ELECTROMAGNETIC EFFECTS IN $K_{\ell 3}$ DECAYS

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$K(p_K) \to \pi(p_\pi)\, \ell^+(p_\ell)\, \nu_\ell(p_\nu)$

Invariant amplitude

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{us}^* \, \ell^\mu \, C_K \left[f_+^{K\pi}(t)\,(p_K + p_\pi)_\mu + f_-^{K\pi}(t)\,(p_K - p_\pi)_\mu \right]$$

$$\ell^\mu = \bar{u}(p_\nu)\,\gamma^\mu\,(1-\gamma_5)\,v(p_\ell) \qquad t = (p_K - p_\pi)^2$$

$$C_K = \begin{cases} 1 & \text{for } K_{e3}^0 \\ \frac{1}{\sqrt{2}} & \text{for } K_{e3}^+ \end{cases}$$

Form factor parametrization

vector form factor $f_+^{K\pi}(t)$

scalar form factor $f_0^{K\pi}(t) = f_+^{K\pi}(t) + t/(M_K^2 - M_\pi^2) f_-^{K\pi}(t)$

$$\bar{f}_+^{K\pi}(t) \equiv \frac{f_+^{K\pi}(t)}{f_+^{K\pi}(0)} = 1 + \lambda_+ \frac{t}{M_{\pi^\pm}^2} + \frac{1}{2}\lambda_+'' \frac{t^2}{M_{\pi^\pm}^4}$$

$$\bar{f}_-^{K\pi}(t) \equiv \frac{f_-^{K\pi}(t)}{f_+^{K\pi}(0)} = \frac{M_K^2 - M_\pi^2}{M_{\pi^\pm}^2} \left(\lambda_0 - \lambda_+ - \frac{\lambda_+''}{2} \frac{t}{M_{\pi^\pm}^2} \right)$$

Decay distribution

$$\frac{d\Gamma^{(0)}}{dy\,dz} = \frac{G_F^2\,|V_{us}|^2\,M_K^5\,C_K^2}{128\,\pi^3}\,|f_+^{K\pi}(0)|^2\,\bar{\rho}^{(0)}(y,z)$$

$$\bar{\rho}^{(0)}(y,z) = A_1^{(0)}(y,z)\,|\bar{f}_+^{K\pi}(t)|^2 + A_2^{(0)}(y,z)\,\bar{f}_+^{K\pi}(t)\bar{f}_-^{K\pi}(t) + A_3^{(0)}(y,z)\,|\bar{f}_-^{K\pi}(t)|^2$$

$$\begin{aligned}
A_1^{(0)}(y,z) &= 4(z+y-1)(1-y) + r_\ell(4y+3z-3) - 4r_\pi + r_\ell(r_\pi - r_\ell) \\
A_2^{(0)}(y,z) &= 2r_\ell(3-2y-z+r_\ell-r_\pi) \\
A_3^{(0)}(y,z) &= r_\ell(1+r_\pi-z-r_\ell)
\end{aligned}$$

$$z = \frac{2p_\pi\cdot p_K}{M_K^2} = \frac{2E_\pi}{M_K}\,, \quad y = \frac{2p_K\cdot p_\ell}{M_K^2} = \frac{2E_\ell}{M_K}\,, \quad r_\ell = \frac{m_\ell}{M_K^2}\,, \quad r_\pi = \frac{m_\pi}{M_K^2}$$

Decay rate

$$\Gamma^{(0)}(K_{\ell 3}) = \frac{G_{\rm F}^2\,|V_{us}|^2\,M_K^5\,C_K^2}{128\,\pi^3}\,|f_+^{K\pi}(0)|^2\,I_{K\ell}^{(0)}(\lambda_i)$$

$$I_{K\ell}^{(0)}(\lambda_i) = \int_{\mathcal{D}_3} dy\,dz\,\bar{\rho}^{(0)}(y,z)$$

