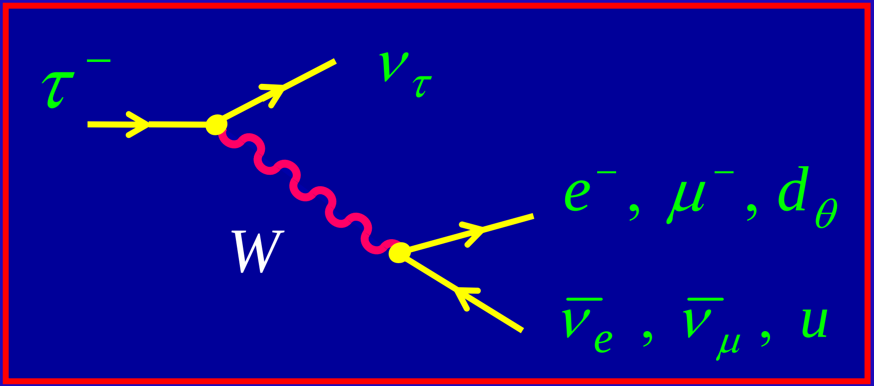
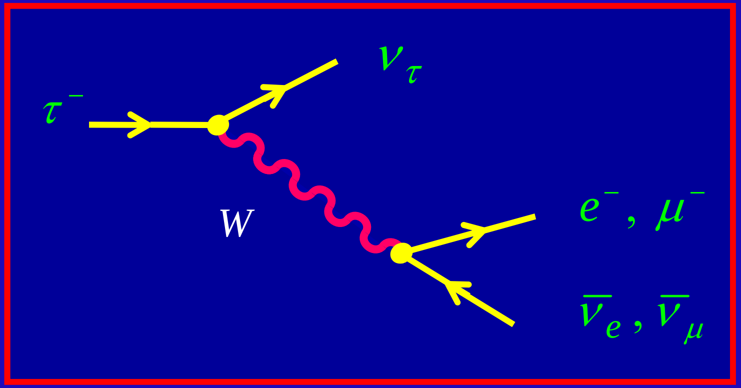
Recent Progress on Tau Lepton Physics



A. Pich

IFIC, Valencia

Euroflavour 08, IPPP, Durham 22-26 September 2008



$$\Gamma(\tau \to \nu_\tau\, l\, \bar{\nu}_l) = \frac{G_F^2\, m_\tau^5}{192\,\pi^3}\; f(m_l^2/m_\tau^2)\; r_{\text{EW}}$$

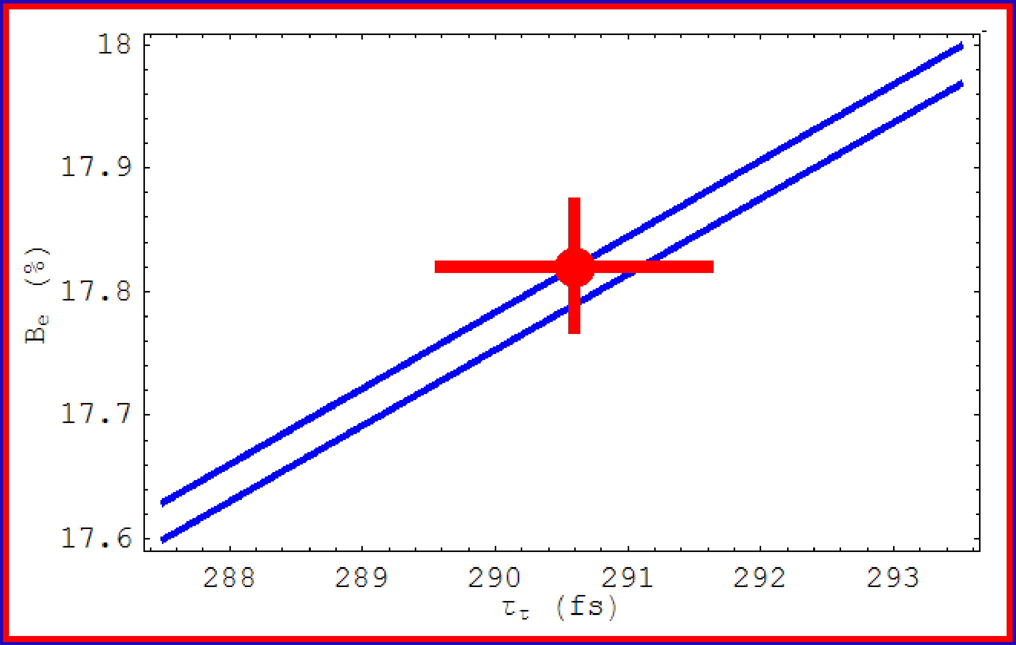
$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$$

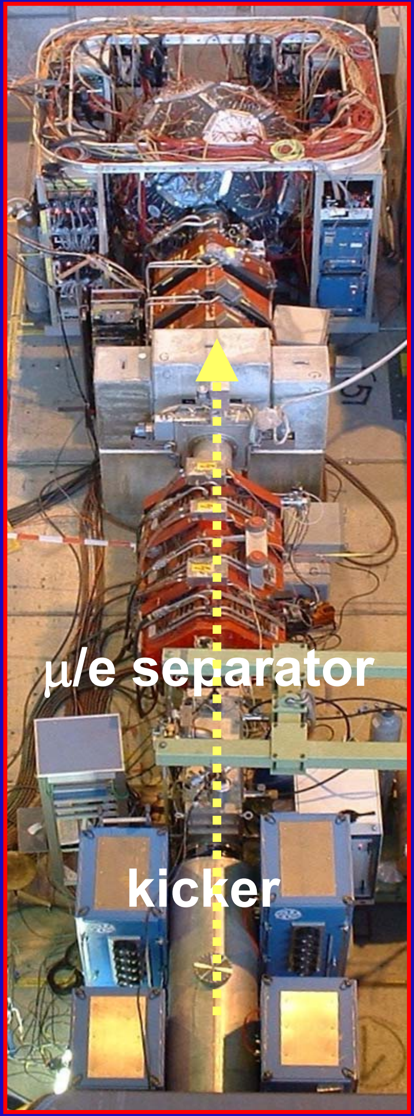
$$r_{\text{EW}} = 0.9960$$

(Marciano-Sirlin)

$$B_e = \frac{B_\mu}{0.972564 \pm 0.000010} = \frac{\tau_\tau}{(1632.1 \pm 1.4) \times 10^{-15}\,\text{s}}$$

$$\left(B_\mu / B_e\right)_{\text{exp}} = 0.9725 \pm 0.0039$$

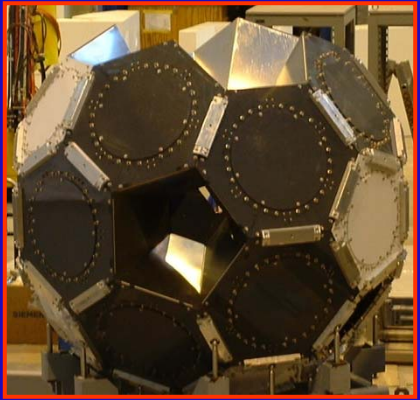


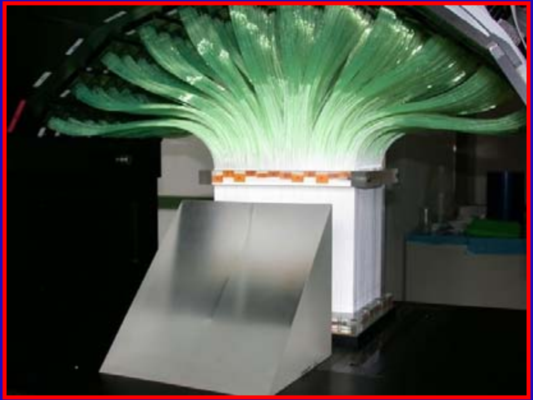


Muon Lifetime

$$\tau_\mu\,(\mu s) = \begin{cases} 2.19703 \pm 0.00004 & \text{PDG '06} \\ 2.197013 \pm 0.000024 & \text{MuLan '07} \\ 2.197083 \pm 0.000035 & \text{FAST '08} \end{cases}$$

MULAN





FAST

$$\frac{1}{\tau_\mu} = \frac{G_F^2\, m_\mu^5}{192\,\pi^3}\,(1+\delta_{\mathrm{QED}})$$

δ_{QED} known to 0.3 ppm

(van-Ritbergen & Stuart)

New World Average:

τ_μ = 2.197 034 (18) μs → **G_F = 1.166 367 (6) x 10^{-5} GeV^{-2}** **(5 ppm)**

τ_τ , Br($\tau \to \mu$) , Br($\tau \to$ e) $\sim$ 0.3% precision

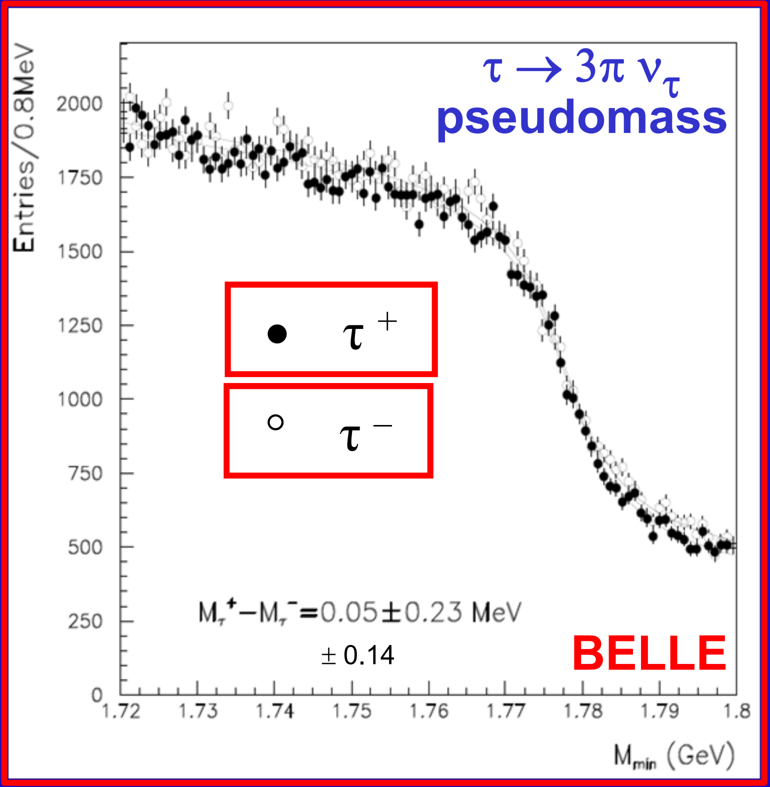
Future Improvements: BABAR , BELLE , KEDR , BESIII , SuperB , …

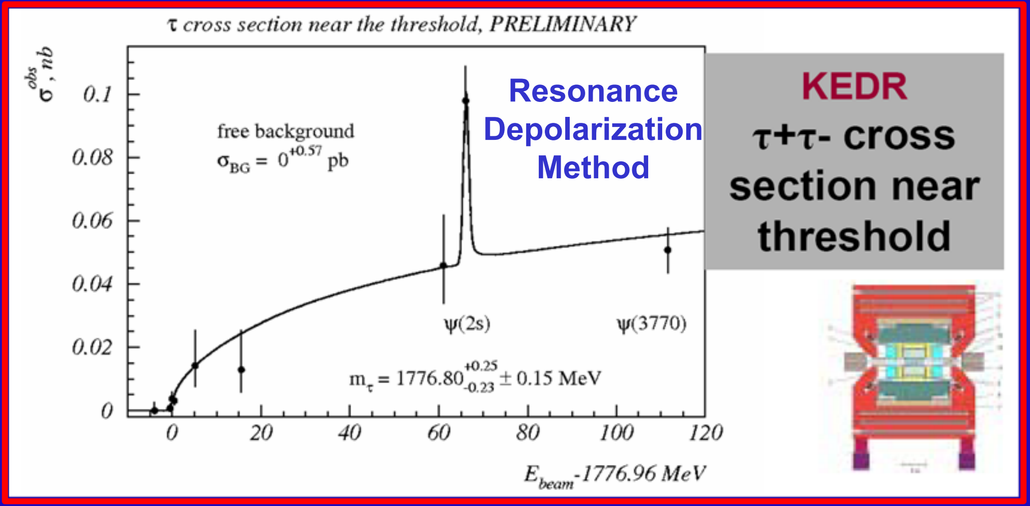
δm_τ ~ 0.023 MeV (12.7 ppm) BESIII

$$m_\tau = 1776.99\, {}^{+0.29}_{-0.26} \text{ MeV} \qquad \text{(PDG06)}$$

$$1776.61 \pm 0.13 \pm 0.35 \text{ MeV} \qquad \text{(BELLE)}$$

$$1776.81 \pm {}^{0.25}_{0.23} \pm 0.15 \text{ MeV} \qquad \text{(KEDR)}$$

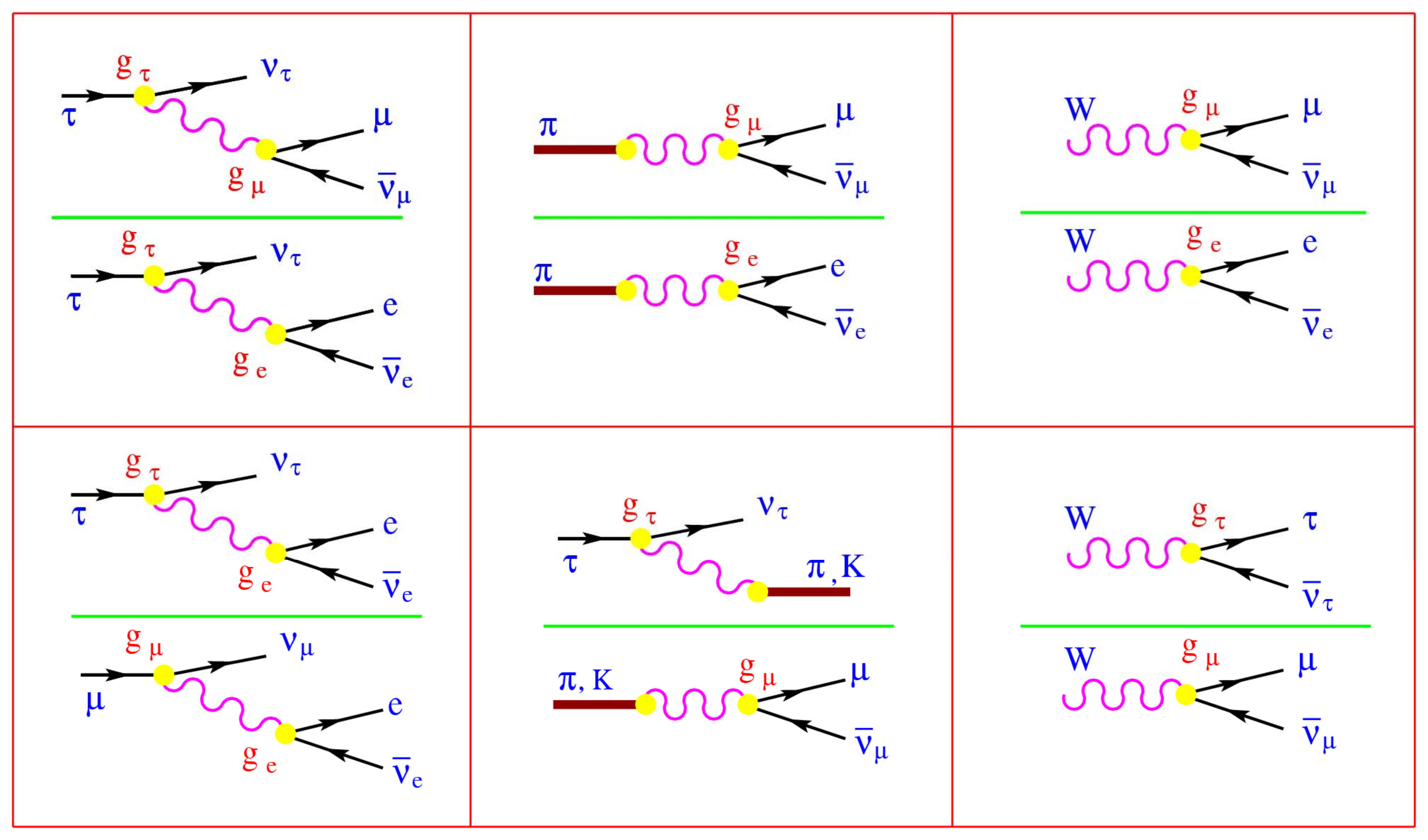




LEPTON UNIVERSALITY

$$\frac{g_\mu}{g_e}$$

$$\frac{g_\tau}{g_\mu}$$



CHARGED CURRENT UNIVERSALITY

$$\left| g_\mu / g_e \right|$$

$B_{\tau\to\mu} / B_{\tau\to e}$	1.0000 ± 0.0020
$B_{\pi\to\mu} / B_{\pi\to e}$	1.0021 ± 0.0016
$B_{K\to\mu} / B_{K\to e}$	1.004 ± 0.007
$B_{K\to\pi\mu} / B_{K\to\pi e}$	1.002 ± 0.002
$B_{W\to\mu} / B_{W\to e}$	0.997 ± 0.010

$$\left| g_\tau / g_\mu \right|$$

$B_{\tau\to e} \;\; \tau_\mu / \tau_\tau$	1.0006 ± 0.0022
$\Gamma_{\tau\to\pi} / \Gamma_{\pi\to\mu}$	0.996 ± 0.005
$\Gamma_{\tau\to K} / \Gamma_{K\to\mu}$	0.979 ± 0.017
$B_{W\to\tau} / B_{W\to\mu}$	1.039 ± 0.013

$$\left| g_\tau / g_e \right|$$

$B_{\tau\to\mu} \;\; \tau_\mu / \tau_\tau$	1.0005 ± 0.0023
$B_{W\to\tau} / B_{W\to e}$	1.036 ± 0.014

