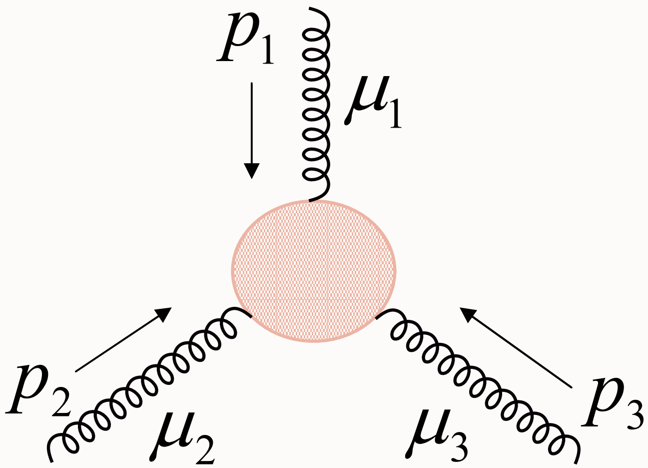
The Renormalization Scale Problem

Stan Brodsky, SLAC/IPPP

$$\hat{\Gamma}_{\mu_1\mu_2\mu_3} =$$



PHYSICAL REVIEW D **74**, 054016 (2006)

Form factors of the gauge-invariant three-gluon vertex

Michael Binger* and Stanley J. Brodsky†

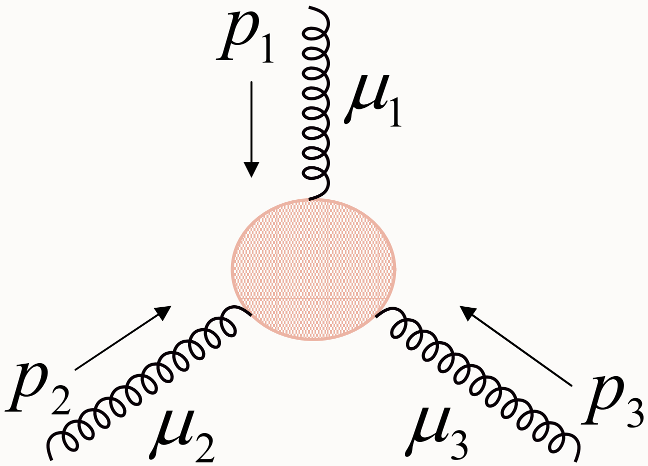
The Renormalization Scale Problem

$$\rho(Q^2) = C_0 + C_1\alpha_s(\mu_R) + C_2\alpha_s^2(\mu_R) + \cdots$$

$$\mu_R^2 = CQ^2$$

Is there a way to set the renormalization scale μ_R ?

What happens if there are multiple physical scales ?



PHYSICAL REVIEW D VOLUME 28, NUMBER 1 1 JULY 1983

On the elimination of scale ambiguities in perturbative quantum chromodynamics

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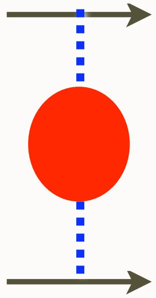
(Received 23 November 1982)

We present a new method for resolving the scheme-scale ambiguity that has plagued perturbative analyses in quantum chromodynamics (QCD) and other gauge theories. For Abelian theories the method reduces to the standard criterion that only vacuum-polarization insertions contribute to the effective coupling constant. Given a scheme, our procedure automatically determines the coupling-constant scale appropriate to a particular process. This leads to a new criterion for the convergence of perturbative expansions in QCD. We examine a number of well known reactions in QCD, and find that perturbation theory converges well for all processes other than the gluonic width of the Υ. Our analysis calls into question recent determinations of the QCD coupling constant based upon Υ decay.

Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$

t



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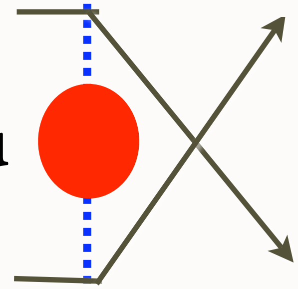
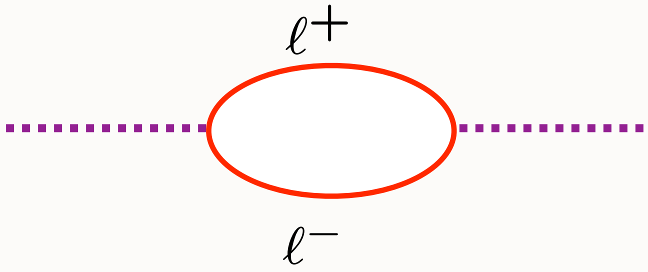


$$\alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

Gell Mann-Low Effective Charge

QED One-Loop Vacuum Polarization



$t = -Q^2 < 0$

(t spacelike)

$$\Pi(Q^2) = \frac{\alpha(0)}{3\pi}[\frac{5}{3} - \frac{4m^2}{Q^2} - (1 - \frac{2m^2}{Q^2})\sqrt{1 + \frac{4m^2}{Q^2}} \log \frac{1 + \sqrt{1 + \frac{4m^2}{Q^2}}}{|1 - \sqrt{1 + \frac{4m^2}{Q^2}}|}]$$

Analytically continue to timelike t: Complex

$$\Pi(Q^2) = \frac{\alpha(0)}{15\pi} \frac{Q^2}{m^2}$$ $Q^2 << 4M^2$ **Serber-Uehling**

$$\Pi(Q^2) = \frac{\alpha(0)}{3\pi} \frac{\log Q^2}{m^2}$$ $Q^2 >> 4M^2$ **Landau Pole**

$$\beta = \frac{d(\frac{\alpha}{4\pi})}{d \log Q^2} = \frac{4}{3}(\frac{\alpha}{4\pi})^2 n_\ell > 0$$

$$\beta_{\mathrm{MS}}(\alpha)=\sum_{i=1}^{4}\beta_i\left(\frac{\alpha}{4\pi}\right)^{i+1}$$

$$=\tfrac{4}{3}N\left(\frac{\alpha}{4\pi}\right)^2+4N\left(\frac{\alpha}{4\pi}\right)^3-(2N+\tfrac{44}{9}N^2)\left(\frac{\alpha}{4\pi}\right)^4$$

$$-\{46N+[-\tfrac{760}{27}+\tfrac{832}{9}\zeta(3)]N^2+\tfrac{1232}{243}N^3\}\left(\frac{\alpha}{4\pi}\right)^5$$

The analytic four-loop corrections
to the QED β-function in the MS scheme
and to the QED ψ-function.
Total reevaluation

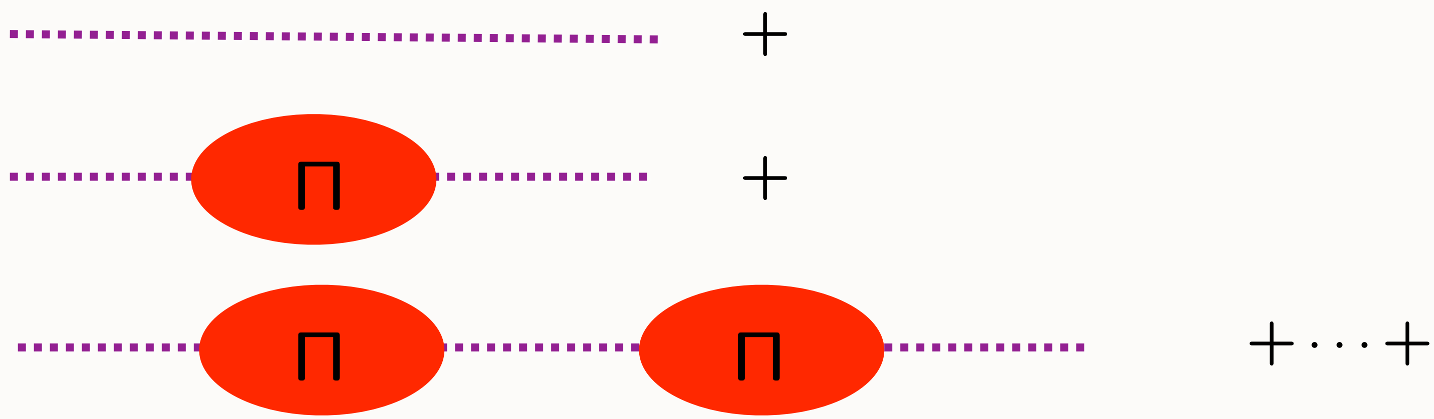
S.G. Gorishny [1], A.L. Kataev, S.A. Larin and L.R. Surguladze [2]
Institute of Nuclear Research, Academy of Sciences of the USSR, SU-117 312 Moscow, USSR

Phys.Lett.B256:81-86,1991

QED Effective Charge

$$\alpha(t) = \frac{\alpha(0)}{1 - \Pi(t)}$$

All-orders lepton loop corrections to dressed photon propagator



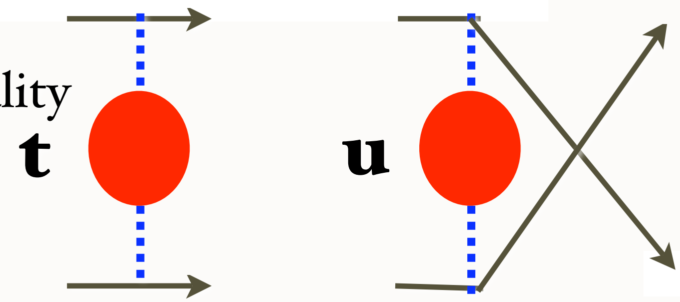
$$\alpha(t) = \frac{\alpha(t_0)}{1 - \Pi(t, t_0)} \qquad \Pi(t, t_0) = \frac{\Pi(t) - \Pi(t_0)}{1 - \Pi(t_0)}$$

Initial scale t_o is arbitrary -- Variation gives RGE Equations

Physical renormalization scale t not arbitrary

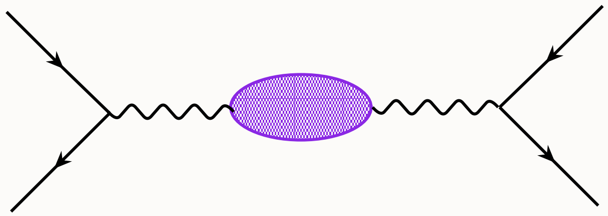
Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$



- Two separate physical scales: t, u = photon virtuality
- Gauge Invariant. Dressed photon propagator
- Sums all vacuum polarization, non-zero beta terms into running coupling.
- If one chooses a different scale, one can sum an infinite number of graphs -- but always recover same result!
- Number of active leptons correctly set
- Analytic: reproduces correct behavior at lepton mass thresholds
- ~~No renormalization scale ambiguity!~~

$$e^+e^- \to \mu^+\mu^-$$



$$\mu_R^2 = s$$

Scale of $\alpha(\mu_r)$ unique !

$$M \propto \alpha(s)$$

The QED Effective Charge

- Complex
- Analytic through mass thresholds
- Distinguishes between timelike and spacelike momenta

Analyticity essential !

$$M(e^+e^- \to e^+e^-) \propto \alpha(s)$$

Has correct analytic / unitarity thresholds for ImM at $s = 4m^2_{\ell^+\ell^-}$

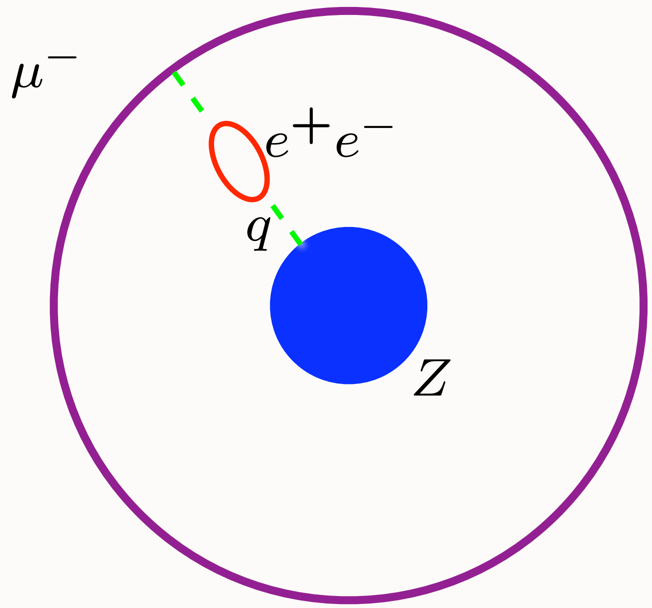
No other scale correct. If one chooses another scale, e.g.,

$$\mu_R^2 = 0.9s,$$

then must resum infinite number of vacuum polarization diagrams.

Recover $\alpha(s)$.

Example in QED: Muonic Atoms



$$V(q^2) = -\frac{Z\alpha_{QED}(q^2)}{q^2}$$

$$\mu_R^2 \equiv q^2$$

$$\alpha_{QED}(q^2) = \frac{\alpha_{QED}(0)}{1-\Pi(q^2)}$$

Scale is unique: Tested to ppm

Gyulassy: Higher Order VP verified to 0.1% precision in μ Pb

The Renormalization Scale Problem

- No renormalization scale ambiguity in QED
- Gell Mann-Low QED Coupling can be defined from physical observable
- Sums all Vacuum Polarization Contributions
- Recover conformal series
- Renormalization Scale in QED scheme: Identical to Photon Virtuality
- Analytic: Reproduces lepton-pair thresholds
- Examples: muonic atoms, g-2, Lamb Shift

Gyulassy: Higher Order VP verified to 0.1% precision in μ Pb

- Time-like and Space-like QED Coupling related by analyticity
- Uses Dressed Skeleton Expansion

Conventional wisdom in QCD concerning scale setting

- Renormalization scale "unphysical": No optimal physical scale
- Can ignore possibility of multiple physical scales
- Accuracy of PQCD prediction can be judged by taking arbitrary guess $\mu_R = Q$ with an arbitrary range $Q/2 < \mu_R < 2Q$
- Factorization scale should be taken equal to renormalization scale $\mu_F = \mu_R$

These assumptions are untrue in QED and thus they cannot be true for QCD

Scale and Scheme Ambiguity

In any perturbative series $$R(Q) = \sum_{n=0}^{N} R_n(Q,\mu)\alpha_s^n(\mu)$$

You can change the scale of the last term :

$$\alpha_s(\tilde{\mu}) = \alpha_s(\mu) - \frac{(\alpha_s(\mu))^2}{2\pi}\beta_0 \log(\tilde{\mu}/\mu)$$

Or the scheme of the last term :

$$\tilde{\alpha}_s(\mu) = \alpha_s(\mu) + C(\alpha_s(\mu))^2$$

The result is formally the same to the order calculated

⇒ The prediction is ambiguous

Convergence of the Series ?

It is commonly believed that the series diverges!

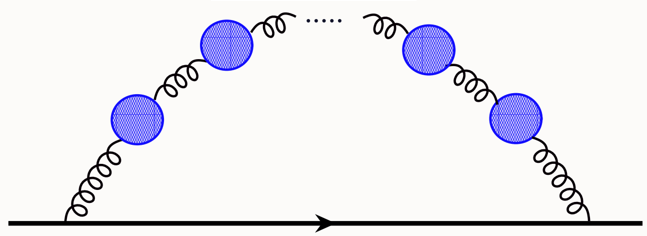
$\Longleftrightarrow$ ***Renormalons***

$$R(Q) = \sum_{n=0}^{N} R_n(Q,\mu)\alpha_s^n(\mu) \qquad R_n \propto n!$$

$$\int d^4k\,\alpha_s(k^2) f(k^\mu, p_i^\mu) \to \infty$$

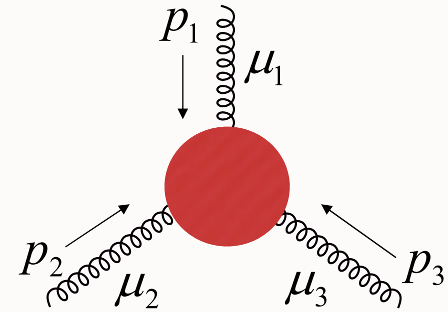
From the $k^2 \approx 0$ region

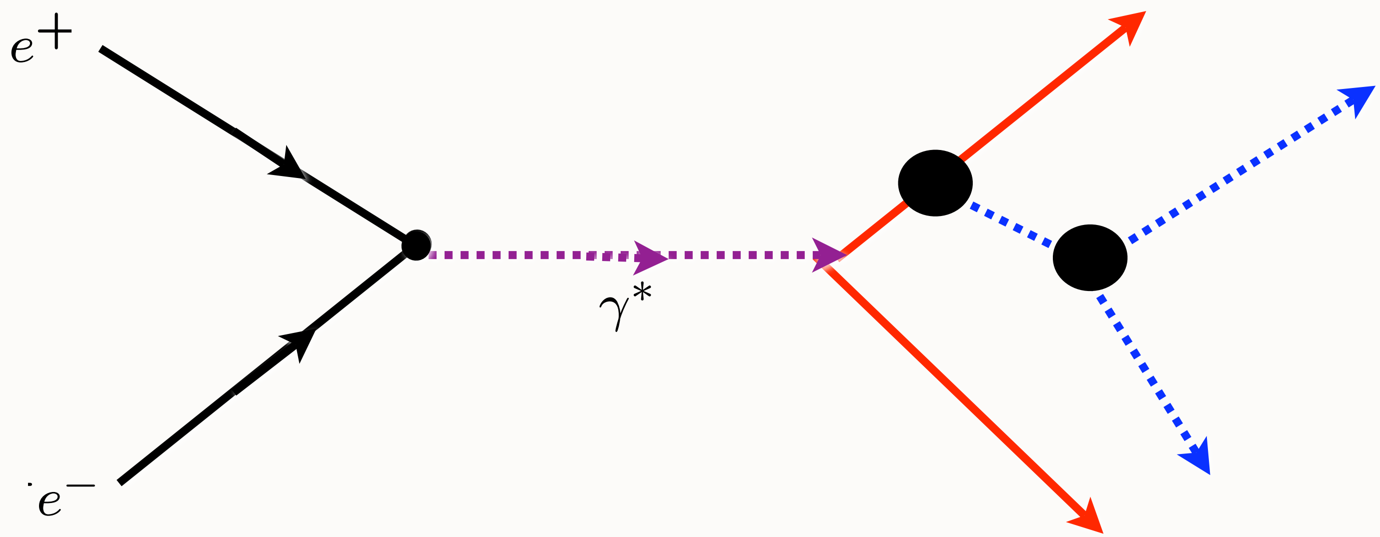
'n' insertions

$$\sum_{n=0}^{\infty}$$ 

15

$$e^+e^- \to \gamma^* \to \text{4jets}$$





Measurement of the strong coupling α_S from the four-jet rate in e^+e^- annihilation using JADE data

J. Schieck[1,a], S. Bethke[1], O. Biebel[2], S. Kluth[1], P.A.M. Fernández[3], C. Pahl[1],
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Eur. Phys. J. C 48, 3–13 (2006)

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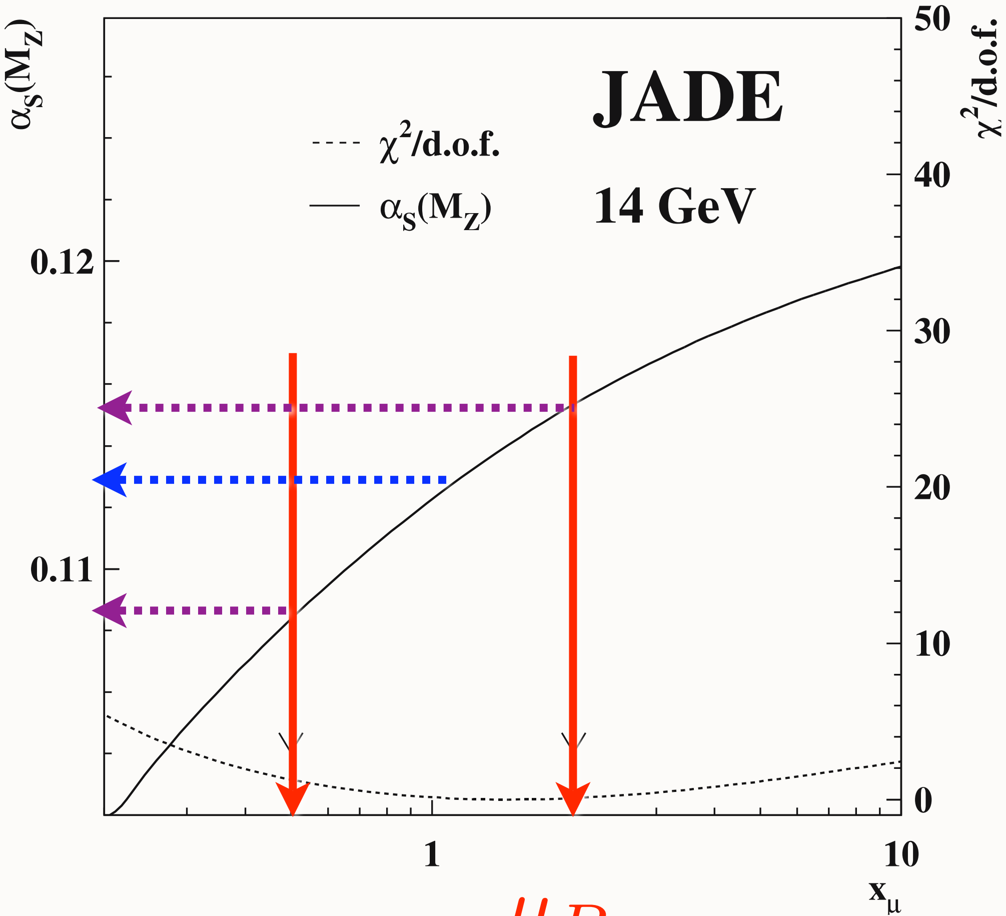


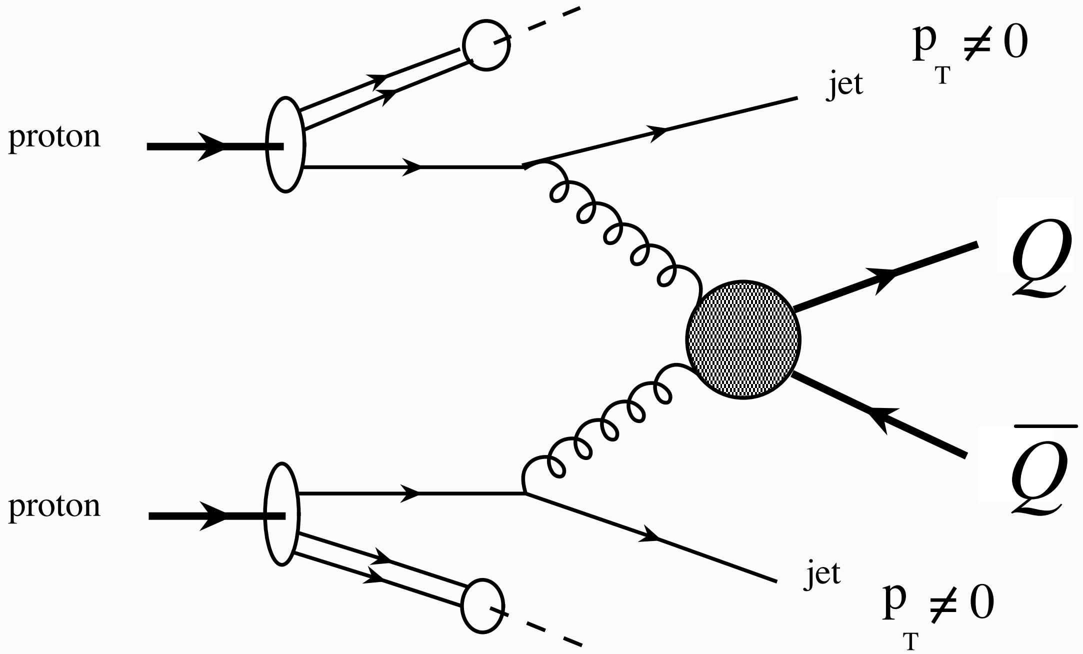
The theoretical uncertainty, associated with missing higher order terms in the theoretical prediction, is assessed by varying the renormalization scale factor x_μ. The predictions of a complete QCD calculation would be independent of x_μ, but a finite-order calculation such as that used here retains some dependence on x_μ. The renormalization scale factor x_μ is set to 0.5 and two. The larger deviation from the default value of α_S is taken as systematic uncertainty.

$$x_\mu = \frac{\mu_R}{\sqrt{s}}$$

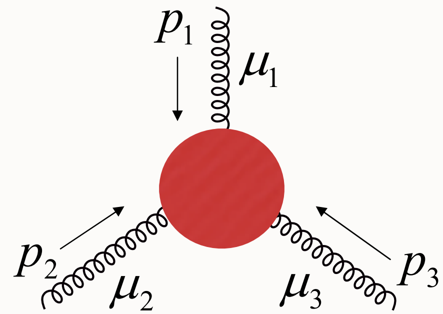
$\alpha_S(M_{Z^0})$ and the χ^2/d.o.f. of the fit to the four-jet rate as a function of the renormalization scale x_μ for $\sqrt{s} = 14$ GeV to 43.8 GeV. The arrows indicate the variation of the renormalization scale factor used for the determination of the systematic uncertainties

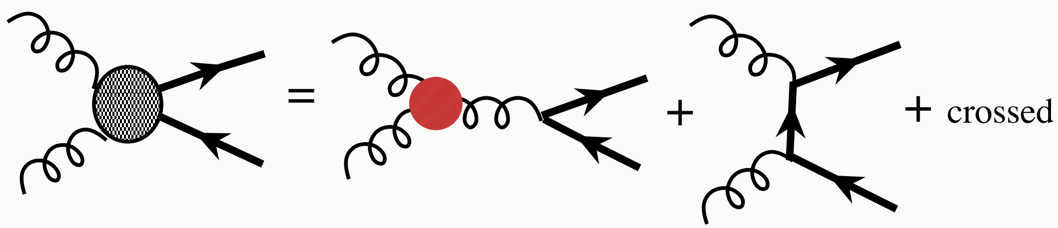
PMS & FAC inapplicable

Heavy Quark Hadroproduction



3-gluon coupling depends on 3 physical scales

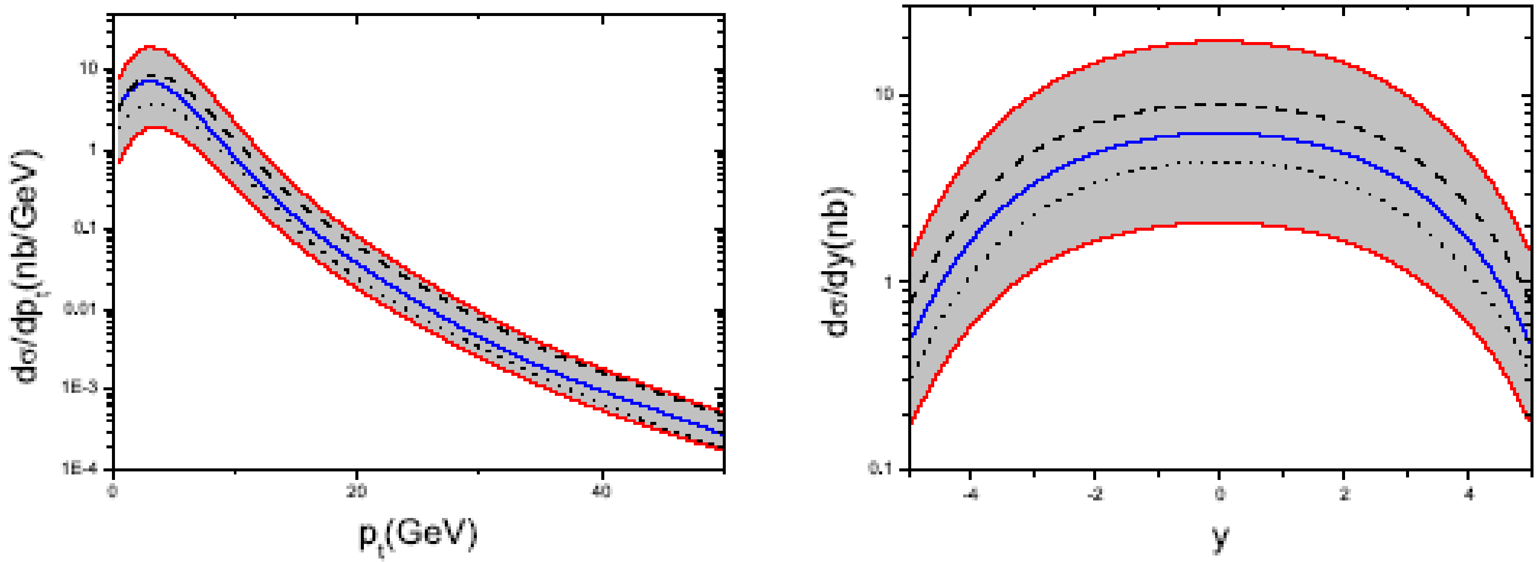




Chao-Hsi Chang

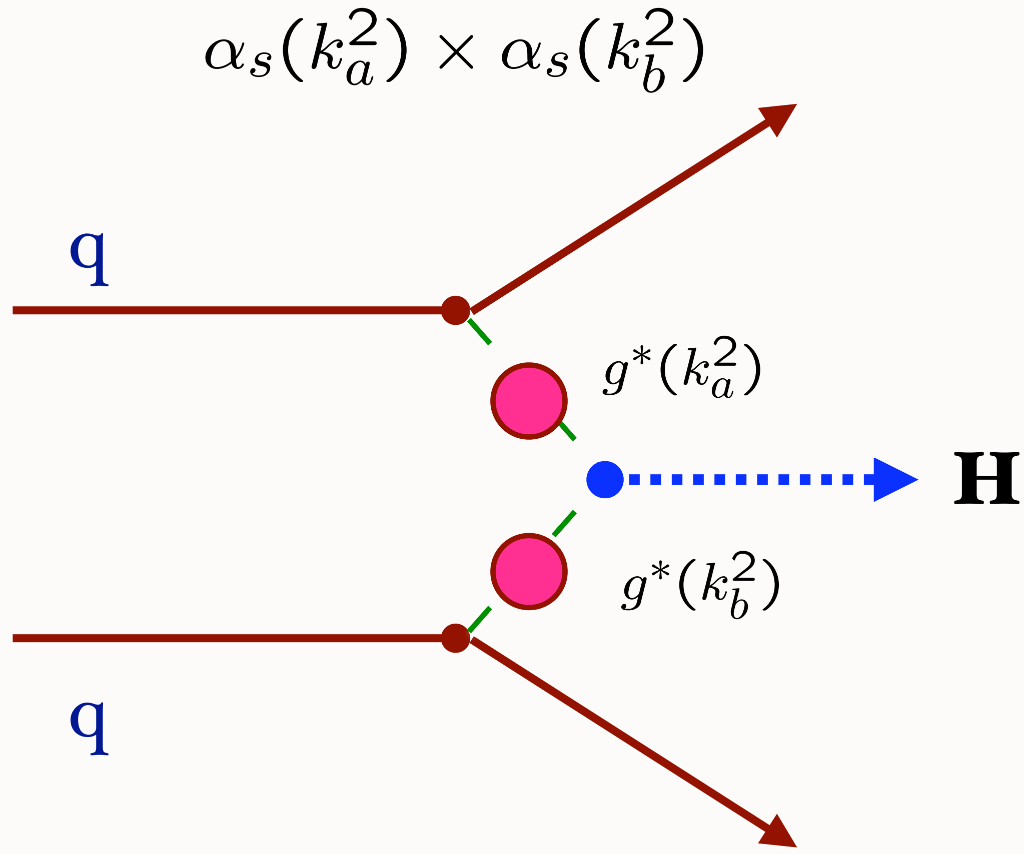
Uncertainties in P-wave Bc Production due to factorization energy scale

The summed P_t distribution and y distribution of all the P-wave states for different factorization scale μ^2_F and renormalization scale μ^2 at LHC

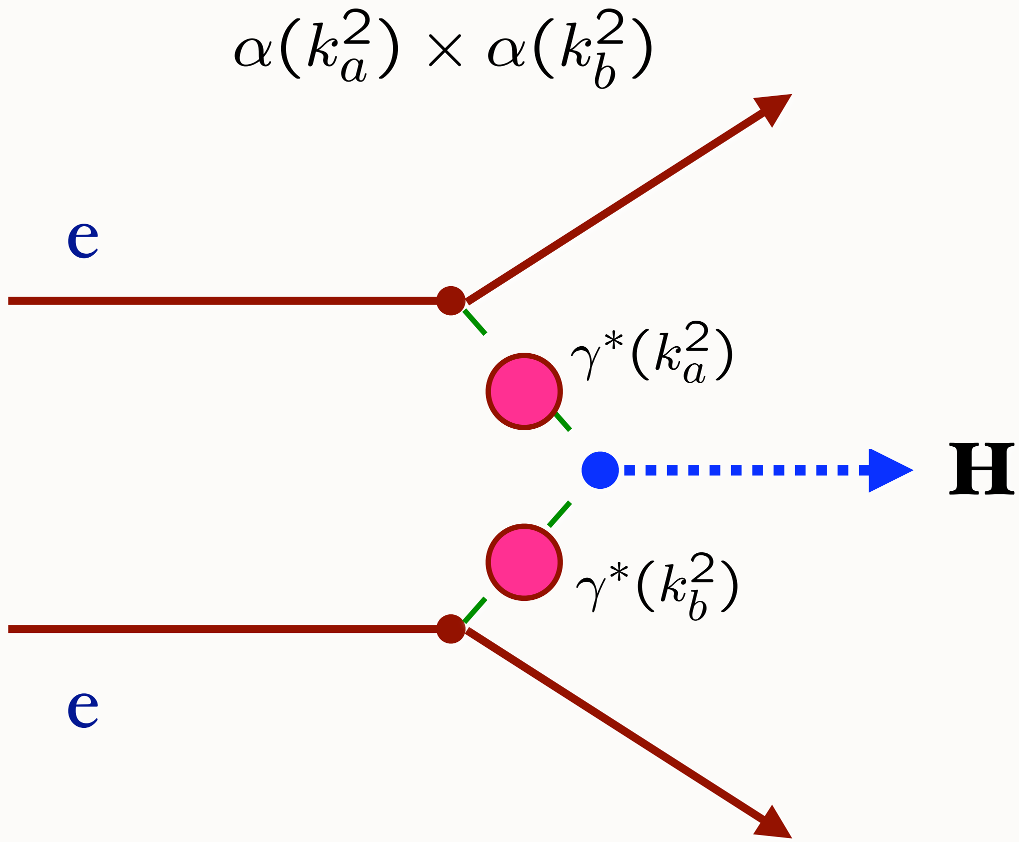


The upper edge of the band corresponds to $\mu^2_F=4M_{Pt}^2$; $\mu^2=M_{Pt}^2/4$; and the lower edge corresponds to that of $\mu^2_F=M_{Pt}^2/4$; $\mu^2=4M_{Pt}^2$. The solid line, the dotted line and the dashed line corresponds to that of $\mu^2_F=\mu^2=M_{Pt}^2$; $\mu^2_F=\mu^2=4M_{Pt}^2$; $\mu^2_F=\mu^2=M_{Pt}^2/4$.