3-body Dalitz plot $\mathcal{D}_3$:

$$2\sqrt{r_\ell} \le y \le 1 + r_\ell - r_\pi, \quad a(y) - b(y) \le z \le a(y) + b(y)$$

$$a(y) = \frac{(2-y)(1+r_\ell+r_\pi-y)}{2(1+r_\ell-y)}, \quad b(y) = \frac{\sqrt{y^2-4r_\ell}(1+r_\ell+r_\pi-y)}{2(1+r_\ell-y)}$$

Radiative corrections

Short distance electroweak corrections

universal factor $S_{\rm ew} = 1 + \frac{2\alpha}{\pi}\left(1 - \frac{\alpha_s}{4\pi}\right) \times \log\frac{M_Z}{M_\rho} + \mathcal{O}(\frac{\alpha\alpha_s}{\pi^2})$ **Sirlin**

Long distance EM corrections

CHPT with virtual photons and leptons **Knecht, N., Rupertsberger, Talavera 2000**

$K_{\ell 3}$ **(general formulae): Cirigliano, Knecht, N., Rupertsberger, Talavera 2002**

numerics for K_{e3}**: Cirigliano, N., Pichl 2004**

numerics for K_{e3} **(update) and** $K_{\mu 3}$ **(new): Cirigliano, Giannotti, N. 2008**

analysis at $\mathcal{O}(e^2 p^2)$

$$\bar{u}(p_\nu)\,\gamma^\mu\,(1-\gamma_5)\,v(p_\ell)\cdot(p_K \pm p_\pi)_\mu \sim O(p^2)$$

$\longrightarrow$ **EM corrections to $f_\pm^{K\pi}$ start at $\mathcal{O}(e^2 p^0)$**

$$y, z, r_\ell, r_\pi \sim \mathcal{O}(1) \;\longrightarrow\; A^{(0)}_{1,2,3} \sim \mathcal{O}(1)$$

$\longrightarrow$ **EM corrections to decay distributions and rates start at $\mathcal{O}(e^2 p^0)$**

Virtual photons

$$f_+^{K\pi}(t) \;\longrightarrow\; f_+^{K\pi}(t) + \delta f_+^{K\pi}(v) + \frac{\alpha}{4\pi}\,\Gamma_c(v, m_\ell^2, M_c^2; M_\gamma^2)$$
$$f_-^{K\pi}(t) \;\longrightarrow\; f_-^{K\pi}(t) + \delta f_-^{K\pi}(v) \qquad (1)$$

charged meson mass M_c; $v = u \equiv (p_K - p_\ell)^2$ $(K_{\ell 3}^\pm)$; $v = s \equiv (p_\pi + p_\ell)^2$ $(K_{\ell 3}^0)$

$\Gamma_c(v, m_\ell^2, M_c^2; M_\gamma^2)$: universal soft photon corrections (IR divergent)

$\delta f_\pm^{K\pi}$: structure dependent terms

$\longrightarrow$ shift to differential distribution of $\mathcal{O}(e^2 p^0)$:

$$\delta\bar{\rho}^{\mathrm{EM-virtual}}(y,z) = A_1^{(0)}(y,z)\cdot\left[2\,\delta f_+^{K\pi}(v) + \frac{\alpha}{2\pi}\Gamma_c(v, m_\ell^2, M_c^2; M_\gamma^2)\right] + A_2^{(0)}(y,z)\cdot\delta f_-^{K\pi}(v)$$

Real photons

only inclusive sum of $K \to \pi\ell\nu$ and $K \to \pi\ell\nu + n\gamma$ IR finite and observable

need only $K \to \pi\ell\nu\gamma$ for our analysis

$$\sum_{\text{pol}} |\mathcal{M}_\gamma(K \to \pi\ell\nu_\ell\gamma)|^2 = \frac{e^2 G_F^2 |V_{us}|^2 C_K^2}{2} \left[T_{\text{IR}}^{K\ell} + T_{\text{IB}}^{K\ell} \right]$$

integrate over all variables except $y,\ z \ \longrightarrow$

$$\delta\bar{\rho}^{\text{EM}-\text{real}}(y,z) = A_1^{(0)}(y,z) \cdot \frac{\alpha}{\pi} I_0(y,z;M_\gamma) + \Delta_1^{\text{IB}}(y,z)$$

