Progress in cubic string field theory and nonlocal cosmology

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Based on

- G.C., *Cosmological tachyon from cubic string field theory*, JHEP 05 (2006) 012 [hep-th/0512259].
- G.C., G. Nardelli, *Tachyon solutions in boundary and cubic string field theory*, 0708.0366 [hep-th].
- G.C., M. Montobbio, G. Nardelli, *A route to nonlocal cosmology*, 0705.3043 [hep-th].

Outline

The many faces of the string tachyon

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- Cubic string field theory (CSFT) [Witten 1986a,b; Preitschopf et al. 1990; Aref'eva et al. 2002; Aref'eva et al. 1990a,b; Berkovits 1996,1999].

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For reviews see Sen (hep-th/0410103) and Ohmori (hep-th/0102085).

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where $\lambda = 3^{9/2}/2^6 \approx 2.19$. Theories with an ∞ number of derivatives are called nonlocal.

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   - Three questions
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   - Cosmology
   - Problems in constructing nonlocal solutions

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   - Localization
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- Nonlocal theories at odds with the inflationary paradigm: while the latter tends to erase any memory of the initial conditions, the formers do preserve this memory. SR approximation unclear: The cosmological eom's are nonlinear and involve the whole infinite SR tower.

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- Undetermined convergence properties of perturbative solutions [Coletti et al. 2005; Forini et al. 2006].

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- Reheating can be achieved with a negative KKLT-like Λ. Anisotropies adjusted with small warp factor [Garousi et al. 2004].
- With phenomenological potentials, good inflation and non-Gaussianity but non-characteristic predictions.

DBI tachyon as dark energy

with Andrew R. Liddle – PRD **74**, 043528 (2006), astro-ph/0606003

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- For a wide choice of potentials (ad-hoc or motivated), fine tuning on either the i.c. or the parameters of the potential.
- High-precision observational cosmology allows to constrain the theory in a remarkable way.
- The tachyon cannot decay faster than dust matter, $\rho_T \sim a^{-3(1+w_T)} > \rho_m \sim a^{-3} \Rightarrow$ cannot be used as quintessential inflaton.

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See Joukovskaya 2007 for a recent numerical method.

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$$e^{r_*\Box} = \sum_{\ell=0}^{+\infty}\frac{r_*^\ell}{\ell!}\Box^\ell \equiv \sum_{\ell=0}^{+\infty} c_\ell \Box^\ell$$

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In terms of $\tilde{\phi}$:

$$-(\Box - m^2)e^{-2r_* \Box}\tilde{\phi} + \tilde{U}' = 0$$

Skip energy-momentum tensor

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$$\tilde{T}^{(s)}_{\mu\nu} \equiv g_{\mu\nu}\left[(e^{s\Box}\tilde{U}')(\Box e^{-s\Box}\tilde{\phi}) + (\partial_\alpha e^{s\Box}\tilde{U}')(\partial^\alpha e^{-s\Box}\tilde{\phi})\right] -2(\partial_\mu e^{s\Box}\tilde{U}')(\partial_\nu e^{-s\Box}\tilde{\phi})$$

Outline

Friedmann–Robertson–Walker background

Flat FRW metric:

$$ds^2 = -dt^2 + a^2(t)\, dx_i dx^i .$$

The Hubble parameter is defined as $H \equiv \dot{a}/a = d_t a/a$.

Energy density and pressure

$$
\begin{aligned}
\rho &= -T_0{}^0 = \frac{\dot{\phi}^2}{2}(1-\mathcal{O}_2) + \tilde{V} - \mathcal{O}_1 \\
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\mathcal{O}_2 &= \frac{2}{\dot{\phi}^2}\int_0^{r_*} ds\, (e^{s\Box}\tilde{U}')^{\cdot}(e^{-s\Box}\tilde{\phi})^{\cdot}.
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Finite series if p is a positive even number (then $\tilde{\phi} \sim t^p$) or $p - 1 + 3H_0 = 2n$.

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NO!

What is not defined is the nonlocal solution expressed as an infinite series of powers of the d'Alembertian.

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- Define

$$\boxed{\phi(r,t)\equiv e^{r(\beta+\Box/\alpha)}\phi_{\rm loc}(t)}$$

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Valid for general nonlocal operators and ‘diffusion equations’.

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4. Check that $\phi(r,t)$ is a solution of the nonlocal e.o.m.s for some α, β.

Outline

G.C., G. Nardelli, arXiv:0708.0366 [hep-th]

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- The same result can also be found via a new powerful technique (valid only on Minkowski) based on harmonic functions.

Spike and oscillatory solutions

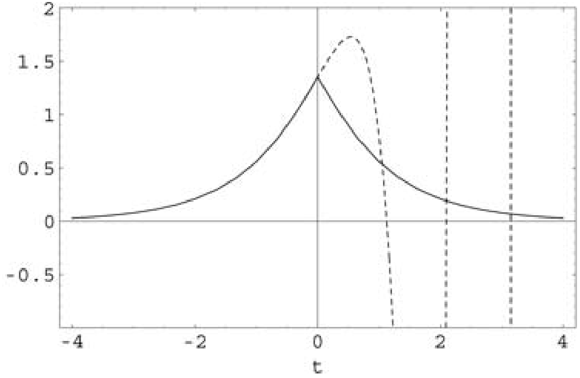

Figure: Solid: global solution. Dashed: asymptotic solution.

The analytic properties of these solutions are all under control.

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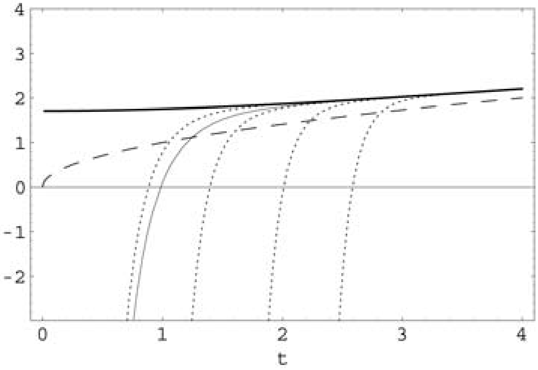

Figure: $p = 1/2$, $\nu = -3/2$, $r = -1$

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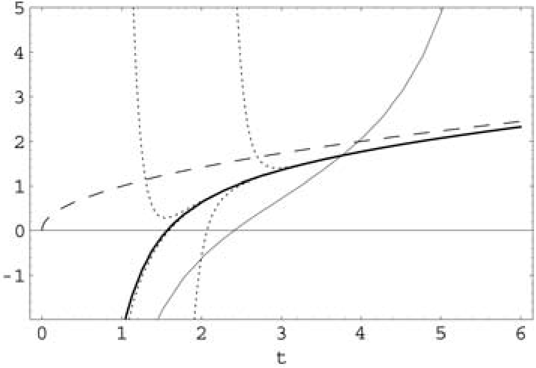

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- Theoretical foundations of the formalism are not completely assessed (to appear).