Universitat Autònoma de Barcelona

IFAE - Institut de Física d'Altes Energies

$K\pi$ vector form factor, dispersive constraints and $\tau \to \nu_\tau K\pi$ decays

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in coll. with Rafel Escribano and Matthias Jamin

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FlaviA net Annual Workshop

1. General Introduction.
2. Dispersion Relations and Unitarity.
3. Fits to data.
4. Results and conclusions.

Form factors: introduction

- CKM matrix: test of the Standard Model

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

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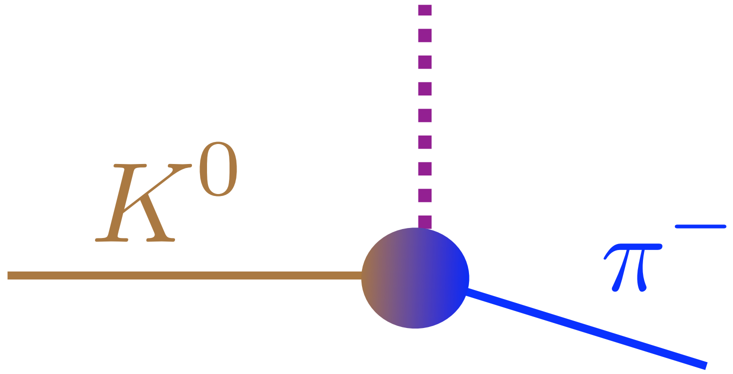
K_{l3} Decays

$$K^0 \to \pi^- \bar{l} \nu$$

We need the $K\pi$ form factors



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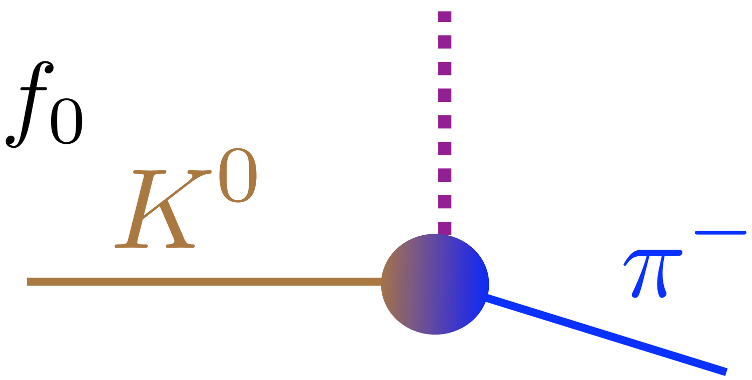
- Parametrization in terms of f_+ and f_0



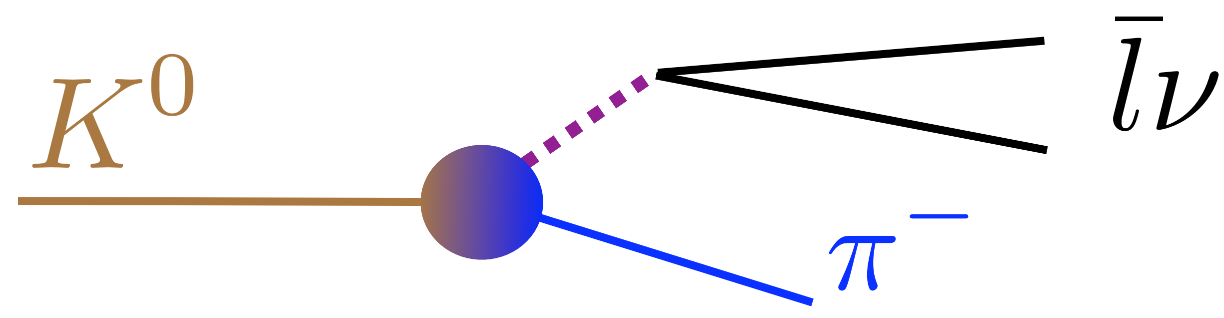
$$\langle \pi^-(k)|\bar{s}\gamma^\mu u|K^0(p)\rangle$$

$$||$$

$$\left[(p+k)^\mu - \frac{\Delta_{K\pi}}{t}(p-k)^\mu\right] f_+(t) + (p-k)^\mu \frac{\Delta_{K\pi}}{t} f_0(t)$$

Form factors

$K^0 \longrightarrow \pi^- \bar{l}\nu$



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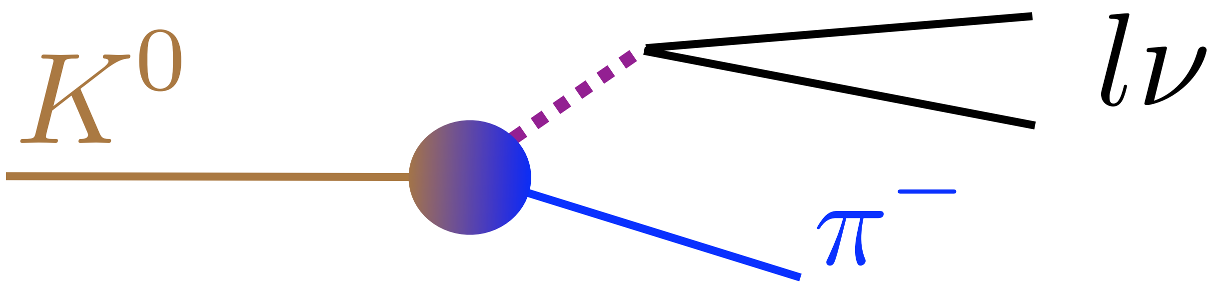

- $f_{+,0}(0)$ ← ChPT, Lattice

Normalization ⟶ $|V_{us}|f_+^{K^0\pi^+}(0)$

Form factors

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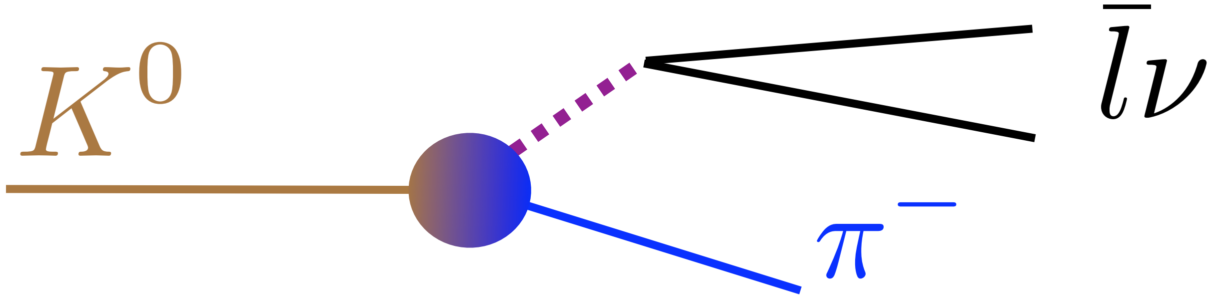

- $f_{+,0}(0) \leftarrow$ ChPT, Lattice
- $f_{+,0}(q^2) \leftarrow$ ChPT, DR, RChPT, Lattice

Normalization $\longrightarrow$ $|V_{us}|f_+^{K^0\pi^+}(0)$

Energy dependence: $\tilde{f}_+(t) = 1 + \frac{\lambda'_+}{m_\pi^2}t + \frac{1}{2}\frac{\lambda''_+}{m_\pi^4}t^2 + \mathcal{O}(t^3)$

Form factors

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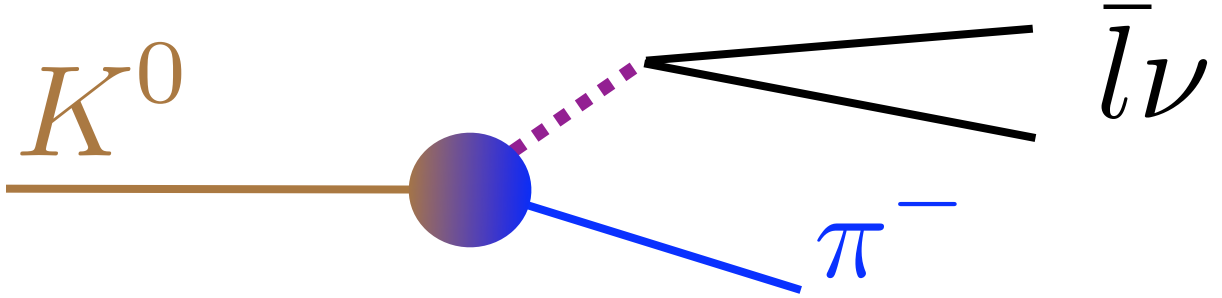

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This work

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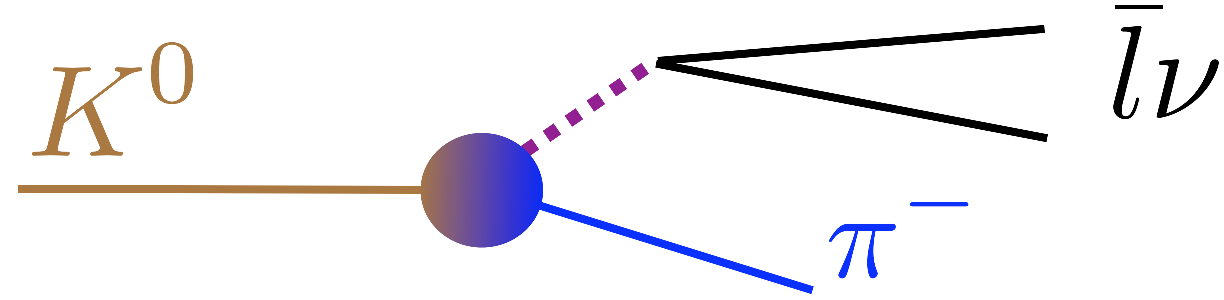

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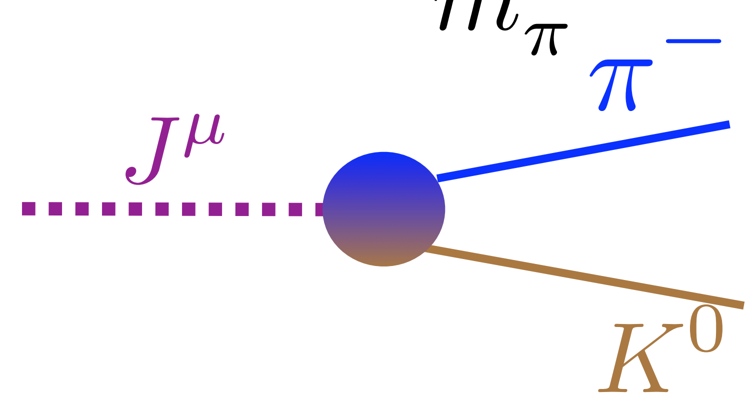
- Crossed channel.



$$\langle K\pi|\bar{s}\gamma^\mu u|0\rangle$$
$$||$$
$$\left[(k-p)^\mu + \frac{\Delta_{K\pi}}{s}(p+k)^\mu\right] f_+(s) - (p+k)^\mu \frac{\Delta_{K\pi}}{s} f_0(s)$$