$T_{\text{IR}} \ \longrightarrow \ I_0(y,z;M_\gamma)$

$T_{\text{IB}} \ \longrightarrow \ \Delta_1^{\text{IB}}$

combine virtual and real photon contributions $\longrightarrow$ **IR finite function**

$$\Delta^{\mathrm{IR}}(y,z) = \frac{\alpha}{\pi}\left[I_0(y,z;M_\gamma) + \frac{1}{2}\Gamma_c(v,m_\ell^2,M^2;M_\gamma^2)\right]$$

$\longrightarrow$ **IR finite shift to differential distribution:**

$$\delta\bar{\rho}^{\mathrm{EM}}(y,z) = A_1^{(0)}\left[\Delta^{\mathrm{IR}} + 2\delta f_+^{K\pi}\right] + \Delta_1^{\mathrm{IB}} + A_2^{(0)}\delta f_-^{K\pi}$$

Decay distribution with EM corrections

$$\frac{d\Gamma}{dy\,dz} = \frac{G_F^2\,|V_{us}|^2\,M_K^5\,C_K^2}{128\,\pi^3}\,S_{ew}\,|f_+^{K\pi}(0)|^2\left[\bar{\rho}^{(0)}(y,z) + \delta\bar{\rho}^{EM}(y,z)\right]$$

Fully inclusive decay rate $\Gamma(K_{\ell 3[\gamma]})$

$$\Gamma = \frac{G_F^2\,|V_{us}|^2\,M_K^5\,C_K^2}{128\,\pi^3}\,S_{ew}\,|f_+^{K^0\pi^-}(0)|^2\,I_{K\ell}^{(0)}(\lambda_i)\left[1 + \delta_{EM}^{K\ell} + \delta_{SU(2)}^{K\pi}\right]$$

$$\delta_{EM}^{K\ell} = \delta_{EM}^{K\ell}(\mathcal{D}_3) + \delta_{EM}^{K\ell}(\mathcal{D}_{4-3}), \qquad \delta_{SU(2)}^{K\pi} = \left(\frac{f_+^{K\pi}(0)}{f_+^{K^0\pi^-}(0)}\right)^2 - 1$$

results Cirigliano, Giannotti, Neufeld 2008

	$I^{(0)}_{K\ell}(\lambda_i)$	$\delta^{K\ell}_{\rm EM}(\mathcal{D}_3)(\%)$	$\delta^{K\ell}_{\rm EM}(\mathcal{D}_{4-3})(\%)$	$\delta^{K\ell}_{\rm EM}(\%)$
K^0_{e3}	0.103070	0.50	0.49	0.99 ± 0.22
$K^\pm_{e3}$	0.105972	-0.35	0.45	0.10 ± 0.25
$K^0_{\mu 3}$	0.068467	1.38	0.02	1.40 ± 0.22
$K^\pm_{\mu 3}$	0.070324	0.007	0.009	0.016 ± 0.25

