Soft gluon effects in the production of coloured sparticles at the LHC

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IPPP, University of Durham, 07.11.2008

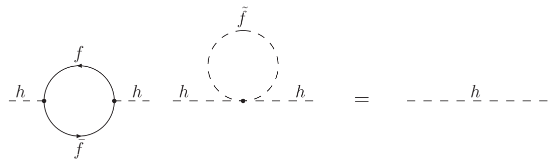
Overview

- Motivation
- Theoretical status
- Soft gluon effects
  - appearence in theoretical expressions
  - systematic treatment to any order in α_s (resummation)
- Application of resummation to coloured sparticle hadroproduction
- Predictions for corrections due to soft gluon emission for $\tilde{g}\tilde{g}$ and $\tilde{q}\bar{\tilde{q}}$ at the LHC

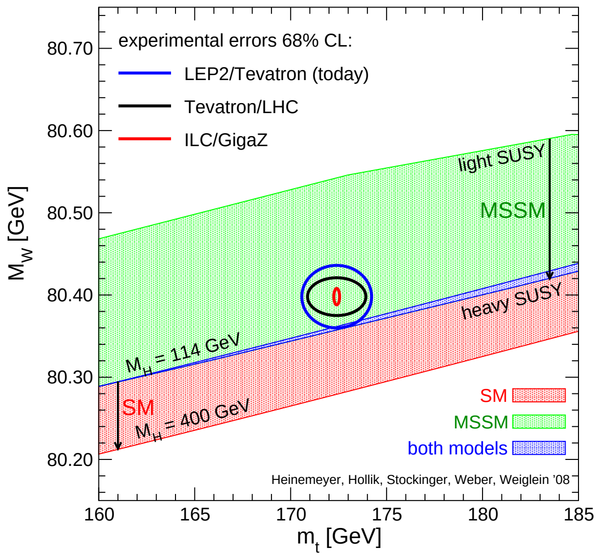
SUSY

Supersymmetry is one the best theoretically motivated extensions of the Standard Model

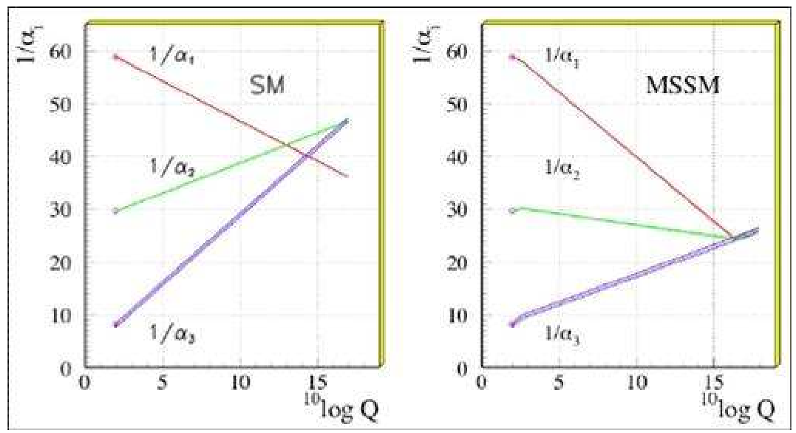
→ Solves the hierarchy problem: SUSY loop corrections cancel SM corrections



→ “Fits like a glove” to EW precision measurements



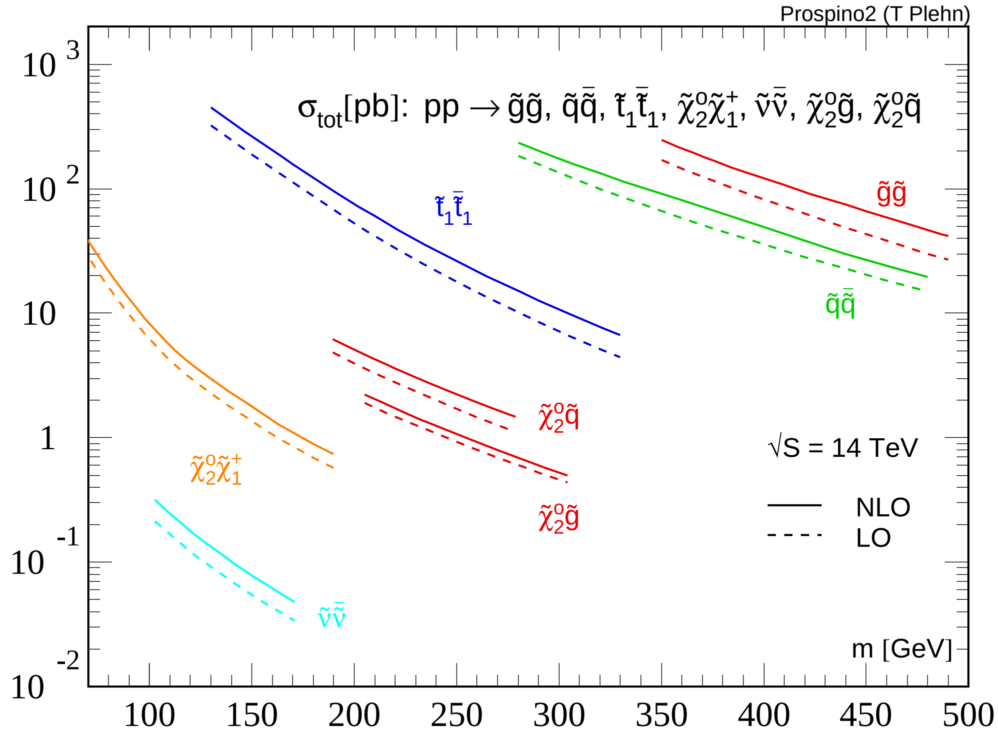
→ Modifies running of the SM gauge couplings (unification of forces)



→ Provides a dark matter candidate

SUSY particle pair-production at the LHC

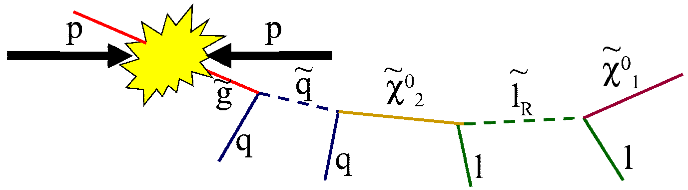
- MSSM: minimal content of SUSY particles + $R-$parity conservation
- At the LHC dominant sparticle production channels involve squarks ($\tilde{q}$) and gluinos ($\tilde{g}$) in the final state ($\tilde{q}\bar{\tilde{q}}$, $\tilde{q}\tilde{q}$, $\tilde{q}\tilde{g}$, $\tilde{g}\tilde{g}$ pairs)



[*Plehn, Prospino2*]

Coloured sparticle production at the LHC

- Cascade decays to LSPs, e.g:

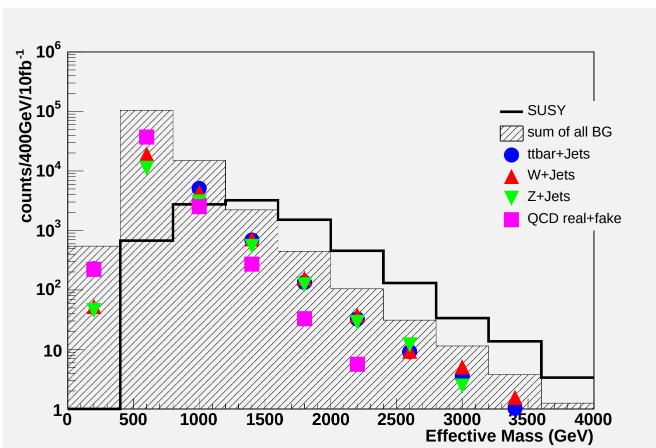


- Long decay chains and large mass differences

  $\Rightarrow$ many energetic particles (decay products) observed

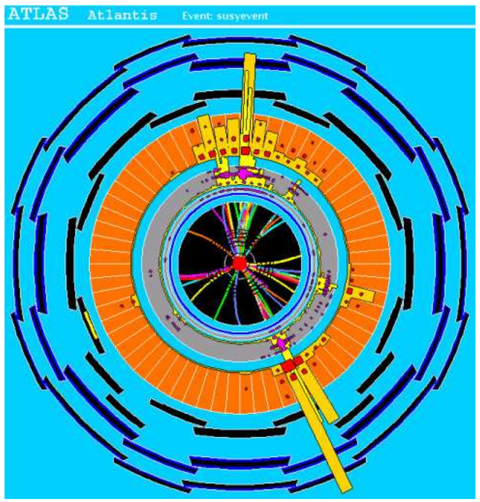
- If $R-$parity conserved, LSP stable, escapes detection $\Rightarrow$ observed as imbalance of energy measured in the transverse direction to the beam

SUSY searches: events with at least 4 jets and missing transverse energy $(\not\!\!E_T)$



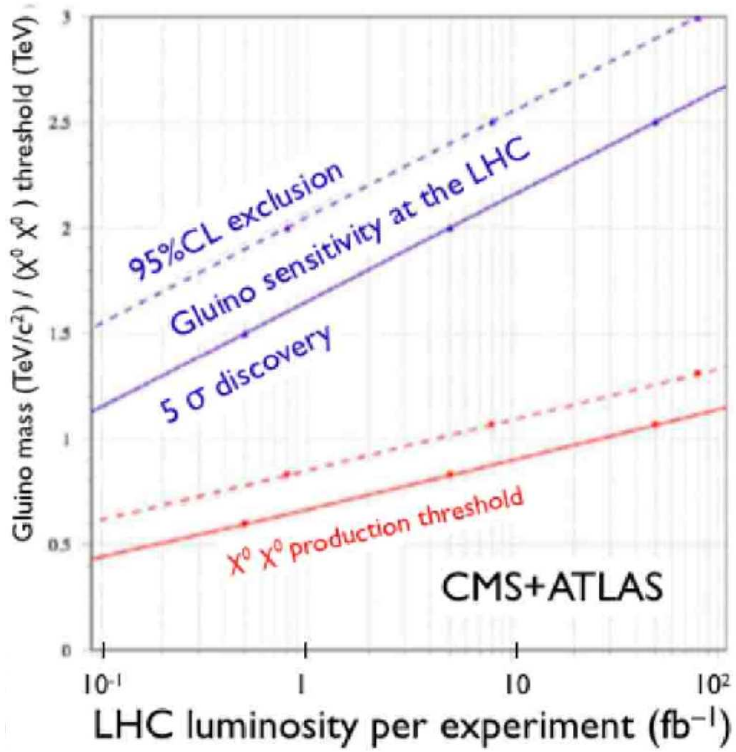
$(m_{\tilde{q}} \sim m_{\tilde{g}} \sim 1\text{TeV})$

$$M_{\text{eff}} = \not\!\!E_T + \sum_{\text{jets}} p_T^{\text{jet}}$$



Coloured sparticle production at the LHC

Production cross sections large ⇒ “easy” SUSY discovery



- SUSY to be seen at the LHC in 2010?
- squarks and gluino discovery possible for masses up to $\sim$ 2 TeV