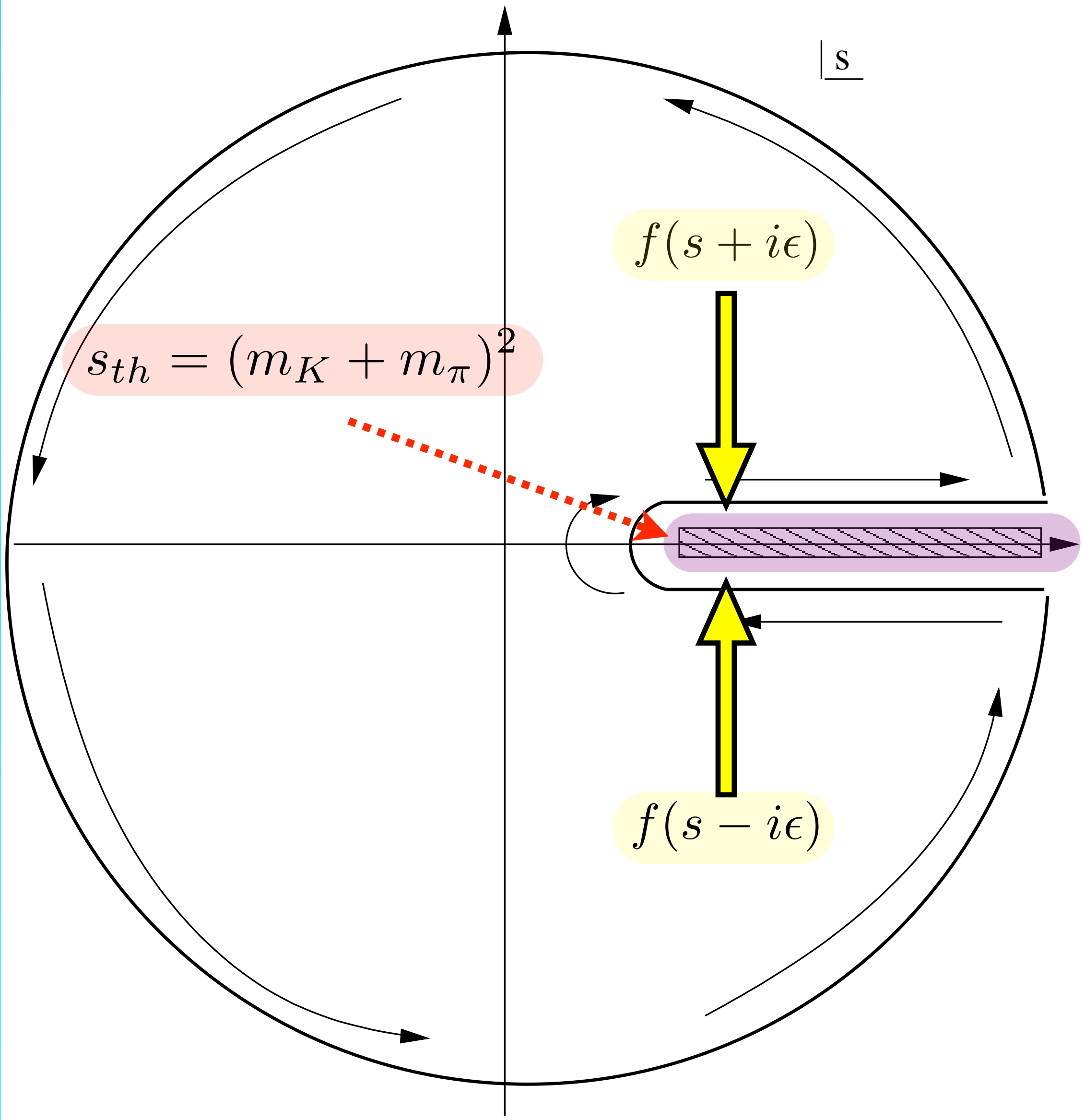
2 Form factor from analyticity and unitarity.

- Dispersion relation (DR) for $f_{+,0}(s)$
- Unitarity (U)

DR + U = Muskhelishvili-Omnès Eq.

- Eq. can be solved.
- Fit to data. → $\lambda'_+ \quad \lambda''_+ \quad \lambda'''_+$ $K^*(892)$ pole

Dispersion Relation for $f_{+,0}(s)$



$$f(s) = \frac{1}{\pi} \int_{s_{th}}^{\infty} ds' \frac{\text{Im}\, f(s')}{s' - s - i\epsilon}$$

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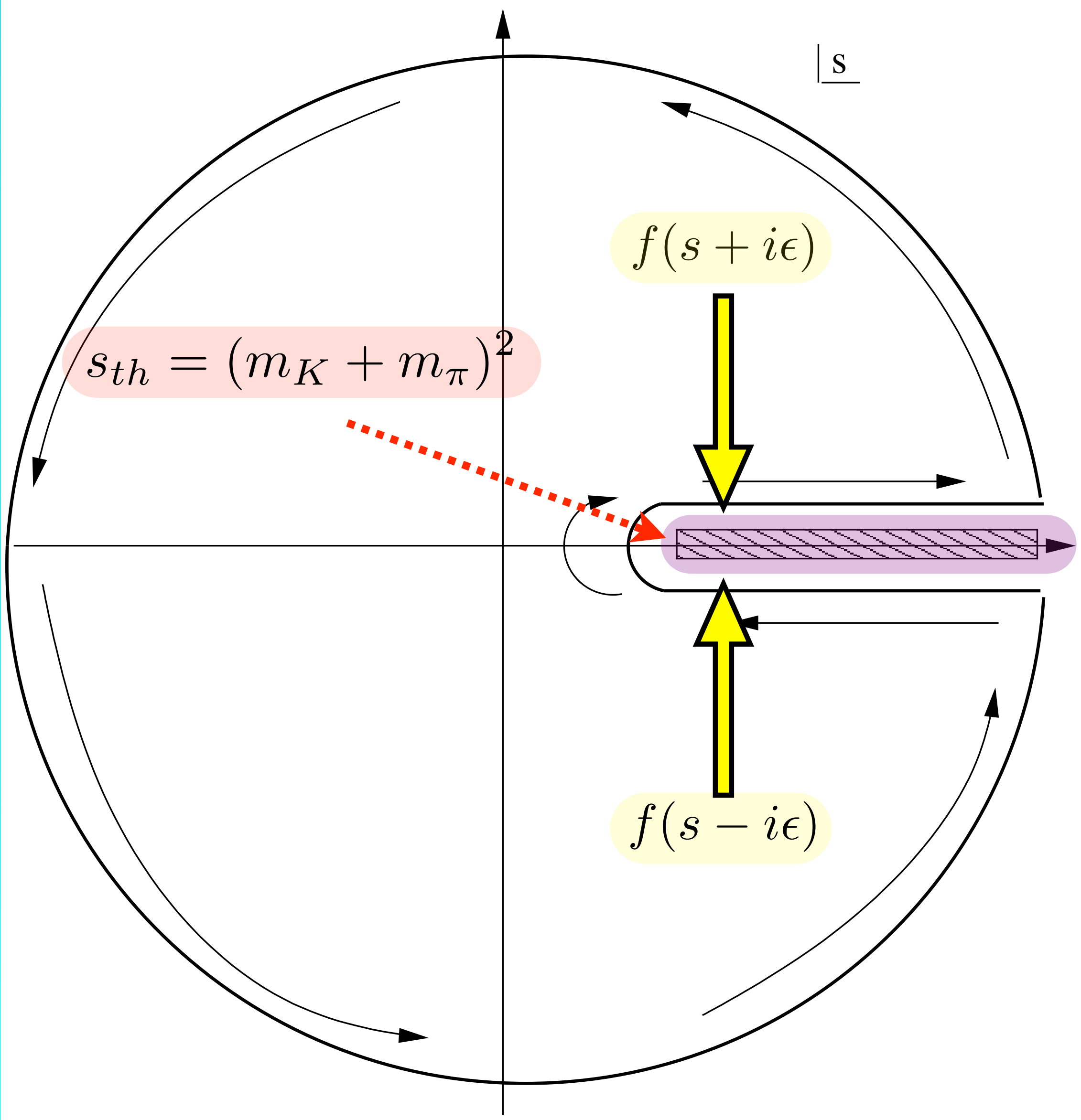

$$f(s)=\frac{1}{\pi}\int_{s_{th}}^{\infty}ds'\,\frac{\mathrm{Im}\,f(s')}{s'-s-i\epsilon}$$

Unitarity

Watson's theorem

$$f(s)=|f(s)|e^{i\delta(s)}$$

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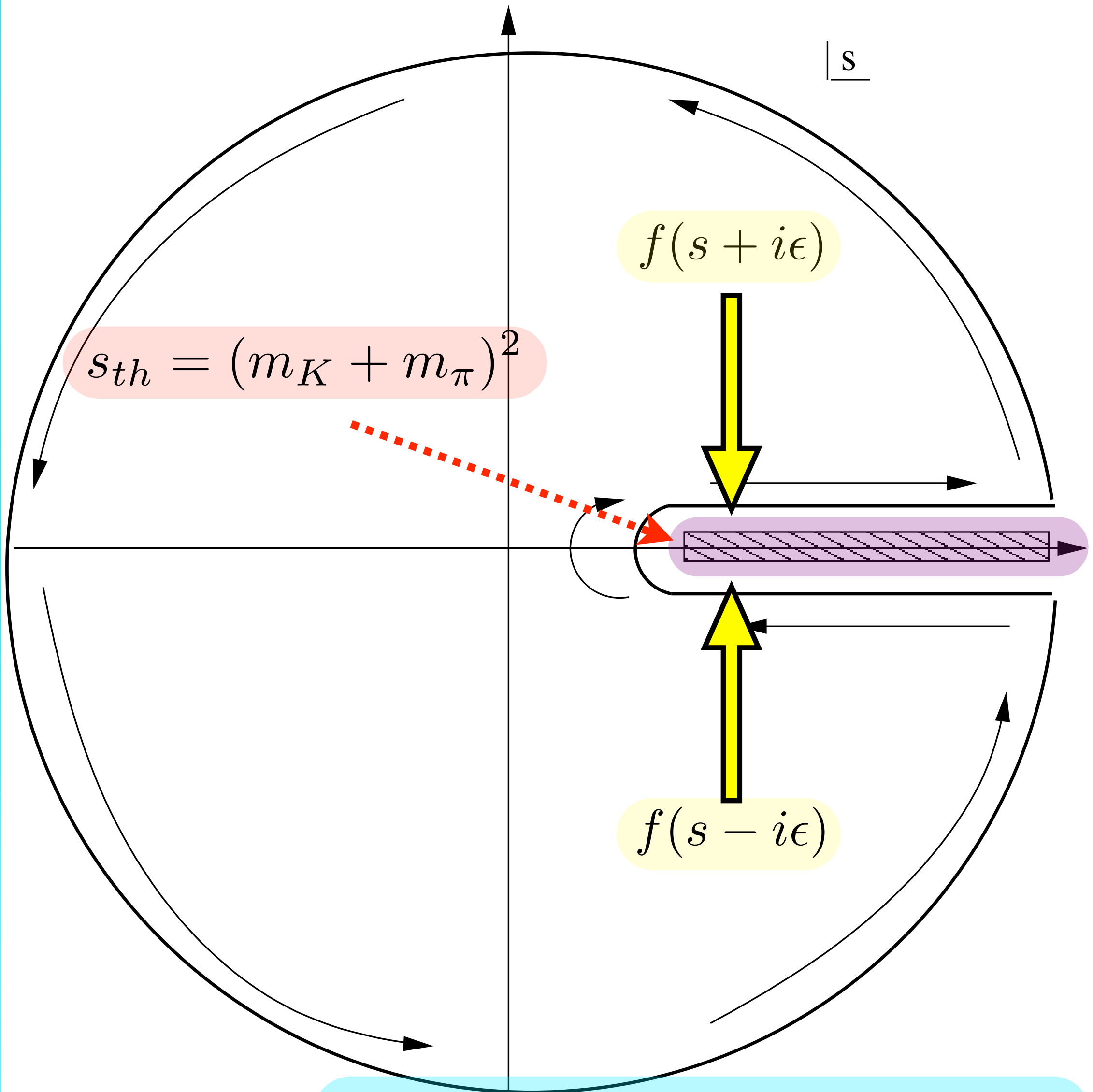

