Large Quark Mixing in SO (10)

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in collaboration with

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Beyond the Standard Model

Standard Model (SM) $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ successfully describes electromagnetic, weak and strong interactions; however, it is only an effective theory.

The symmetries and the particle content of the standard model (SM) point towards Grand Unified Theories (GUTs) as the next step in the unification of all forces.

SM group:

$$SU(3)_C \times SU(2)_L \times U(1)_Y \;\subset\; SU(5) \;\subset\; SO(10)\;,$$

quark and lepton fields:

$$\underbrace{Q\left(3,2,\tfrac{1}{6}\right) + u^c\left(\overline{3},1,-\tfrac{2}{3}\right) + d^c\left(\overline{3},1,\tfrac{1}{3}\right) + L\left(1,2,-\tfrac{1}{2}\right) + e^c\,(1,1,1) + N\,(1,1,0)}_{\text{16-dimensional representation of SO(10)}}$$

- SM fermion representations are fixed;
- the SM-singlet is the singlet neutrino. $\longrightarrow$ Neutrinos are massive.

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- embed G_{SM} in simple group
- explain SM charge assignments
- require right-handed neutrinos (all but SU(5))

  $\longrightarrow$ neutrinos are massive!

  small masses are explained by seesaw mechanism

  $\longrightarrow$ set natural scale for heavy Majorana masses.

$$m^\nu \sim \frac{(100\ \text{GeV})^2}{M_N} \sim 0.1\ \text{eV} \quad \longrightarrow \ M_N \sim 10^{14}\ \text{GeV}\,.$$

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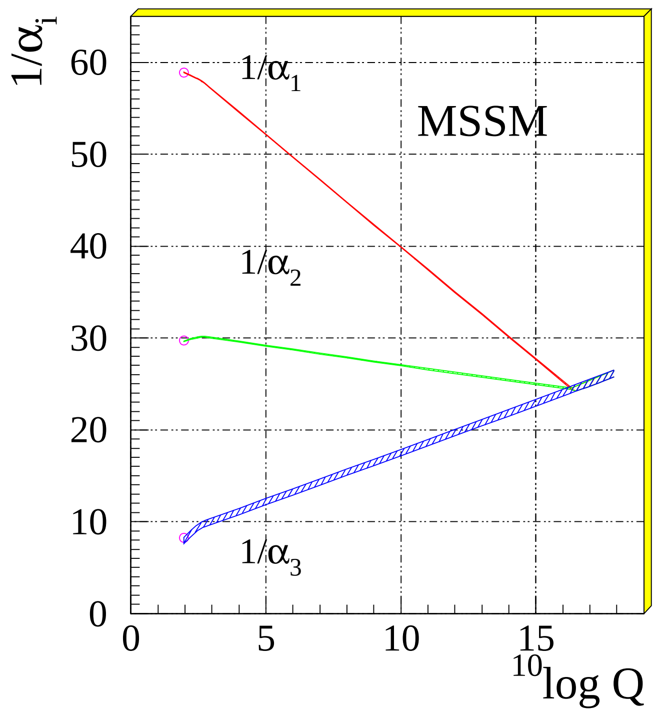
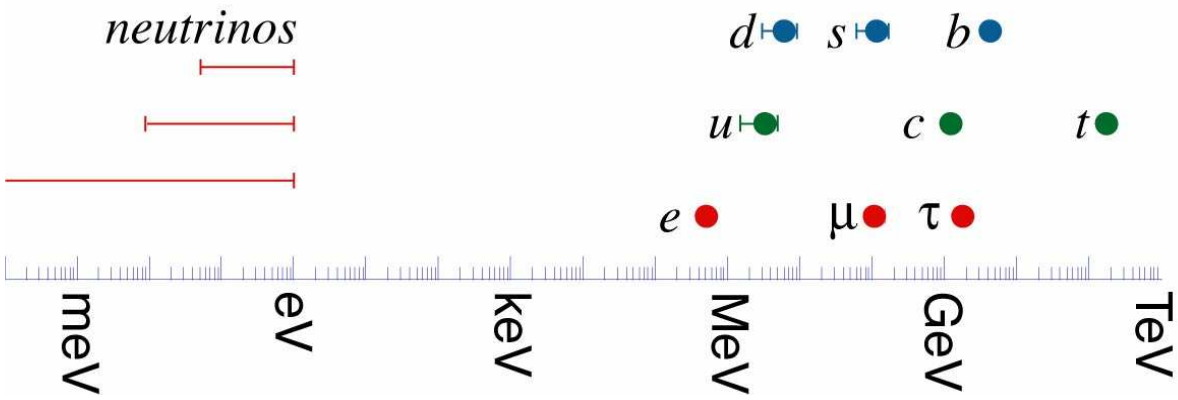
  $\rightarrow$ set natural scale for heavy Majorana masses.
- very successful with low-energy supersymmetry
  - controls the quadratic divergence of Higgs mass
  - symmetry between fermions and bosons
  - only possible extension of Poincaré algebra