Coloured sparticle production at the LHC

- If SUSY is discovered, need to determine SUSY parameters to discriminate between models
- Otherwise, need to derive limits

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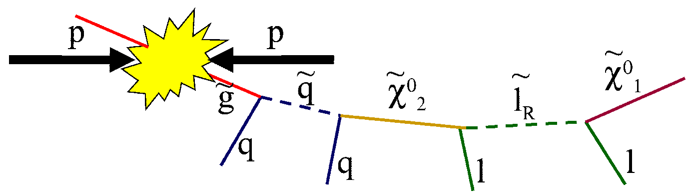
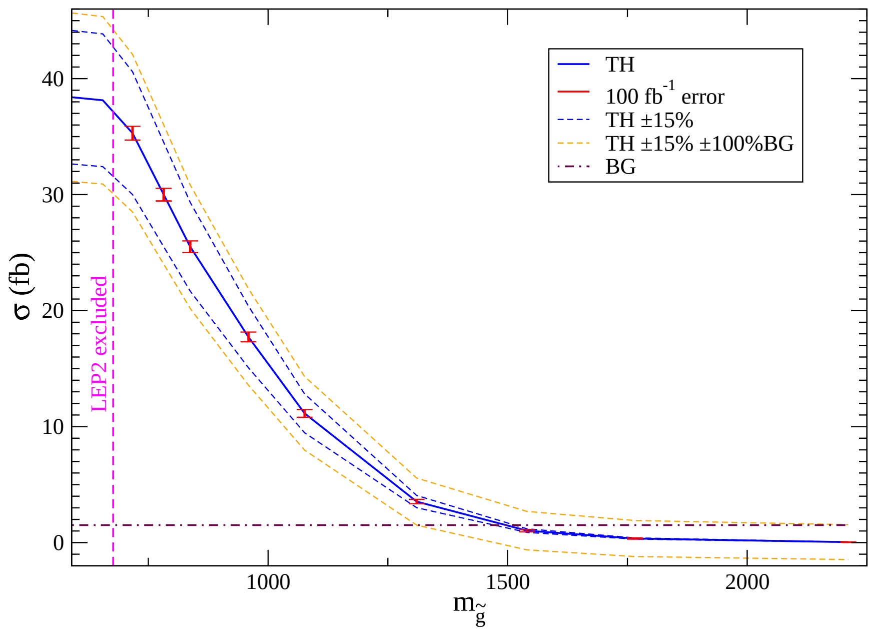


- Mass spectrum determination
  - through measurement of the invariant mass distributions (endpoints, resonance peaks) or just all visible momenta for every SUSY event [*Bachacou, Hinchcliffe, Paige'00*][*Allanach, Lester, Parker, Webber'00*][*Gjelsten, Miller, Osland'04'05*][*Gjelsten, Miller, Osland, Raklev'06*][*Butterworth, Ellis, Raklev'06*][*Lester, Parker, White'06*][*Tovey'08*][*Nojiri, Polesello, Tovey '03'08*][*Kawagoe, Nojiri, Polesello'05*]
  - can be difficult in long decay chains [*Baer et al.'07*]

Gluino mass determination



[*Baer et al.'07*]

Points from the FP/HB region of MSUGRA parameter space
$(\tan\beta = 30,\ A_0 = 0,\ \Omega_{\tilde{Z}_1} h^2 \sim 0.11)$

Sets of cuts ($n_{\rm jets} \geq 7,\ n_{\rm b-jets} \geq 2,$
$\not{E}_T + \sum_{\rm lept+jets} \geq 1400\,{\rm GeV}$)

Theoretical uncertainty:

- renormalization/factorization scale dependence
- variations in the squark masses $(2 - 5\,{\rm TeV})$

$\Rightarrow$ 8 % error on gluino mass determination claimed

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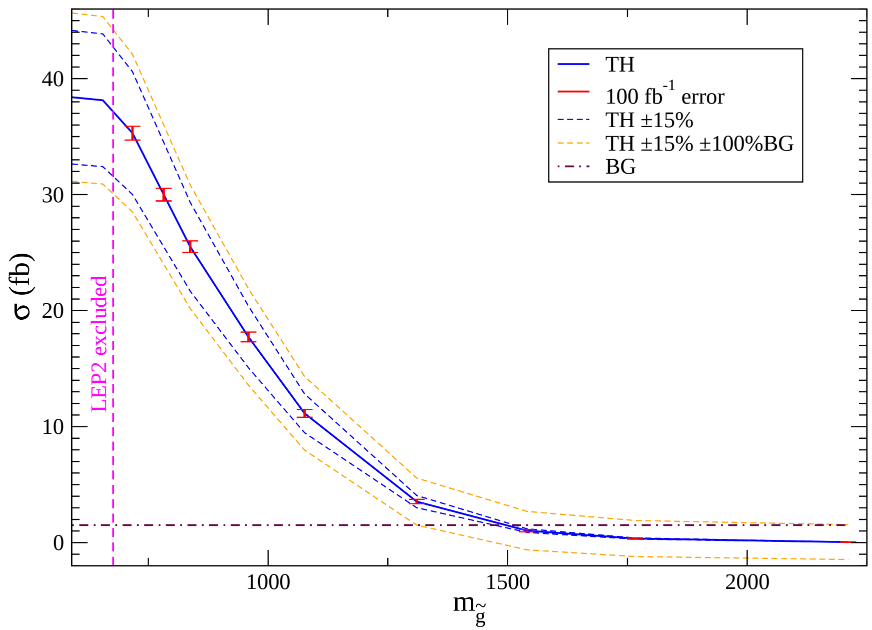

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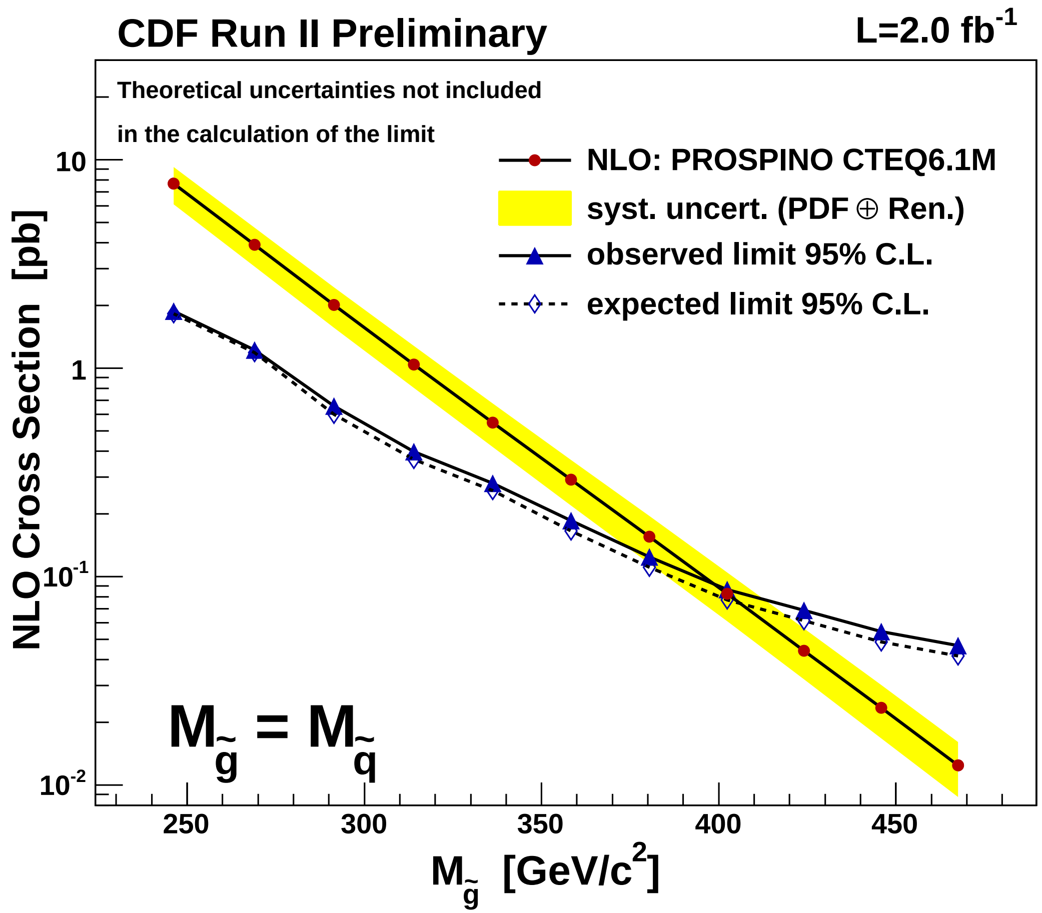
"The precision will increase if an NNLO computation of gluino pair production is made"

Gluino mass exclusion limits



mSUGRA with $A_0 = 0$, $sgn(\mu) = -1$, $\tan\beta = 5$

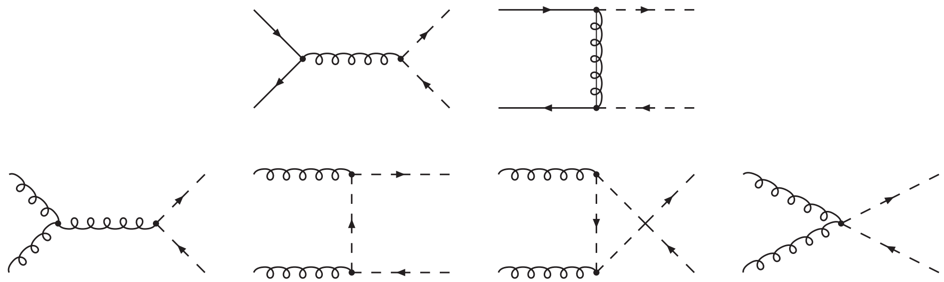
- Total cross sections for sparticle production useful for mass determination / crucial for exclusion limits
- Important to know them with high precision