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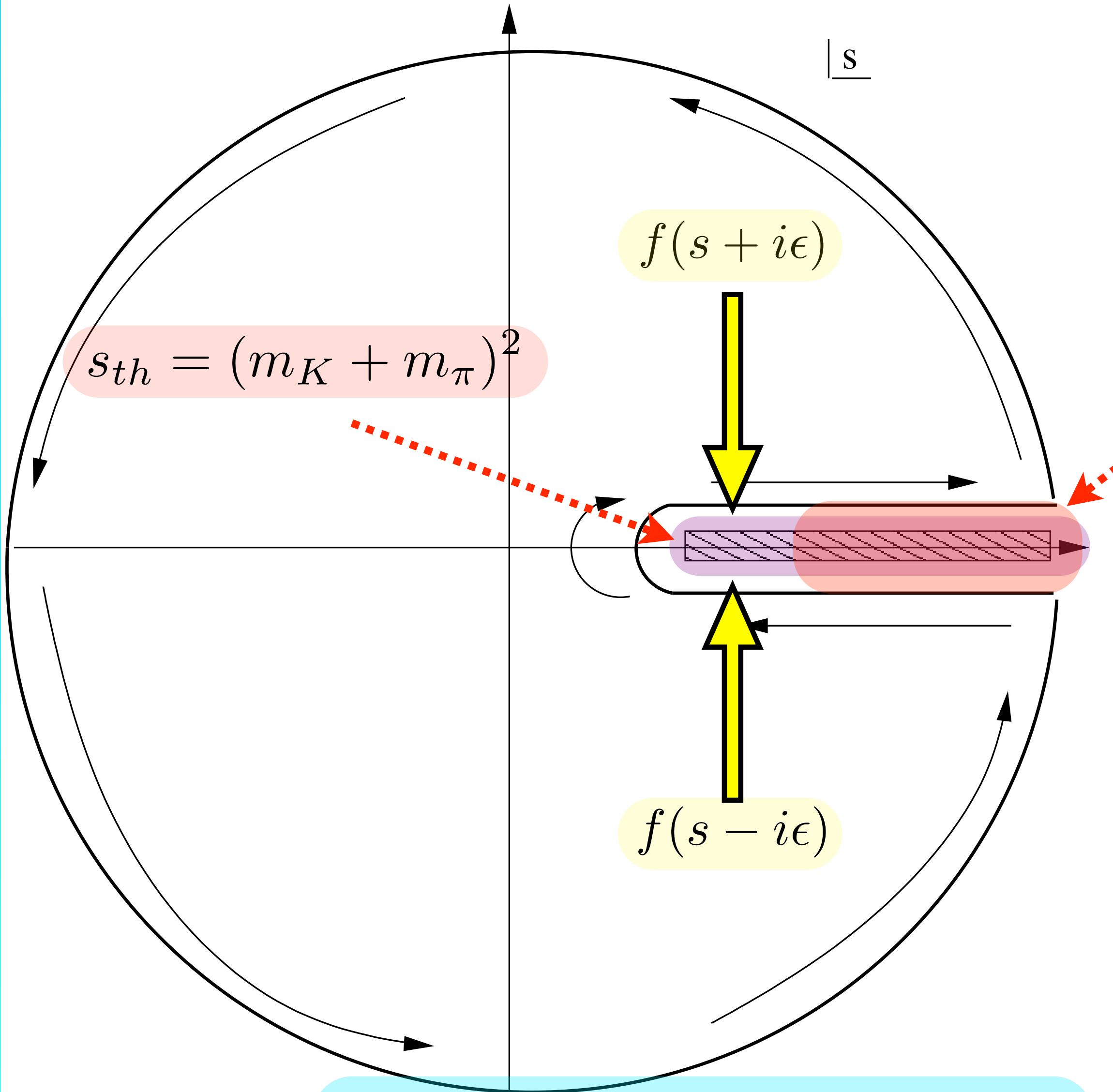
Omnès Equation

Omnès, Nuovo Cim. **8** (1958).

$$f(s) = \frac{1}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\tan\delta(s')\mathrm{Re}\, f(s')}{s' - s - i\epsilon}$$

Dispersion Relation for $f_{+,0}(s)$



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Other cuts!

Unitarity

Watson's theorem

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$$f(s) = \frac{1}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\tan\delta(s')\mathrm{Re}\, f(s')}{s' - s - i\epsilon}$$

Elastic approximation!

Omnès Eq.: solution

$$f(s) = \frac{1}{\pi}\int_{s_{th}}^{\infty} ds' \, \frac{\tan\delta(s')\mathrm{Re}\, f(s')}{s'-s-i\epsilon}$$

- Solution:

$$f(s) = f(0)\exp\left[\frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\,\frac{\delta(s')}{s'-s-i\epsilon}\right]$$

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- Strategy: subtracted dispersion relations.

- Generalized solution: n subtractions at $s_0 = 0$

$$f(s) = \exp\left[\alpha_1 + \alpha_2 s \cdots + \alpha_{n-1}s^{n-1} + \frac{s^n}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{(s')^n}\,\frac{\delta(s')}{s'-s-i\epsilon}\right]$$

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Related to $\lambda_+^{(n)}$

Helps the convergency!

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3 Extracting the form factor from $\tau \to \nu_\tau K\pi$.

- We have $f_0(s)$ — Jamin, Oller, Pich NP **B587** (2002); PRD **74** (2006).
- Omnès solution for $f_+(s)$
- We can fit the data and extract → λ'_+ λ''_+ λ'''_+ $K^*(892)$ pole

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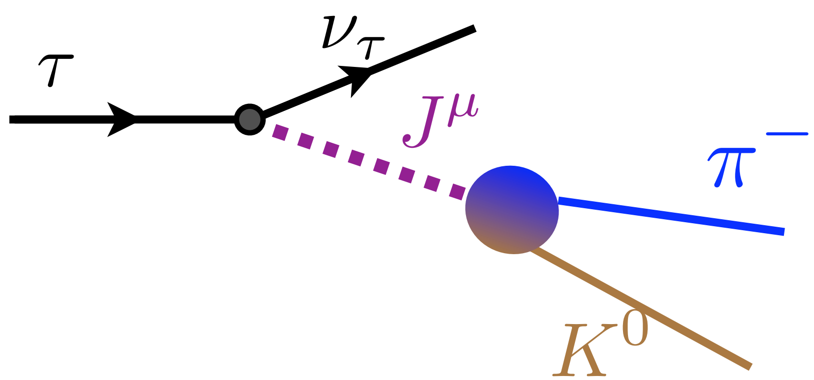
- First results without DR:

  Jamin, Pich and Portolés, PL **B664** (2008)
- Belle data:

  D. Epifanov et. al., PL **B654** (2007)

$\tau \to \nu_\tau K\pi$

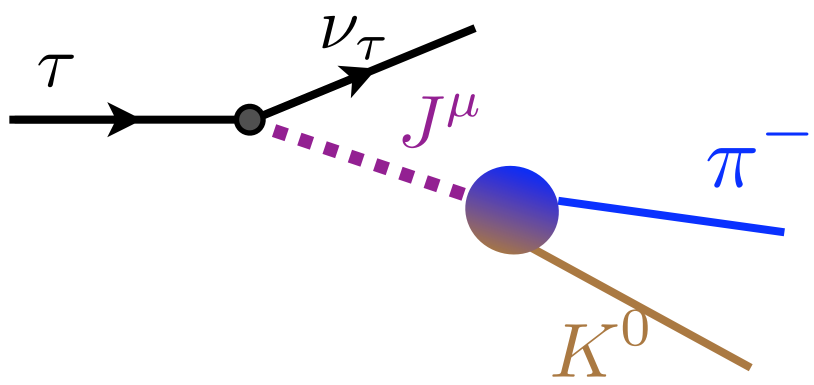


Finkemeier and Mirkes, ZP **C72** (1996)

$$\frac{d\Gamma_{K\pi}}{d\sqrt{s}} = \phi(s,m_\tau)G_F^2|V_{us}|^2|f^{K^0\pi^-}(0)|^2 S_{EW}\left[q_1(s,m_\tau)|\tilde{f}_+(s)|^2 + q_2(s,m_\tau)|\tilde{f}_0(s)|^2\right]$$

$K\pi$ vector form factor

$\tau \to \nu_\tau K \pi$



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Antonelli et. al. [FlaviaNet Kaon Working Group]
arXiv:0801.1817

$$|V_{us}| f_+^{K^0\pi^-}(0) = 0.21664 \pm 0.00048$$

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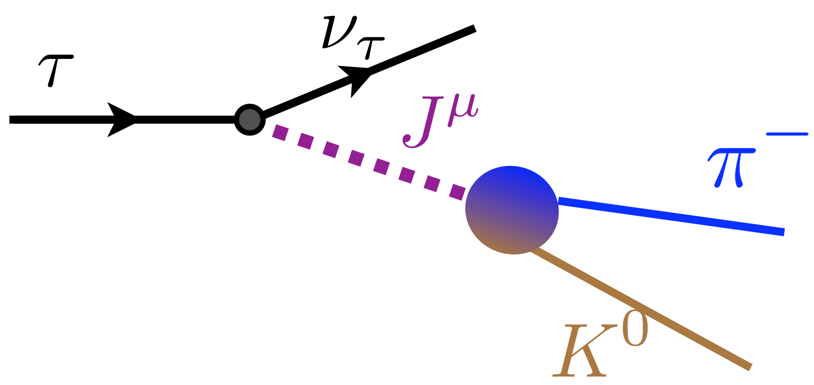

