α_s from tau decay: evidence for duality violations(?)

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ongoing work done in collaboration with

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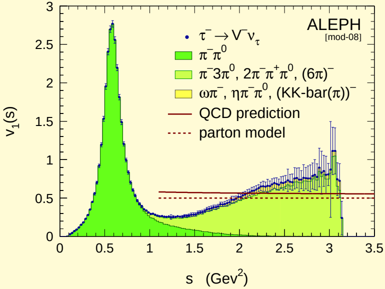
Motivation

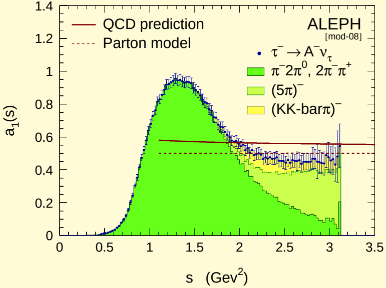
Consider the vector correlator

$$\Pi_{AB}(q) = \int d^4x \; e^{iq\cdot x} < 0|T\{\mathcal{O}_A(x)\mathcal{O}_B(0)\}|0 >$$

$$\mathcal{O}_A(x) = [\bar{u}\gamma_\mu d](x), \qquad \mathcal{O}_A(x) = [\bar{u}\gamma_\mu\gamma_5 d](x)$$

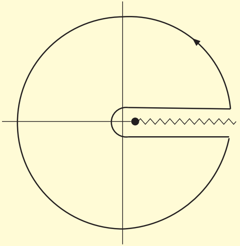
whose spectral function is known experimentally





$$R_\tau = \frac{\Gamma(\tau \to \text{hadrons})}{\Gamma(\tau \to \nu_\tau e \bar{\nu}_e)}$$

$$= 12\pi S_{EW} |V_{ud}|^2 \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \mathrm{Im}\,\Pi^{(1)}(s) + \mathrm{Im}\,\Pi^{(0)}(s)\right]$$



$$\int_0^{m_\tau^2} ds W(s) \frac{1}{\pi} \mathrm{Im}\,\Pi(s) = -\frac{1}{2\pi i} \oint_{|q^2|=m_\tau^2} dq^2 W(q^2) \Pi(q^2)$$

$$= -\frac{1}{2\pi i} \oint_{|q^2|=m_\tau^2} dq^2 W(q^2) \left\{\Pi(q^2)^{OPE} + \mathcal{D}(q^2)\right\}$$

- Issue already discussed in the past [Bigi, Uraltsev, Shifman, Blok, ...] in different contexts.
- However, our approach is the first phenomenological analysis (deals with experimental data).
- α_s from tau decay one of the most precise determinations, extracted from

$$R_{\tau,V/A} = \frac{N_c}{2} S_{EW} |V_{ud}|^2 \left[1 + \delta^{(0)} + \delta'_{EW} + \sum_{D\geq 2} \delta^{(D)}_{ud,V/A} + \Delta^{DV}_{V/A} \right]$$

so far neglecting $\Delta^{DV}_{V/A}$...

- What do we know about different contributions?

$$\begin{aligned} \delta^{(0)} &\longrightarrow \text{systematic } (\alpha_s) \\ \delta^{(D)}_{ud,V/A} &\longrightarrow \text{systematic } \left(\frac{1}{Q^2}\right) \\ \Delta^{DV}_{V/A} &\longrightarrow \text{non} - \text{systematic } (?) \end{aligned}$$

Standard analysis

Selection of moments like R_τ with different weights

$$W(s)_{k,l} = \left(1 + 2\frac{s}{m_\tau^2}\right)\left(1 - \frac{s}{m_\tau^2}\right)^{2+k}\left(\frac{s}{m_\tau^2}\right)^{l}$$

to suppress (even further) duality violations.

