χPT in the mixed regime

based on the papers: 0808.1986 and 0707.3887

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χ PT & LQCD

χPT is an effective theory of QCD:

- that determines which operators are relevant at low energies
- that allows to calculate the correlators at a given order in p/Λ_χ in terms of a finite number of coupling constants L_i

The L_is:

- encode information due to quark dynamics which is relevant at low energies
- $\Rightarrow$ can be determined from matching correlation functions calculated in χPT and in LQCD

χPT in finite volume

Provided $L\Lambda_\chi \sim 4\pi LF \gg 1$, χPT is a good description of QCD at large distances:

1. if $m_q \Sigma V \gg 1$ (p regime)
   - finite size effects are $\sim e^{-LM_\pi}$
   - 12 L_is at NLO
2. if $m_q \Sigma V \lesssim 1$ (ϵ regime)
   - finite size effects are NOT exponentially suppressed
   - the result depends only on F and Σ at NLO
   - the dependence on the topological sector ν is enhanced
3. mixed regime both:

$$m_h \Sigma V \gg 1 \qquad m_l \Sigma V \lesssim 1$$

Why consider the mixed regime?

It is necessary to consider the mixed regime in the following situations:

1. Split the s and u/d quarks in full theory simulations:

$$m_s \Sigma V \gg 1 \quad m_{u,d} \Sigma V \lesssim 1$$

2. Push just the valence quarks to the chiral limit, in PQ simulations with mixed actions [Baer, Rupak, Shoresh]:

$$m_{sea} \Sigma V \gg 1 \quad m_{valence} \Sigma V \lesssim 1$$

The ϵ regime

The zero momentum modes contribution to pion propagators:

$$\sim \frac{1}{M_\pi^2 V} \sim \frac{1}{m_q \Sigma V} \sim O(1)$$

is nonperturbative.

Solution: factorize zero modes $U = U_0 e^{\frac{2i\xi}{F}}$

- integrate U_0 over all $SU(N_f)$
- $\int dx^4 \xi(x) = 0 \Rightarrow \xi \sim L^{-1}$

[Gasser & Leutwyler 1987, Hansen 1990]

Non perturbative integration and fixed topology

To fix topology, one has to Fourier transform with respect to the vacuum angle θ:

$$\langle O \rangle_\nu = \int_0^{2\pi} d\theta \, e^{-i\nu\theta} \langle O(\theta) \rangle$$

The zero modes partition functional is ($U \equiv U_0 e^{\frac{i\theta}{N_f}}$):

$$\mathcal{Z}^\nu_{N_f} = \int_{U(N_f)} dU \, (\det U)^\nu \exp\left(\frac{\Sigma V}{2} \mathrm{Tr}\left[\mathcal{M} U + U^\dagger \mathcal{M} \right] \right)$$

The solution is known in terms of Bessel functions. [Brower et al. 1982, Leutwyler & Smilga 1992, Jackson et al. 1996]
The other integrals we have needed can be obtained by deriving with respect to the quark masses.

Notation in mixed regime

Start from:

$$\mathcal{M} = \begin{pmatrix} \mathcal{M}_l & 0 \\ \underbrace{0}_{N_l} & \underbrace{\mathcal{M}_h}_{N_h} \end{pmatrix} \qquad \mathcal{M}_h \sim L^{-2} \;\; \mathcal{M}_l \sim L^{-4}$$

Calculating the propagator one sees that:

- pions corresponding to $SU(N_l)$ are light $(M^2 \sim L^{-4})$
- other pions are heavy $(M^2 \sim L^{-2})$

$$M^2_{ab} = \frac{\Sigma}{F^2}(m_a + m_b) \qquad M^2_{ll} \sim L^{-4} \quad M^2_{hl} \sim L^{-2} \quad M^2_{hh} \sim L^{-2}$$

Factorization for mixed regime

We used a new parametrization:

$$U = \begin{pmatrix} U_0 & 0 \\ 0 & \mathbf{1} \end{pmatrix} e^{\frac{2i\xi}{F}} \qquad\qquad U_0 \in SU(N_l)$$

$$\int dx^4 \ \mathrm{Tr}[T^a \xi(x)] = 0 \qquad\qquad T^a \in SU(N_l)$$

This affects the propagator and the measure $d\mu$ through the nontrivial Jacobian $J(\xi)$:

$$d\mu = d\mu_0 d\xi J(\xi)$$

At NLO there is factorization of zero and non-zero modes.

