Flavour physics with lattice QCD

EUROFLAVOUR 2008
IPPP Durham

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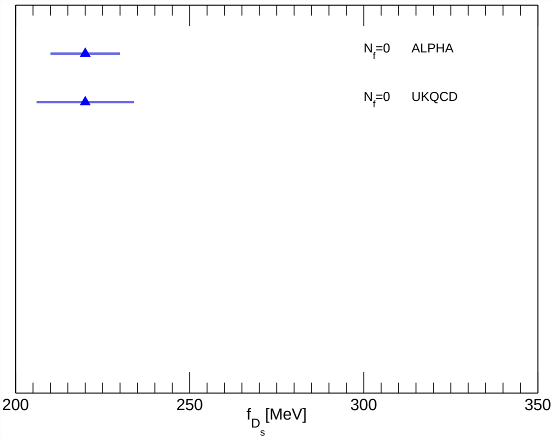
Overview over recent developments and results from lattice QCD

this talk: review last 12 months in lattice QCD

- discuss problems and possible solutions, mostly concerned with control of systematic effects
- selection of interesting recent developments in lattice QCD
- for comprehensive discussions of flavour physics related lattice results:
  Plenary talks at Lattice 2008: E. Gamiz, L. Lellouch and e.g. Della Morte, A.J. at Lattice 2007

Recent excitement

- 5 years ago the leptonic decay constant f_{D_s} was considered a bench mark test for lattice QCD
- recent results for f_{D_s} from lattice QCD and experiment



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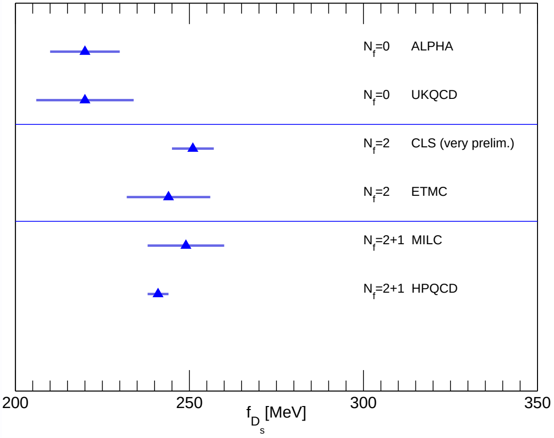

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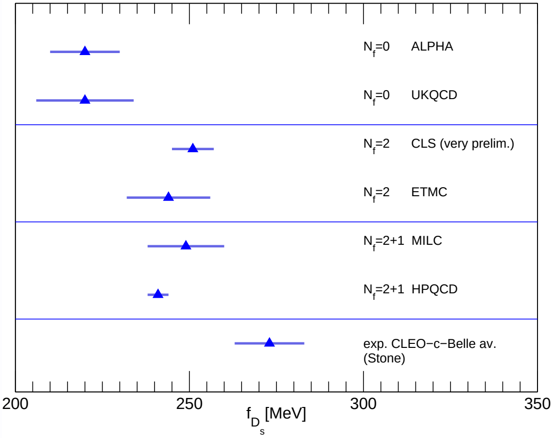


- in the wake of these results:
  paper by *Dobrescu, Kronfeld, PRD 100, 241802 (2008)* :
  "Accumulating evidence for Nonstandard Leptonic Decays of D_s Mesons"

Contributions to flavour phyiscs from lattice QCD

- CKM matrix elements recent work on the lattice

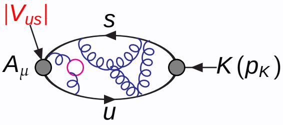
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

  best either from leptonic or semi-leptonic decays
  e.g. $K \rightarrow \pi l\nu$, $D \rightarrow K(\pi)l\nu$, $B \rightarrow \pi l\nu$, $B \rightarrow D^{*} l\nu$
  or from leptonic decays K, $D_{(s)}$, $B_{(s)}$
- meson mixing computation of bag parameters
- quark masses $M_{u,d}$, M_s, M_c, M_b various methods