$\rightarrow$ Supersymmetric (SUSY) GUTs

Flavor Physics and Grand Unification



Neutrino masses have
weak hierarchy.

Masses of charged fermions
are strongly hierarchical.

Mixing is close to ‘tribimaximal’,

$$V_{\text{PMNS}} \sim \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & \sqrt{2} & 0 \\ -1 & \sqrt{2} & \sqrt{3} \\ 1 & -\sqrt{2} & \sqrt{3} \end{pmatrix}$$

Small mixing among (left-handed) quarks,

$$V_{\text{CKM}} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} ; \quad \lambda \simeq \theta_C$$

GUT: Are large mixing angles observable in the quark sector?

$\longrightarrow$ Large mixings among right-handed quarks?

Grand Unification

$$\text{GUT} \times \text{SUSY} \xrightarrow{M_{\text{GUT}}} \text{G}_{\text{SM}} \times \text{SUSY} \xrightarrow{M_{\text{SUSY}}} \text{G}_{\text{SM}} \xrightarrow{M_{\text{EW}}} \text{SU(3)}_C \times \text{U(1)}_{\text{EM}}$$

Simplest choice: SU(5)

Fermion fields:

$$\psi\left(\bar{5}\right) = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ \hline e^- \\ -\nu_e \end{pmatrix}, \qquad \chi\left(10\right) = \frac{1}{\sqrt{2}} \left(\begin{array}{ccc|cc} 0 & u_3^c & -u_2^c & -u^1 & -d^1 \\ -u_3^c & 0 & u_1^c & -u^2 & -d^2 \\ u_2^c & -u_1^c & 0 & -u^3 & -d^3 \\ \hline u^1 & u^2 & u^3 & 0 & -e^c \\ d^1 & d^2 & d^3 & e^c & 0 \end{array}\right).$$

If the large atmospheric neutrino mixing angle stems from a rotation of $\bar{5}_2$ and $\bar{5}_3$, it will induce a large $\tilde{b}_R - \tilde{s}_R$-mixing. [Moroi 2000]

$\rightarrow$ new $b_R - s_R$ transitions from gluino–squark loops

SU(5) does not provide a single fermion representation and does not require massive neutrinos. $\rightarrow$ SU(5) is disfavored.

Grand Unification

$$\text{GUT} \times \text{SUSY} \xrightarrow{M_{\text{GUT}}} \text{G}_{\text{SM}} \times \text{SUSY} \xrightarrow{M_{\text{SUSY}}} \text{G}_{\text{SM}} \xrightarrow{M_{\text{EW}}} \text{SU(3)}_C \times \text{U(1)}_{\text{EM}}$$

SO(10) is a very attractive candidate:

- unification of gauge groups and fermions;
- anomaly free;
- gauged $\text{U(1)}_{\text{B}-\text{L}}$: B$-$L breaking gives rise automatically to Majorana masses for the singlet neutrinos and the seesaw mechanism.

Combine the attractive features of SU(5) and SO(10) and look at

$$\text{SO(10)} \times \text{SUSY} \to \text{SU(5)} \times \text{SUSY} \to \text{G}_{\text{SM}} \times \text{SUSY} \to \text{G}_{\text{SM}} \to \text{SU(3)}_C \times \text{U(1)}_{\text{EM}}$$

Grand Unification

- Fermions are assigned to the spinor representation, 16,

$$Q\,(3,2,\tfrac{1}{6}) + L\,(1,2,-\tfrac{1}{2}) + u^c\,(\overline{3},1,-\tfrac{2}{3}) + d^c\,(\overline{3},1,\tfrac{1}{3}) + e^c\,(1,1,1) + N\,(1,1,0)$$

The singlet is the right-handed neutrino!