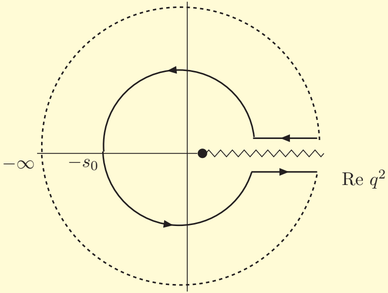
- $$\begin{aligned}\alpha_s(m_\tau^2) &= 0.344 \pm 0.005_{\mathrm{exp}} \pm 0.007_{\mathrm{th}} \quad (\mathrm{Davier\ et\ al.\ \ arXiv\ :\ 0803.0979}) \\ &= 0.316 \pm 0.003_{\mathrm{exp}} \pm 0.005_{\mathrm{th}} \quad (\mathrm{Beneke\&Jamin,\ arXiv\ :\ 0806.3156}) \\ &= 0.321 \pm 0.005_{\mathrm{exp}} \pm 0.012_{\mathrm{th}} \quad (\mathrm{Maltman\&Yavin,\ arXiv\ :\ 0807.0650})\end{aligned}$$

- $$\begin{aligned}< \alpha_s G_{\mu\nu} G^{\mu\nu} >_V &= (-0.8 \pm 0.4) \cdot 10^{-2}\,\mathrm{GeV}^4 \quad (\mathrm{Davier\ et\ al.\ '08}) \\ < \alpha_s G_{\mu\nu} G^{\mu\nu} >_A &= (-2.2 \pm 0.4) \cdot 10^{-2}\,\mathrm{GeV}^4\end{aligned}$$

Not fully under control ...

Duality violations

- Not systematic $\Rightarrow$ a certain degree of arbitrariness.
- Educated assumptions:
  - *(a)* Suppressed for large values of $|s|$.
  - *(b)* Only substantial on the Minkowski axis.



Then one can show that [O.C., Golterman, Peris'05]

$$\Delta R_\tau = -\frac{12\pi^2}{m_\tau^2} S_{EW} |V_{ud}|^2 \int_{m_\tau^2}^{\infty} ds\, W(s)\, \frac{1}{\pi} \mathrm{Im}\, \Delta_V(s)$$

- An educated ansatz:

$$\frac{1}{\pi}\mathrm{Im}\,\Delta_V(s) = \kappa e^{-\gamma s}\sin(\alpha+\beta s)$$

  (a) If the OPE is asymptotic, one should expect exponential damping [conservative].

  (b) Oscillatory behaviour suggested by pattern of resonances in the spectrum.

  (c) Realized in the asymptotic regime of a toy model [Blok et al.'98] of large-N_c inspiration, with finite widths, Regge behaviour, analyticity,...

- Strategy: fit

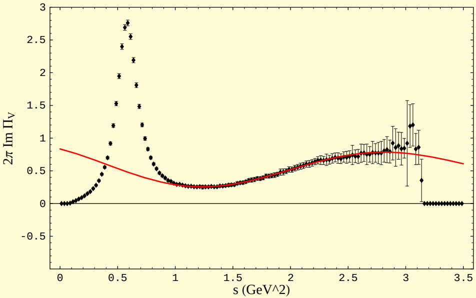
$$\frac{1}{\pi}\mathrm{Im}\Pi_V(s) = \frac{N_c}{12\pi^2}\left[1+\hat{\rho}(s)\right] + \kappa e^{-\gamma s}\sin(\alpha+\beta s)$$

  to experimental data and extract values for $\kappa, \gamma, \alpha, \beta$.

  (a) Assumption 1: The formula is valid as low as $s \simeq 1$ GeV^2.

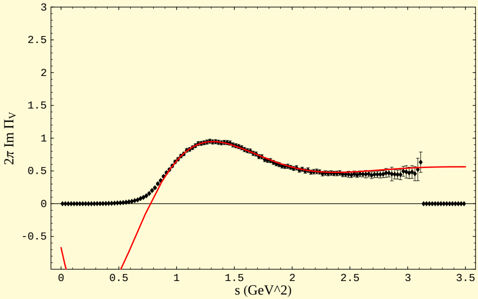
  (b) Assumption 2: The result can be extrapolated to arbitrary large s, i.e., some regularity pattern of the spectrum is assumed beyond the tau mass.