Example: CKM elements from lattice QCD

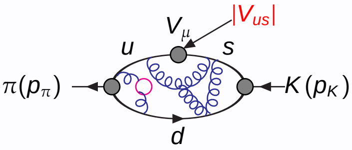
- typical processes e.g.:

K_{l2}-decay



$\langle 0|A_\mu(0)|K(p_K)\rangle$

K_{l3}-decay



$\langle\pi(p_\pi)|V_\mu(0)|K(p_K)\rangle$

$q_\mu = (p_K - p_\pi)_\mu$

- in practice:
  - measure decay rates $\Gamma(i \to j)$
  - compute process in SM (FF, RC, $SU(2)$)
  - $\Gamma(i \to j) = const. \times G_F^2 |V_{ij}|^2 \times FF \times RC$

Non-perturbative regime → Lattice Field Theory

- Correlation functions in terms of **Euclidean** path integral

  $\langle O[\bar{\psi},\psi,A]\rangle_{\mathrm{QCD}} = \frac{1}{Z}\int D\bar{\psi}D\psi DA\, O(\bar{\psi},\psi,A)\, e^{-S_G(U)-S_q(\bar{\psi},\psi,U)}$
- discretisation → space time lattice
- path integral now finite-(high)-dimensional
  calculate by Monte Carlo method (statistical sampling)
- from first principles:
  tune bare parameters (coupling and quark masses)
  and compute properties of bound states

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- discretisation → space time lattice → regulator π/a

  not unique, e.g. QCD:

Glue:	Fermions:
Wilson,	Wilson,
IWASAKI,	DWF,
DBW2, ...	overlap,
	staggered, ...

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- discretisation → space time lattice
- path integral now finite-(high)-dimensional
  calculate by Monte Carlo method (statistical sampling)

- from first principles:
  tune bare parameters (coupling and quark masses)
  - lattice spacing: $a^{-1} = \frac{f_\pi^{\rm exp}}{af_\pi}$
  - quark masses: $\frac{am_H}{am_V} = \frac{m_H^{\rm exp}}{m_V^{\rm exp}}$ $(H = \pi, K, D, \ldots)$

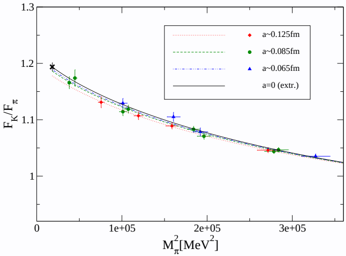
  and compute properties of bound states
  - spectrum
  - matrix elements (decay constants, form factors, scattering phase shifts)
  - quark masses
  - renormalised coupling
  - ...

Errors

- statistical
- light quark mass (in particular u, d, s is usually ok)
- discretisation errors (cut-off effects)
- heavy quark masses (related to discretisation errors)
- renormalisation: perturbative or non-perturbative (the latter allows far better control of the systematics) $\rightarrow$ *Michael Donnelan's talk*
- finite volume errors

Errors

- statistical
- light quark mass (in particular u, d, s is usually ok)
  - state of the art is $m_\pi \approx 200 - 300$MeV
  - physical point through extrapolation in the light quark mass using chiral perturbation theory



BMW collaboration 2008

  - interesting/fruitful interplay between χPT and lattice has just started
- discretisation errors (cut-off effects)
- heavy quark masses (related to discretisation errors)
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Errors

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- light quark mass (in particular u, d, s is usually ok)
- discretisation errors (cut-off effects)
  - $a \approx 0.1fm \rightarrow 1/a \approx 2\text{GeV}$
    naive estimate of cut-off effects $O(a\Lambda_{\rm QCD}) \approx 13\%$
  - $O(a)$-improvement, χ symmetry $O(a^2\Lambda^2_{\rm QCD}) \approx 1.5\%$
  - continuum extrapolation
- heavy quark masses (related to discretisation errors)
- renormalisation: perturbative or non-perturbative (the latter allows far better control of the systematics) $\rightarrow$ *Michael Donnelan's talk*
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Errors