LEPTON FLAVOUR VIOLATION

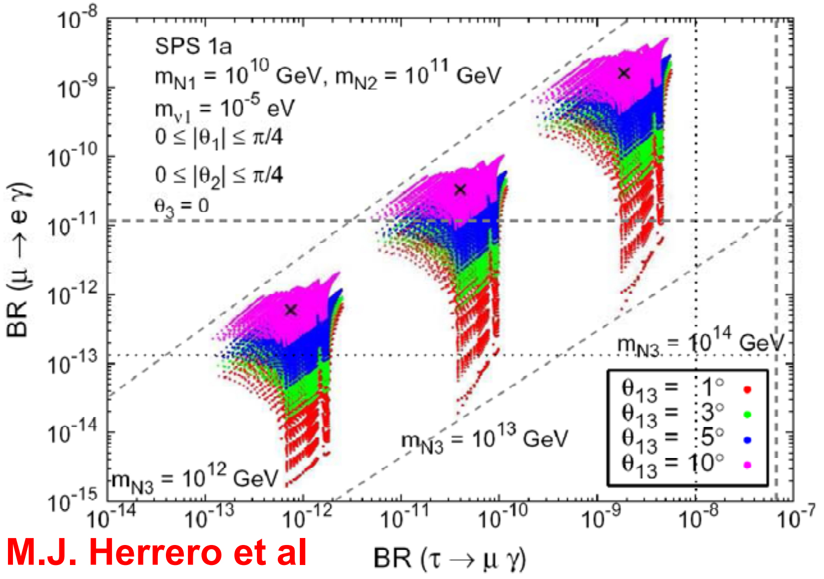
90 % CL Upper Limits on $Br(l^- \to X^-)$ [BABAR / BELLE]

Decay	U.L.	Decay	U.L.	Decay	U.L.
$\mu^- \to e^-\gamma$	$1.2 \cdot 10^{-11}$	$\mu^- \to e^-e^+e^-$	$1.0 \cdot 10^{-12}$	$\mu^- \to e^-\gamma\gamma$	$7.2 \cdot 10^{-11}$
$\tau^- \to e^-\gamma$	$1.1 \cdot 10^{-7}$	$\tau^- \to e^-e^+e^-$	$3.6 \cdot 10^{-8}$	$\tau^- \to e^-e^+\mu^-$	$2.7 \cdot 10^{-8}$
$\tau^- \to \mu^-\gamma$	$4.5 \cdot 10^{-8}$	$\tau^- \to e^-\mu^+\mu^-$	$3.7 \cdot 10^{-8}$	$\tau^- \to \mu^-e^+\mu^-$	$2.3 \cdot 10^{-8}$
$\tau^- \to e^-e^-\mu^+$	$2.0 \cdot 10^{-8}$	$\tau^- \to \mu^-\mu^+\mu^-$	$3.2 \cdot 10^{-8}$	$\tau^- \to e^-\pi^0$	$8.0 \cdot 10^{-8}$
$\tau^- \to \mu^-\pi^0$	$1.1 \cdot 10^{-7}$	$\tau^- \to e^-\eta'$	$1.6 \cdot 10^{-7}$	$\tau^- \to \mu^-\eta'$	$1.3 \cdot 10^{-7}$
$\tau^- \to e^-\eta$	$9.2 \cdot 10^{-8}$	$\tau^- \to \mu^-\eta$	$6.5 \cdot 10^{-8}$	$\tau^- \to e^-K^{*0}$	$7.8 \cdot 10^{-8}$
$\tau^- \to e^-K_S$	$5.6 \cdot 10^{-8}$	$\tau^- \to \mu^-K_S$	$4.9 \cdot 10^{-8}$	$\tau^- \to \mu^-\rho^0$	$6.8 \cdot 10^{-8}$
$\tau^- \to e^-K^+K^-$	$1.4 \cdot 10^{-7}$	$\tau^- \to e^-K^+\pi^-$	$1.6 \cdot 10^{-7}$	$\tau^- \to e^-\pi^+K^-$	$3.2 \cdot 10^{-7}$
$\tau^- \to \mu^-K^+K^-$	$2.5 \cdot 10^{-7}$	$\tau^- \to \mu^-K^+\pi^-$	$3.2 \cdot 10^{-7}$	$\tau^- \to \mu^-\pi^+K^-$	$2.6 \cdot 10^{-7}$
$\tau^- \to e^-\pi^+\pi^-$	$1.2 \cdot 10^{-7}$	$\tau^- \to \mu^-\pi^+\pi^-$	$2.9 \cdot 10^{-7}$	$\tau^- \to \Lambda\pi^-$	$7.2 \cdot 10^{-8}$
$\tau^- \to e^+K^-K^-$	$1.5 \cdot 10^{-7}$	$\tau^- \to e^+K^-\pi^-$	$1.8 \cdot 10^{-7}$	$\tau^- \to e^+\pi^-\pi^-$	$2.0 \cdot 10^{-7}$
$\tau^- \to \mu^-K^{*0}$	$5.9 \cdot 10^{-8}$	$\tau^- \to e^-\phi$	$7.3 \cdot 10^{-8}$	$\tau^- \to \mu^-\omega$	$8.9 \cdot 10^{-8}$
$\tau^- \to \mu^+K^-K^-$	$4.4 \cdot 10^{-7}$	$\tau^- \to \mu^+K^-\pi^-$	$2.2 \cdot 10^{-7}$	$\tau^- \to \mu^+\pi^-\pi^-$	$0.7 \cdot 10^{-7}$

Impact of θ_{13} on LFV processes

(All plotted points lead to 'viable BAU' and respect EDM bounds)

$(-\pi/4 \lesssim \arg\theta_1 \lesssim \pi/4,\ 0 \lesssim \arg\theta_2 \lesssim \pi/4)$

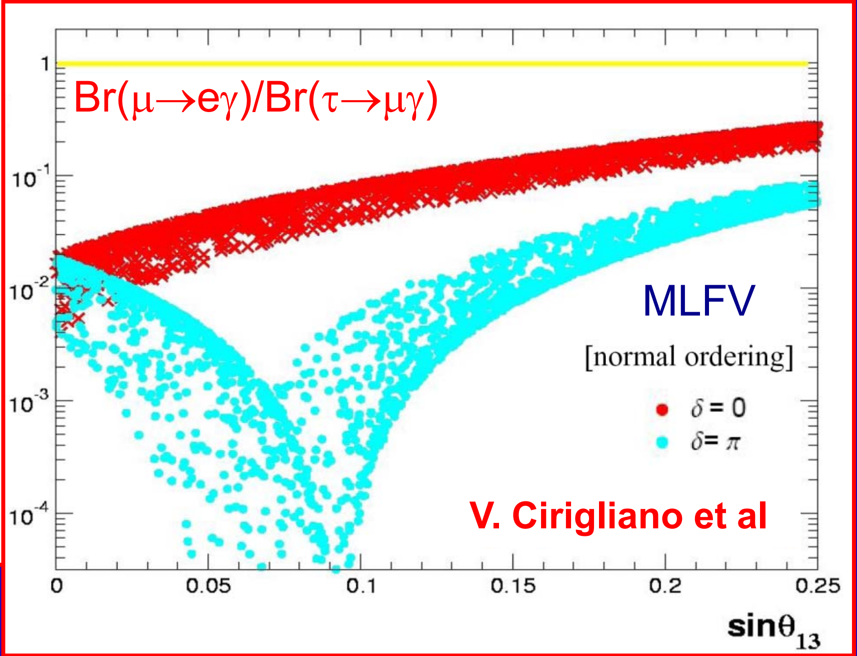


M.J. Herrero et al

MEG: $Br(\mu\to e\gamma) \sim 10^{-13}$

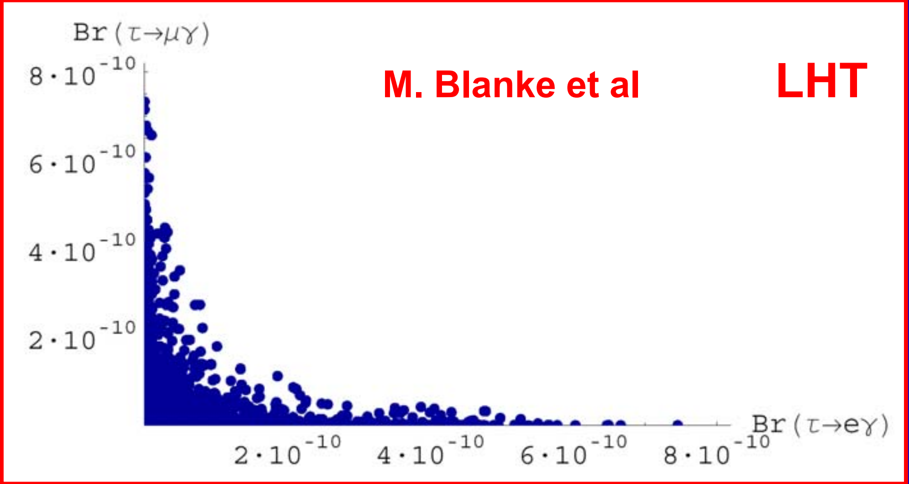
Prism: $Pr(\mu\to e) \sim 10^{-18}$

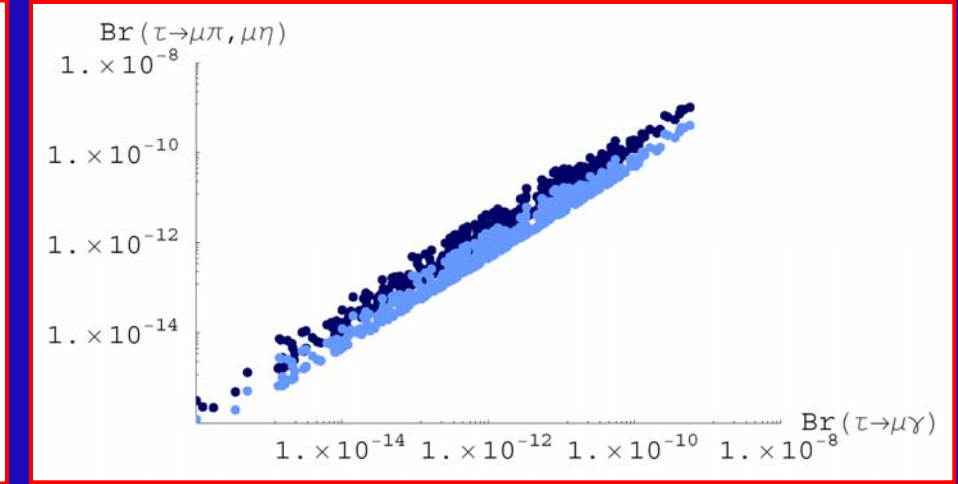
SuperB: $Br(\tau \to \mu\gamma) \sim 10^{-9}$