We have calculated the vector, axial vector, scalar, pseudoscalar charge correlators in the most general case of non degenerate masses. External particles are restricted to correspond to $SU(N_l)$ generators.

Vector charge correlator in epsilon regime

The ϵ regime result for N_l quarks is at NLO:

$$\mathcal{V}^c_{ud}(t) = -\frac{F^2_{NLO}}{2T}\mathcal{J}^{0(N_l)}_{-}((\mu_l)_{NLO}) + \frac{T}{2V}N_l k_{00}\mathcal{J}^{0(N_l)}_{+}(\mu_l) - \frac{\Sigma}{2}(m_u - m_d)(\mathcal{S}_u - \mathcal{S}_d)\ T h_1\left(\frac{t}{T}\right)$$

- The $\mathcal{J}$ and $\mathcal{S}$ come from the non perturbative integrations over $SU(N_l)$ ($\mu_l = \Sigma V \mathcal{M}_l$)

$$\mathcal{S}_l \equiv \frac{\partial}{\partial \mu_l} \ln \mathcal{Z}^{\nu}_{N_l}(\mu)$$

- the $\mathcal{J}$s come from second derivatives of $\ln \mathcal{Z}^{\nu}_{N_l}(\mu)$...
- $h_1(\frac{t}{T}) \equiv \frac{1}{2}\left(\frac{t}{T} - \frac{1}{2}\right)^2 - \frac{1}{24}$

Vector charge correlator in mixed regime

Going to the mixed regime the form of the result does not change:

$$\mathcal{V}^c_{ud}(t) = -\frac{F^2_{NLO}}{2T}\mathcal{J}_-^{0(N_l)}((\mu_l)_{NLO}) + \frac{T}{2V}N_l k_{00}\mathcal{J}_+^{0(N_l)}(\mu_l) - \frac{\Sigma}{2}(m_u - m_d)(\mathcal{S}_u - \mathcal{S}_d)\; T h_1\left(\frac{t}{T}\right) + O(e^{-M^2_{hh}L})$$

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- In agreement with the Decoupling theorem [Appelquist & Carazzone 1975], the effect of the heavy quarks is all contained:
  - in the renormalization of the L_is
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  - in the renormalization of the L_is
  - in exponentially suppressed volume effects
- As expected the time dependent contribution vanishes in the isospin limit.[F.B. et al. 2008]

Q & PQ χ PT

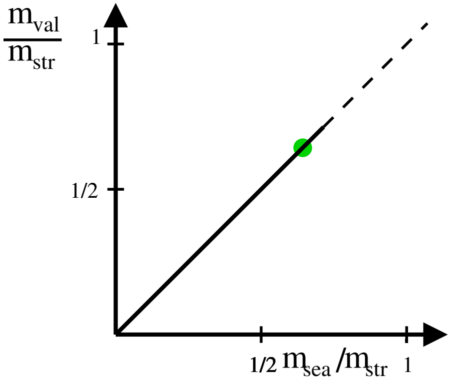
1. SUSY method to quench quarks (add quarks of opposite statistics): [Sharpe, Bernard & Goltermann 1990]

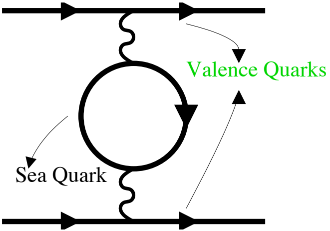
$$\mathcal{Z}_E[J] = \int [dA_\mu] \frac{[\det(\gamma_\mu D_\mu + m_h)]^{N_h} [\det(\gamma_\mu D_\mu + m_l)]^{N_l+N_q}}{[\det(\gamma_\mu D_\mu + m_l)]^{N_q}} e^{-S_g[A_\mu]}$$

