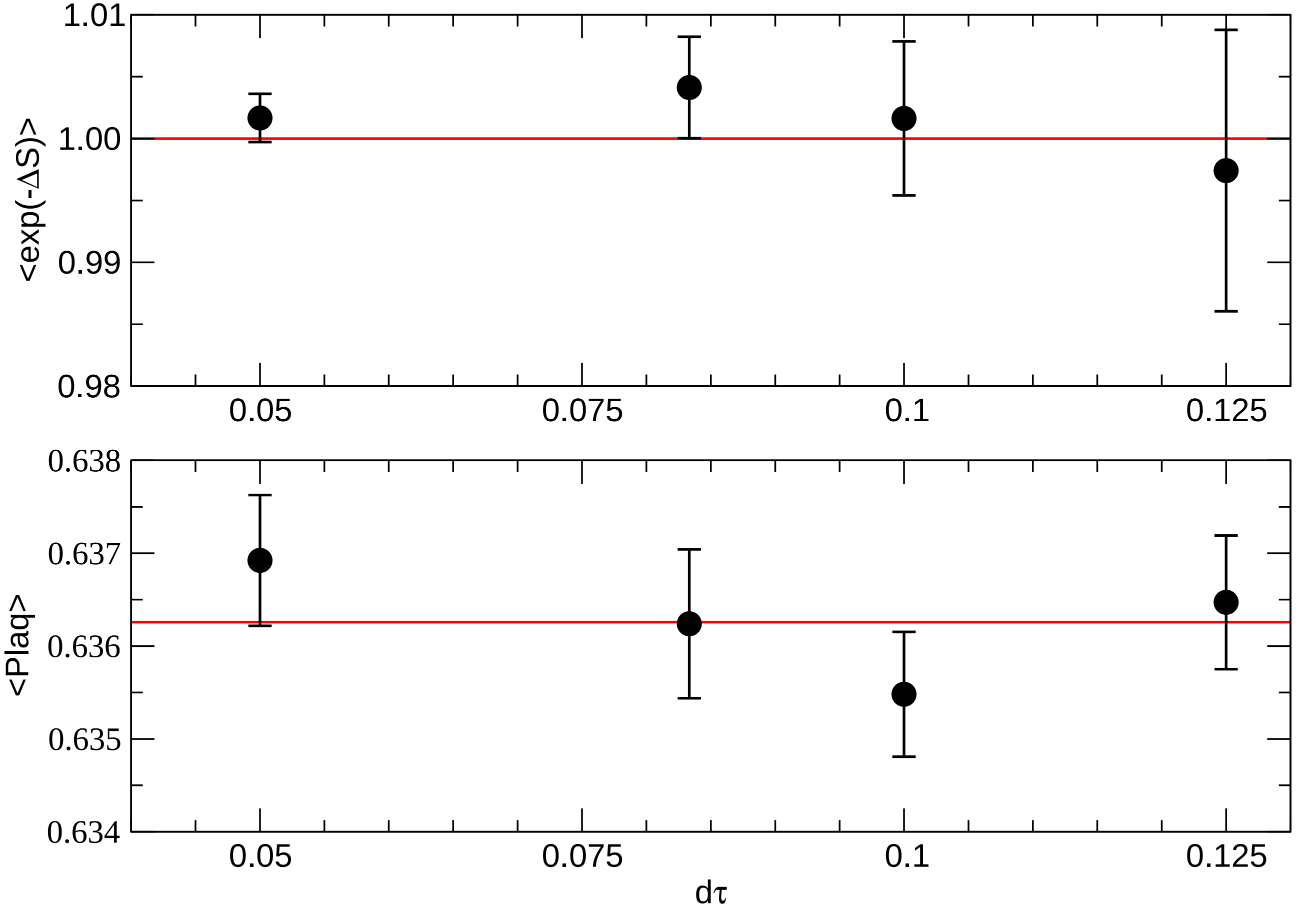


SU(2) 2S/adjoint repr

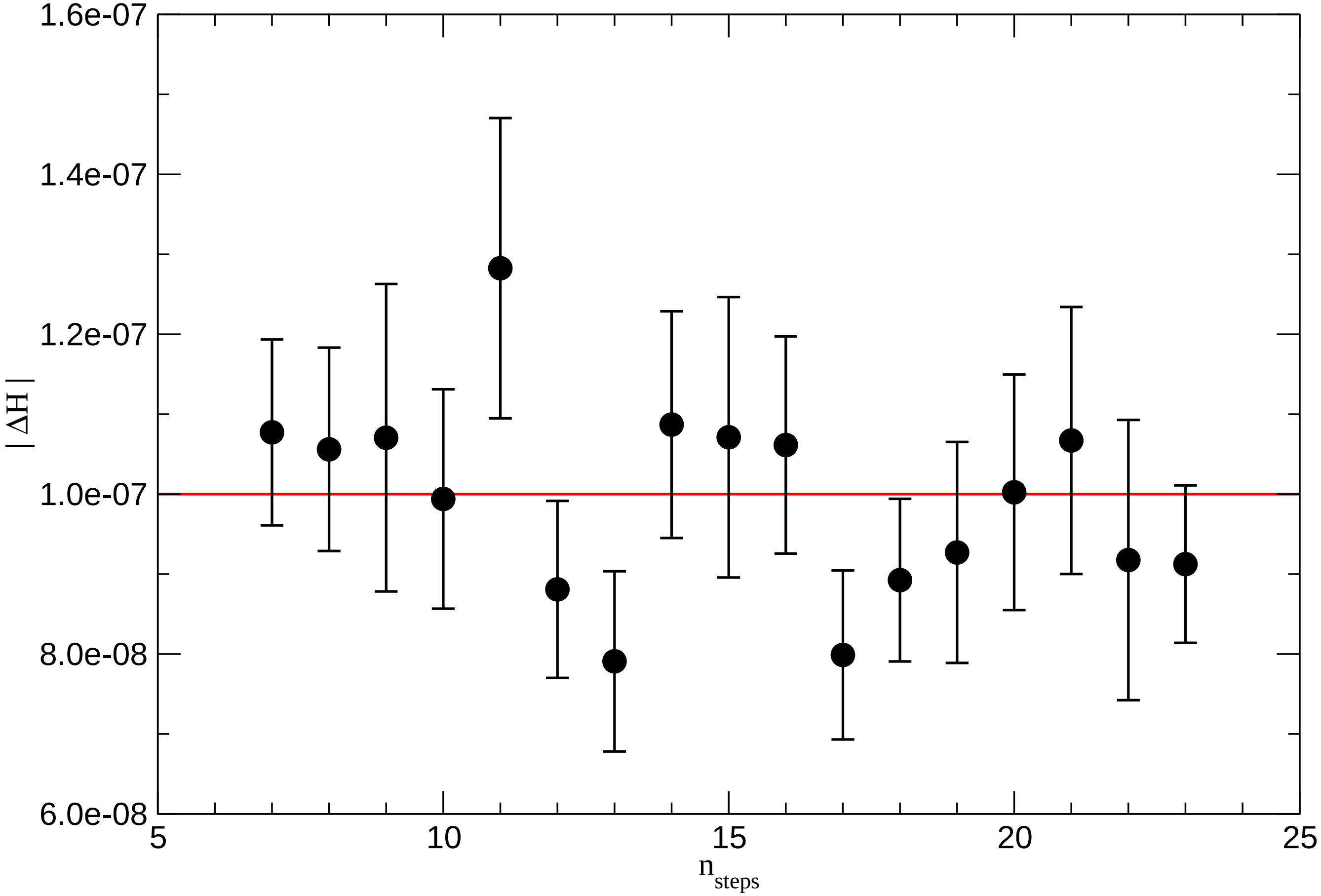
$4^3 \times 8, \beta = 2.0, \kappa = 0.12500$