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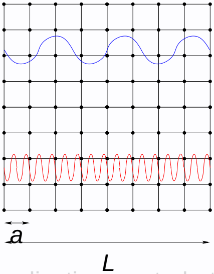
  typical lattice parameters: $L \approx 3\text{fm}$, $a^{-1} \approx 2 - 3\text{GeV}$



effective theory treatment of b-quark necessary:

- NRQCD
- Fermilab approach
- HQET
- step scaling
- combinations thereof

- renormalisation: perturbative or non-perturbative (the latter allows far better control of the systematics) → *Michael Donnelan's talk*
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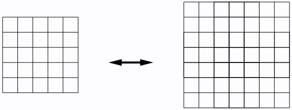
- finite volume errors
  - correct using chiral perturbation theory or
  - compare two simulations:



Status of simulations

Recent dynamical lattice simulations

collaboration	N_f	action	a/fm	Lm_π	m_π/MeV
QCDSF+UKQCD	2	clover (NP)	$\gtrsim 0.06$	$\gtrsim 4.2$	$\gtrsim 300$
ETM	2	max. tmQCD	$\gtrsim 0.09$	$\gtrsim 3.2$	$\gtrsim 270$
CLS	2	clover (NP)	$\gtrsim 0.04$	$\gtrsim 3.2$	$\gtrsim 260$
JLQCD	2	Neuberger	0.12	$\gtrsim 2.7$	$\gtrsim 280$
MILC	2+1	staggered	$\gtrsim 0.06$	$\gtrsim 4$	$\gtrsim 240$
RBC+UKQCD	2+1	DWF	$\gtrsim 0.08$	$\gtrsim 4.6$	$\gtrsim 330$
BMW	2+1	Stout-link Wilson	$\gtrsim 0.07$	$\gtrsim 4.0$	$\gtrsim 200$
PACS-CS	2+1	clover (NP)	0.09	$\gtrsim 2.3$	$\gtrsim 160$

large number of groups involved in phenomenology from lattice QCD

- results from a variety of formulations with increasingly good quality
- groups are really independent (in most cases)

Recent developments (only a selection, sorry)

- chiral extrapolations of lattice data at NLO
  how to treat the strange quark
- momentum resolution - better control over form factors
  how to avoid/constrain phenomenological ansätze for the q^2 dependence
- tackling the heavy quark on the lattice - some new ideas
  how to reduce/control discretisation effects in the full theory

Chiral perturbation theory

- masses of pions on the lattice currently $m_\pi \gtrsim 200\text{MeV}$
- extrapolate to physical point guided by chiral perturbation theory f_π, f_K, B_K, ...

	SU(3)	SU(2)
dof	π, K, η	π
LEC's	$f(m_{c,b,t}, \Lambda_{\rm QCD})$	$f(m_s, m_{c,b,t}, \Lambda_{\rm QCD})$

cf. Lellouch Lattice 2008

- Example: $SU(3)$ NLO χPT for pion decay constant:

$$f_\pi = f_0 \left\{ 1 + \frac{24}{f_0^2} L_4 \bar{\chi} + \frac{8}{f_0^2} L_5 \chi_{ud} - \frac{1}{16\pi^2 f_0^2} \left(2\chi_{ud} \log \frac{\chi_{ud}}{\Lambda_\chi^2} + \frac{\chi_{ud} + \chi_s}{2} \log \frac{\chi_{ud} + \chi_s}{2\Lambda_\chi^2} \right) \right\}$$

$(\chi_i = 2B_0 m_i,\ \bar{\chi} = (2\chi_{ud} + \chi_s)/3)$

Chiral perturbation theory

Study of m_π, f_π, m_K, f_K by *RBC+UKQCD arXiv:0804.0473*

- data set:
  16^3 and 24^3, $a \approx 0.11$,
  $m_\pi \approx 330, 415, 555\,670$MeV
  $am_s \approx am_h$ “strange a bit too heavy”,
- $m_l \to m_{u,d}$ using NLO chiral peturbation theory
  (partial quenching *Sharpe & Shoresh PRD62 094503 2000*: lightest pion am_π240MeV)
- questions:
  - how reliable?
  - $SU(3)_L \times SU(3)_R$ or $SU(2)_L \times SU(2)_R$
  - values of the LEC's ($\to$ Gilberto Colangelo)