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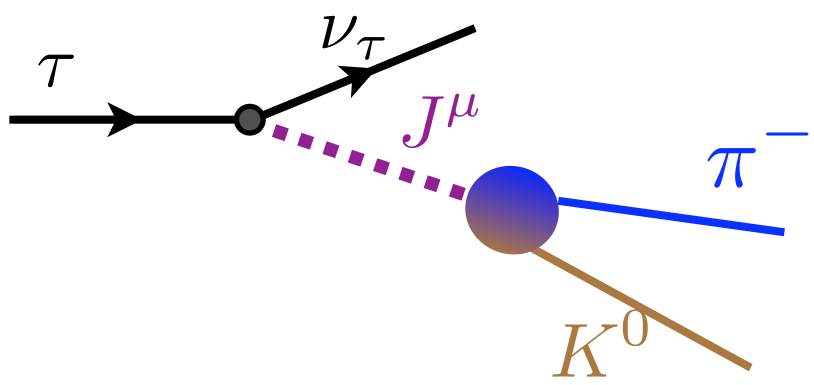

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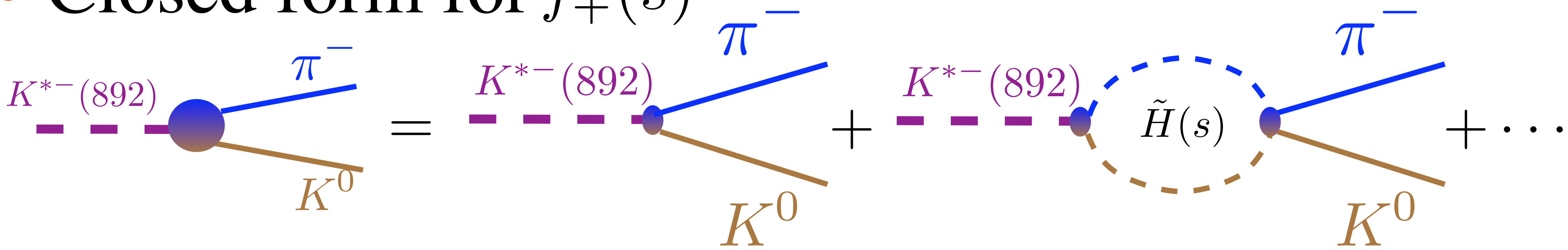
$$\frac{1}{2} \cdot \frac{2}{3} 0.0115 \, [\mathrm{GeV/bin}] \, \mathcal{N}_T \frac{1}{\Gamma_\tau \bar{B}_{K\pi}} \frac{d\Gamma_{K\pi}}{d\sqrt{s}}$$

$\tau \to \nu_\tau K\pi$: one resonance.

- Closed form for $f_+(s)$



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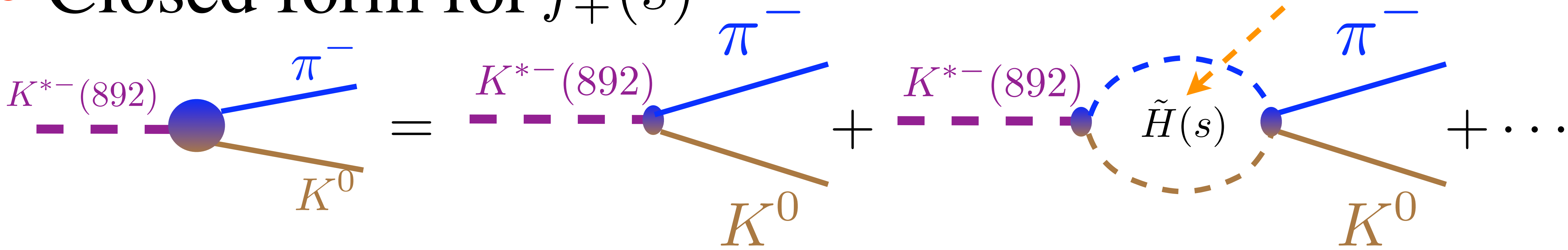

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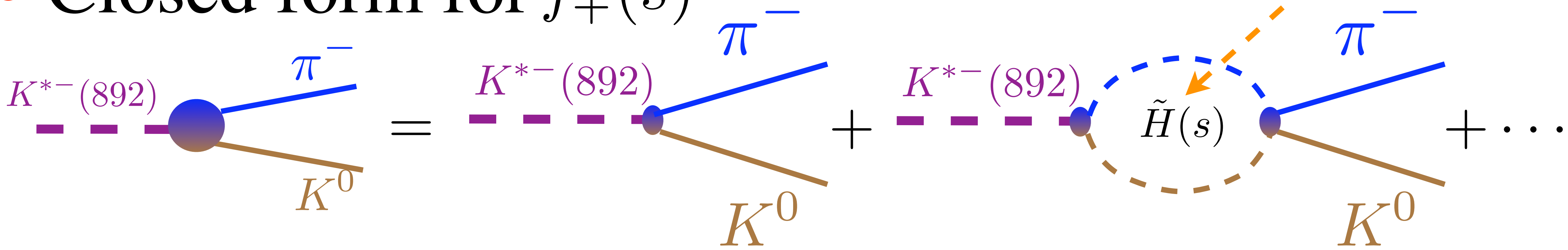

Omnès

$$\tilde{f}_+(s) = \frac{1}{f_+(0)} \frac{m_{K^*}^2}{m_{K^*}^2 - s - \kappa \mathrm{Re}\tilde{H}_{K\pi}(s) - i m_{K^*}\Gamma(s)}$$

$$= \exp\left[\frac{s}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\delta(s')}{s'(s'-s-i\epsilon)}\right]$$

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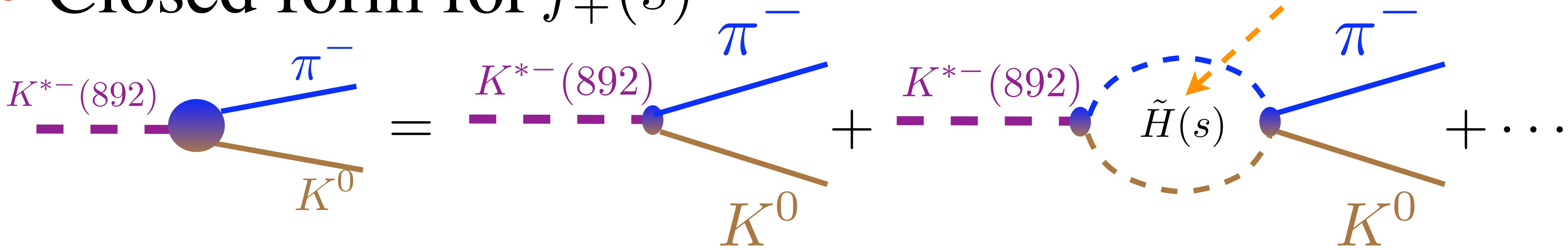

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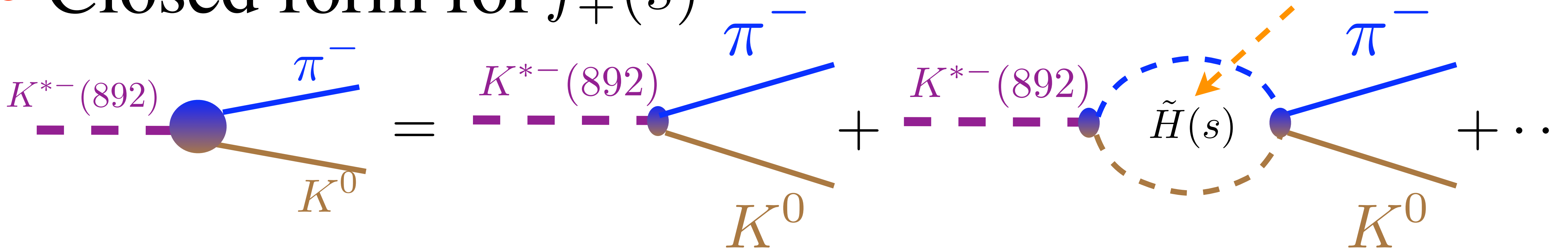

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$$\delta(s) = \tan^{-1}\left[\frac{m_{K^*}\Gamma(s)}{m_{K^*}^2 - s - \kappa \mathrm{Re}\tilde{H}_{K\pi}(s)}\right]$$

$$\lambda'_+ = [1 + N g^2 \tilde{H}'(0)] f_+(0) m_\pi^2$$

Omnès

$$= \exp\left[\frac{s}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\delta(s')}{s'(s'-s-i\epsilon)}\right]$$

$$= \delta(s)$$

$$= \frac{m_\pi^2}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\delta(s')}{s'^2}.$$

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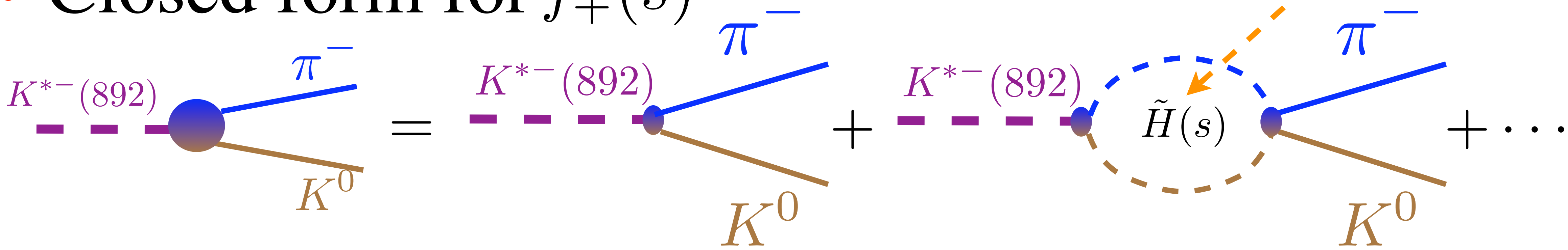

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$$\lambda'_+ = [1 + Ng^2 \tilde{H}'(0)] f_+(0) m_\pi^2$$

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- Model with two parameters: m_{K^*}, Γ_{K^*}
- $\lambda_+^{(n)}$ determined by m_{K^*}, Γ_{K^*}

Pole positions:

