Studies in Holographic QCD

Stefano Nicotri

Università degli Studi di Bari, & INFN Sezione di Bari, Italy

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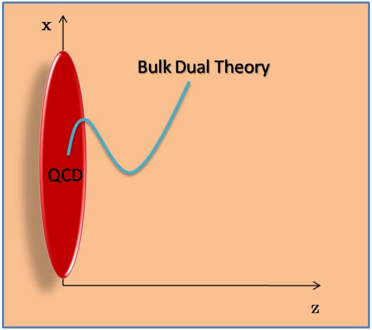
in collaboration with P. Colangelo, F. De Fazio, F. Giannuzzi and F. Jugeau

Outline

1. Intro
   - AdS/CFT correspondence conjecture
   - QCD as "quasi-conformal theory"

2. Phenomenology
   - Chiral Symmetry (Breaking)
   - Scalar Mesons
   - Glueballs



Introduction

QCD: a difficult theory to deal with

Two regimes of QCD and tools for theoretical analysis

Non-Perturbative

- Lattice QCD
- QCD sum rules
- Effective theories (χPT, HQET, SCET, NRQCD,...)
- ...

Perturbative

- Perturbation theory

Old dream: "QCD: a solvable theory"

Holographic QCD → new analytical attempt to approach QCD → new promising tool for computing low-energy observables

What is holography?
How to construct a holographic description of QCD?
(To have an idea just look outside the door, on your right...)

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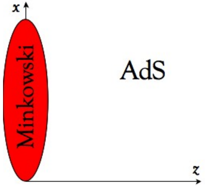
AdS/CFT

Anti-de Sitter space

Conformally flat spacetime, solution of the equation of motion derived from the Einstein-Hilbert action with negative cosmological constant:

$$ds^2 = \frac{R^2}{z^2}(dx^2 + dz^2) \text{ (Poincaré coordinates)}$$

$$g_{MN} = \frac{R^2}{z^2}\text{diag}(-1, +1, +1, +1, +1)$$



- Isomorphism between the isometry group of AdS_{d+1} and the conformal group in d dimensions
- The boundary ($z = 0$) of an AdS_{d+1} space is a flat (Minkowski) space
- The isometry group of AdS_{d+1} acts as the conformal group on the boundary $\partial\text{AdS}_{d+1} = \mathcal{M}_d$

Conjectured duality (J. Maldacena)

SUGRA in $\text{AdS}_5 \times S^5$ $\leftrightarrows$ $\text{SU}(N)$ $\mathcal{N} = 4$ SYM, $N \to \infty$

Extension: gauge/gravity duality (E. Witten, A. Polyakov, I. Klebanov...)

Gravity theory in $d + 1$-dim		Quantum conformal field theory in d-dim
Curved spacetime (AdS)		Flat spacetime (∂AdS)
Weak coupling	$\leftrightarrows$	Strong coupling
Field $\varphi(x, y, z)$		Gauge invariant local operator $\mathcal{O}(x)$

Gauge/gravity duality (precise meaning)

The partition function of the two theories are equal

Being $Z[\varphi]$ the partition function of the 5-dimensional (semiclassical) gravity theory:

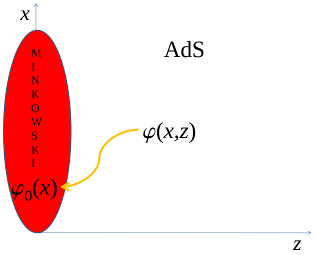
$$\left\langle \exp \int \varphi_0 \mathcal{O} \right\rangle_{\text{CFT}} = Z[\varphi] \sim \exp(-iS_{\text{eff}}[\varphi])$$

with the boundary condition:

$$\varphi(x,z) = \int d^4x' \, K(x-x',z) \, \varphi_0(x')$$

$$K(x-x',z) \xrightarrow[z \to 0]{} \delta^4(x-x')$$

$K(x-x',z)$ is a bulk-to-boundary evolution operator.



The correlation functions of the operator $\mathcal{O}(x)$ in the strongly coupled theory can be calculated by functionally deriving the (classical) partition function $\exp(-iS_{\text{eff}}[\varphi(x,z)])$ with respect to its source $\varphi_0(x)$ in the generating functional.