SO(10) can be broken to SU(5) via $\langle (1,1,0)_{16_H} \rangle$.

- The adjoint representation is 45-dimensional.

SU(5) can be broken to G_{SM} via an adjoint Higgs field, 45_H.

i.e., $\langle 45_H \rangle \propto Y$.

$$\mathrm{SO(10)} \xrightarrow[\sim 10^{17}\ \mathrm{GeV}]{\langle 16_H \rangle} \mathrm{SU(5)} \xrightarrow[\sim 10^{16}\ \mathrm{GeV}]{\langle 45_H \rangle} G_{SM}$$

Chang–Masiero–Murayama (CMM) Model

[Chang, Masiero and Murayama 2003]

The CMM model is based on the symmetry breaking chain SO(10) $\to$ SU(5) $\to$ G_{SM}.

- The SUSY-breaking terms are universal at the Planck scale.
- Renormalization effects from the top coupling destroy the universality at M_{GUT}.
- Rotating $\bar{5}_2$ and $\bar{5}_3$ into mass eigenstates generates a $\tilde{b}_R - \tilde{s}_R$ element in the mass matrix of right-handed squarks.

Phenomenological effect : leads to MSSM with

- new loop-induced $b_R \to s_R$ and $b_L \to s_R$ transitions, while all other FCNC transitions are CKM-like,
- all MSSM masses and couplings fixed in terms of a few GUT parameters.

$\to$ well-motivated falsifiable version of the MSSM without minimal flavour violation (MFV), largest effects into $b_R \to s_R$.

CMM Model: Yukawa Couplings

$$W_Y = \frac{1}{2}\left\{ 16_i\, Y_1^{ij}\, 16_j\, 10_H + 16_i\, Y_2^{ij}\, 16_j\, \frac{45_H\, 10'_H}{M_{\rm Pl}} + 16_i\, Y_N^{ij}\, 16_j\, \frac{\overline{16}_H \overline{16}_H}{M_{\rm Pl}} \right\}$$

Masses for up quarks and neutrinos; masses for down quarks and charged fermions; Majorana masses for right-handed neutrinos.

- MSSM Higgs doublets H_u and H_d are contained in 10_H and $10'_H$, respectively; two fields, 10_H and $10'_H$, allow for non-trivial CKM matrix.
- The effective coupling Y_2 is asymmetric.

$$\text{SO(10)} \xrightarrow[\sim 10^{17}\ \text{GeV}]{\langle 16_H \rangle} \text{SU(5)} \xrightarrow[M_{\rm GUT} \sim 10^{16}\ \text{GeV}]{\langle 45_H \rangle} \text{G}_{\rm SM} \xrightarrow[M_{\rm EW}]{\langle 10_H \rangle,\, \langle 10'_H \rangle} \text{SU(3)}_C \times \text{U(1)}_{\rm em}$$

CMM Model: Yukawa Couplings

Choose up-basis and assume that Y_1 and Y_N are simultaneously diagonal:

$$W_Y = \frac{1}{2}\left\{ 16^\top \mathsf{D}_1 P_1\, 16\, 10_H + 16^\top V_q^* \mathsf{D}_2 V_\ell\, 16\, \frac{45_H\, 10'_H}{M_{\mathsf{Pl}}} + 16^\top \mathsf{D}_N P_N\, 16\, \frac{\overline{16}_H \overline{16}_H}{M_{\mathsf{Pl}}} \right\}$$

Masses for up quarks and neutrinos; masses for down quarks and charged fermions; Majorana masses for right-handed neutrinos.

$$\underbrace{(Q,\, e^c)}_{\chi(10)}^\top V_q^*\, \mathsf{D}_2\, V_\ell\, \underbrace{(d^c,\, L)}_{\psi(\bar{5})}\, \frac{45_H\, 10'_H}{M_{\mathsf{Pl}}}$$

Apart from SUSY breaking terms, the flavor structure is fully contained in Y_2; up to phases, V_q and V_ℓ coincide with V_{CKM} and V_{PMNS}.