Chiral perturbation theory fits - results

Pions:

- NLO $SU(3)$ and $SU(2)$ χPT fit the data well for $m_{PS} < 400$MeV
- large (50% of LO) corrections in $SU(3)$, less for $SU(2)$
- would need more data for NNLO - more fit parameters (some collaborations are doing this)

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Kaons:

- NLO $SU(3)$ does not fit data
- $K\chi PT$ *Roessl NPB555 507 1999*: $SU(2)_L \times SU(2)_R$ for u, d + matter fields for kaons,

  m_K, f_K, B_K *RBC+UKQCD arXiv:0804.0473*
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- adopted by other collaborations (PACS-CS, ETMC, BMW)

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Outlook:

- It is not entirely clear which is the right way to go - further tests necessary
- estimate systematics by comparing $SU(3)$ and $SU(2)$ and polynomial fits
- possibly include NNLO terms

another field where χPT has helped : observables with $\vec{p}$ dependence . . .

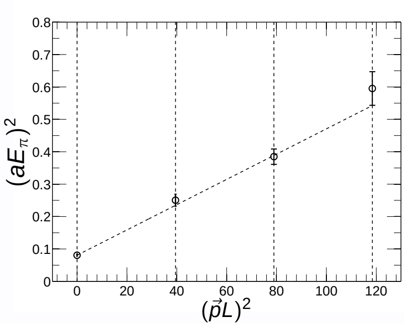
Partially twisted boundary conditions

periodic bc's

$$\psi(x_i + L) = \psi(x_i)$$

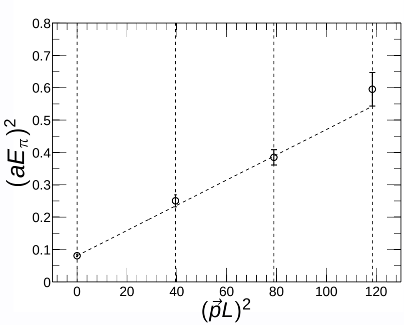
$$\vec{p}_{quark} = \vec{n}\frac{2\pi}{L}$$

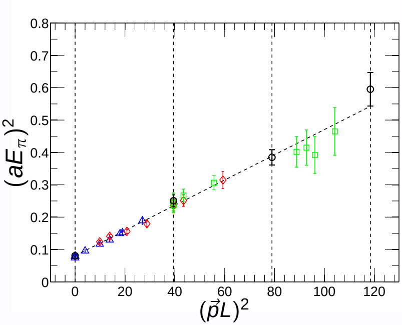
$$E_\pi = \sqrt{m_\pi^2 + (\vec{n}\frac{2\pi}{L})^2}$$



Partially twisted boundary conditions

periodic bc's	twisted bc's
$\psi(x_i+L)=\psi(x_i)$	$\psi(x_i+L)=e^{i\theta_i}\psi(x_i)$
$\vec{p}_{quark}=\vec{n}\frac{2\pi}{L}$	$\vec{p}_{quark}=\vec{n}\frac{2\pi}{L}+\frac{\vec{\theta}}{L}$
$E_\pi=\sqrt{m_\pi^2+(\vec{n}\frac{2\pi}{L})^2}$	$E_\pi=\sqrt{m_\pi^2+(\vec{n}\frac{2\pi}{L}+\frac{\vec{\theta}_u-\vec{\theta}_d}{L})^2}$





de Divitiis et al. PLB 595 (2004) 408, Bedaque PLB 593 (2004) 82, Sachrajda and Villadoro PLB 609 (2005) 73, UKQCD PLB 632 (2006) 313