- Perturbative χ PT: $SU(N_h + N_l) \rightarrow SU(N_h + N_l + N_q|N_q)$
- Non perturbative integrations of ϵ regime over Riemannian maximal submanifold of $\mathcal{G}(N_h + N_l + N_q|N_q)$ [Zirnbauer, Damgaard et al. 1998]

2. Replica method: $\lim_{N_q \rightarrow 0} \Pi_i \frac{\delta}{\delta J_i} \log \mathcal{Z}_E[J]$
   - correlators have a dependence in N_q such that the analytic continuation in the complex plane is unique
   - equivalent to SUSY at NLO [Splittorff & Damgaard 1992]

What is and why consider PQQCD? 1





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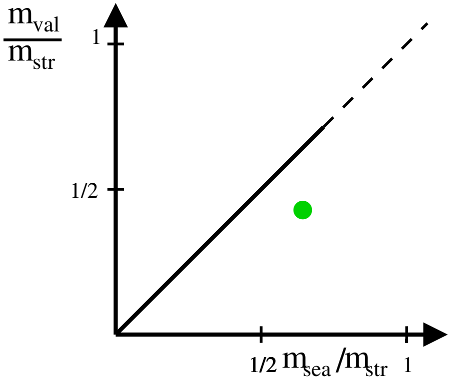

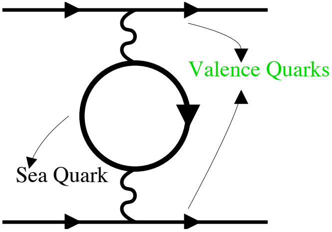

$$m_{valence} = \frac{2}{3} m_{sea}$$

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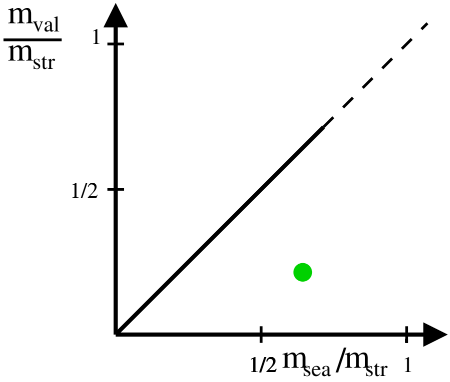

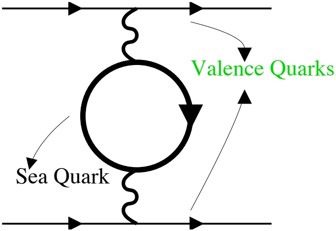

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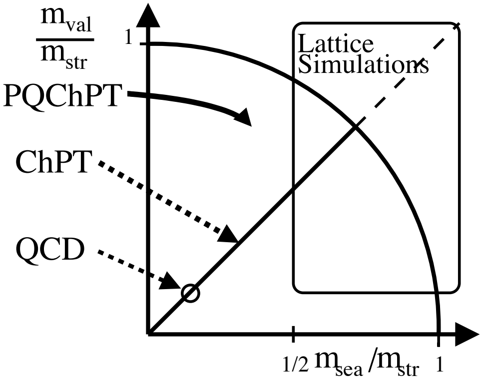
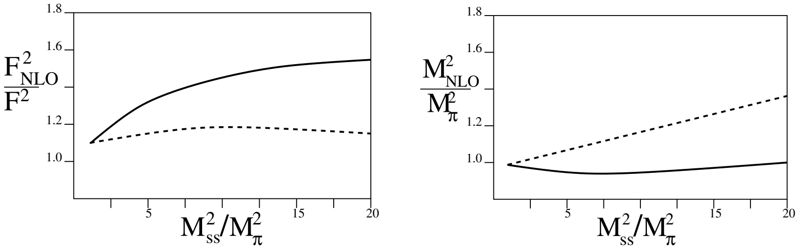
What is and why consider PQQCD? 2