Vector channel



$$
\begin{aligned}
\kappa &= (0.018 \pm 0.004)\,\mathrm{GeV}^2 \\
\gamma &= (0.15 \pm 0.15)\,\mathrm{GeV}^{-2} \\
\alpha &= 2.19 \pm 0.26 \\
\beta &= (1.97 \pm 0.13)\,\mathrm{GeV}^{-2} \\
\chi^2/dof &= 0.13
\end{aligned}
$$

Axial channel



$$
\begin{aligned}
\kappa &= (0.20 \pm 0.06)\,\mathrm{GeV}^2 \\
\gamma &= (1.7 \pm 0.2)\,\mathrm{GeV}^{-2} \\
\alpha &= -0.27 \pm 0.12 \\
\beta &= (-2.98 \pm 0.08)\,\mathrm{GeV}^{-2} \\
\chi^2/dof &= 0.22
\end{aligned}
$$

Impact on α_s

- Use

$$R_{\tau,V/A} = \frac{N_c}{2} S_{EW} |V_{ud}|^2 \left[1 + \delta^{(0)} + \cdots \right] + \Delta R_{\tau,V/A}$$

with duality violations given by

$$\Delta R_{\tau,V/A} = -\frac{12\pi^2}{m_\tau^2} S_{EW} |V_{ud}|^2 \int_{m_\tau^2}^{\infty} ds\, W(s)\, \frac{1}{\pi} \mathrm{Im}\, \Delta_{V/A}(s)$$

and compare it with phenomenological numbers for R_τ.

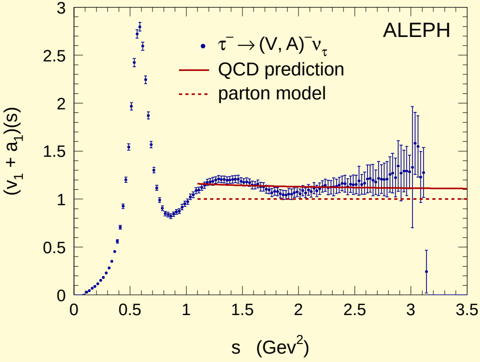
- Results:

$$\Delta R_{\tau,V} \simeq -0.022 \pm 0.014 (-1.5\%); \qquad \Delta R_{\tau,A} \simeq (3 \pm 2) \cdot 10^{-4} (0.02\%)$$

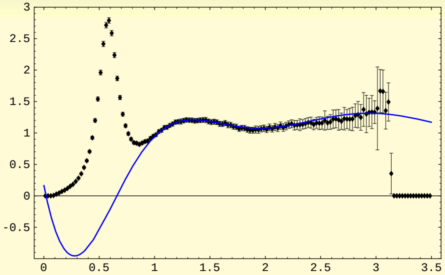
$$\delta\alpha_s(m_\tau^2)|_{th} \simeq 0.01$$

Comment on V+A

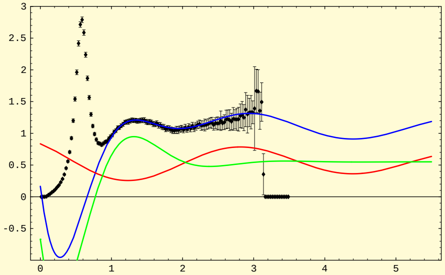
There are some claims that V+A may be insensitive to duality violations due to cancellations in the V and A channels.



Indeed we see such cancellation...



but only below the tau mass



Conclusions

- Experimental data does not rule out the impact of duality violations in α_s.
- We cannot claim a number for α_s, but $\delta\alpha_s(m_\tau^2)|_{th} \lesssim 0.01$ too optimistic.
- Contrary to common lore, V+A may not be protected against duality violations. Separate understanding of V and A is essential.
- Data on e^+e^- may be useful to extrapolate our ansatz beyond the tau mass. However, one should first understand better duality violations with the strange quark.
- *Nihilists* and *conservatists* may both be wrong: maybe the way out is to find optimal weights to kill duality violations.