$\tilde{q}\bar{\tilde{q}}$ and $\tilde{g}\tilde{g}$ production at LO

[*Dawson, Eichten, Quigg'85*]

- $h_1 h_2 \to \tilde{q}\bar{\tilde{q}}$

  LO partonic level: $q\bar{q} \to \tilde{q}\bar{\tilde{q}}$, $\bar{q}q \to \tilde{q}\bar{\tilde{q}}$, $gg \to \tilde{q}\bar{\tilde{q}}$



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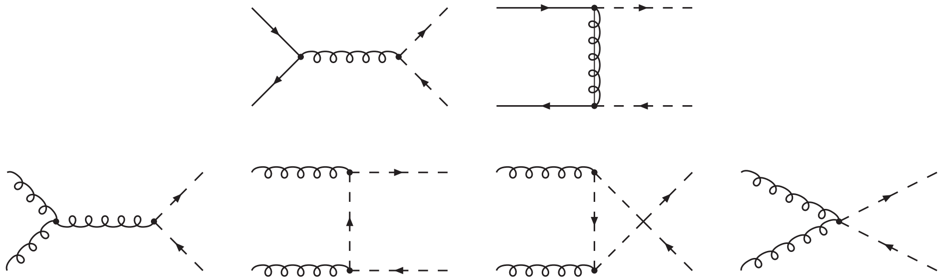

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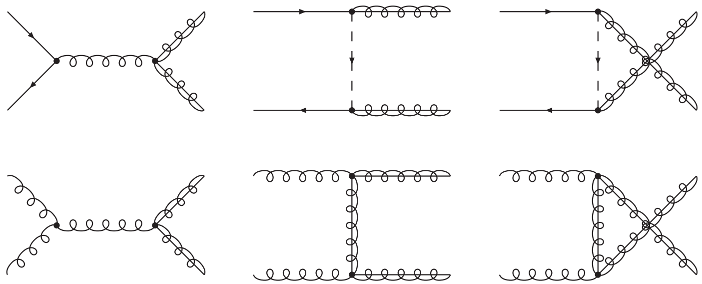

Beyond $\mathcal{O}(\alpha_s^2)$

- NLO corrections
  - SUSY-QCD corrections $\rightarrow \mathcal{O}(\alpha_s^3)$ [*Beenakker, Höpker, Spira, Zerwas'96*]
  - EW corrections $\rightarrow \mathcal{O}(\alpha_s^2 \alpha)$ [*Hollik, Kollar, Trenkel'07*][*Hollik, Mirabella'08*]

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- NLO corrections
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  - EW corrections $\longrightarrow \mathcal{O}(\alpha_s^2\alpha)$ [*Hollik, Kollar, Trenkel'07*][*Hollik, Mirabella'08*]
- For $\tilde{q}\bar{\tilde{q}}$ production, tree-level EW effects:
  - Tree-level QCD-EW interference $\longrightarrow \mathcal{O}(\alpha\alpha_s)$ [*Bornhauser et al.'07*] [*Alan, Cankocak, Demir'07*]
  - Tree-level photon-induced ($\gamma g \longrightarrow \tilde{q}\bar{\tilde{q}}$) contributions $\longrightarrow \mathcal{O}(\alpha\alpha_s)$ [*Hollik, Kollar, Trenkel'07*]
  - Tree-level EW $\longrightarrow \mathcal{O}(\alpha^2)$ [*Bornhauser et al.'07*] [*Alan, Cankocak, Demir'07*]

$\tilde{q}\bar{\tilde{q}}$ and $\tilde{g}\tilde{g}$ production at NLO (SUSY-QCD) I

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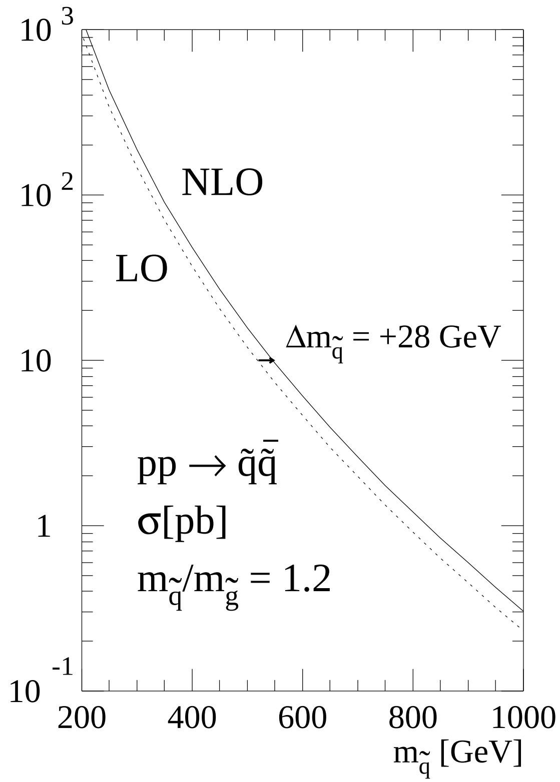

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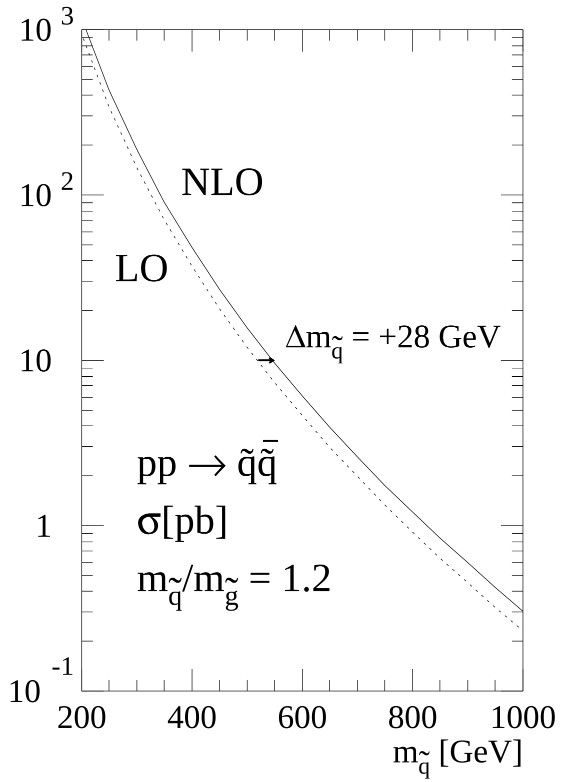

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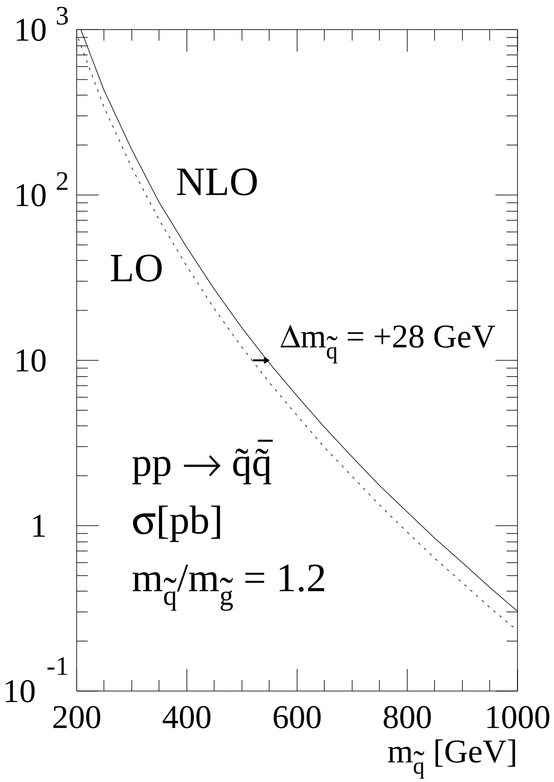
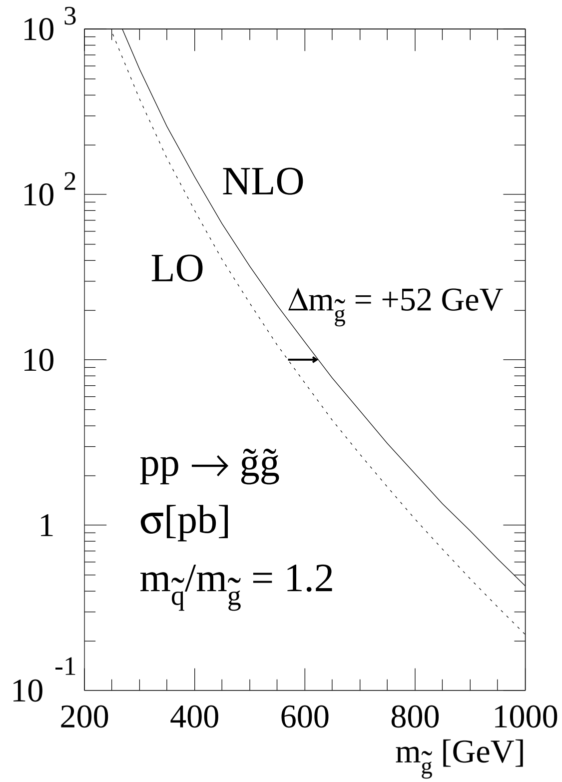
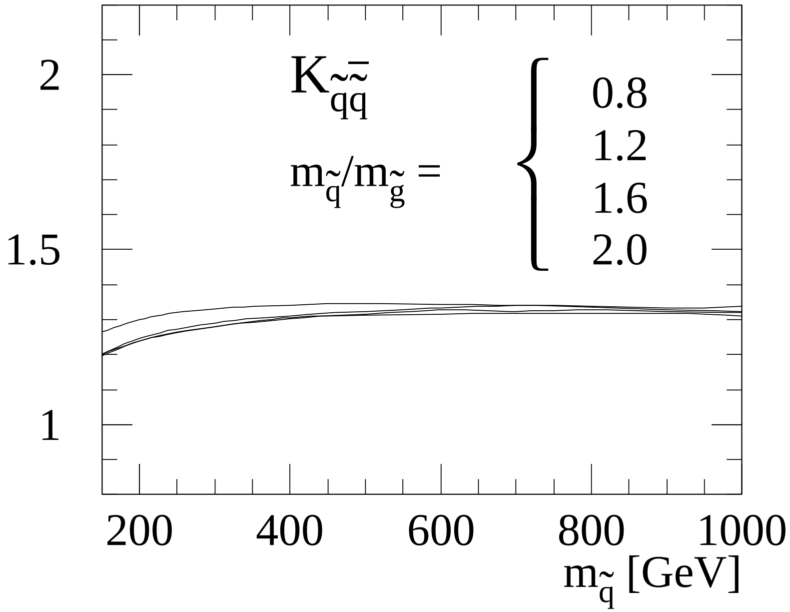