Tests of the algorithm (2)

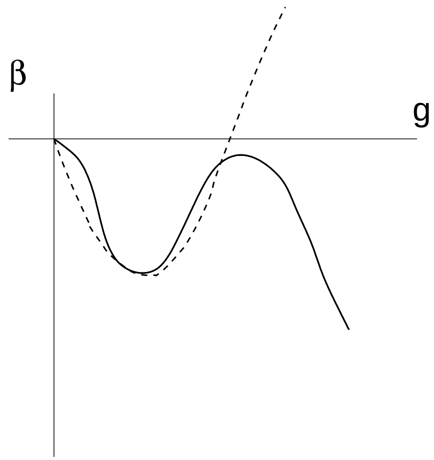


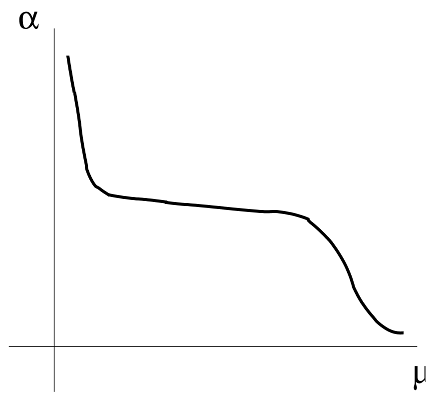
Tests of the algorithm (3)



Minimal Walking Technicolor

- arguments in favour of (near) conformal behaviour for $n_f = 2$ [sannino et al 05ff]





- candidate for minimal walking technicolor model
- lattice simulations can provide NP input catterall et al 07, shamir et al 08]

> - phase structure in bare parameter space
> - spectroscopy
> - NP study of near–conformal behaviour

Perturbation theory

[ldd, frandsen, panagopoulos, sannino 08]

Perturbative computations can be easily generalized:

- vertices with fermions and gluons involve generators T_R^a
- vertices with 3 and 4 gluons involve generators T_F^a
- at one–loop replace Casimir operators as necessary
- at two–loop the structure of each diagram needs to be inspected
- numerical integral are independent of the color structure

$$
\begin{aligned}
am_c(g_0) &= g_0^2\Sigma^{(1)} + g_0^4\Sigma^{(2)} \\
\Sigma^{(1)} &= 2C_2(R)\left[c_1 + c_2\right] \\
\Sigma^{(2)} &= C_2(R)Nd_1 + 2C_2(R)T_R n_f d_2 + C_2(R)C_2(F)d_3 + C_2(r)^2 d_4
\end{aligned}
$$

$$
Z_A = 1 - \frac{g_0^2}{16\pi^2}C_2(R) \times 15.7
$$

Chiral symmetry

For a generic N the 2–index representations are complex:

$$\mathrm{SU}(n_f) \times \mathrm{SU}(n_f) \longrightarrow \mathrm{SU}(n_f)$$

$\Longrightarrow$ usual pattern of chiral symmetry breaking

SU(2) is a special case: $\square\!\square$ is the adjoint representation

chiral symmetry breaking pattern for $n_f = 2$:

$$\mathrm{SU}(4) \longrightarrow \mathrm{SO}(4)$$

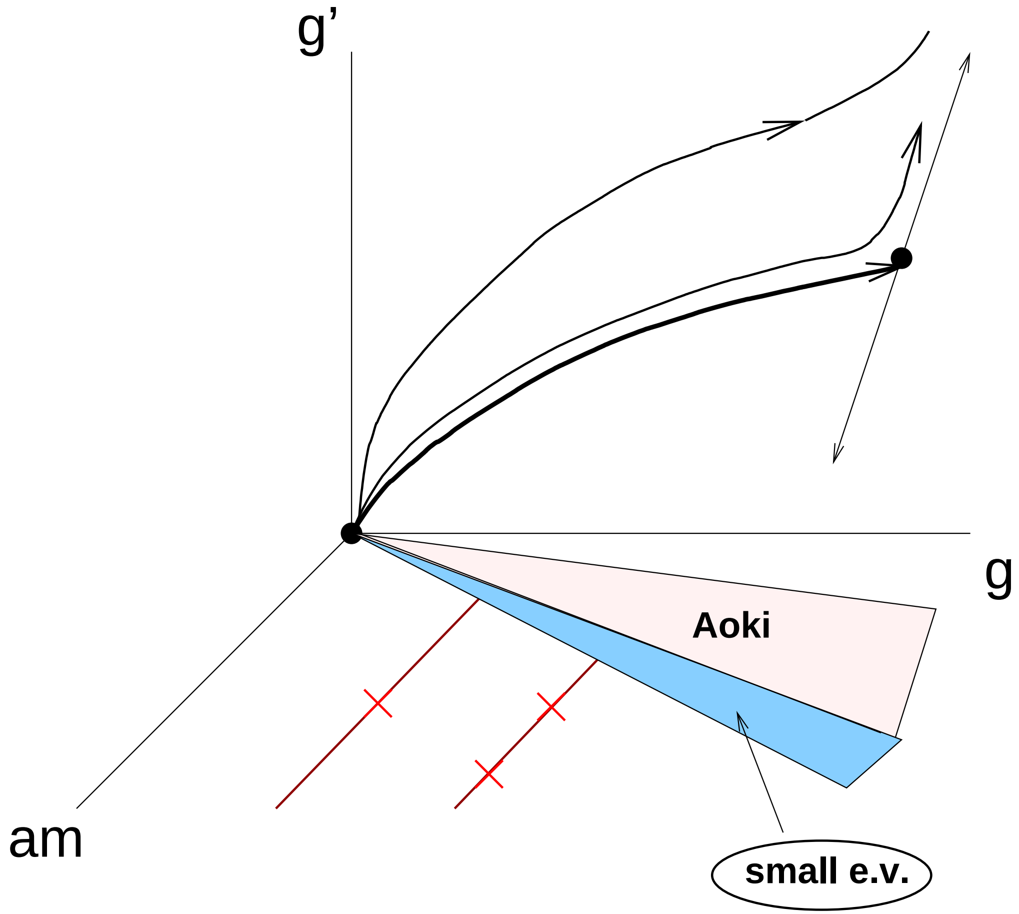
Goldstone bosons are coupled to EW gauge bosons:

$$\boxed{F_{\mathrm{PS}} = v \approx 250 \text{ GeV}}$$

$\hookrightarrow$ used to set the scale

Conformal fixed point?