Both the SU(5) singlet and adjoint of 45_H have non-vanishing vevs:

- vev of the SU(5) adjoint, $\langle \Sigma_{24}\,(45_H)\rangle \equiv \sigma$, breaks SU(5) to G_{SM},
- singlet component acquires a vev, when SO(10) is broken, $\langle S\,(45_H)\rangle \equiv v_0$.

Supersymmetry Breaking Terms

Assume universal parameters at $M_{\rm Pl}$.

RGE running : The soft mass matrix for the right-handed down squarks, $\mathsf{m}^2_{\tilde{d}}$, keeps its diagonal form but the third generation gets significant corrections from the large top Yukawa coupling,

$$\mathsf{m}^2_{\tilde{d}}(M_Z) = \mathrm{diag}\left(m^2_{\tilde{d}},\, m^2_{\tilde{d}},\, m^2_{\tilde{d}} - \Delta_{\tilde{d}}\right).$$

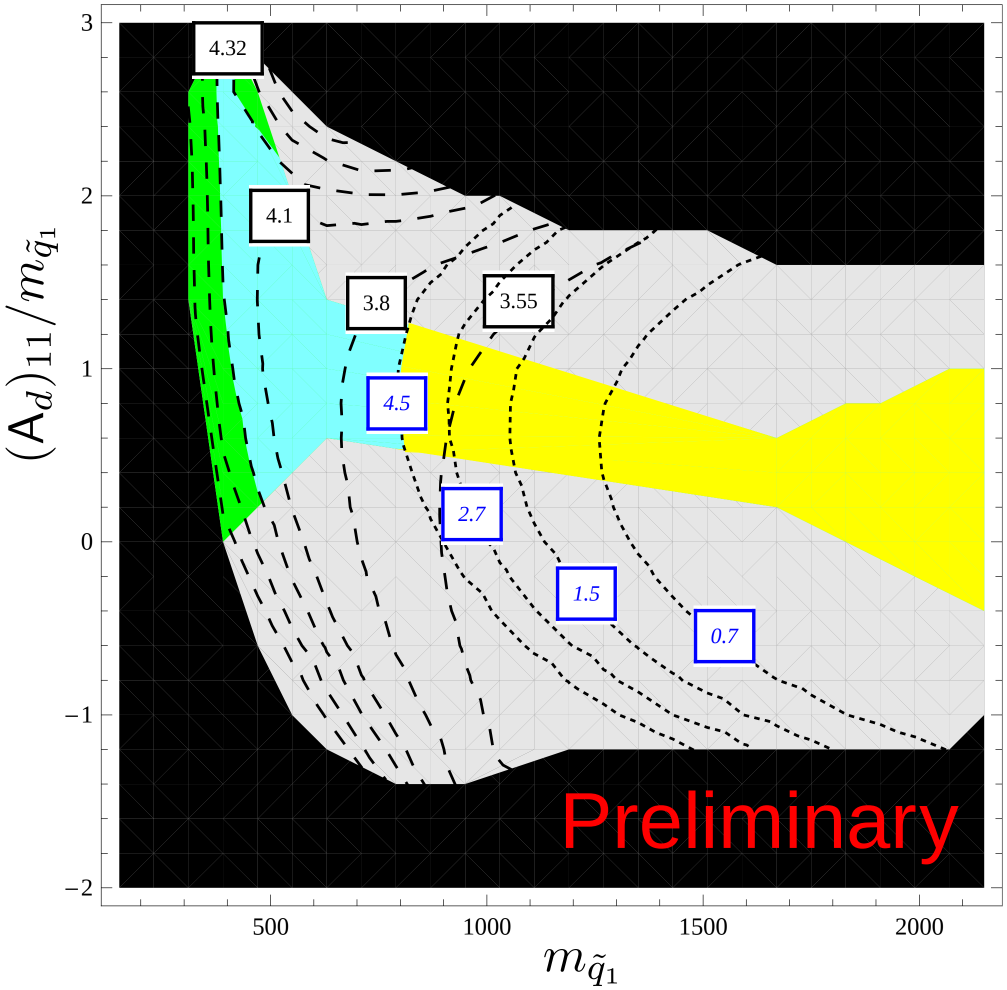
Super-CKM basis :