- Complex zeros of the denominator:

$$D(m_n,\Gamma_n) = m_n^2 - s - \kappa_n \,\mathrm{Re}\tilde{H}(s) - i m_n \Gamma_n(s)$$

$$s = x + iy$$

$$\sqrt{s_p} = M_R - \frac{i}{2}\Gamma_R$$

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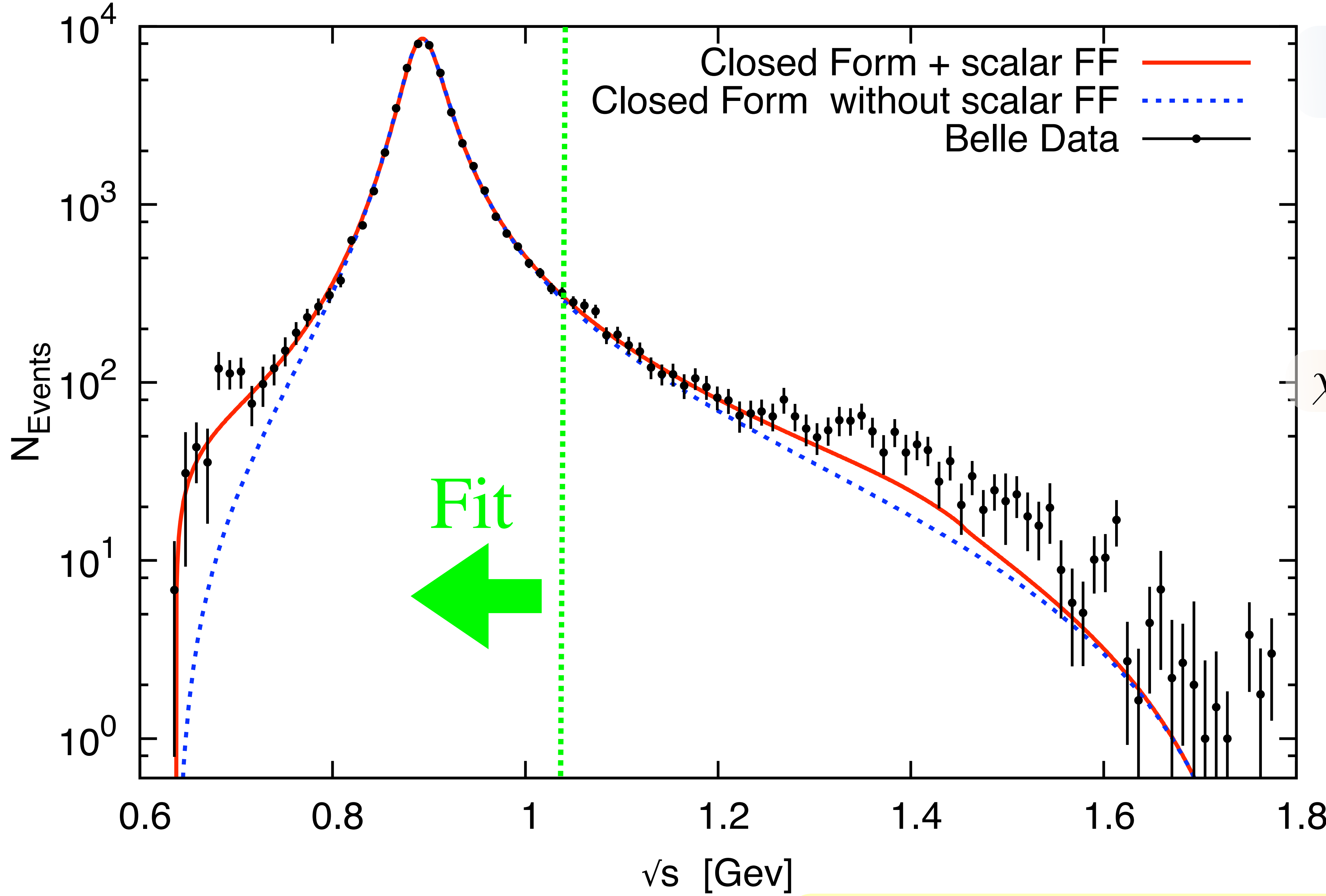
Important:

- $m_n \approx 940\ \mathrm{MeV}$ Model parameter

$\neq$

- $M_R \approx 892\ \mathrm{MeV}$ Physical Value

$K\pi$ vector form factor



Results 1

$\chi^2/d.o.f. = 40/27$

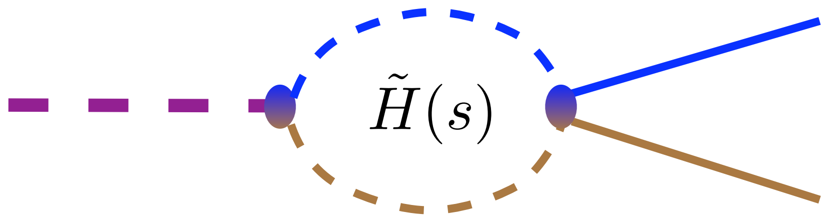
$K^*(892)$ pole → $\sqrt{s_p} = (892.3 \pm 1.2) - \frac{i}{2}(46.01^{+0.34}_{-0.29})$ MeV

$\bar{B}_{K\pi}(B_{K\pi})$	$0.3611 \pm 0.0042\%\ (0.3563\%)$
m_{K^*}	943.35 ± 0.52 MeV
Γ_{K^*}	66.29 ± 0.79 MeV
$\chi^2/d.o.f.$	$39.4/27$

λ'_+	$(22.975 \pm 0.030) \times 10^{-3}$
λ''_+	$(11.062 \pm 0.032) \times 10^{-4}$
λ'''_+	$(8.163 \pm 0.037) \times 10^{-5}$

$K\pi$ vector form factor

$\tau \to \nu_\tau K\pi$: 2 resonances 3 subtractions

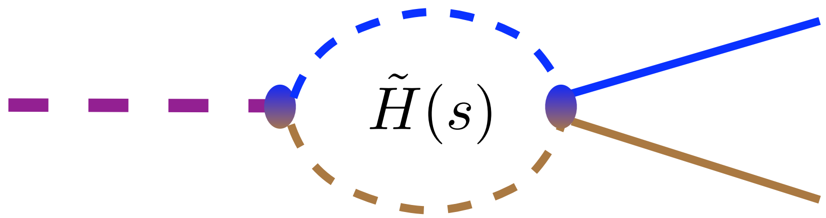


Jamin, Pich and Portolés, PL **B664** (2008)

$$\tilde{f}_+(s) = \left[\frac{m_{K^*}^2 - Ng^2 \mathrm{Re}\tilde{H}_{K\pi}(0) + \gamma s}{D(m_{K^*}, \Gamma_{K^*})} - \frac{\gamma s}{D(m_{K'}, \Gamma_{K'})}\right]$$

$$D(m_n, \Gamma_n) = m_n^2 - s - \kappa_n \,\mathrm{Re}\tilde{H}(s) - i m_n \Gamma_n(s)$$

$\tau \to \nu_\tau K\pi$: 2 resonances 3 subtractions



Jamin, Pich and Portolés, PL **B664** (2008)

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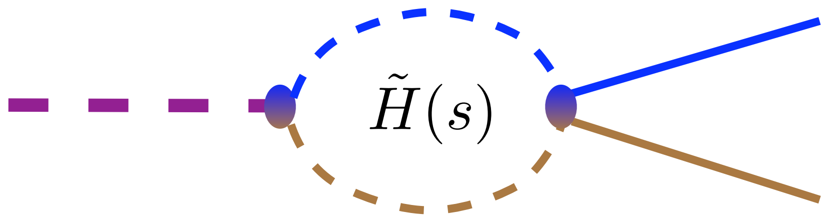

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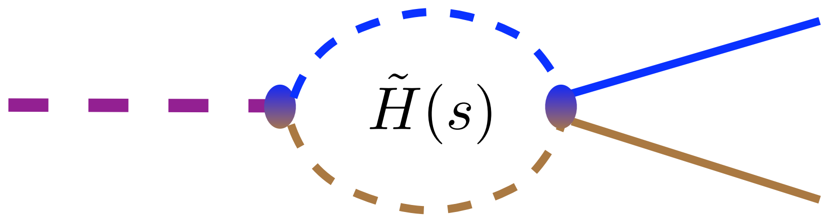

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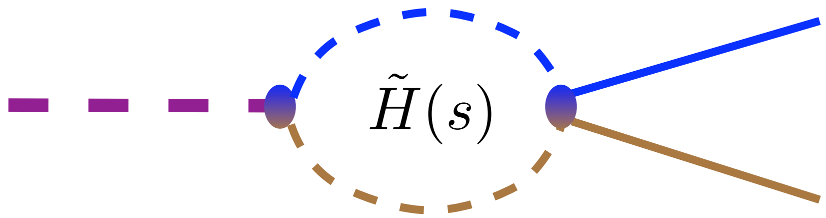
- Same strategy used in Pich and Portolés PRD **63** (2001) for the pion vector form factor.

$K\pi$ vector form factor

$\tau \to \nu_\tau K\pi$: 2 resonances 3 subtractions



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See also Passemar's talk

$\tau \to \nu_\tau K\pi$: 2 resonances 3 subtractions

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Pich and Portolés PRD **63** (2001)
for the pion vector form factor

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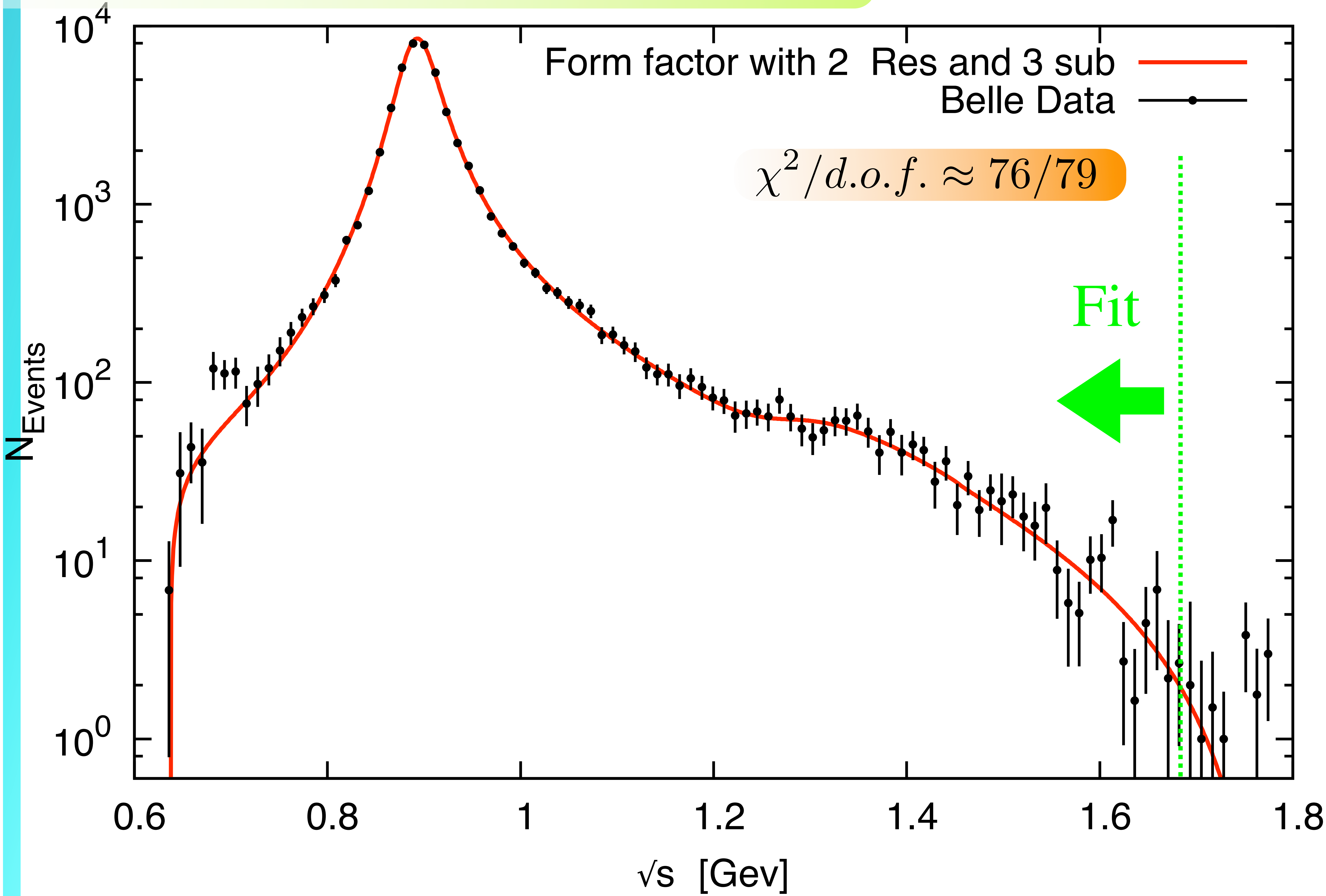
Cut-off to check the stability

