

Momentum dependence of $a_0(980)$ and $f_0(980)$
meson interactions in $R_{\chi T}$ face the KLOE data

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Focus on

- light scalar mesons (S):

$$a_0(980) (I^G(J^{PC}) = 1^-(0^{++})) \text{ and } f_0(980) (0^+(0^{++}))$$

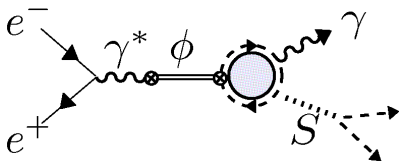
Employ

- Resonance chiral perturbation theory ($R\chi T$)

Study

- $\frac{dB_{\phi \rightarrow \gamma \pi^0 \pi^0}}{dm_{\pi^0 \pi^0}}$ and $\frac{dB_{\phi \rightarrow \gamma \pi^0 \eta}}{dm_{\pi^0 \eta}}$
- important influence from pseudoscalar meson dynamics

Experiments with scalar mesons: $\phi \rightarrow \gamma a_0/f_0$



$$\frac{dB}{dm} = \frac{d\sigma}{dm} \frac{m_\phi^2}{12\pi} \frac{\Gamma_{\phi, tot}}{\Gamma_{\phi \rightarrow e^+ e^-}}$$



$$\frac{dB_{\phi \rightarrow \gamma \pi^0 \pi^0}}{dm_{\pi^0 \pi^0}}, \quad \frac{dB_{\phi \rightarrow \gamma \pi^0 \eta}}{dm_{\pi^0 \eta}}$$

THE MODEL

- non-linear realization of spontaneous chiral symmetry breaking
- not assuming any given internal structure

$R\chi T$ features

- ChPT + V, A, S resonances \leftrightarrow QCD
- reliable below and even near 1 GeV
- momentum-dependent vertices, decoupling in the chiral limit

Resonance chiral perturbation theory Lagrangian

$$L_{\text{scalar,pseudoscalar}} = c_d \langle S^{\text{oct}} u_\mu u^\mu \rangle + c_m \langle S^{\text{oct}} \chi_+ \rangle \\ + \tilde{c}_d S^{\text{sing}} \langle u_\mu u^\mu \rangle + \tilde{c}_m S^{\text{sing}} \langle \chi_+ \rangle,$$

$$L_{\text{vector,pseudoscalar}} = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{iG_V}{\sqrt{2}} \langle V_{\mu\nu} u^\mu u^\nu \rangle$$

" $\langle \dots \rangle$ " – trace in the flavor space

$$u_\mu = i u^\dagger D_\mu u^\dagger$$

$u = \exp(i\sqrt{2}\Phi/F_\pi)$ – non-linear parametrization of the pseudoscalar fields

D_μ – chiral covariant derivative

F_π – the pion decay constant

$S^{\text{oct}}, S^{\text{sing}}, V_{\mu\nu}$ – scalar and vector resonances

$c_{d,m}, \tilde{c}_{d,m}, F_V, G_V$ – effective couplings

$$f_+^{\mu\nu} = e F^{\mu\nu} (u Q u^\dagger + u^\dagger Q u), \quad Q \equiv \text{diag}(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$$

$$\chi_+ = u^\dagger \chi u^\dagger + u \chi u,$$

$$\chi = 2B_0 \text{diag}(m_u, m_d, m_s) \approx \chi = \text{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2),$$

Ecker et al. NP B **321**, 311 (1989)

Resonance chiral perturbation theory Lagrangian

$$L_{\text{scalar,pseudoscalar}} = c_d \langle S^{\text{oct}} u_\mu u^\mu \rangle + c_m \langle S^{\text{oct}} \chi_+ \rangle \\ + \tilde{c}_d S^{\text{sing}} \langle u_\mu u^\mu \rangle + \tilde{c}_m S^{\text{sing}} \langle \chi_+ \rangle,$$

