Pion observables in lattice QCD

and activities of the FLAG

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in memoriam of Jan Stern

Outline

Introduction

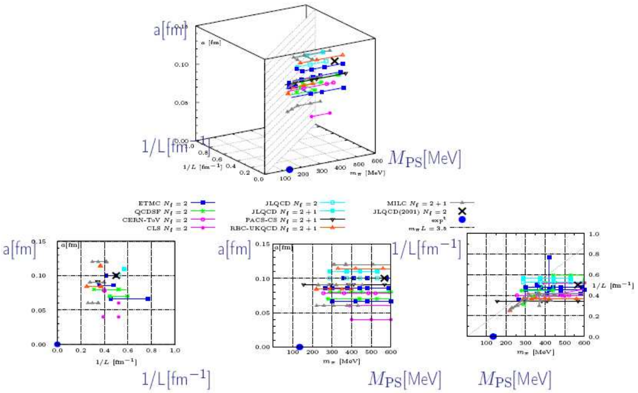
Highlights of recent developments

SU(2) low energy constants

FLAG activities

The current lattice landscape

The parameter landscape (thanks to G. Herdoiza)



From K. Jansen @ Lattice08

Remarkable recent developments (my picks)

- Simulations with two flavours (and soon three) of dynamical quarks respecting exact chiral symmetry

  JLQCD (overlap) – [Bern (perfect action)]

- $N_f = 2 + 1$ simulations close to the physical mass point

  PACS-CS – [BMW]

Exact chiral symmetry on the lattice

- exact chiral symmetry on the lattice (*i.e.* Ward identities have the same form as in the continuum) now theoretically understood Ginsparg-Wilson, Neuberger-Narayanan, Hasenfratz, Lüscher
- beautiful theoretical construction but very expensive to implement in simulations
- advantages:
  protection against powerlike divergences, *e.g.*

$$(\bar{q}q)_{\mathrm{continuum}} = Z_{qq}(\bar{q}q)_{\mathrm{lattice}} + \frac{Z_{\mathrm{mixing}}}{a^3}\mathbf{1}_{\mathrm{lattice}}$$

  chiral symmetry $\Rightarrow\ Z_{\mathrm{mixing}} = 0$
- difficulties:
  topology is a problem – the solution adopted by JLQCD leads to additional (powerlike) finite volume effects

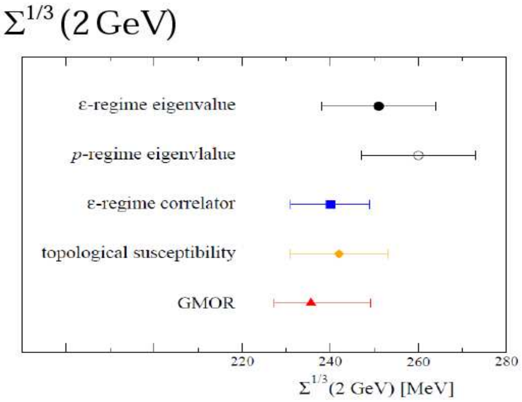
Exact chiral symmetry on the lattice

Why do not all the others do the same?

- it is a matter of how/where to invest resources: affording a "de luxe" solution concerning chiral symmetry implies investing less in taking the continuum/physical mass/large volume limits:
  - RBC/UKQCD have an almost perfect chiral symmetry (domain wall fermions) but have $N_f = 2 + 1$ simulations in larger volumes and smaller lattice spacings
  - PACS-CS (Wilson, NP improved) are doing $N_f = 2 + 1$ simulations almost at the physical point in larger volumes and smaller lattice spacings
- not all the quantities require having an exact chiral symmetry on the lattice (everybody wants to take the continuum limit eventually)
- and sometimes a clever idea may circumvent the absence of exact chiral symmetry: quark condensate extracted from the spectral density (Giusti, Lüscher)