Pich and Portolés PRD **63** (2001) for the pion vector form factor

$$\lambda'_+ = \alpha_1 \qquad \lambda''_+ = \alpha_2 + \alpha_1^2$$

$$\tilde{f}_+(t) = 1 + \frac{\lambda'_+}{m_\pi^2} t + \frac{1}{2}\frac{\lambda''_+}{m_\pi^4} t^2 + \mathcal{O}(t^3)$$

$\tau \to \nu_\tau K\pi$: 2 resonances 3 subtractions

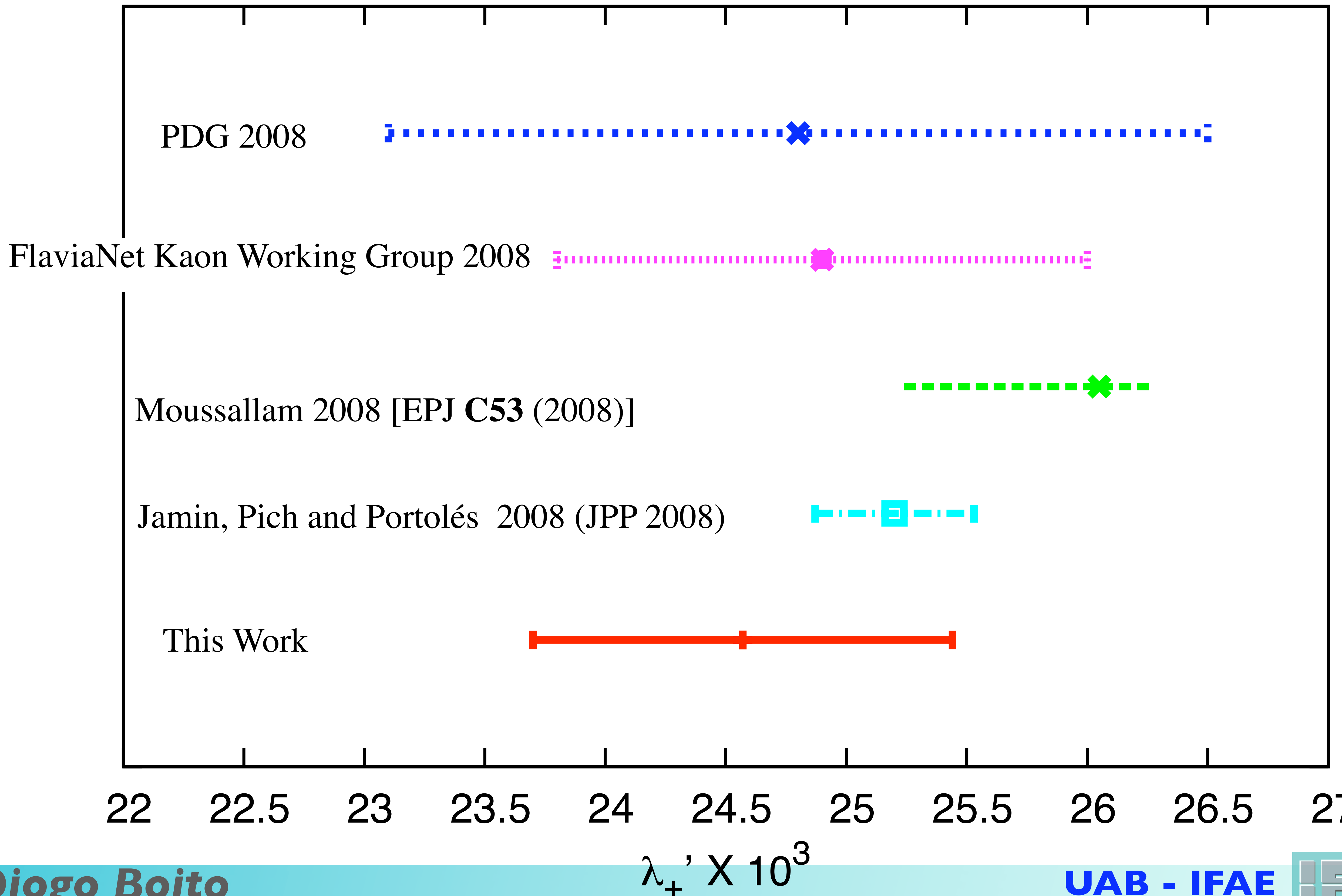


4 Final Results

$K\pi$ vector form factor

$$\lambda'_+ = (24.6 \pm 0.9) \times 10^{-3}$$

$$\tilde{f}_+(t) = 1 + \frac{\lambda'_+}{m_\pi^2} t + \frac{1}{2}\frac{\lambda''_+}{m_\pi^4} t^2 + \mathcal{O}(t^3)$$



$$\lambda''_+ = (11.96 \pm 0.25) \times 10^{-4}$$

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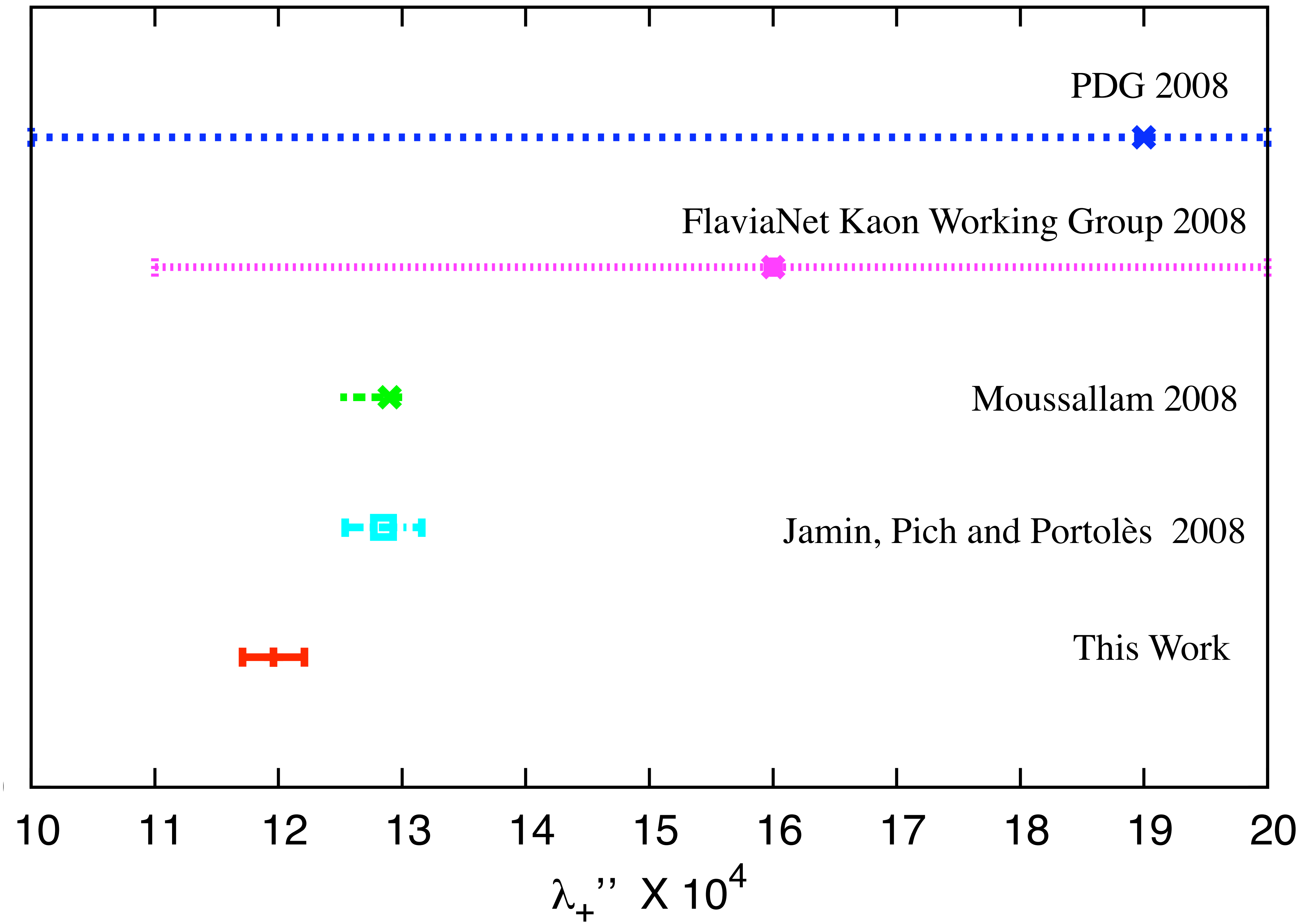