$$V_\ell^* \, \mathsf{m}^2_{\tilde{d}} \, V_\ell^\dagger = \begin{pmatrix} m^2_{\tilde{d}} & 0 & 0 \\ 0 & m^2_{\tilde{d}} - \frac{1}{2}\Delta_{\tilde{d}} & -\frac{1}{2}\Delta_{\tilde{d}}\, e^{i\xi} \\ 0 & -\frac{1}{2}\Delta_{\tilde{d}}\, e^{-i\xi} & m^2_{\tilde{d}} - \frac{1}{2}\Delta_{\tilde{d}} \end{pmatrix}$$

$\longrightarrow$ flavor-changing quark-squark-gluino and quark-squark-neutralino vertices.

Constraints from Heavy Generations

Constraints from $B_s - \overline{B}_s$ mixing, $\tau \to \mu\gamma$ and $b \to s\gamma$ on $m_{\tilde{q}_1}$ and $(\mathrm{A}_d)_{11}/m_{\tilde{q}_1}$:



$m_{\tilde{g}} = 350$ GeV, $\arg\mu = 0$

Black: negative $(\text{soft mass})^2$

Green: excluded by $\tau \to \mu\gamma$ and $b \to s\gamma$

Blue: excluded by $\tau \to \mu\gamma$

Gray: excluded by $B_s - \overline{B}_s$ mixing

Yellow: allowed

dashed lines: $10^4 \cdot \text{Br}\,(b \to s\gamma)$;

dotted lines: $10^8 \cdot \text{Br}\,(\tau \to \mu\gamma)$.

Extension to the Lighter Generations

$$W_Y \ni \frac{1}{2}\, \mathsf{16}_i\, \mathsf{Y}_2^{ij}\, \mathsf{16}_j\, \frac{\mathsf{45}_H\, \mathsf{10}'_H}{M_{\mathsf{Pl}}}$$

- Dominant contributions from terms with SU(5) singlet vev, suppressed by $v_0/M_{\mathsf{Pl}} \sim 10^{-2}$ (small $\tan\beta$ scenario).
- These contribute equally to down quark and charged fermion masses.

  $\longrightarrow$ Yukawa unification, $\mathsf{Y}_d = \mathsf{Y}_e^\top$.

- Relation works remarkably well for the third generation but not for the lighter ones.

  $\longrightarrow$ Corrections from terms with vev of the SU(5) adjoint, $\sigma/M_{\mathsf{Pl}} \sim 10^{-3}$.

- $\langle \Sigma\, (\mathsf{45}_H) \rangle \propto Y \longrightarrow$ modifies Yukawa relation to

$$\mathsf{Y}_d = \mathsf{Y}_e^\top + \underbrace{5}_{Y_{d^c} - Y_L} \frac{\sigma}{M_{\mathsf{Pl}}} \mathsf{Y}_\sigma\ .$$

- Modification affects mostly lighter generations.

Rotation of Lighter Down (S)Quarks

Ansatz : Rotation of the down-quark singlet fields differ from that of the lepton doublets by a unitary matrix U,

$$R_d = U\,V_\ell^*, \quad U = \begin{pmatrix} U_{11} & U_{12} & 0 \\ U_{21} & U_{22} & 0 \\ 0 & 0 & e^{i\phi_4} \end{pmatrix} = \begin{pmatrix} \cos\theta\, e^{i\phi_1} & -\sin\theta\, e^{i(\phi_1-\phi_2+\phi_3)} & 0 \\ \sin\theta\, e^{i\phi_2} & \cos\theta\, e^{i\phi_3} & 0 \\ 0 & 0 & e^{i\phi_4} \end{pmatrix}$$

Good bottom-tau unification implies that the (33)-entry of U should be close to one, up to a phase, and the remaining entries of the third row and column should be small.

