Moments of Light-Cone Distribution Amplitudes from Lattice QCD

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Collaborators

This work is part of the UKQCD/RBC Collaborations' $N_f = 2 + 1$ **domain-wall fermion** phenomenology programme.

Distribution amplitudes work is primarily done by the following UKQCD subset:

- Southampton:
  - Dirk Brömmel
  - Michael Donnellan ($\rightarrow$ Zeuthen)
  - Jonathan Flynn
  - Andreas Jüttner ($\rightarrow$ Mainz)
  - Chris Sachrajda
- Edinburgh:
  - Peter Boyle
  - Chris Kelly

Introduction

Distribution amplitudes (DAs):

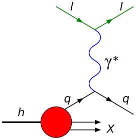
- are hadronic properties, encoding the non-perturbative QCD effects that show up in the factorization of **hard exclusive processes**.
- originated in **form factors** at high-Q^2, but now: flavour physics interest due to applicability to **B-decays** (so CKM, UT, CPv, NP).
- are accessible on the lattice via their **lowest moments**.

This talk will cover:

- **context -** quick review of DAs' importance, physical meaning, status.
- the most important points needed for a fair appraisal of the **lattice calculation**.
- PRELIMINARY **results** for 1st, 2nd moments of pseudoscalar mesons π, K and (longitudinally-polarized) vector mesons ρ, K^*, ϕ.

Factorization: Inclusive vs. Exclusive Case

- **DIS**: $e^- h \to e^- X$. Destroy, but ignore h. $d\sigma/d\Omega$ cares only about charge, momentum of struck parton **- PDFs**. Its **relation to rest of $\mathbf{h}$** irrelevant.



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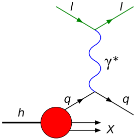

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- PDFs are **single-particle** probabilities, **not amplitudes**.
  Correlations between quarks and gluons, roles of different Fock states?
  *Inclusive processes depend on hadrons' partonic **content**, not **structure**.*

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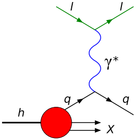

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- **Elastic**: $d\sigma/d\Omega$ **dominated by select partonic configurations**. Why? Partons must reform $h \implies$ all must be turned to final direction. Need **hard gluon** exchanges $\implies$ **valence Fock state** dominates. (**"Collinear factorization"**, *but*: soft effects).

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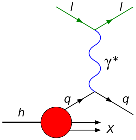

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- DA $\sim$ **overlap with valence Fock state**. Mesons: collinear $q\bar{q}$ pair, small transverse separation, momentum fractions u and $\bar{u}(= 1-u)$:

$$\phi_\pi(u, Q^2) = \int^{Q^2} d^2k_\perp \; \psi_{q\bar{q}/\pi}(u, \vec{k}_\perp, S=0).$$

Distribution Amplitudes

- So DAs originated in **collinear factorization applied to form factors** like: $e^- \pi \to e^- \pi$, $\gamma\gamma^* \to \pi$. At large Q^2, can write:

$$F_\pi(Q^2) = \int_0^1 dx \int_0^1 dy\, \phi_\pi(y, Q^2) T_H(x, y, Q^2) \phi_\pi(x, Q^2)$$

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- Now, needed for **flavour physics**. Important in both main approaches to factorization for B-decays: **'QCD Factorization'** (BBNS) and **Soft Collinear Effective Theory**. Heavy-meson case more difficult (e.g. soft effects more important, want inverse moment), but **light-meson** DAs still needed for decays like $B \to \pi K$.

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- DAs **defined** through hadronic light-cone matrix elements, e.g.:

$$\langle 0|\bar{q}(z)\gamma_\rho\gamma_5 \mathscr{P}(z,-z)q(-z)|\pi(p)\rangle|_{z^2=0} \equiv f_\pi(ip_\rho)\int_0^1 du\, e^{i(u-\bar{u})p.z}\phi_\pi(u,\mu)$$

  which reduce to the decay constants as $z \to 0$.