A few physics results from JLQCD

The condensate has been extracted in five different ways



S. Hashimoto, talk@Lattice08

A few physics results from JLQCD

Comparison of lattice determinations of Σ

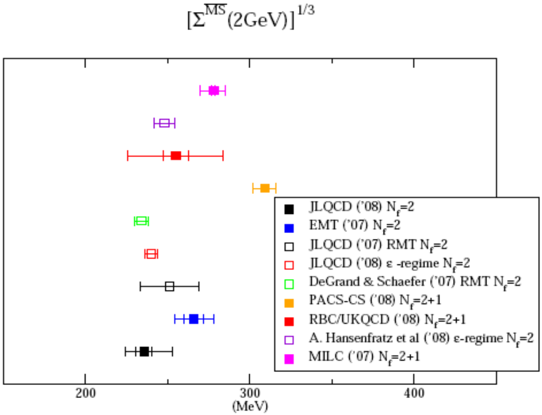


Fig. courtesy of S. Necco

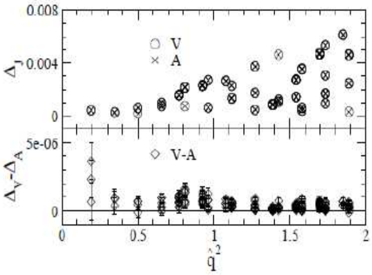
A few physics results from JLQCD

Vector and axial vector correlators:

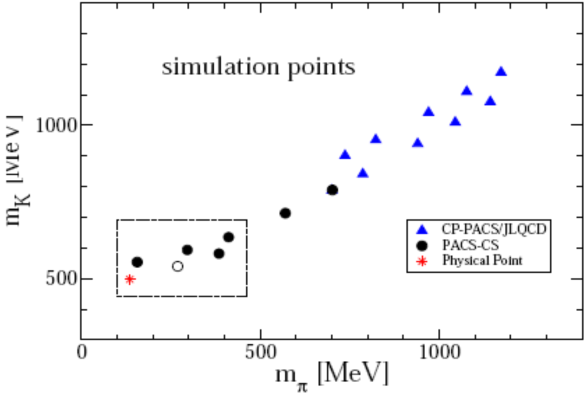
$$\int dx e^{iqx} \langle T J_\mu(x) J_\nu(0) \rangle = (g_{\mu\nu} q^2 - q_\mu q_\nu) \Pi_J^{(1)}(q^2) - q_\mu q_\nu \Pi_J^{(0)}(q^2)$$

Weinberg sum rule: $F_\pi^2 = -\lim_{q^2 \to 0} q^2 \left[\Pi_V^{(1+0)}(q^2) - \Pi_A^{(1+0)}(q^2) \right]$

On the lattice, these correlators contain extra contributions violating Lorentz invariance and current conservation $\equiv \Delta_J$



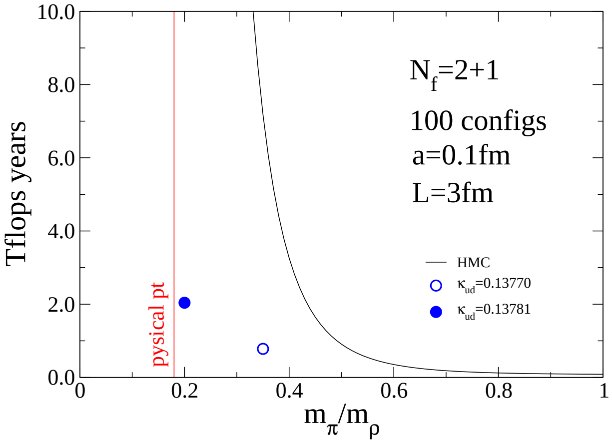
Approaching the physical mass point – PACS-CS



unitary points with $m_\pi \lesssim 400\text{MeV}$

Y. Kuramashi, talk@Lattice08

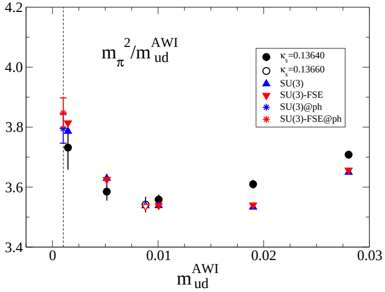
Approaching the physical mass point – PACS-CS