Gluon-Fusion : Higgs Production

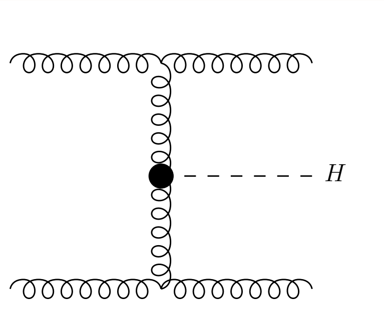


QED Analog: Two-Photon Higgs Production



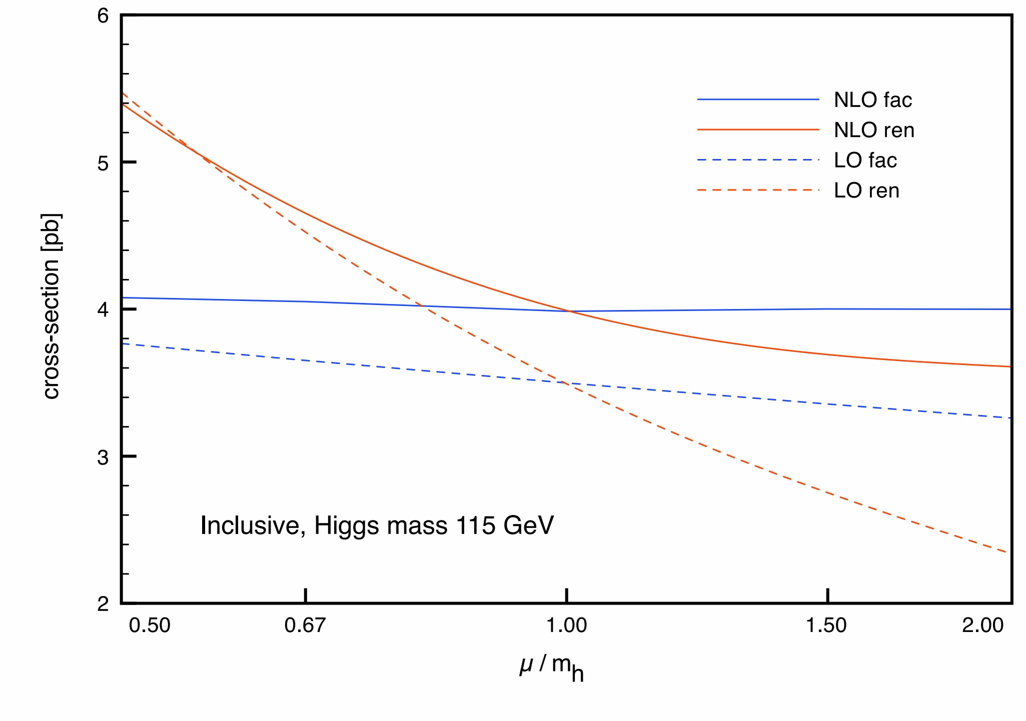
Campbell, Ellis, Zanderighi

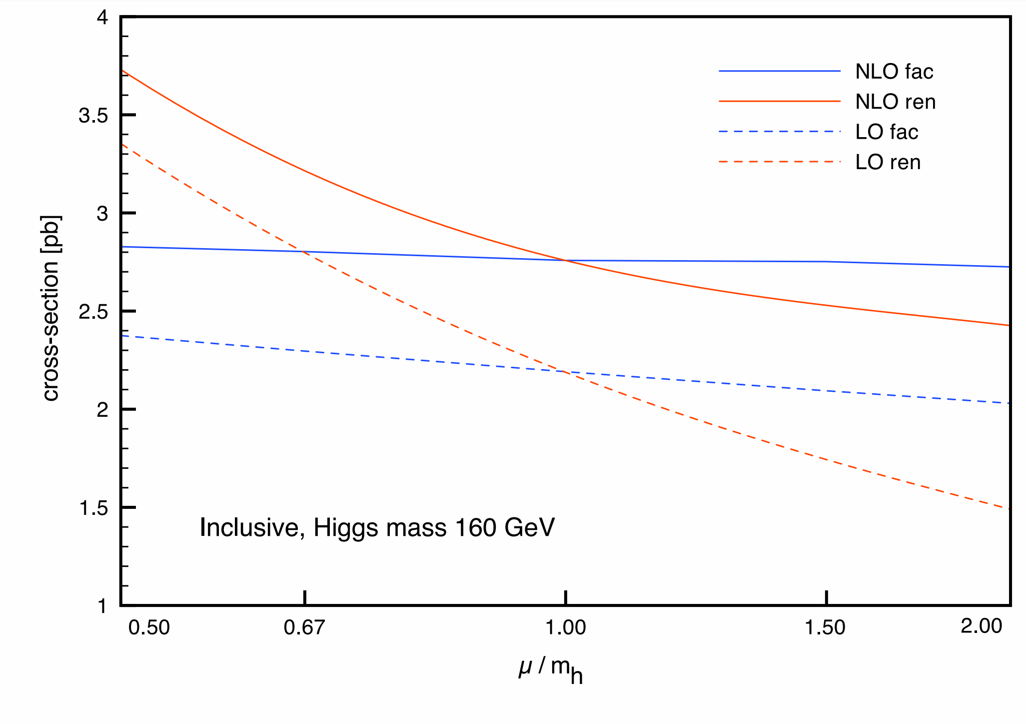
Next-to-Leading order Higgs + 2 jet production via gluon fusion.



$$p_t(\text{jet}) > 40 \text{ GeV}, \qquad |\eta_{\text{jet}}| < 4.5$$

Virtuality of gluons controlled by p_t(jet)





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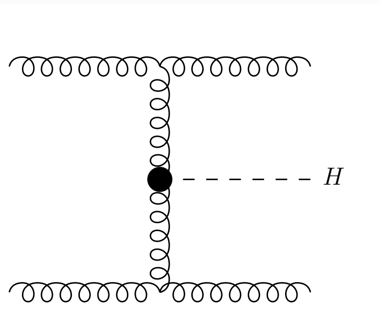
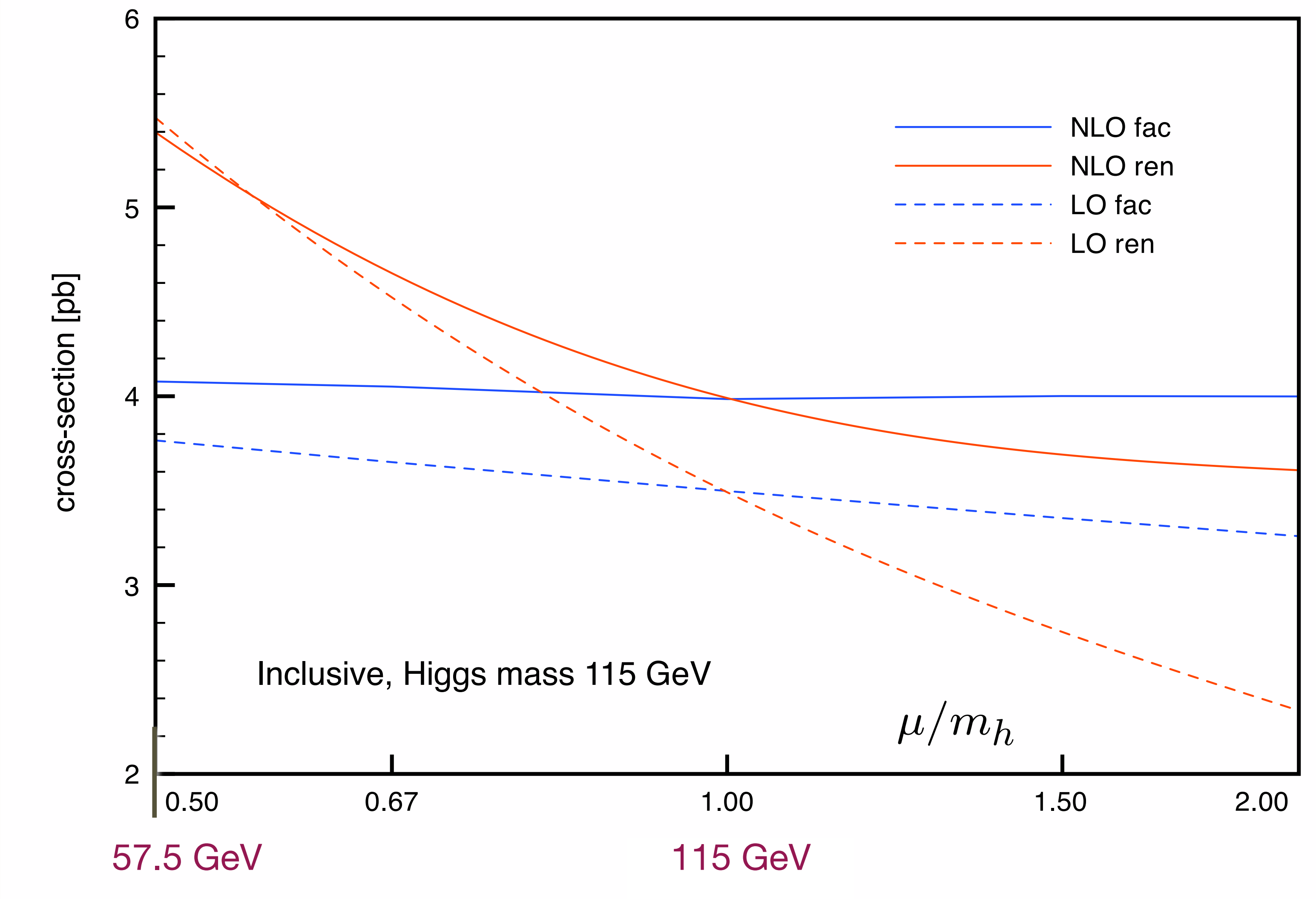
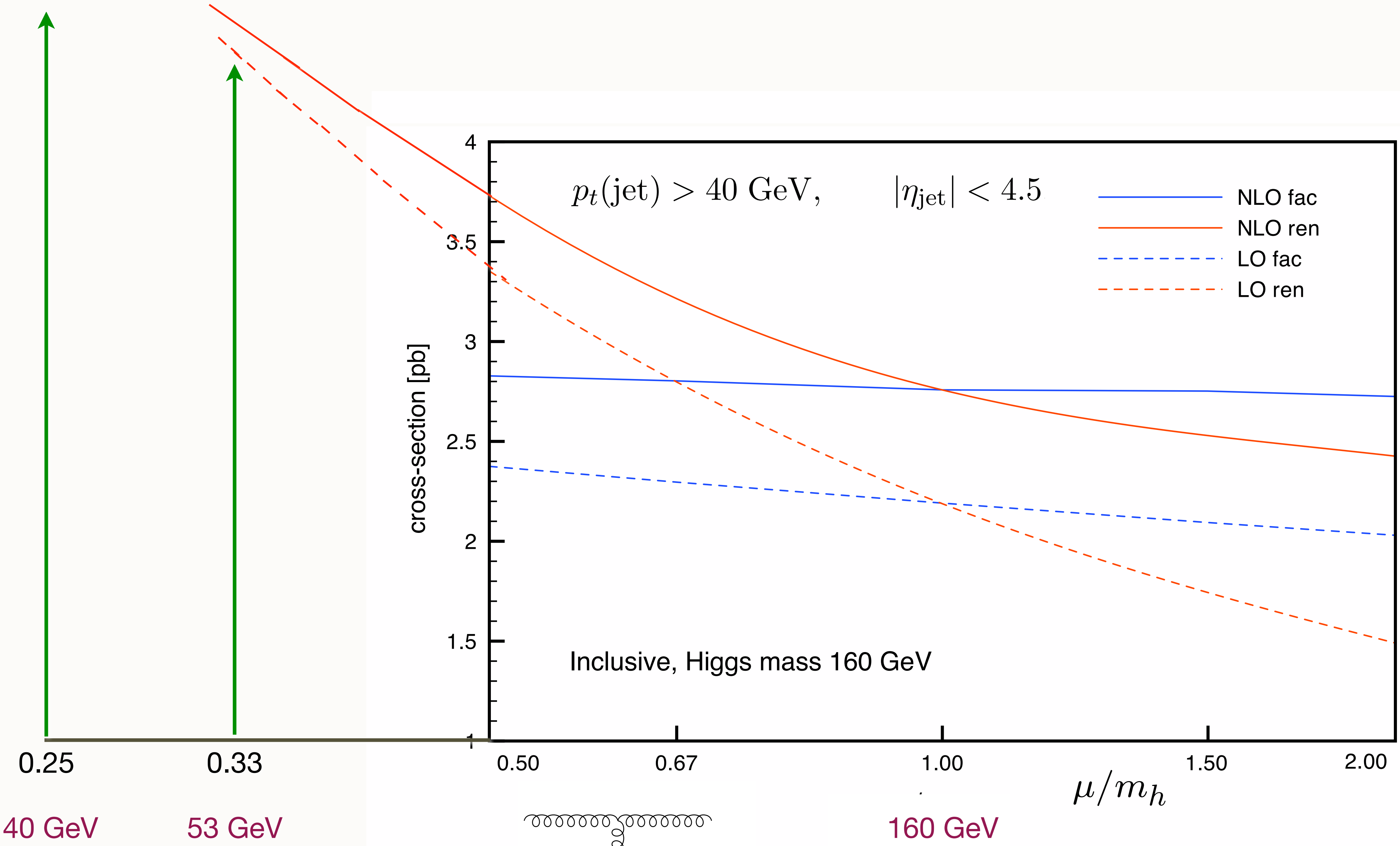
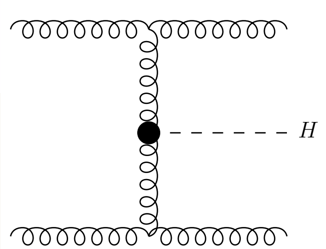
Campbell, Ellis, Zanderighi





$$p_t(\text{jet}) > 40 \text{ GeV}, \qquad |\eta_{\text{jet}}| < 4.5$$





Campbell, Ellis, Zanderighi

Next-to-Leading order Higgs + 2 jet production via gluon fusion.

Campbell, Ellis, Zanderighi

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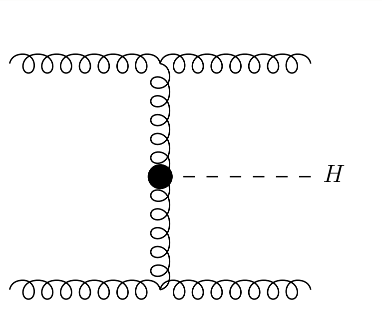
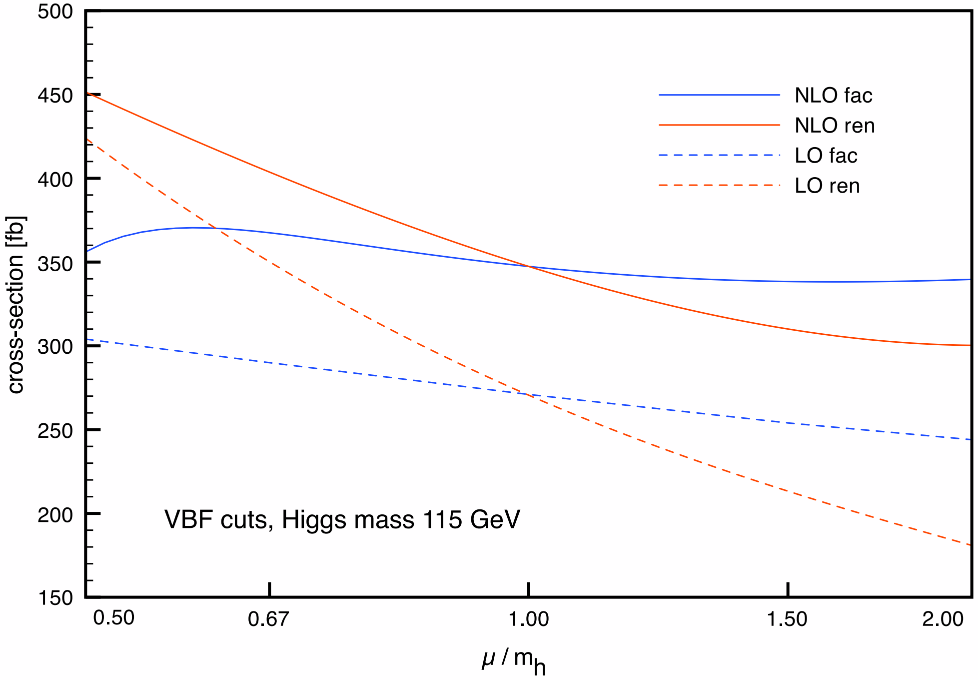
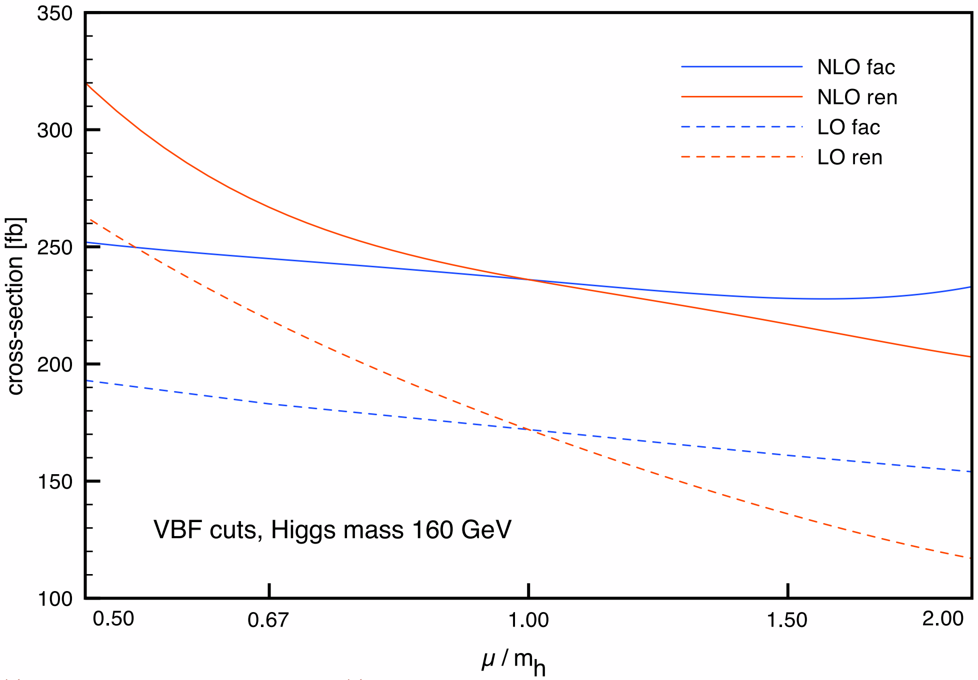






$$p_t(\text{jet}) > 40 \text{ GeV}, \qquad |\eta_{\text{jet}}| < 4.5$$

Jets well-separated in rapidity

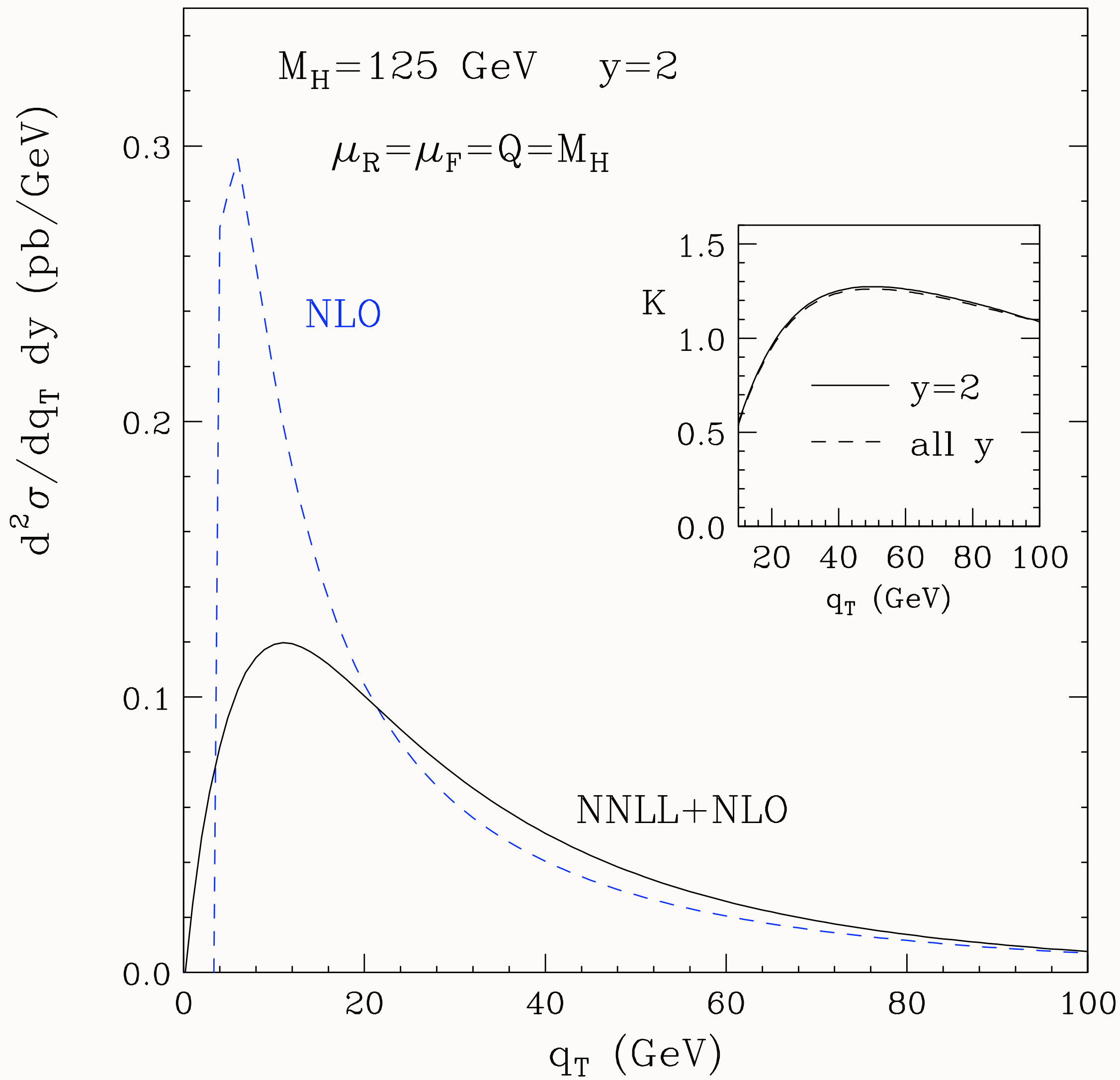
$$|\eta_{j_1} - \eta_{j_2}| > 4.2,$$

Small gluon virtuality:

Even smaller physical renormalization scale

G. Bozzi, S. Catani, D. de Florian, M. Grazzini   arXiv:0705.3887

Transverse Momentum of Higgs in QCD



Lessons from Higgs calculation

- Renormalization scale not set by Higgs Mass
- No reason to take $Q = M_H$
- Physical renormalization scale related to gluon virtuality -- minimum jet p_T
- Similar to QED analog; analytic limit $N_C \to 0$
- PMS inapplicable
- No sign that sensitivity to renormalization scale is reduced at NLO

QCD Lagrangian

gluon dynamics

quark kinetic energy +
quark-gluon dynamics

mass term

$$L_{QCD} = -\frac{1}{4g^2} Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{nf} i\overline{\psi_f} D_\mu \gamma^\mu \psi_f + \sum_{f=1}^{nf} m_f \overline{\psi_f} \psi_f$$

QCD color charge

field strength tensor

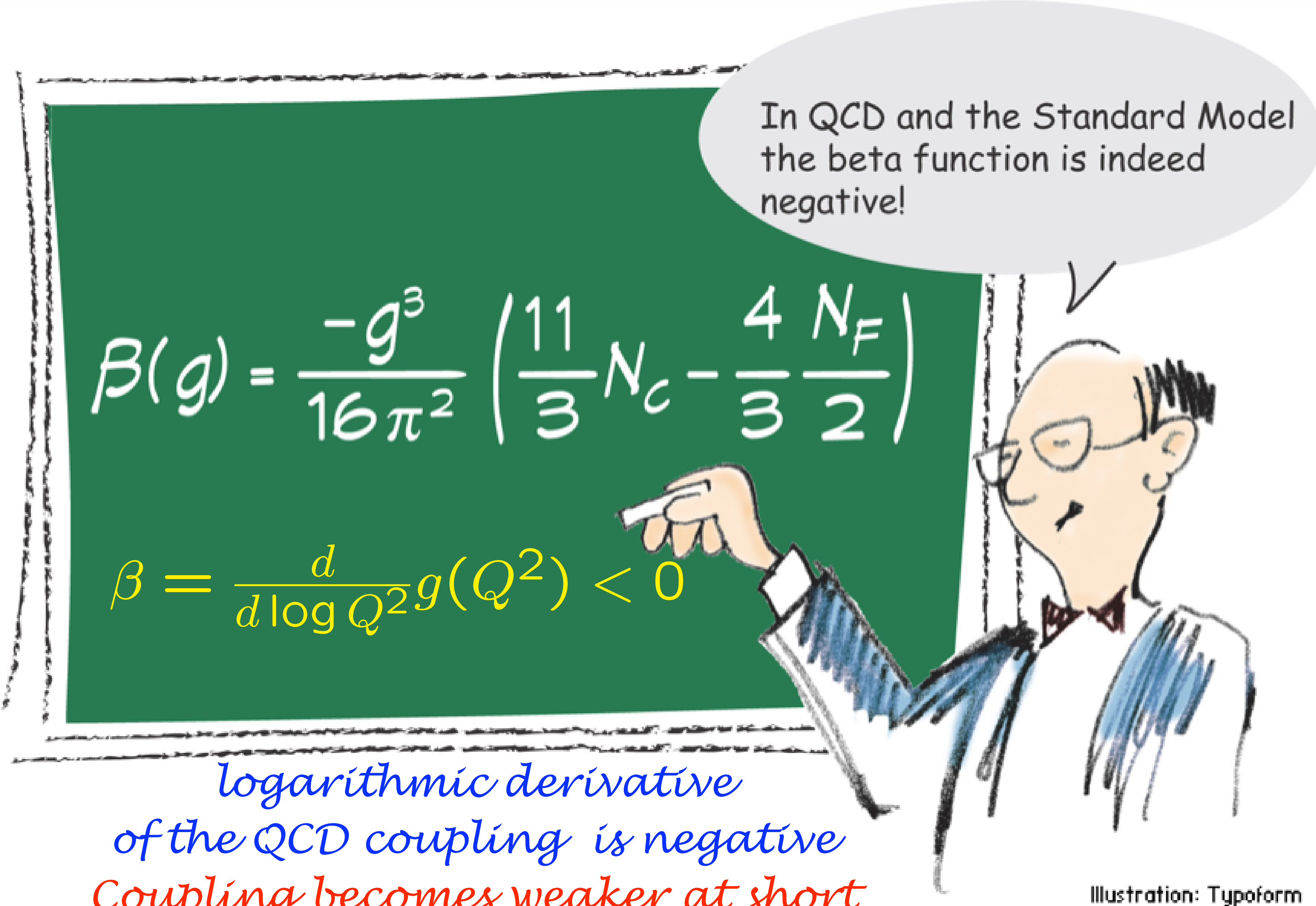
covariant derivative

quark field

$\lim N_C \to 0$ at fixed $\alpha = C_F \alpha_s, n_\ell = n_F/C_F$ $\quad [C_F = \frac{N_C^2-1}{2N_C}]$

Analytic limit of QCD: Abelian Gauge Theory

P. Huet, sjb

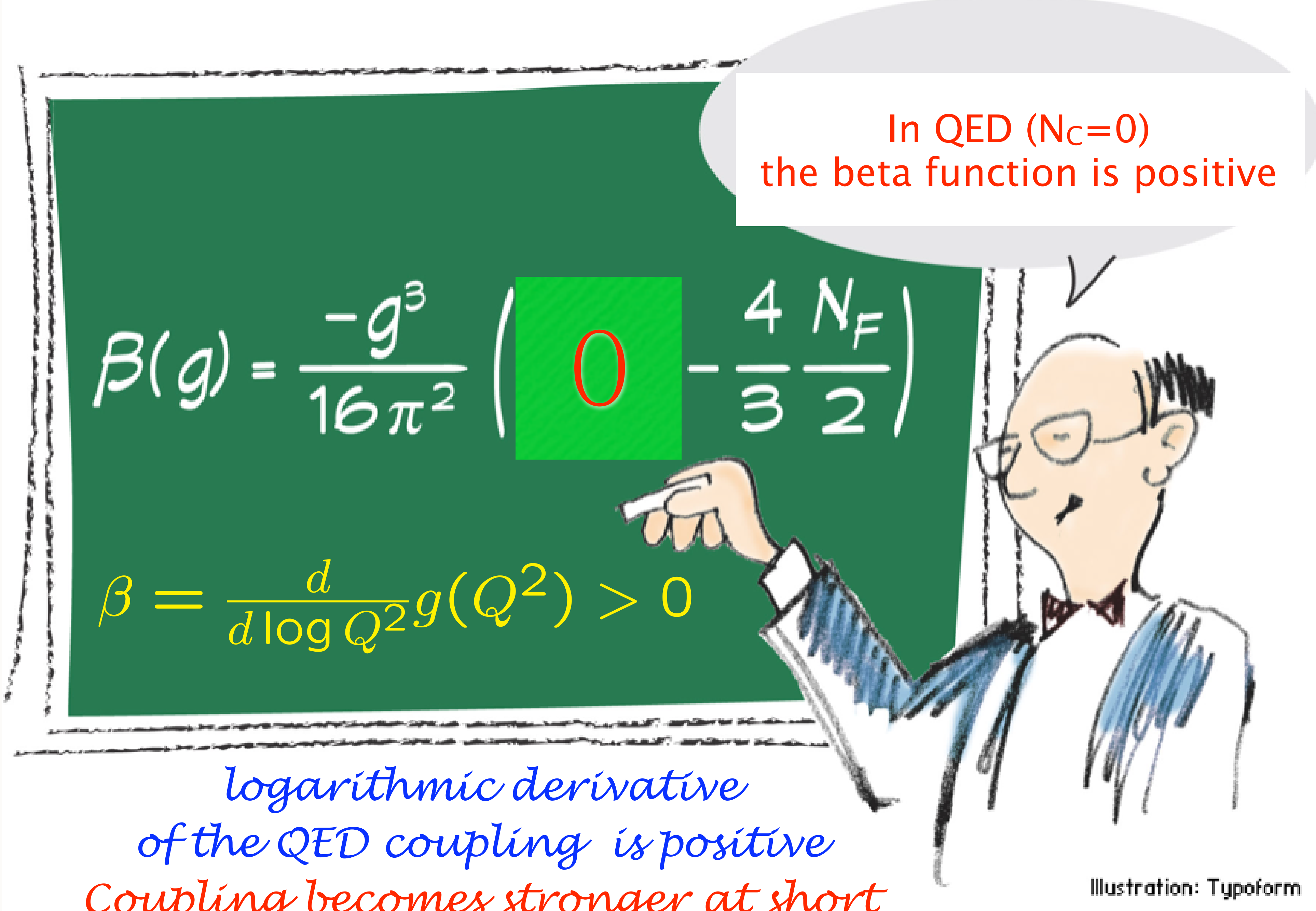


$$\beta = \frac{d}{d \log Q^2} g(Q^2) < 0$$

logarithmic derivative of the QCD coupling is negative

Coupling becomes weaker at short distances or high momentum transfer

Gross, Wilczek, Politzer
Khriplovich, `t Hooft



logarithmic derivative of the QED coupling is positive

Coupling becomes stronger at short distances or high momentum transfer

Huet, sjb

$$\lim N_C \to 0 \text{ at fixed } \alpha = C_F \alpha_s, n_\ell = n_F / C_F$$

$$\text{QCD} \to \text{Abelian Gauge Theory}$$

Analytic Feature of SU(Nc) Gauge Theory

Scale-Setting procedure for QCD must be applicable to QED

Lessons from QED : Summary

- Effective couplings are complex analytic functions with the correct threshold structure expected from unitarity
- Multiple "renormalization" scales appear
- The scales are unambiguous since they are physical kinematic invariants
- Optimal improvement of perturbation theory

Features of BLM Scale Setting

On The Elimination Of Scale Ambiguities In Perturbative Quantum Chromodynamics.

Lepage, Mackenzie, sjb **Phys.Rev.D28:228,1983**

- All terms associated with nonzero beta function summed into running coupling
- BLM Scale Q* sets the number of active flavors
- Only n_f dependence required to determine renormalization scale at NLO
- Result is scheme independent: Q* has exactly the correct dependence to compensate for change of scheme
- Correct Abelian limit
- **Resulting series identical to conformal series!**
- Renormalon n! growth of PQCD coefficients from beta function eliminated!
- In general, BLM scale depends on all invariants

BLM Scale Setting

$$\beta_0 = 11 - \tfrac{2}{3} n_f$$

$$\rho = C_0 \alpha_{\overline{MS}}(Q) \left[1 + \frac{\alpha_{\overline{MS}}(Q)}{\pi} \left(-\tfrac{3}{2} \beta_0 A_{VP} + \tfrac{33}{2} A_{VP} + B \right) + \cdots \right]$$

n_f dependent coefficient identifies quark loop VP contribution

by

$$\rho = C_0 \alpha_{\overline{MS}}(Q^*) \left[1 + \frac{\alpha_{\overline{MS}}(Q^*)}{\pi} C_1^* + \cdots \right] ,$$

Conformal coefficient - independent of β

where

$$Q^* = Q \exp(3A_{VP}) ,$$

$$C_1^* = \tfrac{33}{2} A_{VP} + B .$$

The term $33A_{VP}/2$ in C_1^* serves to remove that part of the constant B which renormalizes the leading-order coupling. The ratio of these gluonic corrections to the light-quark corrections is fixed by $\beta_0 = 11 - \frac{2}{3} n_f$.

Use skeleton expansion:
Gardi, Grunberg, Rathsman, sjb

$$R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q{}^2 \left[1 + \frac{\alpha_R(Q)}{\pi} \right] .$$

$$R_{e^+e^-}(Q^2) = 3 \sum_q e_q{}^2 \left[1 + \frac{\alpha_{\overline{MS}}(Q)}{\pi} + \frac{\alpha_{\overline{MS}}{}^2}{\pi^2}(1.98 - 0.115 n_f) + \cdots \right]$$

n_f dependent coefficient identifies quark loop VP contribution to

$\beta_0 = 11 - \frac{2}{3} n_f$

$$\rightarrow 3 \sum_q e_q{}^2 \left[1 + \frac{\alpha_{\overline{MS}}(Q^*)}{\pi} + \frac{\alpha_{\overline{MS}}{}^2(Q^*)}{\pi^2} 0.08 + \cdots \right],$$

Conformal coefficient ! Independent of β

$\boxed{Q^* = 0.710 Q.}$ Notice that $\alpha_R(Q)$ differs from $\alpha_{\overline{MS}}(Q^*)$ by only $0.08 \alpha_{\overline{MS}}/\pi$, so that $\alpha_R(Q)$ and $\alpha_{\overline{MS}}(0.71 Q)$ are effectively interchangeable (for any value of n_f).

Features of BLM Scale Setting

On The Elimination Of Scale Ambiguities In Perturbative Quantum Chromodynamics.