errors: estimates of higher-order contributions

K_i^r **from Ananthanarayan, Moussallam 2004**

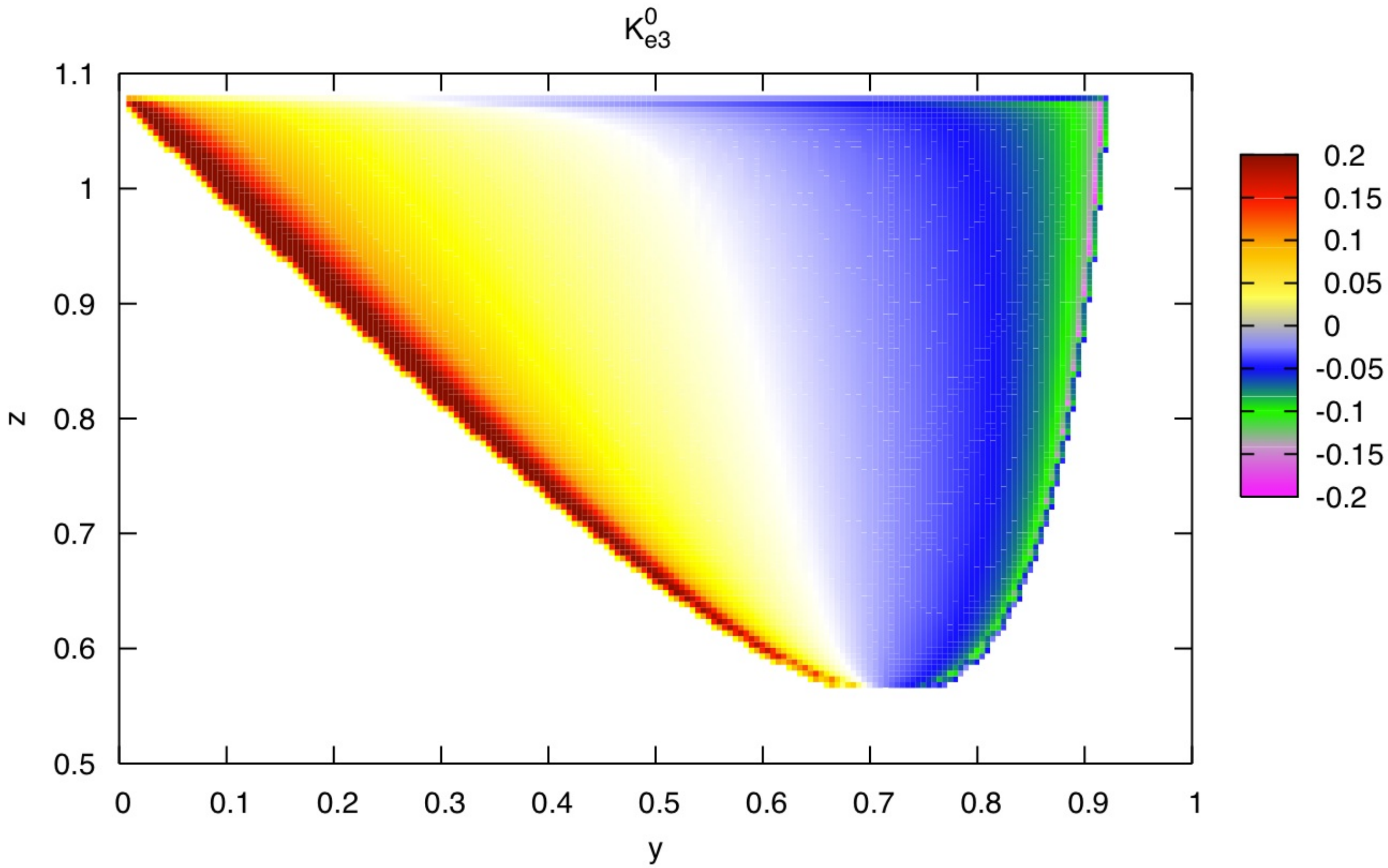
X_i^r **from Descotes-Genon, Moussallam 2005**

“soft-photon factorization”