Moments

- Ordinary **moments**: $\langle \xi^n \rangle \equiv \int d\xi \, \xi^n \phi(\xi, Q^2)$ ($\xi \equiv u - \bar{u} = 2u - 1$).
  Get **local matrix elements** by Taylor expanding around $z = 0$, e.g.:

$$\langle 0 | \bar{q} \gamma_{\{\rho} \gamma_5 \overleftrightarrow{D}_{\mu\}} q | \pi(p) \rangle = f_\pi \langle \xi \rangle p_\rho p_\mu$$

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- Today: **conformal expansion** ($\sim$ partial-wave expansion).
  Asymptotic $Q^2 \to \infty$ behaviour known from pQCD: $\phi_{as}(u) = 6u\bar{u}$.
  Expand in **Gegenbauer** polynomials $C_n^{3/2}(2u-1)$, moments $a_n(Q^2)$:

$$\phi(u, Q^2) = \phi_{as}(u) \left[1 + \sum_{n=1} a_n(Q^2) C_n^{3/2}(2u-1) \right]$$

  Simple algebraic relations: $a_1 = \frac{5}{3} \langle \xi^1 \rangle \quad a_2 = \frac{7}{12} \left(5 \langle \xi^2 \rangle - 1 \right)$.

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  Simple algebraic relations: $a_1 = \frac{5}{3} \langle \xi^1 \rangle \quad a_2 = \frac{7}{12} \left(5 \langle \xi^2 \rangle - 1 \right)$.
- Single anomalous dimension (at LO). Hierarchy: $\gamma_{n+1} > \gamma_n > 0$.
  So **lowest moments** dominate at high enough scales.

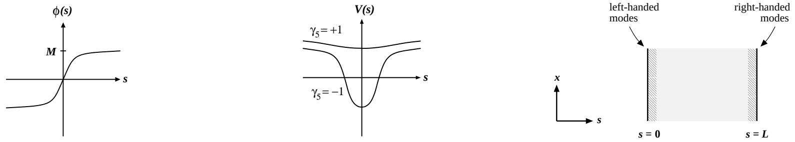
Status

- Mesons of definite G-parity: $u \leftrightarrow \bar{u}$ symmetry.
  So **odd moments $\mathbf{a_{2n+1}}$ vanish** for π, ρ, ϕ ...
  ... and $\langle \xi^1 \rangle_K$, $\langle \xi^1 \rangle^{\parallel}_{K^*}$ are (important) **SU(3)-breaking** effects.
- DA normalizations are the **decay constants** - partly accessible by theory, partly by experiment (f_V^T/f_V results: arXiv:0804.0473).
  DA **shape** not well-constrained experimentally (lack of accurate data, higher-twist contamination).
- Main tool was **QCD Sum Rules**, but: irreducible $\sim 10-15\%$ error.
- Early lattice attempts ($\sim$ 1990) are best termed 'exploratory'.
  Now producing **phenomenologically useful numbers**.
  Also QCDSF/UKQCD $N_f = 2$ improved Wilson programme (with recent results for nucleons)
- Representative numbers for $\langle \xi^1 \rangle_K$:

$$a_1^K(1\ \mathrm{GeV}) = 0.05(2) \quad 0.10(12) \quad 0.050(25) \quad 0.06(3)$$

Formulation & Simulation Details

- Calculations mostly performed on custom-made **QCDOC**.
  Full QCD with 2+1 flavours of **domain-wall fermions**
  (fictitious 5th dimension $\rightarrow$ tunable **lattice chiral symmetry**).



- **Lattice dimensions**: $16^3 \times 32(\times 16)$ and $24^3 \times 64(\times 16)$.
  Fixed spacing $a^{-1} = 1.73(3)\text{GeV} \implies$ vol.s (1.83fm)^3, (2.74fm)^3.
- **Quark masses**: $am_s = 0.04$, $am_{u/d} = 0.03,\ 0.02,\ 0.01,\ (0.005)$.
  Giving $\mathbf{m}_\pi$: 672 MeV, 557 MeV, 419 MeV, 331 MeV.
- **Too-heavy strange quark**: Physically, $am_s = 0.0343(16)$.
  Actually not so bad...