$\Rightarrow$ Increase of the cross sections due to NLO SUSY-QCD corrections over the whole range of masses covered by the LHC $\Rightarrow$ mass shifts from $\sim$10 to $\sim$100 GeV

$\tilde{q}\bar{\tilde{q}}$ and $\tilde{g}\tilde{g}$ production at NLO (SUSY-QCD) II

[Beenakker, Höpker, Spira, Zerwas'96]



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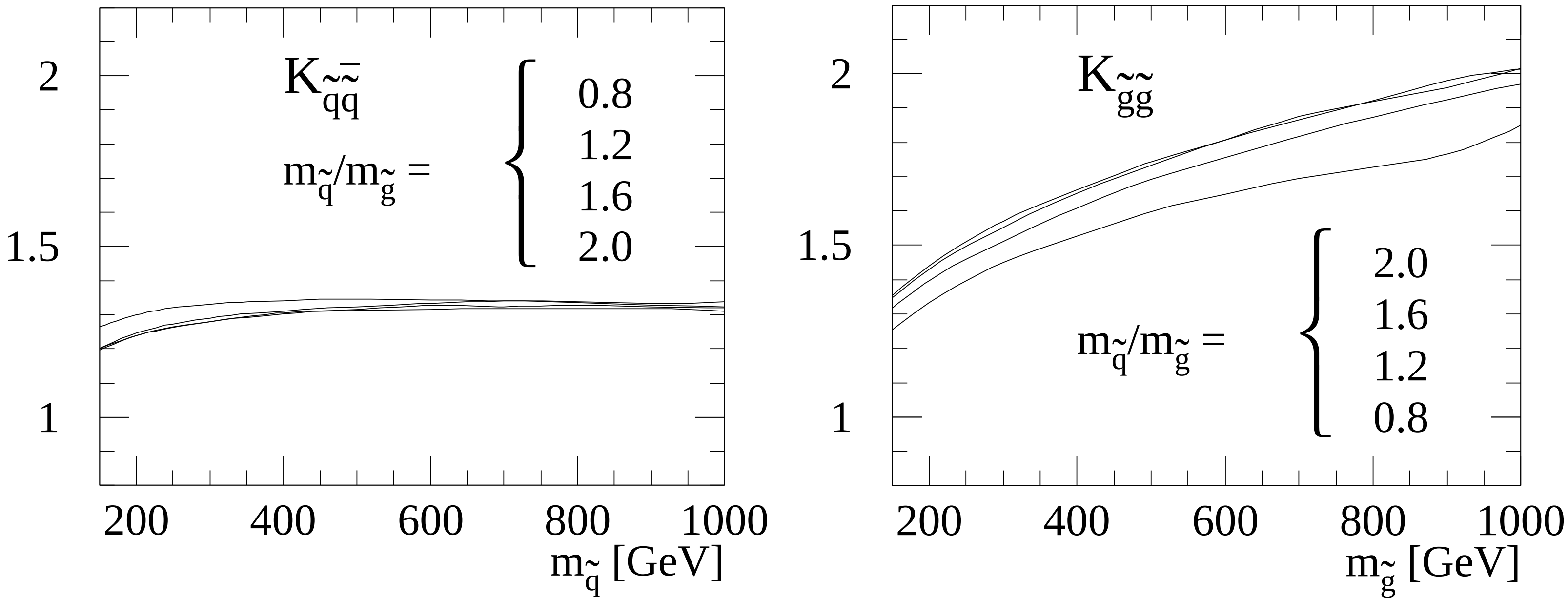

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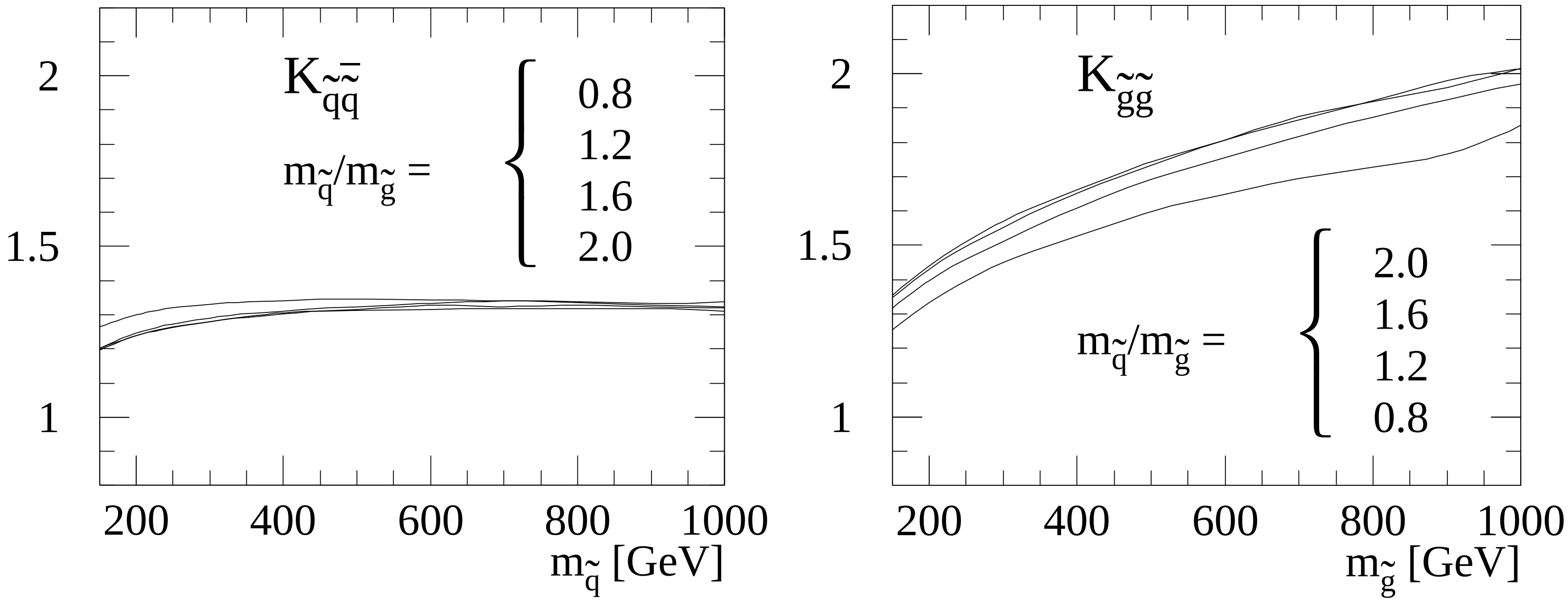

$\Rightarrow$ Large K factors, specially for $\tilde{g}\tilde{g}$

Note: assume all squarks ($\tilde{q}_L$,$\tilde{q}_R$) mass degenerate; no final state stops $\Rightarrow$

[*Beenakker, Krämer, Plehn, Spira, Zerwas'98*]

Higher-order effects in $\tilde{q}\bar{\tilde{q}}$ and $\tilde{g}\tilde{g}$

[*Beenakker, Höpker, Spira, Zerwas'96*]

- 100 % correction to $\sigma_{\tilde{g}\tilde{g}}$ at $m_{\tilde{g}} = 1\,\mathrm{TeV}$; 30% correction to $\sigma_{\tilde{q}\bar{\tilde{q}}}$ at $m_{\tilde{q}} = 1\,\mathrm{TeV}$

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  $\longrightarrow$ the closer to threshold the more important the logarithmic terms
- Additionally, for $\tilde{g}\tilde{g}$ production
  - both gg initial state (prevalent contribution) and $\tilde{g}\tilde{g}$ final state radiate strongly: C_A colour charge

$\longrightarrow$ expect a lot of (soft) gluon radiation

Soft-gluon corrections to $\tilde{q}\bar{\tilde{q}}$ and $\tilde{g}\tilde{g}$ production

- Seen already at NLO: at threshold [Beenakker, Höpker, Spira, Zerwas'96]

$$\hat{\sigma}^{\rm NLO}_{gg\to\tilde{g}\tilde{g}} \sim 4\pi\alpha_{\rm s}\hat{\sigma}^{\rm LO}_{gg\to\tilde{g}\tilde{g}}\left\{\frac{3}{2\pi^2}\log^2(8\beta^2) - \frac{29}{4\pi^2}\log(8\beta^2) - \frac{3}{2\pi^2}\log(8\beta^2)\log\left(\frac{\mu}{m_{\tilde{g}}}\right) + \frac{1}{16}\frac{1}{\beta}\right\}$$

with $\beta^2 = 1 - \frac{4m^2_{\tilde{g}}}{\hat{s}}$ and $\hat{\sigma}^{\rm LO}_{gg\to\tilde{g}\tilde{g}} \sim \frac{27}{64}\alpha_{\rm s}^2\pi\frac{\beta}{m^2_{\tilde{g}}}$

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In the limit of $\beta \rightarrow 0$ convergence of fixed-order expansion spoiled

Origin of the logarithmic terms

In general, in the IR region

$$\hat{\sigma}^{NLO} \sim \sum_{i,j} \int d\Phi^{LO} |M^{LO}|^2 \int d\Phi^{(1)} V_{i,j} \left[\Theta^{R}_{PS}(\ldots, p_i, p_j, \ldots) - \Theta^{V}_{PS}(\ldots, p_i + p_j, \ldots) \right]$$

Born level

singular
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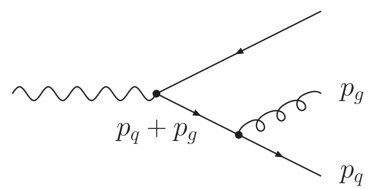
Born level singular process independent phase-space conditions

- $V_{i,j}$ singular in the soft and colinear limit



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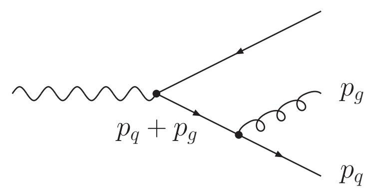

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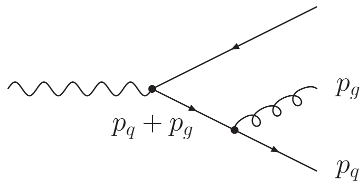