V. Cirigliano et al

M. Blanke et al **LHT**



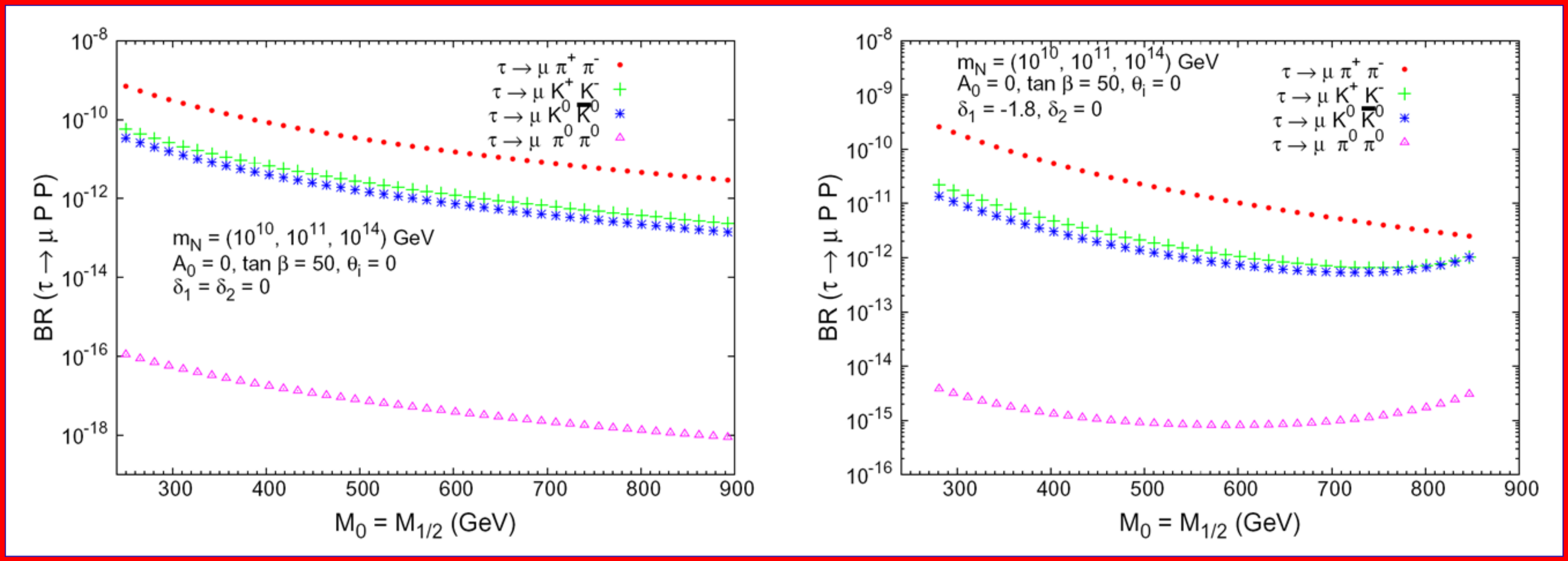


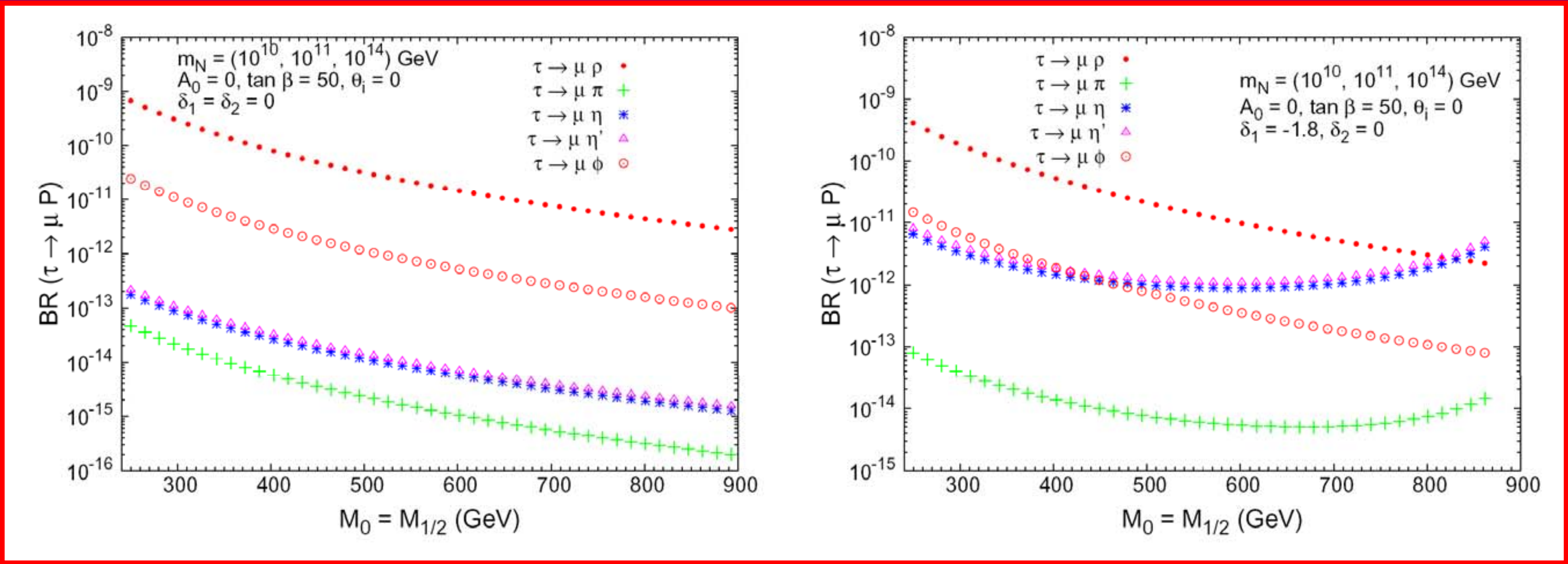
Arganda-Herrero-Portolés 08

CMSSM

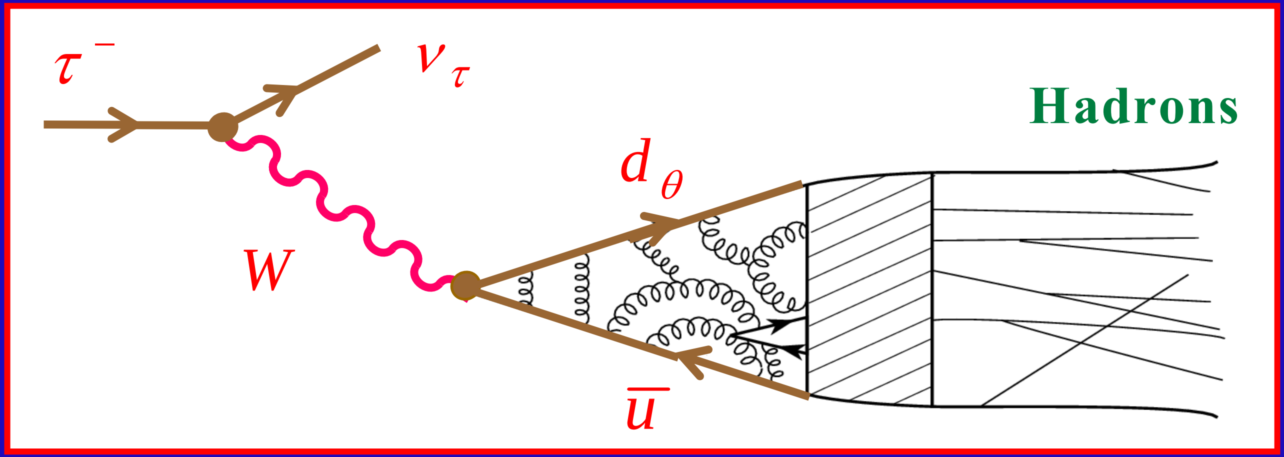
Constrained MSSM–Seesaw Scenarios

NUHM





HADRONIC TAU DECAY

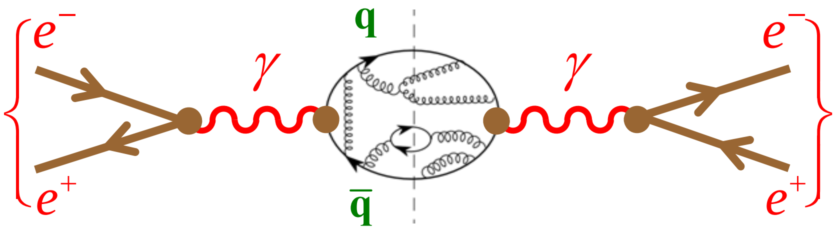


$$d_\theta = V_{ud}\; d + V_{us}\; s$$

Only lepton massive enough to decay into hadrons

$$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{Hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau\, e^-\, \bar{\nu}_e\right)} \approx N_C \qquad ; \qquad R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.639 \pm 0.011$$

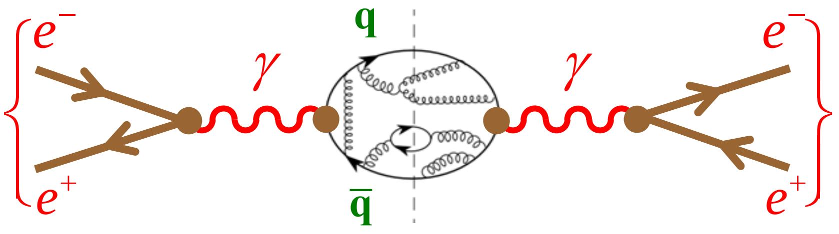
$\sigma \sim \mathrm{Im}$



$$\frac{\sigma(e^+e^- \to \mathrm{had})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 12\pi\, \mathrm{Im}\,\Pi_{\mathrm{em}}(s)$$

$$\Pi^{\mu\nu}_{\mathrm{em}}(q) \equiv i \int d^4x\; e^{iqx} \left\langle 0 \right| T[J^{\mu}_{\mathrm{em}}(x)\, J^{\nu}_{\mathrm{em}}(0)] \left| 0 \right\rangle = \left(-g^{\mu\nu} q^2 + q^{\mu} q^{\nu}\right) \Pi_{\mathrm{em}}(q^2)$$

$\sigma \sim \text{Im}$



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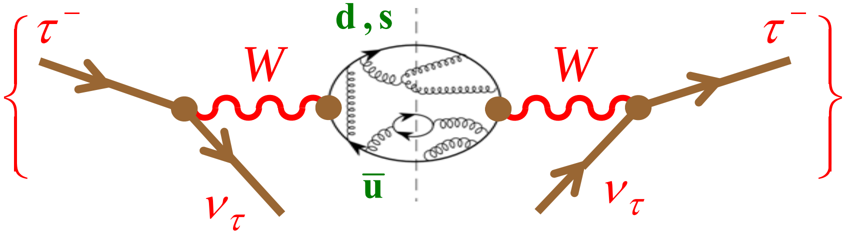
$\Gamma_{\tau \to \nu_\tau + \text{had}} \sim \text{Im}$



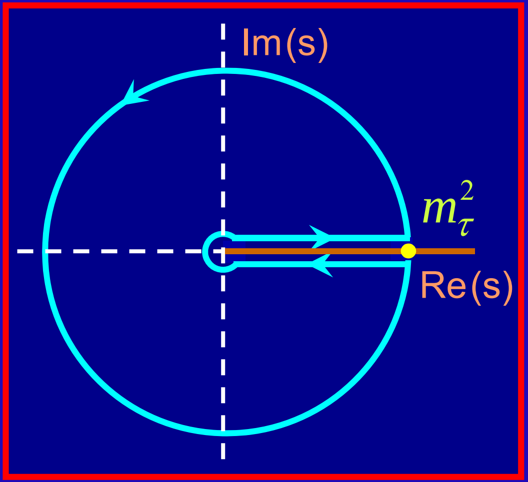
$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{had})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} \;=\; 12\pi \int_0^{m_\tau^2} dx \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \text{Im}\,\Pi^{(1)}(s) + \text{Im}\,\Pi^{(0)}(s)\right]$$

$$\Pi^{(J)}(s) \equiv |V_{ud}|^2 \left[\Pi^{(J)}_{ud,V}(s) + \Pi^{(J)}_{ud,A}(s)\right] \;+\; |V_{us}|^2 \left[\Pi^{(J)}_{us,V}(s) + \Pi^{(J)}_{us,A}(s)\right]$$

$$\Pi^{\mu\nu}_{ij,J}(q) \equiv i \int d^4x\; e^{iqx}\, \langle 0 | T[J^{\mu}_{ij}(x)\, J^{\nu}_{ij}(0)^\dagger] | 0 \rangle = \left(-g^{\mu\nu} q^2 + q^\mu q^\nu\right) \Pi^{(1)}_{ij,J}(q^2) + q^\mu q^\nu\; \Pi^{(0)}_{ij,J}(q^2)$$

Braaten-Narison-Pich

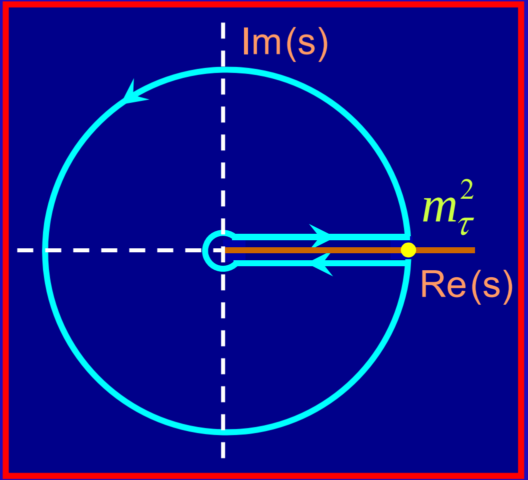
$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \mathrm{had})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = 12\pi \int_0^1 dx\,(1-x)^2 \left[(1+2x)\,\mathrm{Im}\,\Pi^{(1)}(x m_\tau^2) + \mathrm{Im}\,\Pi^{(0)}(x m_\tau^2)\right]$$



$$R_\tau = 6\pi i \oint_{|x|=1} dx\,(1-x)^2 \left[(1+2x)\,\Pi^{(0+1)}(x m_\tau^2) - 2x\,\Pi^{(0)}(x m_\tau^2)\right]$$

Braaten-Narison-Pich

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{had})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = 12\pi \int_0^1 dx\,(1-x)^2 \left[(1+2x)\,\text{Im}\,\Pi^{(1)}(x m_\tau^2) + \text{Im}\,\Pi^{(0)}(x m_\tau^2)\right]$$



$$\Big\downarrow$$

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$$\Pi^{(J)}(s) = \sum_{D=2n} \frac{C_D^{(J)}(s,\mu)\,\langle O_D(\mu)\rangle}{(-s)^{D/2}}$$

OPE

Braaten-Narison-Pich

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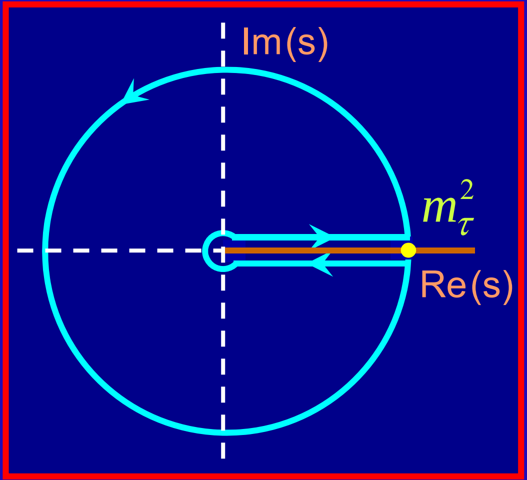

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OPE

$$R_\tau = N_C\, S_{\mathrm{EW}}\left(1+\delta_{\mathrm{P}}+\delta_{\mathrm{NP}}\right) = R_{\tau,V}+R_{\tau,A}+R_{\tau,S}$$

$S_{\mathrm{EW}} = 1.0201\ (3)$; $\delta_{\mathrm{NP}} = -0.0059 \pm 0.0014$

Marciano-Sirlin, Braaten-Li, Erler

Fitted from data (Davier et al)

$$\delta_{\mathrm{P}} = a_\tau + 5.20\ a_\tau^2 + 26\ a_\tau^3 + \ldots \approx\ 20\% \quad ; \quad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

Perturbative: $(m_q=0)$

$K_4 = 49.07570$ (Baikov-Chetyrkin-Kühn ‘08)

$$-s\,\frac{d}{ds}\,\Pi^{(0+1)}(s) = \frac{1}{4\pi^2}\sum_{n=0} K_n \left(\frac{\alpha_s(-s)}{\pi}\right)^n \quad ; \quad K_0 = K_1 = 1 \quad , \quad K_2 = 1.63982 \quad , \quad K_3 = 6.37101$$

$$\delta_{\mathrm{P}} = \sum_{n=1} K_n\, A^{(n)}(\alpha_s) = a_\tau + 5.20\; a_\tau^2 + 26\; a_\tau^3 + 127\; a_\tau^4 + \cdots$$

Le Diberder- Pich ‘92

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}\,(1-2x+2x^3-x^4)\left(\frac{\alpha_s(-s)}{\pi}\right)^n = a_\tau^n + \cdots \quad ; \quad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

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➡

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Le Diberder- Pich '92

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}\,(1-2x+2x^3-x^4)\left(\frac{\alpha_s(-s)}{\pi}\right)^n = a_\tau^n + \cdots \quad ; \quad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

Power Corrections:

Braaten-Narison-Pich

$$\Pi_{\mathrm{OPE}}^{(0+1)}(s) \approx \frac{1}{4\pi^2}\sum_{n\geq 2}\frac{C_{2n}\langle O_{2n}\rangle}{(-s)^n}$$

$$C_4\langle O_4\rangle \approx \frac{2\pi}{3}\langle 0|\alpha_s G^{\mu\nu}G_{\mu\nu}|0\rangle$$

$$\delta_{\mathrm{NP}} \approx \frac{-1}{2\pi i}\oint_{|x|=1} dx\;\; (1-3x^2+2x^3)\sum_{n\geq 2}\frac{C_{2n}\langle O_{2n}\rangle}{(-x m_\tau^2)^n} = -3\,\frac{C_6\langle O_6\rangle}{m_\tau^6} - 2\,\frac{C_8\langle O_8\rangle}{m_\tau^8}$$

Suppressed by m_τ^6 [additional chiral suppression in $C_6\langle O_6\rangle^{V+A}$]

Similar predictions for $R_{\tau,V}$, $R_{\tau,A}$, $R_{\tau,S}$ and the moments

$$R_\tau^{kl}(s_0) \equiv \int_0^{s_0} ds \left(1-\frac{s}{s_0}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{dR_\tau}{ds}$$

Different sensitivity to power corrections through k , l

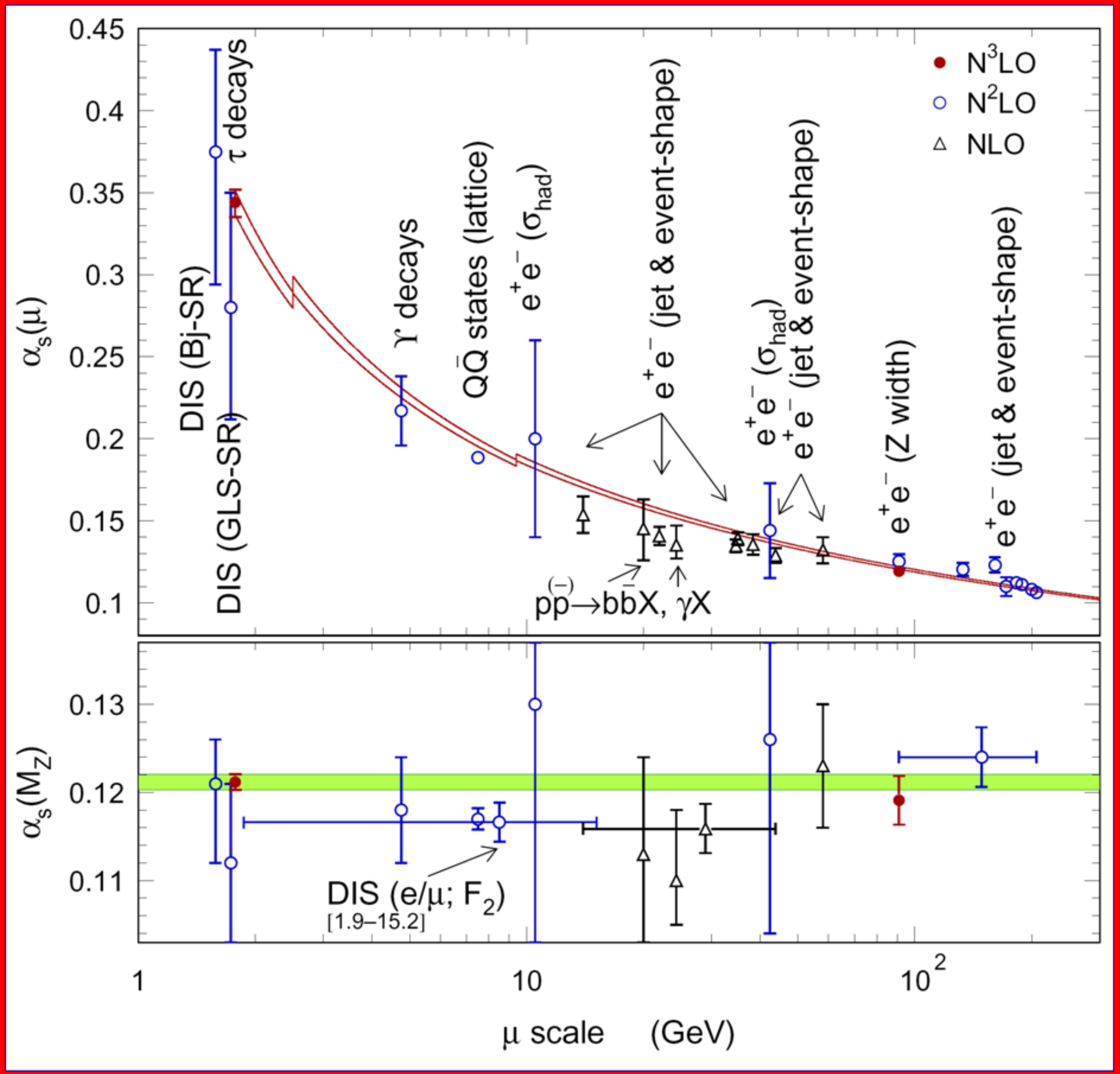
The non-perturbative contribution to R_τ can be obtained from the invariant-mass distribution of the final hadrons:

$$\delta_{\mathrm{NP}} = -0.0059 \pm 0.0014$$ Davier et al. (ALEPH data)

$$R_{\tau,V} = 1.783 \pm 0.011 \quad ; \quad R_{\tau,A} = 1.695 \pm 0.011 \quad ; \quad R_{\tau,V+A} = 3.478 \pm 0.010$$