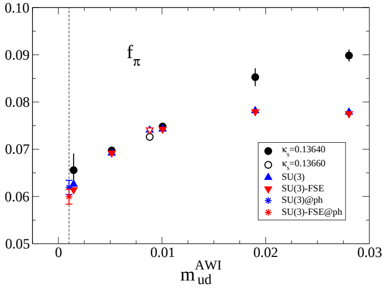


Y. Kuramashi, talk@Lattice08

Chiral fits

CHPT SU(3) NLO fits

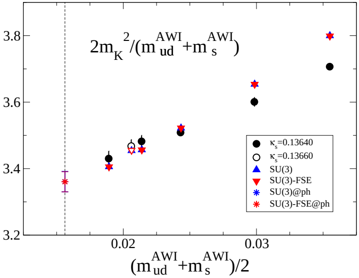


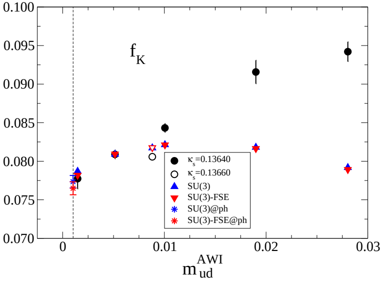


PACS-CS arXiv:0807.1661

Chiral fits

CHPT SU(3) NLO fits





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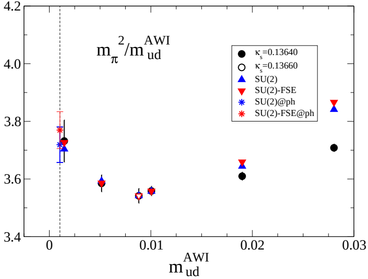
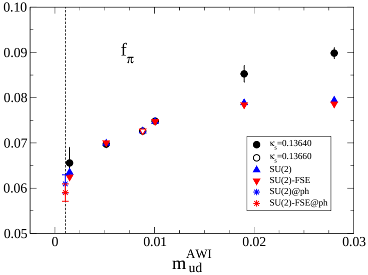
Chiral fits

CHPT SU(2) NLO fits





PACS-CS arXiv:0807.1661

Chiral fits

- even when reaching such small up and down quark masses, SU(3) fits do not work well
- similar conclusions had been reached by RBC/UKQCD
- to check the convergence of the SU(3) chiral expansion, it would be interesting to reduce the strange quark mass even below the physical value
- SU(2) fits work well – the extraction of the LEC's close to the physical point should be very reliable

... we have come a long way from Myint and Rebbi (94)

$\bar{\ell}_3$ and $\bar{\ell}_4$

group	ChPT	$\bar{\ell}_3$	$\bar{\ell}_4$
		$N_f = 2+1$	
PACS-CS	SU(2) no FV	3.23(21)	4.10(20)
	SU(2) FV	3.14(23)	4.04(19)
	SU(3) no FV	3.50(11)	4.22(10)
	SU(3) FV	3.47(11)	4.21(11)
RBC/UKQCD	SU(3)	2.87(28)	4.10(5)
	SU(2)	3.13(33)(24)	4.43(14)(77)
MILC	SU(3)	$1.1(6)\left(^{+1.0}_{-1.5}\right)$	$4.4(4)\left(^{+4}_{-1}\right)$
		$N_f = 2$	
JLQCD	SU(2)	$3.44(57)\left(^{+0}_{-68}\right)\left(^{+32}_{-0}\right)$	$4.14(26)\left(^{+49}_{-0}\right)\left(^{+32}_{-0}\right)$
ETM	SU(2)	3.44(8)(35)	4.61(4)(11)
CERN	SU(2)	3.0(5)(1)	—
		pheno	
CGL	SU(2)	2.9(2.4)	4.4(2)