- $\Theta^R_{PS} \sim \Theta^V_{PS}$ in the soft and collinear region (IR and collinear safety)
- In other regions of phase space, real and virtual contributions can be highly unbalanced
- Single-gluon emission probability ($1 - z =$ fraction of energy carried out by the gluon):

$$\frac{d\omega(z)}{dz} \sim \alpha_s \left[\frac{1}{1-z} \ln \frac{1}{1-z} \right]_+ \;\Rightarrow\; \int_x^1 dz \frac{d\omega(z)}{dz} \sim \alpha_s \log^2(1-x)$$

$\Rightarrow$ if $x \to 1$ then double logarithmic divergences (soft and collinear limit)

- Factorization properties in the IR limit: double logarithmic structure carries to all orders

Resummation concept

At n-th order in α_s (wrt. LO) logarithmic contributions at threshold of the form

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Schematically

$$\begin{array}{lll} \text{NLO} & : & \alpha_s \Big(\boxed{c_{12} \log^2(\beta^2)} + \boxed{c_{11}\log(\beta^2)} + \boxed{c_{10}} \Big) \\ \text{NNLO} & : & \alpha_s^2 \Big(\boxed{c_{24} \log^4(\beta^2)} + \boxed{c_{13}\log^3(\beta^2)} + \boxed{c_{22}\log^2(\beta^2)} + \dots \Big) \\ & & \quad \dots \qquad\qquad \dots \qquad\qquad \dots \end{array}$$

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Schematically

$$\begin{array}{lll} \text{NLO} & : \alpha_s \Big(& c_{12}\log^2(\beta^2) + c_{11}\log(\beta^2) + c_{10} \Big) \\ \text{NNLO} & : \alpha_s^2 \Big(& c_{24}\log^4(\beta^2) + c_{13}\log^3(\beta^2) + c_{22}\log^2(\beta^2) + \ldots \Big) \\ & & \ldots \qquad\qquad \ldots \qquad\qquad \ldots \end{array}$$

LL	NLL	NNLL
$\alpha_s^n \log^{2n}$	$\alpha_s^n \log^{2n-1}$	$\alpha_s^n \log^{2n-2}$

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reorganization of the perturbative series = resummation

Schematically

$$\begin{array}{lll} \text{NLO} & : \alpha_s \Big(& c_{12}\log^2(\beta^2) + c_{11}\log(\beta^2) + c_{10} \Big) \\ \text{NNLO} & : \alpha_s^2 \Big(& c_{24}\log^4(\beta^2) + c_{13}\log^3(\beta^2) + c_{22}\log^2(\beta^2) + \ldots \Big) \\ & & \ldots \qquad\qquad \ldots \qquad\qquad \ldots \end{array}$$

LL	NLL	NNLL
$\alpha_s^n \log^{2n}$	$\alpha_s^n \log^{2n-1}$	$\alpha_s^n \log^{2n-2}$

- Each “order”: infinite number of terms
- Proven: Such reorganized series of terms sums up $\rightarrow$ exponentiate

  [*Dokshitzer et al'78*][*Parisi, Petronzio'79*][*Collins, Soper'81-83*][*Altarelli et al'84*][*Collins, Soper, Sterman'85*]

Resummation

Generic hard scattering process:

$$\hat{\sigma}(z) \sim \hat{\sigma}_0 \left[1 + \sum_{n=1}^{\infty} \int_0^1 dz_1 \ldots dz_n \frac{d\omega_n(z_1, \ldots, z_n)}{dz_1 \ldots dz_n} \Theta_{PS}^{(n)}(z, z_1, \ldots, z_n) \right]$$

- Dynamical factorization (universal, process independent)

  QCD: gluon emission correlated (colour charge) but in the soft limit QED-like factorization

  [*Ermolaev, Fadin '81*][*Bassetto, Ciafaloni, Marchesini '83*]

$$\frac{d\omega_n(z_1, \ldots, z_n)}{dz_1 \ldots dz_n} = \frac{1}{n!} \prod_{i=1}^{n} \frac{d\omega(z_i)}{dz_i} .$$

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- Phase–space factorization depends on the process: Θ_{PS} contains kinematical constraints defining physical cross section

$$\Theta_{PS}^{(n)}(z, z_1, ..., z_n) = \prod_{i=1}^{n} \Theta_{PS}(z, z_i)$$

$$\hat{\sigma}(z) \sim \hat{\sigma}_0 \left\{ 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \left[\int_0^1 dz_i \frac{d\omega(z_i)}{dz_i} \Theta_{PS}(z, z_i) \right]^n \right\} \sim \hat{\sigma}_0 \exp \left[\int_z^1 dz' \frac{d\omega(z')}{dz'} \Theta_{PS}(z, z') \right] \sim \hat{\sigma}_0 \exp \left[\alpha_s L^2 + \ldots \right]$$

- In practice phase–space factorization often occurs in the space conjugate to the space of kinematic variables

Threshold resummation

Resummation of threshold logarithms is carried out in the Mellin moment space, where the cross section factorizes:

$\left(g^{(N)} = \int_0^1 dz\, z^{N-1}\, g(z)\right)$

$$\sigma^{(N)}_{h_1 h_2} = \sum_{i,j} f^{(N+1)}_{i/h_1}(\mu_F) f^{(N+1)}_{j/h_2}(\mu_F) \hat{\sigma}^{(N)}_{ij}(\mu_F)$$

and the logarithmic terms exponentiate

$$\hat{\sigma}^{(N)} = \hat{\sigma}^{(N)}_0 \mathcal{C} \exp(\mathcal{S})$$

($\mathcal{C}$ contains finite contributions)

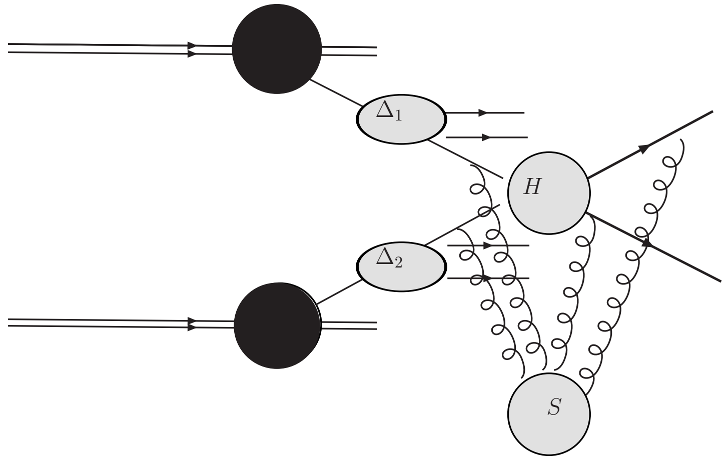
$$\mathcal{S} = L f_1(\alpha_s L) + f_2(\alpha_s L) + \alpha_s f_3(\alpha_s L) + \ldots \qquad L = \ln(N)$$

$$LL \qquad NLL \qquad NNLL \qquad \ldots$$

Resummation

$\Rightarrow$ In general, resummation follows from factorization

(factorization $\Rightarrow$ evolution eq. $\Rightarrow$ exponentiation)



Schematically, in the space of Mellin moments:

$$\begin{aligned}
\sigma(N) &= H(p_1/\mu, p_2/\mu, \zeta_i) S(Q/\mu N, \zeta_i) \Delta_1(p_1\zeta_1/\mu, Q/\mu N) \Delta_2(p_2\zeta_2/\mu, Q/\mu N) \\
\mu \frac{d}{d\mu}\sigma &= 0 \qquad \Rightarrow \qquad \mu\frac{d}{d\mu}\ln H = -\gamma_H \qquad \mu\frac{d}{d\mu}\ln\Delta = -\gamma_\Delta \qquad \mu\frac{d}{d\mu}\ln S = -\gamma_S \\
(\gamma_H + \gamma_S + \sum_i \gamma_\Delta &= 0)
\end{aligned}$$

$$S(Q_S/\mu) = S(1)\exp\left[-\int_{Q_S}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_S(\bar{\mu})\right]$$

Resummation for colour singlet production

Drell-Yan, Higgs, ... [*Catani, Trentadue'89*] [*Sterman'87*]

$$\sigma_{ij}^{(N)} = \hat{\sigma}_{0,\, ij}^{(N)} C_{ij} \Delta_i^{(N)} \Delta_j^{(N)} \Delta_{\mathrm{int},ij}^{(N)}$$

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$\Delta_i^{(N)}$ soft gluons collinear to initial state partons (universal)

$$\Delta_i^{(N)} = \exp\left\{ \int_0^1 dz \frac{z^{N-1} - 1}{1-z} \int_{\mu_F^2}^{(1-z)^2 Q^2} \frac{d\mu^2}{\mu^2} A_i(\alpha_s(\mu^2)) \right\}$$

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Also process dependent:

C_{ij} N-independent finite coefficients

$\hat{\sigma}_{0,\, ij}^{(N)}$ LO partonic cross sections in N space

Resummation for non-trivial colour flow

- Soft wide-angle gluon emission sensitive to colour flow of the underlying hard-scattering
  - for $\geq$ 4 partons: $\hat{\sigma}_0^{(N)} \Delta_{\rm int}^{(N)}$ has to be replaced by $\sum_{IJ} H_{0,IJ}^{(N)} S_{JI}^{(N)}$

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$$\left(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g}\right) S_{JI}^{(N)} = -\Gamma^{\dagger}_{JK} S(N)_{KI} - S(N)_{JL} \Gamma_{LI}$$

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- The general solution for the soft function $S_{JI}^{(N)}$

$$\begin{aligned}\mathrm{Tr}\left(H^{(N)}\left(\frac{Q}{\mu}\right)S^{(N)}\left(\frac{Q}{\mu}\right)\right) = \; & \mathrm{Tr}\quad\left[H^{(N)}(\frac{Q}{\mu})\bar{P}\exp\left(\int_{\mu}^{Q/N}\frac{dq}{q}\Gamma^{\dagger}(\alpha_{\rm s}(q^2))\right)\right.\\ & \times\quad\left. S^{(N)}(1)P\exp\left(\int_{\mu}^{Q/N}\frac{dq}{q}\Gamma(\alpha_{\rm s}(q^2))\right)\right]\end{aligned}$$

with $\Gamma(g) = -\frac{g}{2}\frac{\partial}{\partial g}{\rm Res}_{\epsilon\to 0}Z(g,\epsilon)$ $\qquad$ Z = renormalization constant for S

Resummation for $2 \to 2$ with colour flow

Simplification:

In orthogonal basis in colour space for which $\Gamma^{ij\to kl}$ is diagonal [*Kidonakis, Sterman'96-97*][*Bonciani, Catani, Mangano, Nason'98*]

$$\hat{\sigma}^{(N)}_{ij\to kl} = \sum_I \hat{\sigma}^{(N)}_{0,ij\to kl,I}\, \widetilde{C}_{ij\to kl,I}\, \Delta^i_{(N+1)}\, \Delta^j_{(N+1)}\, \Delta^{(N+1)}_{\rm (int),ij\to kl,I}$$

→ I corresponds to different colour channels

→ assume massive final state (no final state jet functions)

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Radiative factor for soft non-collinear emission

$$\Delta^{(N+1)}_{\rm (int),ij\rightarrow kl,I} = \exp\left\{\int_0^1 dz\, \frac{z^{N-1}-1}{1-z} D_{ij\rightarrow kl,I}(\alpha_{\rm s}((1-z)^2Q^2))\right\}$$

related to Γ by

$$D_{ij\rightarrow kl,I} = 2{\rm Re}(\lambda_I) \qquad \text{for } \Gamma^{ij\rightarrow kl} = {\rm diag}(\lambda_1,\dots)$$

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In general need to know $\hat{\sigma}^{(N)}_{0,ij\to kl}$, $D^{(1)}_{ij\to kl}$, $\widetilde{C}_{ij\to kl}$ coefficients in each colour channel

Resummation for $\tilde{q}\bar{\tilde{q}}$ production

Colour structure same as for the heavy quark production $\Rightarrow$ $\Gamma^{ij\to\tilde{q}\bar{\tilde{q}}} = \Gamma^{ij\to Q\bar{Q}}$

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- $C_{ij\to\tilde{q}\bar{\tilde{q}}}$ coefficients contain N-independent terms and Coulomb corrections (also possible to resum); for this calculation keep $\widetilde{C}^{(1)}_{ij\to\tilde{q}\bar{\tilde{q}},I} = 1$

Anomalous dimension for massive colour-octet pair

NLL anomalous dimensions known for all $2 \rightarrow 2$ massless QCD processes

[*Kidonakis, Oderda, Sterman'98*][*Bonciani, Catani, Mangano, Nason'03*]

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- Analogously to $ij \to gg$, in the space of colour exchanges
  - $qq \to \tilde{g}\tilde{g}$ colour basis c_I consists of 3 tensors $\Rightarrow \Gamma q\bar{q} \to \tilde{g}\tilde{g}$ is a 3×3 matrix
  - $gg \to \tilde{g}\tilde{g}$ colour basis c_I consists of 8 tensors $\Rightarrow \Gamma gg \to \tilde{g}\tilde{g}$ is a 8×8 matrix

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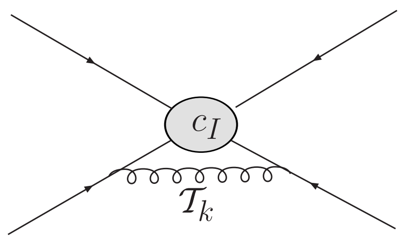
- Evaluation of $\Gamma^{ij \to \tilde{g}\tilde{g}}$ requires one-loop integrals for gluon exchanges between all legs (vertex corrections) + self-energies; calculated in the eikonal approximation [*Kidonakis, Sterman'96*]

  Schematically:



$$\Gamma_{JI} = \sum_k \mathcal{T}_k(c_I)c_J^\dagger \left(-\frac{g}{2}\frac{\partial}{\partial g} I_k \Big|_{\frac{1}{\epsilon}\ \mathrm{pole}} \right)$$

Example: anomalous dimension $\Gamma^{q\bar{q}\to\tilde{g}\tilde{g}}$

[*AK, L.Motyka'08*]

- Orthogonal s-channel basis ($\{c_I^q\}$ correspond to $\mathbf{1}$, $\mathbf{8}_S$ and $\mathbf{8}_A$ representations)

$$c_1^q = \delta^{\alpha_1\alpha_2}\,\delta^{a_3a_4}, \quad c_2^q = T^b_{\alpha_2\alpha_1} d^{ba_3a_4}, \quad c_3^q = iT^b_{\alpha_2\alpha_1} f^{ba_3a_4},$$

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- In this basis

$$\Gamma^{q\bar{q}\to\tilde{g}\tilde{g}} = \frac{\alpha_s}{\pi}\left[\begin{pmatrix} 6\bar{S} & 0 & -\Omega \\ 0 & 3\bar{S}+\frac{3}{2}\Lambda & -\frac{3}{2}\Omega \\ -2\Omega & -\frac{5}{6}\Omega & 3\bar{S}+\frac{3}{2}\Lambda \end{pmatrix} - \frac{4}{3}i\pi\,\hat{\mathbf{I}}\right]$$

$$\text{with} \quad \Lambda \equiv \bar{T}+\bar{U} \qquad \Omega \equiv \bar{T}-\bar{U}$$

$$\bar{T} \equiv \ln\left(\frac{m^2-\hat{t}}{\sqrt{m^2\hat{s}}}\right) - \frac{1-i\pi}{2}, \qquad \bar{U} \equiv \ln\left(\frac{m^2-\hat{u}}{\sqrt{m^2\hat{s}}}\right) - \frac{1-i\pi}{2}, \qquad \bar{S} \equiv -\frac{L_\beta+1}{2}$$

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- Similar procedure to obtain $\Gamma^{gg\to\tilde{g}\tilde{g}}$

Threshold limit for $\Gamma^{ij\to\tilde{g}\tilde{g}}$

- At the threshold $\hat{s} \to 4m^2$, $\Gamma^{ij\to\tilde{g}\tilde{g}}$ matrices for the s-channel colour bases become diagonal

$$\Gamma^{gg\to\tilde{g}\tilde{g}} \quad \to \quad \frac{\alpha_s}{\pi}\,\mathrm{diag}\,(\gamma_1^g,\,\gamma_2^g,\,\ldots,\,\gamma_8^g)\,,$$

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- The resummation formula simplifies with [*Kidonakis, Oderda, Sterman'98*]

$$D^{(1)}_{gg\to\tilde{g}\tilde{g},\,I} \quad = \quad 2\mathrm{Re}(\gamma_I^g)$$

$$D^{(1)}_{q\bar{q}\to\tilde{g}\tilde{g},\,I} \quad = \quad 2\mathrm{Re}(\gamma_I^q)$$

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$$\begin{aligned} \Gamma^{gg\to\tilde{g}\tilde{g}} &\to \frac{\alpha_s}{\pi}\,\mathrm{diag}\,(\gamma_1^g,\, \gamma_2^g,\, \ldots,\, \gamma_8^g)\,, \\ \Gamma^{q\bar{q}\to\tilde{q}\tilde{q}} &\to \frac{\alpha_s}{\pi}\,\mathrm{diag}\,(\gamma_1^q,\, \gamma_2^q,\, \gamma_3^q) \end{aligned}$$

- The resummation formula simplifies with [*Kidonakis, Oderda, Sterman'98*]

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- Values of the quadratic Casimir operators for the SU(3) representations for the outgoing state $\to$ soft gluon radiation from the total colour charge of the heavy-particle pair produced at threshold
- Need $\hat{\sigma}^{(N)}_{0,ij\to\tilde{g}\tilde{g},I}$, coefficient $\widetilde{C}^{(1)}_{ij\to\tilde{g}\tilde{g},I} = 1$

Resummation-improved NLL+NLO total cross section

NLL resummed expression has to be matched with the full NLO result

$$
\begin{aligned}
\sigma^{\rm (match)}_{h_1 h_2 \to kl}(\rho, m^2, \{\mu^2\}) \;&=\; \sum_{i,j=q,\bar{q},g} \int_{C_{\rm M}P-i\infty}^{C_{\rm M}P+i\infty} \frac{dN}{2\pi i}\, \rho^{-N}\, f^{(N+1)}_{i/h_1}(\mu_F^2)\, f^{(N+1)}_{j/h_2}(\mu_F^2) \\
&\times\; \left[\hat{\sigma}^{\rm (res)}_{ij\to kl,N}(m^2,\{\mu^2\}) \;-\; \hat{\sigma}^{\rm (res)}_{ij\to kl,N}(m^2,\{\mu^2\})\Big|_{\rm NLO} \right] \\
&+\; \sigma^{\rm NLO}_{h_1 h_2 \to kl}(\rho, m^2, \{\mu^2\})\,,
\end{aligned}
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$$\times \left[\hat{\sigma}^{(\text{res})}_{ij\to kl,N}(m^2, \{\mu^2\}) - \hat{\sigma}^{(\text{res})}_{ij\to kl,N}(m^2, \{\mu^2\})\Big|_{\text{NLO}} \right]$$
$$+ \sigma^{\text{NLO}}_{h_1 h_2 \to kl}(\rho, m^2, \{\mu^2\}),$$

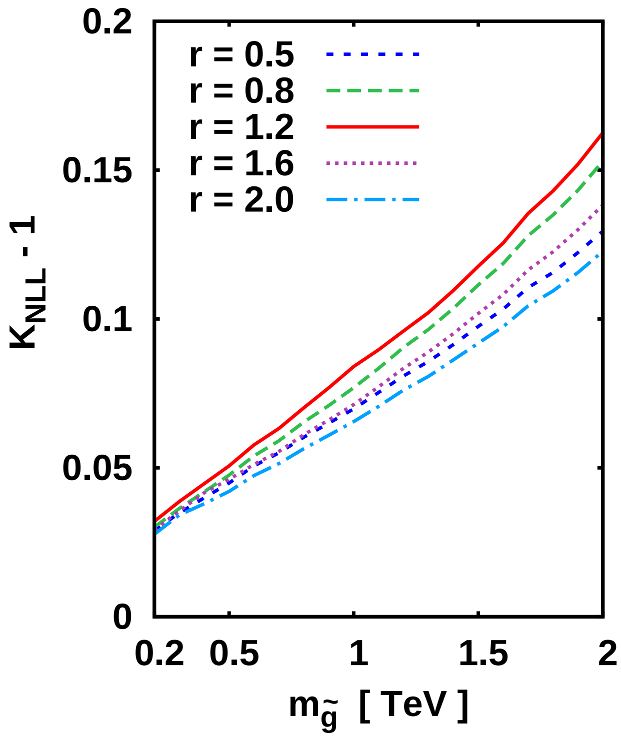
- Inverse Mellin transform evaluated using a contour in the complex N space according to 'Minimal Prescription' [*Catani, Mangano, Nason Trentadue'96*]
- NLO cross sections evaluated with publicly available code PROSPINO