Lepage, Mackenzie, sjb **Phys.Rev.D28:228,1983**

- All terms associated with nonzero beta function summed into running coupling
- Identical procedure in QED
- Resulting series identical to conformal series
- Renormalon n! growth of PQCD coefficients from beta function eliminated!
- In general, BLM scale depends on all invariants

Deep-inelastic scattering. The moments of the nonsinglet structure function $F_2(x,Q^2)$ obey the evolution equation[12]

$$\beta_0 = 11 - \tfrac{2}{3}n_f$$

$$
\begin{aligned}
Q^2\frac{a}{dQ^2}\ln M_n(Q^2)
&= -\frac{\gamma_n^{(0)}}{8\pi}\alpha_{\overline{\rm MS}}(Q)\left[1+\frac{\alpha_{\overline{\rm MS}}}{4\pi}\frac{2\beta_0\beta_n+\gamma_n^{(1)}}{\gamma_n^{(0)}}+\cdots\right.\\
&\to -\frac{\gamma_n^{(0)}}{8\pi}\alpha_{\overline{\rm MS}}(Q_n^*)\left[1-\frac{\alpha_{\overline{\rm MS}}(Q_n^*)}{\pi}C_n+\cdots\right],
\end{aligned}
$$

where, for example,

$$Q_2^* = 0.48Q,\quad C_2 = 0.27\ ,$$

$$Q_{10}^* = 0.21Q,\quad C_{10} = 1.1\ .$$

For n very large, the effective scale here becomes $Q_n^* \sim Q/\sqrt{n}$.

BLM scales for DIS moments

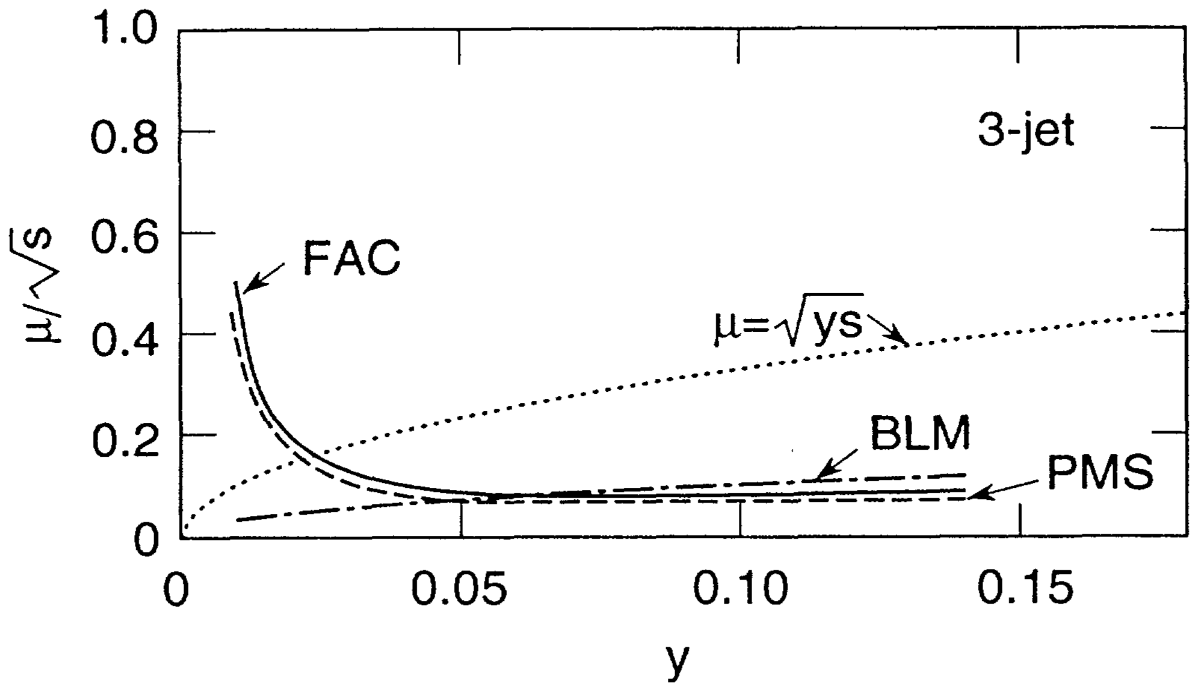
$$V(Q^2) = -\frac{C_F 4\pi\alpha_{\overline{\rm MS}}(Q)}{Q^2}\left[1+\frac{\alpha_{\overline{\rm MS}}}{\pi}\left(\tfrac{5}{12}\beta_0-2\right)+\cdots\right]$$

$$\rightarrow -\frac{C_F 4\pi\alpha_{\overline{\rm MS}}(Q^*)}{Q^2}\left[1-\frac{\alpha_{\overline{\rm MS}}(Q^*)}{\pi}2+\cdots\right].$$

where $Q^* = e^{-5/6}\, Q \cong 0.43Q$. This result shows that the effective scale of the $\overline{\rm MS}$ scheme should generally be about half of the true momentum transfer occurring in the interaction. In parallel to QED, the effective potential $V(Q^2)$ gives a particularly intuitive scheme for defining the QCD coupling constant

$$V(Q^2) \equiv -4\pi C_F \frac{\alpha_V(Q^2)}{Q^2}$$

Similar to pinch scheme

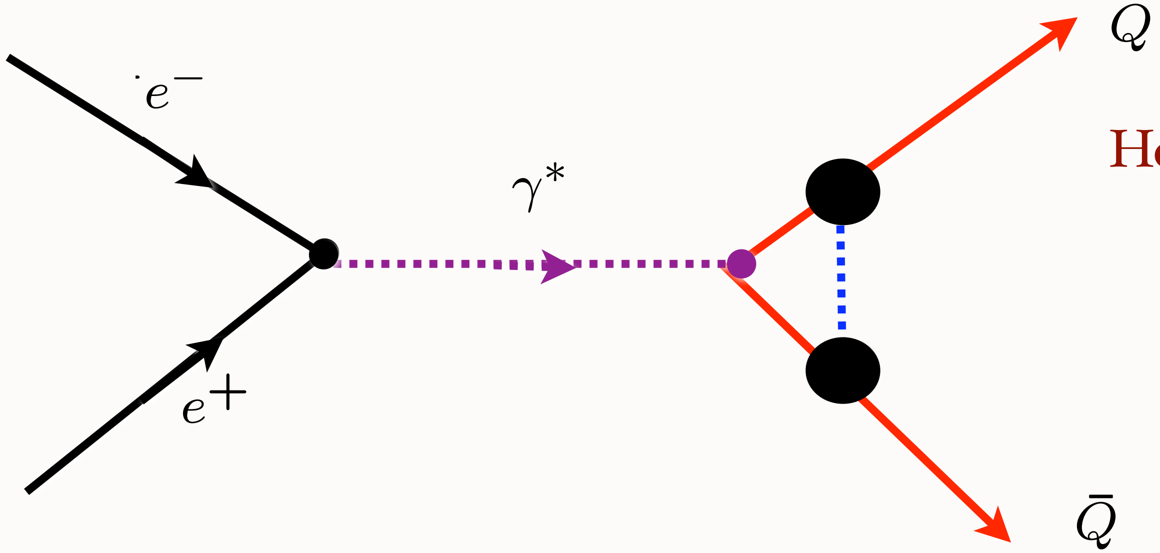


Kramer & Lampe

Three-Jet rate in electron-positron annihilation

The scale $\mu/\sqrt{s}$ according to the BLM (dashed-dotted), PMS (dashed), FAC (full), and $\sqrt{y}$ (dotted) procedures for the three-jet rate in e^+e^- annihilation, as computed by Kramer and Lampe [10]. Notice the strikingly different behavior of the BLM scale from the PMS and FAC scales at low y. In particular, the latter two methods predict increasing values of μ as the jet invariant mass $\mathcal{M} < \sqrt{(ys)}$ decreases.

Other Jet Observables: Rathsman



Hoang, Kuhn, Teubner, sjb

$$
\begin{aligned}
F_1 + F_2 &= 1 + \frac{\alpha(s\,\beta^2)\,\pi}{4\,\beta} - 2\,\frac{\alpha(s\,e^{3/4}/4)}{\pi} \\
&\cong \left(1 - 2\,\frac{\alpha(s\,e^{3/4}/4)}{\pi}\right)\left(1 + \frac{\alpha(s\,\beta^2)\,\pi}{4\,\beta}\right)
\end{aligned}
$$

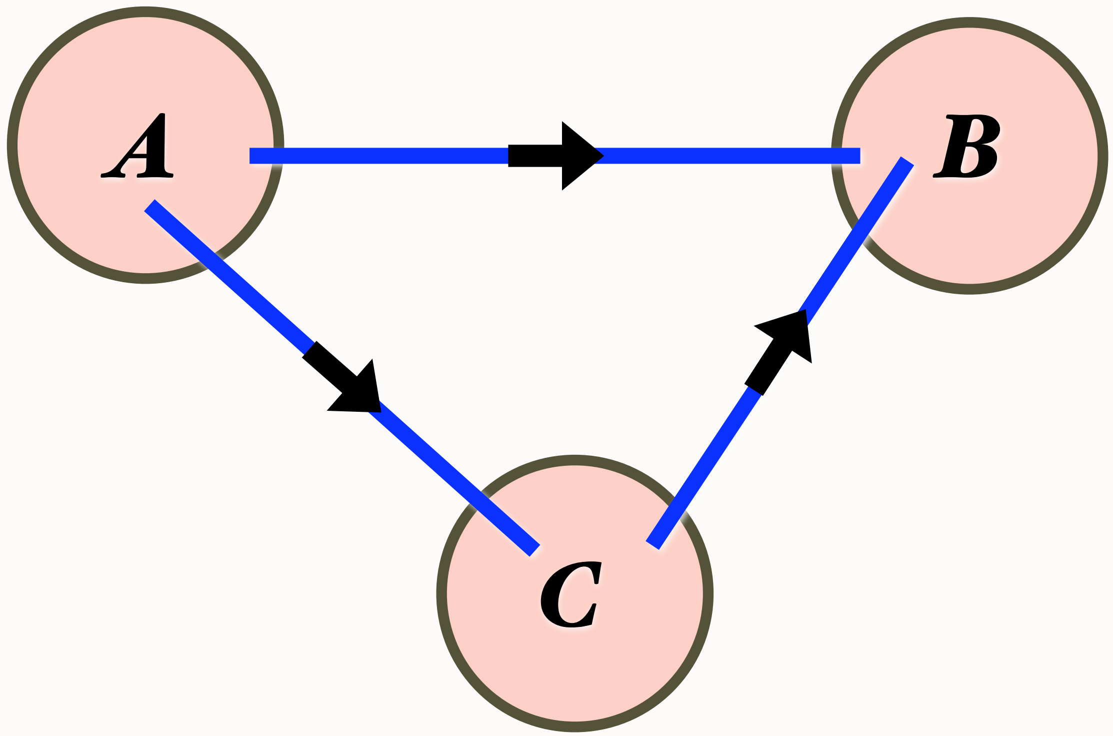
Example of Multiple BLM Scales

Angular distributions of massive quarks and leptons close to threshold.

Relate Observables to Each Other

- Eliminate intermediate scheme
- No scale ambiguity
- Transitive!
- Commensurate Scale Relations
- Example: Generalized Crewther Relation

Transitivity Property of Renormalization Group



A ➔ C   C ➔ B   *identical to*   A ➔ B

Relation of observables independent of intermediate scheme C

Relate Observables to Each Other

- Eliminate intermediate scheme
- No scale ambiguity
- Transitive!
- Commensurate Scale Relations
- Example: Generalized Crewther Relation

$$R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q{}^2 \left[1 + \frac{\alpha_R(Q)}{\pi}\right].$$

$$\int_0^1 dx \left[g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2)\right] \equiv \frac{1}{3}\left|\frac{g_A}{g_V}\right| \left[1 - \frac{\alpha_{g_1}(Q)}{\pi}\right]$$

$$\frac{\alpha_R(Q)}{\pi} = \frac{\alpha_{\overline{MS}}(Q)}{\pi} + \left(\frac{\alpha_{\overline{MS}}(Q)}{\pi}\right)^2 \left[\left(\frac{41}{8} - \frac{11}{3}\zeta_3\right) C_A - \frac{1}{8}C_F + \left(-\frac{11}{12} + \frac{2}{3}\zeta_3\right) f\right]$$
$$+ \left(\frac{\alpha_{\overline{MS}}(Q)}{\pi}\right)^3 \left\{\left(\frac{90445}{2592} - \frac{2737}{108}\zeta_3 - \frac{55}{18}\zeta_5 - \frac{121}{432}\pi^2\right) C_A^2 + \left(-\frac{127}{48} - \frac{143}{12}\zeta_3 + \frac{55}{3}\zeta_5\right) C_A C_F - \frac{23}{32}C_F^2\right.$$
$$+ \left[\left(-\frac{970}{81} + \frac{224}{27}\zeta_3 + \frac{5}{9}\zeta_5 + \frac{11}{108}\pi^2\right) C_A + \left(-\frac{29}{96} + \frac{19}{6}\zeta_3 - \frac{10}{3}\zeta_5\right) C_F\right] f$$
$$\left. + \left(\frac{151}{162} - \frac{19}{27}\zeta_3 - \frac{1}{108}\pi^2\right) f^2 + \left(\frac{11}{144} - \frac{1}{6}\zeta_3\right) \frac{d^{abc}d^{abc}}{C_F d(R)} \frac{\left(\sum_f Q_f\right)^2}{\sum_f Q_f^2}\right\}.$$

$$\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_{\overline{MS}}(Q)}{\pi} + \left(\frac{\alpha_{\overline{MS}}(Q)}{\pi}\right)^2 \left[\frac{23}{12}C_A - \frac{7}{8}C_F - \frac{1}{3}f\right]$$
$$+ \left(\frac{\alpha_{\overline{MS}}(Q)}{\pi}\right)^3 \left\{\left(\frac{5437}{648} - \frac{55}{18}\zeta_5\right) C_A^2 + \left(-\frac{1241}{432} + \frac{11}{9}\zeta_3\right) C_A C_F + \frac{1}{32}C_F^2\right.$$
$$\left. + \left[\left(-\frac{3535}{1296} - \frac{1}{2}\zeta_3 + \frac{5}{9}\zeta_5\right) C_A + \left(\frac{133}{864} + \frac{5}{18}\zeta_3\right) C_F\right] f + \frac{115}{648}f^2\right\}.$$

Eliminate MSbar, Find Amazing Simplification

$$R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q^{\,2} \left[1 + \frac{\alpha_R(Q)}{\pi} \right] .$$

$$\int_0^1 dx \left[g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2) \right] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[1 - \frac{\alpha_{g_1}(Q)}{\pi} \right]$$

$$\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_R(Q^*)}{\pi} - \left(\frac{\alpha_R(Q^{**})}{\pi} \right)^2 + \left(\frac{\alpha_R(Q^{***})}{\pi} \right)^3$$

Geometric Series in Conformal QCD

Generalized Crewther Relation

Lu, Kataev, Gabadadze, Sjb

Lu, Kataev, Gabadadze, Sjb

Generalized Crewther Relation

$$[1 + \frac{\alpha_R(s^*)}{\pi}][1 - \frac{\alpha_{g_1}(q^2)}{\pi}] = 1$$

$$\sqrt{s^*} \simeq 0.52Q$$

Conformal relation true to all orders in perturbation theory

No radiative corrections to axial anomaly

Nonconformal terms set relative scales (BLM)

Analytic matching at quark thresholds

No renormalization scale ambiguity!

* why is the relation between

α_R and α_{g_1}

so simple?

Gabadadze
Kataev
H.J. Lu
S.J.B.

Consider conformal limit $\beta_0 \Rightarrow 0$, $\beta_1 \Rightarrow 0$ ...

$$CSR \Rightarrow \left(1+\hat{\alpha}_R\right)\left(1-\hat{\alpha}_{g_1}\right) = 1$$

* Follows from Crewther relation!

$\beta=0$
chiral

$$3S = KR' = K\left(\tfrac{4}{3}R\right)$$

$\pi^0 \to \gamma\gamma$
Adler Bardeen

β_g
GLS

$R_{e^+e^-}$

$\hat{R}_{e^+e^-}$

Deviations from Crewther Relation

proportional to β

Kataev
Broadhurst

Commensurate Scale Relation:

$$* \quad \alpha_B(Q_B) = \alpha_A(Q_A)\left[1 + C^{(1)}_{A/B}\frac{\alpha_A}{\pi} + \cdots\right]$$

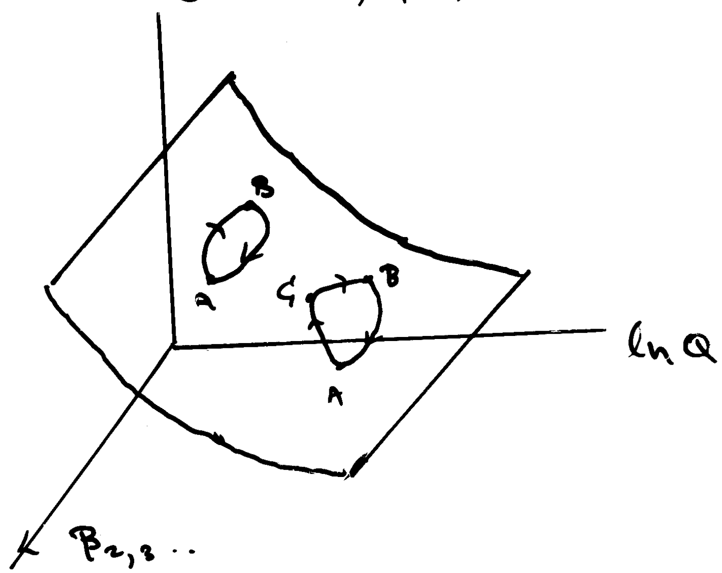
conformal coeff.

$$Q_B/Q_A = \lambda_{B/A}$$

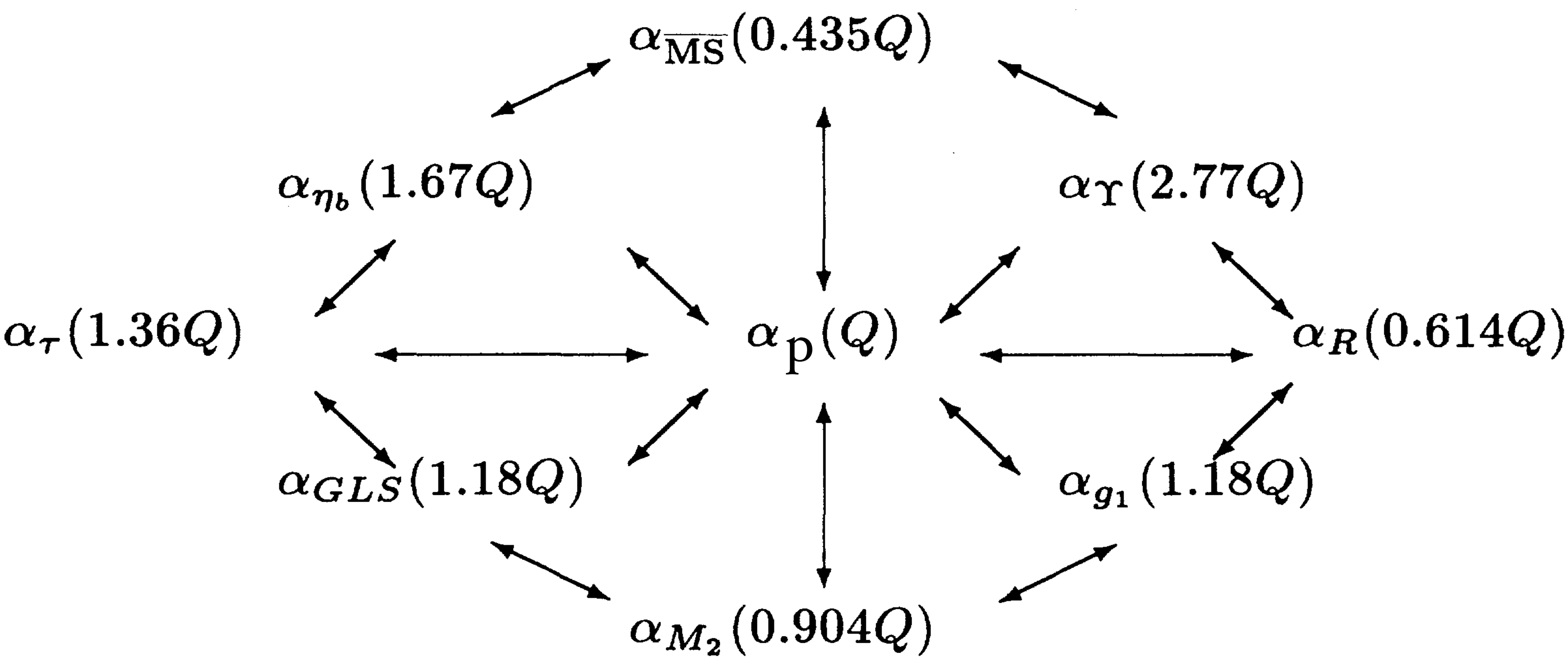
Peterman
Stückelberg
Renormalization
"Group"

$\lambda_{B/A} = \lambda_{B/C}/\lambda_{A/C}$		transitive
$\lambda_{B/A} = \lambda^{-1}_{A/B}$		symmetry
$\lambda_{A/A} = I$	.	identity

$\alpha_s(\ln Q, \beta_n)$



Leading Order Commensurate Scales



Translation between schemes at LO

Analyticity and Mass Thresholds

$\overline{MS}$ does not have automatic decoupling of heavy particles

⇨ Must define a set of schemes in each desert region and match

$$\alpha_s^{(f)}(M_Q) = \alpha_s^{(f+1)}(M_Q)$$

- The coupling has discontinuous derivative at the matching point
- At higher orders the coupling itself becomes discontinuous!
- Does not distinguish between spacelike and timelike momenta

"AN ANALYTIC EXTENSION OF THE MS-BAR RENORMALIZATION SCHEME"
S. Brodsky, M. Gill, M. Melles, J. Rathsman. **Phys.Rev.D58:116006,1998**

Define QCD Coupling from Observable

Grunberg

$$R_{e^+e^- \to X}(s) \equiv 3\Sigma_q e_q^2 \, [1 + \frac{\alpha_R(s)}{\pi}]$$

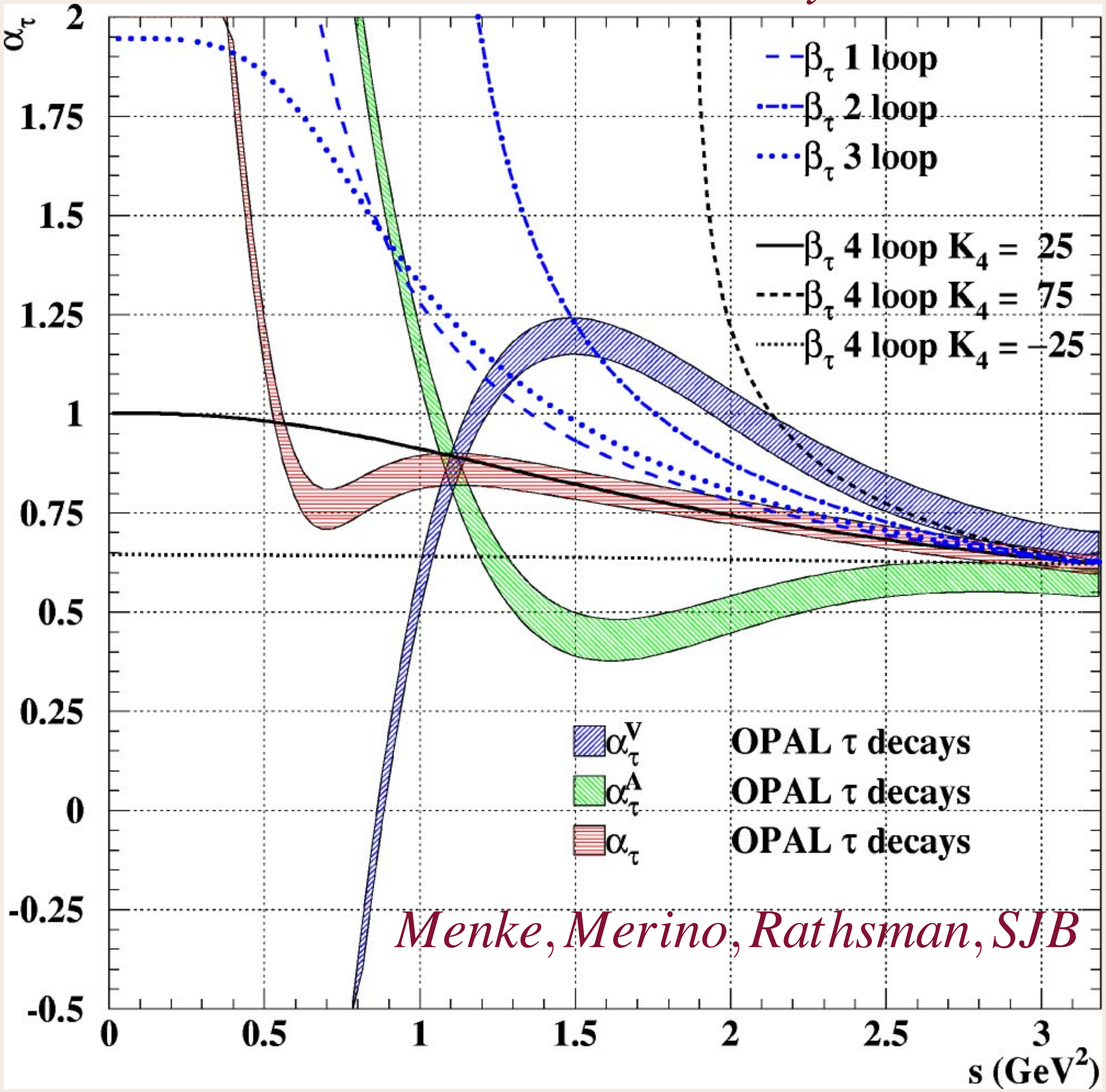
$$\Gamma(\tau \to X e \nu)(m_\tau^2) \equiv \Gamma_0(\tau \to u \bar{d} e \nu) \times [1 + \frac{\alpha_\tau(m_\tau^2)}{\pi}]$$

Commensurate scale relations:
Relate observable to observable at commensurate scales

Effective Charges: analytic at quark mass thresholds, finite at small momenta

H.Lu, Rathsman, sjb

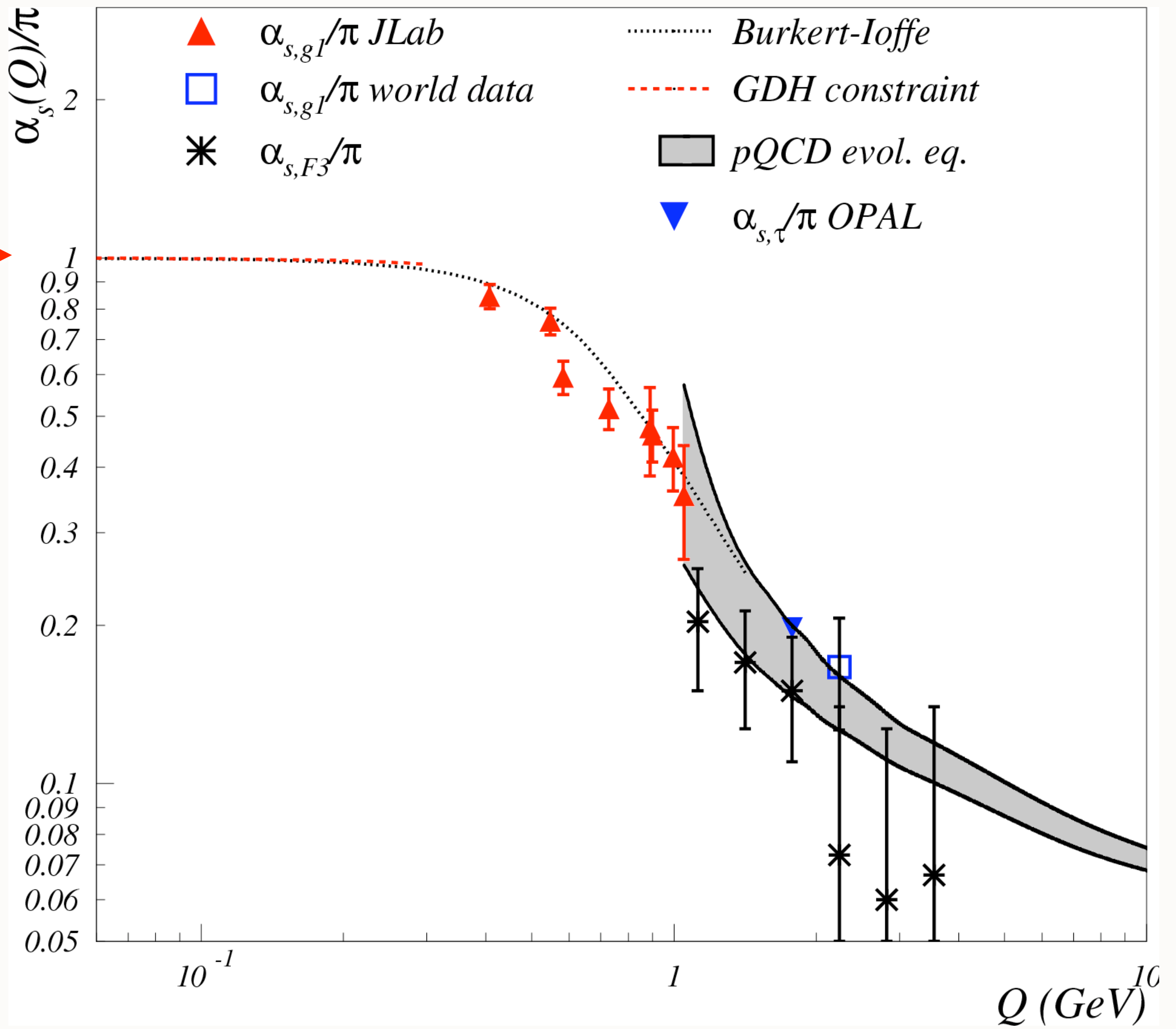
QCD Effective Coupling from *hadronic τ decay*



Deur, Korsch, et al: Effective Charge from Bjorken Sum Rule

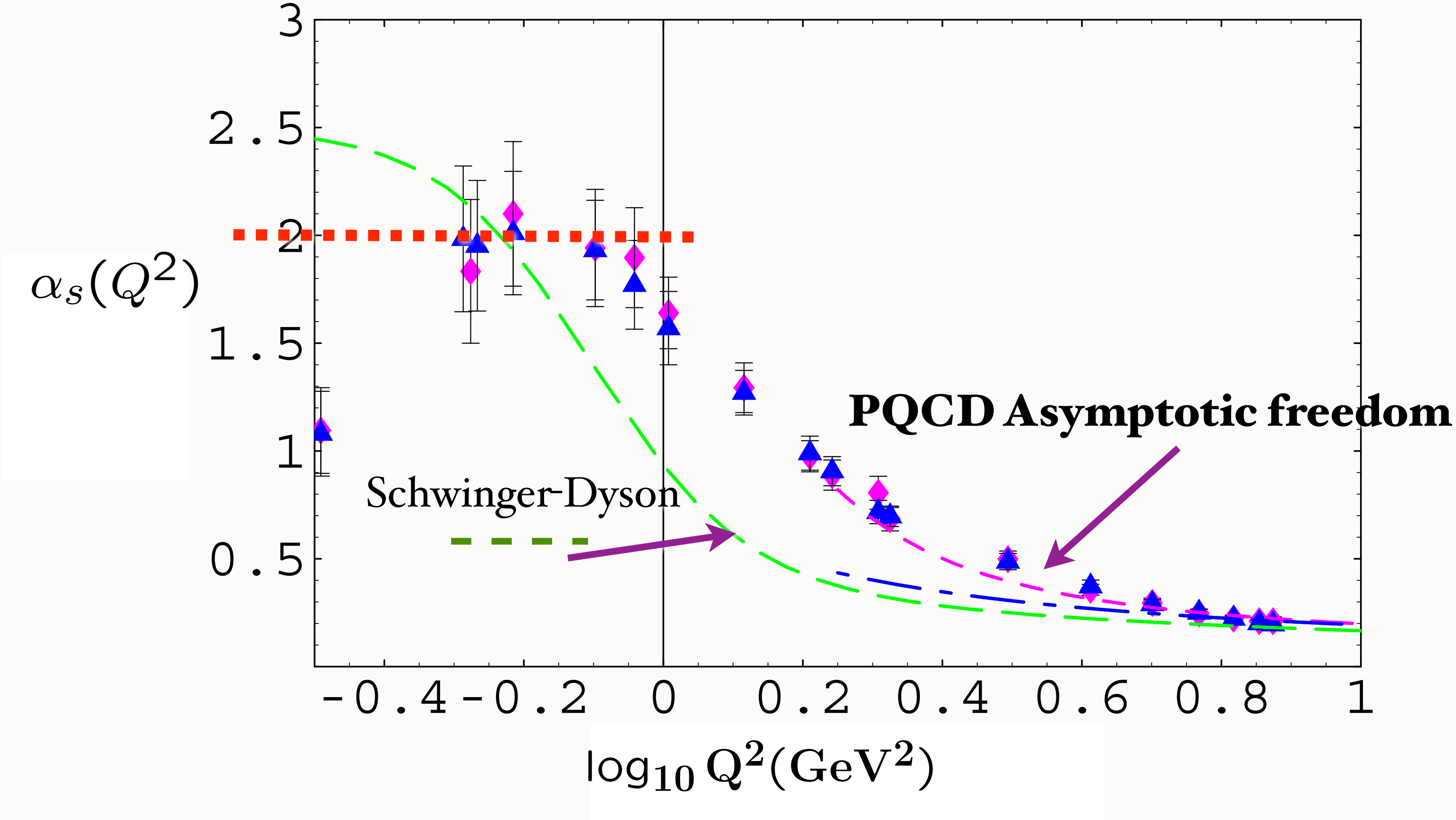
$$\Gamma_{bj}^{p-n}(Q^2) \equiv \frac{g_A}{6}\left[1 - \frac{\alpha_s^{g_1}(Q^2)}{\pi}\right]$$

GDH constraint



Conformal window
Infrared fixed-point

$$\beta(Q^2) = \frac{d\alpha_s(Q^2)}{d \log Q^2} \rightarrow 0$$



Shirkov
Gribov
Dokshitser
Siminov
Maxwell
Cornwall

lattice: Furui, Nakajima (MILC)

DSE: Alkofer, Fischer, von Smekal et al.

IR Fixed Point for QCD?