$\bar{\ell}_3$ and $\bar{\ell}_4$

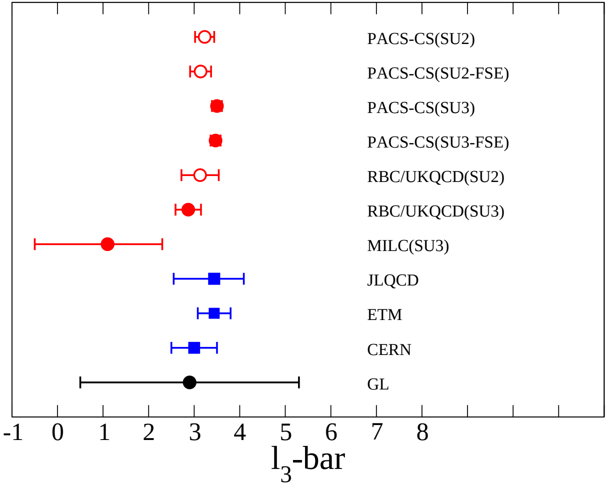


Figure from PACS-CS arXiv/0807.1661

$\bar{\ell}_3$ and $\bar{\ell}_4$

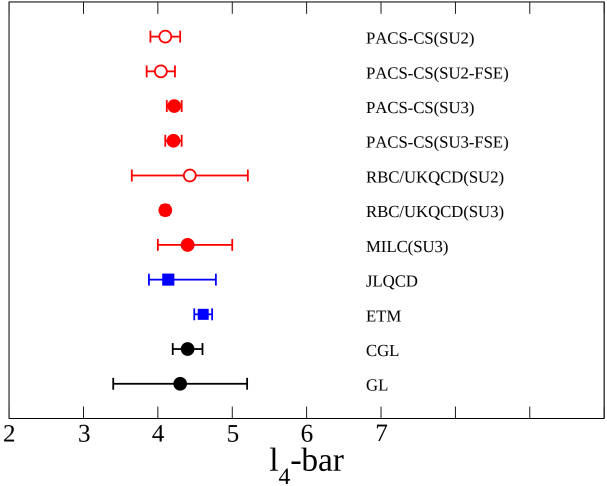


Figure from PACS-CS arXiv/0807.1661

Implications for the $\pi\pi$ scattering lengths

Precise prediction for the $\pi\pi$ S wave scattering lengths

$$\begin{aligned} a_0^0 &= 0.220 \pm 0.001 + 0.027\Delta_{r^2} - 0.0017\Delta\ell_3 \\ 10 \cdot a_0^2 &= -0.444 \pm 0.003 - 0.04\Delta_{r^2} - 0.004\Delta\ell_3 \end{aligned}$$

where

$$\langle r^2\rangle_s = 0.61\mathrm{fm}^2(1+\Delta_{r^2}) \qquad \bar{\ell}_3 = 2.9 + \Delta\ell_3$$

Adding errors in quadrature $[\Delta_{r^2} = 6.5\%,\ \Delta\ell_3 = 2.4]$

$$\begin{aligned} a_0^0 &= 0.220 \pm 0.005 \\ 10 \cdot a_0^2 &= -0.444 \pm 0.01 \\ a_0^0 - a_0^2 &= 0.265 \pm 0.004 \end{aligned}$$

Implications for the $\pi\pi$ scattering lengths

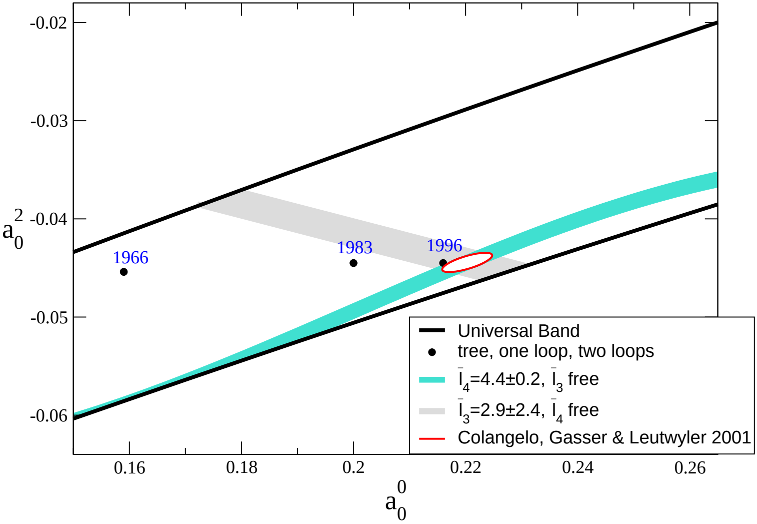
- the theoretical prediction relies on dispersion relations and chiral symmetry for fixing the two subtraction constants
- Low–energy theorem for the $\pi\pi$ scattering amplitude

$$A(s,t,u) = \frac{s - M^2}{F^2} + \mathcal{O}(p^4) \longrightarrow \frac{s - M_\pi^2}{F_\pi^2} + \mathcal{O}(p^4)$$