Davier et al

ALEPH



$$\alpha_s(m_\tau^2) = 0.344 \pm 0.009$$

$$\alpha_s(M_Z^2) = 0.1212 \pm 0.0011$$

$$\alpha_s(M_Z^2)_{Z\,\mathrm{width}} = 0.1191 \pm 0.0027$$

The most precise test of Asymptotic Freedom

$$\alpha_s^\tau(M_Z^2) - \alpha_s^Z(M_Z^2) = 0.0021 \pm 0.0011_\tau \pm 0.0027_Z$$

Recent $\alpha_s(m_\tau)$ Analyses

- Baikov et al ‘08: (CIPT + FOPT) / 2

$$\alpha_s(m_\tau^2) = 0.332 \pm 0.005 \pm 0.015 \quad \Longrightarrow \quad \alpha_s(M_Z^2) = 0.1202 \pm 0.0019$$

- Davier et al ‘08: CIPT

$$\boxed{\alpha_s(m_\tau^2) = 0.344 \pm 0.005 \pm 0.007 \quad \Longrightarrow \quad \alpha_s(M_Z^2) = 0.1212 \pm 0.0011}$$

- Beneke-Jamin ‘08: FOPT + Borel sum of ad-hoc renormalon model

$$\alpha_s(m_\tau^2) = 0.320 \begin{array}{l} +0.012 \\ -0.007 \end{array} \quad \Longrightarrow \quad \alpha_s(M_Z^2) = 0.1180 \pm 0.0008$$

- Maltman-Yavin ‘08: CIPT (FOPT). Hadronic spectral moments only

$$\alpha_s(m_\tau^2) = 0.3209 \pm 0.0046 \pm 0.0118 \quad \Longrightarrow \quad \alpha_s(M_Z^2) = 0.1187 \pm 0.0016$$

Perturbative Uncertainty on $\alpha_s(m_\tau)$

$$-s\,\frac{d}{ds}\,\Pi^{(0+1)}(s) = \frac{1}{4\pi^2}\sum_{n=0} K_n \; a(-s)^n$$

$$\delta_P = \underbrace{\sum_{n=1} K_n \; A^{(n)}(\alpha_s)}_{\text{CIPT}} = \underbrace{\sum_{n=0} r_n \; a_\tau^n}_{\text{FOPT}}$$

$$r_n = K_n + g_n$$

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}\,(1-2x+2x^3-x^4)\,a(-s)^n = a_\tau^n + \cdots \qquad ; \qquad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

n	1	2	3	4
K_n	1	1.640	6.37	49.08
g_n	0	3.563	19.99	78.00
r_n	1	5.203	26.36	127.08

The dominant corrections come from the contour integration

Le Diberder- Pich 1992

Perturbative Uncertainty on $\alpha_s(m_\tau)$

$$-s\,\frac{d}{ds}\,\Pi^{(0+1)}(s) = \frac{1}{4\pi^2}\sum_{n=0} K_n\; a(-s)^n$$

$$\delta_P = \underbrace{\sum_{n=1} K_n\; A^{(n)}(\alpha_s)}_{\textbf{CIPT}} = \underbrace{\sum_{n=0} r_n\; a_\tau^n}_{\textbf{FOPT}}$$

$$r_n = K_n + g_n$$

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}\,(1-2x+2x^3-x^4)\,a(-s)^n = a_\tau^n + \cdots \qquad ; \qquad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

n	1	2	3	4
K_n	1	1.640	6.37	49.08
g_n	0	3.563	19.99	78.00
r_n	1	5.203	26.36	127.08

The dominant corrections come from the contour integration

Le Diberder- Pich 1992

$$a(-s) \simeq \frac{a_\tau}{1-\frac{\beta_1}{2}a_\tau\log\left(-s/m_\tau^2\right)} = \frac{a_\tau}{1-i\,\frac{\beta_1}{2}a_\tau\,\phi} = a_\tau\sum_n\left(i\,\frac{\beta_1}{2}a_\tau\,\phi\right)^n \qquad ; \qquad \phi\in[0,2\pi]$$

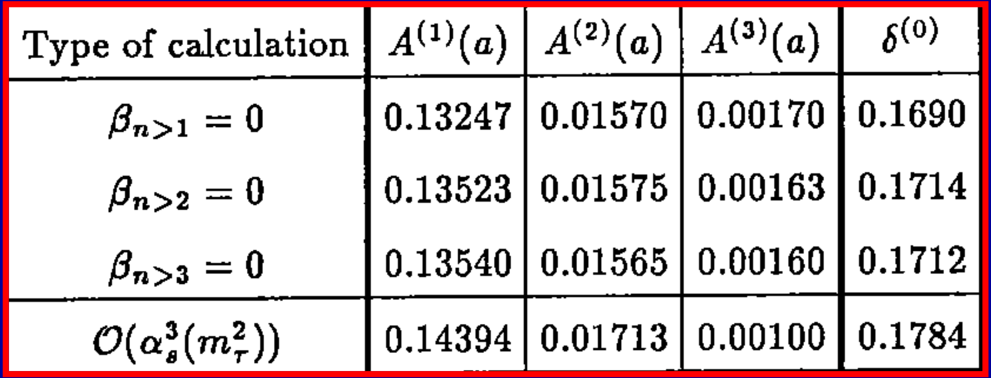
FOPT expansion only convergent if $a_\tau < 0.13$ (0.11) [at 1 (3) loops]

Experimentally $a_\tau \approx 0.11$ → **FOPT should not be used**
(divergent series)

The difference between FOPT and CIPT grows at higher orders

CIPT gives rise to a well-behaved perturbative series:

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}\,(1-2x+2x^3-x^4)\,a(-s)^n \;=\; a_\tau^n + \cdots$$



Type of calculation	$A^{(1)}(a)$	$A^{(2)}(a)$	$A^{(3)}(a)$	$\delta^{(0)}$
$\beta_{n>1}=0$	0.13247	0.01570	0.00170	0.1690
$\beta_{n>2}=0$	0.13523	0.01575	0.00163	0.1714
$\beta_{n>3}=0$	0.13540	0.01565	0.00160	0.1712
$\mathcal{O}(\alpha_s^3(m_\tau^2))$	0.14394	0.01713	0.00100	0.1784

Uncertainty only related to the unknown K_n ($n \geq 5$) coefficients

Modelling a better behaved FOPT

(Beneke – Jamin)

- **Tune large K_n corrections to cancel the g_n ones**

- D = 4 corrections very suppressed in R_τ

➡ **n = 2 IR renormalons can do the job** **($K_n \approx - g_n$)**

- No sign of renormalon behaviour in known coefficients

➡ **n = -1,2,3 renormalons + linear polynomial**

5 unknown constants fitted to K_n ($2 \leq n \leq 5$). Ad-hoc value of K_5

- Borel summation: large renormalon contributions. Smaller α_s

Nice academic model. But too many different possibilities …

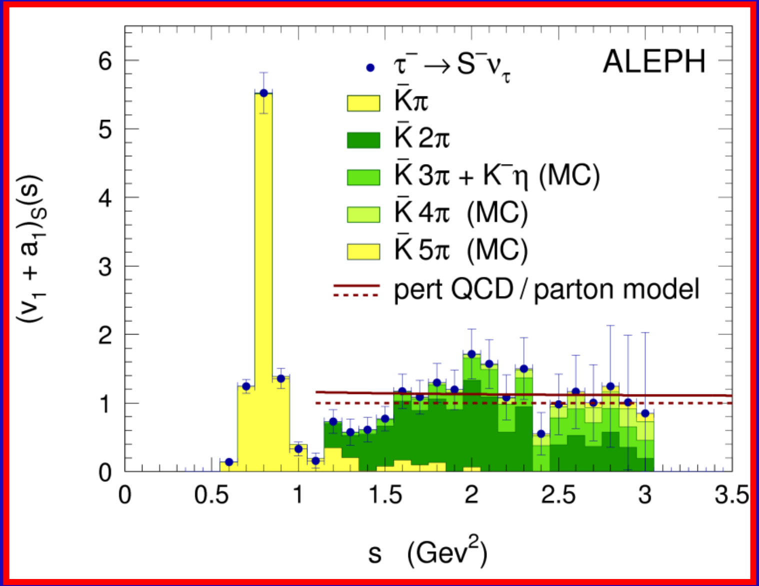
SU(3) Breaking

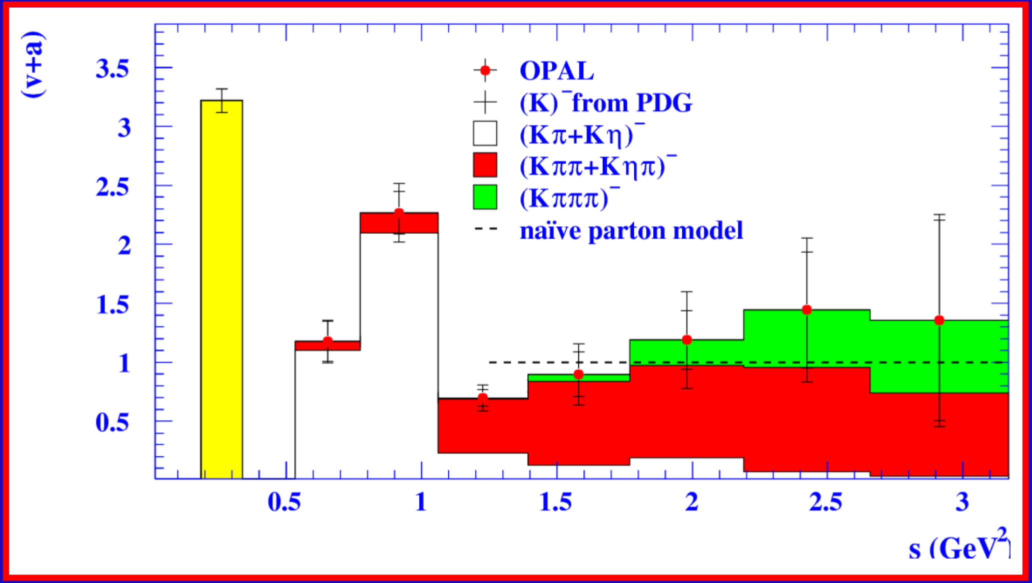
$$R_\tau^{kl} = N_C\; S_{\rm EW} \left\{ \left(|V_{ud}|^2 + |V_{us}|^2 \right) \left[1 + \delta^{kl(0)} \right] + \sum_{D\geq 2} \left[|V_{ud}|^2\, \delta_{ud}^{kl(D)} + |V_{us}|^2\, \delta_{us}^{kl(D)} \right] \right\}$$

$$\Downarrow$$

$$\delta R_\tau^{kl} \equiv \frac{R^{kl}_{\tau,V+A}}{|V_{ud}|^2} - \frac{R^{kl}_{\tau,S}}{|V_{us}|^2} \approx N_C\; S_{\rm EW} \sum_{D\geq 2} \left[\delta_{ud}^{kl(D)} - \delta_{us}^{kl(D)} \right]$$

Strange Spectral Function: SU(3) Breaking





(k,l)	ALEPH	OPAL
(0,0)	0.39 ± 0.14	0.26 ± 0.12
(1,0)	0.38 ± 0.08	0.28 ± 0.09
(2,0)	0.37 ± 0.05	0.30 ± 0.07
(3,0)	0.40 ± 0.04	0.33 ± 0.05
(4,0)	0.40 ± 0.04	0.34 ± 0.04

$$\delta R_\tau^{kl} \equiv \frac{R^{kl}_{\tau,V+A}}{|V_{ud}|^2} - \frac{R^{kl}_{\tau,S}}{|V_{us}|^2} \approx 24\,\frac{m_s^2(m_\tau^2)}{m_\tau^2}\,\Delta_{kl}(\alpha_s)$$

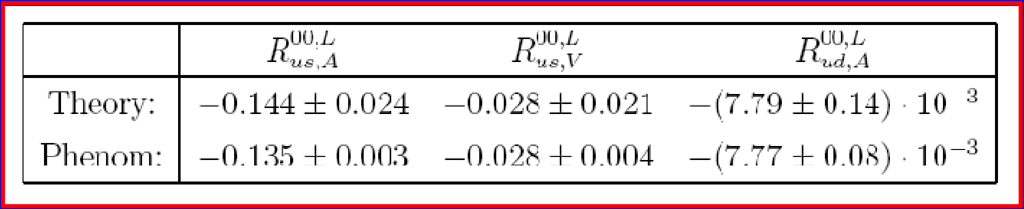
➡ $m_s(m_\tau)$ **determination**

V_{us} and QCD uncertainties

$$\boxed{\delta R_\tau^{kl} \equiv \frac{R_{\tau,V+A}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2} \approx 24\, \frac{m_s^2(m_\tau^2)}{m_\tau^2}\, \Delta_{kl}(\alpha_s)}$$

Known to $O(\alpha_s^3)$

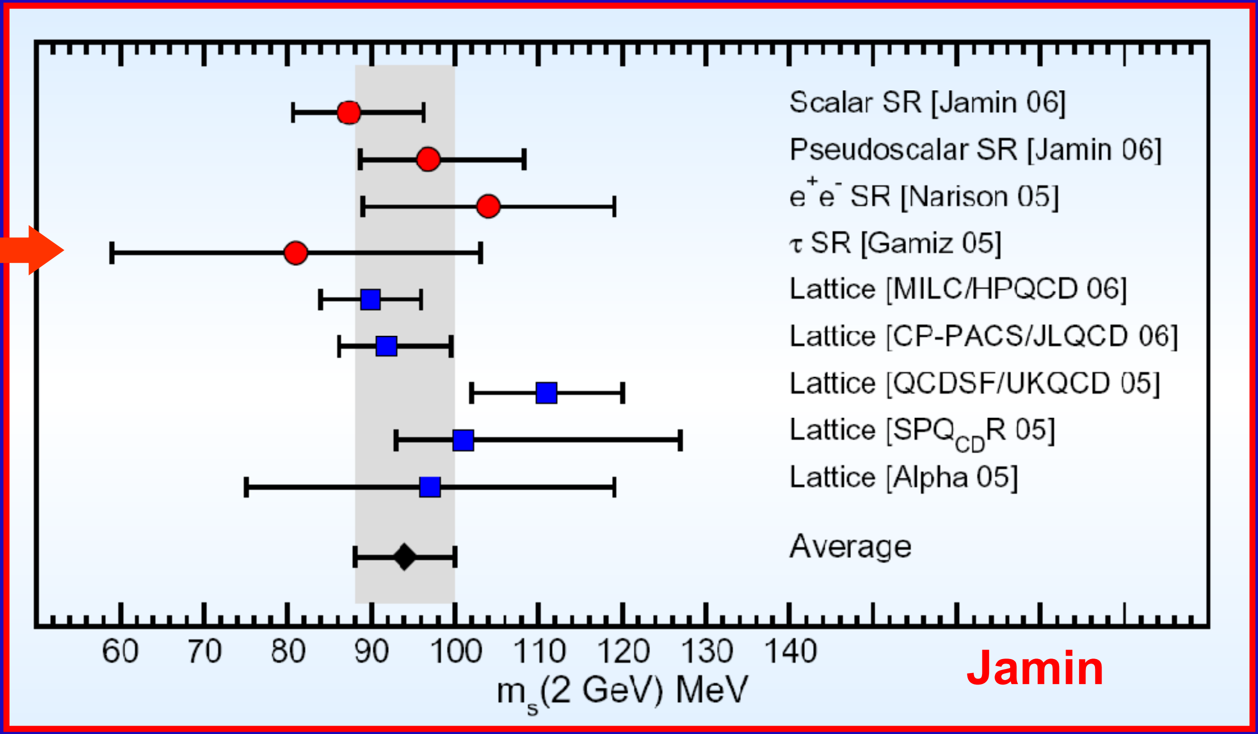
- $\Delta_{kl}(\alpha_s)$ gets **longitudinal (J=0)** and **transverse (J=0+1)** contributions
- Divergent QCD series for J=0
- **Longitudinal contribution determined through data:**
  - Kaon pole (K→μν) (dominant J=0 contribution)
  - Pion pole (π→μν)
  - $(K\pi)_{J=0}$ (S-wave Kπ scattering)
  - …
- Smaller uncertainties