- scale invariance is broken by m AND $1/L$
- HOW to identify the conformal fixed point
- deviations from QCD spectrum
- scaling behaviour of field correlators
- difficult because of the presence of *two* relevant operators
- Schrödinger functional/twisted BC allow $m = 0$



List of runs

β	V	κ	$-m$	$\langle P \rangle$	τ
2.00	8×4^3	0.12500	0.0	0.5093(14)	2.9(0.4)
	8×4^3	0.14286	0.5	0.5163(16)	3.1(0.5)
	8×4^3	0.15385	0.75	0.5235(18)	3.1(0.5)
	8×4^3	0.16667	1.0	0.5373(20)	6.0(1.2)
	8×4^3	0.18182	1.25	0.5742(37)	12.0(3.6)
	8×4^3	0.18382	1.28	0.5850(50)	22.3(9.3)
	8×4^3	0.18587	1.31	0.6013(55)	48.3(23.3)
	8×4^3	0.18657	1.32	0.6159(58)	40.7(16.3)
	16×8^3	0.18587	1.31	0.5951(4)	5.8(3.6)
	16×8^3	0.18657	1.32	0.6040(6)	9.0(9.6)
	16×8^3	0.18692	1.325	0.6107(5)	4.2(2.6)
	16×8^3	0.18727	1.33	0.6168(7)	2.6(1.8)
	16×8^3	0.18748	1.333	0.6250(4)	13.1(5.9)
	16×8^3	0.18769	1.336	0.6296(7)	15.6(10.9)
	16×8^3	0.18797	1.34	0.6347(6)	13.6(11.5)

Two–point functions

1. Correlators of fermion bilinears:

$$f_{\Gamma_1\Gamma_2}(t) = \sum_{\vec{x}} \langle (\bar{u}\Gamma_1 d)^\dagger(\vec{x},t)\, (\bar{u}\Gamma_2 d)(\vec{0},0)\rangle$$

are computed by performing the Wick contractions:

$$f_{\Gamma_1\Gamma_2}(t) = -\sum_{\vec{x}} \langle \mathrm{tr}\left[\Gamma_1' G(x,0)\Gamma_2' G(x,0)^\dagger\right]\rangle$$

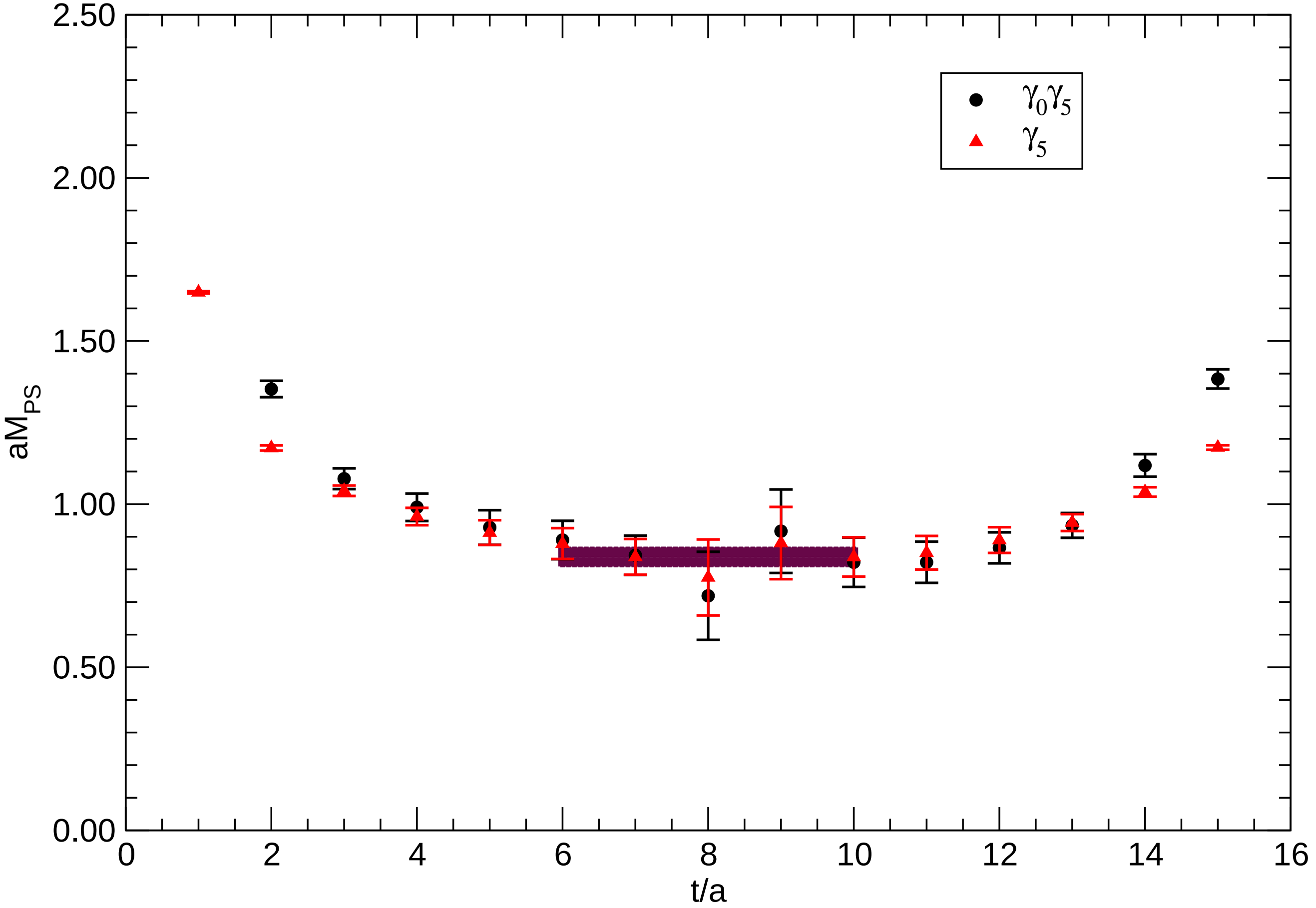
2. Asymptotic behaviour at large Euclidean times:

$$\begin{aligned} f_{\Gamma_1\Gamma_2}(t) &= A_{\Gamma_1\Gamma_2} e^{-mt} + \ldots \\ m_{\mathrm{eff}}(t) &= -\log \frac{f_{\Gamma_1\Gamma_2}(t)}{f_{\Gamma_1\Gamma_2}(t-1)} \longrightarrow m \end{aligned}$$

3. The quark mass is obtained from:

$$m_{\mathrm{pcac}}(t) = \left[\frac{1}{2}\left(\partial_0 + \partial_0^*\right) f_{\mathrm{AP}}(t)\right] / f_{\mathrm{PP}}(t) \longrightarrow m_{\mathrm{pcac}}$$