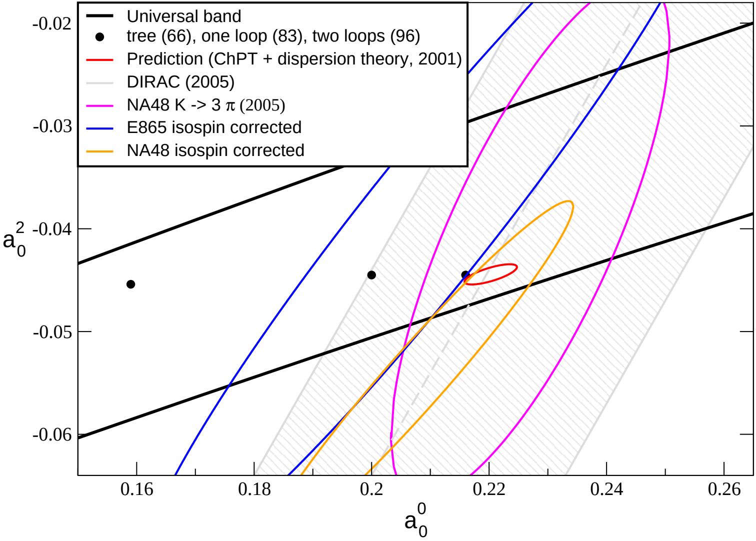
- the two subtraction constants are essentially given by M_π and F_π, up to higher order corrections (which matter and have been taken into account) – the most important ones are:

$$\begin{aligned} M_\pi^2 &= M^2 \left(1 - \frac{M^2}{32\pi^2 F^2}\bar{\ell}_3 + O(M^4)\right) \\ F_\pi &= F \left(1 + \frac{M^2}{16\pi^2 F^2}\bar{\ell}_4 + O(M^4)\right) \end{aligned}$$

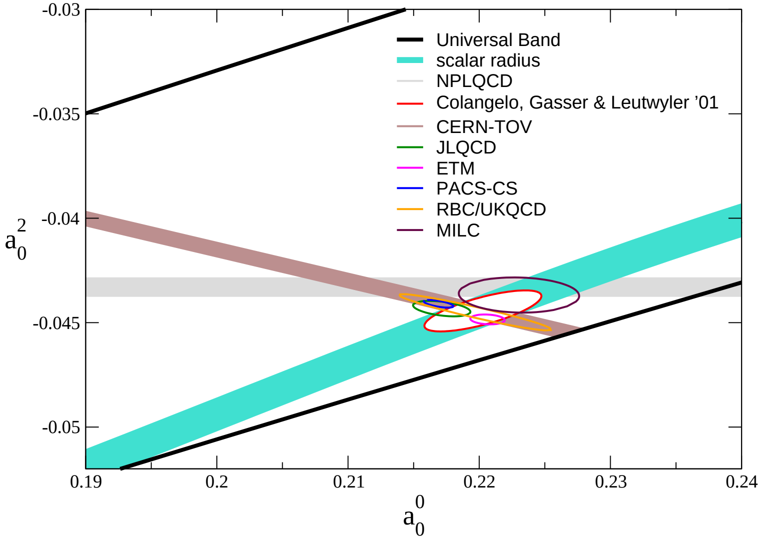
Implications for the $\pi\pi$ scattering lengths



Implications for the $\pi\pi$ scattering lengths



Implications for the $\pi\pi$ scattering lengths



Lattice calculation of a_0^2 (NPLQCD coll.)