[*Beenakker, Hoepker, Krämer, Plehn, Spira, Zerwas*]

[*Plehn, http://www.ph.ed.ac.uk/ tplehn/prospino/*]

NLL gluino-pair production at the LHC

[*AK, L. Motyka'08*]



$$\mathrm{K}^{\mathrm{NLL}} = \frac{\sigma^{\mathrm{match}}}{\sigma^{\mathrm{NLO}}}$$

$$r = \frac{m_{\tilde{g}}}{m_{\tilde{q}}}$$

$$(\mu_F = \mu_R = m_{\tilde{g}})$$

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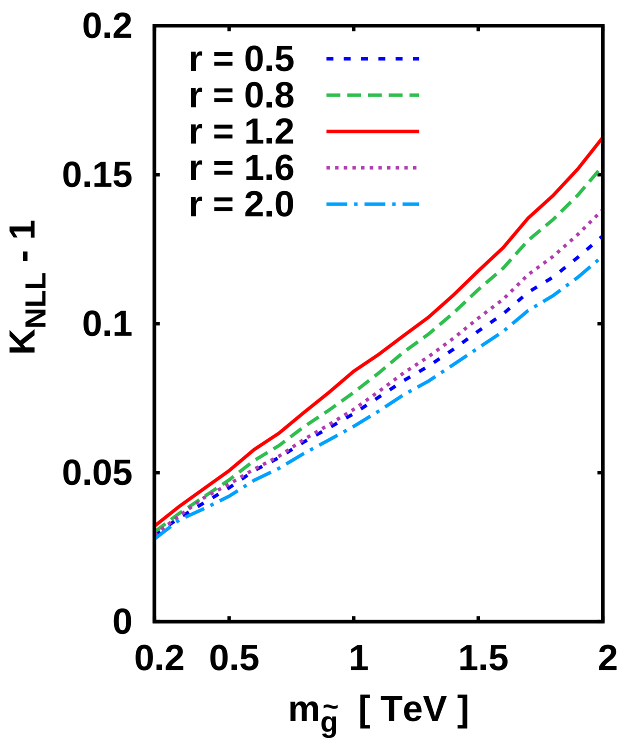

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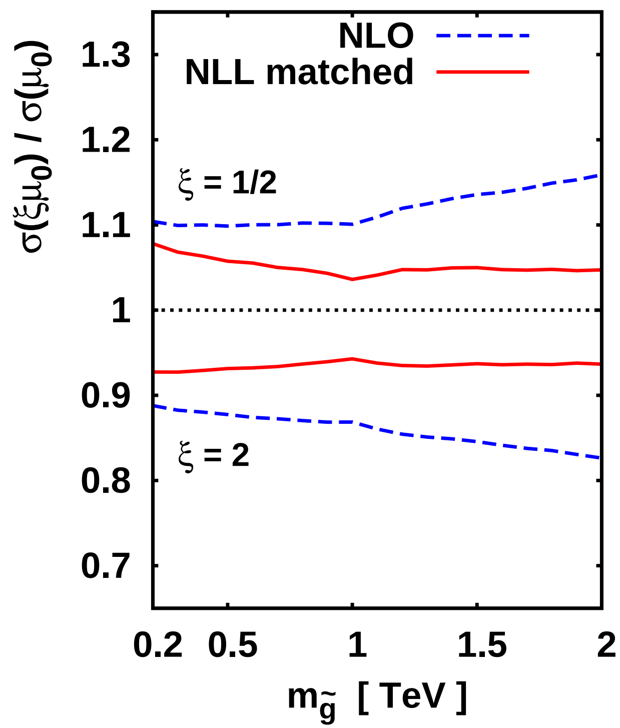


$$\sigma^{\mathrm{NLO}}(\mu = \xi m_{\tilde{g}})/\sigma^{\mathrm{NLO}}(\mu = m_{\tilde{g}})$$

vs.

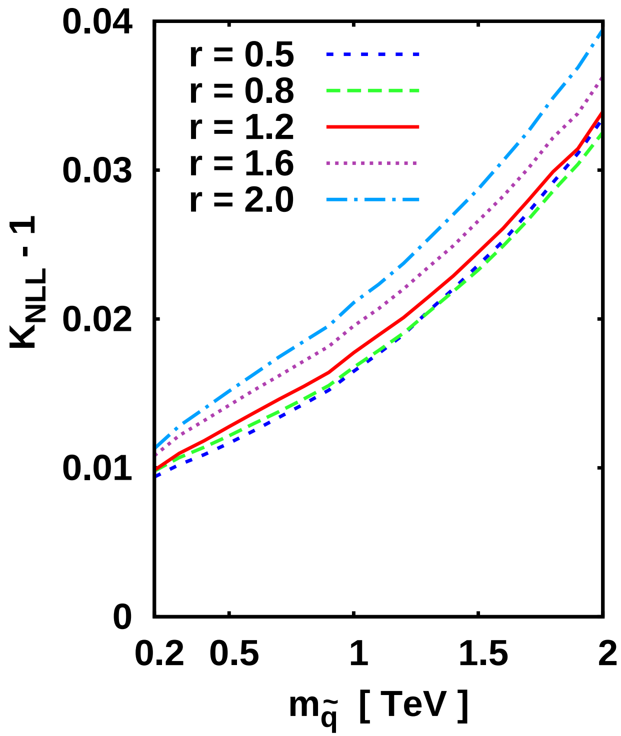
$$\sigma^{(\mathrm{match})}(\mu = \xi m_{\tilde{g}})/\sigma^{(\mathrm{match})}(\mu = m_{\tilde{g}})$$

for $\xi = 0.5$ and $\xi = 2$

$$(\mu = \mu_F = \mu_R; r = 1.2)$$

NLL squark-antisquark production at the LHC

[*AK, L. Motyka'08*]



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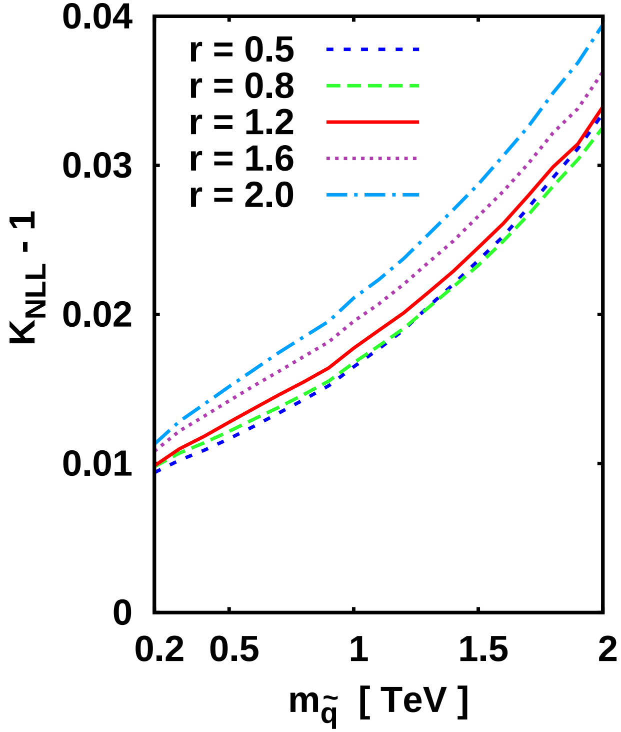