	$R_{us,A}^{00,L}$	$R_{us,V}^{00,L}$	$R_{ud,A}^{00,L}$
Theory:	-0.144 ± 0.024	-0.028 ± 0.021	$-(7.79 \pm 0.14) \cdot 10^{\;3}$
Phenom:	-0.135 ± 0.003	-0.028 ± 0.004	$-(7.77 \pm 0.08) \cdot 10^{-3}$

$$\delta R_{\tau,\mathrm{th}}^{00} \equiv \underbrace{0.1544\,(37)}_{J=0} + \underbrace{0.062\,(15)}_{m_s(m_\tau)\,=\,0.100\,(10)} = 0.216\,(16)$$

OPAL τ data



Jamin

Large uncertainty from V_{us}

Strong sensitivity to V_{us}

$$|V_{us}|^2 = \frac{R_{\tau,S}^{00}}{\frac{R_{\tau,V+A}^{00}}{|V_{ud}|^2} - \delta R_{\tau,\mathrm{th}}^{00}}$$

Gámiz-Jamin-Pich-Prades-Schwab

τ data: $R_{\tau,S}^{00} = 0.1686\ (47)$

$R_{\tau,V+A}^{00} = 3.471\ (11)$

PDG 06: $|V_{ud}| = 0.97377\ (27)$

$$|V_{us}|^2 = \frac{R^{00}_{\tau,S}}{\frac{R^{00}_{\tau,V+A}}{|V_{ud}|^2} - \delta R^{00}_{\tau,\mathrm{th}}}$$

Gámiz-Jamin-Pich-Prades-Schwab

τ data: $R^{00}_{\tau,S} = 0.1686\ (47)$

$R^{00}_{\tau,V+A} = 3.471\ (11)$

PDG 06: $|V_{ud}| = 0.97377\ (27)$

$\delta R^{00}_{\tau,\mathrm{th}} = 0$ ➡ $|V_{us}| = 0.215\ (3)$

$$\boxed{|V_{us}|^2 = \frac{R_{\tau,S}^{00}}{\frac{R_{\tau,V+A}^{00}}{|V_{ud}|^2} - \delta R_{\tau,\text{th}}^{00}}}$$

Gámiz-Jamin-Pich-Prades-Schwab

τ data: $R_{\tau,S}^{00} = 0.1686\ (47)$

$R_{\tau,V+A}^{00} = 3.471\ (11)$

PDG 06: $|V_{ud}| = 0.97377\ (27)$

$\delta R_{\tau,\text{th}}^{00} = 0$ → $|V_{us}| = 0.215\ (3)$

Taking as input (from non τ sources) $m_s(m_\tau) = 100 \pm 10\ \text{MeV}$:

$\delta R_{\tau,\text{th}}^{00} = 0.216\ (16)$ → $\boxed{|V_{us}| = 0.2212 \pm 0.0031_{\text{exp}} \pm 0.0005_{\text{th}}}$

$$|V_{us}|^2 = \frac{R^{00}_{\tau,S}}{\frac{R^{00}_{\tau,V+A}}{|V_{ud}|^2} - \delta R^{00}_{\tau,\mathrm{th}}}$$

Gámiz-Jamin-Pich-Prades-Schwab

τ **data:** $R^{00}_{\tau,S} = 0.1686\ (47)$

$R^{00}_{\tau,V+A} = 3.471\ (11)$

PDG 06: $|V_{ud}| = 0.97377\ (27)$

$\delta R^{00}_{\tau,\mathrm{th}} = 0$ ➡ $|V_{us}| = 0.215\ (3)$

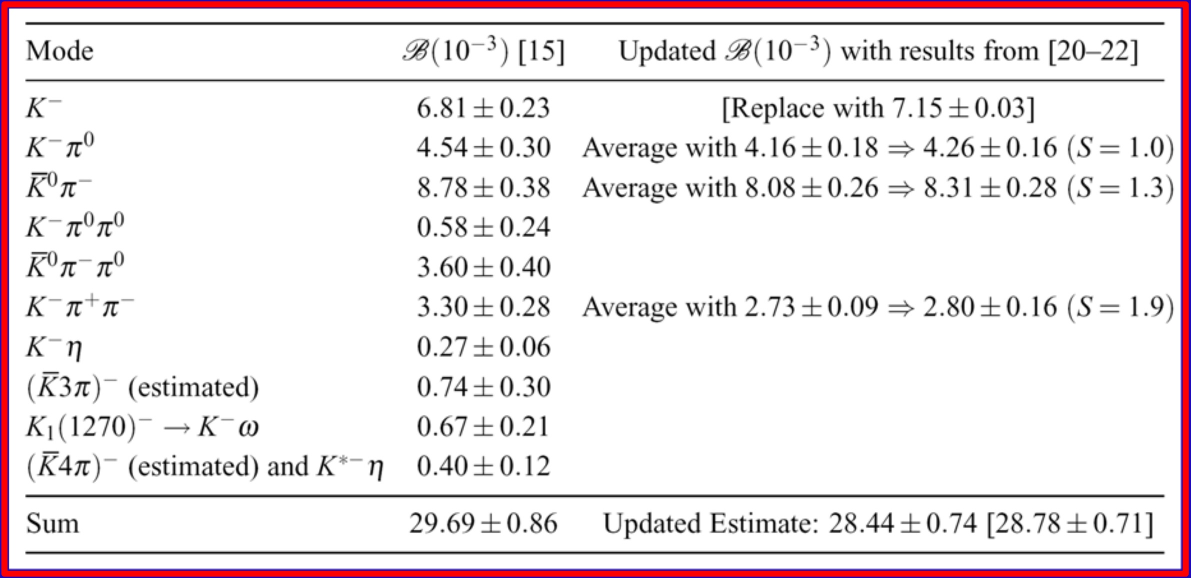
Taking as input (from non τ sources) $m_s(m_\tau) = 100 \pm 10\ \ \mathrm{MeV}$:

$\delta R^{00}_{\tau,\mathrm{th}} = 0.216\ (16)$ ➡ $\boxed{|V_{us}| = 0.2212 \pm 0.0031_{\mathrm{exp}} \pm 0.0005_{\mathrm{th}}}$

K_{l3}: $|V_{us}| = 0.2233 \pm 0.0024$ $[\, f_+(0) = 0.97 \pm 0.01 \,]$

The τ could give the most precise V_{us} determination

First Measurements from Babar and Belle



Mode	$\mathcal{B}(10^{-3})$ [15]	Updated $\mathcal{B}(10^{-3})$ with results from [20–22]
K^-	6.81 ± 0.23	[Replace with 7.15 ± 0.03]
$K^-\pi^0$	4.54 ± 0.30	Average with $4.16 \pm 0.18 \Rightarrow 4.26 \pm 0.16$ ($S = 1.0$)
$\bar{K}^0\pi^-$	8.78 ± 0.38	Average with $8.08 \pm 0.26 \Rightarrow 8.31 \pm 0.28$ ($S = 1.3$)
$K^-\pi^0\pi^0$	0.58 ± 0.24	
$\bar{K}^0\pi^-\pi^0$	3.60 ± 0.40	
$K^-\pi^+\pi^-$	3.30 ± 0.28	Average with $2.73 \pm 0.09 \Rightarrow 2.80 \pm 0.16$ ($S = 1.9$)
$K^-\eta$	0.27 ± 0.06	
$(\bar{K}3\pi)^-$ (estimated)	0.74 ± 0.30	
$K_1(1270)^- \to K^-\omega$	0.67 ± 0.21	
$(\bar{K}4\pi)^-$ (estimated) and $K^{*-}\eta$	0.40 ± 0.12	
Sum	29.69 ± 0.86	Updated Estimate: 28.44 ± 0.74 [28.78 ± 0.71]

S. Banerjee

KAON'07

Smaller τ→K branching ratios ➡ smaller $R_{\tau,S}$ ➡ smaller V_{us}

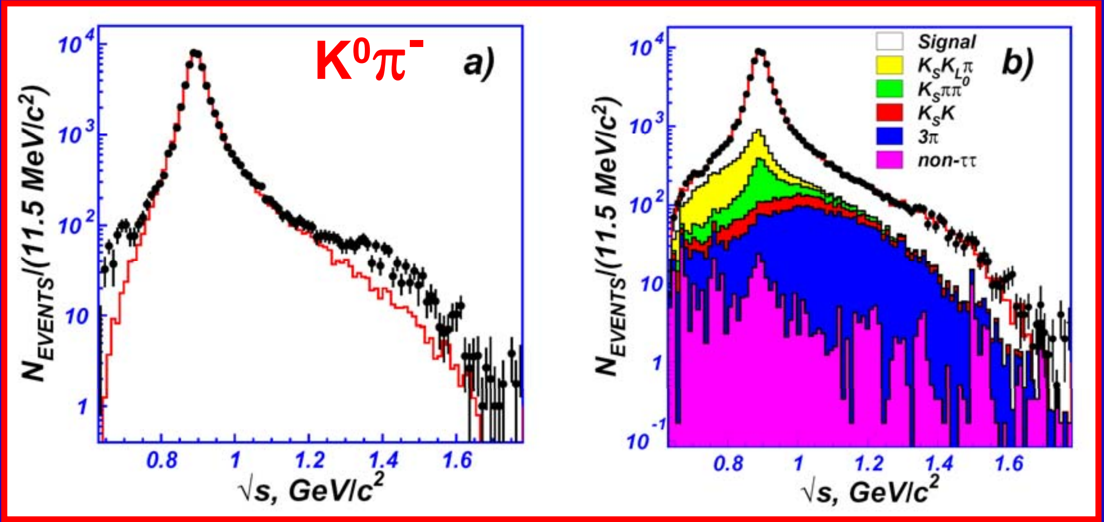
$$R^{00}_{\tau,S}\Big|_{\mathrm{OLD}} = 0.1686\ (47) \quad \to \quad R^{00}_{\tau,S}\Big|_{\mathrm{NEW}} = 0.1617\ (40)$$

$$\boxed{|V_{us}|_{\mathrm{OLD}} = 0.2212 \pm 0.0031_{\mathrm{exp}} \pm 0.0005_{\mathrm{th}}} \quad ➡ \quad \boxed{|V_{us}|_{\mathrm{NEW}} = 0.2165 \pm 0.0026_{\mathrm{exp}} \pm 0.0005_{\mathrm{th}}}$$

Much more data coming. Precise measurement expected soon

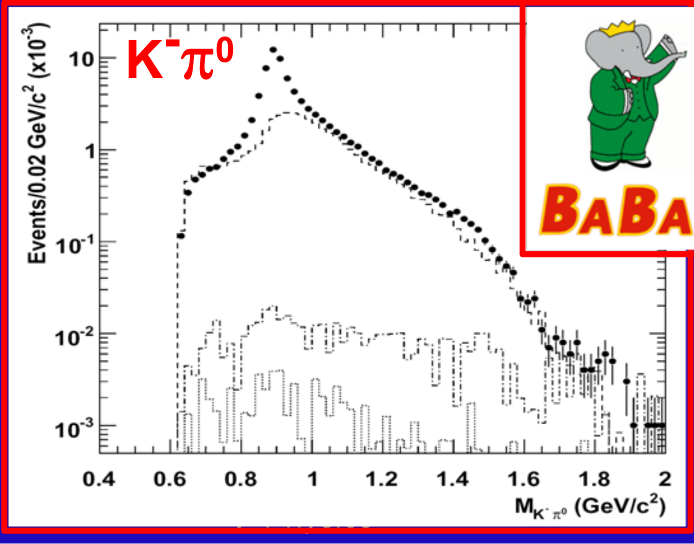
Huge number of $\tau^+\tau^-$ events at the B Factories

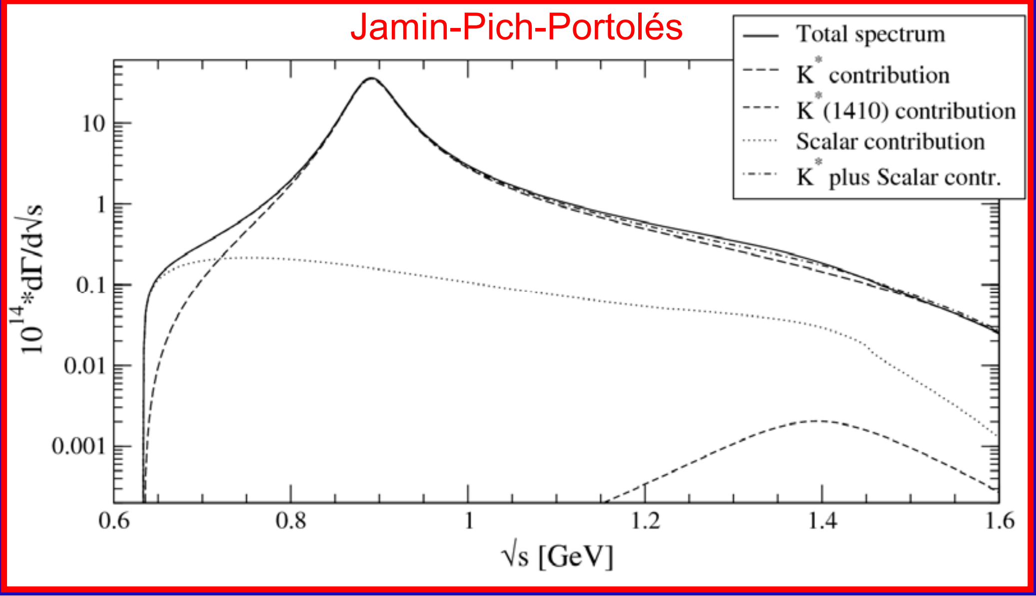
$\mathrm{Br}(\tau^- \to K_S \pi^- \nu_\tau) = (0.404 \pm 0.002_{stat} \pm 0.013_{syst})\ \%$



Ongoing data analysis

$\mathrm{Br}(\tau^- \to K^- \pi^0 \nu_\tau) = (0.416 \pm 0.003_{stat} \pm 0.018_{syst})\ \%$





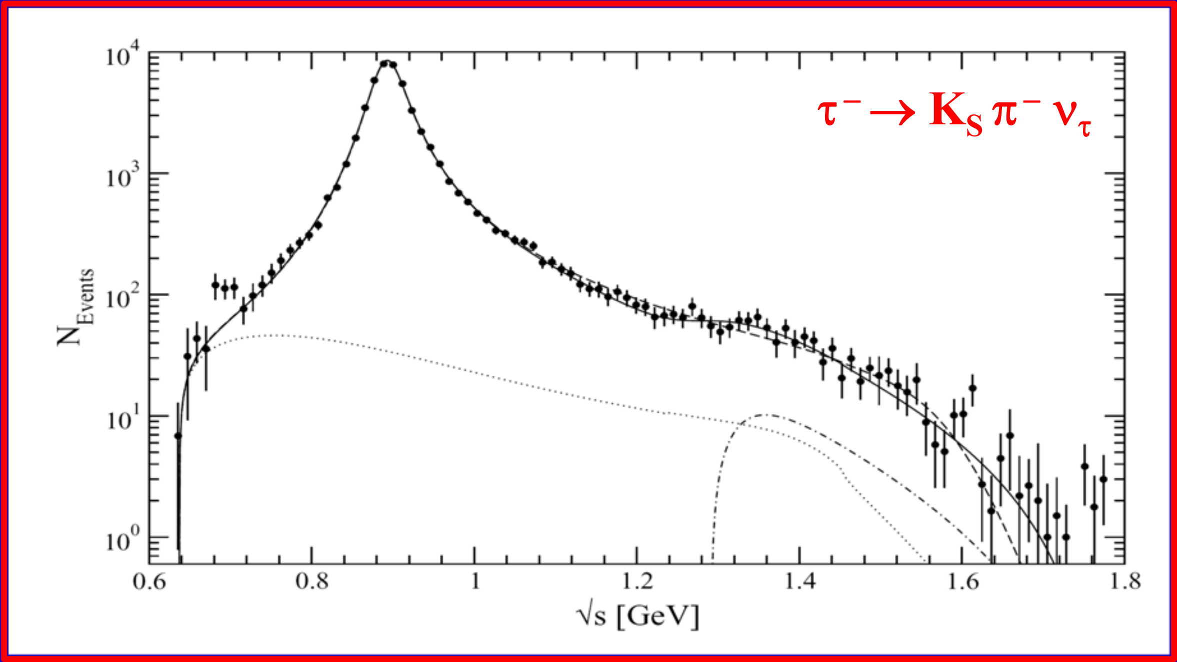
Jamin-Pich-Portolés 08

RχT Description of BELLE data

Shape fit:

M_{K^*} = 895.3 ± 0.2 MeV

Γ_{K^*} = 47.5 ± 0.4 MeV



RχT normalization fixed ➡ $\mathrm{Br}(\tau^- \to K_S \pi^- \nu_\tau)_{th} = (0.427 \pm 0.024)\ \%$

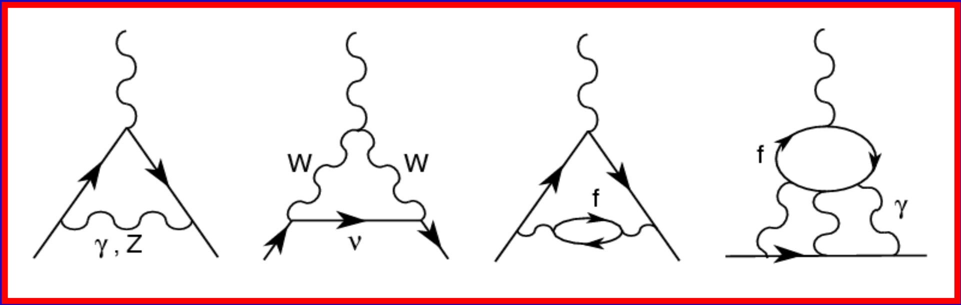
$\mathrm{Br}(\tau^- \to K_S \pi^- \nu_\tau)_{Belle} = (0.404 \pm 0.002_{stat} \pm 0.013_{syst})\ \%$

➡ **Prediction for K_{l3} Form Factor Slopes:**

$$\lambda_+^{'} = (25.20 \pm 0.33)\, 10^{-3} \quad ; \quad \lambda_+^{''} = (12.85 \pm 0.31)\, 10^{-4} \quad ; \quad \lambda_+^{'''} = (9.56 \pm 0.28)\, 10^{-5}$$

EXP: (Flavia*net* Kaon WG) $\lambda_+^{'} = (25.2 \pm 0.9)\, 10^{-3}$; $\lambda_+^{''} = (16 \pm 4)\, 10^{-4}$

Electron Anomalous Magnetic Moment



$$\mu_l \equiv g_l \frac{e}{2\, m_l} \qquad a_l \equiv \frac{1}{2}\,(g_l - 2)$$

$$a_e(\mathrm{QED}) = A_1 + A_2(m_e/m_\mu) + A_2(m_e/m_\tau) + A_3(m_e/m_\mu, m_e/m_\tau)$$

$$A_i = A_i^{(2)} \frac{\alpha}{\pi} + A_i^{(4)} \left(\frac{\alpha}{\pi}\right)^2 + A_i^{(6)} \left(\frac{\alpha}{\pi}\right)^3 + A_i^{(8)} \left(\frac{\alpha}{\pi}\right)^4 + \cdots$$

$$A_1^{(6)} = 1.181\,241\,456\,587 \cdots \qquad A_2^{(6)}(m_e/m_\mu) = -7.373\,941\,64\ (29) \times 10^{-6}$$

(Laporta–Remiddi)

$$A_1^{(8)} = -1.9144\ (35)$$ (Kinoshita–Nio)

$$a_e(\mathrm{QCD}) = 1.671\ (19) \times 10^{-12} \quad ; \quad a_e(\mathrm{Weak}) = 0.030\ (01) \times 10^{-12}$$

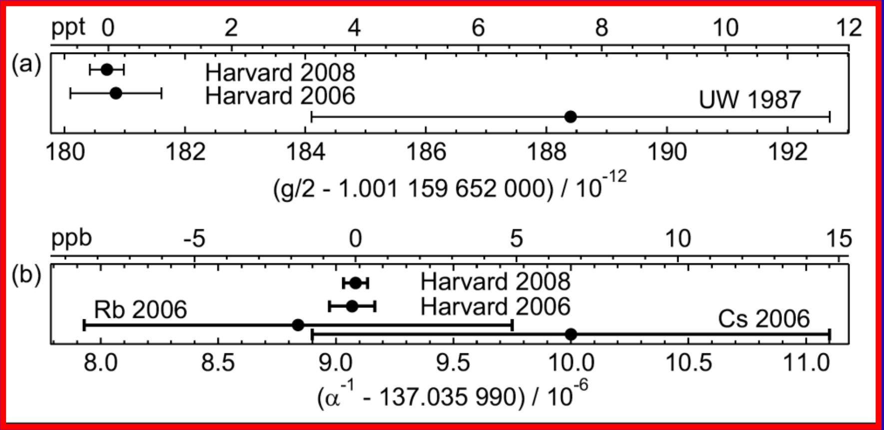
Electron Anomalous Magnetic Moment

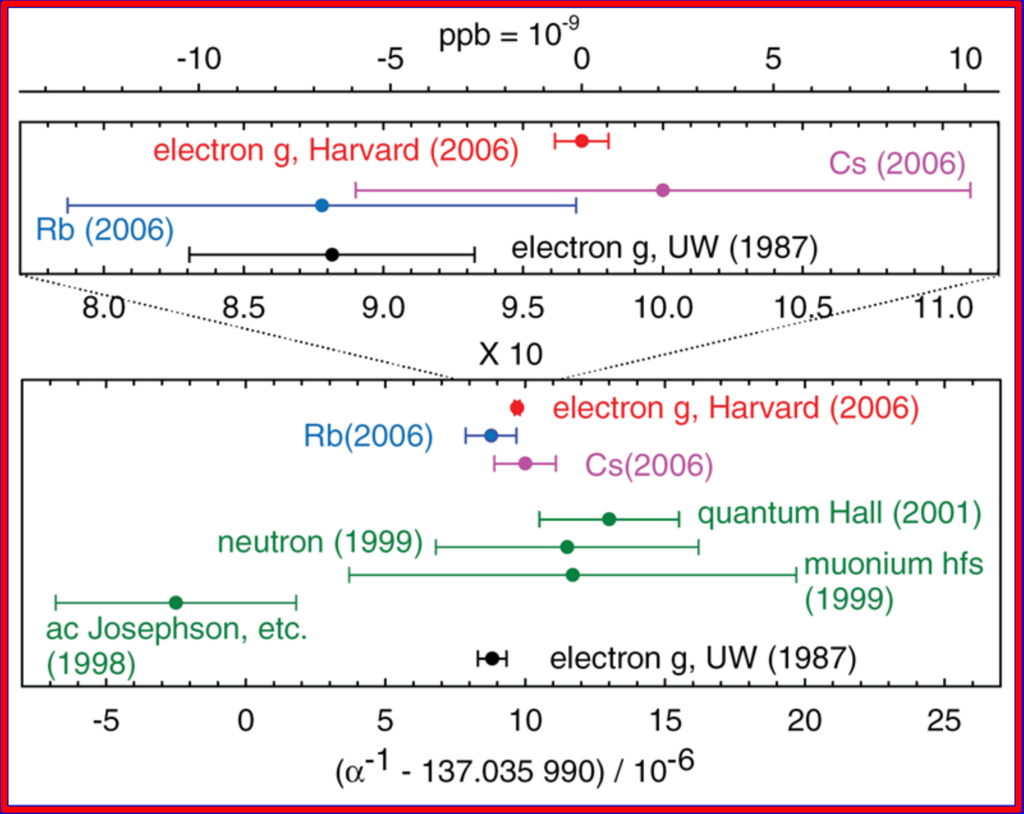
$$a_e = 0.001\,159\,652\,180\,73 \;\; (28)$$

Hanneke-Fogwell-Gabrielse ‘08

$$\alpha^{-1} = 137.035\,999\,084 \quad \underbrace{(51)}_{(33)_{\mathrm{exp}}\,(39)_{\mathrm{th}}}$$

0.37 ppb

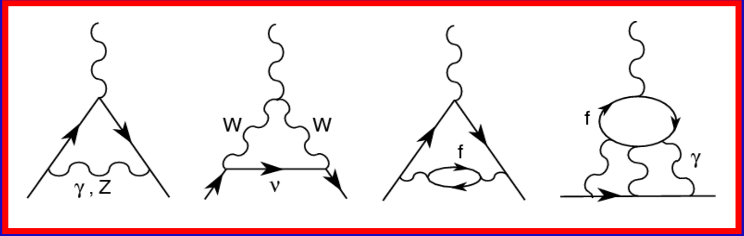


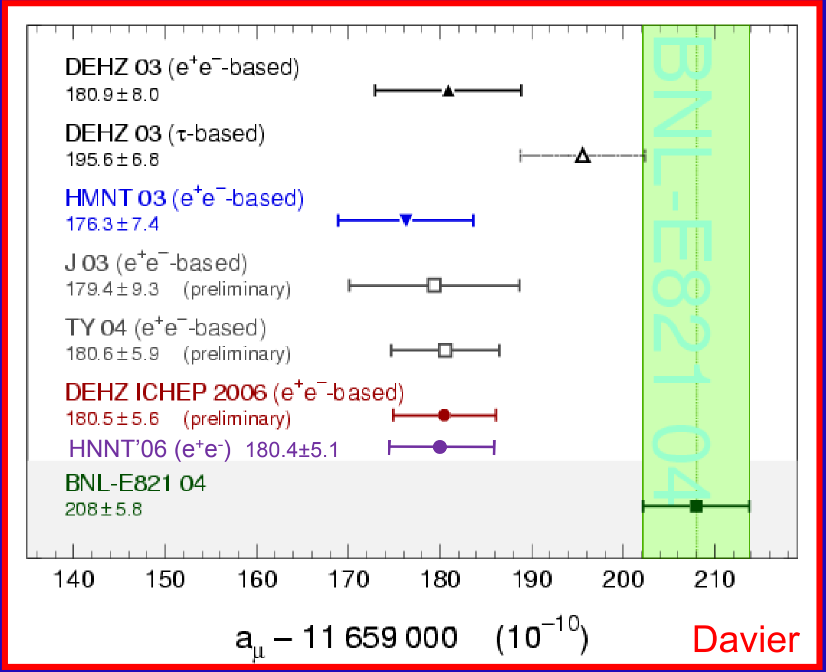


μ Anomalous Magnetic Moment

$$a_\mu^{\mathrm{exp}} = (11\,659\,208.0 \pm 6.0) \times 10^{-10}$$

BNL-E821





$10^{10} \times a_\mu^{\mathrm{th}}$ = 11 658 471.81 ± 0.02 QED — Kinoshita-Nio, Passera

+ 15.4 ± 0.2 EW — Czarnecki-Marciano-Vainshtein

+ 698.2 ± 9.7 hvp $(711.0 \pm 5.8)_\tau$, $(690.9 \pm 4.4)_{e^+e^-}$ — Davier et al

– 9.8 ± 0.1 hvp NLO — Krause, Hagiwara et al

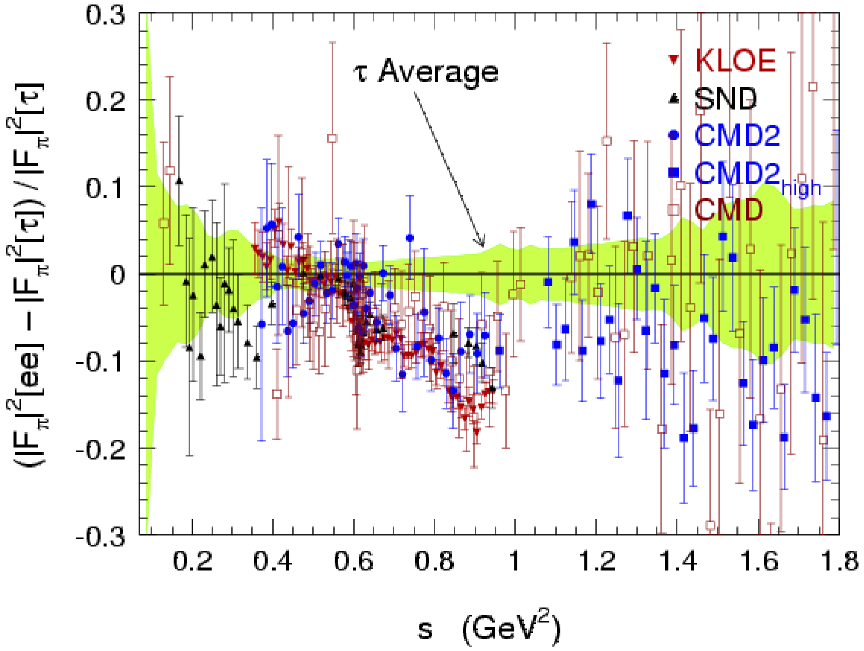
+ 12.0 ± 3.5 light-by-light — Melnikov-Vainshtein, Knech et al, Prades et al

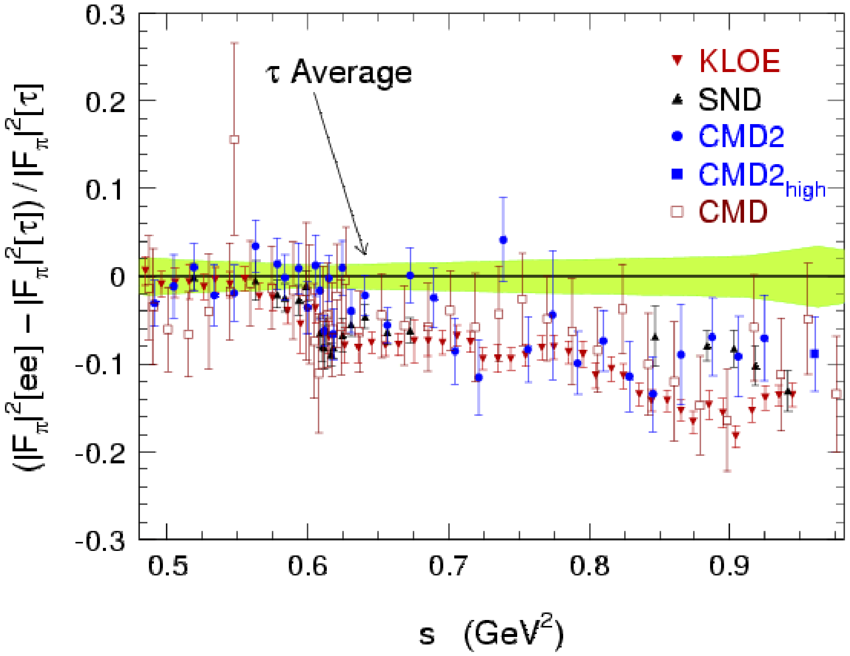
= 11 659 187.6 ± 10.3 $\quad (11\,659\,200.4 \pm 6.8)_\tau$, $(11\,659\,180.3 \pm 5.6)_{e^+e^-}$

$a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{th}}$ = 1.3 σ $\quad$ 0.8 σ $\quad$ 3.4 σ