Lattice Correlation Functions

- Bare moments from **ratios of 2-point correlators** (to cancel f_π,...). Also **reduces statistical fluctuations**.
- **Example -** pseudoscalar first moment:

$$C_{A_\nu P}(t,\vec{p}) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle 0|A_\nu(t,\vec{x})P^\dagger(0)|0\rangle$$

$$C_{\{\rho\mu\}}(t,\vec{p}) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle 0|O_{\{\rho\mu\}}(t,\vec{x})P^\dagger(0)|0\rangle$$

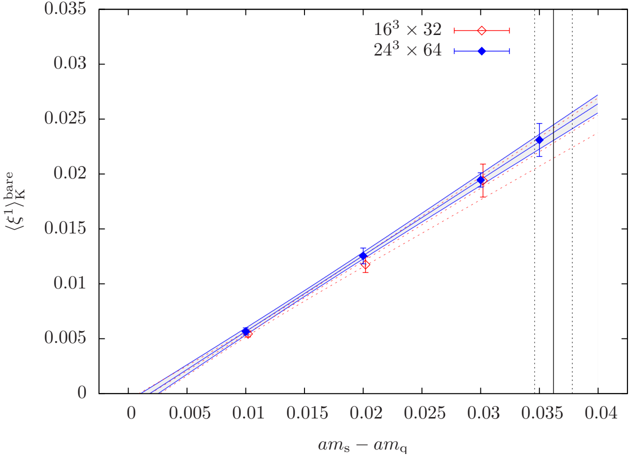
Then:

$$R_{\{\rho\mu\};\nu}(t,\vec{p}) \equiv \frac{C_{\{\rho\mu\}}(t,\vec{p})}{C_{A_\nu P}(t,\vec{p})} \overset{t\to\infty}{=} i\frac{p_\rho p_\mu}{p_\nu}\langle \xi^1\rangle$$

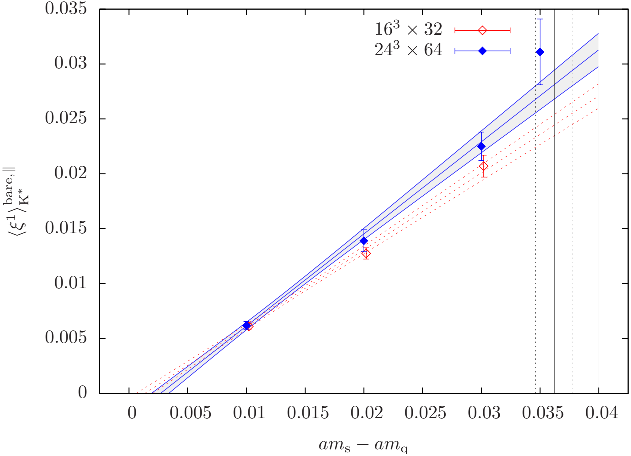
- **Hypercubic group** $\mathscr{H}_4 \implies$ more **operator mixing**. Consider operators' transformations under $\mathscr{H}_4$, P, C. Control mixing, induce **as little momentum as possible**.

Results: $\langle \xi^1 \rangle_K^{\rm bare}$

- **NLO χPT** (Chen & Stewart, 2004) says: $\langle \xi^1 \rangle_K = \frac{8B_0}{f^2}(m_s - m_{u/d})b_{1,2}$.