This additional rotation leads to visible effects in ϵ_K:

$$\epsilon_K^{\text{CMM}} = \mathcal{K}^{\text{had}} \frac{\alpha_s^2}{2m_{\tilde{g}}^2} \,\text{Im}\left[R_d^{23}(R_d^{13})^*\right]^2 \times \text{ loop-fcts}$$

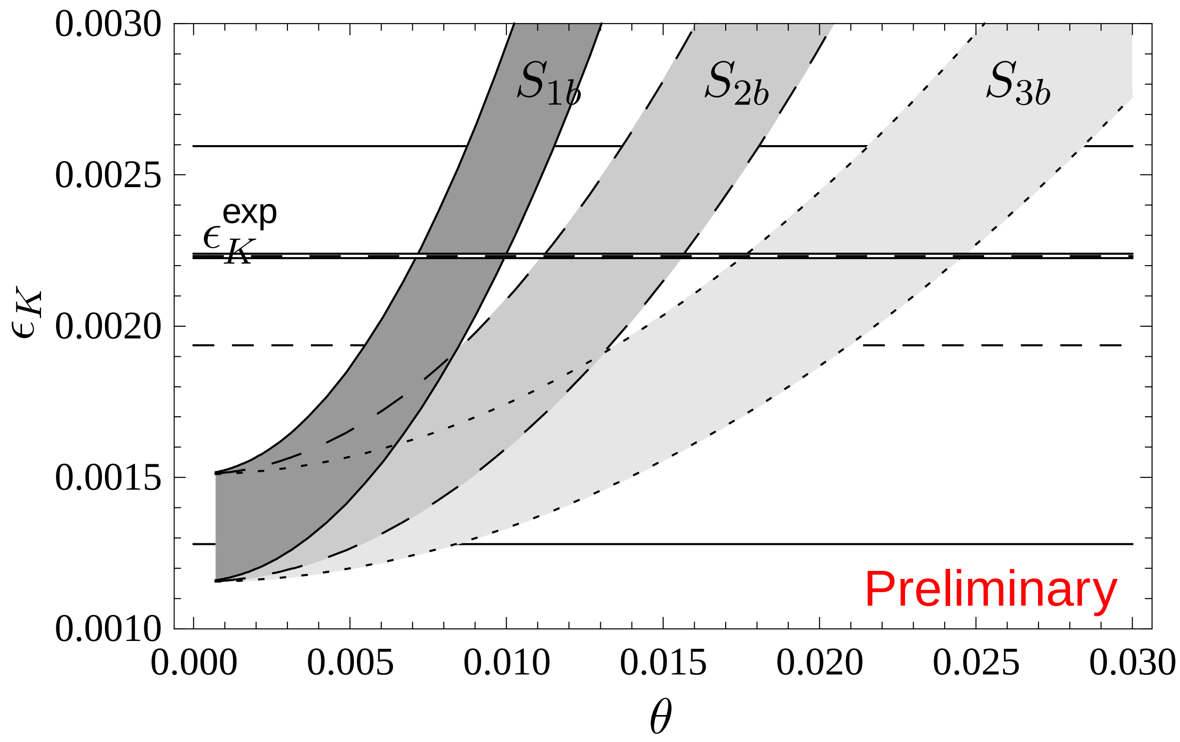
$$(R_d)_{13} = \frac{1}{\sqrt{2}} U_{12}\, e^{i(\alpha_3+\alpha_4-\alpha_1)}$$

$$\propto \sin^2(2\theta)\sin\left(2\left(\phi_2-\phi_1\right)\right)$$

[Phases α_i from V_ℓ cancel out.]

Constraints from Kaon Physics

CMM contributions to ϵ_K:



- S_{ib} refer to three sets of SUSY parameters;
- lower margins correspond to $\widehat{B}_K = 0.65$, upper to $\widehat{B}_K = 0.85$.

Conclusions

SUSY GUTs provide a beautiful framework for theories beyond the SM.

Weak-scale supersymmetry allows us to look for GUT imprints in flavor physics.

- The large leptonic mixing may yield large mixing among right-handed quarks.
- The Chang–Masiero–Murayama (CMM) Model provides a simple but predictive framework and leads to an MSSM without minimal flavour violation.

  It can be falsified by the rich flavor data, in particular $B_s - \overline{B}_s$ mixing and $\tau \to \mu\gamma$.
- We have extended the model to study the lighter generations.

  Here, ϵ_K provides a stringent constraint.
- Further restrictions will come from $B_d - \overline{B}_d$ mixing, proton decay as well as new experimental data.
- Extend the results to other GUT scenarios.