- PQχPT extends the region of parameters that can be used for matching
- to move on the vertical line is cheap in lattice simulations (to change the valence mass does not require a new simulation)

[Sharpe & Shoresh 2000]

What is and why consider PQQCD? 3



- Full lines: full theory with $m_s = m_v$ and $N = 2$. L_i are taken to typical values from [Gasser & Leutwyler 1984].
- Dashed lines: PQ theory with $N_s = 2$
  - The pion squared mass is linear in m_s at NLO.
  - The corrections to F are smaller

Vector charge correlator in mixed regime for PQ theory

The result looks the same as the full theory one:

$$\mathcal{V}^c_{ud}(t) = -\frac{F^2_{NLO}}{2T}\mathcal{J}^{0(N_l)}_{-}((\mu_l)_{NLO}) + \frac{T}{2V}N_l k_{00}\mathcal{J}^{0(N_l)}_{+}(\mu_l) - \frac{\Sigma}{2}(m_u - m_d)(\mathcal{S}_u - \mathcal{S}_d)\ Th_1\left(\frac{t}{T}\right) + O(e^{-M^2_{hh}L})$$

- the dependence in N_q from perturbative contributions is analytic, and is just irrelevant after taking the replica limit.
- what really changes are the non perturbative integrals that are taken on the graded manifold [Zirnbauer]
- for this correlator the heavy quarks decouple but this is not a general fact. The Decoupling theorem does not hold for non unitary theories.

[F.B. et al. 2008]

Reason for non decoupling

The pion η is the one associated to the generator:

$$T_\eta \equiv \sqrt{\frac{N_l N_h}{2(N_l + N_h)}} \mathrm{diag}\{\underbrace{\frac{1}{N_l}, \ldots, \frac{1}{N_l}}_{N_l}, \underbrace{-\frac{1}{N_h}, \ldots, -\frac{1}{N_h}}_{N_h}\}$$

- It has an N_l dependent mass: $M_\eta^2 = \frac{N_l \overbrace{M_{hh}^2}^{\sim L^{-2}} + N_h \overbrace{M_{ll}^2}^{\sim L^{-4}}}{N_h + N_l}$ (here we restrict to: $\mathcal{M}_h = m_h \mathbf{1}$)
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Double poles and non decoupling for PQχPT

$$\mathcal{P}^c_{vv'}(t) = -L^3 \frac{\tilde{\Sigma}^2}{4}\left[\mathcal{K}_-^{0(N_l)}\right]_{NLO} + \frac{\Sigma^2}{2F^2}\left[\mathcal{K}_+^{1(N_l)} T h_1\left(\frac{t}{T}\right) - \mathcal{K}_+^{0(N_l)} r(t)\right]$$

$$r(t) = \frac{1}{T}\sum_{p\neq 0} e^{ipt}\frac{p^2+M_{hh}^2}{N_l p^2(p^2+M_{hh}^2)+N_h p^4} \quad (\text{if } \mathcal{M}_h = m_h \mathbf{1})$$

- if $N_l = 0$ a double pole appears (sign of non unitarity)
- if $N_l = 0$ the limit $M_{hh}^2 \to \infty$ leads to a divergence (non decoupling)
- if $N_l = 0$ we can match with a Quenched theory in the ϵ regime: the η plays the role of the η'

$$\frac{\overbrace{\alpha}^{\text{field strength}}}{N_c} = \frac{1}{N_h} \quad \frac{\overbrace{m_0^2}^{M_{\eta'}}}{N_c} = \frac{\overbrace{M_{hh}^2}^{M_\eta}}{N_h}$$

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Conclusions

- We have studied χPT in a finite volume with non-degenerate masses such that N_l dynamical quarks fall in the ϵ and N_h in the p regime
- We have computed AA, VV, SS, PP with non degenerate masses in the heavy and light sector
- We considered the case in which some or all the quarks are quenched

Outlook

- Apply the results to analyze PQ correlation functions from lattice simulations with mixed actions (overlap on the valence and Wilson O(a) improved on the sea)
- Include discretization effects if necessary

Many thanks for your attention!