The poles

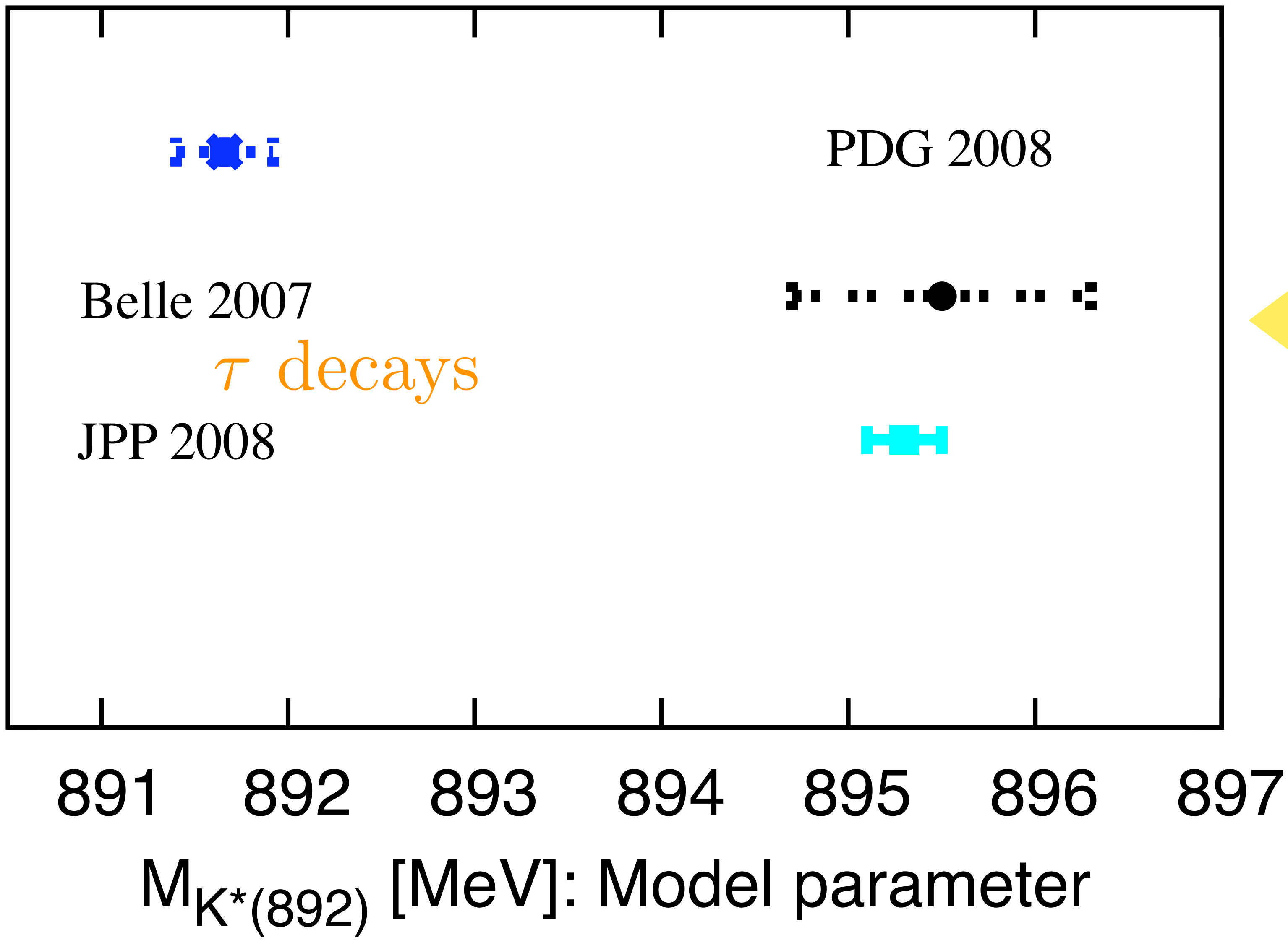
$$\sqrt{s_p} = M_R - \frac{i}{2}\Gamma_R$$

- $K^*(892)$ pole

$$\sqrt{s_{p_1}} = (892.01 \pm 0.91) - \frac{i}{2}(46.20 \pm 0.38)\ \text{MeV}$$

- $K^{*'}$ or $K^*(1410)$

$$\sqrt{s_{p_2}} = (1280^{+73}_{-77}) - \frac{i}{2}(194^{+60}_{-88})\ \text{MeV}$$



$M_{K^*(892)}$

Model Parameters

$M_{K^*(892)}$

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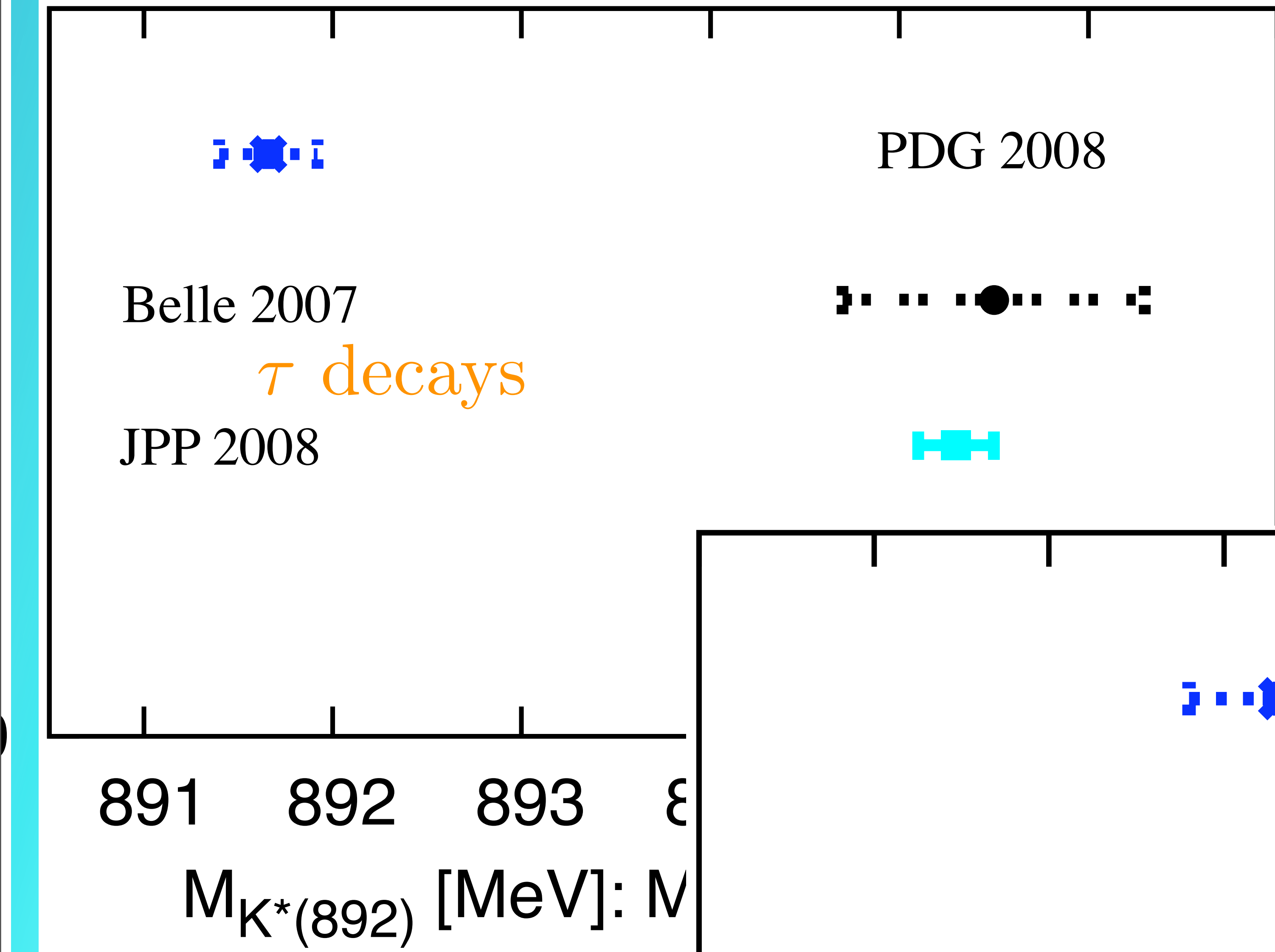
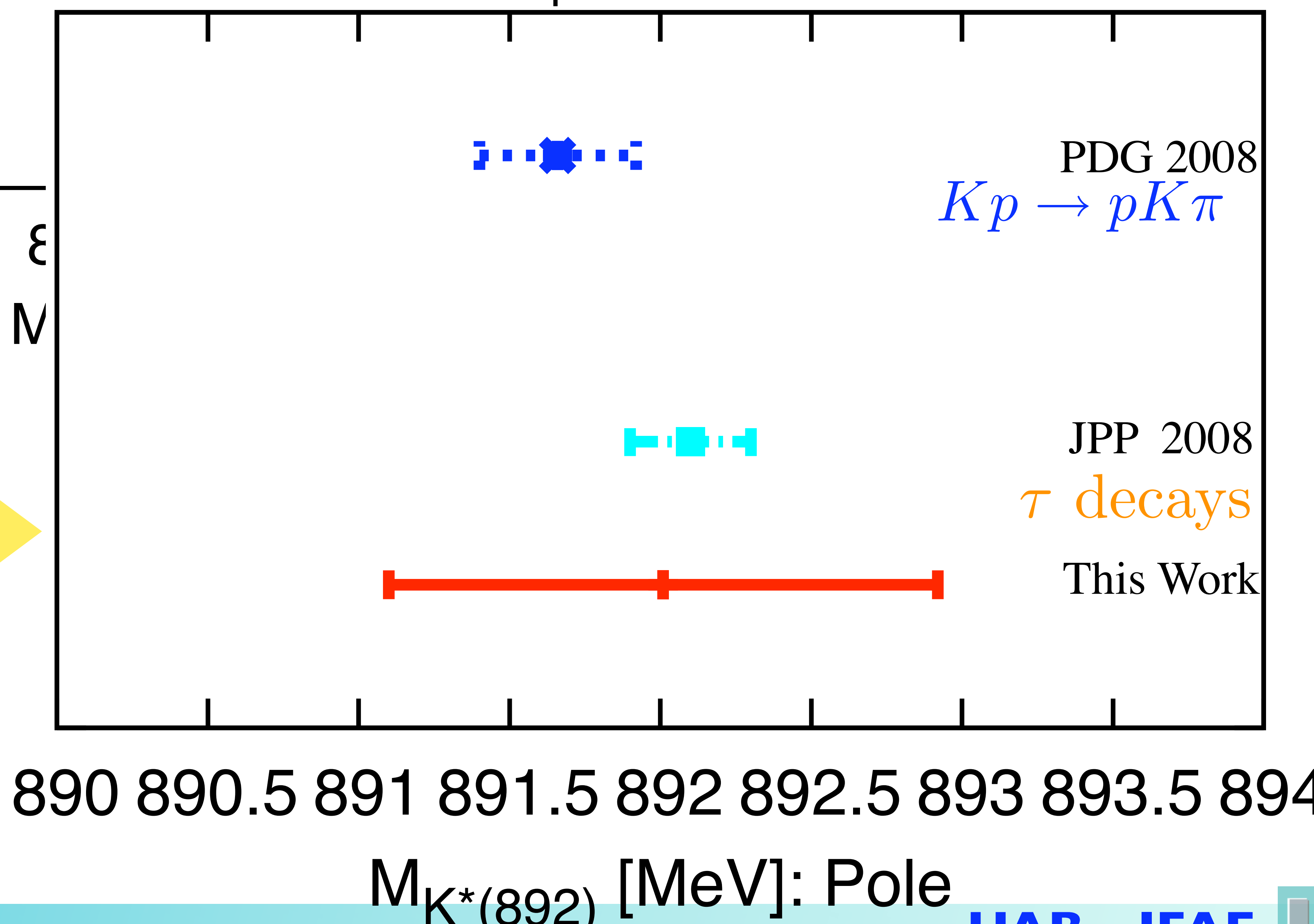
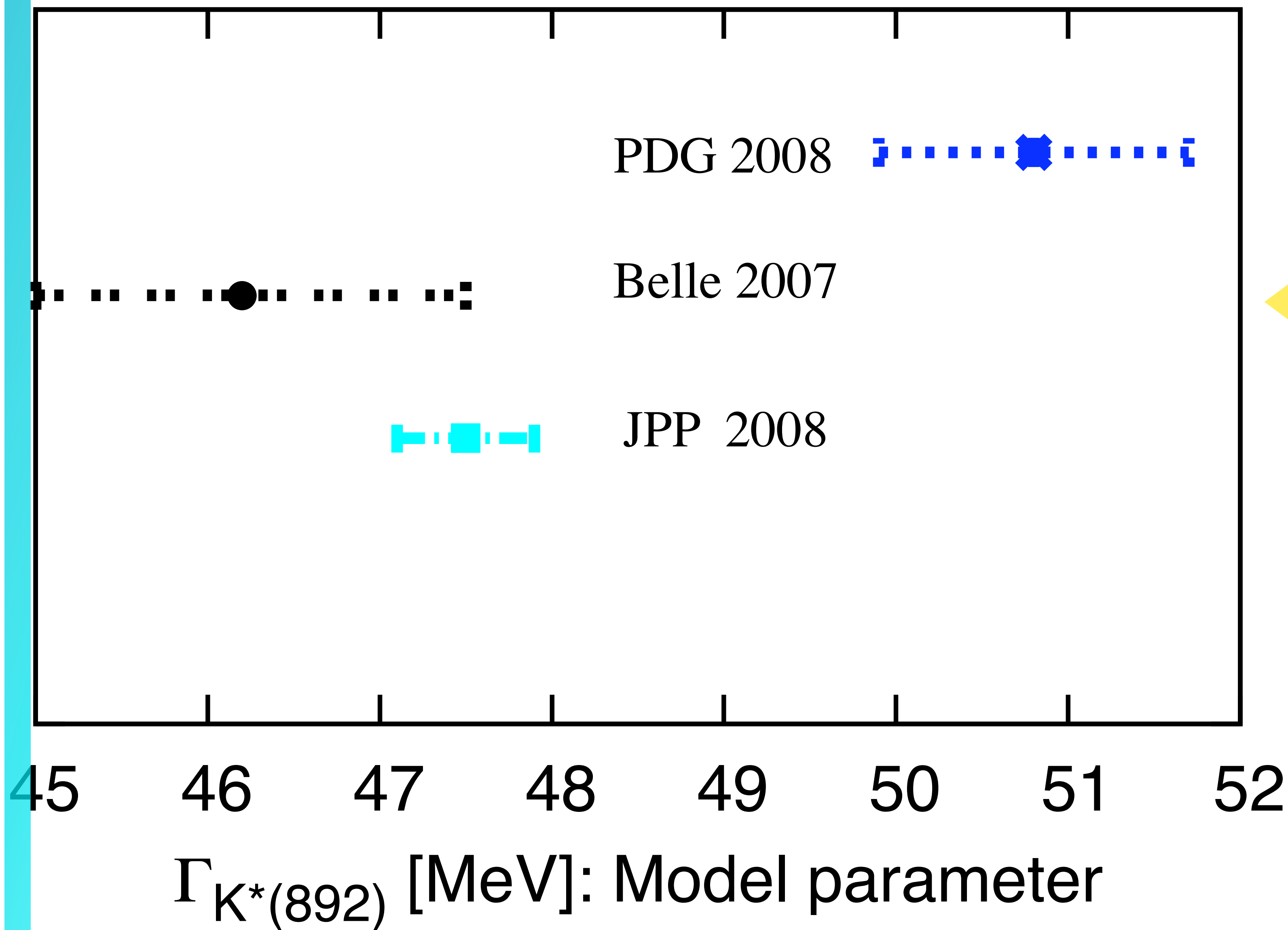