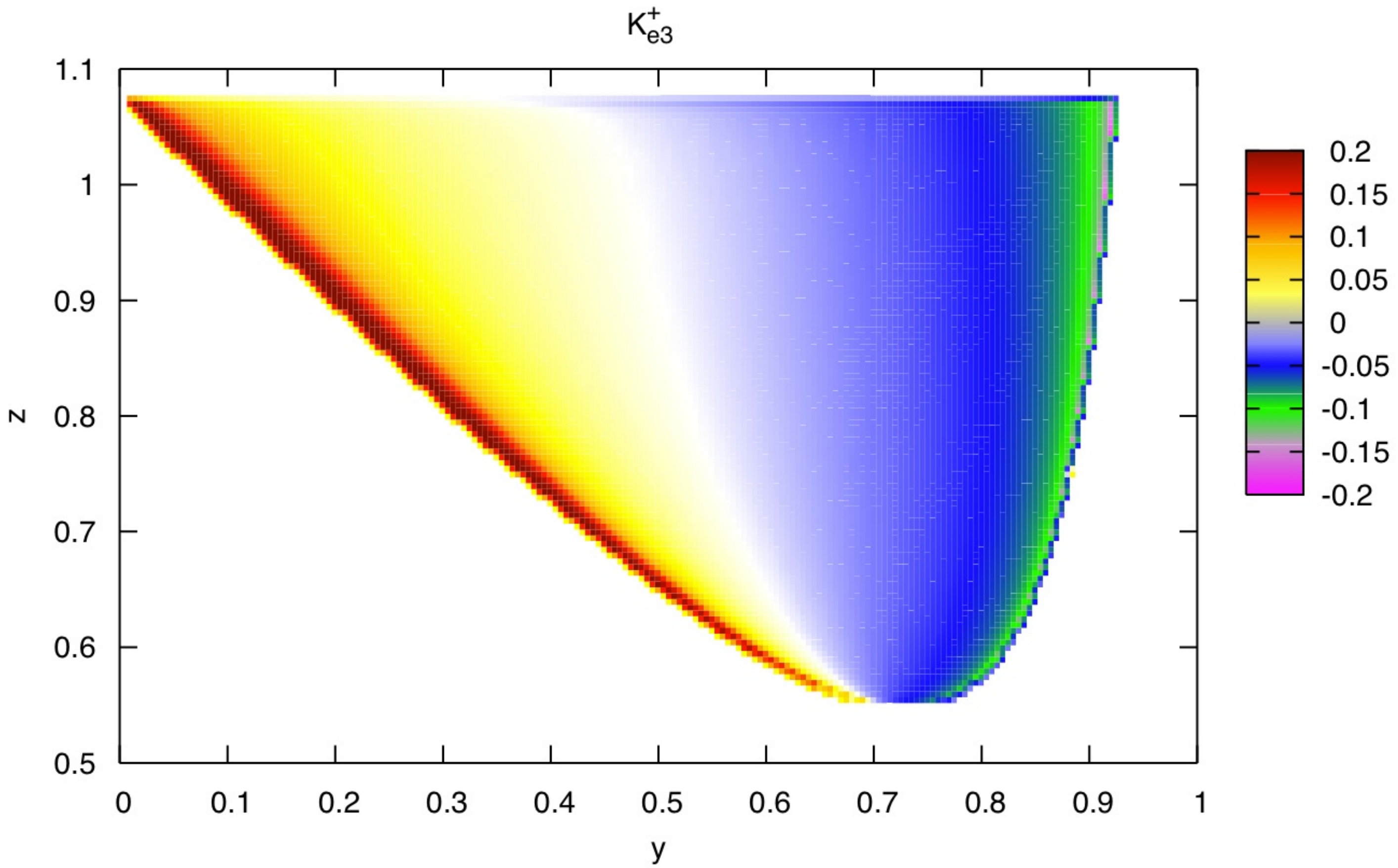
(includes incomplete higher order terms in the chiral expansion)