In practice, the only thing that is needed is the classical action of the field which is dual to the operator that we are studying

QCD and conformal theories

QCD

QCD NOT conformally invariant, but:

- $m_q = 0 \quad \Rightarrow \quad \mathcal{L}_{QCD}^{class}$ invariant under the conformal group
- Asymptotic freedom $\Rightarrow$ conformal invariance in the UV
- $Q^2 \leq 1$ GeV^2: hints of an IR fixed point (Brodsky)

  $\beta(\alpha_s) = 0 \rightarrow \alpha_s(Q^2)$ constant and large $\rightarrow$ conformal symmetry

Window of energies in which QCD is "quasi-conformal"

but

$\mathcal{N} = 4$ SYM is supersymmetric while QCD is not

Approaches

Supposing the existence of a gravity theory dual to QCD:

Top-down approach

- Start from gravity theory in more dimensions
- Get a QCD-like theory through modifications (*e.g.* compactifications of the extra dimensions)

Bottom-up approach

- Start from known features of QCD
- Try to build a higher dimensional model able to reproduce them, getting hints from AdS/CFT
- Hopefully get predictions for experiments

Topics in the bottom-up approach

- Mesons and Glueballs (Erlich, Karch, Katz, Son, Stephanov, Brodsky, De Teramond, Pomarol, Forkel, Giannuzzi, Colangelo, De Fazio, Jugeau, SN, Evans, Csaki...)
- Wilson loop and heavy $Q\bar{Q}$ potential (Andreev, Zakharov, Boschi Filho, Braga...)
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Bottom-up approach: hard wall model

Framework (J. Polchinski, M. Strassler)

Bulk = AdS_5 cut at $z = z_m \sim 1/\Lambda_{QCD}$

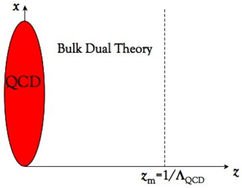
$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2) \quad 0 \leqslant z \leqslant z_m$$

QCD is defined on the boundary $z = 0$: a Minkowski space with metric $\eta_{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1)$

The conformal symmetry is broken by the introduction of the inverse mass scale z_m.

Scale transformation $(x, z) \to (\lambda x, \lambda z)$: $q^2 \to (\lambda)^{-1} q^2 \Leftrightarrow z \to \lambda z$

z = inverse energy scale: $z \leqslant z_m \Rightarrow$ CONFINEMENT



How to construct the action

- Local gauge invariant QCD operator $\mathcal{O}_{\mu\nu\ldots\beta}(x)$ of order p and dim Δ $\leftrightarrow$ free field in the bulk $B_{\mu\nu\ldots\beta}(x, z)$
- $m_5^2 = (\Delta - p)(\Delta + p - 4)$ (from AdS/CFT)
- Global symmetry in the boundary $\leftrightarrow$ local (gauge) symmetry in the bulk

Example: Scalar operator $\mathcal{O}$

$$S = -\frac{1}{2k}\int d^5x\sqrt{|g|}\left(g^{MN}\partial_M X\,\partial_N X + m_5^2 X^2\right) \qquad \partial_M\left[\sqrt{|g|}\,g^{MN}\partial_N X(x,z)\right] - m_5^2 X(x,z) = 0$$

How to calculate observables

- Hadrons with the quantum numbers of $\mathcal{O}$ $\leftrightarrow$ normalizable modes of the field $B_{\mu\nu\ldots\beta}(x, z)$: $B_n(z)$
- Hadron masses $\leftrightarrow$ Eigenvalues of the classical 5-dim EOM

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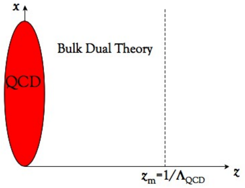

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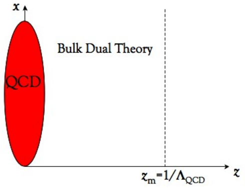

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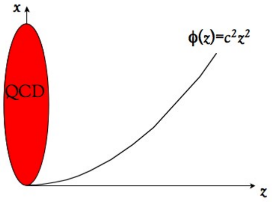
Bottom-up approach: soft wall model