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$$F_V = 2 G_V$$

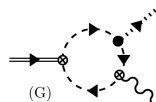
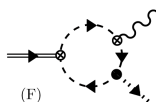
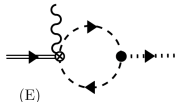
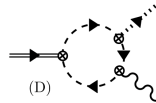
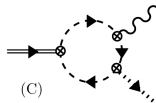
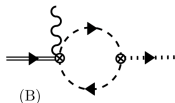
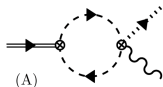
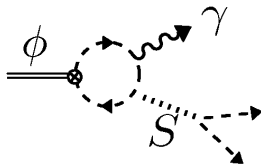
consistency with QCD
asymptotic behavior

$$c_d = c_m = f_\pi/2 = 46.2 \text{ MeV} \quad \text{short-distance constraints}$$

$$\tilde{c}_m = c_m/\sqrt{3}, \tilde{c}_d = c_d/\sqrt{3} \quad \text{large-}N_c \text{ limit}$$

Kaon loop for ϕ decay from $R_{\chi T}$

S.I. and A.Korchin, Eur. Phys. J. C **54** (2008) 89
arXiv: 0707.2700 (hep-ph)



(momentum-dependent \otimes and momentum-independent \bullet vertices for interactions with scalar meson) \equiv \triangleright \equiv \equiv \curvearrowright \curvearrowleft

- **Loop integrals are universal:**

⇒ known analytical expressions $I(a, b)$

e.g. Lucio, Pestieau, PR D **42**, 3253 (1990), *erratum ibid.* **43**, 2447

- no counter-terms needed, provided $F_V = 2 G_V$

(Ecker et al. PL, B **223**, 425)

- momentum-dependent vertex functions enter the game

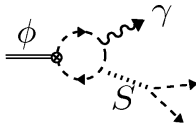
- No contact $V\gamma S$ and $\gamma\gamma S$ coupling

⇒ leading-order diagrams are one-loop

THE MODEL WORKS

Invariant mass distributions

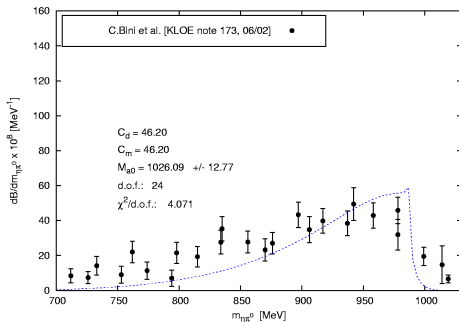
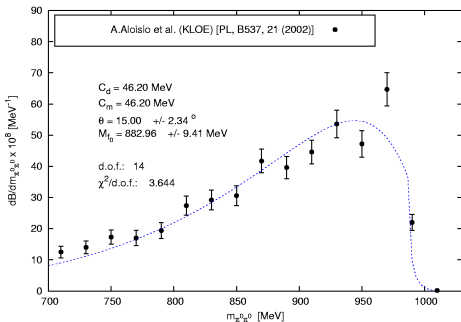
$$\frac{dB_{\phi \rightarrow \gamma \pi^0 \pi^0}}{dm_{\pi^0 \pi^0}}$$



$$\frac{dB_{\phi \rightarrow \gamma \pi^0 \eta}}{dm_{\pi^0 \eta}}$$

2 parameters: M_{f_0}, θ

1 parameter: M_{a_0}



- fit results depend crucially on the **scalar meson propagator** form
 - relativ. BW
 - Flatté - like
 - model - dependent self-energy inclusion
 - mixtures
 - etc.

- to calculate a scalar meson propagator in $R_{\chi T}$ — still a challenge

BEHIND THE CURTAIN:
momentum dependence of effective couplings

Invariant mass distributions: formulae

$$\frac{dB}{dm_{\pi^0\pi^0}} = \frac{\Gamma_{\phi \rightarrow \gamma f_0 \rightarrow \pi^0\pi^0}}{\Gamma_{\phi, \text{tot}}}, \quad m_{\pi^0\pi^0} \equiv \sqrt{p^2}$$