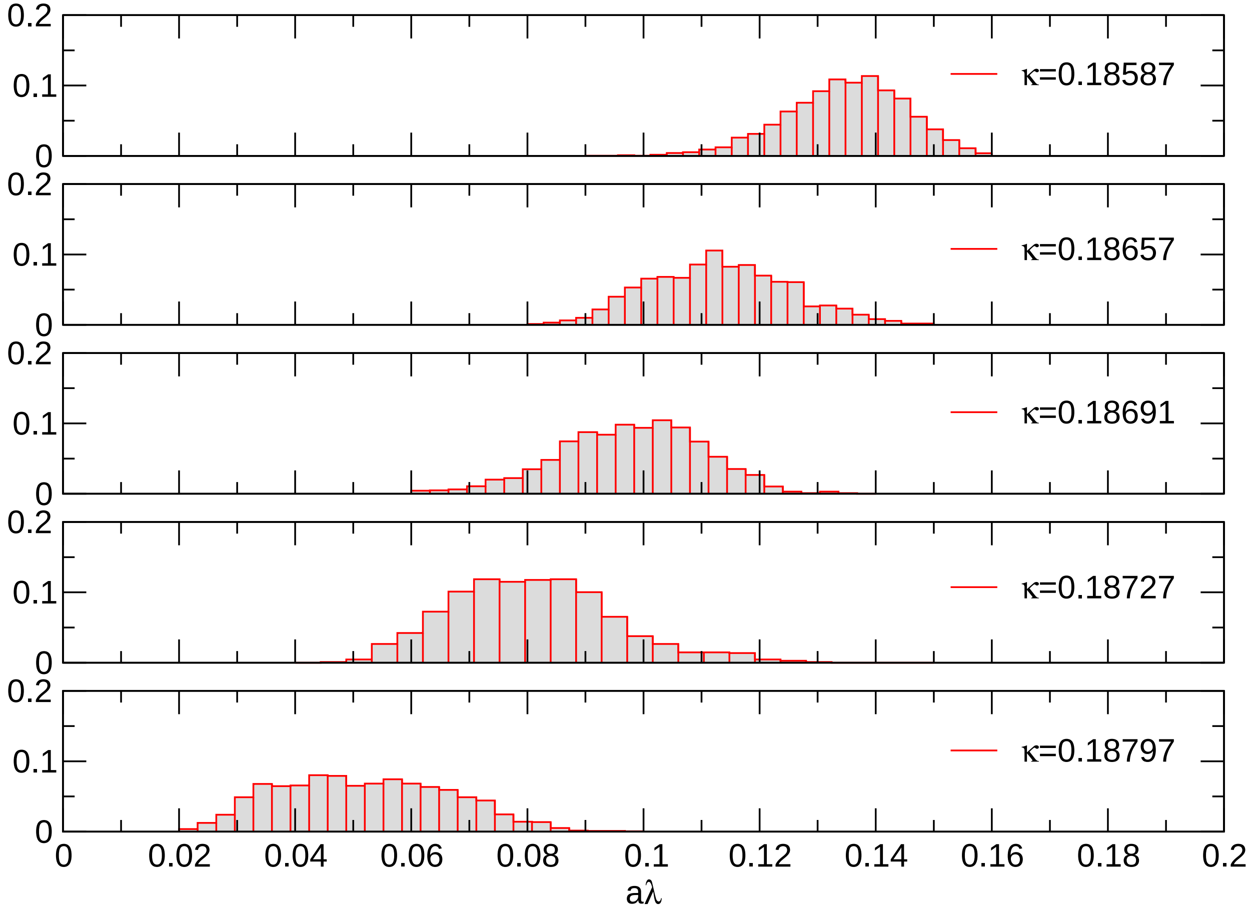
Effective mass



data from 16×8^3 lattice; $\kappa = 0.18727$, $aM_{\mathrm{PS}} = 0.838(29)$

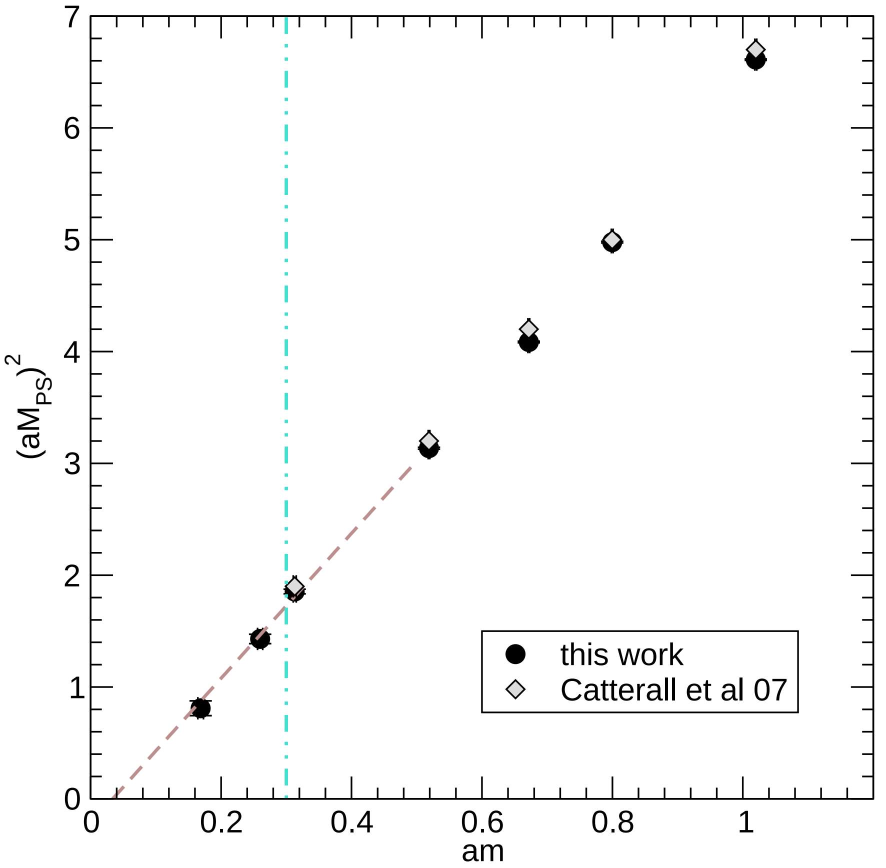
Eigenvalue distributions

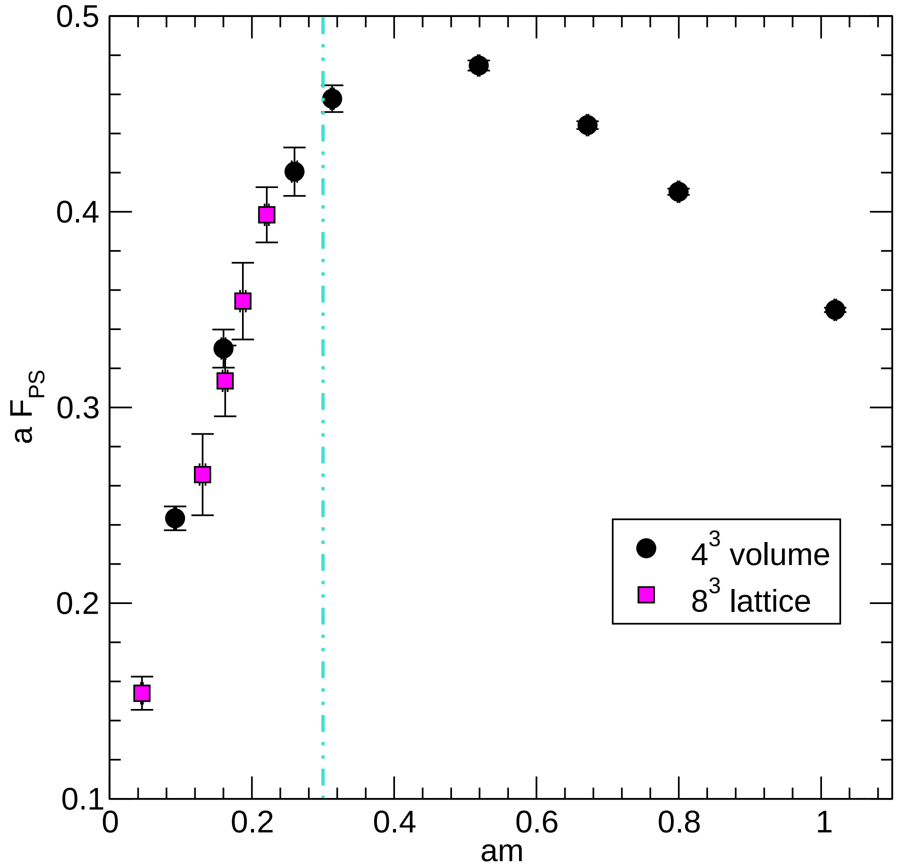
Small eigenvalues of Q lead to instabilities – cfr QCD studies [Idd et al 05]