Framework (A. Karch, E. Katz, D. Son, M. Stephanov)

Bulk = uncut AdS_5 with a background field
$\phi(z) = c^2 z^2$



$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2) \quad 0 \leqslant z \leqslant +\infty$$

$$g_{MN} = \frac{R^2}{z^2}\mathrm{diag}(-1,+1,+1,+1,+1)$$

$$S = -\frac{1}{2k}\int d^5x\sqrt{|g|}\,e^{-\phi(z)}\left(g^{MN}\,\partial_M X\,\partial_N X + m_5^2 X^2\right)$$

$$\partial_M\left[\sqrt{|g|}\,e^{-\phi(z)}g^{MN}\,\partial_N X(x,z)\right] - m_5^2\sqrt{|g|}\,e^{-\phi(z)}X(x,z) = 0$$

c is a parameter with the dimension of $[M]$ responsible of the breaking of conformal symmetry.
All the hadron masses are evaluated in terms of this unique parameter.

This model reproduces Regge behaviour of vector-mesons: $m_n^2 \sim n$

Chiral Symmetry

Holographic model for chiral symmetry breaking (Karch, Katz, Son, Stephanov, Phys. Rev. D **74** (2006) 015005) :

$$S = -\frac{1}{2k}\int d^5x\sqrt{|g|}e^{-\phi(z)}\left[|DX|^2 - 3|X|^2 + \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right], \qquad X = X_0 e^{2i\pi}$$

Same structure as χPT

QCD	AdS	Δ	m_5^2	Symmetry
$\bar{q}_L\gamma_\mu q_L$	$A_{L\mu}$	3	0	$SU(3)_L$
$\bar{q}_R\gamma_\mu q_R$	$A_{R\mu}$	3	0	$SU(3)_R$
$\langle\bar{q}q\rangle$	X_0	3	-3	χ symm breaking

$SU(3)_L \otimes SU(3)_R \rightarrow SU(3)_V$

$\pi(x,z) = \pi^a(x,z)t^a$ χral field

t^a = gen. of SU(3)

$m_5^2 R^2 = (\Delta - p)(\Delta + p - 4)$

$$\partial_z\left(\frac{e^{-\phi}}{z}\partial_z V_\mu^a\right) - \frac{q^2 e^{-\phi}}{z}V_\mu^a = 0$$

$$\partial_z\left(\frac{e^{-\phi}}{z}\partial_z A_{\mu\perp}^a\right) - \frac{q^2 e^{-\phi}}{z}A_{\mu\perp}^a - \frac{4\,g_5^2 X_0^2 e^{-\phi}}{z^3}A_{\mu\perp}^a = 0$$

$X_0 = m_q z + \langle\bar{q}q\rangle z^3$ is responsible of chiral symmetry breaking

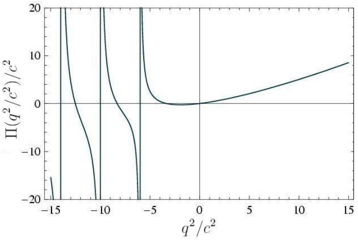
Scalar mesons

To take into account the scalar mesons, described by the operator $\bar{q}_R q_L$, we use the same action, with an additional term:

$$S = -\frac{1}{2k}\int d^5x\sqrt{|g|}e^{-\phi(z)}\left[|DX|^2 - 3|X|^2 + \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right], \qquad X = (X_0 + S)e^{2i\pi}$$

$S(x,z)$ is the field dual to the operator $\bar{q}_R q_L(x)$ describing the scalar mesons

Two-point correlation function of the scalar mesons' field S



$$\Pi(q^2) = i\int d^4x\, e^{iq\cdot x}\langle 0|T[\mathcal{O}_S(x)\mathcal{O}_S(0)]|0\rangle$$

$$\Pi_{AdS}(q^2) = \frac{4c^2R}{k}\left[\left(\frac{q^2}{4c^2} + \frac{1}{2}\right)\ln(c^2z^2) + \left(\gamma_E - \frac{1}{2}\right) + \frac{q^2}{4c^2}\left(2\gamma_E - \frac{1}{2}\right) + \left(\frac{q^2}{4c^2} + \frac{1}{2}\right)\psi\left(\frac{q^2}{4c^2} + \frac{3}{2}\right)\right]\Bigg|_{z=z_{min}}$$