Pole Position



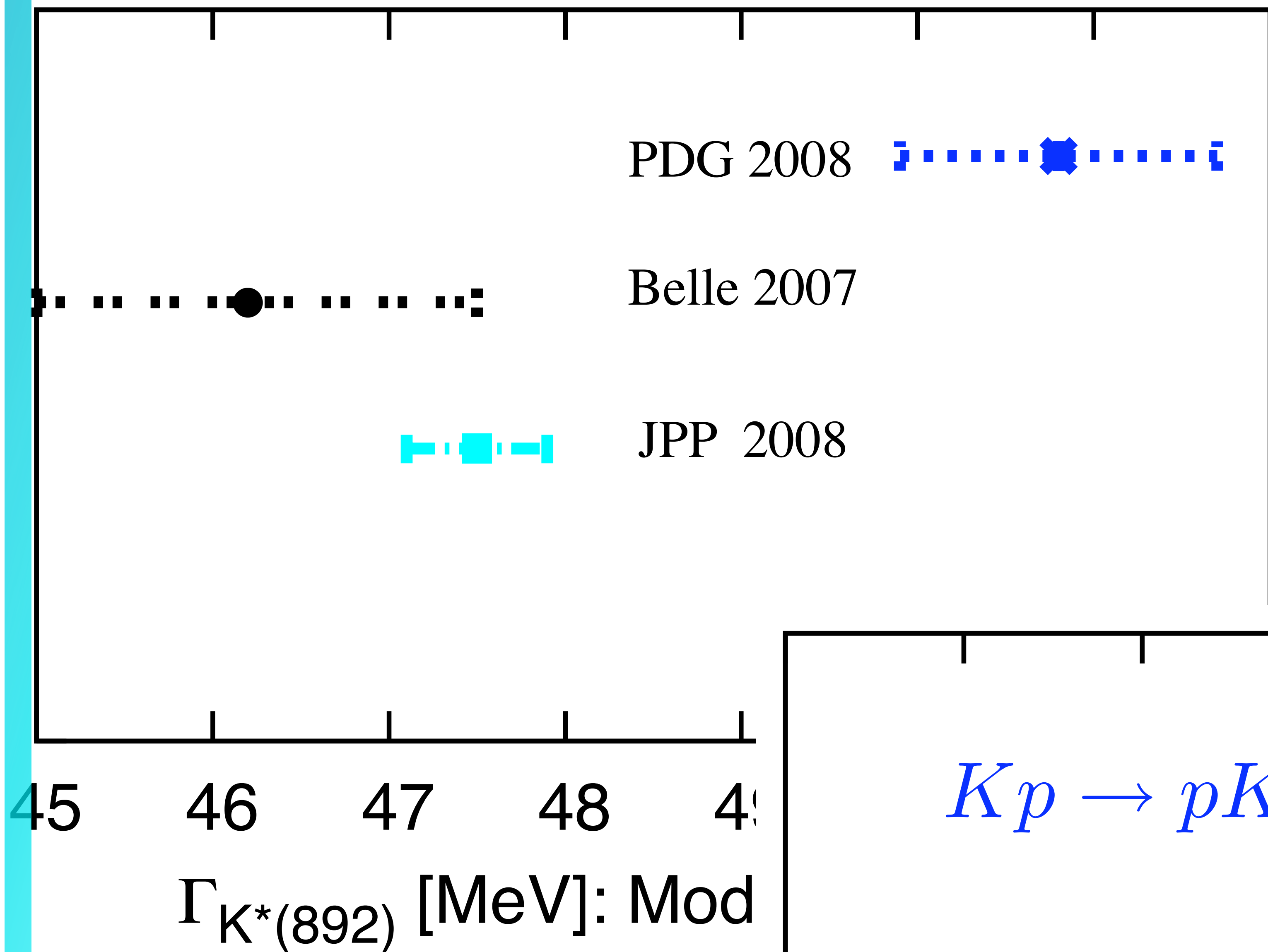


$\Gamma_{K^*(892)}$

Model Parameters

$K\pi$ vector form factor

$\Gamma_{K^*(892)}$



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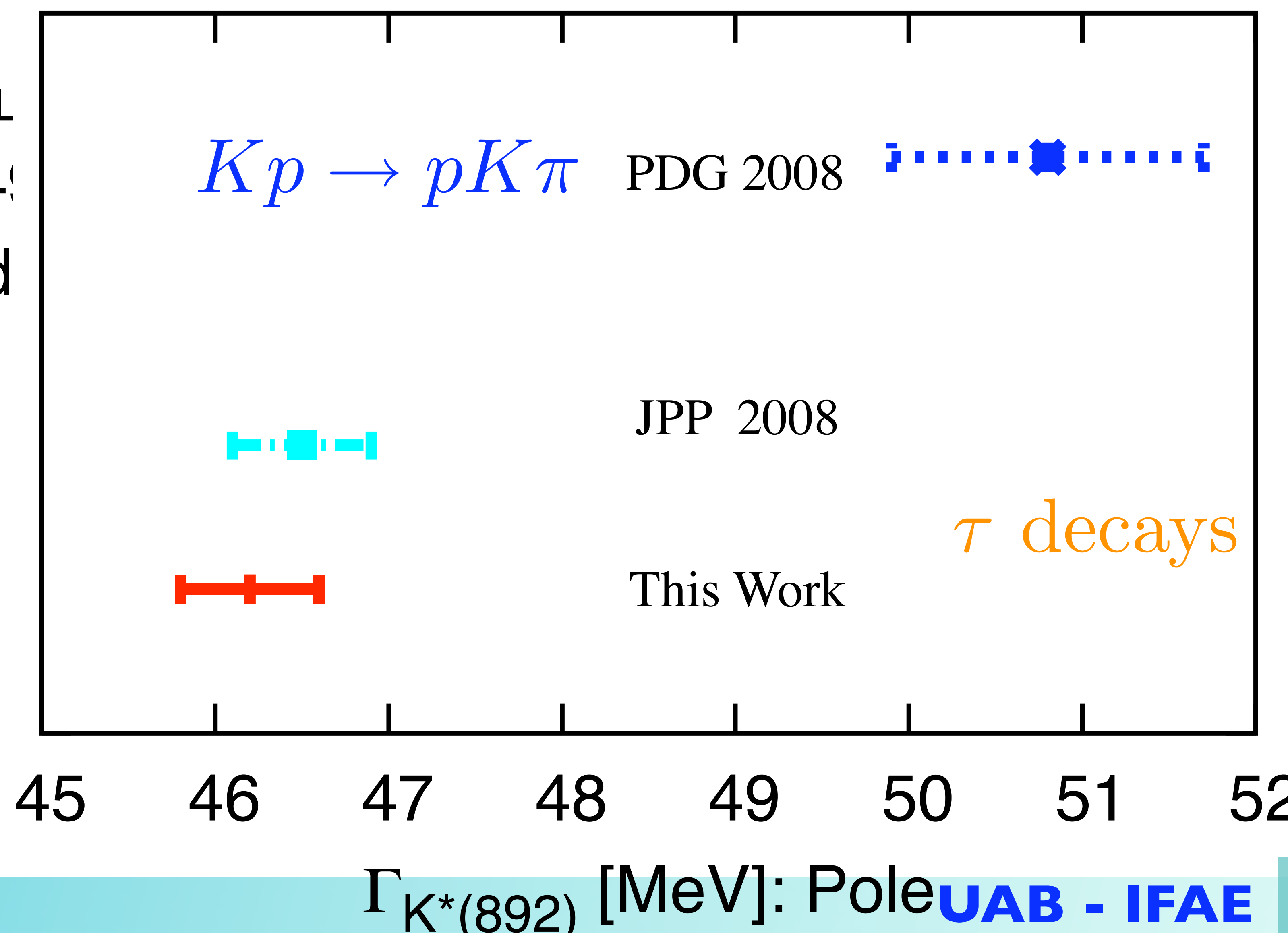

Conclusions

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Less model dependent.

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DRB, R. Escribano and M. Jamin, arXiv:0807.4883