$\longrightarrow$ **validates estimates of theoretical uncertainties**

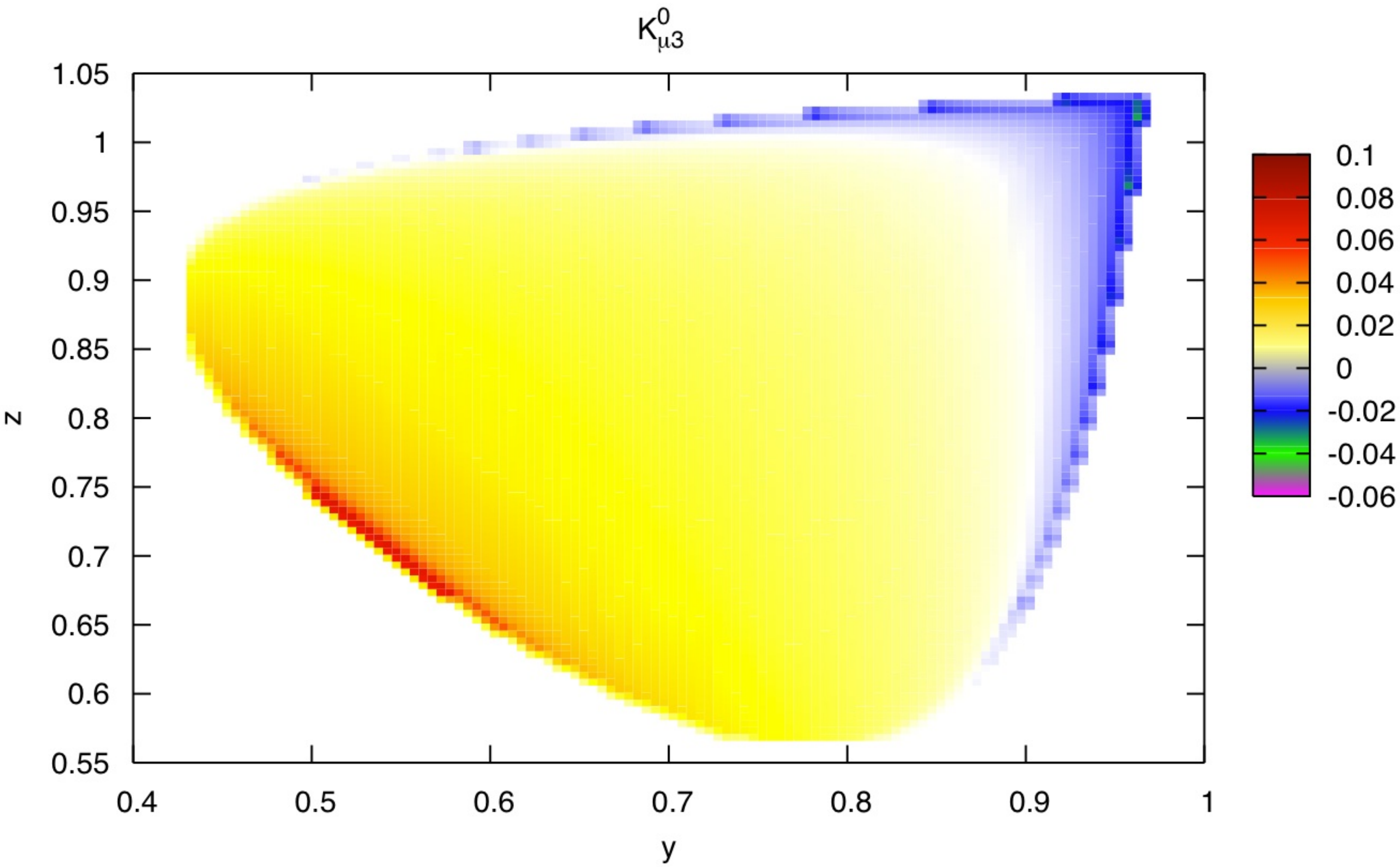
	$\delta_{\mathrm{EM}}^{K\ell}(\mathcal{D}_3)(\%)$	$\delta_{\mathrm{EM}}^{K\ell}(\mathcal{D}_{4-3})(\%)$	$\delta_{\mathrm{EM}}^{K\ell}(\%)$
K_{e3}^0	0.41	0.59	1.0
$K_{e3}^\pm$	-0.564	0.528	-0.04
$K_{\mu3}^0$	1.57	0.04	1.61
$K_{\mu3}^\pm$	-0.006	0.011	0.005