Partially twisted boundary conditions

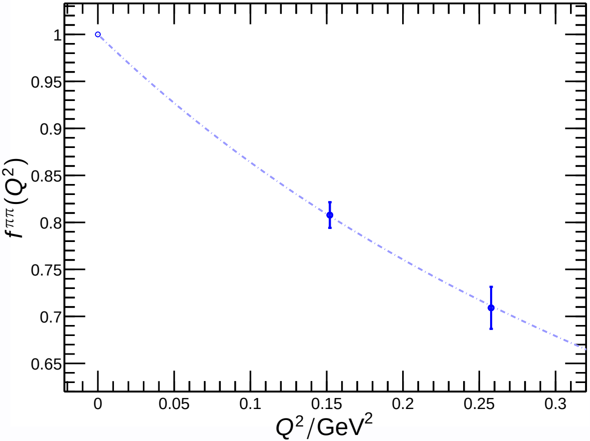
- applications:
  - pion structure (PDA's $\rightarrow$ Michael Donnellan)
  - nucleon form factors
  - electromagnetic $\pi \rightarrow \pi$
  - semi-leptonic $K \rightarrow \pi l \nu$
  - semi-leptonic $B \rightarrow D^{(*)} l \nu$
  - ...
- "partially": change boundary conditions of valence quarks only ($\rightarrow$ checked in χPT that this is a FVE $\propto e^{-m_\pi L}$ for processes with zero/one initial and/or final state *Sachrajda and Villadoro PLB 609 (2005) 73*)

Partially twisted boundary conditions - applications

Example: pion form factor $f_{\pi\pi}(q^2)$ and charge radius $\langle r_\pi^2 \rangle$

- for $\langle \pi(p_f) | V_\mu | \pi(p_i) \rangle$
$$q^2 = (p_i - p_f)^2 = \left\{ [E_i(\vec{p}_i) - E_f(\vec{p}_f)]^2 - \left[(\vec{p}_{\mathrm{FT},i} + \vec{\theta}_i/L) - (\vec{p}_{\mathrm{FT},f} + \vec{\theta}_f/L) \right]^2 \right\}$$

RBC-UKQCD collab.
only $m_\pi = 330$MeV
to be continued
see also ETMC (prelim.)



UKQCD JHEP 05(2007)016, JHEP 0807(2008)112

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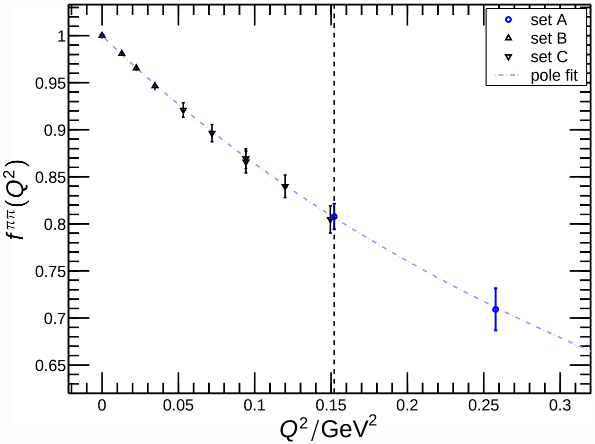

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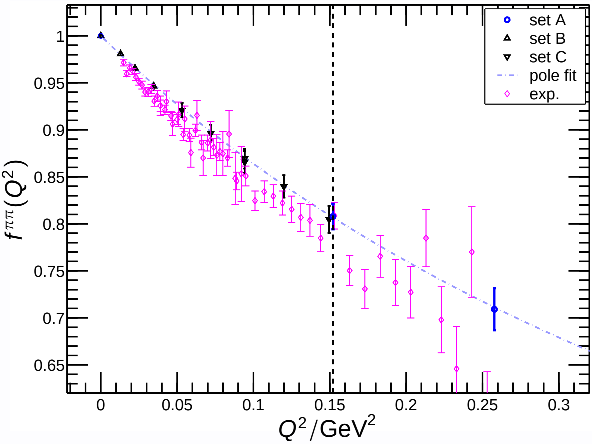