- *Dyson-Schwinger Analysis: QCD coupling (mom scheme) has IR Fixed point!* *Alkofer, Fischer, von Smekal et al.*
- *Lattice Gauge Theory*
- Define coupling from observable, indications of IR fixed point for QCD effective charges
- Confined gluons and quarks: Decoupling of QCD vacuum polarization at small Q^2
- Justifies application of AdS/CFT in strong-coupling conformal window

Conformal symmetry: Template for QCD

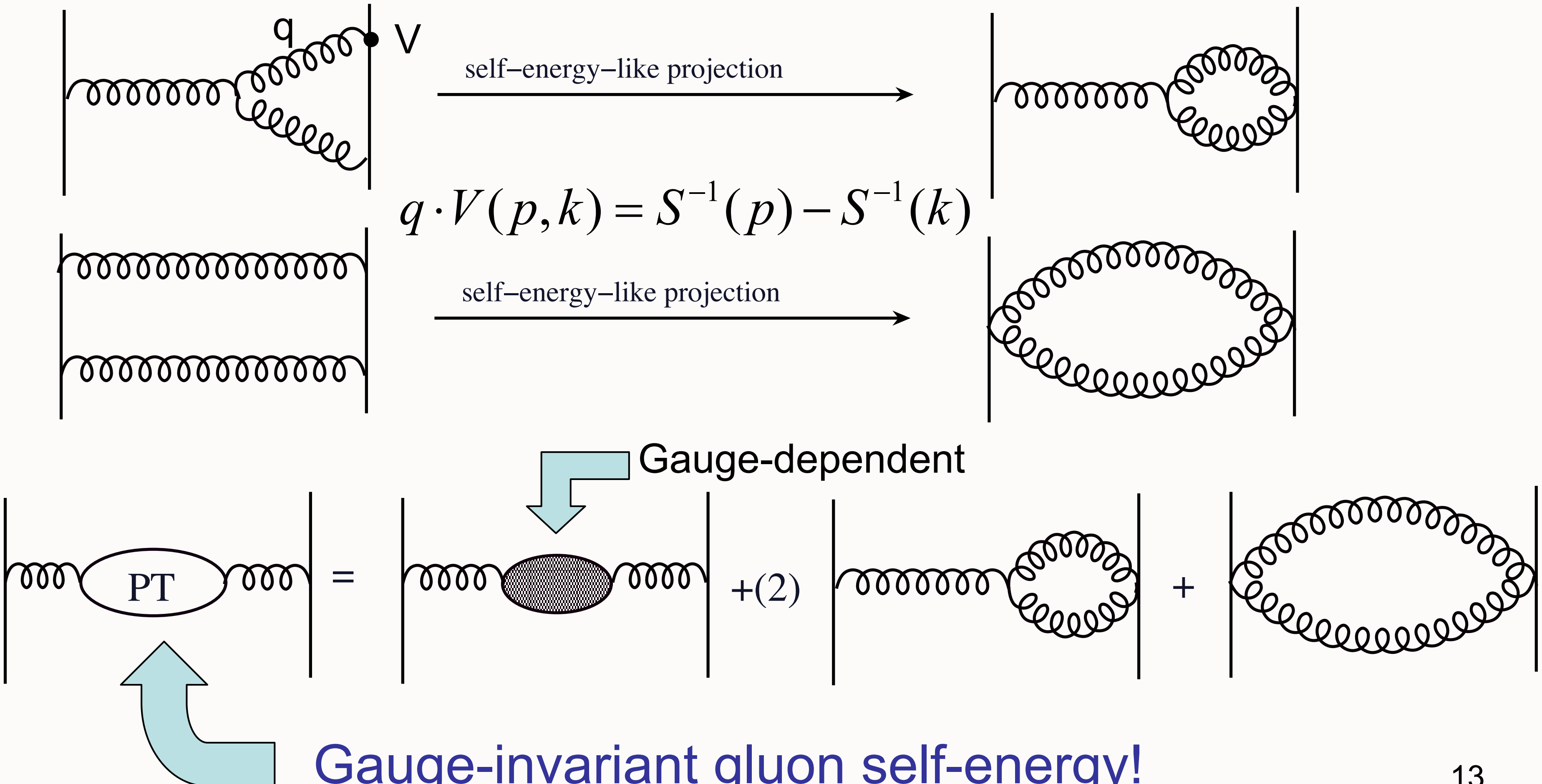
- Initial approximation to PQCD; then correct for non-zero beta function and quark masses: BLM
- Commensurate scale relations: relate observables at corresponding scales: Generalized Crewther Relation
- Arguments for Infrared fixed-point for α_s
- Effective Charges: analytic at quark mass thresholds, finite at small momenta
- Eigensolutions of Evolution Equation of distribution amplitudes

V. Braun et al;
Frishman, Lepage, Sachrajda, sjb

- AdS/QCD

The Pinch Technique

(Cornwall, Papavassiliou)



$$q \cdot V(p,k) = S^{-1}(p) - S^{-1}(k)$$

Gauge-invariant gluon self-energy!

13

natural generalization of QED charge

Pinch Scheme (PT)

- J. M. Cornwall, Phys. Rev. D 26, 345 (1982)
- Equivalent to Background Field Method in Feynman guage
- Effective Lagrangian Scheme of Kennedy & Lynn
- Rearrange Feynman diagrams to satisfy Ward Identities
- Longitudinal momenta from triple-gluon coupling, etc. hit vertices which cancel ("pinch") propagators
- Two-point function: Uniqueness, analyticity, unitarity, optical theorem
- Defines analytic coupling with smooth threshold behavior

Use Physical Scheme to Characterize QCD Coupling

- Use Observable to define QCD coupling or Pinch Scheme
- Analytic: Smooth behavior as one crosses new quark threshold
- New perspective on grand unification

Binger, Sjb

Unification in Physical Schemes

"PHYSICAL RENORMALIZATION SCHEMES AND GRAND UNIFICATION"
M.B. and Stanley J. Brodsky. **Phys.Rev.D69:095007,2004**

$$\alpha_i(Q) = \frac{\alpha_i(Q_0)}{1+\hat{\Pi}_i(Q)-\hat{\Pi}_i(Q_0)}$$

i=1,2,3

$$\hat{\Pi}_i(Q) = \frac{\alpha_i}{4\pi}\sum_p \beta_i^{(p)}\left(L_{s(p)}(Q^2/m_p^2)+\cdots\right)$$

"log-like" function:

$$L_{s(p)} \approx \log(e^{\eta_p}+Q^2/m_p^2)$$

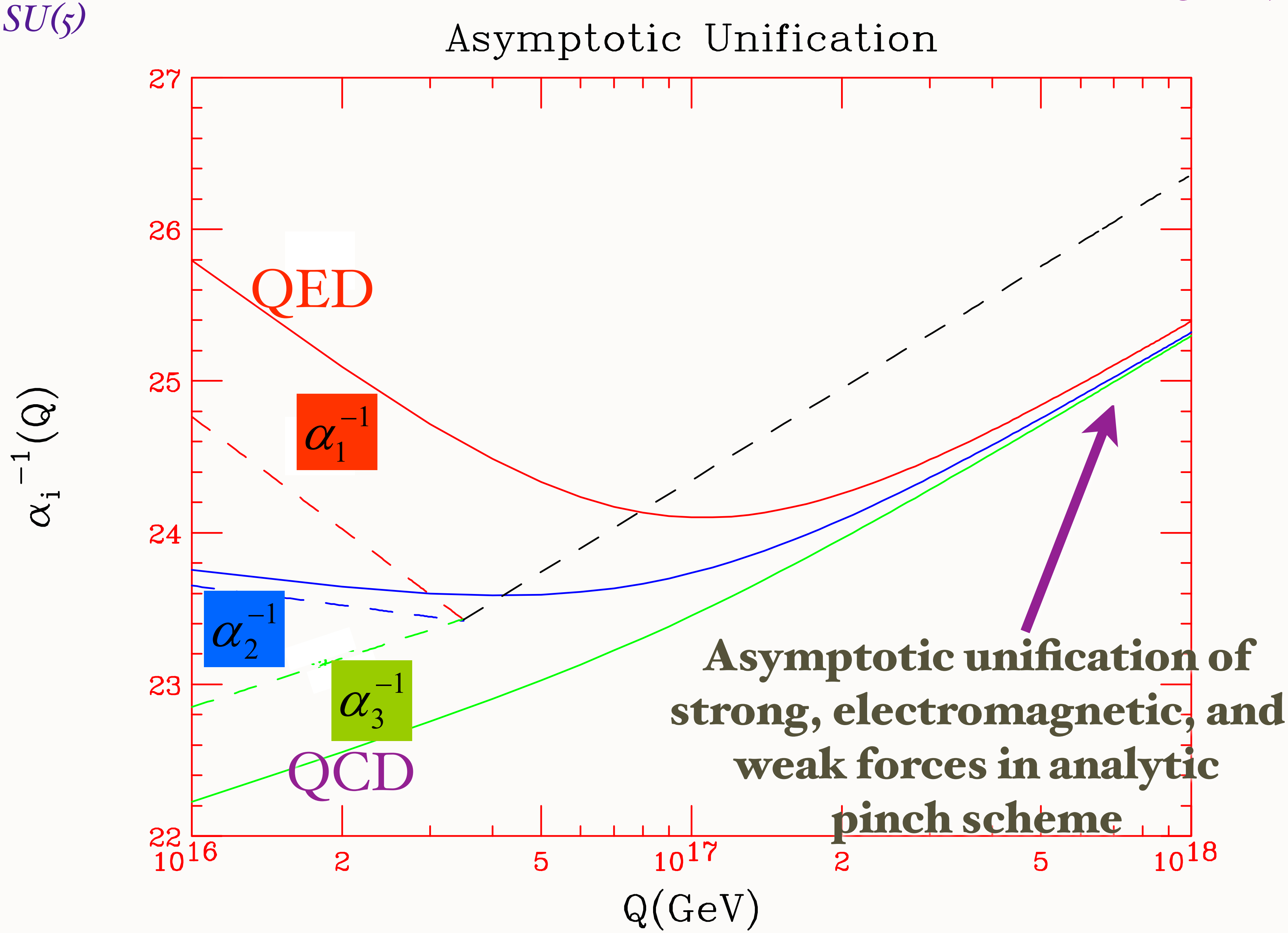
$$\eta_p = 8/3,\, 5/3,\, 40/21$$

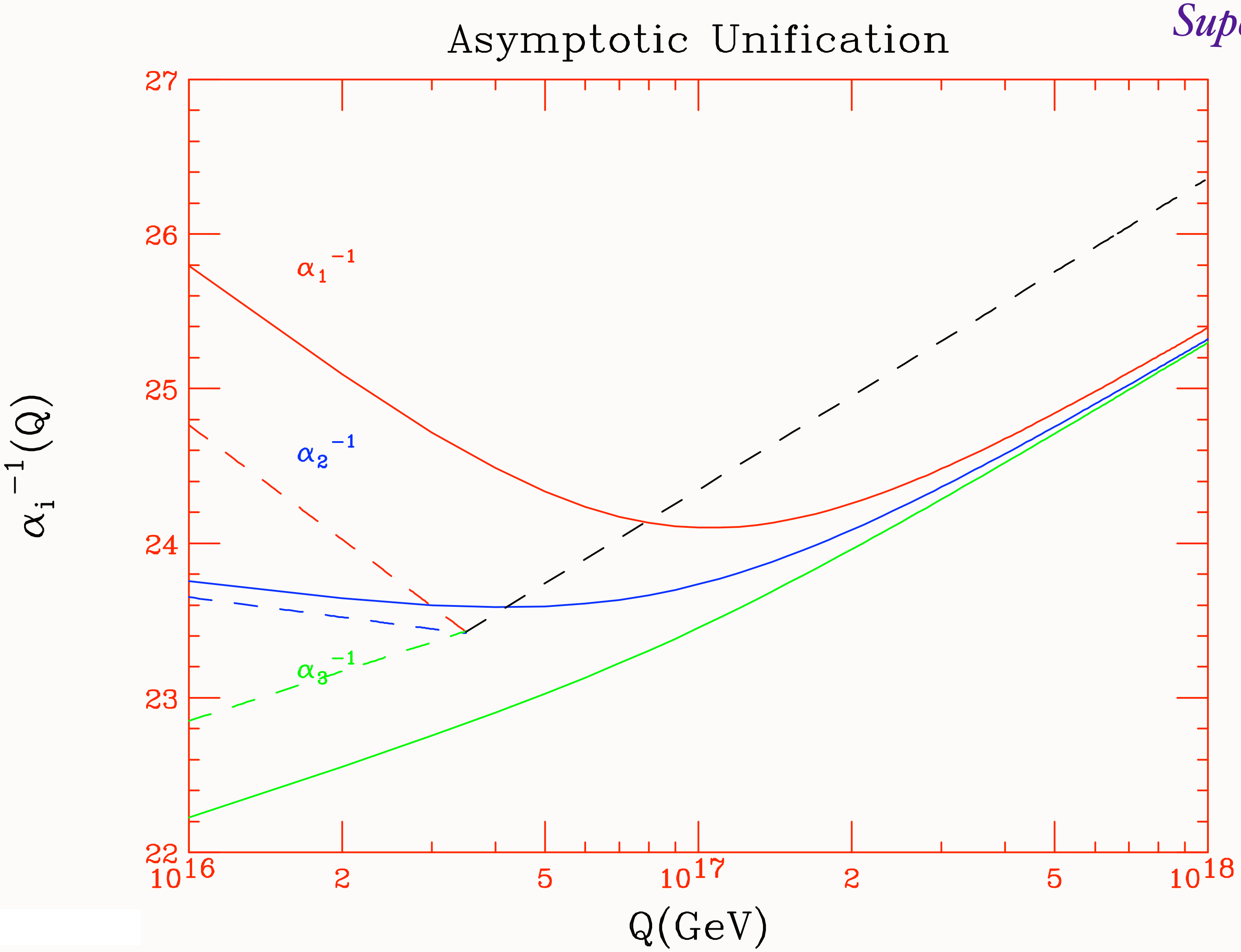
For spin s(p) = 0, ½, and 1

Elegant and natural formalism for all threshold effects

Supersymmetric SU(5)

Binger, sjb





Binger, sjb

Asymptotic Unification. The solid lines are the analytic $\overline{PT}$ effective couplings, while the dashed lines are the $\overline{DR}$ couplings. For illustrative purposes, $\alpha_3(M_Z)$ has been chosen so that unification occurs at a finite scale for $\overline{DR}$ and asymptotically for the $\overline{PT}$ couplings. Here $M_{SUSY} = 200\text{GeV}$ is the mass of all light superpartners except the wino and gluino which have values $\frac{1}{2}m_{\tilde{g}} = M_{SUSY} = 2m_{\tilde{w}}$. For illustrative purposes, we use $SU(5)$.

Unification in Physical Schemes

- Smooth analytic threshold behavior with automatic decoupling

- More directly reflects the unification of the forces

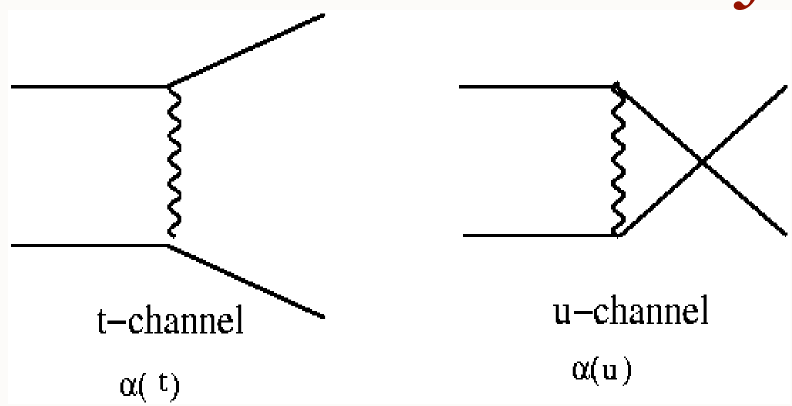
- Higher "unification" scale than usual

Renormalization scale and scheme

- The parameters μ^2 or Λ_{qcd} depend on the details, how the renormalization is done, i.e. which of the final parts are kept...
- Schemes are (Brodsky,Lu PRD 51, 3652 (1995)):
  - Fastest apparent convergence (FAC) choose scale μ^2 such that NLO coefficient vanishes
  - Principle of minimum sensitivity (PMS) chooses μ^2 at a stationary point $\frac{d\rho^{obs}}{d\mu} = 0$
  - BLM scheme (Brodsky,Lepage,Mackenzie) choose scale such that all flavor dependence is put into coupling and coefficients are independent of number of quark flavours renormalising gluon propagators

- What is the relevant scale in QED and QCD ?
  - Apply higher order corrections and hope that changes of the scale do not change much the result .. (standard folklore ..)
  - BLM has clear prescription from QED:

H. Jung



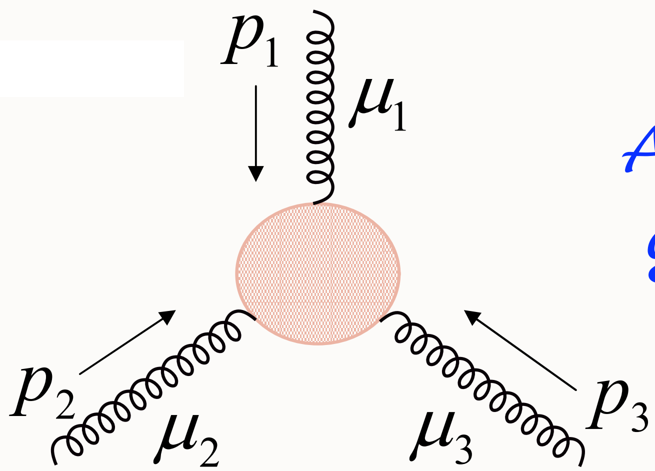
  - From analogy with QED apply no scale uncertainty also for QCD !
  - but what about triple gluon vertex?

General Structure of the Three-Gluon Vertex

"THE FORM-FACTORS OF THE GAUGE-INVARIANT THREE-GLUON VERTEX"

M. Binger, sjb

$$\hat{\Gamma}_{\mu_1\mu_2\mu_3} =$$



Analytic calculation: general masses, spin

3 index tensor $\hat{\Gamma}_{\mu_1\mu_2\mu_3}$ built out of $g_{\mu\nu}$ and p_1, p_2, p_3

with $p_1 + p_2 + p_3 = 0$

⇨ 14 basis tensors and form factors

PokerPages.com The Real Deal

37th Annual World Series of Poker

Event #39 - WSOP No Limit Hold'em Championship WSOP 3rd $4,123,310 Aug 10, 2006

Name: Mike Binger
Location: Atherton, CA, United States
Cashes: 7
Total Winnings: $4,347,767
ProRank 1 Position: 629

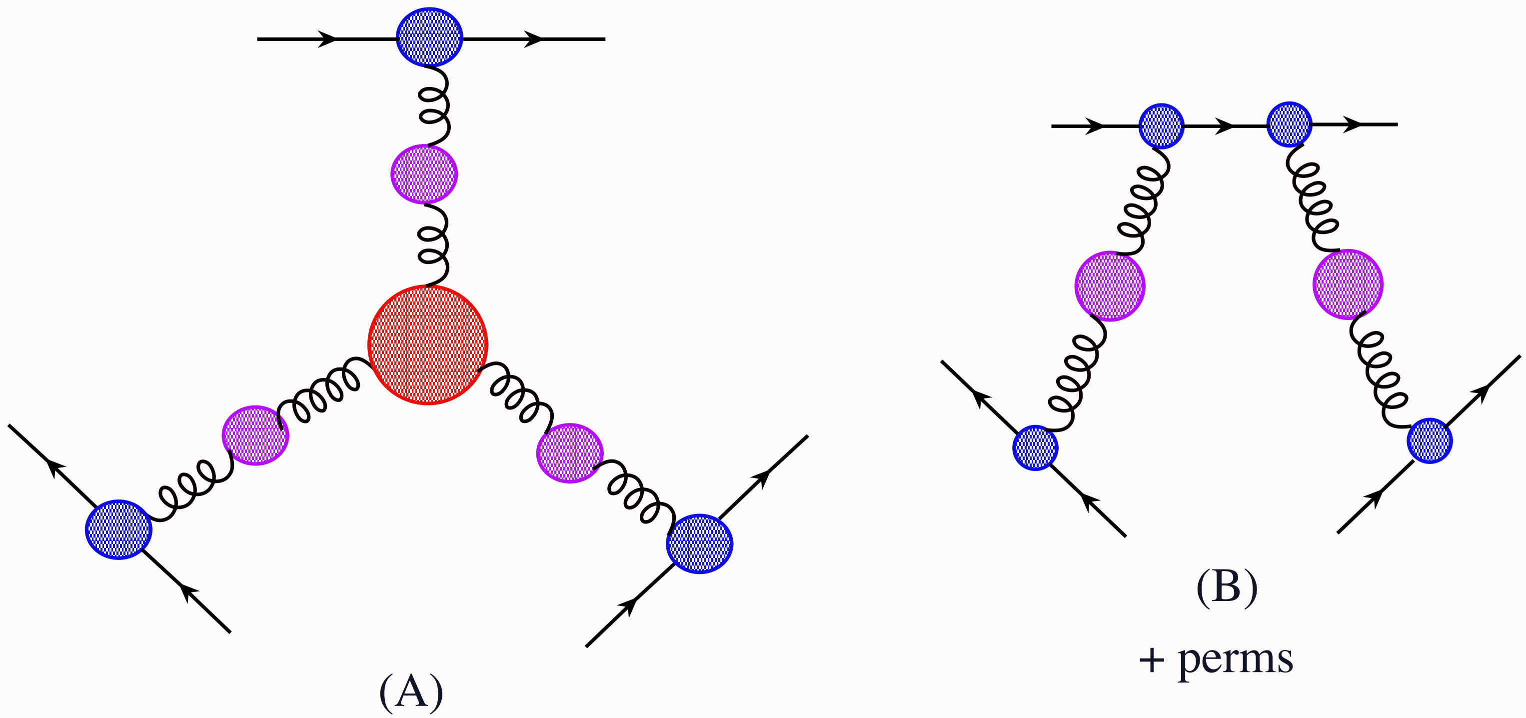
Biggest Poker Accomplishments:

2006 WSOP - Event 39, No-Limit Texas Hold'em Championship Event	3rd	$4,123,310
2007 WSOP - Event 22, No-Limit Hold'em	3rd	$295,245
2006 WSOP - Event 27, No-Limit Hold'em	6th	$101,570
2007 WSOP - Event 8, No-Limit Hold'em w/Re-Buys	14th	$21,278

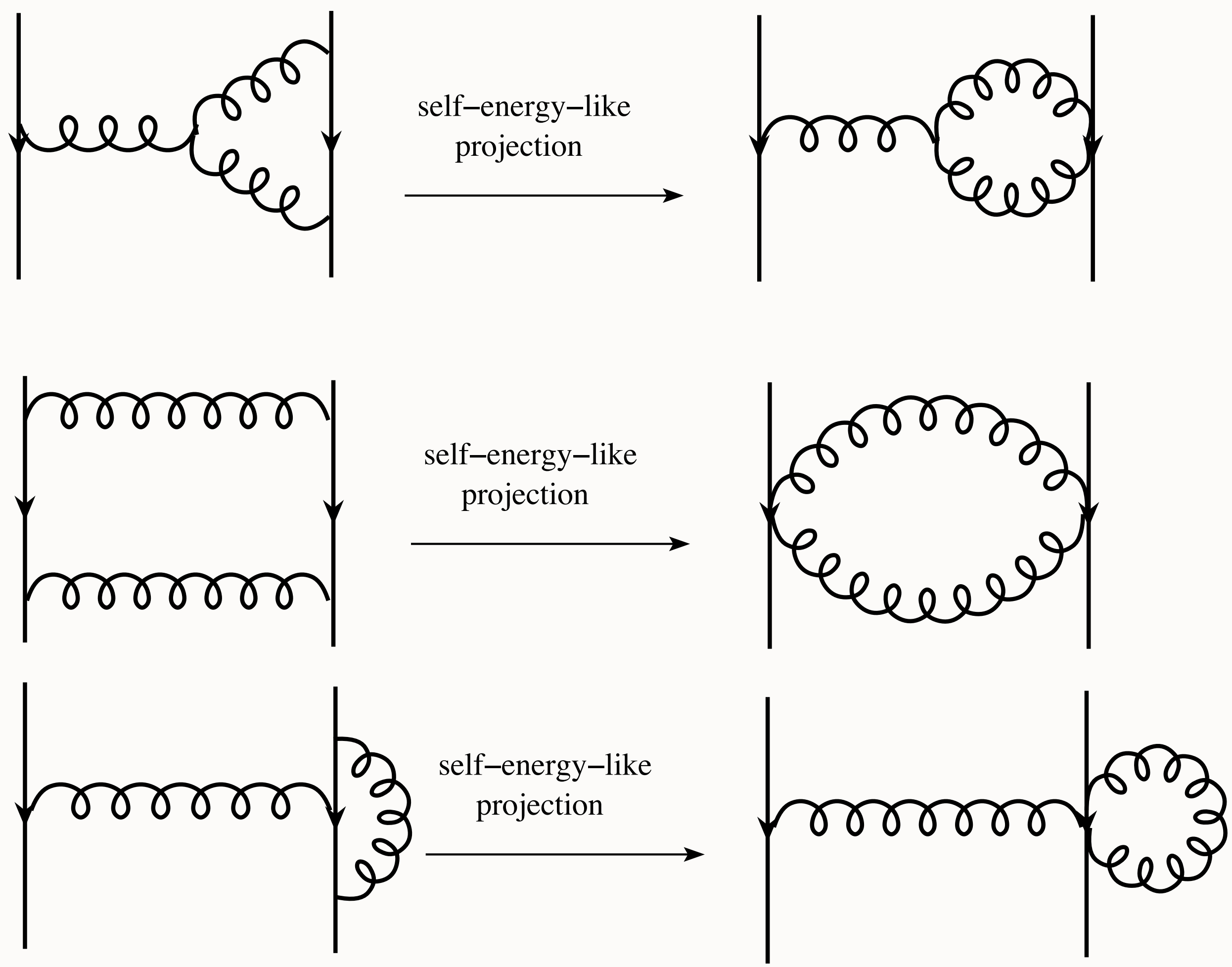
3 Gluon Vertex In Scattering Amplitudes

Pinch-Technique approach :

fully dress with gauge-invariant Green's functions



Pinch Scheme -- Effective Charge



$$< 0|G^{\mu\nu}(x)G^{\sigma\tau}(0)|0 > \qquad G^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu + ig[A^\mu, A^\nu]$$

Background Field Method

- Gauge field is split into quantum (Q) and background (B) parts

$$\underbrace{A_\mu = B_\mu}_{\text{External legs}} + \underbrace{Q_\mu}_{\text{Loops}}$$

PT = BFM in quantum Feynman gauge (BFMFG)

Proven by Binosi and Papavassiliou to all orders

also = star-scheme for electroweak theory at one-loop (Kennedy and Lynn)

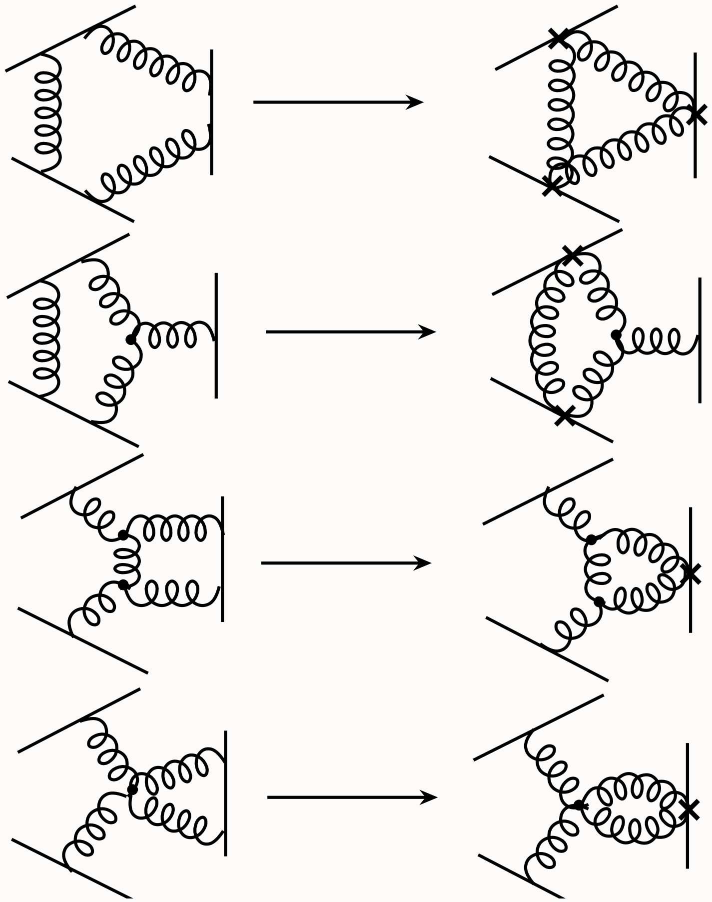
PT/BFMFG Green's functions have excellent properties :

- Non-abelian analogs of QED with simple Ward ID's
- Lead to analytic effective charges
- Can be derived from unitarity (optical theorem)
- Correct asymptotic UV behavior

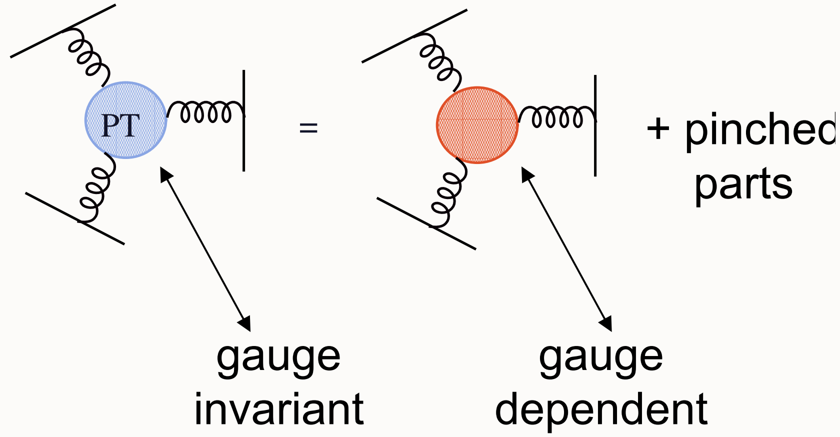
$$\Pi_{PT}(p^2) \propto \beta_0 \log(p^2) + \cdots$$

The Gauge Invariant Three Gluon Vertex

Cornwall and Papavassiliou performed the PT construction :



The “pinched” parts are added to the “regular” 3 gluon vertex



General Structure of the Three-Gluon Vertex

Simple (QED-like) Ward ID

$$p_3^{\mu_3}\hat{\Gamma}_{\mu_1\mu_2\mu_3}(p_1,p_2,p_3) = t_{\mu_1\mu_2}(p_2)\left[1+\hat{\Pi}(p_2)\right] - t_{\mu_1\mu_2}(p_1)\left[1+\hat{\Pi}(p_1)\right]$$

where $t_{\mu\nu}(p) = p^2 g_{\mu\nu} - p_\mu p_\nu$

⇒ One form factor always = 0 (not obvious) ⇒ 13 nonzero form factors

3 Gluon Vertex In Scattering Amplitudes

$$Amplitude = color \times vertices \times g(a)g(b)g(c)$$

$$\times \underbrace{g_{bare}\left[(1+A_0)\right.}_{\widetilde{g}(a,b,c)}\hat{t}_0 + \underbrace{A_+\hat{t}_+ + A_-\hat{t}_- + H\hat{h}\left.\right]}_{\text{Other tensors and form factors}}$$

Tree level tensor structure :

$$\hat{t}_0 = (p_1 - p_2)^{\mu_3} g^{\mu_1\mu_2} + (p_2 - p_3)^{\mu_1} g^{\mu_2\mu_3} + (p_3 - p_1)^{\mu_2} g^{\mu_3\mu_1}$$

Form factors A_0, A_+, A_-, H depend on these $\begin{cases} a = p_1^2 \\ b = p_2^2 \\ c = p_3^2 \end{cases}$

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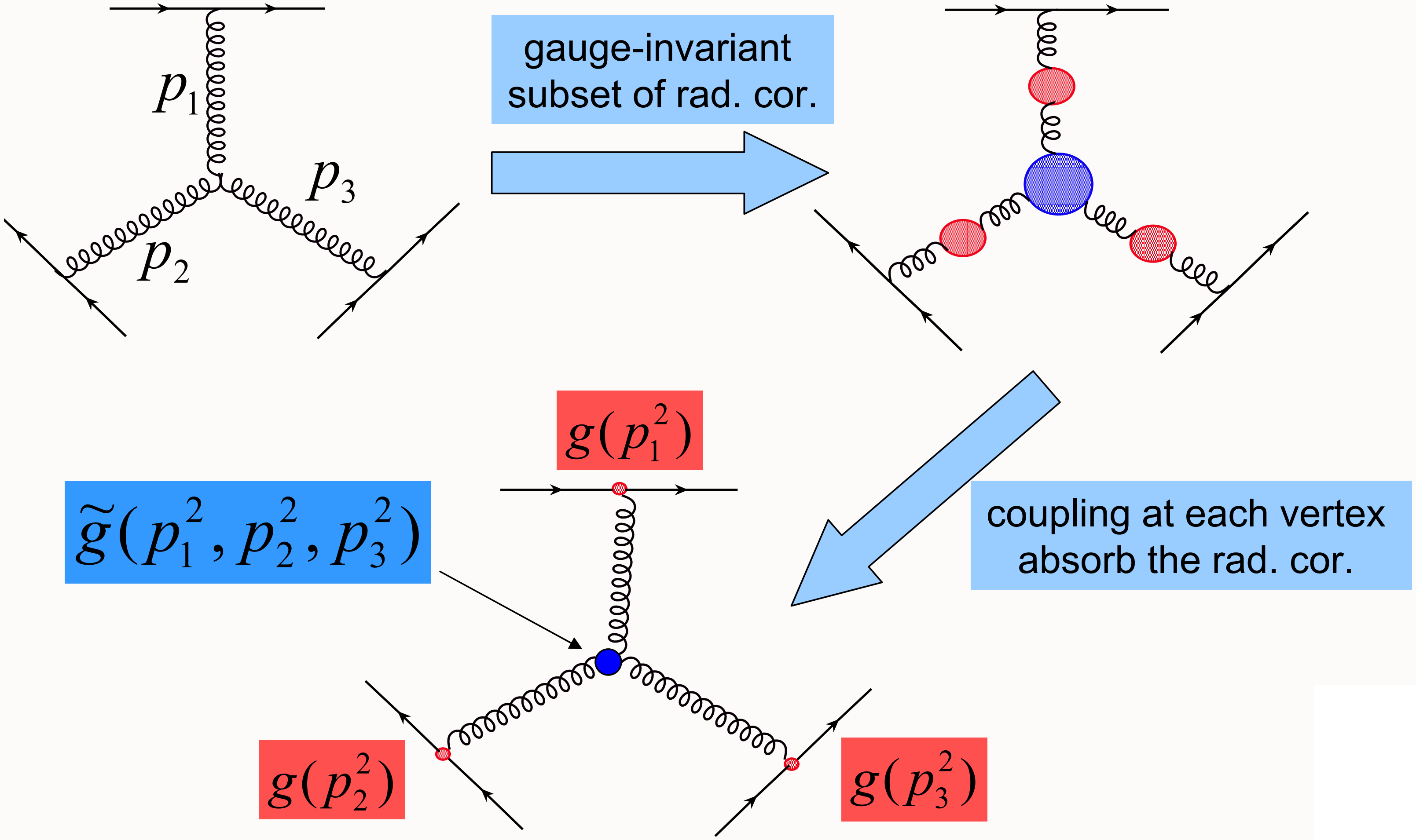
Multi-scale Renormalization of the Three-Gluon Vertex



Convenient Tensor Bases

Physical $\pm$ Basis

- Written in terms of linear combinations of momenta called "+" and "-" momenta such that $p_+ \cdot V_{ext} = 0$

  by elementary Ward IDs

- Maximum # of FF's vanish when in a physical matrix element

- Good for real scattering problems

LT Basis

- Longitudinal (L) FF's :

$$p_3^{\mu_3} \cdot \hat{\Gamma}^{(L)}_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3) \neq 0$$

- Transverse (T) FF's :

$$p_3^{\mu_3} \cdot \hat{\Gamma}^{(T)}_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3) = 0$$

- Good for theoretical work and solving Ward ID

Complementary in their relation to current conservation (Ward ID's) 24

Form Factors : Supersymmetric Relations

- Any form factor can be decomposed :

$$F = C_A F_G + 2\sum_f T_f F_Q + 2\sum_s T_s F_S$$

G = gluons
Q = quarks
S = scalars

C_A, T_f, T_s are color factors

- Individually, F_G, F_Q, F_S are complicated…

Form Factors : Supersymmetric Relations (Massless)

….but certain linear sums are simple :

$$\Sigma_{QG}(F) \equiv \frac{d-2}{2} F_Q + F_G \longrightarrow 0$$

for 7 of the 13 FF's (in physical basis)
$\pm$

Simple N=1 SUSY contribution in d=4

$$F_G + 4F_Q + (10-d)F_S = 0$$

For all FF's !!