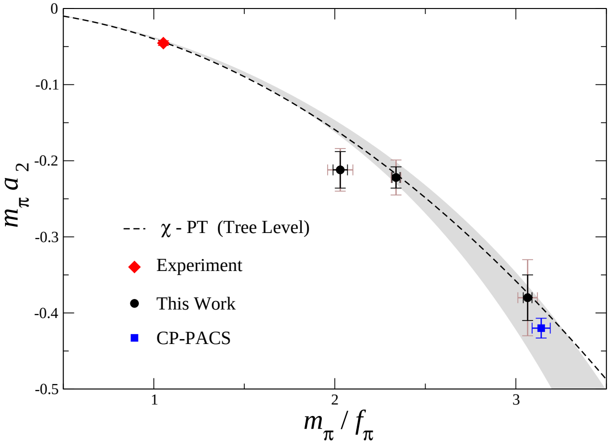
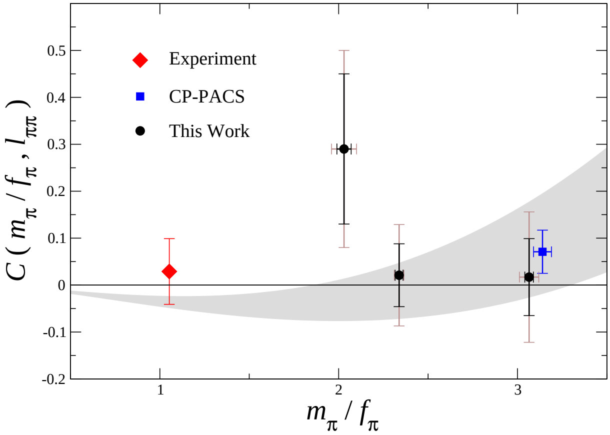


Figure from NPLQCD 06

Lattice calculation of a_0^2 (NPLQCD coll.)



$C \equiv a_0^2/a_0^2(\mathrm{LO}) - 1$

Figure from NPLQCD 06

Charge radius of the pion

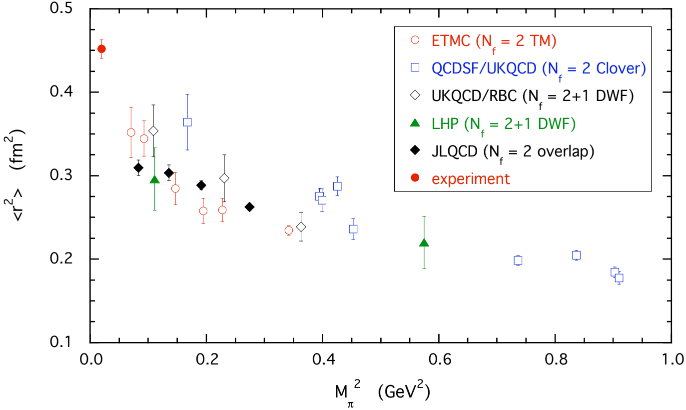
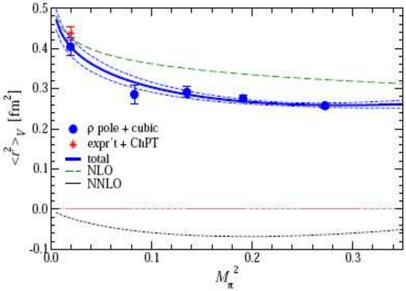
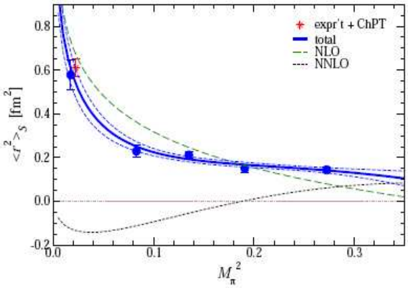