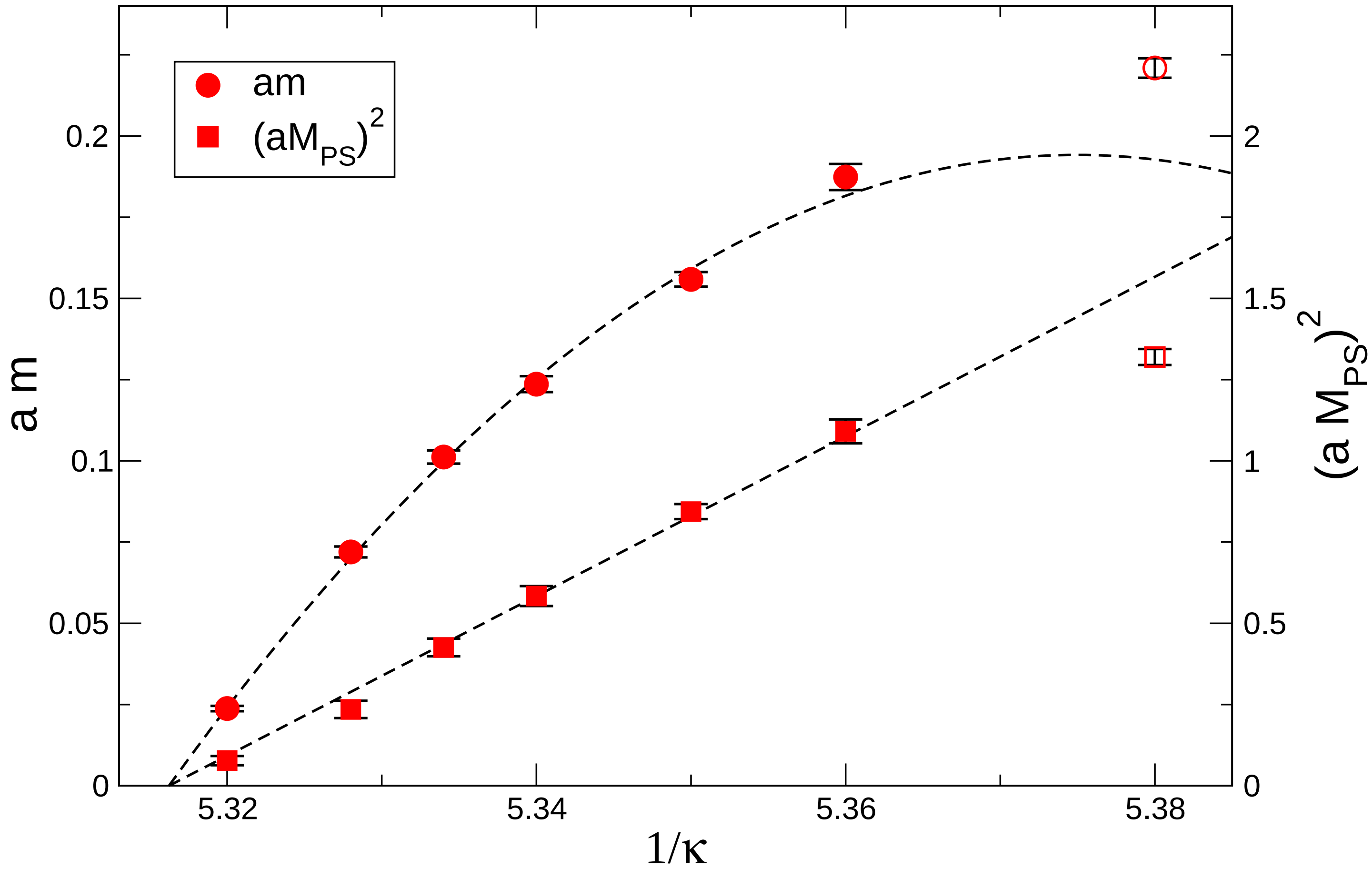
data from 16×8^3 lattice – by comparing with data from 8×4^3: $\sigma \approx 1/\sqrt{V}$