$$\begin{aligned} \frac{dB}{dm_{\pi^0\pi^0}} &= \frac{1}{2} \frac{1}{\Gamma_{\phi, \text{tot}}} \frac{\alpha \sqrt{p^2} \sqrt{1 - 4m_{\pi}^2/p^2}}{4 \times 48\pi^4 m_K^4} \\ &\times \left| \frac{I(a, b)}{D_{f_0}(p^2)} \right|^2 \left(\frac{M_{\phi}^2 - p^2}{M_{\phi}} \right)^3 \\ &\times \frac{G_{f_0\pi\pi}^2(p^2)}{4\pi} \frac{G_{f_0KK}^2(p^2)}{4\pi} \left(\frac{\sqrt{2}G_V M_{\phi}}{f_{\pi}^2} \right)^2 \end{aligned}$$

$a = M_{\phi}^2/m_K^2$, $b = p^2/m_K^2$, $\alpha \approx 1/137$.

$\pi^0\eta$ case is analogous.

1/2 — for identity of the π^0 s.

Flatté-like form of the scalar-meson propagator

$$D_S(p^2) = \left[p^2 - m_S^2 + i \sqrt{p^2} \tilde{\Gamma}_{S, tot}(p^2) \right]^{-1}$$

$$\tilde{\Gamma}_{f_0, tot}(p^2) = \tilde{\Gamma}_{f_0 \rightarrow \pi\pi}(p^2) + \tilde{\Gamma}_{f_0 \rightarrow K\bar{K}}(p^2)$$

$$\tilde{\Gamma}_{a_0, tot}(p^2) = \tilde{\Gamma}_{a_0 \rightarrow \pi^0\eta}(p^2) + \tilde{\Gamma}_{a_0 \rightarrow K\bar{K}}(p^2)$$

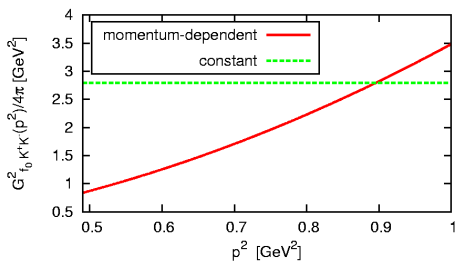
$$\tilde{\Gamma}_{f_0 \rightarrow \pi\pi}(p^2) = \frac{3}{2} \frac{1}{2p^2} \sqrt{\frac{p^2}{4} - m_\pi^2} \times \frac{G_{f_0\pi\pi}^2(p^2)}{4\pi}$$

analytic function of p^2 , defined **above** and **below the threshold** with $\sqrt{p^2/4 - m_\pi^2} = i\sqrt{|p^2/4 - m_\pi^2|}$.

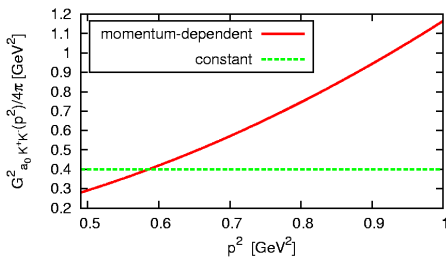
The analogous formulae hold for other decays.

Momentum dependence of effective couplings

$$\frac{G_{f_0 KK}^2(p^2)}{4\pi}$$



$$\frac{G_{a_0 KK}^2(p^2)}{4\pi}$$



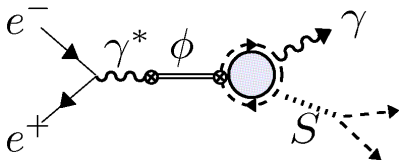
- There is a room for model improvement
- 3 parameters – too few to fit $\frac{dB_{\phi \rightarrow \gamma \pi^0 \pi^0}}{dm_{\pi^0 \pi^0}}$, $\frac{dB_{\phi \rightarrow \gamma \pi^0 \eta}}{dm_{\pi^0 \eta}}$ with good χ^2
- momentum-dependent vertex functions enter the game
- mind the scalar resonance propagator.

Acknowledgements

- H. Czyż for discussion of prospects
- NASU grant for young researchers (contract 8.63/2008)
- Joint NASU-RFFR scientific project N 38/50-2008
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- INTAS grant 05-1000008-8328.