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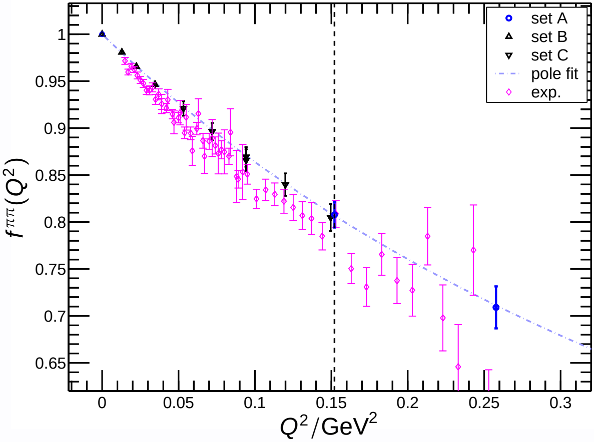

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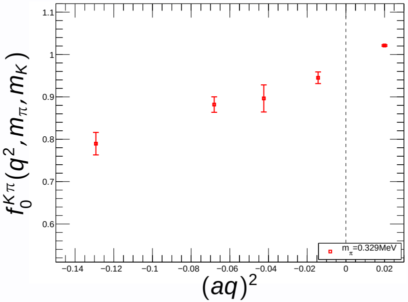
- applications for flavour physics, e.g. $K \to \pi l \nu$ and $B \to D^{(*)} l \nu$

K_{l3}-decay - 3 steps on the lattice

$$\langle\pi(p_\pi)|V_\mu(0)|K(p_K)\rangle = f_+^{K\pi}(q^2)(p_K+p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K-p_\pi)_\mu, \quad q_\mu=(p_K-p_\pi)_\mu$$

Becirevic et al. Nucl.Phys.B, 2005:

1) compute $f_0^{K\pi}(q^2)$
   meson momenta in a finite box:
   $$\vec{p}=\vec{n}\frac{2\pi}{L},\ n_i\in\pm\{0,1,2,\dots\}$$
2) interpolate to $q^2=0$
   phenomenological ansatz (e.g. pole)



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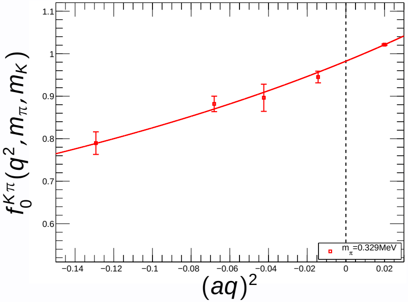

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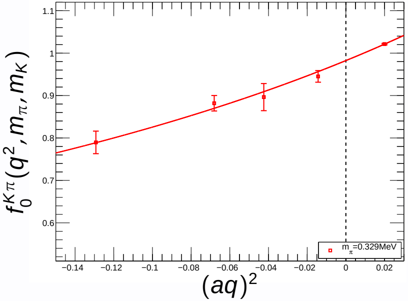


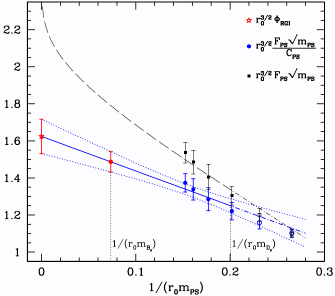
Partially twisted boundary conditions allow to completely remove systematic due to point 2) (shown in *UKQCD JHEP 05(2007)016, UKQCD JHEP 0807(2008)112*)
new: now applied to $K\to\pi$ on large scale (RBC+UKQCD upcoming):
$\to$ cheaper and systematic removed

together with other new ideas now also applied to heavy-light mesons ...

New ideas for heavy-light mesons

let's start with the *ALPHA*-approach, e.g. heavy-light decay constant:

- extra- or interpolate, e.g. heavy-light decay constant:
  - lattice QCD around $M_q \approx M_{\rm charm}$
  - lattice HQET
  - interpolate to M_b guided by HQET: $\Phi(M_h) = \Phi^{(0)} + \frac{1}{M_h}\Phi^{(1)} + O(\frac{1}{M_h^2})$
  - depending on the observable more or less strong M_h-dependence
  - in large volume too far away from $m_{B_{(s)}}$



example heavy-light decay constant
ALPHA JHEP 0802:078, 2008

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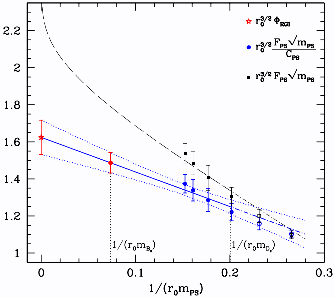
let's start with the *ALPHA*-approach, e.g. heavy-light decay constant:

- extra- or interpolate, e.g. heavy-light decay constant:
  - lattice QCD around $M_q \approx M_{\rm charm}$
  - lattice HQET
  - interpolate to M_b guided by HQET: $\Phi(M_h) = \Phi^{(0)} + \frac{1}{M_h}\Phi^{(1)} + O(\frac{1}{M_h^2})$
  - depending on the observable more or less strong M_h-dependence
  - in large volume too far away from $m_{B_{(s)}}$



example heavy-light decay constant
ALPHA JHEP 0802:078, 2008

- observation: factorise

$$O(m_b, m_l, L_\infty) = \underbrace{O(m_b, m_l; L_0)}_{\text{small volume } L_0 \approx 0.5\text{fm}} \times \underbrace{[\text{finite volume effects}](m_b, m_l; L_0)}_{\text{finite volume corrections}}$$

1) small volume $\rightarrow \frac{1}{a} << m_b$ possible, even continuum limit
2) correct for finite volume corrections $\rightarrow$ step scaling method

New ideas for heavy-light mesons

- step-scaling:
  ALPHA-collaboration,
  Guagnelli et al. PLB 546 237 (2002),
  de Divitiis, Petronzio, Tantalo JHEP 10(2007)062, arXiv:0807.2944

$$\begin{aligned} \mathcal{O}(m_b,m_l) &= \mathcal{O}(m_b,m_l;L_0)\times[\text{finite volume effects}](m_b,m_l;L_0) \\ &= \underbrace{\mathcal{O}(m_b,m_l;L_0)}_{\text{cl. of LQCD}}\times\underbrace{\frac{O(m_b,m_l;2L_0)}{O(m_b,m_l;L_0)}}_{\text{in c.l.:}\sigma(m_b,m_l,L_0)}\underbrace{\frac{O(m_b,m_l;sL_0)}{O(m_b,m_l;2L_0)}}_{\text{in c.l.:}\sigma(m_b,m_l,2L_0)} \end{aligned}$$

- of course sL_0 must be large, old problem $\frac{1}{a}\approx M_b$ appears again, but: instead of extrapolating:

  $$O(m_b,m_l;L)=O^0(m_l;L)\left[1+\frac{O^1(m_l,L)}{m_b}\right]$$

  extrapolate step scaling function with supressed $1/m_b$-corrections:

  $$\sigma(m_b,m_l;L)=\frac{O^0(m_l;2L)}{O^0(m_l;L)}\left[1+\frac{O^1(m_l;2L)-O^1(m_l;L)}{m_b}\right]$$

- in practice 2 steps are sufficient

$B \to D^{(*)} l\nu$ in the fully relativistic theory

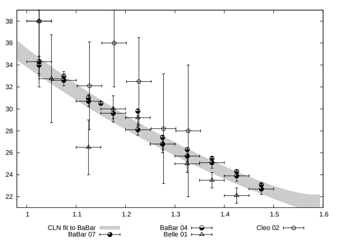
application:

- determining $|V_{cb}|$ in semi-leptonic decays

$$\frac{d\Gamma(B \to Dl\nu)}{d\omega} = (\text{kin. fact})|V_{cb}|^2(\omega^2 - 1)^{\frac{3}{2}} \left(G^{B\to D}(\omega)\right)^2$$

where

$$G^{B\to D}(\omega) = h_+^{B\to D}(\omega) - \frac{M_D - M_B}{M_D + M_B} h_-^{B\to D(\omega)}$$



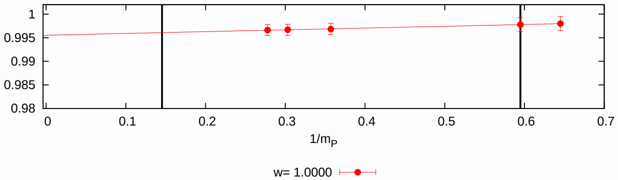
(plot by N. Tantalo at CKM 2008)

- recent work on the lattice: *C. Bernard et al. arXiv:0808.2519, Okamoto et al. Nucl.Phys.Proc.Suppl.140(2005)* used effective theory-descriptions of the heavy quark + computation for $\omega = 1$, only
- experiment bad at 0 recoil (kinematic supression)
- can do better ...