Comparison with previous results



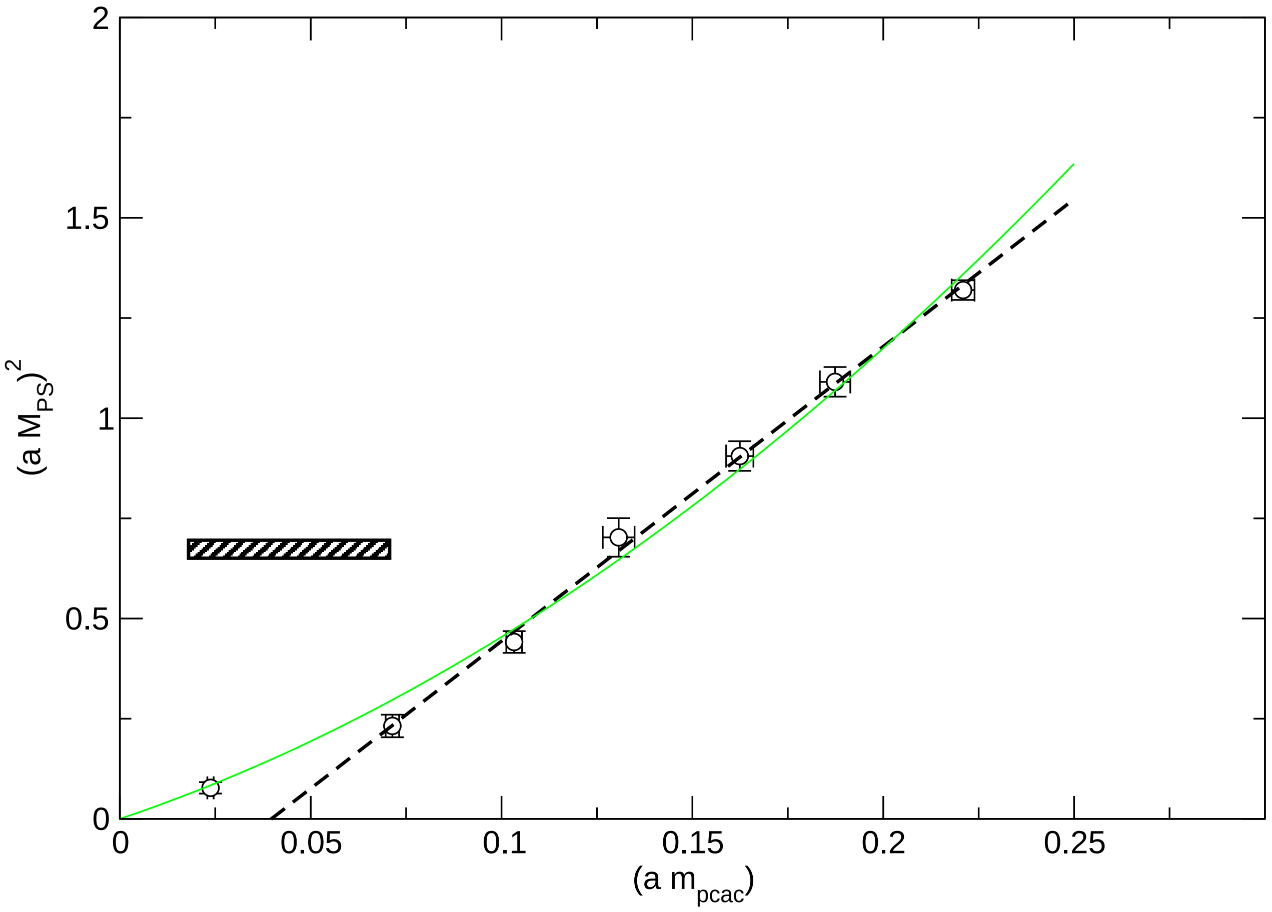


Chiral limit (1)



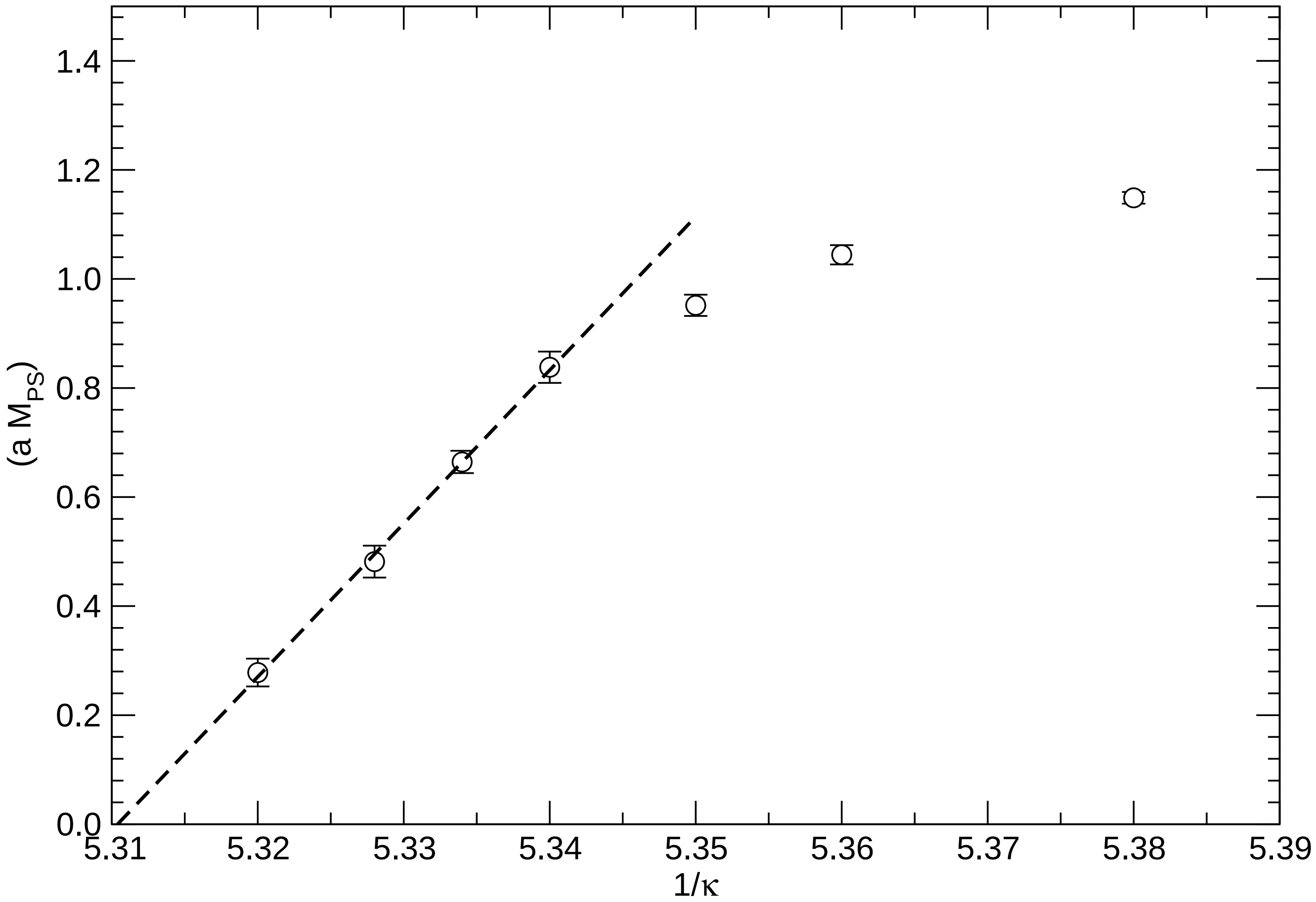
data from 16×8^3 lattice

Chiral limit (2)