SPARES

$$e^+e^- \rightarrow \gamma S \rightarrow \pi\pi\gamma \text{ (or } \pi\eta\gamma)$$



We study the dynamical properties of the decay mechanism exploiting the Chiral Dynamics!

(Energies are within the $R_{\chi T}$ applicability limits)

- in order to study $a_0 \rightarrow \pi\eta$, one cannot avoid the pseudoscalar octet-singlet mixing problem ($\eta - \eta'$ mixing)

$$\begin{cases} \eta = \eta_8 \cos \theta_8 - \eta_1 \sin \theta_1, \\ \eta' = \eta_8 \sin \theta_8 + \eta_1 \cos \theta_1, \end{cases}$$

$$\theta_8 = -9.2^\circ, \quad \theta_1 = -21.2^\circ$$

[Feldmann, Kroll and Stech, PR, D58, 114006]

- a scheme for multiplet decomposition

$$\begin{cases} a_0 = S_3^{oct}, \\ f_0 = S^{sing} \cos \theta - S_8^{oct} \sin \theta, \end{cases}$$

S_3, S_8 – octet S^{oct} components, θ – octet-singlet mixing angle.

- for octet and siglet (\sim) chiral couplings we employ

$$\tilde{c}_m = \frac{c_m}{\sqrt{3}}, \quad \tilde{c}_d = \frac{c_d}{\sqrt{3}},$$

(Can be derived from assumption of large number of quark colors ($N_c \rightarrow \infty$))

- R ChPT contains the $U(1)$ axial anomaly

$\eta - \eta'$ mixing is complicated

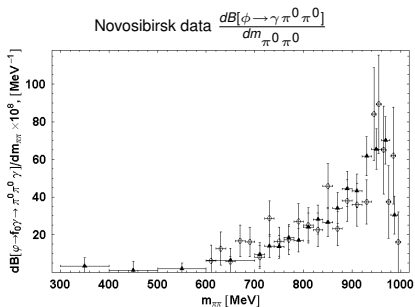
- a priori one does not know whether the light scalars fit into the scalar nonet of R ChPT (how to verify?), do isovector $a_0(980)$ and isoscalar $f_0(980)$ mix with each other?
- see papers by J.R. Peláez for large- N_c analysis:

- ordinary resonances become stable for $N_c \rightarrow \infty$: $\Gamma \rightarrow 0$
- extraordinary ones **disappear** in $N_c \rightarrow \infty$: width grows!

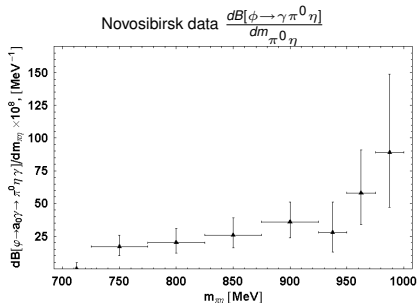
Novosibirsk data

the $e^+e^- \rightarrow \phi(1020) \rightarrow \pi^0\pi^0\gamma, \pi^0\eta\gamma$ events are convenient

The final photon may come only from the final state radiation
(ϕ meson decay)