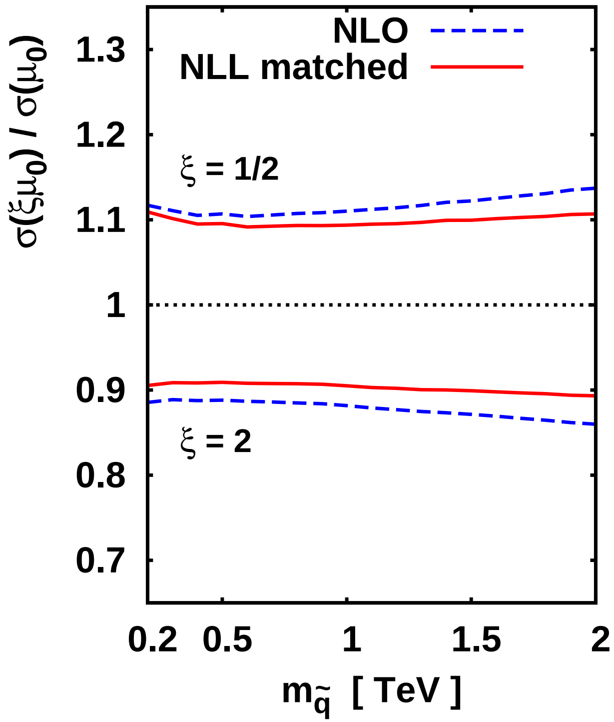

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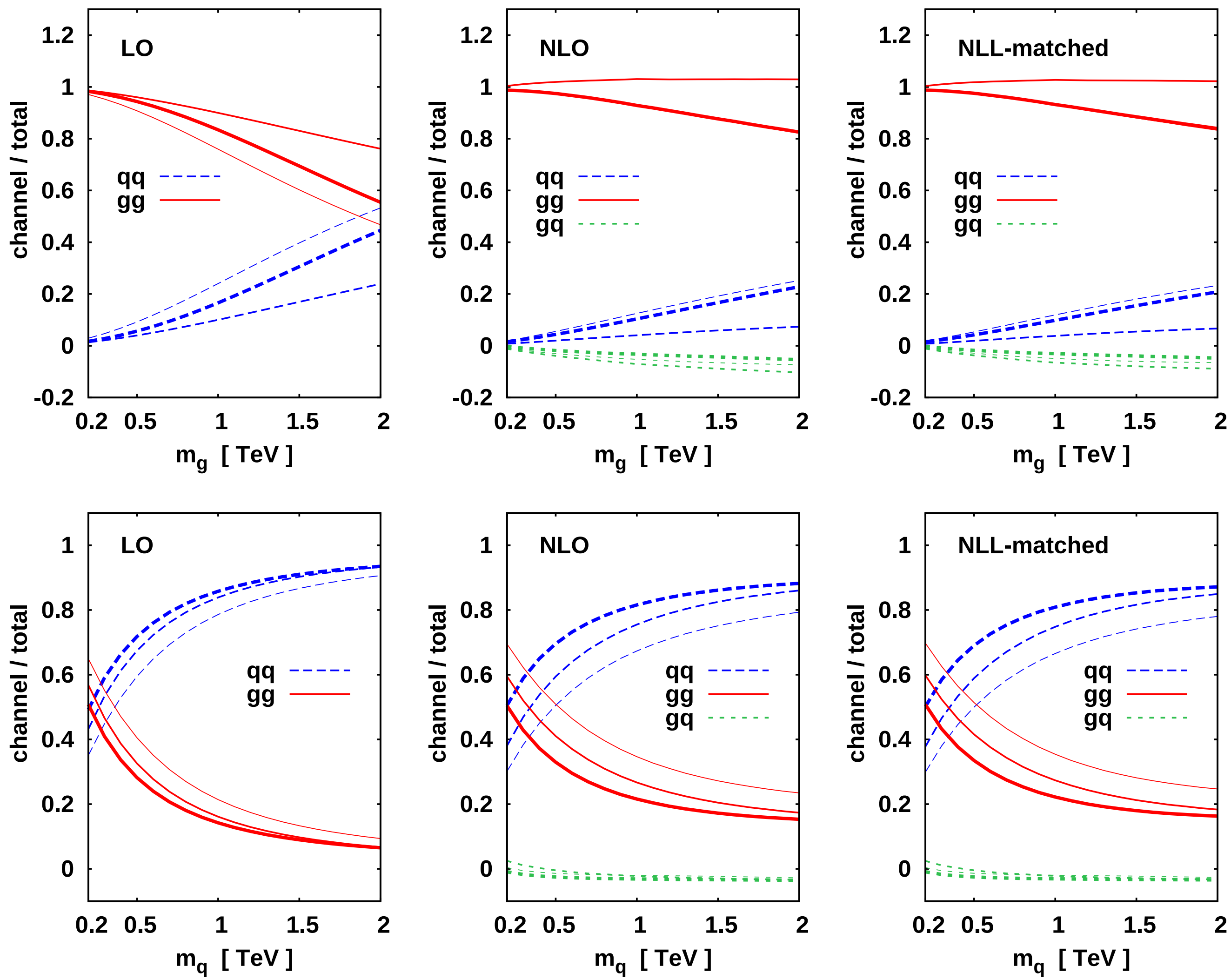
Squark and gluino production at the LHC

$pp \to \tilde{g}\tilde{g}$: [AK, L. Motyka, in prep.]

$pp \to \tilde{q}\bar{\tilde{q}}$:



(thick line: $m_{\tilde{g}}/m_{\tilde{q}} = 0.5$, medium: $m_{\tilde{g}}/m_{\tilde{q}} = 1.2$, thin: $m_{\tilde{g}}/m_{\tilde{q}} = 2$)

Coulomb corrections

Leading Coulomb corrections

$$\alpha_s^n/\beta^n \qquad \text{wrt. LO}$$

can also be resummed [*Fadin, Khoze, Sjöstrand' 90*] [*Catani, Mangano, Nason, Trentadue'96*]

$$\hat{\sigma}^{\text{Coul}}_{ij\to kl} = \sum_I \hat{\sigma}^{\text{LO}}_{ij\to kl,\,I}\;\frac{X_{ij\to kl,\,I}}{1-\exp(-X_{ij\to kl,\,I})}$$

$$X_{ij\to kl,\,I} = \pi\alpha_s C_{ij\to kl,\,I}\,/\beta$$

$C_{ij\to kl,\,I}$ are appropriate colour factors

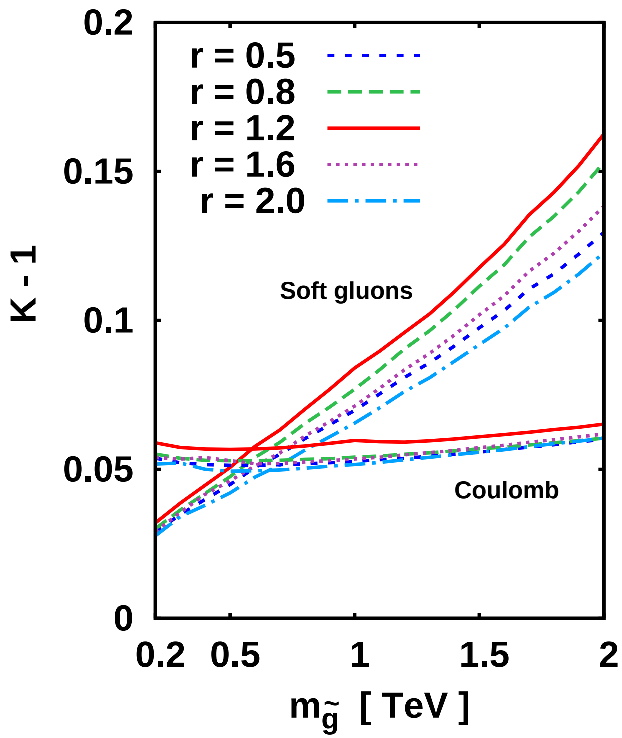
Define the "Coulomb K-factor" as

$$K^{\text{Coul}}_{ij\to kl} = \frac{\hat{\sigma}^{\text{Coul}}_{ij\to kl} - \hat{\sigma}^{\text{Coul}}_{ij\to kl}|_{\text{NLO}}}{\sigma^{\text{NLO}}_{ij\to kl}}$$

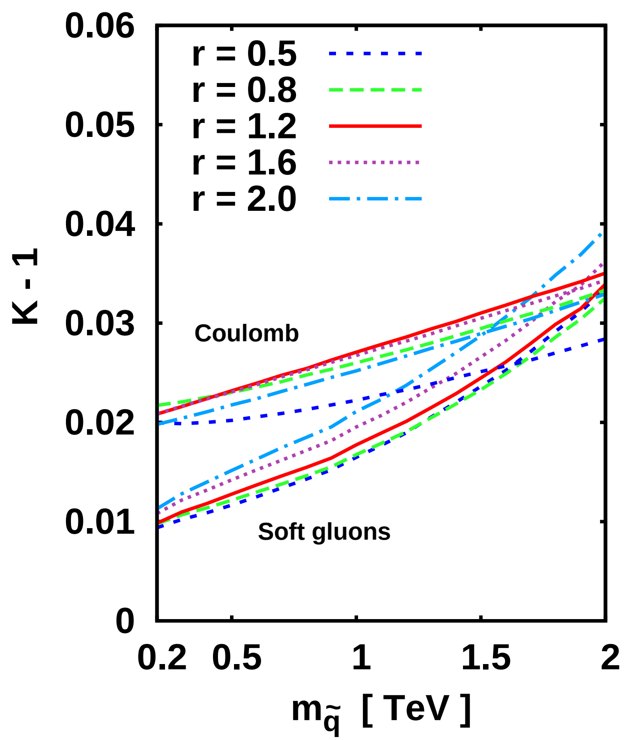
Threshold effects for $\tilde{g}\tilde{g}$ and $\tilde{q}\bar{\tilde{q}}$ production at the LHC

[*AK, L. Motyka, in prep.*]

$pp \to \tilde{g}\tilde{g}$



$pp \to \tilde{q}\bar{\tilde{q}}$

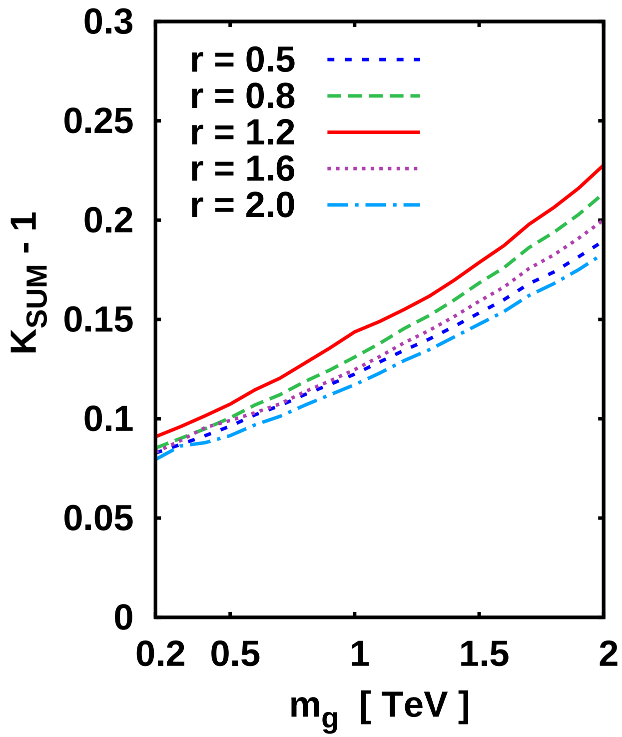


Threshold effects for $\tilde{g}\tilde{g}$ and $\tilde{q}\bar{\tilde{q}}$ production at the LHC

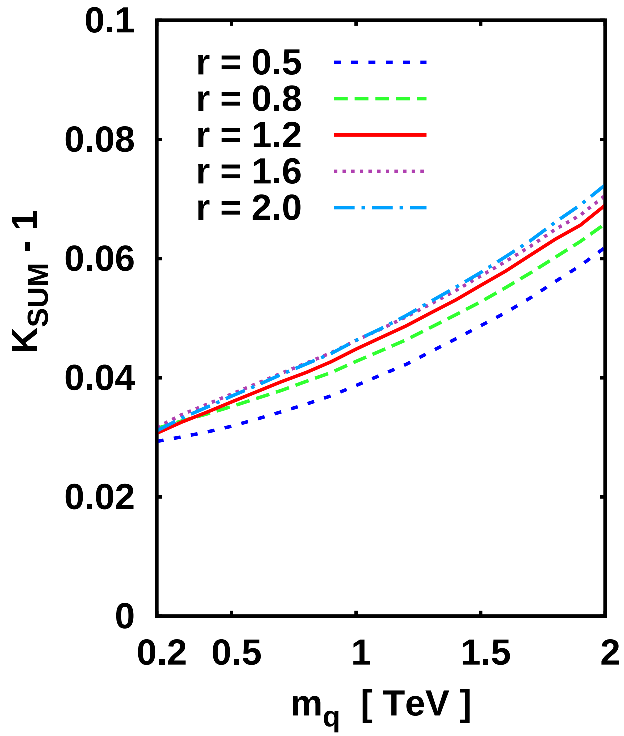
[AK, L. Motyka, in prep.]

Soft + Coulomb corrections

$pp \to \tilde{g}\tilde{g}$



$pp \to \tilde{q}\bar{\tilde{q}}$



Summary

- If SUSY realized in Nature, $\tilde{q}\bar{\tilde{q}}$ and $\tilde{g}\tilde{g}$ production will be among the most dominant mechanisms of sparticle production at the LHC
  - total cross sections will be the first measured quantities

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