N=4 SUSY in d=4 gives 0

These are off-shell generalizations of relations found in
SUSY scattering amplitudes by
Z. Bern, L.J. Dixon, D.C. Dunbar, and D.A. Kosower (NPB 425,435)

Vanishing contribution of the N=4 supermutiplet in d=4 dimensions

Form Factors : Consequences of Supersymmetric Relations

For any SUSY each of the 13 FF's are $\propto \beta_0$ even though only one FF is directly related to coupling renormalization

$$\beta_0(d) = \frac{7d-6}{2(d-1)} C_A - \frac{2(d-2)}{d-1} \sum_f T_f - \frac{1}{d-1} \sum_f T_s$$

$$\xrightarrow{\text{d = 4}} \quad \frac{11}{3} C_A - \frac{4}{3} T_f - \frac{1}{3} T_s$$

⇨ Contributions of gluons, quarks, and scalars have same functional form

33

Form Factors : Supersymmetric Relations (Massive)

Equal masses for massive gauge bosons (MG), quarks (MQ), and scalars (MS)

$$F_{MG} + 4F_{MQ} + (9-d)F_{MS} = 0$$

1 d.o.f. “eaten” by MG

Massive gauge boson (MG) inside of loop might be the X and Y gauge bosons of SU(5), for example

External gluons remain unbroken and massless

$$\Sigma_{MQG}(F) \equiv \frac{d-1}{2} F_{MQ} + F_{MG}$$ is simple

Form Factors : Consequences of Supersymmetric Relations

For any SUSY each of the 13 FF's are $\propto \beta_0$ even though only one FF is directly related to coupling renormalization

$$\beta_0(d) = \frac{7d-6}{2(d-1)} C_A - \frac{2(d-2)}{d-1} \sum_f T_f - \frac{1}{d-1} \sum_f T_s$$

$$\xrightarrow{d\,=\,4} \quad \frac{11}{3} C_A - \frac{4}{3} T_f - \frac{1}{3} T_s$$

⇨ Contributions of gluons, quarks, and scalars have same functional form

Summary of Supersymmetric Relations

Massless	Massive
$F_G + 4F_Q + (10-d)F_S = 0$	$F_{MG} + 4F_{MQ} + (9-d)F_{MS} = 0$
$\Sigma_{QG}(F) \equiv \frac{d-2}{2}F_Q + F_G$	$\Sigma_{MQG}(F) \equiv \frac{d-1}{2}F_{MQ} + F_{MG}$
= simple	= simple

3 Scale Effective Charge

$$\tilde{\alpha}(a,b,c) \equiv \frac{\tilde{g}^2(a,b,c)}{4\pi}$$

(First suggested by H.J. Lu)

$$\frac{1}{\tilde{\alpha}(a,b,c)} = \frac{1}{\alpha_{bare}} + \frac{1}{4\pi}\beta_0\left(L(a,b,c) - \frac{1}{\varepsilon} + \cdots \right)$$

$$\frac{1}{\tilde{\alpha}(a,b,c)} = \frac{1}{\tilde{\alpha}(a_0,b_0,c_0)} + \frac{1}{4\pi}\beta_0\left[L(a,b,c) - L(a_0,b_0,c_0) \right]$$

L(a,b,c) = 3-scale “log-like” function

L(a,a,a) = log(a)

3 Scale Effective Scale

$$L(a,b,c) \equiv \log\left(Q^2_{eff}(a,b,c)\right) + i\,\mathrm{Im}\,L(a,b,c)$$

Governs strength of the three-gluon vertex

$$\frac{1}{\widetilde{\alpha}(a,b,c)} = \frac{1}{\widetilde{\alpha}(a_0,b_0,c_0)} + \frac{1}{4\pi}\beta_0\left[L(a,b,c) - L(a_0,b_0,c_0)\right]$$

$$\hat{\Gamma}_{\mu_1\mu_2\mu_3} \propto \sqrt{\widetilde{\alpha}(a,b,c)}$$

Generalization of BLM Scale to 3-Gluon Vertex

3 Scale Log-Like Function

$$L(a,b,c)=\frac{1}{\mathrm{K}}\left(\alpha\gamma\log a+\alpha\beta\log b+\beta\gamma\log c-abc\bar{J}(a,b,c)\right)+\Omega$$

$$\mathrm{K}=\alpha\beta+\beta\gamma+\gamma\alpha$$

Master triangle integral can be written in terms of Clausen functions

$$Cl_2(\theta)=\mathrm{Im}Li_2\left(e^{i\theta}\right)$$

$$\Omega\approx 3.125$$

$$\alpha=p_1\cdot p_2=\frac{1}{2}(c-a-b)$$

$$\beta=p_2\cdot p_3=\frac{1}{2}(a-b-c)$$

$$\gamma=p_3\cdot p_1=\frac{1}{2}(b-c-a)$$

$$a=p_1^2$$

$$b=p_2^2$$

$$c=p_3^2$$

Properties of the Effective Scale

$$Q^2_{eff}(a,b,c) = Q^2_{eff}(-a,-b,-c)$$

$$Q^2_{eff}(\lambda a, \lambda b, \lambda c) = |\lambda| Q^2_{eff}(a,b,c)$$

$$Q^2_{eff}(a,a,a) = |a|$$

$$Q^2_{eff}(a,-a,-a) \approx 5.54 |a|$$

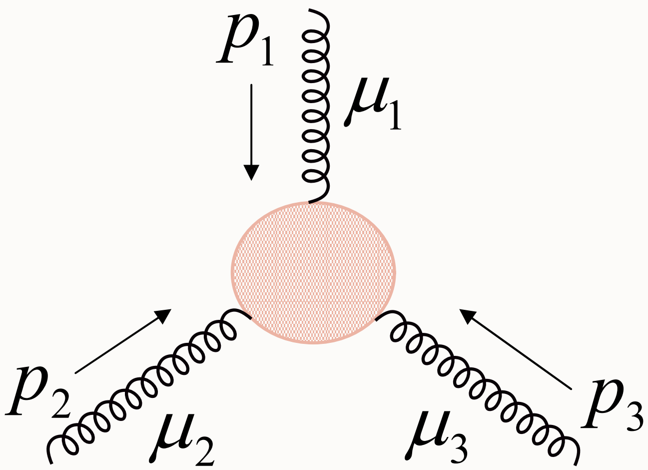
$$Q^2_{eff}(a,a,c) \approx 3.08 |c| \quad \text{for} \quad |a| >> |c|$$

$$Q^2_{eff}(a,-a,c) \approx 22.8 |c| \quad \text{for} \quad |a| >> |c|$$

$$Q^2_{eff}(a,b,c) \approx 22.8 \frac{|bc|}{|a|} \quad \text{for} \quad |a| >> |b|, |c|$$

Surprising dependence on Invariants

$$\hat{\Gamma}_{\mu_1\mu_2\mu_3} =$$



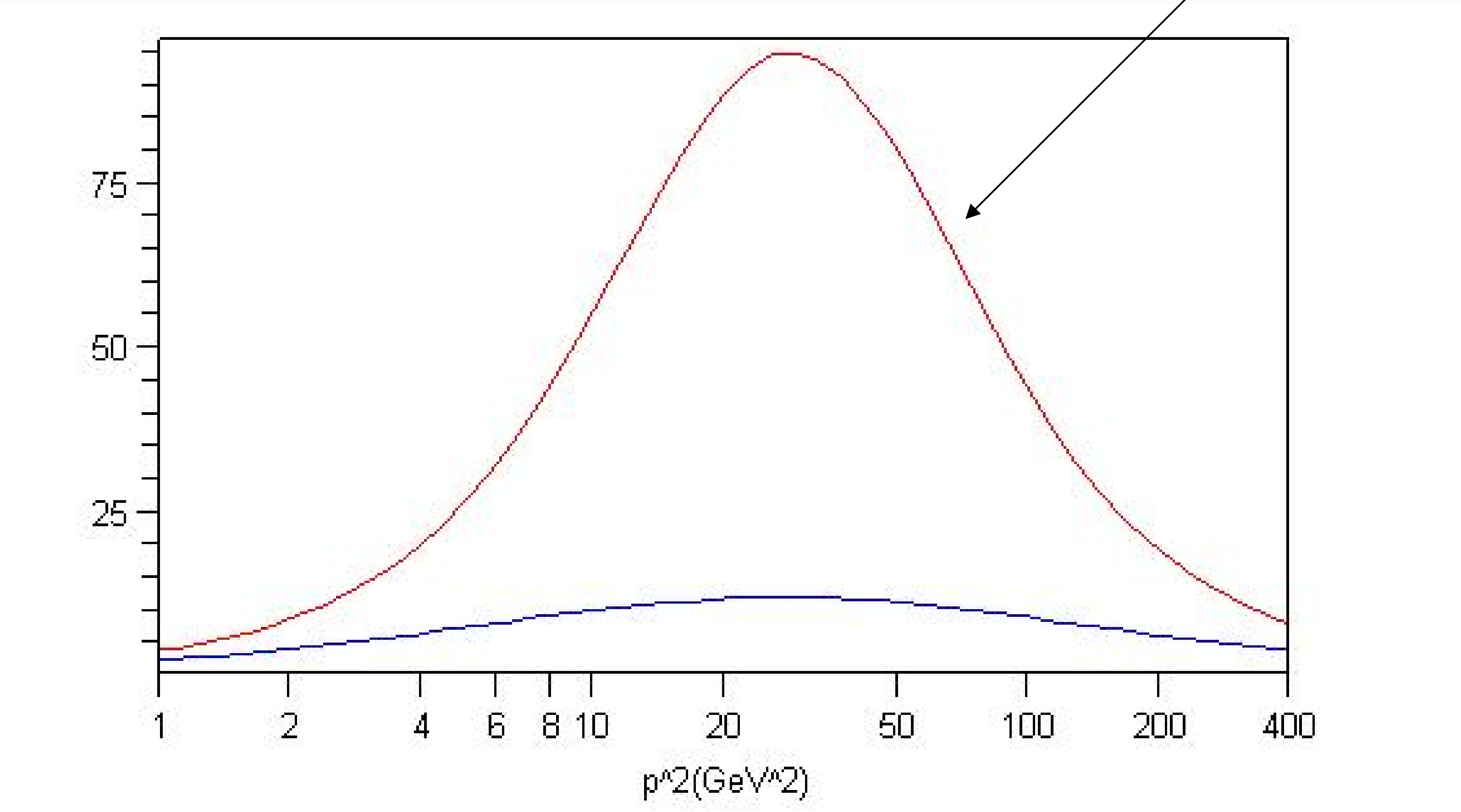
H. J. Lu

$$\mu_R^2 \simeq \frac{p^2_{min} p^2_{med}}{p^2_{max}}$$

The Effective Scale

$Q^2_{eff}\left(10\,\mathrm{GeV}^2, 10\,\mathrm{GeV}^2, p^2\right)$

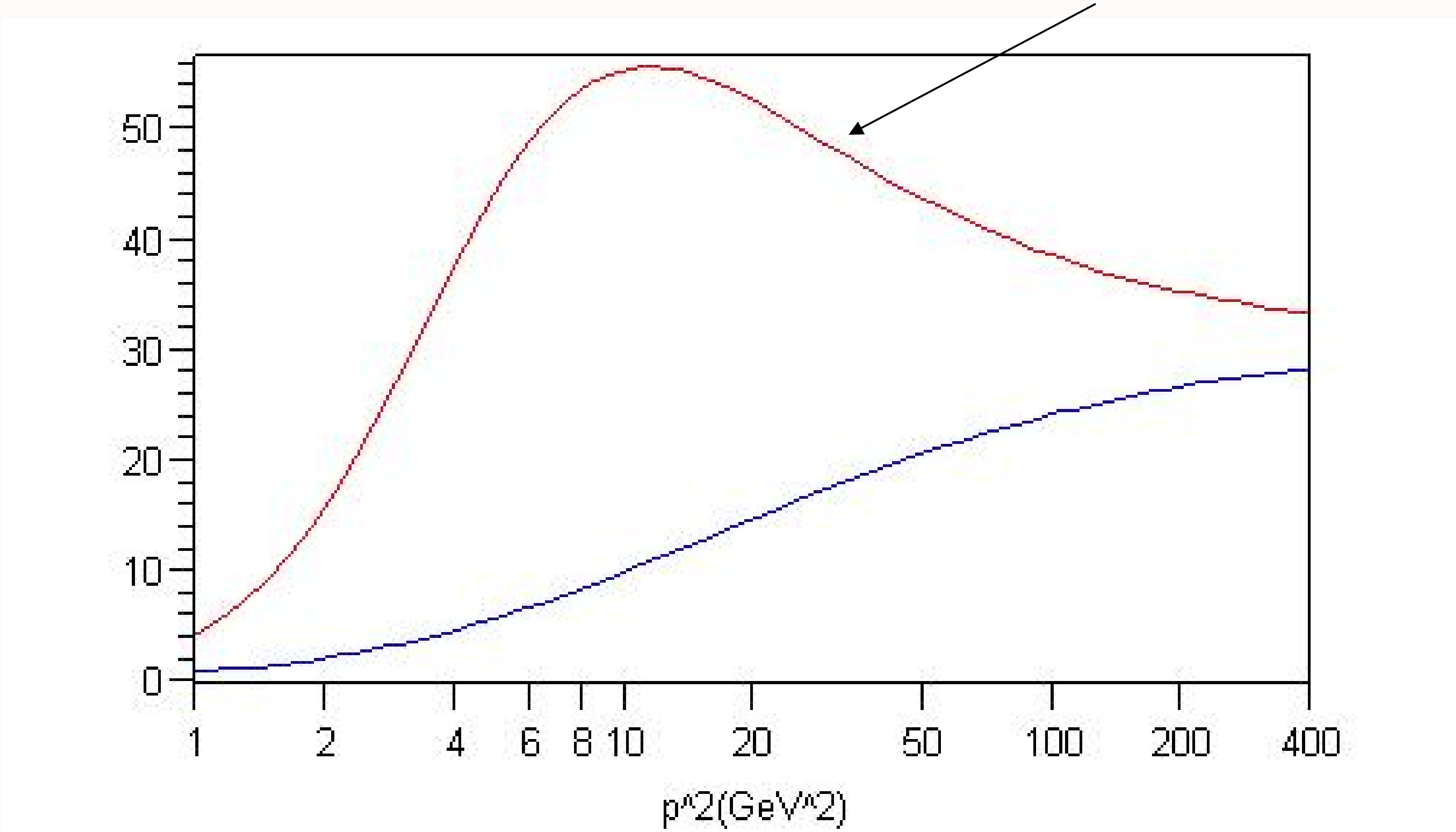
$Q^2_{eff}\left(-10\,\mathrm{GeV}^2, -10\,\mathrm{GeV}^2, p^2\right)$



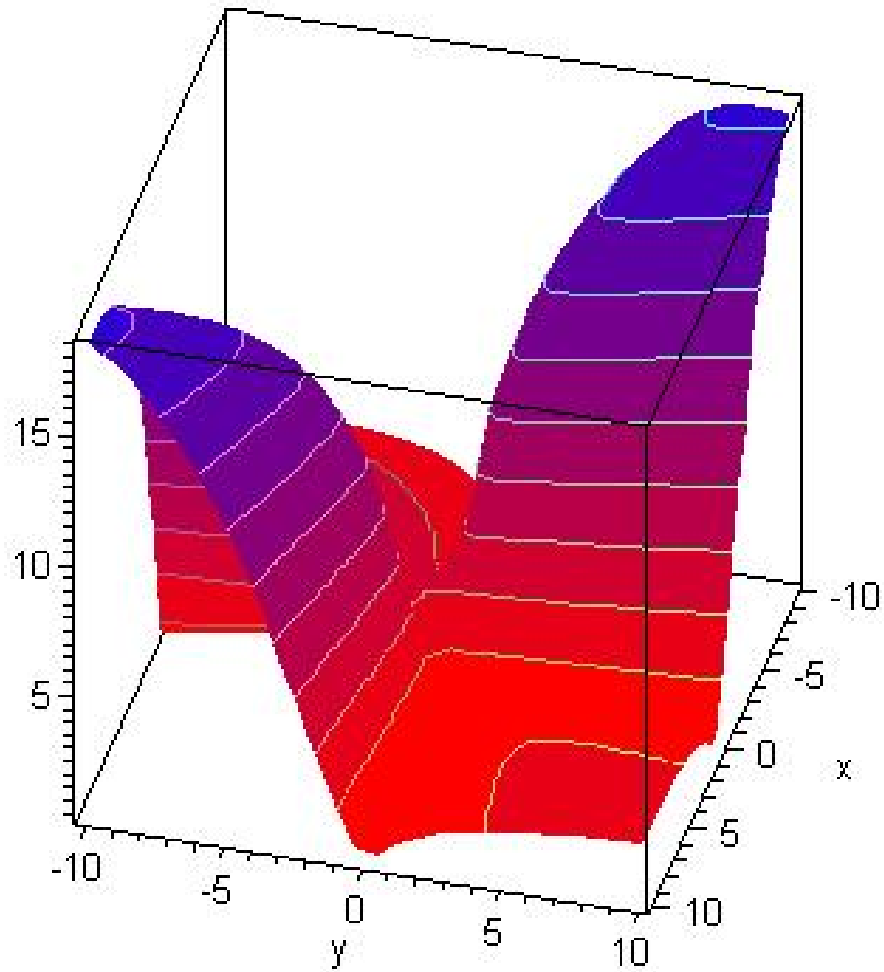
The Effective Scale

$Q^2_{eff}\left(10\,\mathrm{GeV}^2, p^2, p^2\right)$

$Q^2_{eff}\left(-10\,\mathrm{GeV}^2, p^2, p^2\right)$



The Effective Scale



$$Q^2_{eff}(1, x, y)$$

Mass Effects

Calculated for all form factors

SUSY relations $F_{MG} + 4F_{MQ} + (9-d)F_{MS} = 0$

FF of tree level tensor structure ⟷ Effective Charge

Massive “log-like” function : $L_{MQ}\left(\frac{a}{M^2},\frac{b}{M^2},\frac{c}{M^2}\right)$

$$L_{MQ}\left(\frac{a}{M^2},\frac{b}{M^2},\frac{c}{M^2}\right) \approx 5.125 \ \text{for} \ \ M^2 >> |a|,|b|,|c|$$

$$L_{MQ}\left(\frac{a}{M^2},\frac{b}{M^2},\frac{c}{M^2}\right) \approx L(a,b,c) - \log M^2 \ \ \text{for} \ \ M^2 << |a|,|b|,|c|$$

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Massive Log-Like Function

$$L_{MQ}\left(\frac{a}{M^2},\frac{b}{M^2},\frac{c}{M^2}\right)=\frac{1}{\mathrm{K}}\left(\alpha\gamma\Lambda(a)+\alpha\beta\Lambda(b)+\beta\gamma\Lambda(c)-abc\overline{J_M}(a,b,c)\right)+\Omega$$

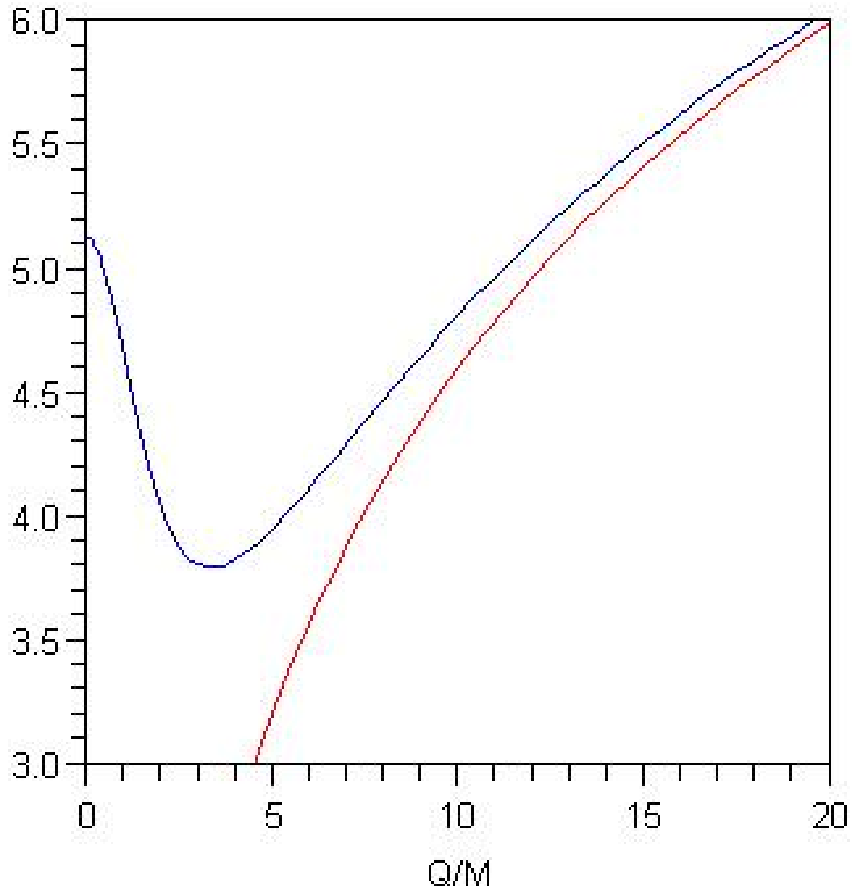
$$+2M^2\left(\frac{\Lambda(a)-2}{a}+\frac{\Lambda(b)-2}{b}+\frac{\Lambda(c)-2}{c}-\overline{J_M}(a,b,c)\right)$$

$$\Lambda(a)=\left\{\begin{array}{l} 2v\tanh^{-1}\left(v^{-1}\right) \\ 2\bar{v}\tan^{-1}\left(\bar{v}^{-1}\right) \\ 2v\tanh^{-1}(v)-iv\pi \end{array}\right\} \text{ for } \left\{\begin{array}{l} a<0 \\ 0<a<4M^2 \\ a>4M^2 \end{array}\right\}$$

$$v=\sqrt{1-\frac{4M^2}{a}} \qquad \bar{v}=\sqrt{\frac{4M^2}{a}-1}$$

Massive Master Triangle Integral (very complicated)

Symmetric Spacelike



$$L_{MQ}\left(-\frac{Q^2}{M^2},-\frac{Q^2}{M^2},-\frac{Q^2}{M^2}\right)$$

$$\log\left(\frac{Q^2}{M^2}\right)$$

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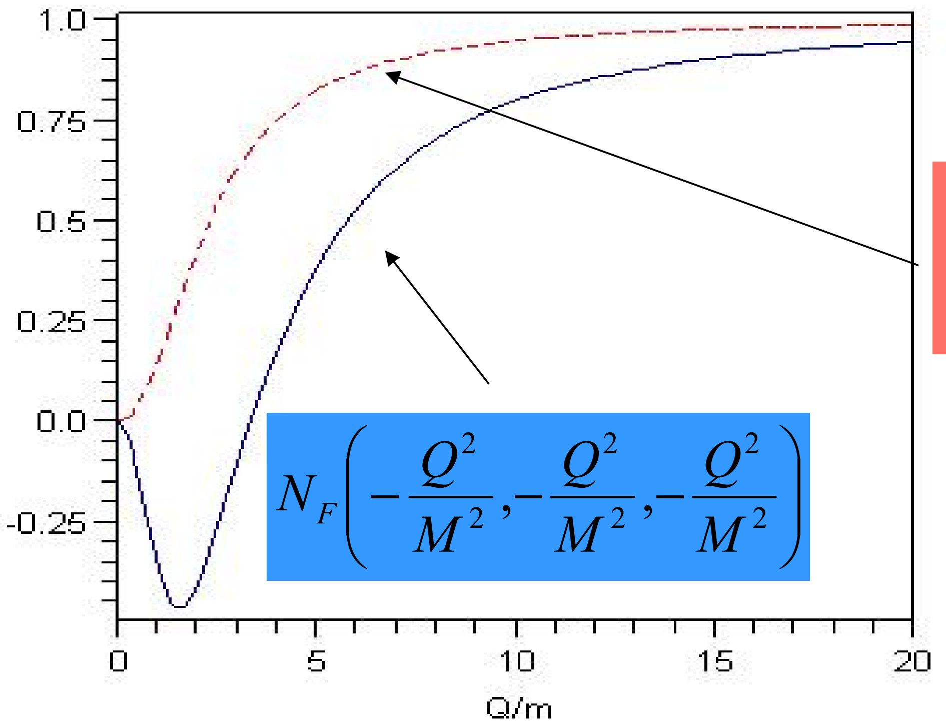
Effective Number of Flavors

$$N_F\left(\frac{a}{M^2},\frac{b}{M^2},\frac{c}{M^2}\right)=-\frac{d}{d\log M^2}L_{MQ}\left(\frac{a}{M^2},\frac{b}{M^2},\frac{c}{M^2}\right)$$



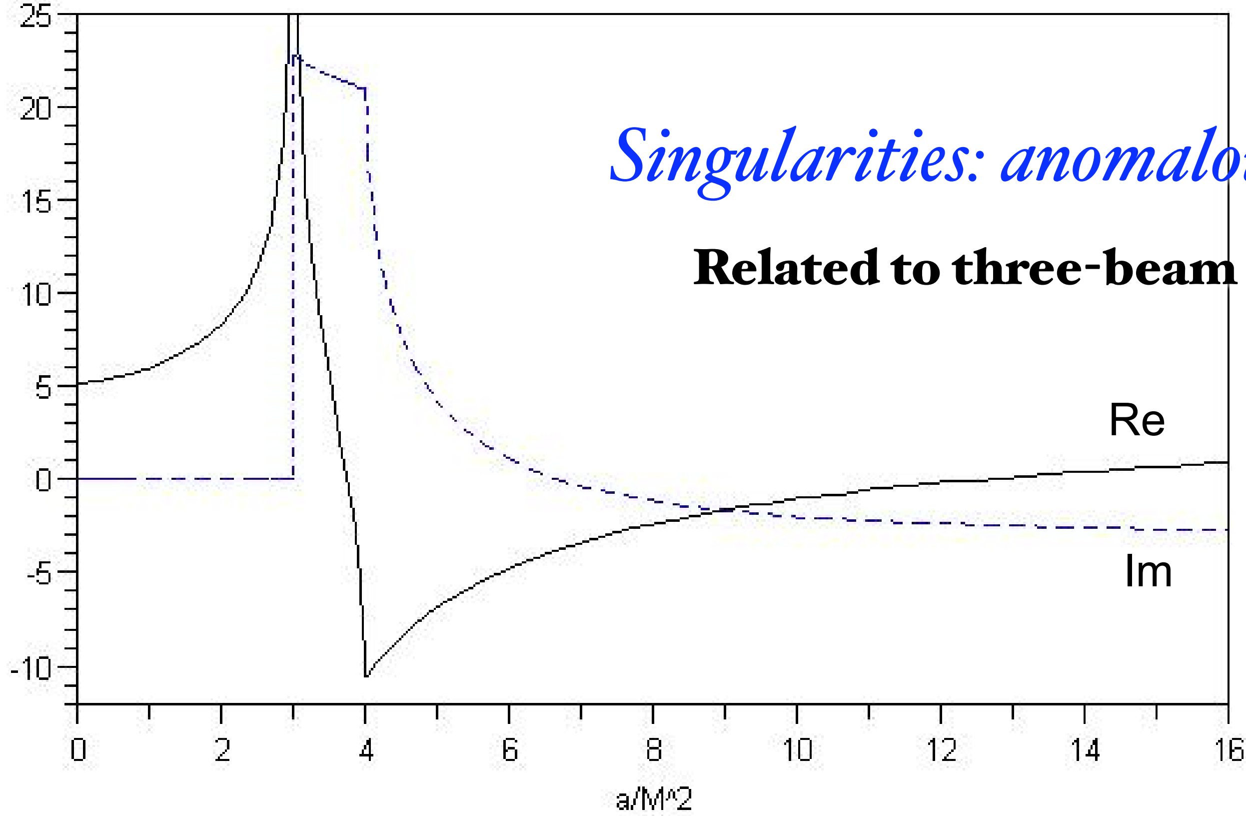
$$N_F\left(-\frac{Q^2}{M^2},-\frac{Q^2}{M^2},-\frac{Q^2}{M^2}\right)$$

$$n_f\left(\frac{Q^2}{M^2}\right)=-\frac{d}{d\log M^2}L_{1/2}\left(\frac{Q^2}{M^2}\right)$$

$$\approx \frac{1}{1+\frac{M^2}{Q^2}e^{5/3}}$$

Symmetric Timelike

$$L_{MQ}\left(\frac{a}{M^2},\frac{a}{M^2},\frac{a}{M^2}\right)$$



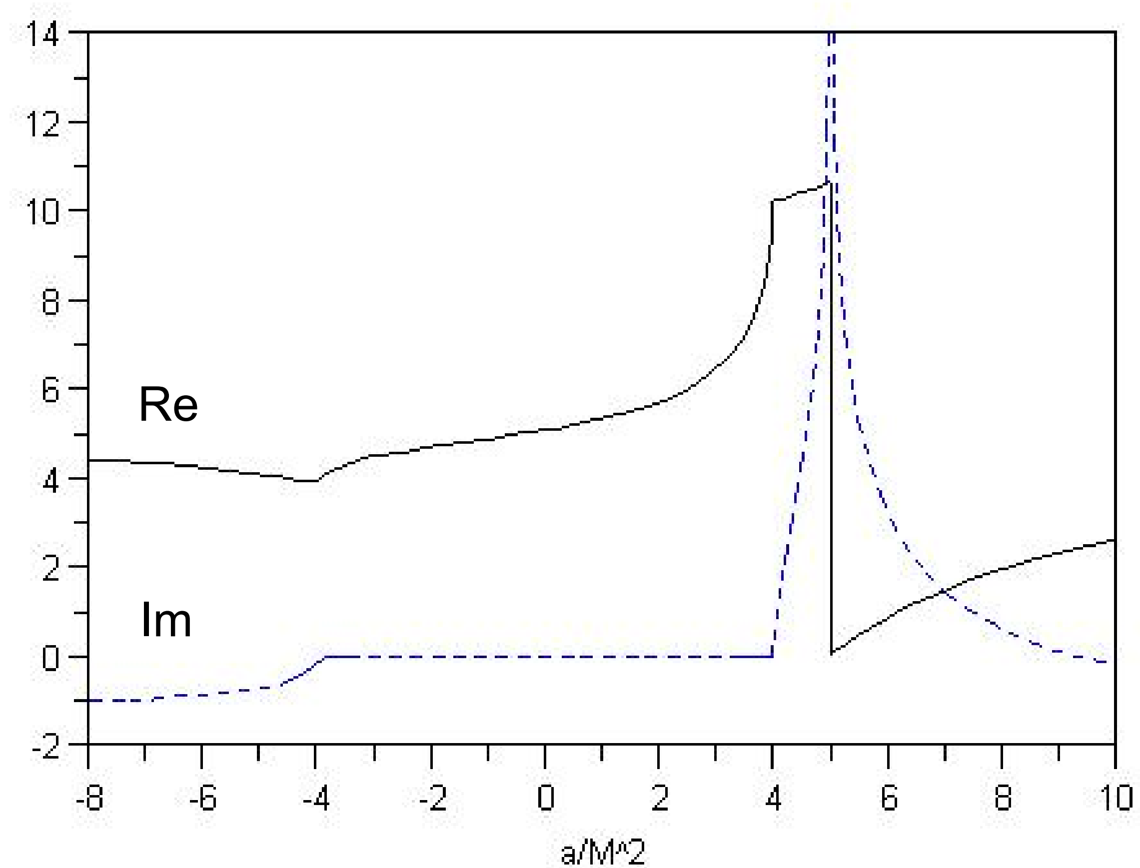
Singularities: anomalous thresholds

Related to three-beam scattering?

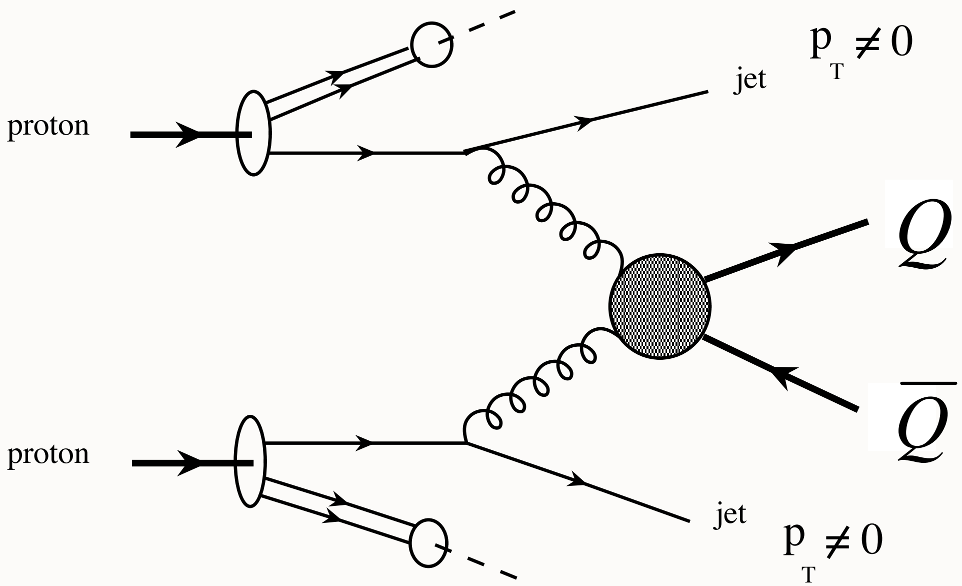
Related to three-beam scattering?