$B \to D^{(*)} l\nu$ in the fully relativistic theory - step scaling method

programme by *de Divitiis, Petronzio, Tantalo, JHEP 10(2007)062, arXiv:0807.2944*
(still quenched):

1) use twisted bc's to simulate $\omega > 1$
2) compute $\sigma(m_h, m_l; L_0)$ and $\sigma(m_h, m_l; L_1 = 2L_0)$ in the continuum limit of lattice QCD
   here $L_0 \approx 0.4$fm; $L_2 \approx 1.4$fm
3) one may have $m_h < m_b$, thus extrapolate to m_b in the continuum:
   in practice flat extrapolation of σ to m_b, e.g. for the $B \to D^{(*)} l\nu$ form factor *de Divitiis, Petronzio, Tantalo arXiv:0807.2944*:
   extrapolation indeed flat, e.g. $\sigma(m_h, m_l; L_0)$:



$B \to D^{(*)} l\nu$ in the fully relativistic theory - step scaling method

programme by *de Divitiis, Petronzio, Tantalo, JHEP 10(2007)062, arXiv:0807.2944* (still quenched):

1) use twisted bc's to simulate $\omega > 1$
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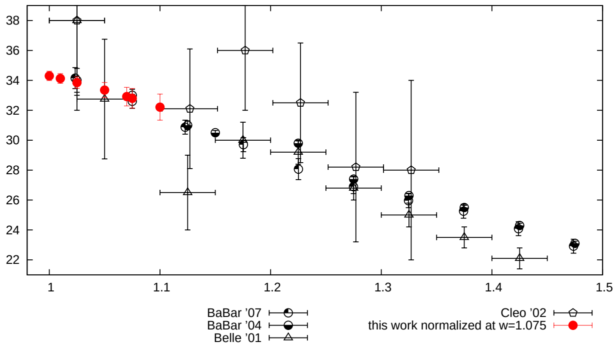
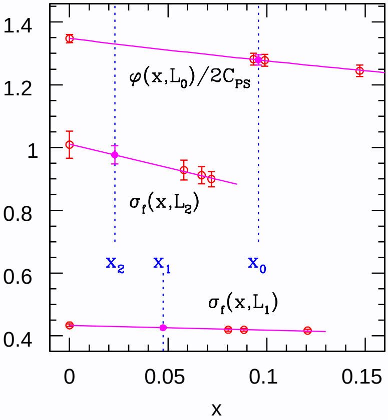
   in practice flat extrapolation of σ to m_b, e.g. for the $B \to D^{(*)} l\nu$ form factor *de Divitiis, Petronzio, Tantalo arXiv:0807.2944*:
   although still quenched, compares well with experiment



combination of step scaling in QCD in HQET

- until now: extrapolation of step scaling functions
- *Guazzini, Sommer, Tantalo, JHEP 01(2008)076*: do step scaling in HQET → constrain $\sigma(m_h, m_l; L)$ at $m_h \to \infty$
  example for decay constant



- small m_h-dependence; no curvature visible
- for some observables including static limit improves result
- no conceptional problems expected with dynamical fermions

Summary, comments

- simulations of lattice QCD are not far from the physical point
- interesting interplay between lattice QCD and effective theories (HQET and χPT)
- continous development of techniques improves understanding and control of systematic effects
  this takes time but is clearly worth the effort
- for summary of latest results: *Plenary talks at Lattice 2008: E. Gamiz, L. Lellouch and e.g. Della Morte, A.J. at Lattice 2007*