Fig. courtesy of S. Simula

Scalar and charge radius of the pion – JLQCD/TWQCD





- dof = 6, $\chi^2/\text{dof} = 1.3$

T. Kaneko, talk@Lattice08

Update on the FLAG

FLAG=Flavianet Lattice Averaging Group

Members:
Gilberto Colangelo (Bern)
Stephan Dürr (Wuppertal, BMW)
Andreas Jüttner (Mainz, RBC/UKQCD)
Laurent Lellouch (Marseille, BMW)
Heiri Leutwyler (Bern)
Vittorio Lubicz (Rome 3, ETM)
Silvia Necco (Valencia, Alpha)
Chris Sachrajda (Southampton, RBC/UKQCD)
Silvano Simula (Rome 3, ETM)
Tassos Vladikas (Rome 2, Alpha)
Hartmut Wittig (Mainz, Alpha)

Update on the FLAG

FLAG=Flavianet Lattice Averaging Group

Aims:

- provide up-to-date information on phenomenologically relevant quantities determined on the lattice in a way which is easily accessible to non-experts
- summarize the most important aspects of the calculations
  - lattice action
  - minimal value and range of quark masses
  - minimal value and range of lattice spacing
  - maximal value and range of lattice volumes
  - renormalization method

  in a unified and easy to read (color coding) manner
- despite the name we may not provide averages, unless this makes sense beyond any doubt
- provide a dictionary on lattice actions and other lattice-specific subtleties for non experts

Update on the FLAG

FLAG=Flavianet Lattice Averaging Group

Status:

- we formed the FLAG in Orsay in November 2007
- had a meeting in Bern in March 2008, discussed and agreed on how to proceed and distributed tasks
- we decided to concentrate on a small set of pion- and kaon-related quantities:
  LEC's, light quark masses, form factors, and B_K
- the goal was to have first drafts ready for this meeting (and we have them) and prepare the next step: go public
- if you have suggestions, let us know, or otherwise stay tuned!

Color coding

Already used sometimes by lattice people in summaries
Our current working hypothesis

Color coding

Already used sometimes by lattice people in summaries
Our current working hypothesis

- chiral extrapolation
  - 🟢 using NLO or NNLO CHPT formulae
  - 🟠 phenomenologically motivated chiral extrap.
  - 🔴 no chiral extrapolation

Color coding

Already used sometimes by lattice people in summaries
Our current working hypothesis

- chiral extrapolation
- smallest pion mass
  - (green) min $M_\pi < 300$ MeV
  - (orange) min $M_\pi < 400$ MeV
  - (red) min $M_\pi > 400$ MeV

Color coding

Already used sometimes by lattice people in summaries
Our current working hypothesis

- chiral extrapolation
- smallest pion mass
- finite volume effects
  - (green) volume scaling study (in combination with CHPT)
  - (orange) CHPT
  - (red) no assessment of finite volume corrections

Color coding

Already used sometimes by lattice people in summaries
Our current working hypothesis

- chiral extrapolation
- smallest pion mass
- finite volume effects
- continuum extrapolation
  - ● (green) $O(a)$-improved or chiral fermions, at least two a's, min $a < 0.1$ fm
  - ● (orange) $O(a)$-improved or chiral fermions, one $a \lesssim 0.1$ fm
  - ● (red) larger lattice spacing or no improvement

Color coding

Already used sometimes by lattice people in summaries

Example

	$\bar{\ell}_3$	N_f	chiral extrapolation	smallest pion mass	finite volume errors	continuum extrapolation
CERN	3.0(5)(1)	2	🟢	🟠	🟠	🟢
ETM	3.44(8)(35)	2	🟢	🟠	🟠	🟠
MILC	$1.1(6)(^{+1.0}_{-0.5})$	2+1	🟢	🟢	🟢	🟢
RBC/UKQCD	3.13(33)(24)	2+1	🟢	🟠	🟠	🟠
JLQCD/TWQCD	$3.44(57)(^{+32}_{-68})$	2	🟢	🟢	🟠	🟠
PACS-CS	3.14(23)(?)	2+1	🟢	🟢	🟠	🟠

Conclusions

- Recent developments in lattice QCD are breathtaking
- The physical mass point is within reach, and with it a precision determination of SU(2) LEC's
- Failure of SU(3) CHPT calls for better understanding (either it will go away or will have implications for the phenomenology)
- The FLAG is working and will go public soon