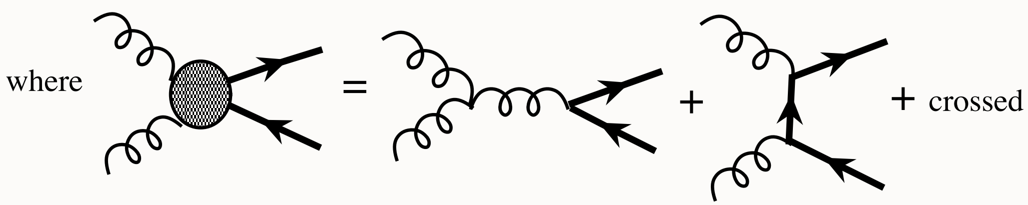
Symmetric Mixed Signature

$$L_{MQ}\left(\frac{a}{M^2},\frac{a}{M^2},-\frac{a}{M^2}\right)$$



Heavy Quark Hadro-production





- Preliminary calculation using (massless) results for tree level form factor
- Very low effective scale

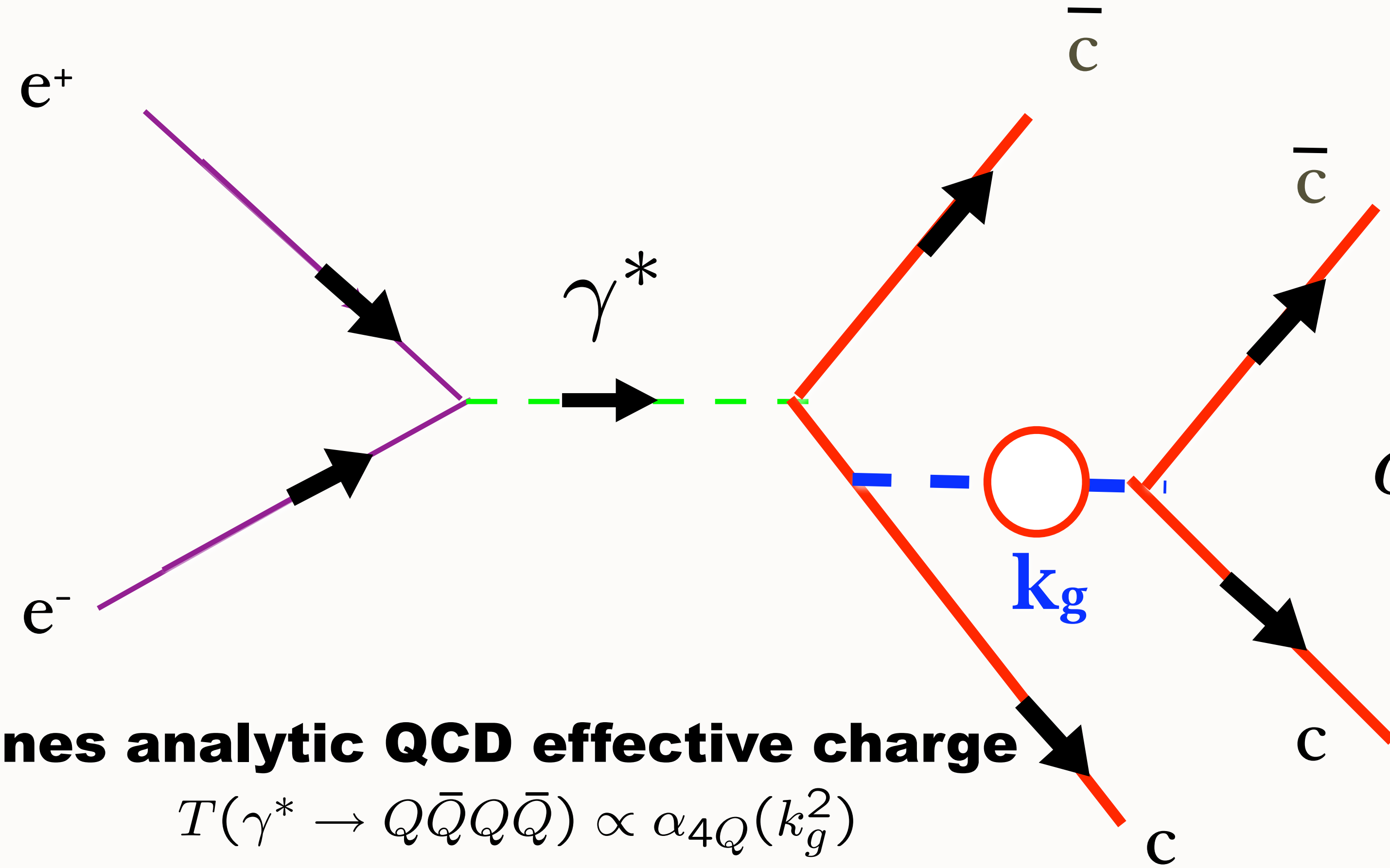
→ much larger cross section than $\overline{MS}$ with scale $\mu_R = M_{Q\overline{Q}}$ or M_Q

- Future : repeat analysis using the full mass-dependent results and include all form factors

Expect that this approach accounts for most of the one-loop corrections

Production of four heavy-quark jets

M. Binger, sjb



$$\alpha_{4Q}(k_g^2)$$

Defines analytic QCD effective charge

$$T(\gamma^* \to Q\bar{Q}Q\bar{Q}) \propto \alpha_{4Q}(k_g^2)$$

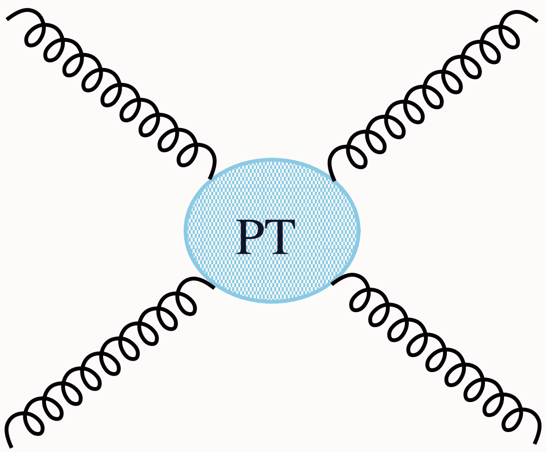
time-like values not same as space-like

coupling similar to "pinch" scheme

complex for time-like argument

Future Directions

Gauge-invariant four gluon vertex

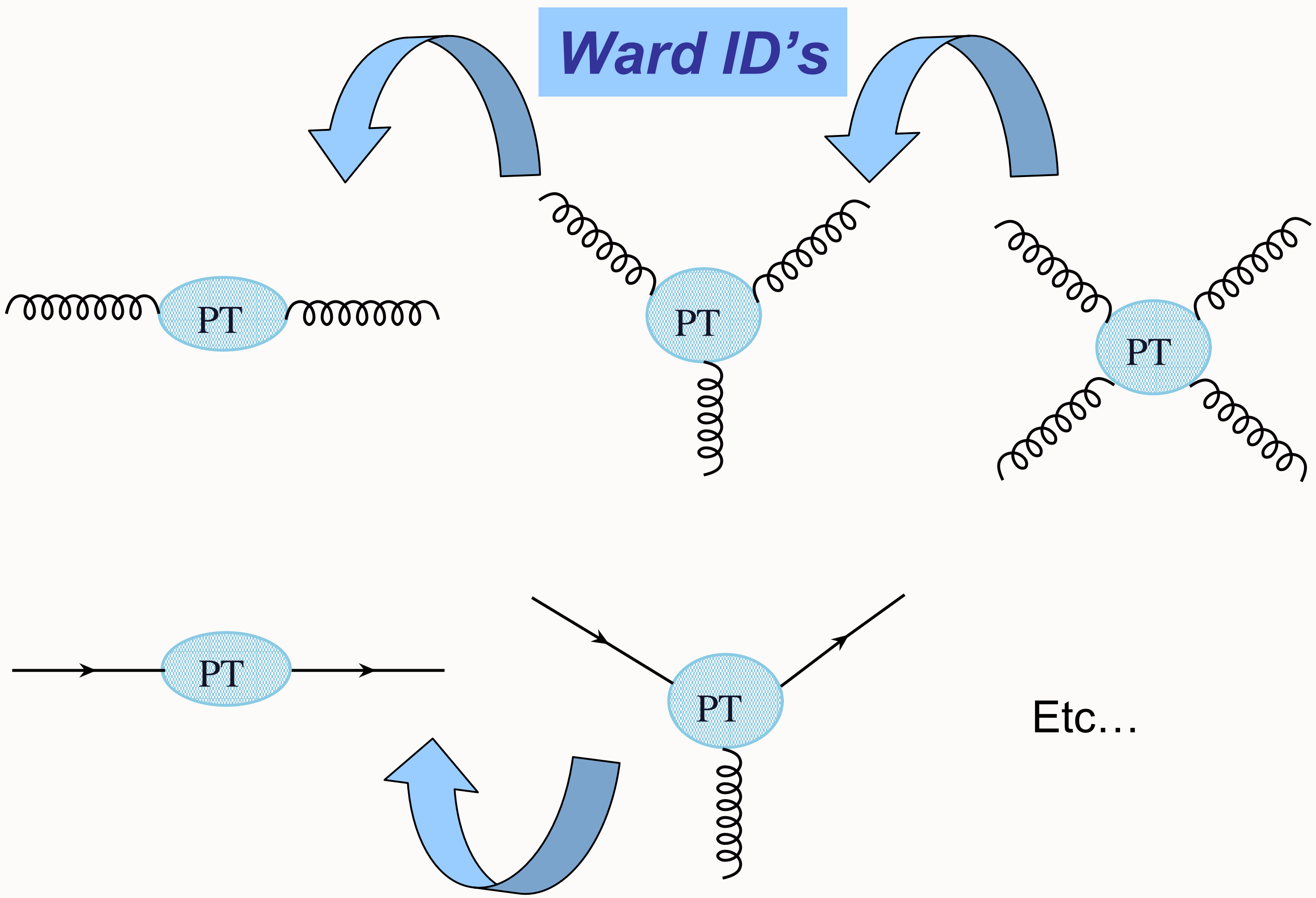


$$L_4(p_1, p_2, p_3, p_4)$$

$$Q^2_{4\,eff}(p_1, p_2, p_3, p_4)$$

Hundreds of form factors!

The Gauge-Invariant Family of Green's Functions



PT Self-Energy at Two-Loops

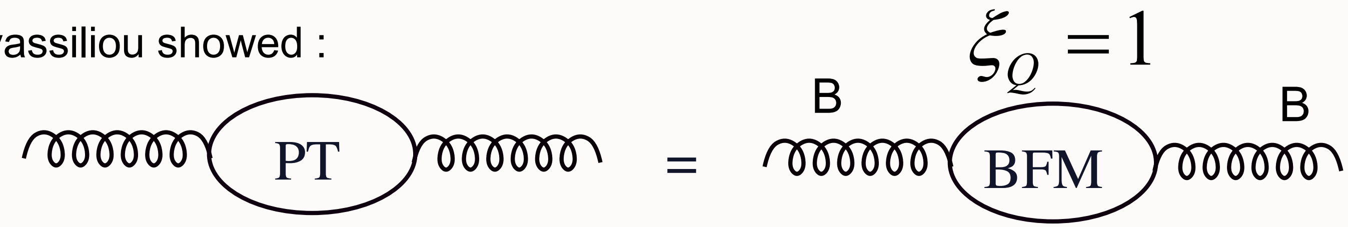
- Finite terms give relation between

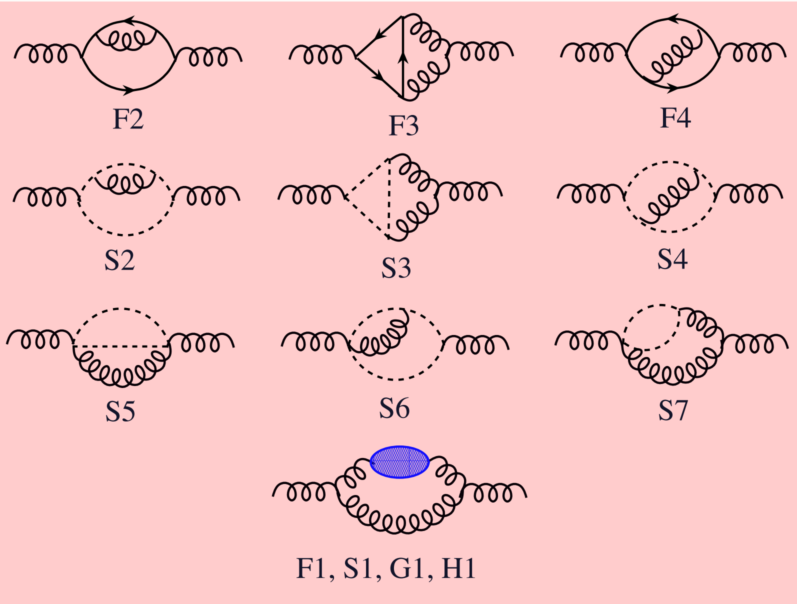
$$\alpha_{PT}(Q^2) \text{ and } \alpha_{\overline{MS}}(Q^2)$$

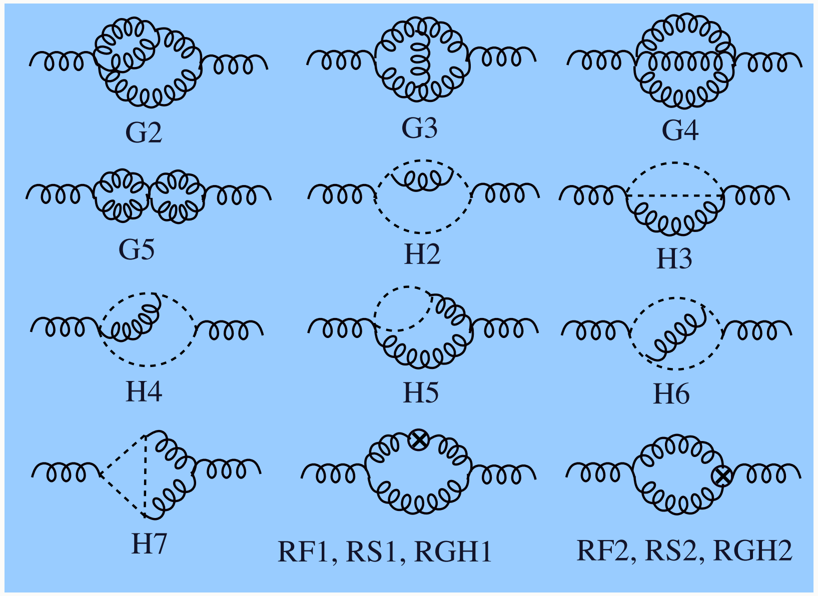
- 3-loop beta function
- 2-loop longitudinal form factors of the three-gluon vertex (via the Ward ID)
- N=4 Supersymmetry gives a non-zero but UV finite contribution

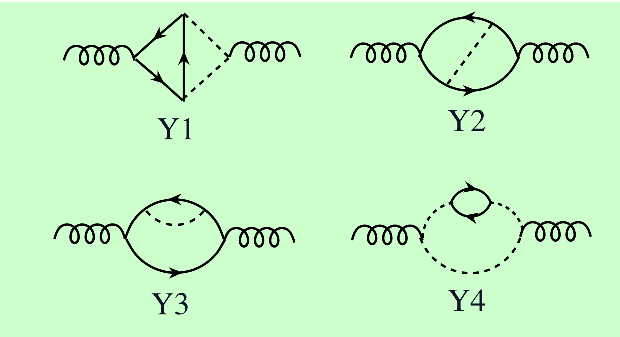
PT Self-Energy at Two-Loops

Papavassiliou showed :









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Summary and Future

- ***Multi-scale analytic*** renormalization based on ***physical, gauge-invariant*** Green's functions

- ***Optimal*** improvement of perturbation theory with ***no scale-ambiguity*** since physical kinematic invariants are the arguments of the (multi-scale) couplings

Factorization scale

$$\mu_{\text{factorization}} \neq \mu_{\text{renormalization}}$$

- Arbitrary separation of soft and hard physics
- Dependence on factorization scale not associated with beta function - present even in conformal theory
- Keep factorization scale separate from renormalization scale

$$\frac{d\mathcal{O}}{d\mu_{\text{factorization}}} = 0$$

- Residual dependence when one works in fixed order in perturbation theory.

New Insights into Hard Inclusive Reactions in QCD

- Elimination of Renormalization Scale Ambiguity
- Heavy quark distributions: severely underestimated at high x_F -- intrinsic charm and bottom
- Higher-twist processes can dominate
- Off-shell effects: DGLAP modified at high x
- Anomalous nuclear effects: hidden color, factorization breaking
- Initial and final-State Interactions: SSA, Diffraction, shadowing, antishadowing, violation of Lam-Tung, breakdown of PQCD factorization formulae
- Hadronization at Amplitude Level: LFWFs, AdS/CFT

Leading Twist Sivers Effect

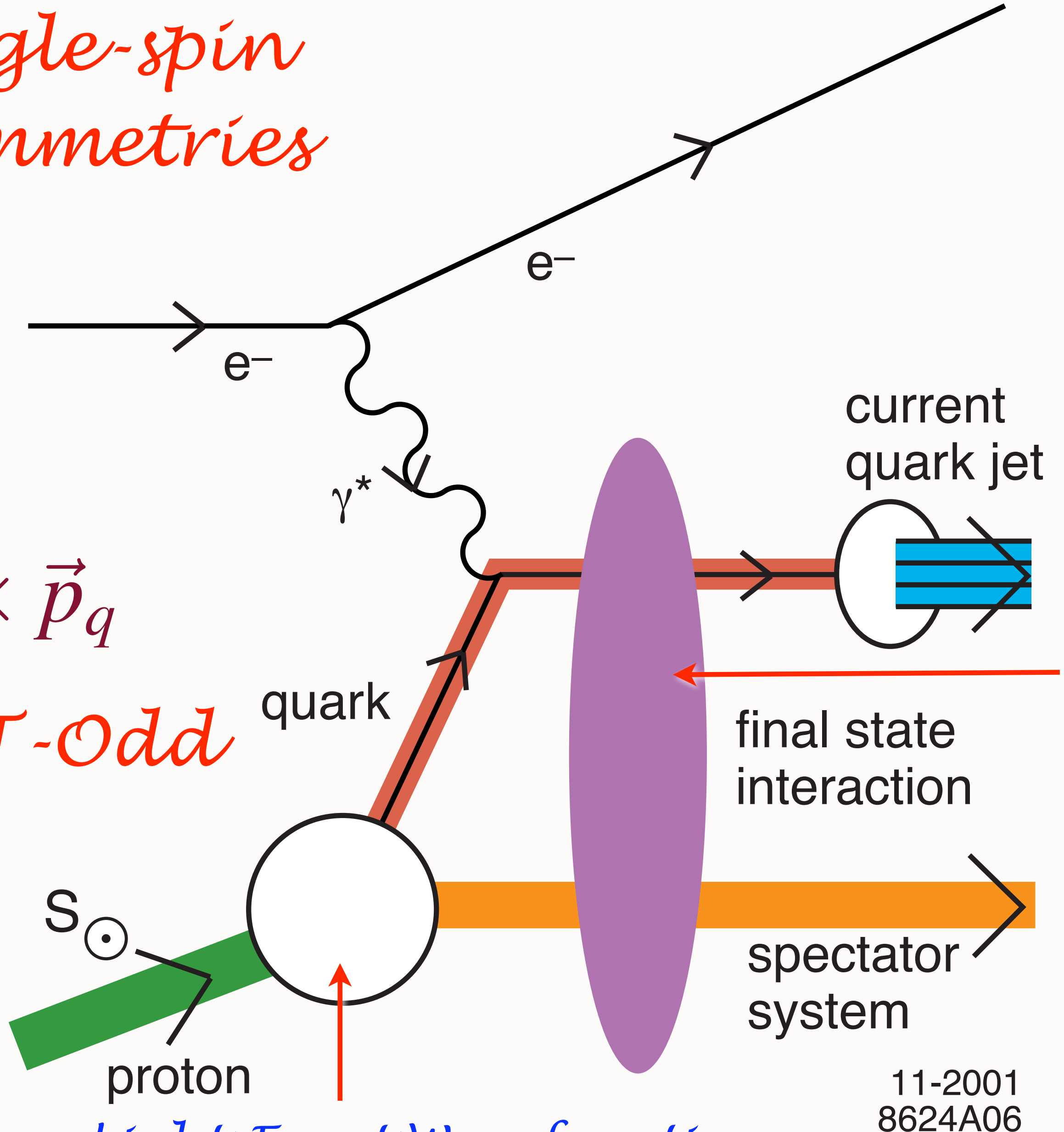
Hwang, Schmidt, sjb

Collins, Burkardt Ji, Yuan

Single-spin asymmetries

$i\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$

Pseudo-T-Odd



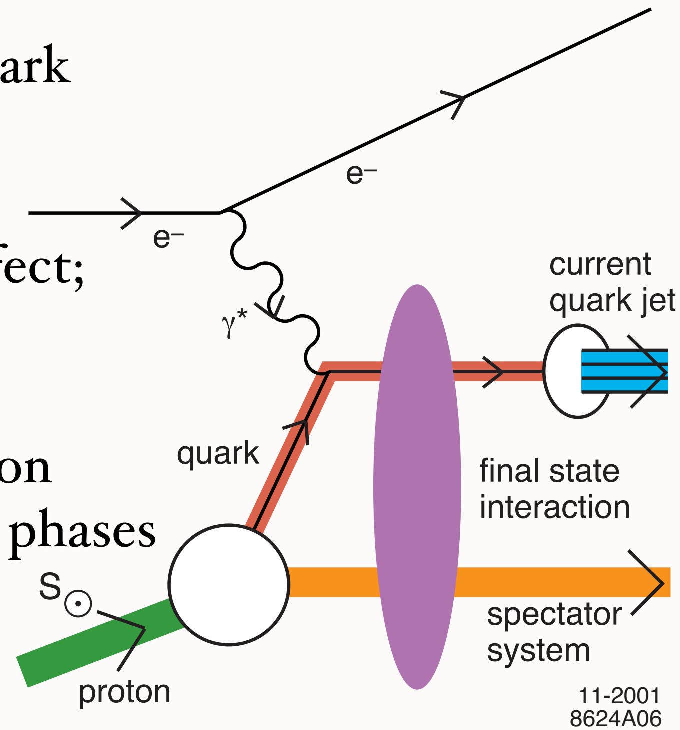
QCD S- and P-Coulomb Phases --Wilson Line

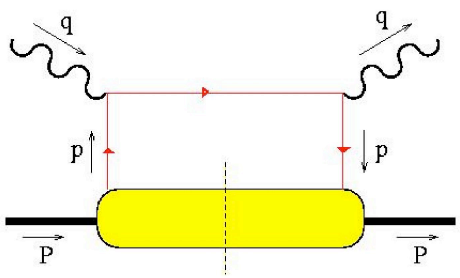
Light-Front Wavefunction S and P- Waves

Final-State Interactions Produce Pseudo T-Odd (Sivers Effect)

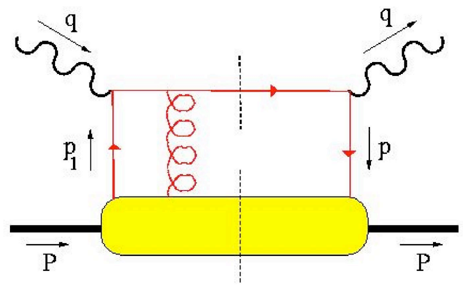
$$\mathbf{i}\ \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$$

- Leading-Twist Bjorken Scaling!
- Requires nonzero orbital angular momentum of quark
- Arises from the interference of Final-State QCD Coulomb phases in S- and P- waves; Wilson line effect; gauge independent
- Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases
- QCD phase at soft scale
- New window to QCD coupling and running gluon mass in the IR
- QED S and P Coulomb phases infinite -- difference of phases finite





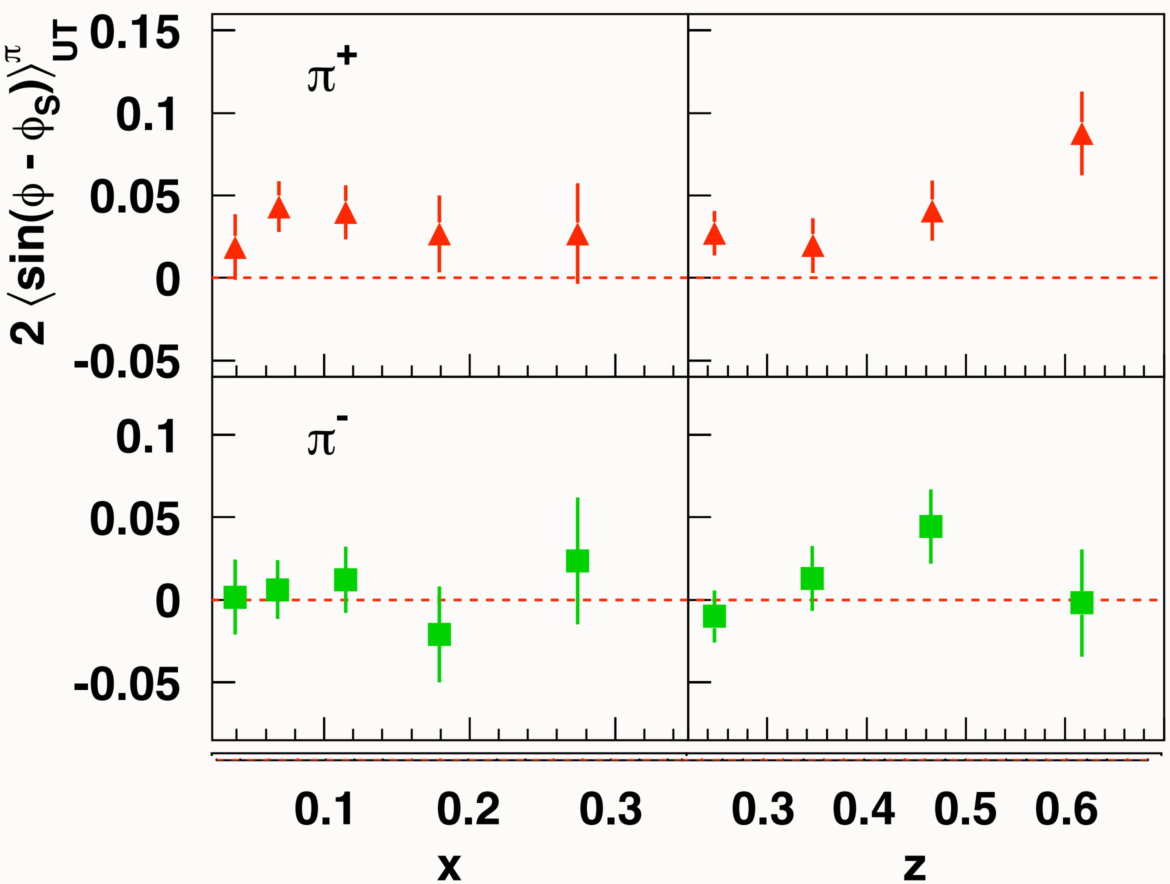
can interfere with



and produce a T-odd effect! (also need $L_z \neq 0$)

HERMES coll., A. Airapetian et al., Phys. Rev. Lett. 94 (2005) 012002.

Sivers asymmetry from HERMES

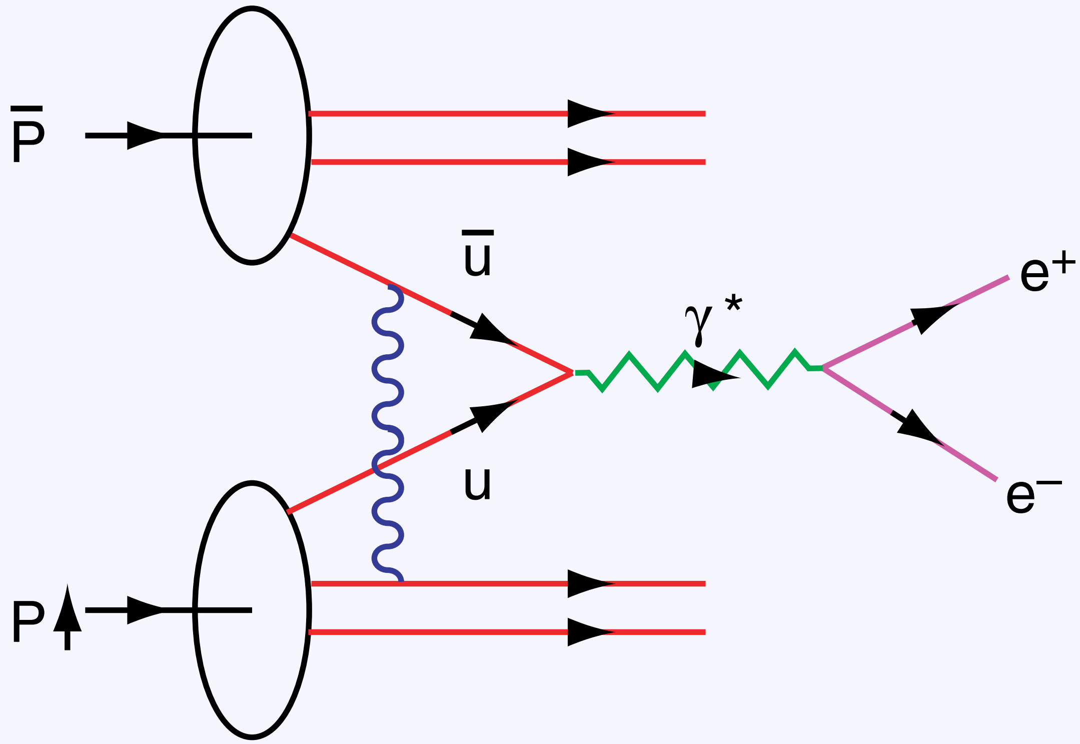


- First evidence for non-zero Sivers function!
- $\Rightarrow$ presence of non-zero **quark orbital angular momentum**!
- Positive for π^+...
  Consistent with zero for π^-...

Gamberg: Hermes data compatible with BHS model

Schmidt, Lu: Hermes charge pattern follow quark contributions to anomalous moment

Predict Opposite Sign SSA in DY !



Collins;
Hwang, Schmidt.
sjb

Single Spin Asymmetry In the Drell Yan Process

$\vec{S}_p \cdot \vec{\bar{p}} \times \vec{q}_{\gamma^*}$

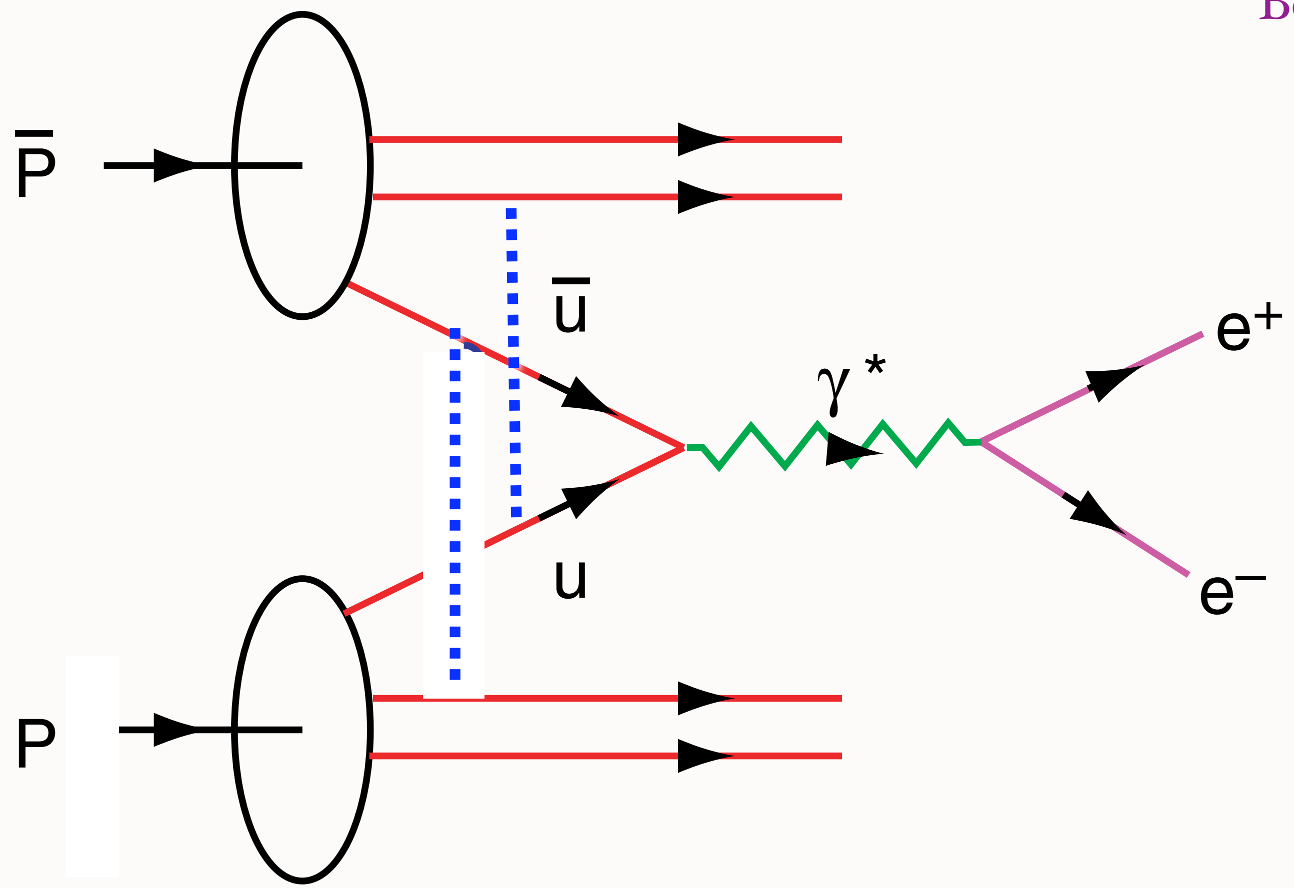
Quarks Interact in the Initial State

Interference of Coulomb Phases for S and P states

Produce Single Spin Asymmetry [Siver's Effect]Proportional to the Proton Anomalous Moment and α_s.