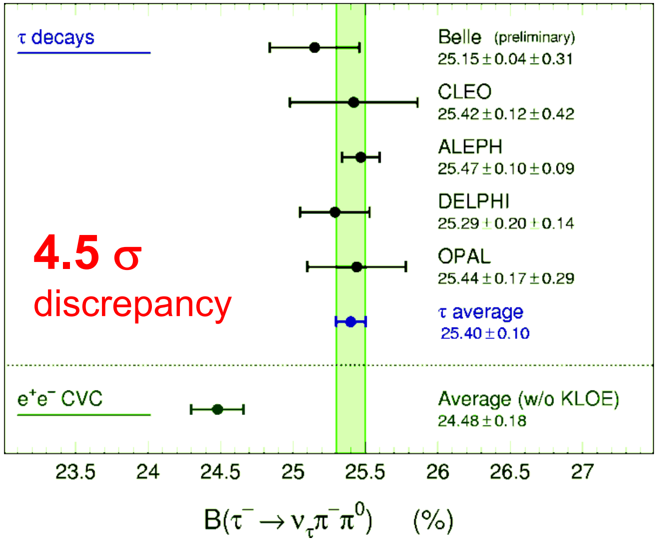
Today's e^+e^- - τ data comparison

M. Davier TAU'06





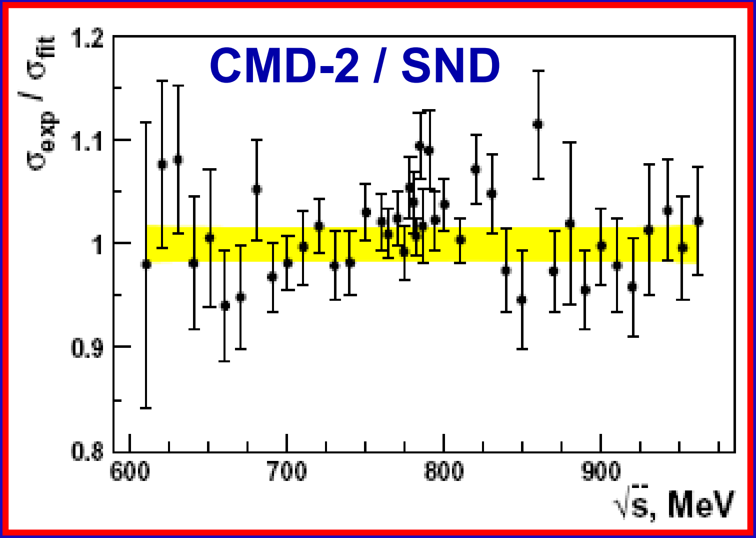
CMD2: 0.6% syst. error

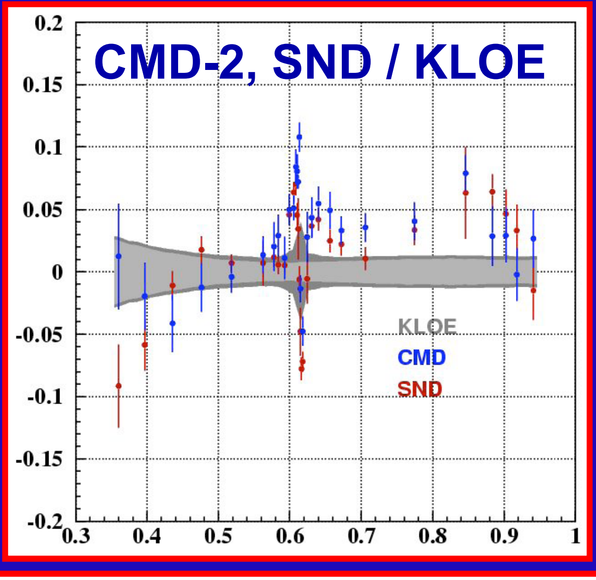


4.5 σ
discrepancy

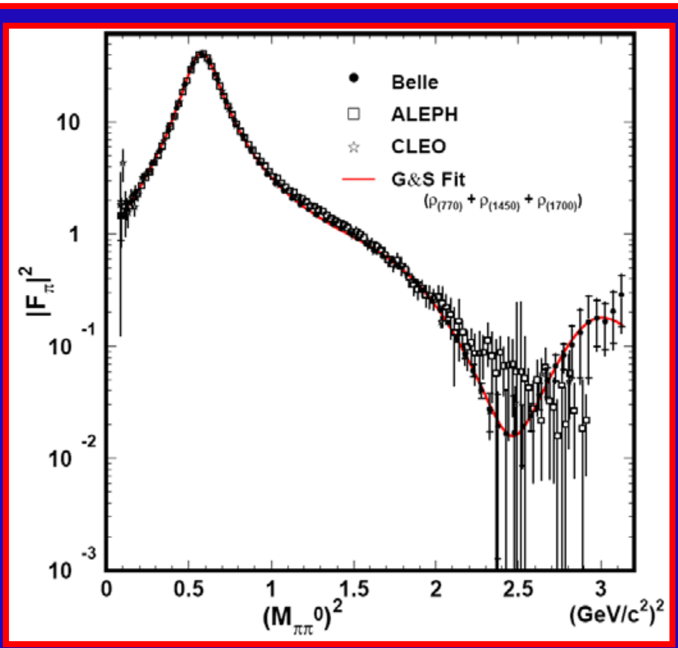
Belle '08 (final):
25.12 ± 0.01 ± 038

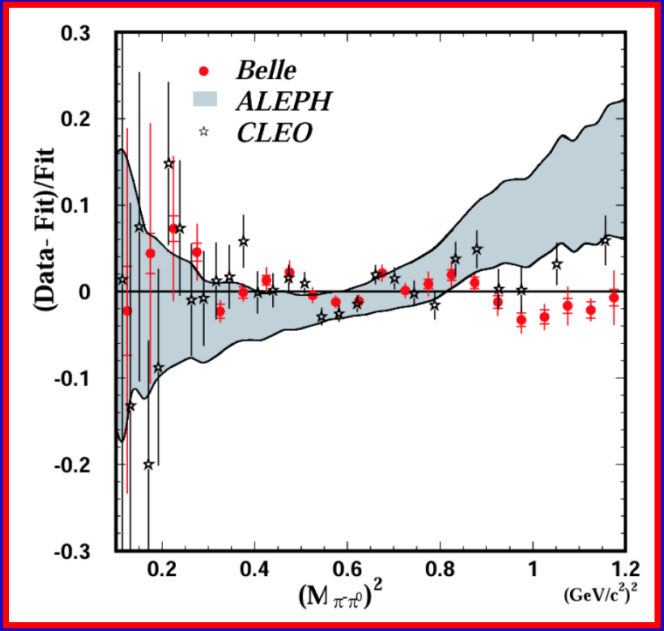
e^+e^- data





τ data





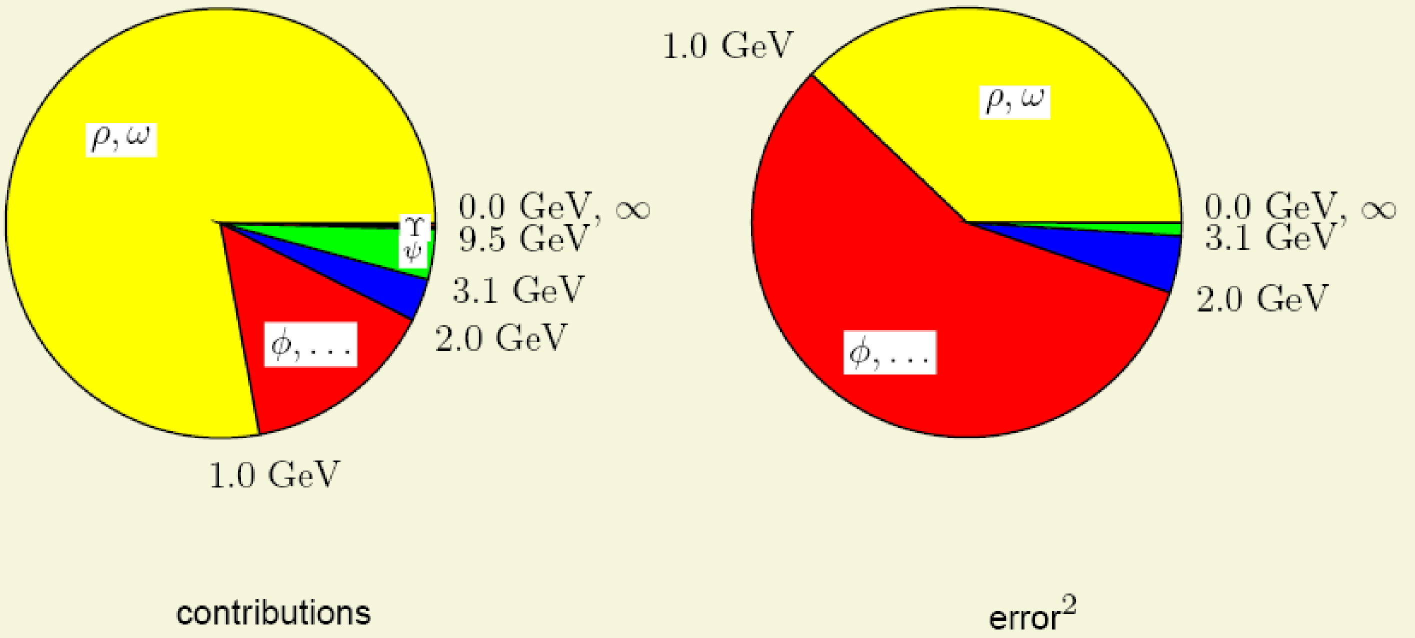
τ versus e^+e^- : Isospin violation $\Delta a_\mu =$ (-9.3 ± 2.4) 10^{-10} [(-1.8 ± 0.5)%]

OPE contraints satisfied better by τ data Maltman

- Experimental error implies theoretical uncertainty!
- Low energy contributions enhanced: $\sim 67\%$ of error on $a_\mu^{\rm had}$ comes from region $4m_\pi^2 < m_{\pi\pi}^2 < M_\Phi^2$

$$a_\mu^{\rm had(1)} = (692.3 \pm 6.0)\ 10^{-10}$$

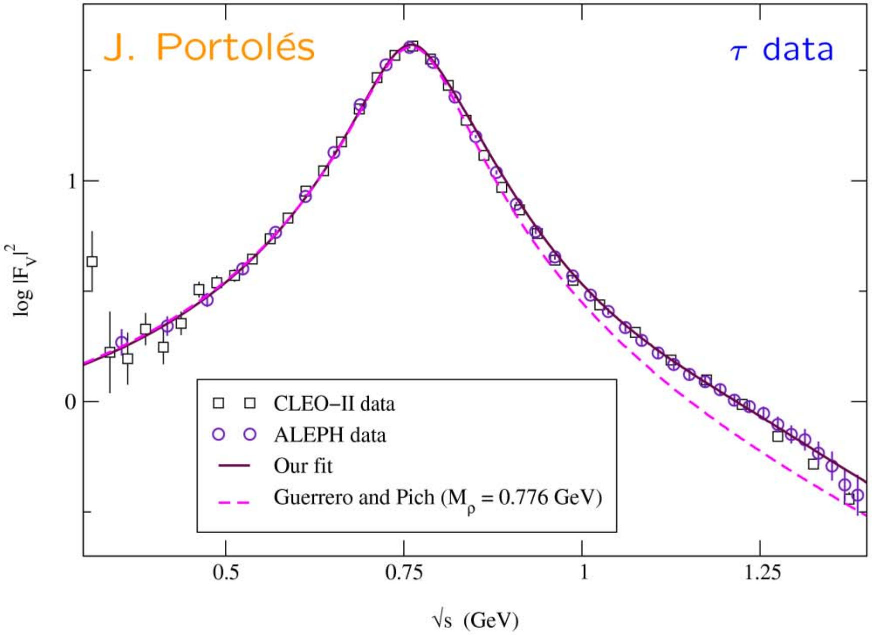
e^+e^-–data based

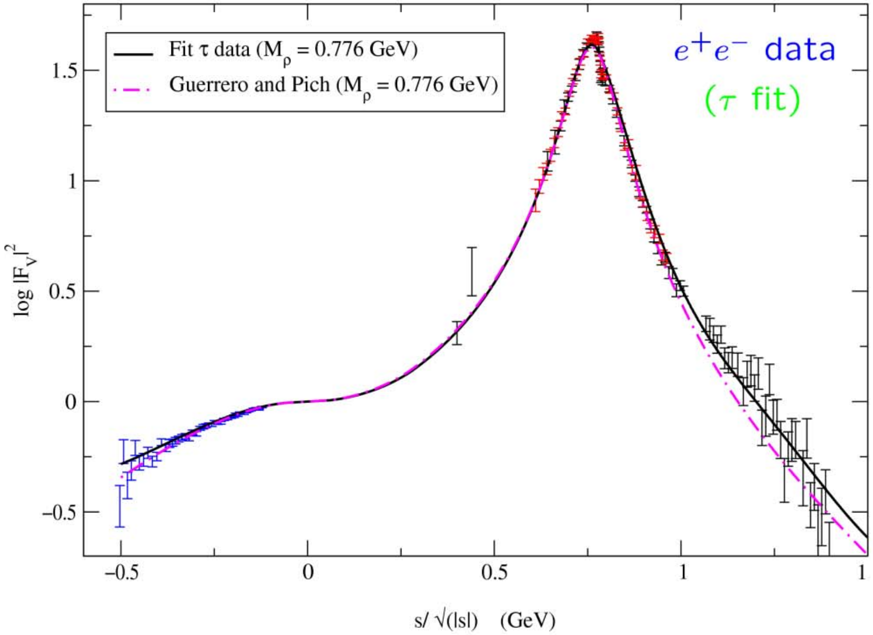


present distribution of contributions and errors

PION FORM FACTOR

Portolés





Th: Analyticity, Unitarity, Chiral Symmetry (χPT, RχT), QCD, $N_C \to \infty$

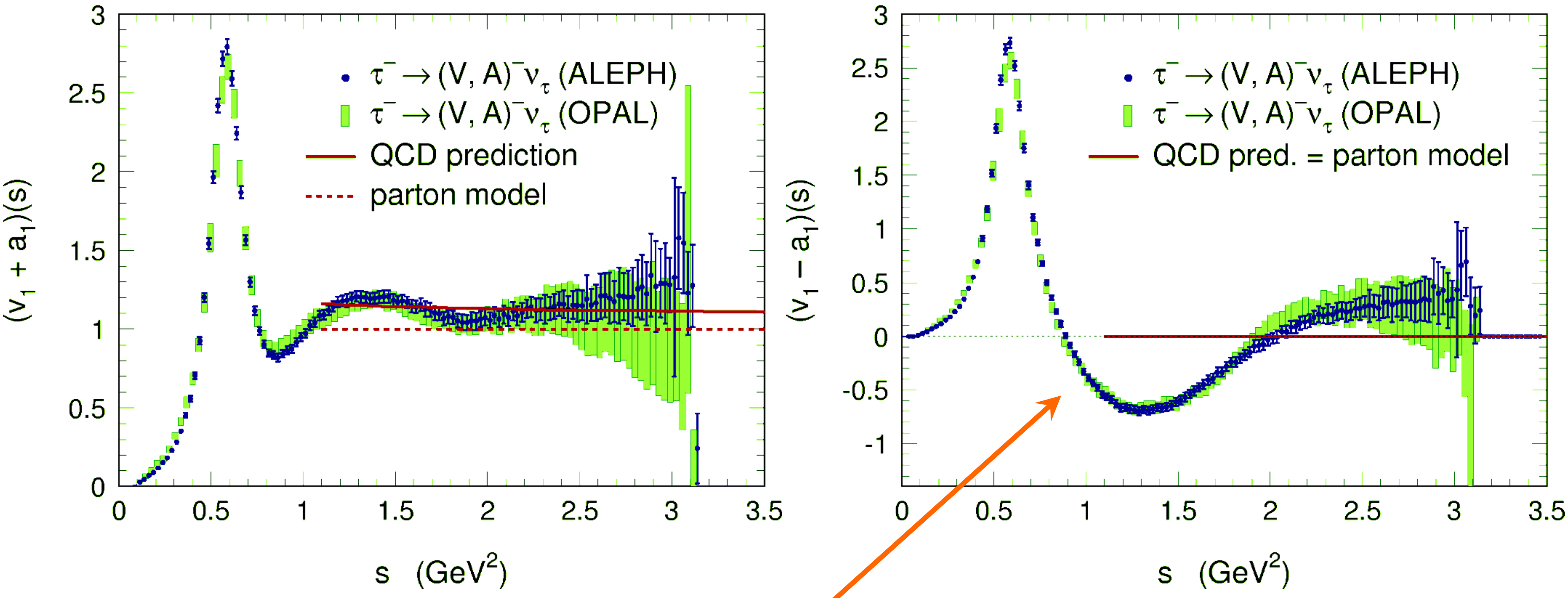
$$\boxed{F_\pi(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \;\exp\left\{-\frac{s}{96\pi^2 f_\pi^2}\,\mathrm{Re}\left[A\left(\frac{M_\pi^2}{s},\frac{M_\pi^2}{M_\rho^2}\right)\right]\right\}}$$

Guerrero–Pich '97

$$A(x,y) \equiv \log y + 8x - \frac{5}{3} + \sigma_x^3 \log\left(\frac{\sigma_x+1}{\sigma_x-1}\right) \quad ; \quad \sigma_x \equiv \sqrt{1-4x} \quad ; \quad \Gamma_\rho(s) = \theta(s-4M_\pi^2)\,\sigma_\pi^3\,\frac{M_\rho\, s}{96\pi f_\pi^2}$$

Inclusive *V+A* and *V–A* Spectral Functions

- Results from ALEPH and OPAL — and their comparison

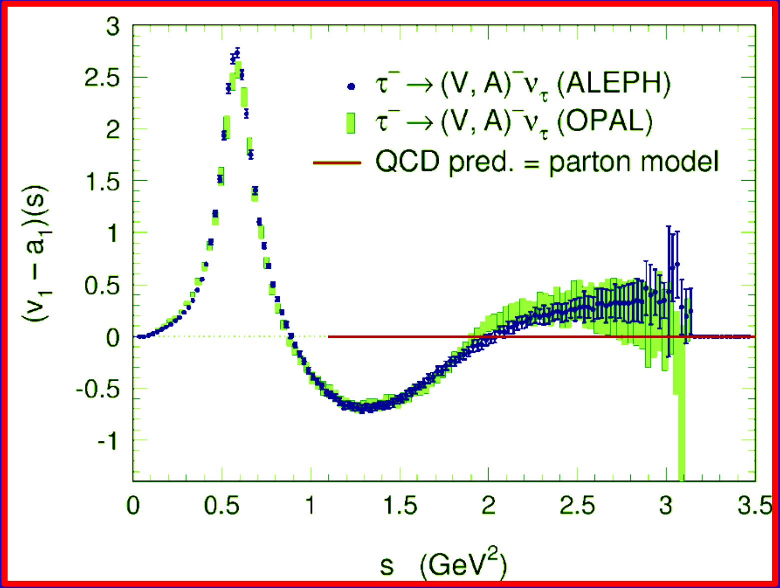


Of purely nonperturbative origin

ALEPH, Phys. Rep. 421 (2005)
OPAL, EPJ, C7, 571 (1999)