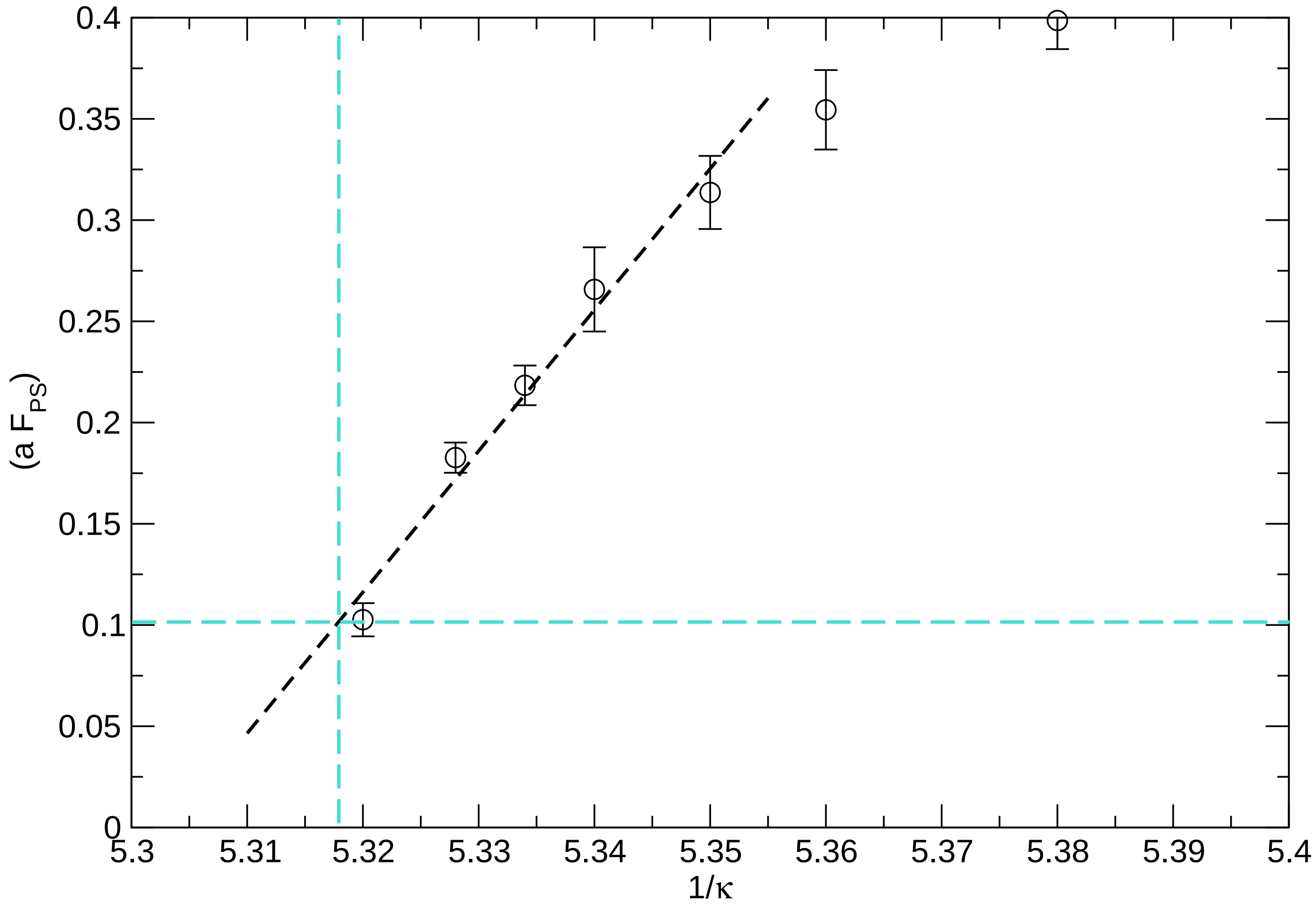
data from 16×8^3 lattice

Chiral limit (3)