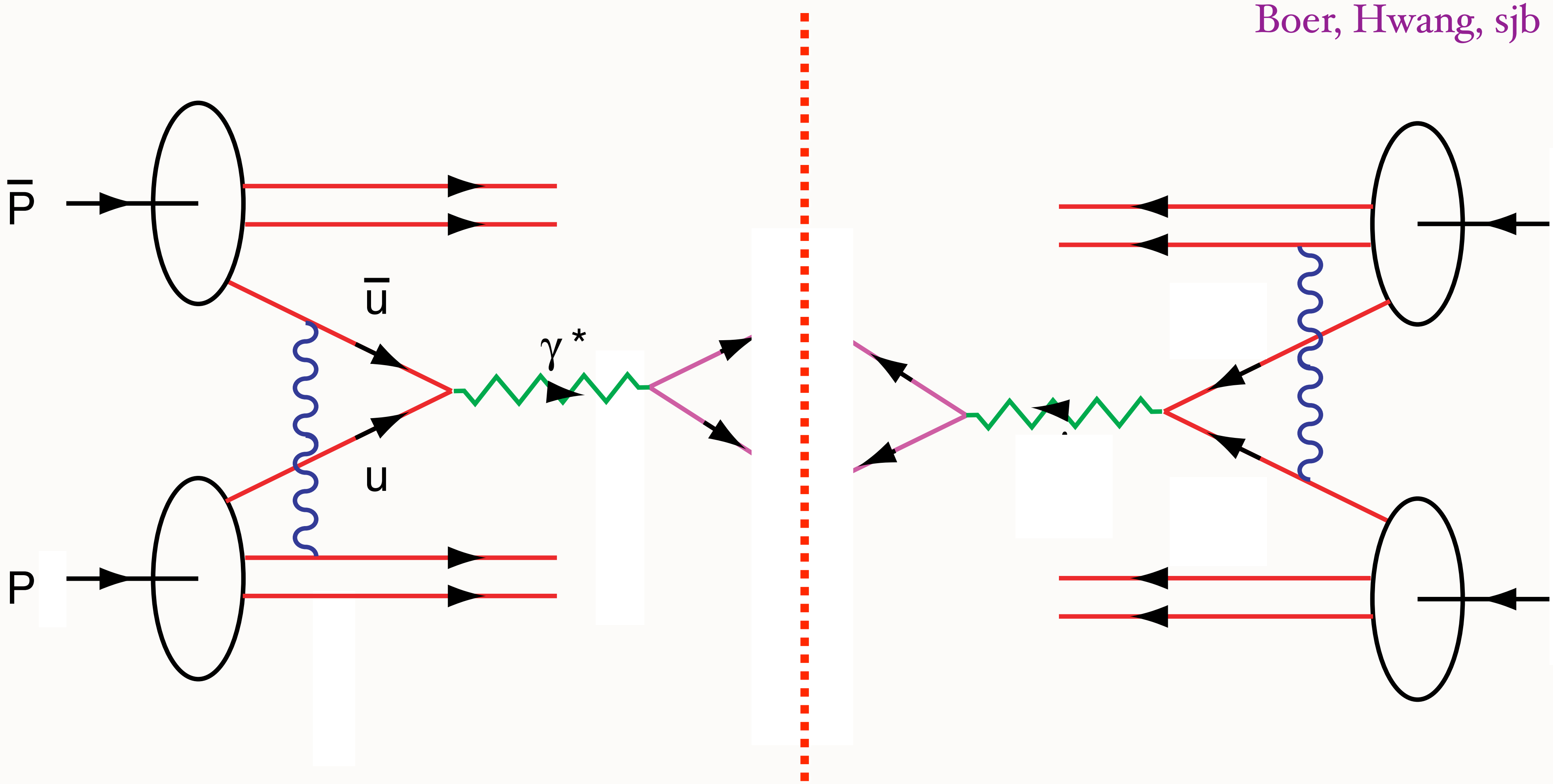
Opposite Sign to DIS! No Factorization

Boer, Hwang, sjb



DY $\cos 2\phi$ **correlation at leading twist from double ISI**

Boer, Hwang, sjb

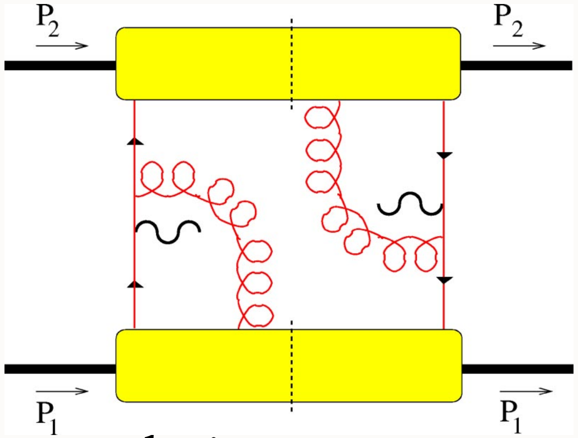


DY $\cos 2\phi$ correlation at leading twist from double ISI

Anomalous effect from Double ISI in Massive Lepton Production

Boer, Hwang, sjb

$\cos 2\phi$ correlation



- Leading Twist, valence quark dominated
- Violates Lam-Tung Relation!
- Not obtained from standard PQCD subprocess analysis
- Normalized to the square of the single spin asymmetry in semi-inclusive DIS
- No polarization required
- Challenge to standard picture of PQCD Factorization

Double Initial-State Interactions generate anomalous $\cos 2\phi$

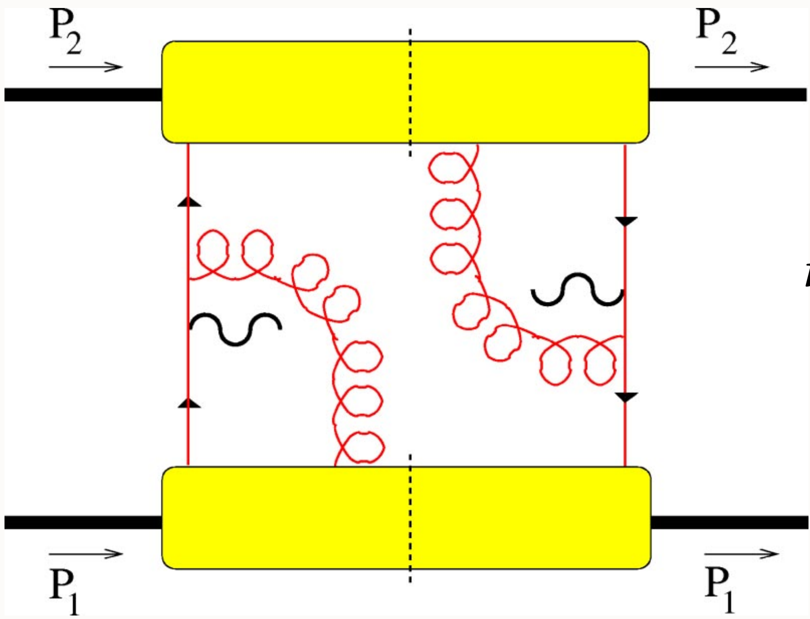
Boer, Hwang, sjb

Drell-Yan planar correlations

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} \propto \left(1 + \lambda\cos^2\theta + \mu\sin 2\theta\,\cos\phi + \frac{\nu}{2}\sin^2\theta\cos 2\phi\right)$$

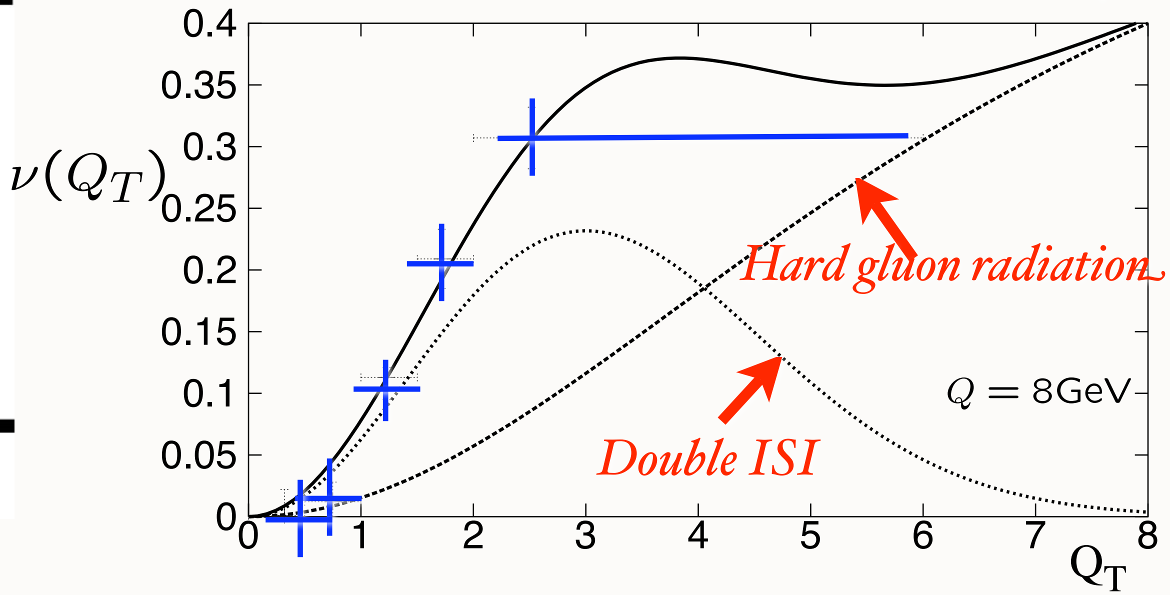
PQCD Factorization (Lam Tung): $1 - \lambda - 2\nu = 0$

$$\frac{\nu}{2} \propto h_1^\perp(\pi)\, h_1^\perp(N)$$

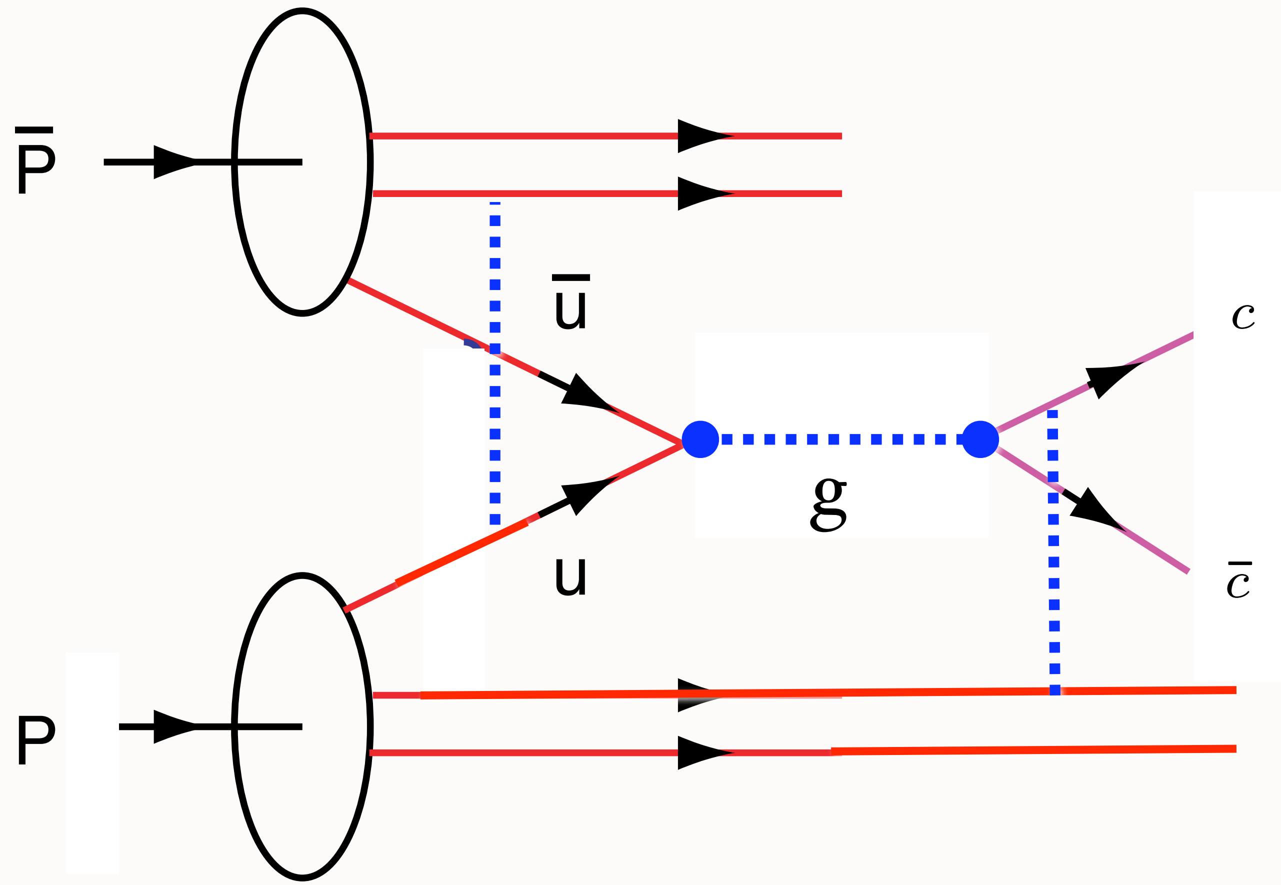


Violates Lam-Tung relation!

$\pi N \to \mu^+\mu^- X$ NA10



Model: Boer,

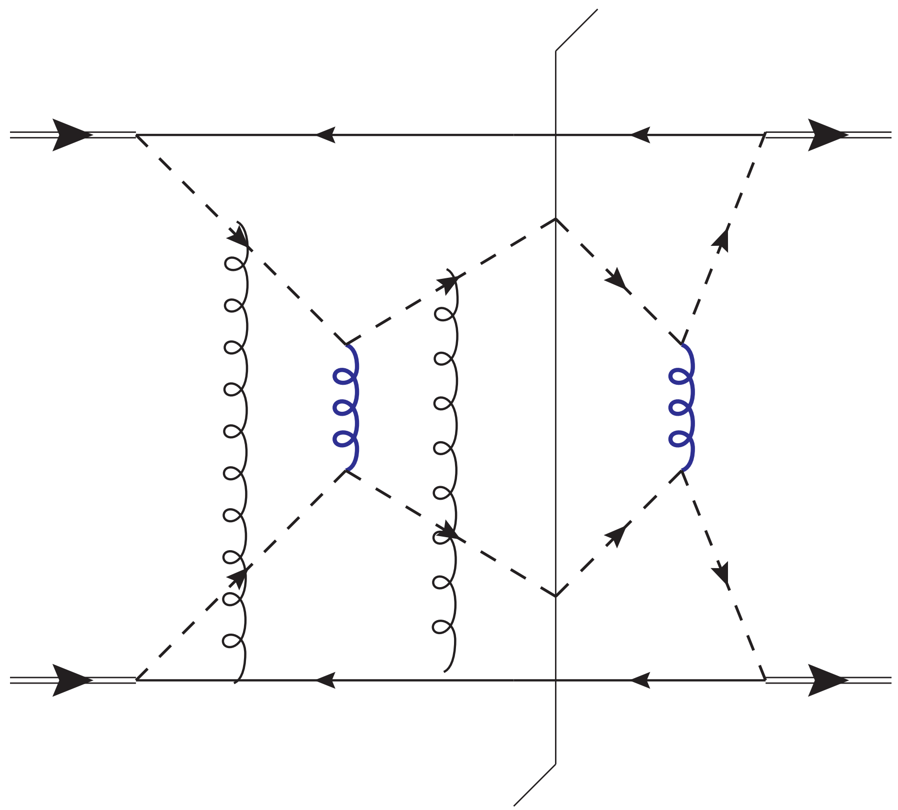


Problem for factorization when both ISI and FSI occur

Factorization is violated in production of high-transverse-momentum particles in hadron-hadron collisions

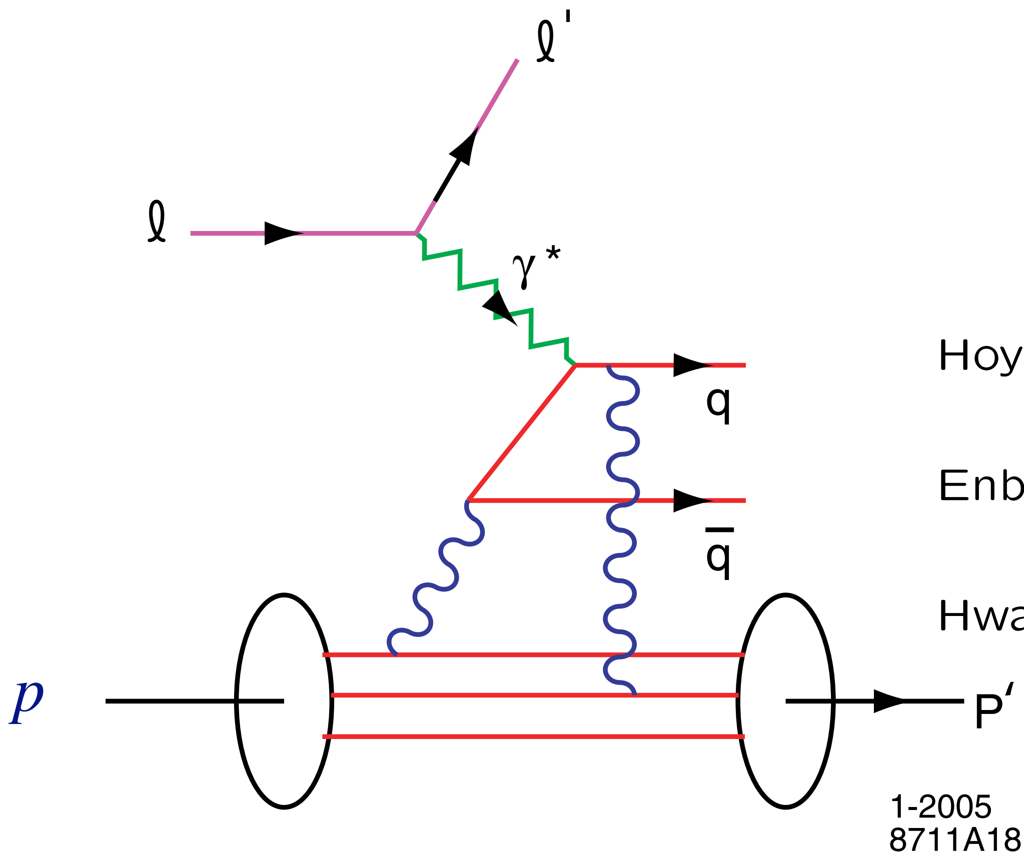
John Collins, Jian-Wei Qiu . ANL-HEP-PR-07-25, May 2007.

e-Print: **arXiv:0705.2141** [hep-ph]



The exchange of two extra gluons, as in this graph, will tend to give non-factorization in unpolarized cross sections.

Final-State Interaction Produces Diffractive DIS



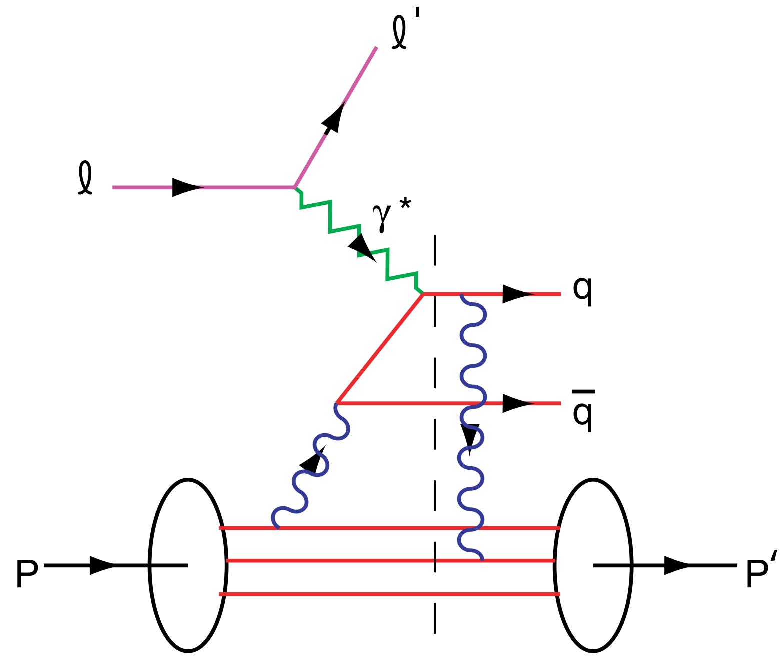
Quark Rescattering

Hoyer, Marchal, Peigne, Sannino, SJB (BHM

Enberg, Hoyer, Ingelman, SJB

Hwang, Schmidt, SJB

Low-Nussinov model of Pomeron



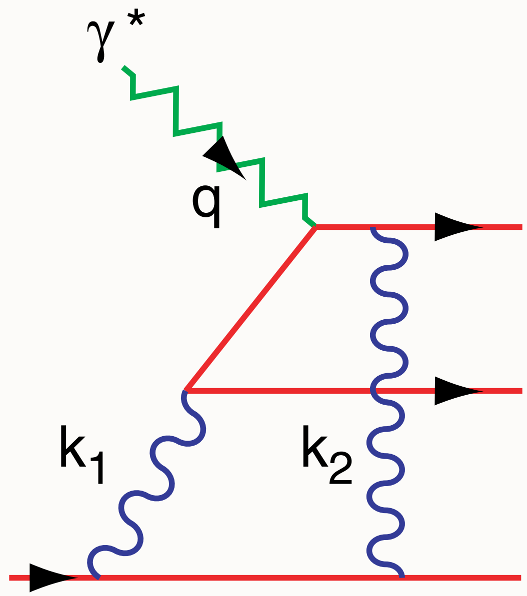
Integration over on-shell domain produces phase i

Need Imaginary Phase to Generate Pomeron

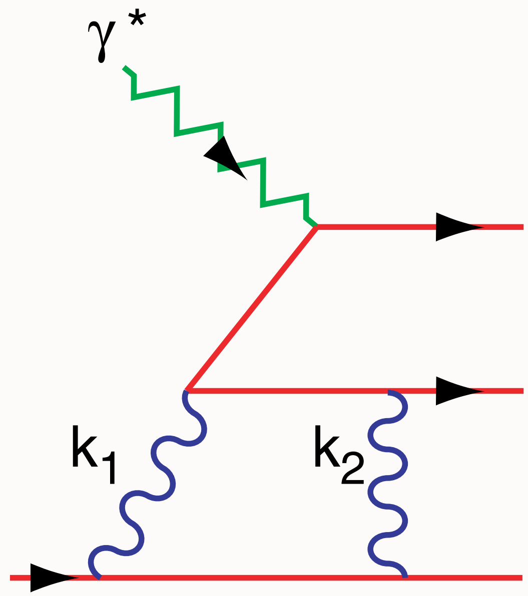
Need Imaginary Phase to Generate
T-Odd Single-Spin Asymmetry

Physics of FSI not in Wavefunction of Target

Final State Interactions in QCD



Feynman Gauge



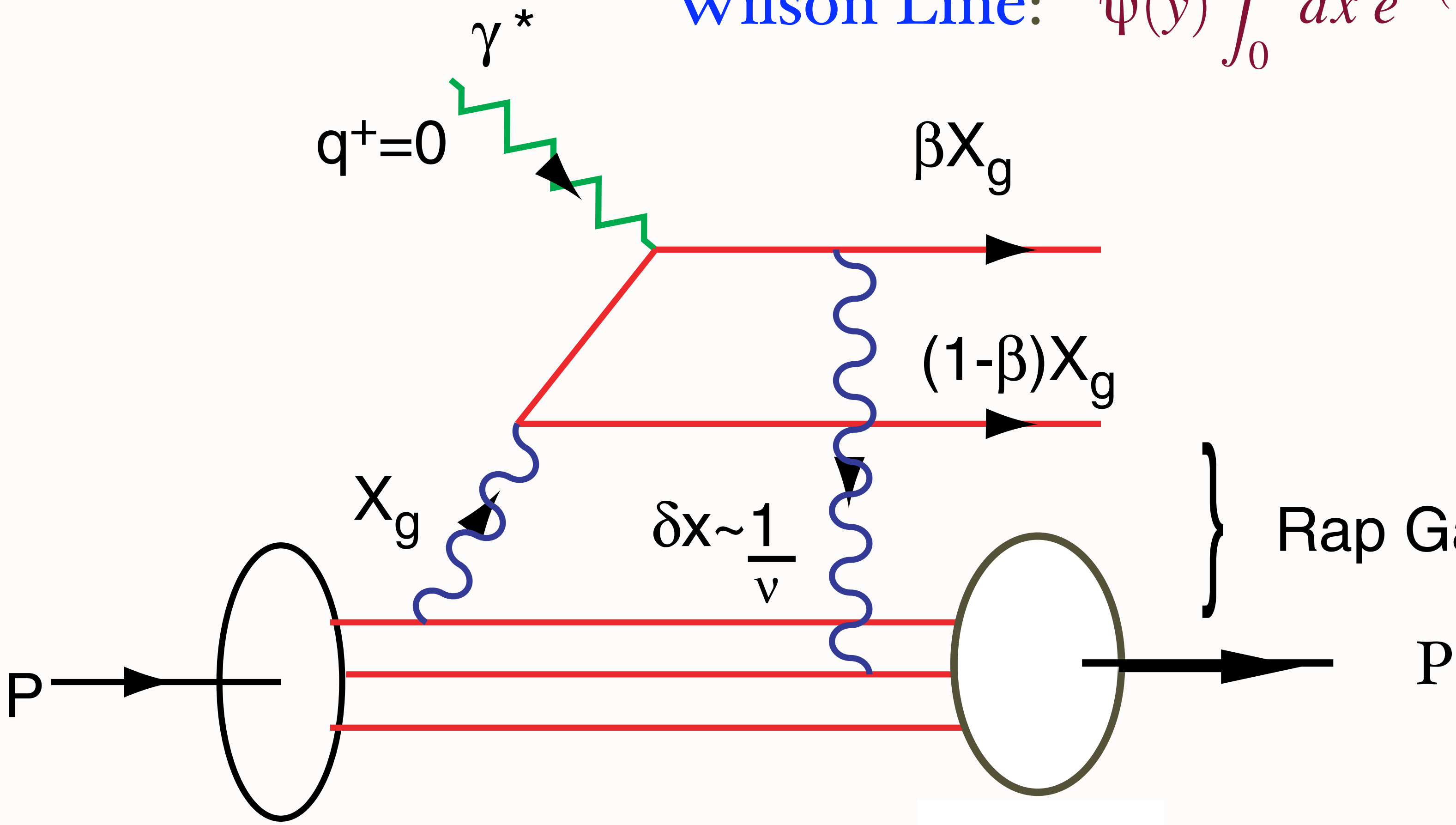
Light-Cone Gauge

Result is Gauge Independent

Hoyer, Marchal, Peigne, Sannino, sjb

QCD Mechanism for Rapidity Gaps

Wilson Line: $\overline{\psi}(y) \int_0^y dx\, e^{iA(x)\cdot dx}\, \psi(0)$



Reproduces lab-frame color dipole approach

Physics of Rescattering

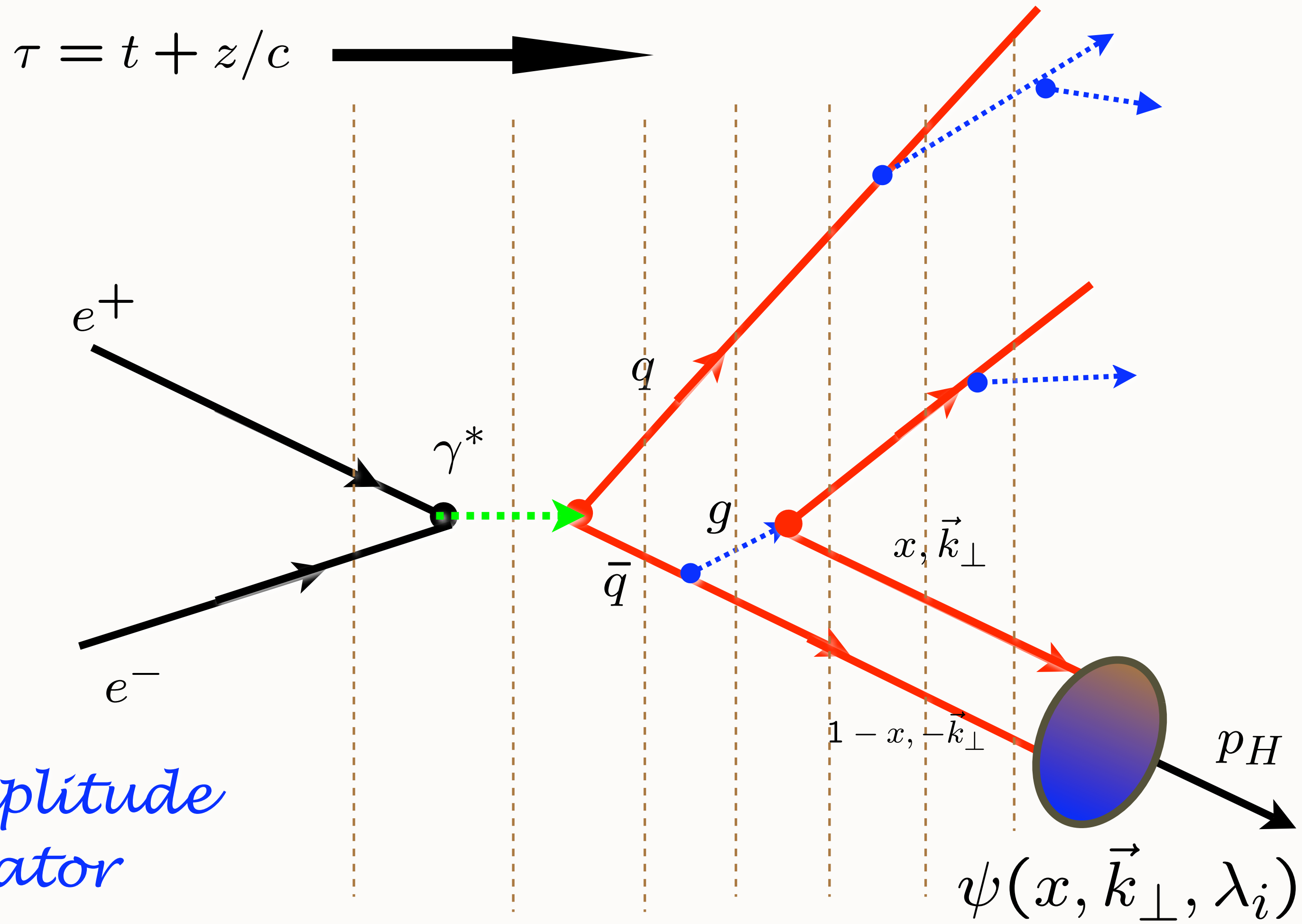
- Diffractive DIS: New Insights into Final State Interactions in QCD
- Origin of Hard Pomeron
- Structure Functions not Probability Distributions!
- T-odd SSAs, Shadowing, Antishadowing
- Diffractive dijets/ trijets, doubly diffractive Higgs
- Novel Effects: Color Transparency, Color Opaqueness, Intrinsic Charm, Odderon

"Dangling Gluons"

- Diffractive DIS
- Non-Unitary Correction to DIS: Structure functions are not probability distributions
- Nuclear Shadowing, Antishadowing- Not in Target WF
- Single Spin Asymmetries -- opposite sign in DY and DIS
- DY $\cos 2\phi$ distribution at leading twist from double ISI-- not given by PQCD factorization -- breakdown of factorization!
- Wilson Line Effects not 1 even in LCG
- Must correct hard subprocesses for initial and final-state soft gluon attachments
- Corrections to Handbag Approximation in DVCS!

Hoyer, Marchal, Peigne, Sannino, sjb

Hadronization at the Amplitude Level



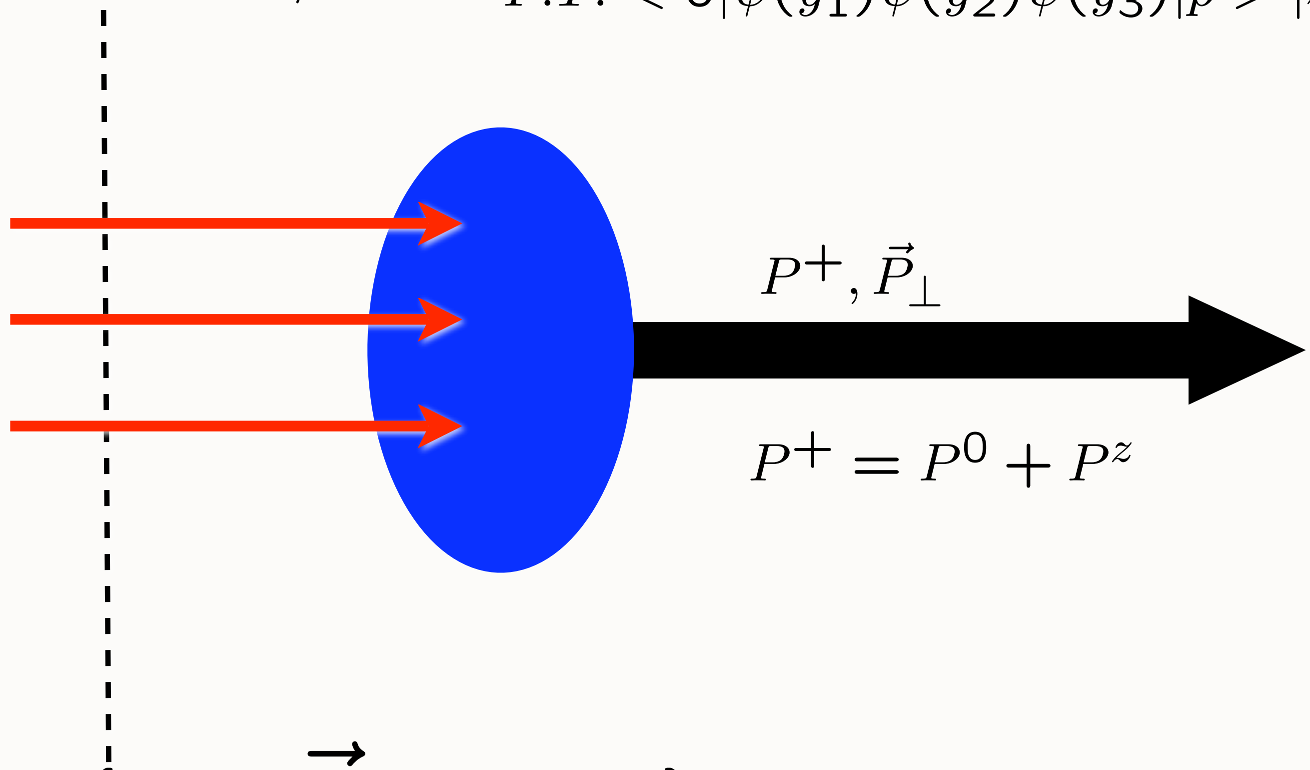
Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via Light-Front Wavefunctions

Light-Front Wavefunctions

Fixed $\tau = t + z/c$

$F.T. < 0|\psi(y_1)\psi(y_2)\psi(y_3)|p > |_{\tau_i=0}$



$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$P^+, \vec{P}_\perp$

$\sum_i^n x_i = 1$

$P^+ = P^0 + P^z$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Invariant under boosts! Independent of P^μ

New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental description of hadrons at amplitude level
- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.
- Quark Interchange dominant force at short distances