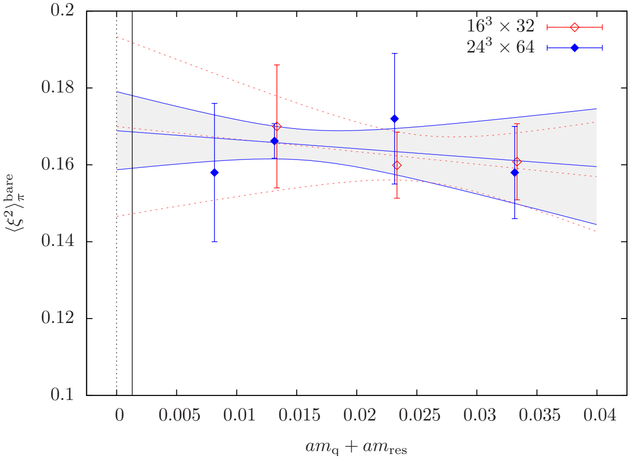


Results: $\langle \xi^1 \rangle_{K^*}^{\text{bare},\parallel}$

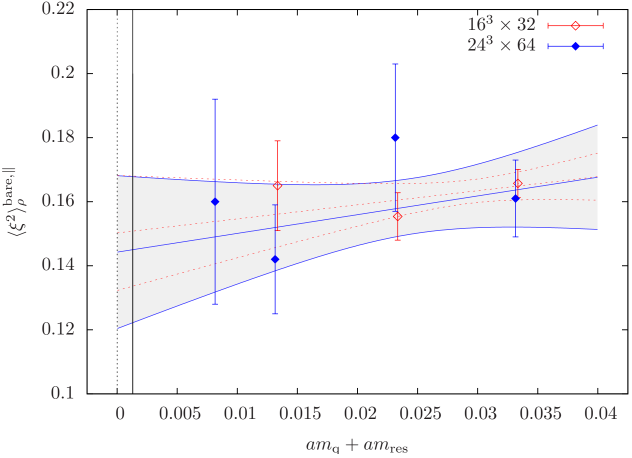


Results: $\langle \xi^2 \rangle_\pi^{\mathrm{bare}}$

- Can't really distinguish quark mass dependence - fit linearly. χPT says no non-analytic dependence to 2-loops, linear in m_π^2.



Results: $\langle \xi^2 \rangle_\rho^{\text{bare},\parallel}$



Perturbative Matching vs. NPR

- Can **'match'** at $\mu = 1/a$, by comparing lattice and continuum perturbative calculations of amputated vertices. **Done at 1-loop**.

  $$O^{\overline{\text{MS}}}_{\{\rho\mu\}}(\mu) = Z_{O_{\{\rho\mu\}}} O^{\text{latt}}_{\{\rho\mu\}}(a) \quad \mathscr{O}^{\overline{\text{MS}}}_{DD}(\mu) = Z_{DD,DD}\mathscr{O}^{\text{latt}}_{DD}(a) + Z_{DD,\partial\partial}\mathscr{O}^{\text{latt}}_{\partial\partial}(a)$$

  But: poor convergence, hard-to-estimate uncertainty, cannot disentangle μ- from a-dependence.
- Better: **intermediate nonperturbative scheme**. 2 approaches: Schrödinger Functional-based **finite volume** schemes ($\mu = L^{-1}$), and **RI/MOM** ($\sim$ mimics the perturbative approach).
- **RI/MOM condition** imposed on amputated, far-off-shell quark correlators (in fixed gauge), calculated on lattice:

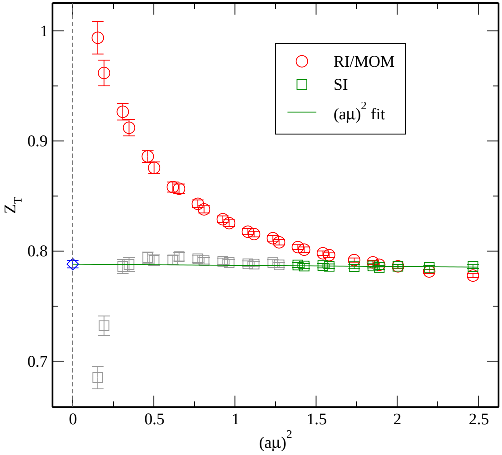
  $$\Gamma_R(p)|_{p^2=\mu^2} = \Gamma(p)^{\text{tree}}$$

  Need **window** in lattice momenta:

  $$\Lambda_{\text{QCD}} \ll |p| \ll a^{-1}$$

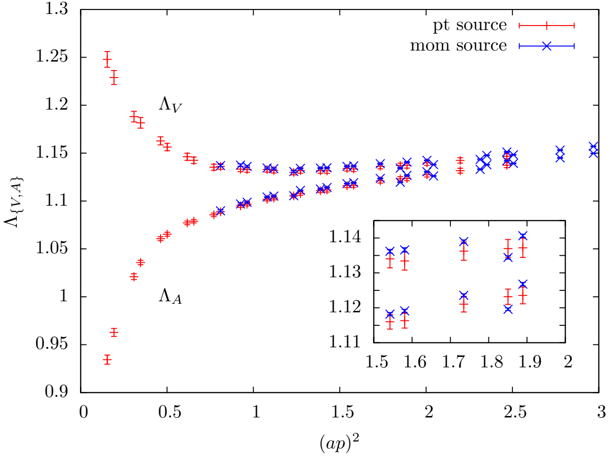
‘Point Source’ Results

- Organize calculation so operator **position is fixed**, but data is easily obtained for **many momenta**. Seems to work reasonably well.