- position of the poles $\longrightarrow$ mass spectrum $m_n^2 = c^2(4n+6)$
- residues $\longrightarrow$ decay constants $F_n^2 = \frac{16Rc^4}{k}(n+1)$

Three-point correlation function

Coupling with two pseudoscalars:

$$g_{S_0PP} = \frac{\sqrt{N_c}}{2\pi}\frac{m_{S_0}^2}{f_\pi^2}Rc\int_0^\infty d\hat{z}\, e^{-\hat{z}^2}X_0(\hat{z}) \quad \sim \quad \mathcal{O}(10\text{ MeV}) \quad (!!!)$$

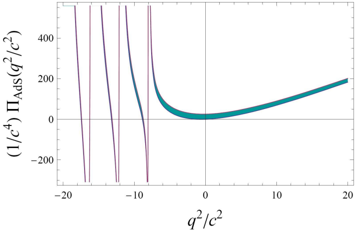
Scalar glueball

QCD	AdS	Δ	m_5^2
$\beta(\alpha_s)\, G^a_{\mu\nu}\, G^{\mu\nu\, a}$	$Y(x,z)$	4	0

$$m_5^2 R^2 = (\Delta - p)(\Delta + p - 4) = 0$$

$$S = -\frac{1}{2k}\int d^5x \sqrt{|g|}\, e^{-\phi} g^{\mu\nu}(\partial_\mu Y)(\partial_\nu Y)$$

Two-point correlation function



$$\Pi(q^2) = i\int d^4x\, e^{iq\cdot x}\langle 0|T[\mathcal{O}_G(x)\mathcal{O}_G(0)]|0\rangle$$

- Poles $\longrightarrow$ spectrum: $m_n^2 = 4c^2(n+1)$
- Residues $\longrightarrow$ decay constants:
  $f_n^2 = \langle 0|\mathcal{O}_G(0)|n\rangle^2 = \frac{8R^3c^6}{k}(n+1)(n+2)$

Matching to QCD

QCD	AdS
$C_0\, q^4\left(-\ln(q^2/\nu^2) + 2 - \frac{1}{\varepsilon'}\right)$ + condensates	$\frac{R^3}{8k} q^4\left(-\ln(q^2/\nu^2) + 2 - 2\gamma_E + \ln 4\right)$ + condensates

Results

- $M_{0^+} = 1.1$ GeV
- $f_{0^+} = \langle 0|\mathcal{O}_G(0)|n=0\rangle = 0.8$ GeV^3
- Difficulties in evaluating $\langle(\alpha_s/\pi)G^2\rangle$ (!!!)
- Dimension two condensate (!!!)

Numerical results

AdS

1. Scalar glueball
   - $M_{0^+} = 1.1$ GeV
2. Scalar Mesons
   - $M_{S_0} = 943$ MeV
   - $F_{S_0} = 0.08$ GeV^2
   - $g_{SPP} \sim \mathcal{O}(10$ MeV$)$
   - $\langle(\alpha_s/\pi)G^2\rangle \simeq 0.004$ GeV^4

QCD

1. Scalar glueball
   - $M_{0^+} \sim 1.5$ GeV (lattice)
2. Scalar Mesons
   - $M_{a_0} = 980$ MeV
   - $F_{a_0} = 0.21 \pm 0.05$ GeV^2
   - $g_{a_0\eta\pi} = 12 \pm 6$ GeV
   - $\langle(\alpha_s/\pi)G^2\rangle \simeq 0.012$ GeV^4

Conclusions

Holographic QCD is a new promising tool to try to analytically investigate the non-perturbative regime of QCD.

We have tested a particular model, the soft wall model, in the scalar mesons and scalar glueball sector, comparing the results with current phenomenology and finding results close to the QCD ones.

We think that this is a quite interesting path to follow, and a subject that needs to be more deeply investigated.

Thanks to P. Colangelo and F. Giannuzzi

Thank you for your attention

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