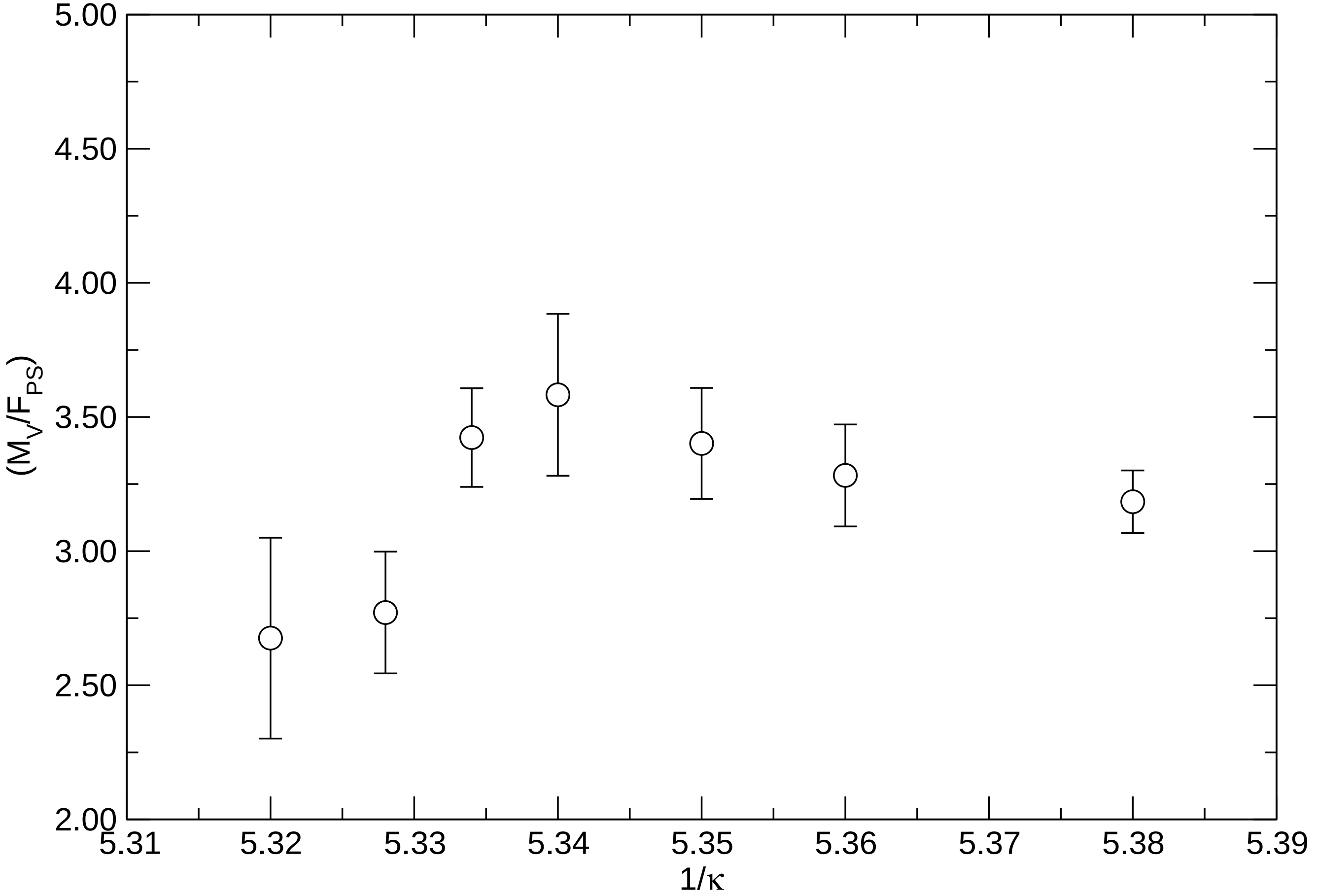
data from 16×8^3 lattice

Decay constant



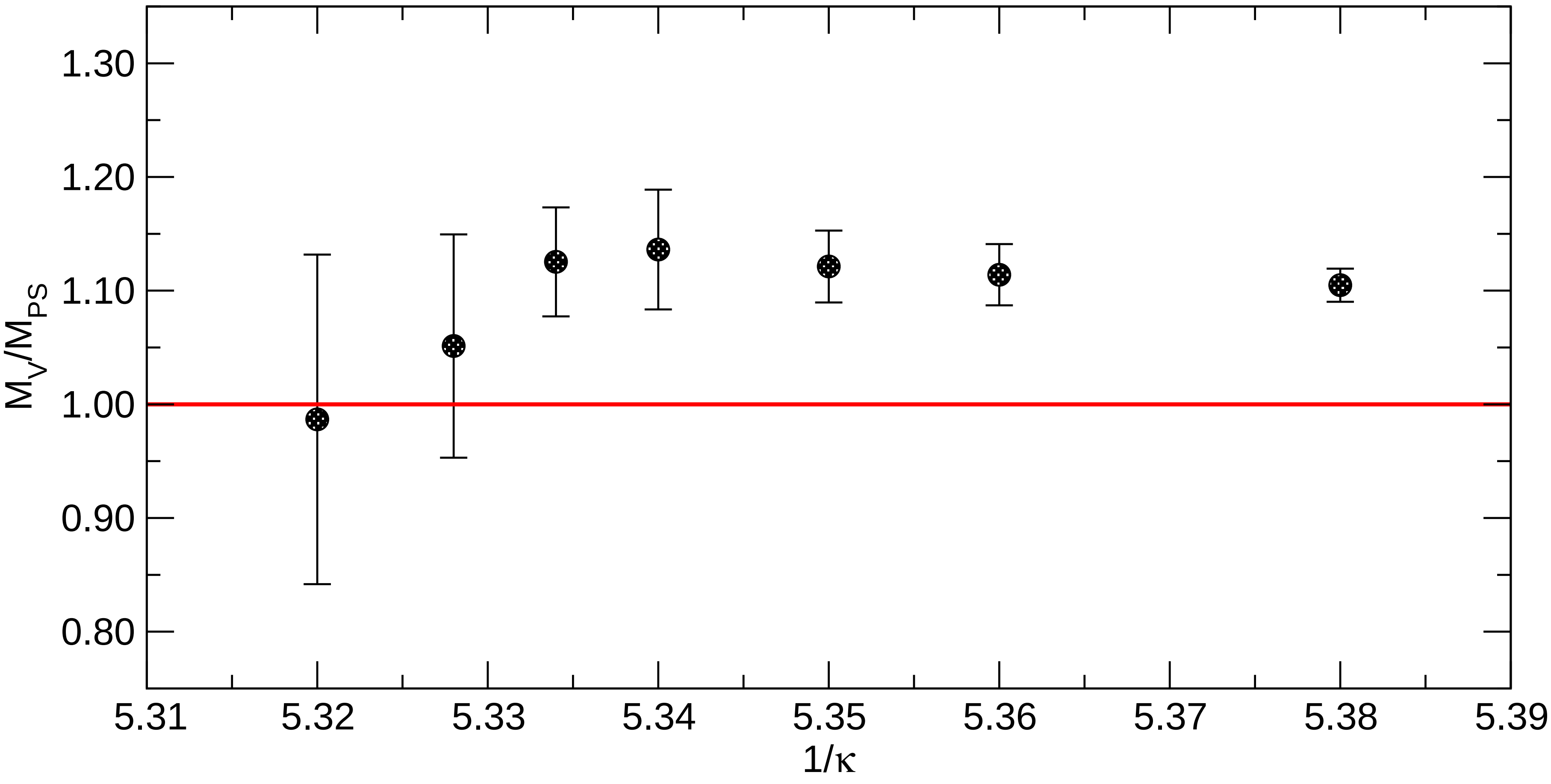
$$\boxed{Z_A = 0.602 \Longrightarrow aF_{\mathrm{PS}} = 0.0612}$$

Vector Mass (1)

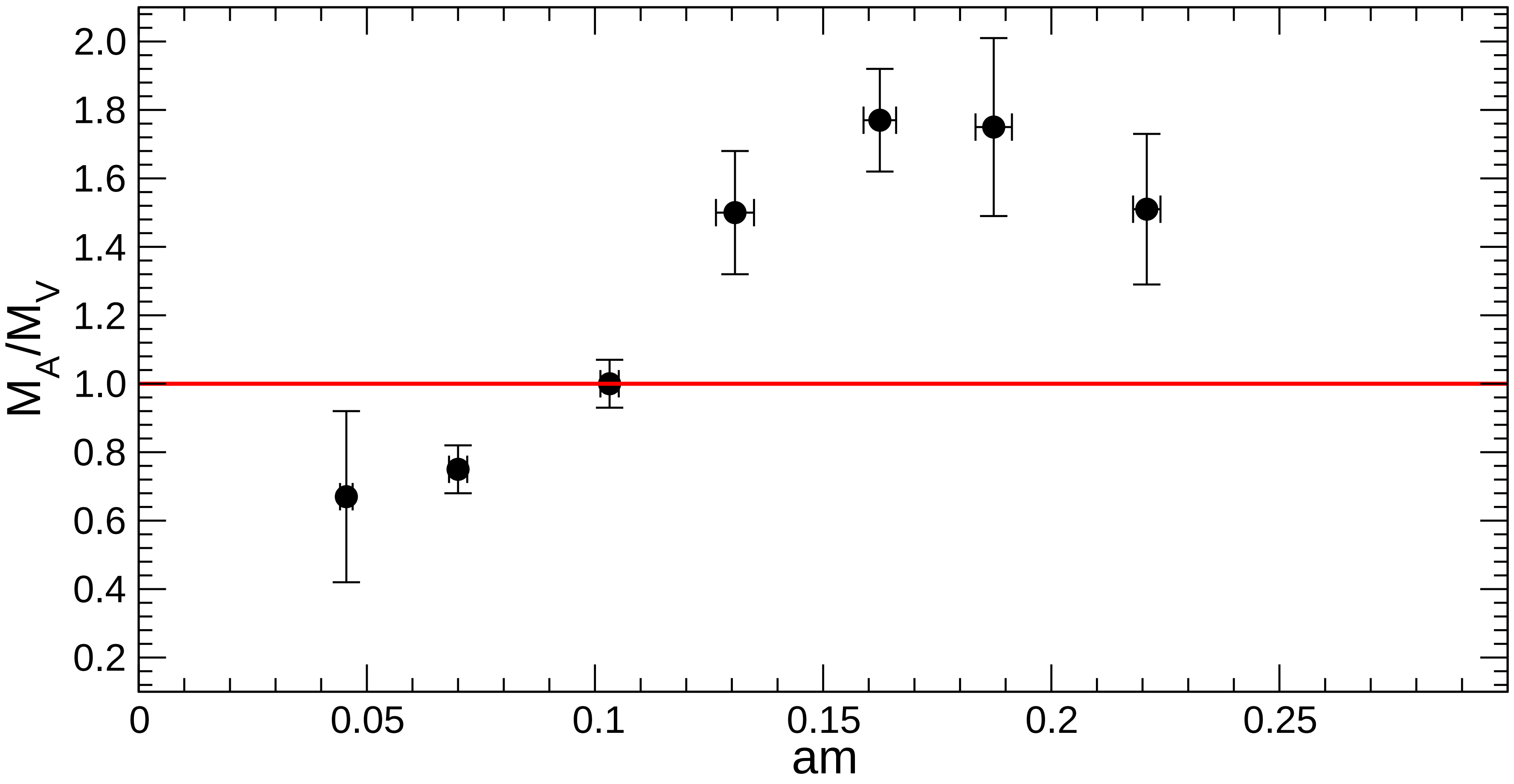


$$\boxed{M_{\rm V} \approx 3 \times F_{\rm PS}}$$

Vector Mass (2)



Axial Mass



$$\boxed{M_{\mathrm{A}} \approx M_{\mathrm{V}}}$$

Conclusions

- generalization of the Dirac operator to higher–dim representations
- quenched large–N: spectrum, comparison with string theory
- $\longrightarrow$ continuum limit, scalar meson under study

- simulations with dynamical quarks [catterall et al., shamir et al., ldd et al.]
- even/odd preconditioning, RHMC: arbitrary N and n_f

- agreement with spectrum observed by Svetitsky et al.
- spectroscopy $\longrightarrow$ scalar
- first preliminary results $\longrightarrow$ larger volumes, scaling to control systematics
- $M_V \approx 750 \text{ GeV}$...
- NP walking behaviour: Schrödinger functional [appelquist et al] , Wilson loop [lin et al]