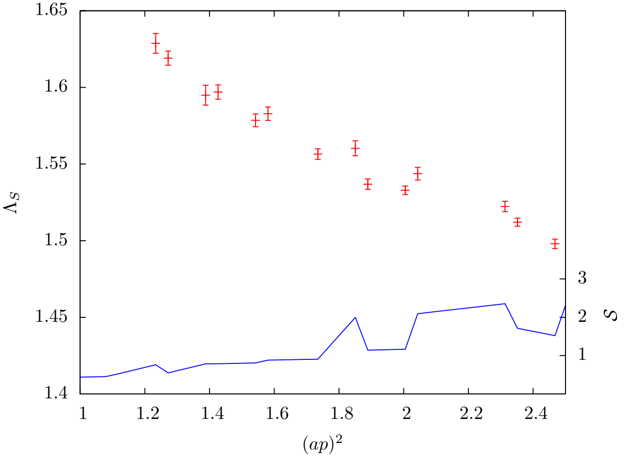
'Momentum Source' vs. 'Point Source'

- **Volume-averaging** for position, with **4-momentum fixed**. Drastic reduction in statistical errors. But... there are wiggles.



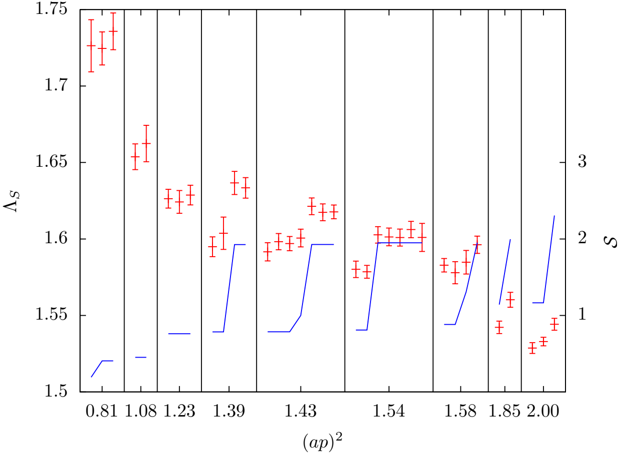
Discretization Effects I

- Define $S \equiv \sum_\mu \frac{2\pi}{L_\mu} p_\mu^4$. It starts to make sense:



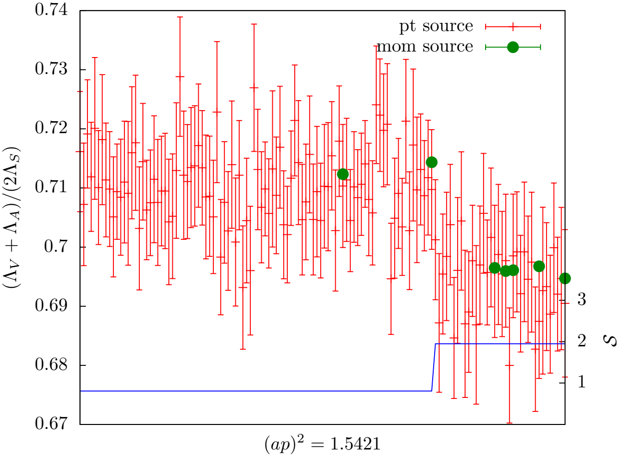
Discretization Effects II

- Reasonably clear **splittings** are seen:



Discretization Effects III

- Why we could neglect all this with the ‘point source’ approach:



Summary and Outlook

- PRELIMINARY: with **perturbative renormalization**, in the $\overline{\mathrm{MS}}$ scheme at 2 GeV, and with (stat)(sys) errors:

	16^3	24^3
$\langle \xi^2 \rangle_\pi$	0.277(35)(13)	0.275(15)(13)
$\langle \xi^1 \rangle_K$	0.0274(17)(16)	0.02857(86)(164)
$\langle \xi^2 \rangle_K$	0.285(17)(14)	0.269(11)(13)
$\langle \xi^2 \rangle_\rho^\parallel$	0.246(27)(12)	0.237(36)(12)
$\langle \xi^1 \rangle_{K^*}^\parallel$	0.0293(11)(16)	0.0337(16)(21)
$\langle \xi^2 \rangle_{K^*}^\parallel$	0.257(13)(13)	0.249(17)(12)
$\langle \xi^2 \rangle_\phi^\parallel$	0.246(11)(12)	0.247(10)(12)

- Plus discretization error of $\sim 5\%$.
- Compatible with Sum Rules results (but with smaller errors) and with QCDSF where comparison is possible.
- In progress: NPR. Finer lattices. Partial-twisting.
- In future: Transverse vector mesons. Baryons.