PL,B 440 (1998) 442; PL, B 485 (2000) 349

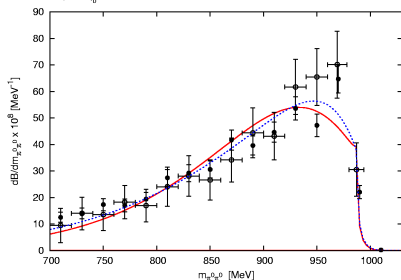


PL,B 479 (2000) 53

The analyses favor the kaon-loop mechanism for the ϕ decay

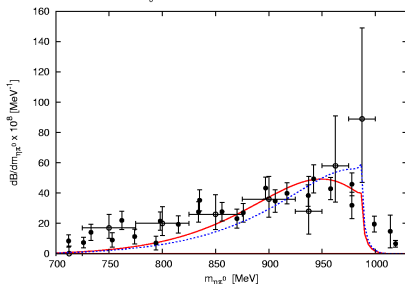
Our previous fit

M.N.Achasov et al. [PL, B485, 349 (2000)] \circ
 A.Aloisio et al. (KLOE) [PL, B537, 21 (2002)] \bullet
 ReII(ρ^2)=0, $M_{\rho_0}=(1078.45 \pm 7.46)$ MeV, $\theta=27.38^\circ \pm 1.68^\circ$, $\chi^2/d.o.f.=86.536/29$ —
 Flatte ReII(ρ^2), $M_{\rho_0}=(881.27 \pm 6.38)$ MeV, $\theta=15.92^\circ \pm 1.51^\circ$, $\chi^2/d.o.f.=59.39/29$ - - -



data: [PL, B 440 (1998) 442; PL, B 485 (2000) 349]

M.N.Achasov et al. [PL, B479, 53 (2000)] \circ
 C.Bini et al. [KLOE note 173, 06/02] \bullet
 ReII(ρ^2)=0, $M_{\rho_0}=(1089.61 \pm 9.06)$ MeV, $\chi^2/d.o.f.=80.064/32$ —
 Flatte ReII(ρ^2), $M_{\rho_0}=(1025.48 \pm 11.15)$ MeV, $\chi^2/d.o.f.=106.08/32$ - - -



data: [PL, B 479 (2000) 53]

$$C_d = C_m = f_\pi/2 = 46.2 \text{ MeV}$$

to appear in Nucl. Phys. Proc. Suppl.

see also AIP Conf. Proc. **1030** (2008) 123; arXiv:0805.4088

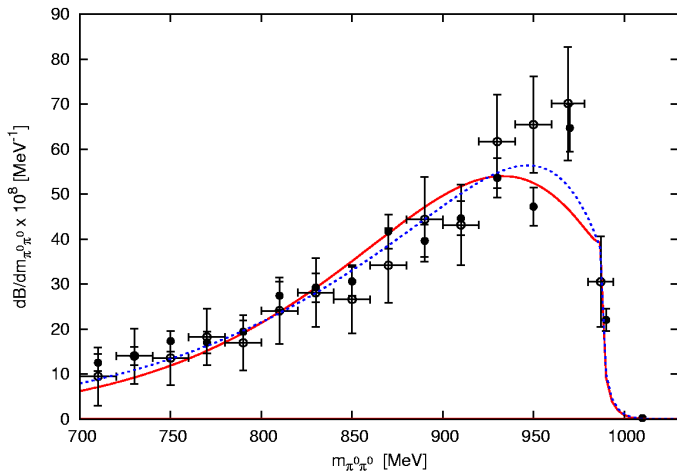
$\pi^0\pi^0$ distribution: zoom

M.N.Achasov et al. [PL, B485, 349 (2000)] \circ

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Re $\Pi(p^2)=0$, $M_{f_0}=(1078.45 \pm 7.46)$ MeV, $\theta=27.38^\circ \pm 1.68^\circ$, $\chi^2/\text{d.o.f.}=86.536/29$ —

Flatte Re $\Pi(p^2)$, $M_{f_0}=(881.27 \pm 6.38)$ MeV, $\theta=15.92^\circ \pm 1.51^\circ$, $\chi^2/\text{d.o.f.}=59.39/29$ - - -



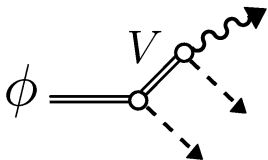
Pros of the distributions +

- much more sensitive to parameters of the model
- sensitive to momentum dependence of the vertices
- more reliable
(widths depend on the model used in the data analysis)

Cons of the distributions -

- often available only as plots

Background is complicated!



Background channels

- $\phi \rightarrow \rho^0 \pi^0 \rightarrow \pi^0 \pi^0 \gamma$,
- $\phi \rightarrow \omega \eta \rightarrow \eta \pi^0 \gamma$,
- $\phi \rightarrow \rho^0 \pi^0 \rightarrow \eta \pi^0 \gamma$,
- other contributions ...

A good option to fix model parameters and to test a model – to put the model into MC generator for Dalitz plots and distributions analysis.