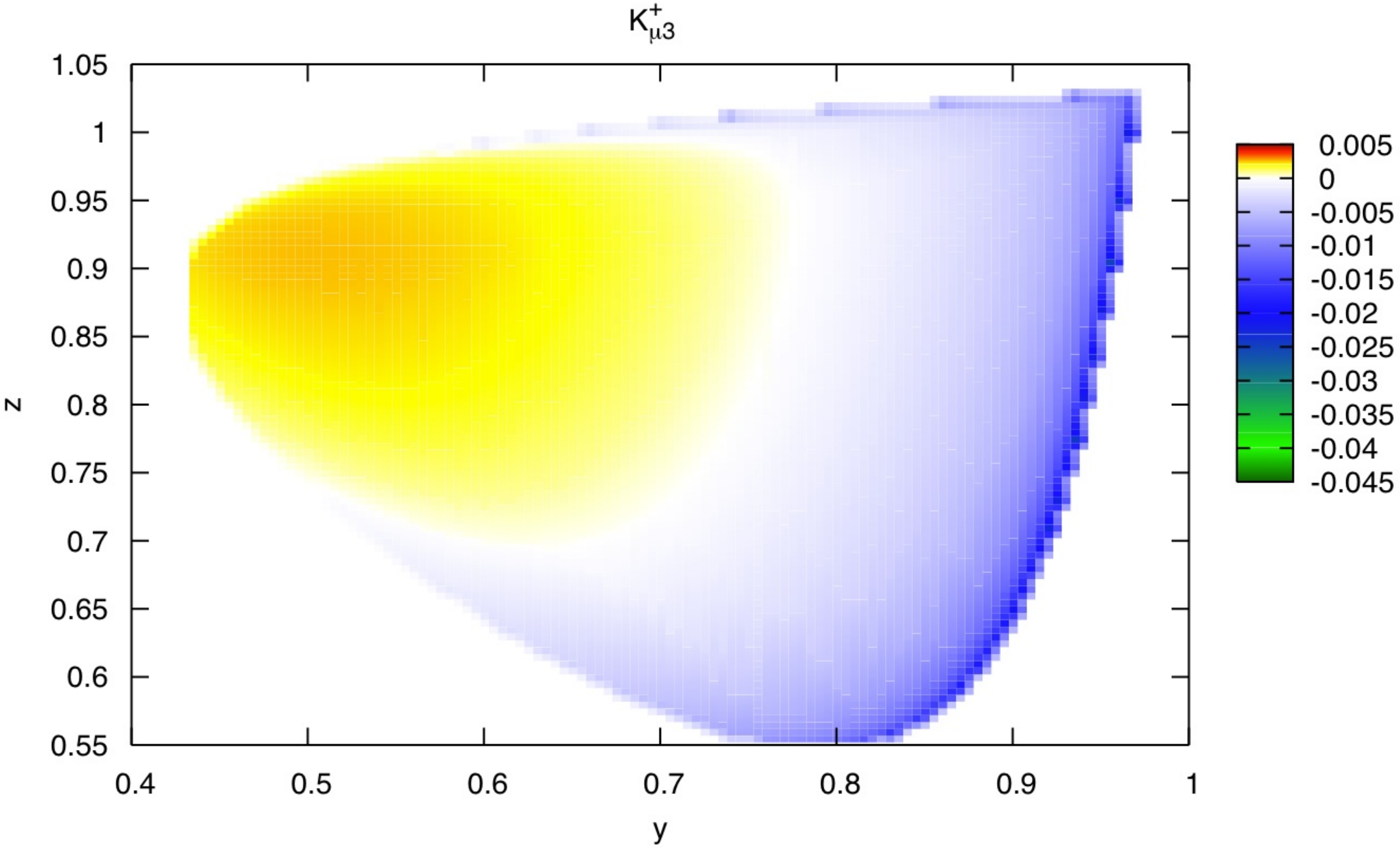
EM correction to differential distribution of K^0_{e3}: $\delta\bar{\rho}^{\mathrm{EM}}(y,z)/\bar{\rho}^{(0)}(y,z)$



EM correction to differential distribution of K^+_{e3}: $\delta\bar{\rho}^{\rm EM}(y,z)/\bar{\rho}^{(0)}(y,z)$



EM correction to differential distribution of $K^0_{\mu 3}$: $\delta\bar{\rho}^{\mathrm{EM}}(y,z)/\bar{\rho}^{(0)}(y,z)$



EM correction to differential distribution of $K^+_{\mu 3}$: $\delta\bar{\rho}^{\mathrm{EM}}(y,z)/\bar{\rho}^{(0)}(y,z)$

Summary

- **CHPT suitable framework for EM corrections in semileptonic decays**
- **EM corrections for all K_{l3} decay modes at fixed chiral order e^2p^2**
- **fully inclusive prescription of real photon emission**
- **update of structure dependent EM contributions**
- **proper treatment of EM corrections mandatory in analysis of $K_{\ell 3}$ data**