M. Davier *et al.*: ALEPH Tau Spectral Functions and QCD — revisited

Non-Perturbative Physics



Low-energy QCD engineering

- ☐ **ChPT :** Chiral sum rules $f_\pi \; , m^2_{\pi^\pm} - m^2_{\pi^0} \; , F_A / \langle r^2_\pi \rangle \; \ldots$

$$L^r_{10}(M_\rho) = -(4.06 \pm 0.40) \cdot 10^{-3} \quad , \quad C^r_{87}(M_\rho) = (4.89 \pm 0.19) \cdot 10^{-3} \text{ GeV}^{-2}$$

(M. González-Alonso '08)

- ☐ **RChT , 1/N$_C$, ...**

- ☐ **Electromagnetic Penguins (Q$_{7,8}$):** ε'/ε

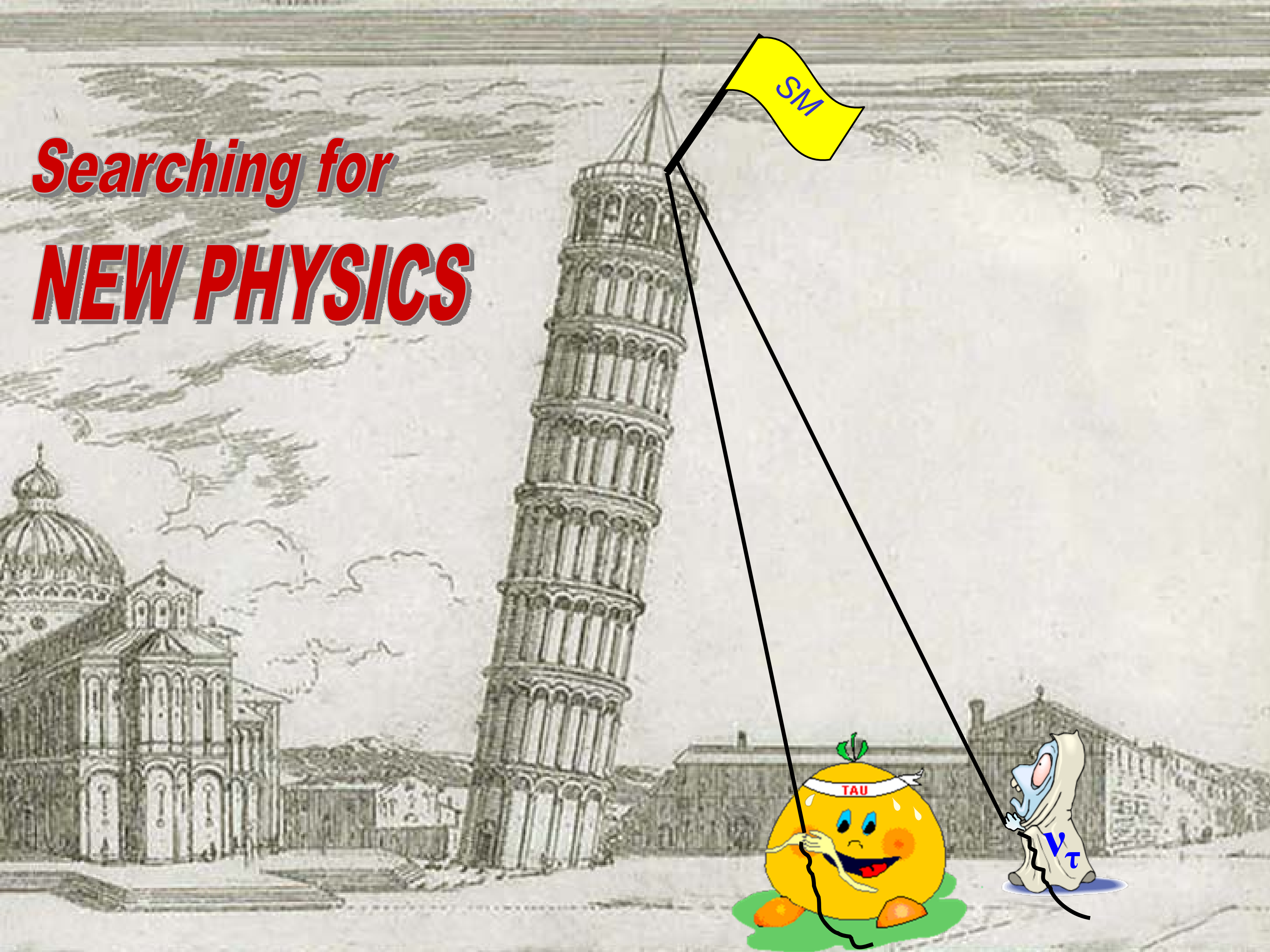
SUMMARY

The τ is a clean laboratory to test the Standard Model

- Precision physics
- QCD tests
- Open questions: $(g\text{-}2)_\mu$
- Hints of new physics: ν's
- Future facilities: τcF, SuperB
- Tool for NP searches: LHC

There is an exciting future ahead

Searching for NEW PHYSICS



BACKUP SLIDES

The τ could give the most precise V_{us} determination

- From present τ data one gets:

$$\boxed{|V_{us}| = 0.2165 \pm 0.0026_{\text{exp}} \pm 0.0005_{\text{th}}}$$

- Accuracy similar already to K_{l3}:

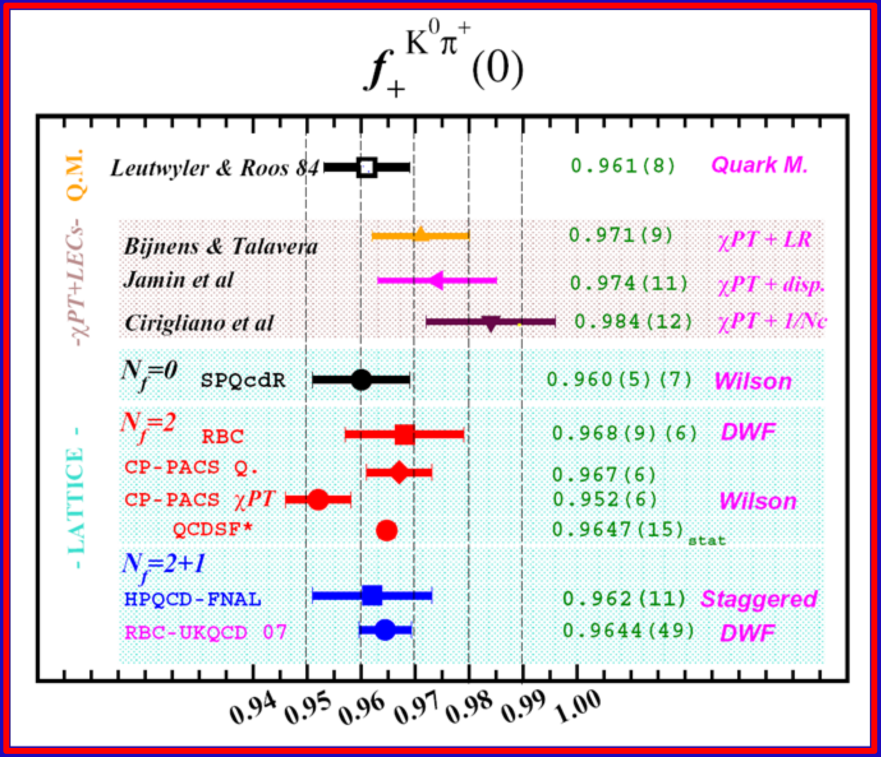
$$|V_{us}| = 0.2233 \pm 0.0024 \qquad [\, f_+(0) = 0.97 \pm 0.01]$$

Interesting challenge for the B Factories & BESIII

K_{l3} Decays

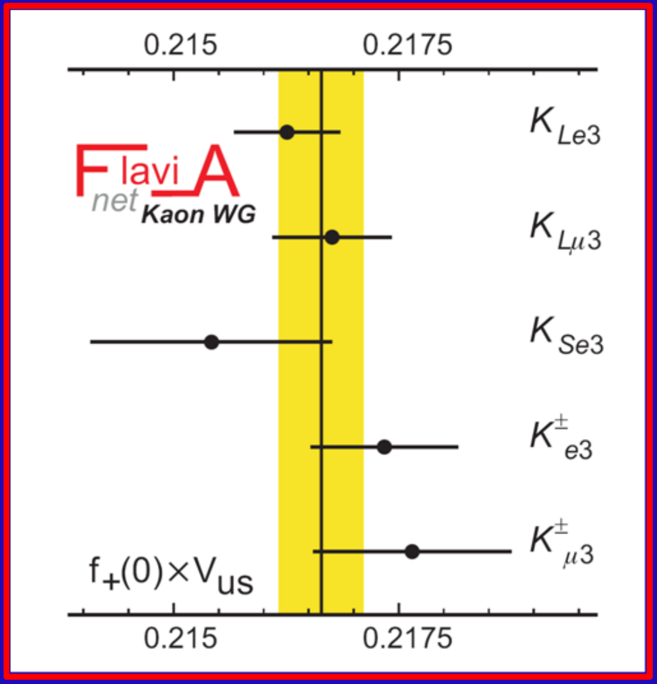
arXiv:0801.1817 [hep-ph]

Large $O(p^6)$ ChPT correction (Bijnens-Talavera)



$O(p^4)$

$O(p^6)$



$$\left|f_+(0)\, V_{us}\right| = 0.2166 \pm 0.0005$$

$$f_+(0) = 0.97 \pm 0.01 \qquad \Longrightarrow \qquad \left|\mathbf{V_{us}}\right| = \mathbf{0.2233 \pm 0.0024}$$

A simultaneous m_s & V_{us} fit could be possible

However:

- ☐ **Perturbative QCD corrections need to be better understood** (CIPT)

$$\Delta_{00}(\alpha_s)^{L+T} = 0.753 + 0.214 + 0.065 - 0.063 + \ldots$$

$$\Delta_{10}(\alpha_s)^{L+T} = 0.912 + 0.334 + 0.192 + 0.069 + \ldots$$

$$\Delta_{20}(\alpha_s)^{L+T} = 1.055 + 0.451 + 0.330 + 0.232 + \ldots$$

$$\Delta_{30}(\alpha_s)^{L+T} = 1.190 + 0.571 + 0.484 + 0.432 + \ldots$$

$$\Delta_{40}(\alpha_s)^{L+T} = 1.324 + 0.697 + 0.657 + 0.676 + \ldots$$

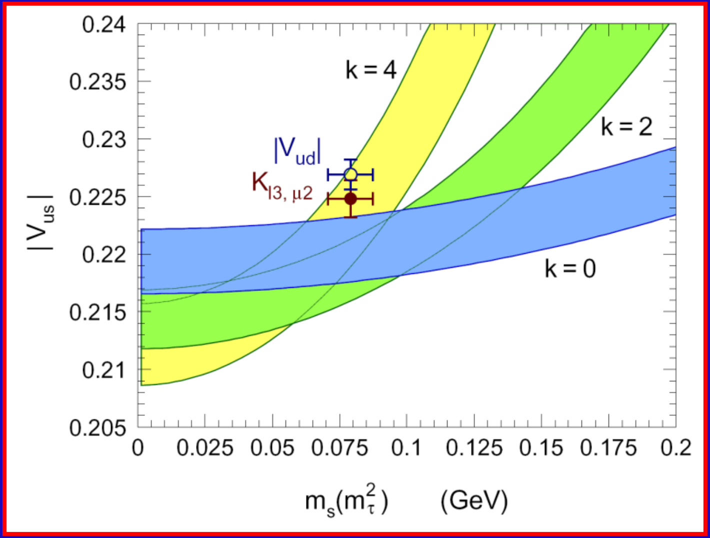
Sizeable theoretical uncertainties

Resummations, pinched weights (Maltman & Wolfe), …

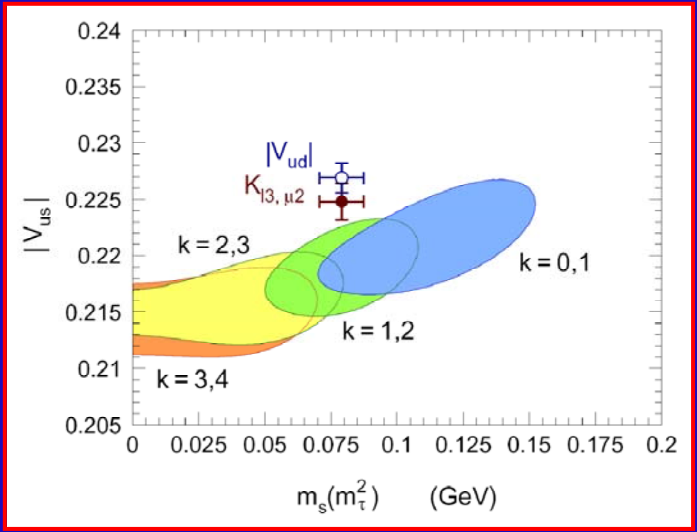
- ☐ **Not enough sensitivity with present data**

Large correlations. Low statistics. Missing decay modes …

ALEPH

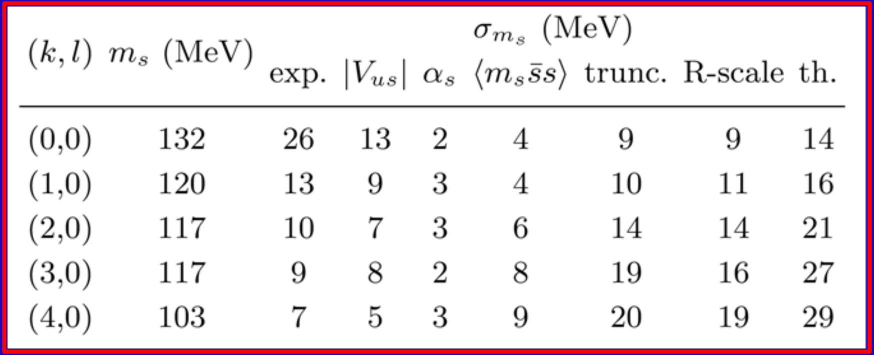


Davier-Höcker-Zhang ‘05



Taking **V_{us} = 0.2225 (21) :**

Chen et al ’01 , J=0 included



(k,l)	m_s (MeV)	σ_{m_s} (MeV)						
		exp.	$\lvert V_{us}\rvert$	α_s	$\langle m_s\bar{s}s\rangle$	trunc.	R-scale	th.
(0,0)	132	26	13	2	4	9	9	14
(1,0)	120	13	9	3	4	10	11	16
(2,0)	117	10	7	3	6	14	14	21
(3,0)	117	9	8	2	8	19	16	27
(4,0)	103	7	5	3	9	20	19	29

$$m_s(m_\tau) = \left(120\,^{+21}_{-26}\right)\ \mathrm{MeV}$$

$$m_s(2\,\mathrm{GeV}) = \left(116\,^{+20}_{-25}\right)\ \mathrm{MeV}$$

Gámiz et al ’03 , J=0 excluded

Moment	$m_s(m_\tau)$ [MeV]
(0,0)	192 ± 72
(1,0)	164 ± 31
(2,0)	137 ± 20
(3,0)	115 ± 17
(4,0)	100 ± 17

$$m_s(m_\tau) = (122 \pm 17)\ \mathrm{MeV}$$

$$m_s(2\,\mathrm{GeV}) = (117 \pm 17)\ \mathrm{MeV}$$

- Strong k dependence with ALEPH data (m_s decreases with increasing k)

Spectral function underestimated at large invariant masses

→ Missing events / modes (Kππ, Kπππ, …)

- Much better behaviour with OPAL data:

Gámiz et al ’05 , J=0 excluded

(0,0) → V_{us} **= 0.2208 (34)** →

Moment	$m_s(m_\tau)$ [MeV]
(2,0)	89 ± 39
(3,0)	84 ± 27
(4,0)	78 ± 22

$$m_s(m_\tau) = \left(84 \pm 23\right)\,\mathrm{MeV} \quad , \quad m_s(2\,\mathrm{GeV}) = \left(81 \pm 22\right)\,\mathrm{MeV}$$

- $\tau \to K\nu$ from $K \to \mu\nu$ + OPAL: V_{us} **= 0.2220 (33)**