Holography: Map AdS/CFT to 3+1 LF Theory

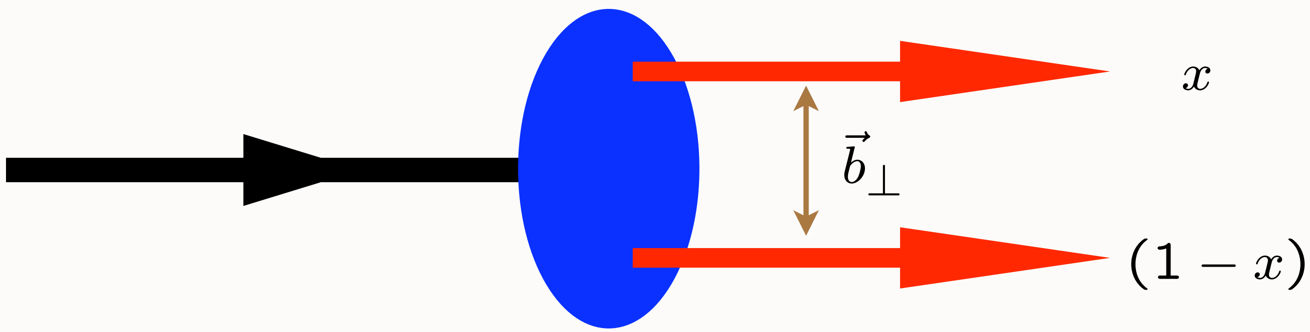
Relativistic LF radial equation

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

G. de Teramond, sjb

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2 .$$

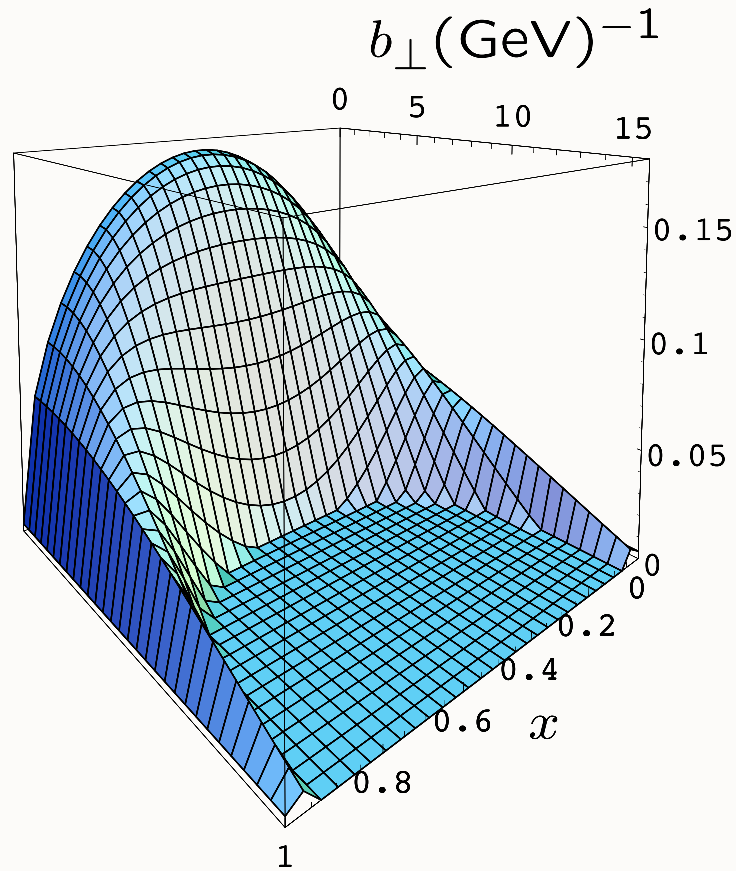


Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2} .$$

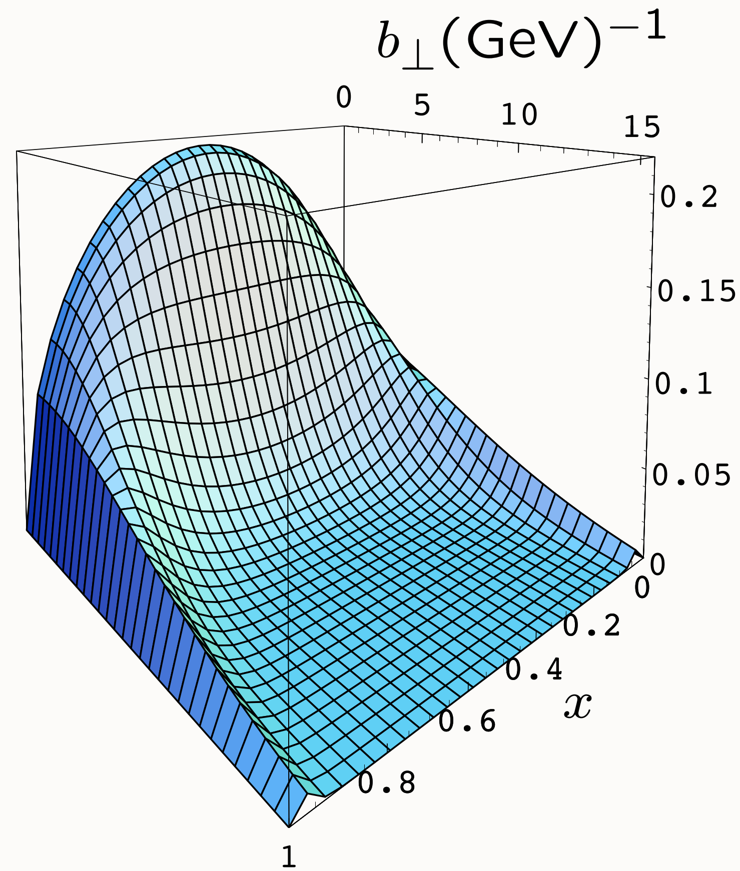
Induced by conformal metric

AdS/CFT Predictions for Meson LFWF $\psi(x, b_\perp)$



$\Lambda_{\rm QCD} = 0.32$ GeV

Truncated Space

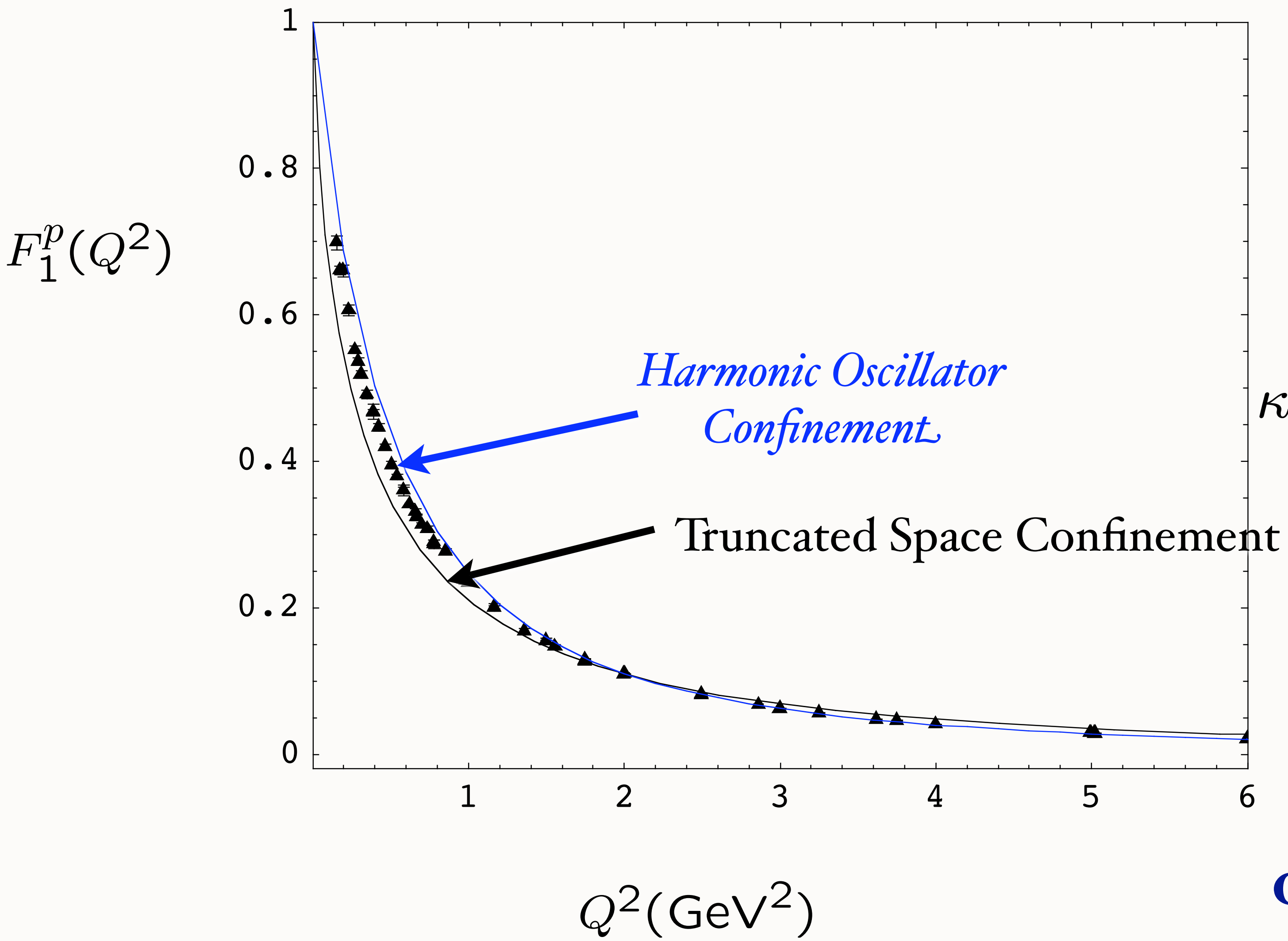


$\kappa = 0.76$ GeV

Harmonic Oscillator

G. de Teramond, sjb

Preliminary



$\kappa = 0.424\ \mathrm{GeV}$

$\Lambda = 0.2\ \mathrm{GeV}$

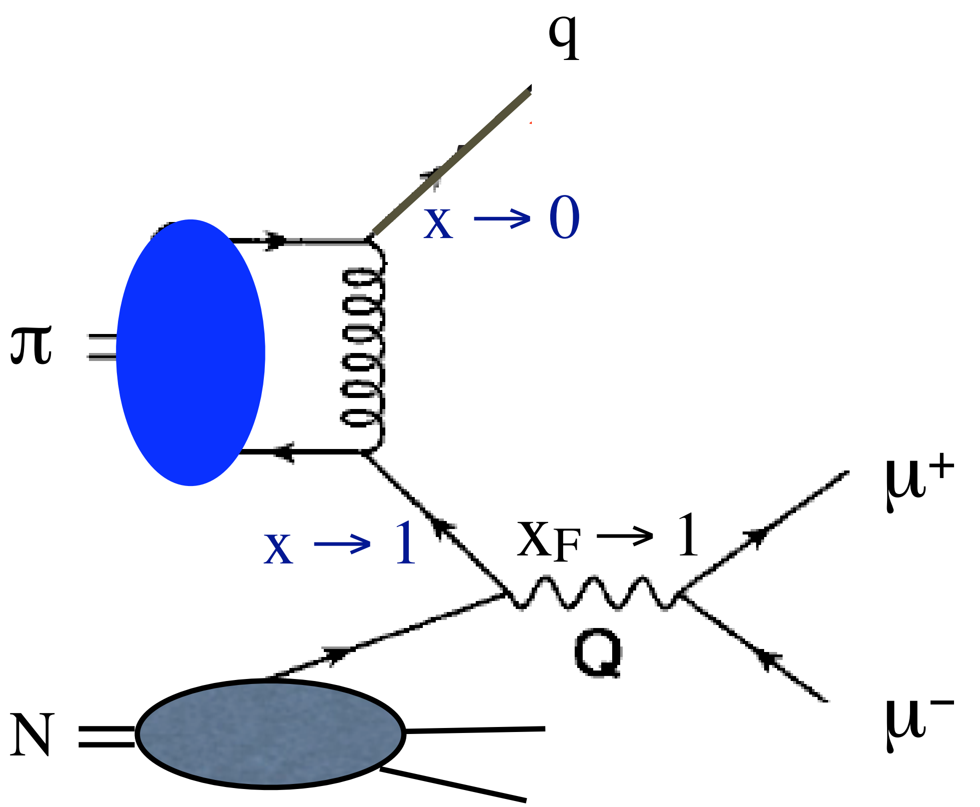
Current modified by metric

$$F_1(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F^{\uparrow}(z)J(Q,z)\Phi_I^{\uparrow}(z)$$

$\pi N \to \mu^+ \mu^- X$ at high x_F

In the limit where $(1-x_F)Q^2$ is fixed as $Q^2 \to \infty$

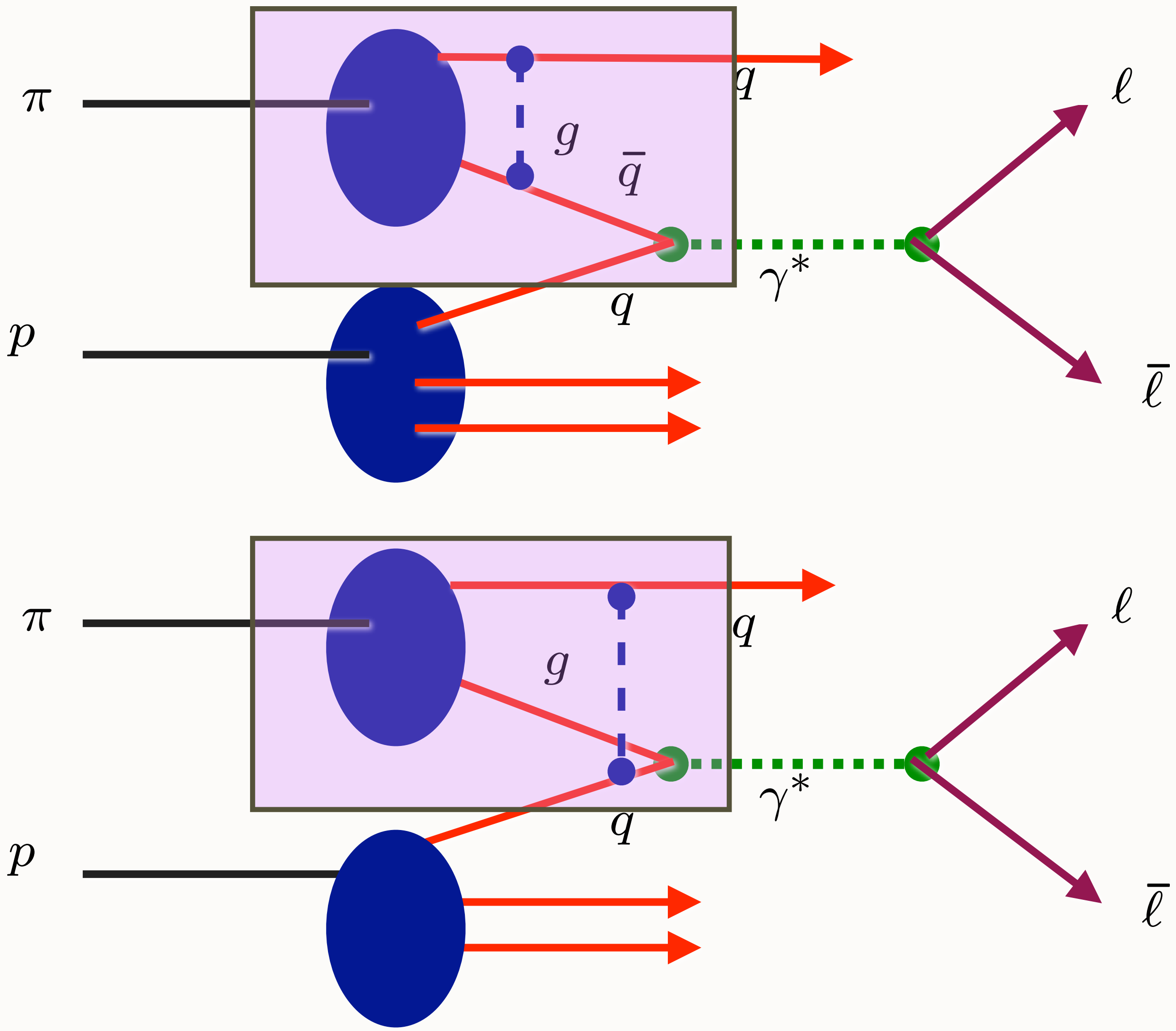
Entire pion wf contributes to hard process



Virtual photon is longitudinally polarized

Berger and Brodsky, PRL 42 (1979) 940

Berger, Lepage, sjb



$$\pi q \to \gamma^* q$$

Pion appears directly in subprocess at large x_F

All of the pion's momentum is transferred to the lepton pair

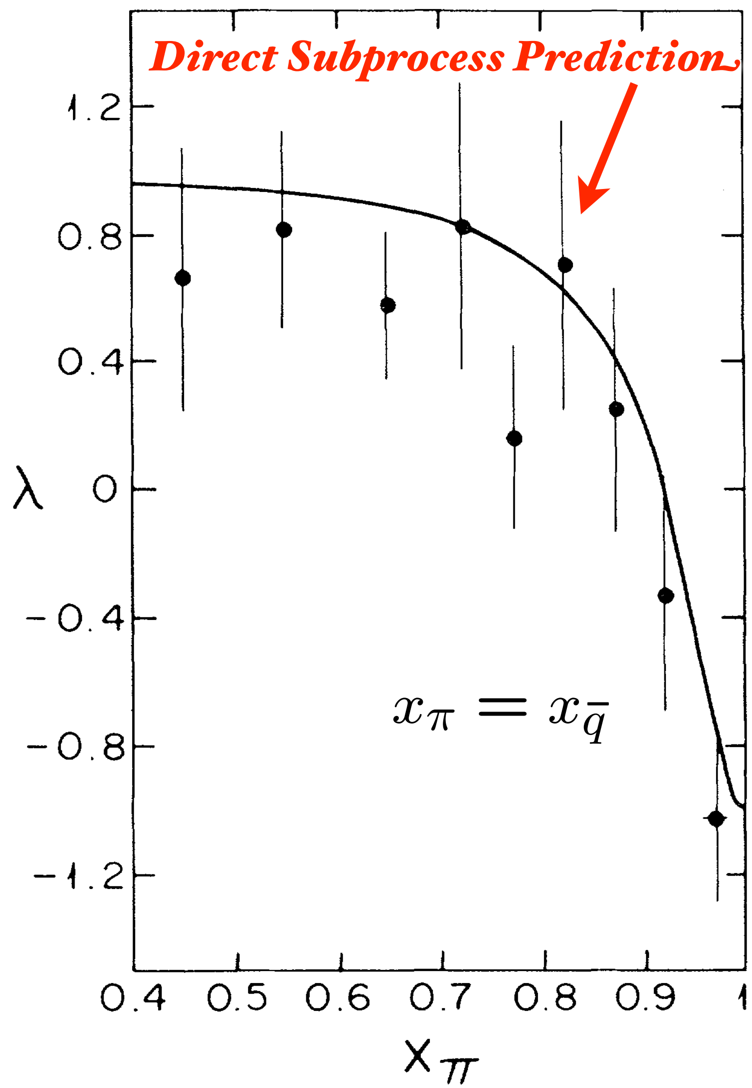
Lepton Pair is produced longitudinally polarized

$$\pi^- N \to \mu^+ \mu^- X \text{ at } 80 \text{ GeV}/c$$

$$\frac{d\sigma}{d\Omega} \propto 1 + \lambda \cos^2\theta + \rho \sin 2\theta \cos\phi + \omega \sin^2\theta \cos 2\phi.$$

$$\frac{d^2\sigma}{dx_\pi d\cos\theta} \propto x_\pi \left[(1 - x_\pi)^2 (1 + \cos^2\theta) + \frac{4}{9} \frac{\langle k_T^2 \rangle}{M^2} \sin^2\theta \right]$$

$$\langle k_T^2 \rangle = 0.62 \pm 0.16 \ \text{GeV}^2/c^2$$



Dramatic change in angular distribution at large x_F

Example of a higher-twist direct subprocess

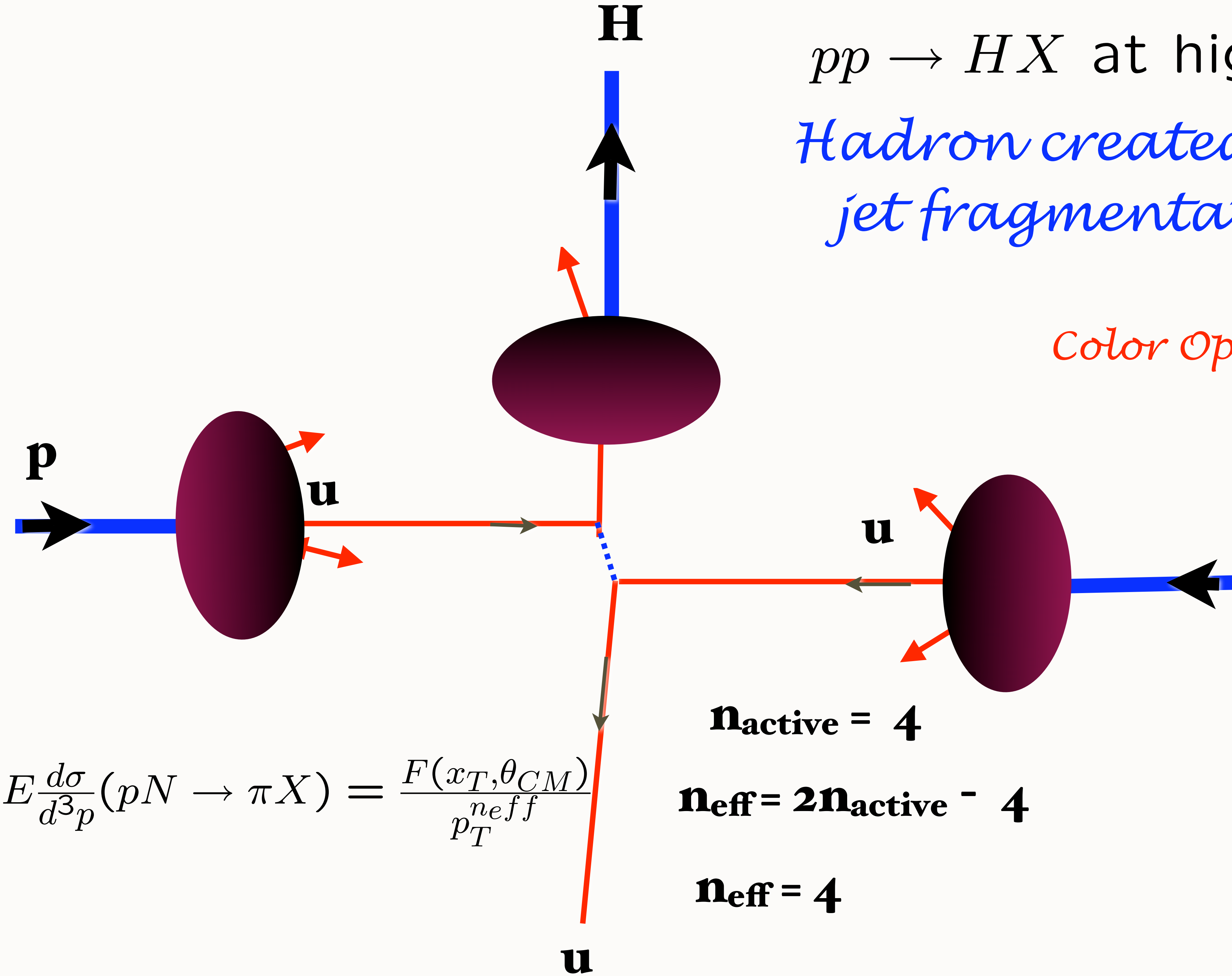
Chicago-Princeton Collaboration

Phys.Rev.Lett.55:2649,1985

$pp \to HX$ at high p_T

Hadron created from jet fragmentation

Color Opaque



$$E\frac{d\sigma}{d^3p}(pN \to \pi X) = \frac{F(x_T, \theta_{CM})}{p_T^{n_{eff}}}$$

$n_{active} = 4$

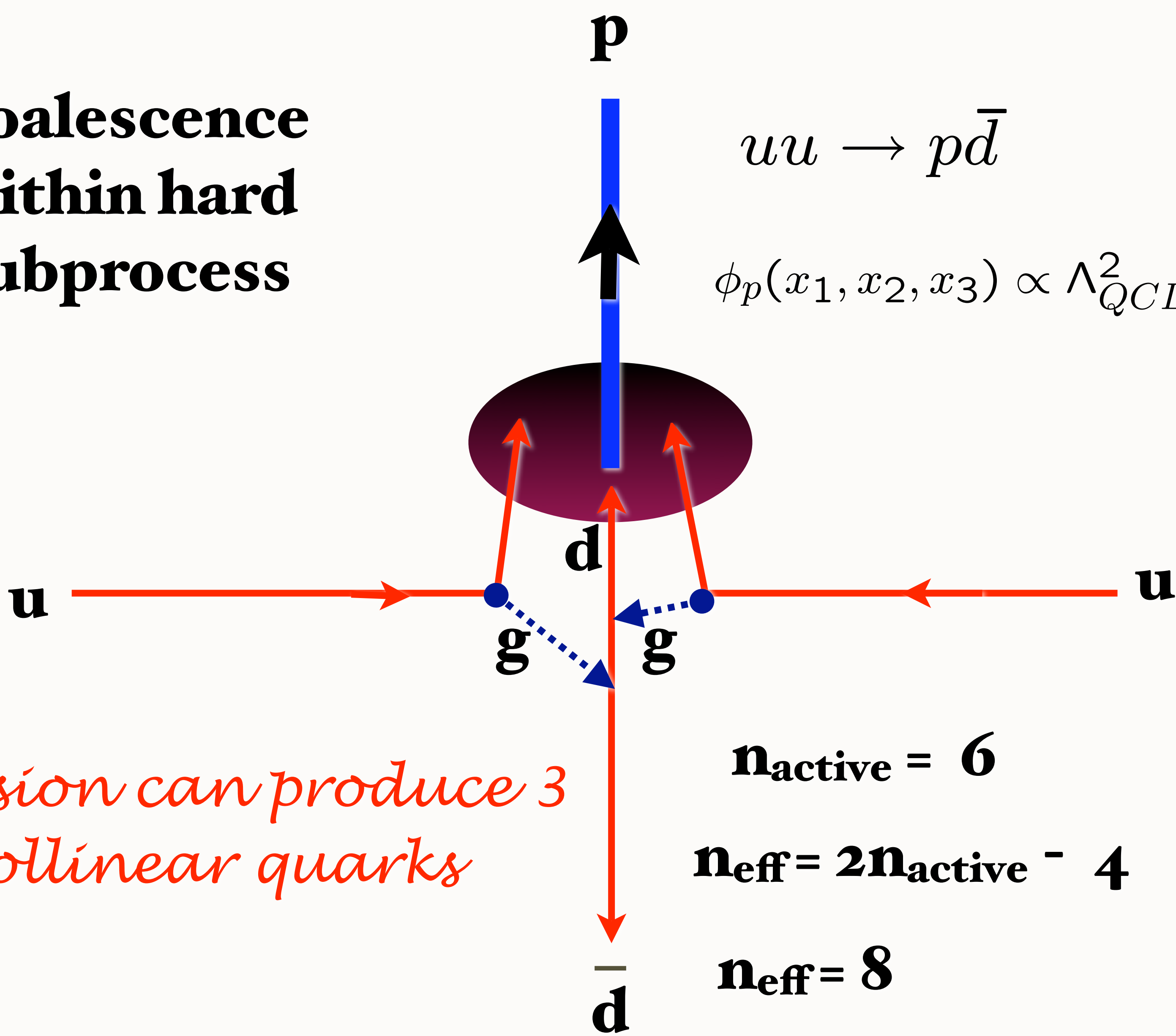
$n_{eff} = 2n_{active} - 4$

$n_{eff} = 4$

Baryon can be made directly within hard subprocess

Bjorken
Blankenbecler, Gunion, sjb
Berger, sjb
Hoyer, et al: Semi-Exclusive

Coalescence within hard subprocess



$$uu \to p\bar{d}$$

$$\phi_p(x_1, x_2, x_3) \propto \Lambda^2_{QCD}$$

Collision can produce 3 collinear quarks

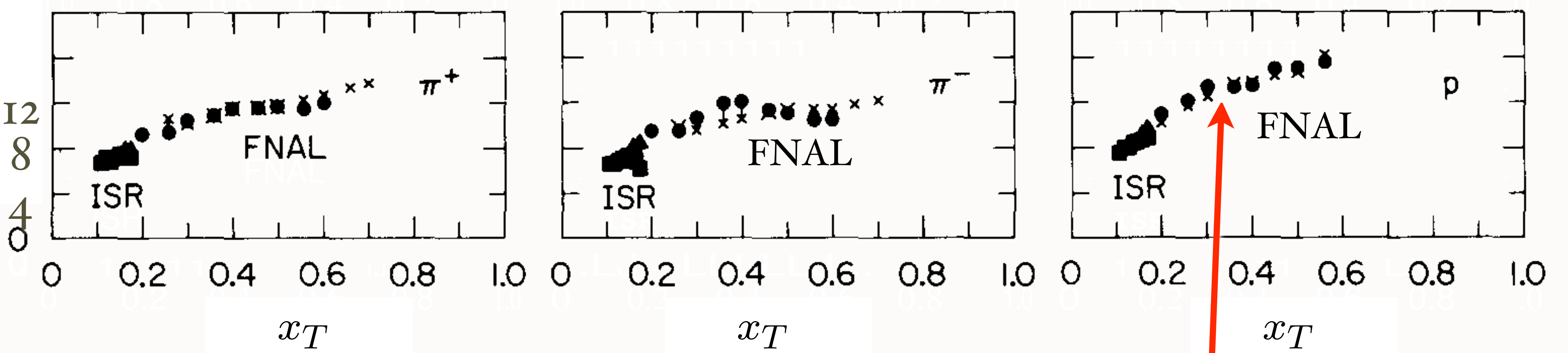
$n_{active} = 6$

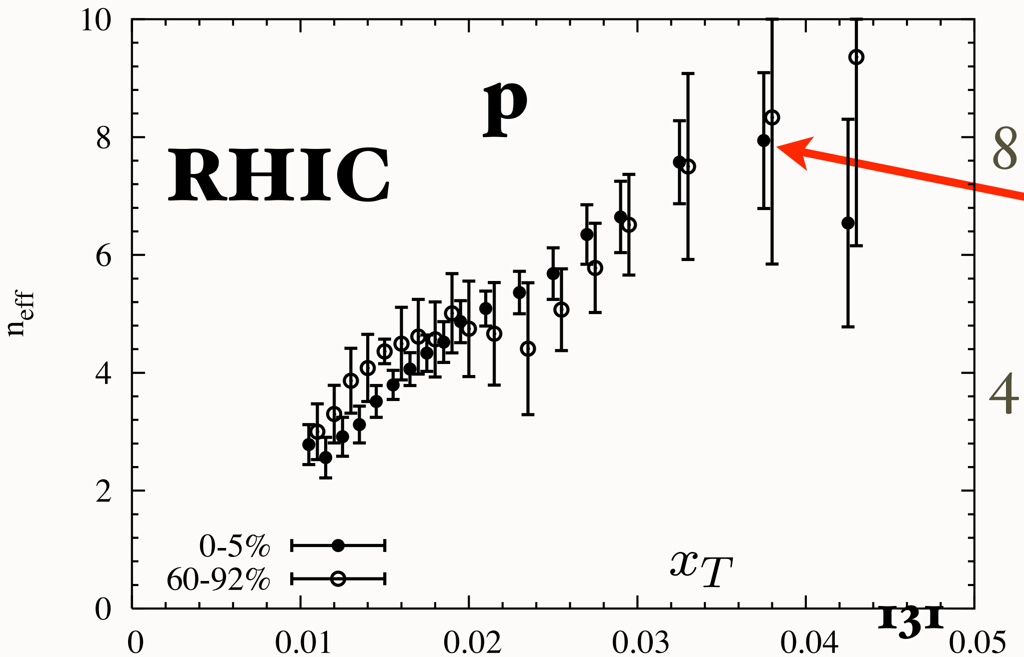
$n_{eff} = 2n_{active} - 4$

$n_{eff} = 8$

$$qq \to B\bar{q}$$

$$E\frac{d\sigma}{d^3p}(pp \rightarrow HX) = \frac{F(x_T,\theta_{CM})}{p_T^{n_{eff}}}$$





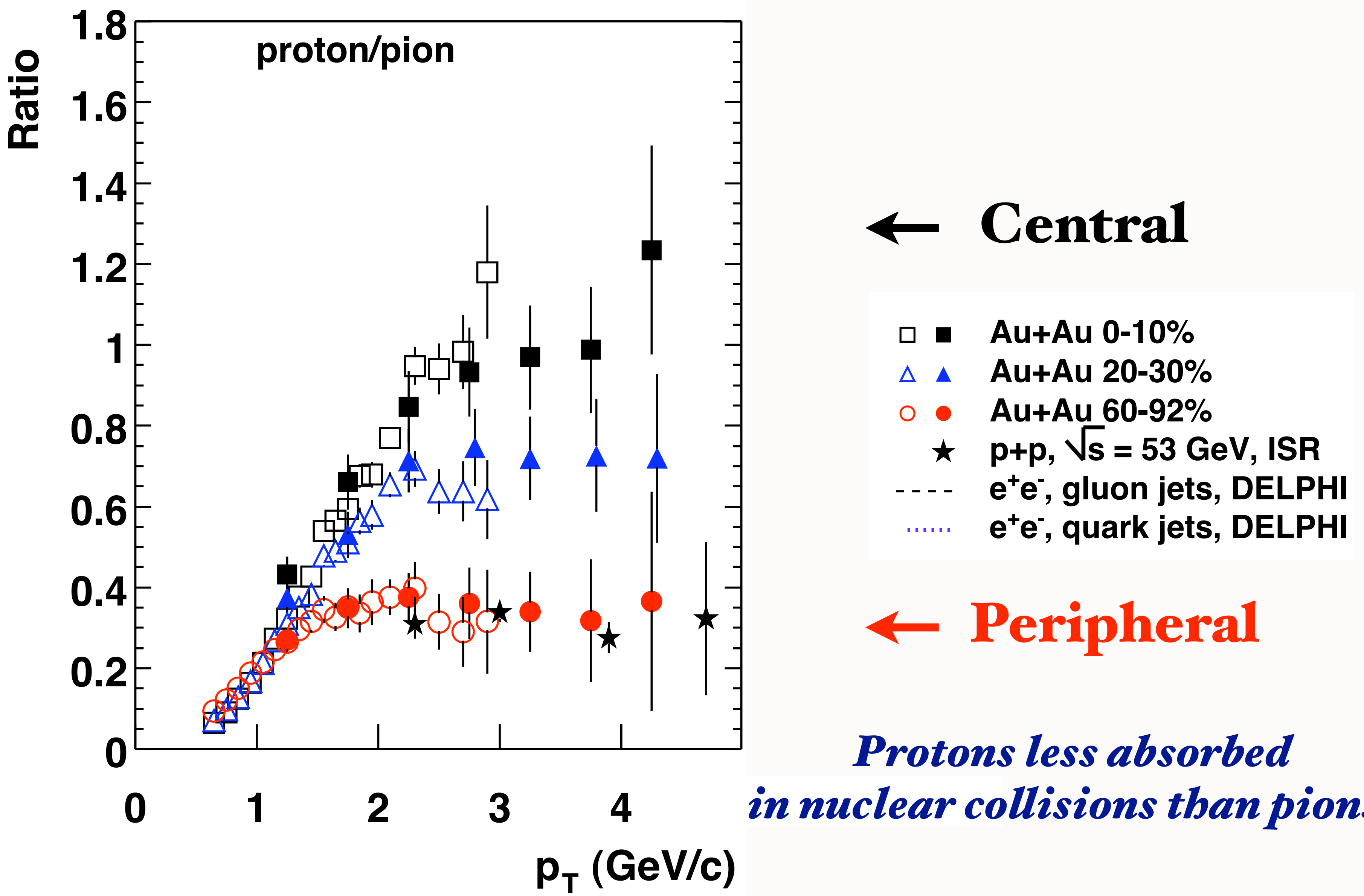
$$E\frac{d\sigma}{d^3p}(pp \rightarrow pX) = \frac{F(x_T,\theta_{CM})}{p_T^{12}}$$

$$E\frac{d\sigma}{d^3p}(pp \rightarrow pX) = \frac{F(x_T,\theta_{CM})}{p_T^{8}}$$

Trend consistent with RHIC at small x_T

S. S. Adler *et al*. PHENIX Collaboration *Phys. Rev. Lett*. **91**, 172301 (2003).

Particle ratio changes with centrality!



← **Central**

← **Peripheral**

Protons less absorbed in nuclear collisions than pions!

Open (filled) points are for $\pi^{\pm}$ (π^0), respectively.

Evidence for Direct, Higher-Twist Subprocesses

- Anomalous power behavior at fixed x_T
- Protons more likely to come from direct subprocess than pions
- Protons less absorbed than pions in central nuclear collisions because of **color transparency**
- Predicts increasing proton to pion ratio in central collisions
- Exclusive-inclusive connection at $x_T = 1$

Conventional renormalization scale-setting method :

- Guess arbitrary renormalization scale and take arbitrary range. Wrong for QED and Precision Electroweak.
- Prediction depends on choice of renormalization scheme
- Variation of result with respect to renormalization scale only sensitive to nonconformal terms; no information on genuine (conformal) higher order terms
- FAC and PMS give unphysical results.
- Renormalization scale not arbitrary: Analytic constraint from flavor thresholds

Features of BLM Scale Setting

On The Elimination Of Scale Ambiguities In Perturbative Quantum Chromodynamics.

Lepage, Mackenzie, sjb

Phys.Rev.D28:228,1983

- All terms associated with nonzero beta function summed into running coupling
- BLM Scale Q* sets the number of active flavors
- Only n_f dependence required to determine renormalization scale at NLO
- Result is scheme independent: Q* has exactly the correct dependence to compensate for change of scheme
- Correct Abelian limit
- **Resulting series identical to conformal series!**
- Renormalon n! growth of PQCD coefficients from beta function eliminated!
- In general, BLM scale depends on all invariants

Use BLM!

- Satisfies Transitivity, all aspects of Renormalization Group; scheme independent
- Analytic at Flavor Thresholds
- Preserves Underlying Conformal Template
- Physical Interpretation of Scales; Multiple Scales
- Correct Abelian Limit ($N_C = 0$)
- Eliminates unnecessary source of imprecision of PQCD predictions
- Commensurate Scale Relations: Fundamental Tests of QCD free of renormalization scale and scheme ambiguities
- BLM used in many applications, QED, LGTH, BFKL, ...